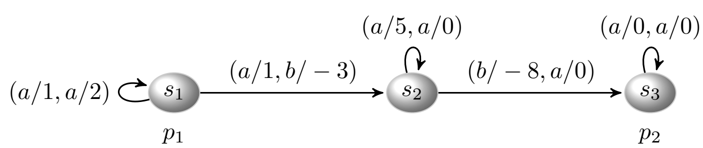
TD: Logical Aspects of Artificial Intelligence
ATL$^+$ + ATL-like logics with resources (26/10/2022)

Exercise 1. Let $\mathfrak{M}$ be a CGS with resources, $s \in S$ and $\langle\langle A^{\vec{b}} \rangle\rangle \Phi$ be a formula in ATL(Agt, r) (ATL with r resources). Show that the set $\{\vec{b} \in \mathbb{N}^r \mid \mathfrak{M}, s \models \langle\langle A^{\vec{b}} \rangle\rangle \Phi\}$ is upward-closed.

Exercise 2. Below, we present a CGS one resource $\mathfrak{M}$ with two agents, the transitions are labelled by pairs of actions with the respective weights. For instance, the total weight of the transition from s_1 to s_2 is -2.



1. Characterise the set $\{n \in \mathbb{N} \mid \mathfrak{M}, s_1 \models \langle\langle \{1\}^n \rangle\rangle (\mathsf{G} p_1 \vee \mathsf{F} p_2)\}$.
2. Characterise the set of natural numbers $n \in \mathbb{N}$ such that there is a positional and n-consistent strategy σ w.r.t. s_1 for the coalition $\{1\}$ such that for all $\lambda \in \mathtt{Comp}(s_1, \sigma)$, we have $\mathfrak{M}, \lambda \models (\mathsf{G} p_1) \vee (\mathsf{F} p_2)$.

Exercise 3. Design a CGS with one ressource $\mathfrak{M}$, $s \in S$ and $b \in \mathbb{N}$ such that $\mathfrak{M}, s \models \langle\langle A^b \rangle\rangle \mathsf{F} p$ but there is no *positional* b-strategy σ such that for all computations $\lambda \in \mathtt{Comp}(s, \sigma)$, we have $\mathfrak{M}, \lambda \models \mathsf{F} p$.

Exercise 4. Let $\mathfrak{M}$ be a CGS with one resource, $s \in S$, a coalition $A \subseteq Agt$, $b \in \mathbb{N}$ and σ be a b-consistent strategy from s for the coalition A such that for all computations $\lambda \in \mathtt{Comp}(s, \sigma)$, we have $\mathfrak{M}, \lambda \models \mathsf{G} p$.

1. Show that for all $\lambda \in \mathtt{Comp}(s, \sigma)$ with

$$RAV(b, \lambda) = (s_0, v_0) \rightarrow (s_1, v_1) \cdots (s_i, v_i) \cdots,$$

   for all $i \geq 0$, we have $\mathfrak{M}, s_i \models p$ and $v_i \geq 0$.

2. We assume a total ordering on $\bigcup_{s \in S} D_A(s)$ (unspecified here). We write $D_A(s)$ to denote the set of joint actions for the coalition A from the state s. Let us define a positional strategy σ' as follows.
   - If a state s' does not occur in any computation of $\mathtt{Comp}(s, \sigma)$, then $\sigma'(s') \stackrel{\text{def}}{=} \min D_A(s')$ (dummy value).
   - Now, assume that s' occurs in some computation of $\mathtt{Comp}(s, \sigma)$. Let $min_{s'}$ be

$$min_{s'} \stackrel{\text{def}}{=} \min\{v \mid \exists\, \lambda \in \mathtt{Comp}(s, \sigma) \text{ s.t. } (s', v) \text{ occurs in } RAV(b, \lambda)\}$$

Then,

$$\sigma'(s') \stackrel{\text{def}}{=} \min\{(\mathfrak{f}_j)_{|A} \mid \exists\, \lambda = s_0 \xrightarrow{\mathfrak{f}_0} s_1 \cdots \in \texttt{Comp}(s,\sigma) \text{ with}$$

$$RAV(b,\lambda) = (s_0, v_0) \to (s_1, v_1) \cdots,\ j \in \mathbb{N} \text{ s.t. } (s_j, v_j) = (s', min_{s'})\}$$

$(\mathfrak{f}_j)_{|A}$ denotes the restriction of the joint action $\mathfrak{f}_j$ to the agents in A. (considering the minimal value among all the $(\mathfrak{f}_j)_{|A}$'s is just a technical means to pick an arbitrary joint action) Show that all the states occurring in some computation in $\texttt{Comp}(s,\sigma')$ satisfy the propositional variable p.

3. Show that for all computations $\lambda' = s'_0 \xrightarrow{\mathfrak{f}'_0} s'_1 \cdots \in \texttt{Comp}(s,\sigma')$ with $RAV(b,\lambda') = (s'_0, v'_0) \to (s'_1, v'_1) \cdots$, for all $i' \in \mathbb{N}$, there is a computation $\lambda = s_0 \xrightarrow{\mathfrak{f}_0} s_1 \cdots \in \texttt{Comp}(s,\sigma)$ with $RAV(b,\lambda) = (s_0, v_0) \to (s_1, v_1) \cdots$ and $i \in \mathbb{N}$ such that $s'_{i'} = s_i$ and $v'_{i'} \geq v_i$.

4. Conclude $\mathfrak{M}, s \models \langle\!\langle A^b \rangle\!\rangle \mathsf{G} p$ iff there is a *positional* b-strategy σ such that for all computations $\lambda \in \texttt{Comp}(s,\sigma)$, we have $\mathfrak{M}, \lambda \models \mathsf{G} p$.

Exercise 5. Complete the proof of undecidability of $\mathsf{ATL}'(2)$ by showing that M reaches the instruction n iff $\mathfrak{M}, 1 \models \langle\!\langle \{1\}^{\vec{0}} \rangle\!\rangle \mathsf{F}\, p$.