Modal Logics for Updating, Sharing or Composing

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The role of updates in non-classical logics

- Behavioural properties of transition systems expressed in temporal logics.

- Separation logics: extensions of Hoare-Floyd logic for (concurrent) programs with mutable data structures.

- Logics of public announcements can update the knowledge states in view of announcements made in the logical language.
Modal logics updating models are popular!

- Second-order modal logics ($\forall p$) [Fine, Theoria 1970]
- Logics of public announcements ($[[\phi]]$) [Plaza, ISMIS’89]
- Sabotage modal logics ($\Diamond$) [van Benthem, 2002]
- Relation-changing modal logics ($\langle s w \rangle$) [Fervari, PhD 2014]
- Logic with separating modalities LSM ($\ast$) [Courtault & Galmiche & Pym, TCS 2016]
This talk

Recent developments on modal logics
• with built-in update mechanisms based on composition.

• Relationships with other logical formalisms such as second-order modal logics, separation logics, team logics, ... See e.g. [Grädel et al., 2020] relating separation and team logics.

• Results about decidability, computational complexity, expressive power from joint works with
B. Bednarczyk  M. Deters  R. Fervari  A. Mansutti
Plan of the talk

1. Modal logics for Updating or Composing
2. Foundations: the Logic of Bunched Implications $\mathbf{Bl}$
3. Second-Order Modal Logics (with Tree Semantics)
4. Modal Separation Logics
5. New Proposal: Description Logics and Updates
Modal logics in a nutshell

• Formulae: \( \phi ::= p \mid \neg \phi \mid \phi \land \phi \mid \Box \phi \mid \Diamond \phi. \)

• Kripke-style structures \( \mathcal{M} = (W, R, V) \):
  - \( W \): non-empty set of worlds.
  - \( R \subseteq W \times W \): accessibility relation.
  - \( V: \text{PROP} \rightarrow \mathcal{P}(W) \): valuation.

```
\begin{align*}
\models \Diamond \Diamond p & \land \Diamond \neg p \land \Box \neg p
\end{align*}
```

• Satisfaction relation:
  - \( \mathcal{M}, w \models p \iff w \in V(p). \)
  - \( \mathcal{M}, w \models \Diamond \phi \iff \) there is \( w' \) s.t. \((w, w') \in R \) and \( \mathcal{M}, w' \models \phi. \)
  - \( \mathcal{M}, w \models \Box \phi \iff \) for all \( w' \) s.t. \((w, w') \in R \), \( \mathcal{M}, w' \models \phi. \)
How to update pointed Kripke-style structures?

- Bottom line: changing the pointed model with \( \Diamond \).

- Each element from \((W, R, V)\) could potentially be changed.
  (approach advocated in [Aucher et al. ENTCS 2009])

- Changing
  - \( W \) requires the power of some 2nd logic.
  - \( R \) requires the power of some dyadic 2nd logic.
  - \( V \) requires the power of some monadic 2nd logic.
Examples: sabotage and announcement

- Saboting the model with $\diamond$ (deleting exactly one edge).

  See e.g., [van Benthem, 2005; Löding & Rohde, FST&TCS’03]

- Removing states with the public announcement $[\phi]$.

  See e.g., [Plaza, ISMIS’89]
Other logical formalisms

- Propositional quantification $\forall p$ in modal/temporal logics.
  - Second-order modal logics. [Bull, JSL 1969; Fine, Theoria 1970]
  - Quantified CTL with tree semantics.
    See [Laroussinie & Markey, LMCS 2014]

- Tree-like models and compositions.
  - Static ambient logics with composition operator $|$. [Cardelli & Gordon, POPL’00]
  - Modal separation logic for resource trees. [Biri & Galmiche, JLC 2006]

- Modalities and abstract models based on resources.
  - Modal relevant logics of processes. [Dams, PhD thesis 90]
  - Exploitation of a modality for BI in [Pym, Book 2002], see also modal BI in [Pym & Tofts, FAC 2006].
  - Modal extensions of BI. [Courtault & Galmiche, LFCS’13]
Foundations: Logic of Bunched Implications BI
An abstract view based on resources

- Logic of bunched implications BI introduced in
  \[ \text{O’Hearn & Pym, BSL 99} \]

- Boolean BI has classical additive connectives.

- BI, Boolean BI and bunched logics defined proof-theoretically but completeness with different types of resource models.
  \[ \text{Pym, Book 2002; Galmiche et al., MSCS 2005; Docherty, PhD 2019} \]
  \[ \text{Jipsen & Litak, arXiv 2018} \]

- Ingredients for a simple model of resources
  \[ \text{Pym & Tofts, FAC 2006} \]

- a set \( R \) of resource elements,
- partial composition \( \circ : R \times R \rightarrow R \),
- comparing resource elements with \( \sqsubseteq \),
- zero resource element \( e \).
Boolean $\mathsf{BI}$ – the semantics side

- Abstract models with composition: BBI-frame $(M, \circ, e)$
  - $M$ is a non-empty set,
  - binary function $\circ : M \times M \to \mathcal{P}(M)$ such that $\circ$ is commutative and associative,
  - $e \in M$ and $e \circ m = \{m\}$ for all $m \in M$.

- Formulae
  \[
  \phi, \psi ::= I \mid p \mid \phi \land \psi \mid \neg \phi \mid \phi \ast \psi \mid \phi \rightsquigarrow \psi
  \]

- Satisfaction relation ($m \in M$, $V : \text{PROP} \to \mathcal{P}(M)$).
  \[
  m \models_V I \quad \text{iff} \quad m = e
  \]
  \[
  m \models_V p \quad \text{iff} \quad m \in V(p)
  \]
  \[
  m \models_V \phi_1 \ast \phi_2 \quad \text{iff} \quad \text{for some } m_1, m_2 \in M, \text{ we have } m \in m_1 \circ m_2, m_1 \models_V \phi_1 \text{ and } m_2 \models_V \phi_2
  \]
  \[
  m \models_V \phi_1 \rightsquigarrow \phi_2 \quad \text{iff} \quad \text{for all } m', m'' \in M \text{ such that } m'' \in m \circ m', \text{ if } m' \models_V \phi_1 \text{ then } m'' \models_V \phi_2.
  \]
Abstract view leading to undecidability but . . .

• A formula $\phi$ is valid iff for all BBI-models $(M, \circ, e, V)$ and for all $m \in M$, we have $m \models_V \phi$.

• Validity problem for Boolean BI is undecidable.
  
  [Kurucz & Németi & Sain & Simon, JoLLI 1995]
  [Brotherson & Kanovich; Larchey & Galmiche, LiCS’10]

• Decidable concretisations such as separation logics, modal logics of trees, ambient logics (including modal extensions).

• See related structures from substructural logics.
  • Pieces of information in [Urquhart, JSL 1972].
  • Information frames $(P, \circ, 1, \sqsubseteq)$ for substructural logics.
    [D’Agostino & Gabbay, JAR 1994]
  • Routley-Meyer frames for relevance logics $(W, R)$ with ternary $R$. See e.g. [Meyer APAL 2004; Restall, Handbook 2006]
Classical sharing interpretations [Pym, Book 2002]

- **Separation logics** for the verification of program with pointers. [Reynolds, LiCS'02]
  - Separation logics are concretisations of (Boolean) BI.
  - Memory state \((s, h)\) with \(s : \text{PVAR} \to \text{Val}, h : \text{Loc} \to \text{fin Val}\).
  - Disjoint heaps when \(\text{dom}(h_1) \cap \text{dom}(h_2) = \emptyset\) and disjoint union \(h_1 \uplus h_2\).

\[
\begin{align*}
\text{Disjoint heaps:} & \quad h_1 \uplus h_2 \\
\text{Memory state:} & \quad (s, h) = \phi_1 \ast \phi_2 \iff \exists h_1, h_2 \text{ s.t. } h = h_1 \uplus h_2, (s, h_1) \models \phi_1, (s, h_2) \models \phi_2.
\end{align*}
\]

- **Petri net semantics** for linear logic adjusted to BI’s resource interpretation with \((\mathbb{N}^n, +, \sqsubseteq, \mathbf{0})\) (with \(\vec{n} \sqsubseteq \vec{m}\) iff \(\vec{n} \Rightarrow^* \vec{m}\)). [Engberg & Winskel, APAL 1997; Pym et al., TCS 2004]

- **Static ambient logics** have models that are finite edge-labelled trees with composition [Calcagno et al., TLDI’03].
Second-order modal logics (with tree semantics)
Propositional quantification in modal logics

- Changing valuations with $\exists p$.

- QK formulae: $\phi ::= p \mid \neg \phi \mid \phi \land \phi \mid \Diamond \phi \mid \Box \phi \mid \exists p \phi$.

- $\mathcal{M}, w \models \exists p \phi$ iff there is a $p$-variant $\mathcal{M}'$ s.t. $\mathcal{M}', w \models \phi$.

- Second-order quantification is handy!
  - To design algorithms for ATL with strategy contexts. [Laroussinie & Markey, IC 2015]
  - Relationships with epistemic reasoning. [Belardinelli & van der Hoek, AAAI'16]
  - Enriching the modal $\mu$-calculus for control synthesis. [Riedweg & Pinchinat, MFCS'03]
Undecidable logics $\mathcal{QL}$

- Variants second-order modal logics QS4, QS5, etc.
  See e.g. [Kripke, JSL 1959; Fine, PhD 1969; Kaplan, JSL 1970]

- For any modal logic $\mathcal{L}$ between K and S4, the satisfiability problem for $\mathcal{QL}$ is undecidable.
  [Fine, PhD thesis '69, Theoria 1970]

- The satisfiability problem for QS5 is decidable and QS5 as expressive as graded modal logic GS5.
Moving to tree-like models for QŁ

Modal logic K characterised by finite tree models and QK is undecidable.

• What about complexity of QK on finite tree models (QKt)?

Modal logic S4 characterised by models $(W, R^*, V)$ s.t. $(W, R)$ is a finite-branching tree with all branches infinite.

• What about complexity of QS4 on such tree models (QS4t)?

(QS4 on tree models already considered in [Zach, JPL 2004])
Tree semantics in $\text{QCTL}^t$ [Laroussinie & Markey, LMCS 2014]

- Satisfiability problem for $\text{QCTL}^t$ (tree semantics):
  
  **input:** a $\text{QCTL}$ formula $\phi$.
  
  **output:** 1 iff there is a finite total Kripke structure whose tree unfolding satisfies $\phi$.

- Equivalently, finite-branching trees with all branches infinite.

- $\text{SAT}(\text{QCTL}^t)$ is $\text{TOWER}$-complete.

[Laroussinie & Markey, LMCS 2014]
Tower-completeness

- MSO over the infinite tree $S\omega S$ is decidable.  
  [Rabin, TAMS 1969]

- Tower upper bound for $QK^t$ and $QS4^t$ is a consequence of Rabin’s Theorem.

- Tower-hardness for $\text{SAT}(QK^t)$ and $\text{SAT}(QS4^t)$.  
  [Bednarczyk & Demri, LiCS’19]

- More Tower-hardness results about fragments of $QCTL^t$ and related logics can be found in the papers  
  [LMCS 2014; LiCS’19; Mansutti, FoSSaCS’20]
Local nominals captured in $QK^t$

- Nominals from hybrid (modal) logics: propositional variables that hold true in a unique world.
  
  [Areces & Blackburn & Marx, JSL 2001]

- $Q\phi$: there is a unique world satisfying $\phi$ [Fine, PhD 1969].
  
  See also [Garson, 1984; Kaminski & Tiomkin, NDFL 1996].

Exactly one descendant at depth $k$ satisfies $x$.

$$\text{nom}(x, k) \overset{\text{def}}{=} \Box^k x \land \exists p (\Box^k (x \land p) \land \Box^k (x \land \neg p)).$$

- A toolkit for introducing local nominals $x$.
  
  - $\Box^k x \phi \overset{\text{def}}{=} \Box^k (x \land \phi)$: this unique descendant satisfies $\phi$.
  
  - $\text{diff-nom}(x_1, \ldots, x_\alpha, k)$: $\alpha$ distinct descendants at depth $k$.
    
    $$\text{diff-nom}(x_1, \ldots, x_\alpha, k) \overset{\text{def}}{=} \bigwedge_{i \in [1, \alpha]} \text{nom}(x_i, k) \land \bigwedge_{i<j \in [1, \alpha]} \neg @^k_{x_i} x_j.$$
Enforcing an exponential number of children

- Simulation of first-order quantification on a given set of nodes of bounded depth.

- At most $2^n$ children ($\lozenge_{\leq 2^n} \top$ in graded modal logics):

$$\exists p_0, \ldots, p_{n-1} \left( \forall x, y \text{ diff-nom}(x, y, 1) \rightarrow \neg \left( \bigwedge_{i \in [0, n-1]} p_i \leftrightarrow \diamond x p_i \leftrightarrow \diamond y p_i \right) \right)$$

[David & Laroussinie & Markey, CONCUR’16]
How to prove Tower-hardness for $\text{SAT}(\text{QK}^t)$

• Uniform elementary reduction from $k$-$\text{NEXPTIME}$-complete tiling problems $\text{Tiling}_k$.

• $t(0, n) = n$ and $t(k + 1, n) = 2^{t(k, n)}$.

• $\text{Tiling}_k$:

  input:  
  • $(\mathcal{T}, \mathcal{H}, \mathcal{V})$ (tile types, horizontal and vertical matching relations),
  • $c = t_0, t_1, \ldots, t_{n-1} \in \mathcal{T}^n$: initial condition.

  output: 1 iff the grid $[0, t(k, n) - 1] \times [0, t(k, n) - 1]$ can be tiled (with usual hori/verti. constraints)?
Enforcing large numbers of children

[Bednarczyk & Demri, LiCS’19]
Modal Separation Logics
Modal separation logic \( \text{MSL}(\ast, \Diamond, \langle \neq \rangle) \)

- Modal separation logics: Kripke-style semantics with modal and separating connectives.

- \( \text{MSL}(\ast, \Diamond, \langle \neq \rangle) \) inspired from [Demri & Deters, ToCL 2015].

- Formulae: \( \phi ::= p \mid \text{emp} \mid \neg \phi \mid \phi \land \phi \mid \Diamond \phi \mid \langle \neq \rangle \phi \mid \phi * \phi \).

- Models \( \mathcal{M} = \langle \mathbb{N}, \mathcal{R}, \mathcal{V} \rangle \) are tree-like (heap graphs):
  - \( \mathcal{R} \subseteq \mathbb{N} \times \mathbb{N} \) is finite and weakly functional (deterministic),
  - \( \mathcal{V} : \text{PROP} \rightarrow \mathcal{P}(\mathbb{N}) \).
Semantics

- Disjoint unions $M_1 \uplus M_2$.

\[ M, l \models p \quad \overset{\text{def}}{\iff} \quad l \in V(p) \]

\[ M, l \models \Diamond \phi \quad \overset{\text{def}}{\iff} \quad M, l' \models \phi, \text{ for some } l' \in \mathbb{N} \text{ such that } (l, l') \in R \]

\[ M, l \models \langle\neq\rangle \phi \quad \overset{\text{def}}{\iff} \quad M, l' \models \phi, \text{ for some } l' \in \mathbb{N} \text{ such that } l' \neq l \]

\[ M, l \models \text{emp} \quad \overset{\text{def}}{\iff} \quad R = \emptyset \]

\[ M, l \models \phi_1 \ast \phi_2 \quad \overset{\text{def}}{\iff} \quad \langle \mathbb{N}, R_1, V \rangle, l \models \phi_1 \text{ and } \langle \mathbb{N}, R_2, V \rangle, l \models \phi_2, \text{ for some partition } \{R_1, R_2\} \text{ of } R \]

Separating $\ast$ as prop. quantification: mark the source nodes

\[ \models \phi_1 \ast \phi_2 \]

\[ \models \phi_1^p \ast \phi_2^{\neg p} \]
Expressing simple properties

- $\text{size} \geq k \overset{\text{def}}{=} \neg \text{emp} \ast \cdots \ast \neg \text{emp}$. 

  $k$ times

- The model is a loop of length 2 visiting the current location:

  $\text{size} \geq 2 \land \neg \text{size} \geq 3 \land \diamond \diamond \diamond \top \land \neg (\neg \text{emp} \ast \diamond \diamond \diamond \top) \land \neg \diamond (\neg \text{emp} \ast \diamond \diamond \diamond \top)$

  (not expressible in modal logic Alt$_1$)
The tradition of adding modalities to resource logics

- Modal separation logic first named in [Zeilberger, draft 2005].

- Quantification over heaps. [Nishimura, AMAST’06]
  \( s, h \models \lozenge \phi \) iff for some \( h' \), \( \text{dom}(h') = \text{dom}(h) \) and \( s, h' \models \phi \).

- Modal extensions of BI [Courtault & Galmiche, LFCS’13].

- Modal/temporal/epistemic extensions of bunched logics.
  [Pym & Tofts, FAC 2006; Kamide, TCS 2013; Kimmel, PhD 2018]

- Modal separation logic for resource trees.
  [Biri & Galmiche, JLC 2006], see also [Conforti, PhD 2005]

- Modal Kripke resource models with neighbourhood functions.
  [Porello & Troquard, JANCL 2015]
Tower-completeness of $\text{SAT}(\text{MSL}(\ast, \Diamond, \langle \neq \rangle))$

- **Tower** upper bound for $\text{SAT}(\text{MSL}(\ast, \Diamond, \langle \neq \rangle))$ by reduction into satisfiability for the weak MSO theory of $(\mathcal{Q}, f, =)$.

- Linear model:

  $l_0 \rightarrow l_1 \rightarrow \cdots \rightarrow l_n$

- There is a formula $\phi_{\exists l}s$ s.t. $M \models \phi_{\exists l}s$ iff $M$ is linear.

- Star-free expressions $e ::= a \mid \varepsilon \mid e \cup e \mid ee \mid \sim e$.
  - Nonemptiness problem is **Tower**-complete.
    
    [Meyer & Stockmeyer, STOC'73; Schmitz, ToCT 2016]

  - Encoding words by linear models.

  $a_1 \ a_2 \ a_1 \ \triangleright \quad l_0 \rightarrow l_1 \rightarrow l_2 \rightarrow l_3 \rightarrow l_0$

- $\text{MSL}(\ast, \Diamond, \langle \neq \rangle)$ satisfiability problem is **Tower**-hard.

  [Demri & Fervari, AiML’18]
Results for variant logics

- The satisfiability problems for $\text{MSL}(\ast, \Diamond)$ and $\text{MSL}(\ast, \langle \neq \rangle)$ are NP-complete.

- Undecidable variant logics.
  - $\text{MSL}(\ast, \Diamond, \langle \neq \rangle) + \text{magic wand } \rightarrow$.
  - $\text{MSL}(\ast, \Diamond)$ over all frames.

- More Tower-hardness.
  - Modal logic for heaps $\text{MLH}(\ast)$.
  - $\text{MSL}(\ast, \Diamond, \langle \neq \rangle)$ restricted to $\phi ::= T \mid \neg \phi \mid \phi \land \phi \mid \langle U \rangle \phi \mid \phi * \phi$.
  - $\text{MSL}(\ast, \Diamond^{-1})$.  

[Demri & Fervari, JLC 2019]
[Areces et al., JLC 2018]
[Demri & Deters, TOCL 2015]
[Mansutti, FoSSaCS’20]
[Bednarczyk et al., LiCS’20]
Modal logic $\text{ML}(\ast)$ on finite forests

(sister logic $\text{MSL}(\ast, \Diamond^{-1})$)

- Formulae: $\phi ::= p \mid \phi \land \phi \mid \neg \phi \mid \Diamond \phi \mid \phi \ast \phi$.
- Models are finite Kripke-style forests equipped with the composition from separation logic.

\[
\begin{align*}
M, w \models \phi_1 \ast \phi_2 & \text{ iff there are } M_1, M_2 \text{ such that } \\
M &= M_1 + M_2, & M_1, w \models \phi_1 \text{ and } M_2, w \models \phi_2.
\end{align*}
\]

- Recent results [Bednarczyk et al., LiCS’20]
  - Satisfiability problem for $\text{ML}(\ast)$ is \textsc{Tower}-complete.
  - $\text{ML}(\ast)$ strictly less expressive than graded modal logic (modalities are $\Diamond \geq k$).
  - See also results about $\text{ML}(\mid)$ with $\mid$ from static ambient logic.
A Proposal: Dynamic Axioms in DLs
Description logics and updates

- Description logics are well-known logical formalisms for knowledge representation. [Baader et al., Book 2017]

- Known updates in DLs, mainly at the level of ABoxes. See e.g. [Liu et al., KR’06]

- How to specify the evolution of the satisfaction of GCIs ($C \sqsubseteq C'$) or assertions ($C(a)$) when the current interpretation is updated?

- New framework based on separating connectives from separation logics introduced in [Bednarczyk et al., IJCAI’20].
**ALC and EL in a nutshell**

- **ALC concepts:** $C, C' := \top | A | \neg C | C \cap C' | \exists r.C$

- **Assertions:** $C(a), r(a, b)$.

- **General concept inclusion (GCI):** $C_1 \sqsubseteq C_2$.

- **Knowledge base** $\mathcal{K} = (\mathcal{T}, \mathcal{A})$: $\mathcal{T}$ finite set of GCIs and $\mathcal{A}$ finite set of assertions.

- **Interpretation** $\mathcal{I} = (\Delta^\mathcal{I}, .^\mathcal{I}) \in \mathcal{I}$

\[
\mathcal{I} \models C_1 \sqsubseteq C_2 \iff C_1^\mathcal{I} \subseteq C_2^\mathcal{I}
\]

\[
\mathcal{I} \models C(a) \iff a^\mathcal{I} \in C^\mathcal{I}
\]

\[
\mathcal{I} \models r(a, b) \iff (a^\mathcal{I}, b^\mathcal{I}) \in r^\mathcal{I}
\]

- **EL defined as the restriction of ALC to $\cap, \exists, \top$ and to the concept names.**
A framework with dynamic axioms – Bl stricks back!

- KB with dynamic axioms $\mathcal{K}_{da} = (\mathcal{T}, A, \mathcal{D})$
  $\mathcal{D}$ is a dynamic box (DBox).

- Partial composition operator $\oplus : \Pi \times \Pi \rightarrow \Pi$ with AC $\oplus$.
  In this talk: $\oplus$ deals with the disjointness of (arbitrary) role interpretations only.

- Positive dynamic axioms (PDAs)

  $$U, V := \top | C(a) | r(a, b) | C \sqsubseteq D | U \sqcup V | U \sqcap V | U \ast V | U \ominus V$$

  $$\mathcal{I} \models U_1 \ast U_2 \quad \text{iff} \quad \text{there are } \mathcal{I}_1, \mathcal{I}_2 \text{ s.t. } \mathcal{I} = \mathcal{I}_1 \oplus \mathcal{I}_2,$$
  $$\mathcal{I}_1 \models U_1 \text{ and } \mathcal{I}_2 \models U_2$$

  $$\mathcal{I} \models U_1 \ominus U_2 \quad \text{iff} \quad \text{there is } \mathcal{I}' \text{ s.t. } \mathcal{I} \oplus \mathcal{I}' \text{ is defined},$$
  $$\mathcal{I}' \models U_1 \text{ and } \mathcal{I} \oplus \mathcal{I}' \models U_2.$$  

- Dynamic axioms (closure under Boolean operators)

  $$U, V ::= \top | \neg U | U \sqcup V | U \sqcap V$$
Consistency problem with dynamic axioms

[Bednarczyk et al, IJCAI’20]

- A KB for $\mathcal{EL}$ is always consistent.

- A KB with PDAs for $\mathcal{EL}$ is not always consistent.

- Consistency of a KB with PDAs characterised by the non-derivability of $\bot$ in a simple calculus.

\[
\approx \top
\]

\[
r(a, b) \sqcap (r(a, b) \circ (
\top \sqsubseteq \top)
) \]

- The consistency problem for $\mathcal{ALC}$ with role inclusion axioms $r_1 \circ \cdots \circ r_n \sqsubseteq s$ is undecidable [Baldoni et al., TABLEAUX’98].

- CRIAs encoded by dynamic axioms for $\mathcal{ALC}$ and for $\mathcal{EL}$. 

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<thead>
<tr>
<th>Logic \ Dynamic axioms</th>
<th>Positive DAs</th>
<th>DAs</th>
</tr>
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<tbody>
<tr>
<td>$\mathcal{EL}$</td>
<td>in PTime</td>
<td>undecidable</td>
</tr>
<tr>
<td>$\mathcal{ALC}$</td>
<td>EXPTime-complete</td>
<td>undecidable</td>
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New Proposal: Description Logics and Updates
Concluding remarks

• Rich framework of modal logics with updates based on composition operators.

• Concrete logical formalisms on the theme “Modalities and BI” developed by D. Galmiche, D. Pym and colleagues.

• Recent developments improve our understanding. Must-read forthcoming [Mansutti, PhD 2020]

• Potential research directions.
  – Tractable fragments of modal separation logics.
  – Relationships with team logics.
  – Decidable separation logics with partial use of separating implication.
  – Many more directions, for instance related to DLs and updates.