Verification of security protocols:
from confidentiality to privacy

Stéphanie Delaune

LSV, CNRS & ENS Cachan, Université Paris Saclay, France

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Verification of critical software and systems

Goal: develop the algorithmic foundations for proving correctness and detecting flaws in various types of programs

Applications: computerized systems, databases, security protocols

LSV in figures
- founded in 1997
- around 25 permanents + 15 PhD students
- 6 research teams
Security of Information Systems

- 1 engineer + 1 postdoc
- 3 PhD students
Cryptographic protocols everywhere!

**Goal:** they aim at securing communications over public/insecure networks
A variety of security properties

- **Secrecy**: May an intruder learn some secret message exchanged between two honest participants?

- **Authentication**: Is the agent Alice really talking to Bob?

- **Anonymity**: Is an attacker able to learn something about the identity of the participants who are communicating?

- **Non-repudiation**: Alice sends a message to Bob. Alice cannot later deny having sent this message. Bob cannot deny having received the message.

- ...
How does a cryptographic protocol work (or not)?

**Protocol:** small programs explaining how to exchange messages
How does a cryptographic protocol work (or not)?

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**Protocol:** small programs explaining how to exchange messages

**Cryptographic:** make use of cryptographic primitives

**Examples:** symmetric encryption, asymmetric encryption, signature, hashes, ...
What is a symmetric encryption scheme?

Symmetric encryption

- Encryption
- Decryption

Key

- S. Delaune (LSV)

Verification of security protocols

27th June 2016
What is a symmetric encryption scheme?

Symmetric encryption

Example: This might be as simple as shifting each letter by a number of places in the alphabet (e.g. Caesar cipher)

A famous example

**Enigma machine (1918-1945)**
- electro-mechanical rotor cipher machines used by the German to encrypt during World War II
- permutations and substitutions

**A bit of history**
- **1918**: invention of the Enigma machine
- **1940**: Battle of the Atlantic during which Alan Turing’s Bombe was used to test Enigma settings.

→ Everything about the breaking of the Enigma cipher systems remained secret until the mid-1970s.
What is an asymmetric encryption scheme?

Asymmetric encryption

encryption

public key

decryption

private key
Asymmetric encryption

Examples:

- **1976**: first system published by W. Diffie, and M. Hellman,

→ their security relies on well-known mathematical problems (e.g. factorizing large numbers, computing discrete logarithms)

**Today**: those systems are still in use

Prix Turing 2016
What is a signature scheme?

**Signature**

Example:

The RSA cryptosystem (in fact, most public key cryptosystems) can be used as a signature scheme.
Example: Denning Sacco protocol (1981)

\[ aenc(sign(k_{AB}, priv(A)), pub(B)) \]

Is the Denning Sacco protocol a good key exchange protocol?
Example: Denning Sacco protocol (1981)

\[
aenc(\text{sign}(k_{AB}, \text{priv}(A)), \text{pub}(B))
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Is the Denning Sacco protocol a good key exchange protocol? **No!**
Example: Denning Sacco protocol (1981)

\[ \text{aenc}(\text{sign}(k_{AB}, \text{priv}(A)), \text{pub}(B)) \]

Is the Denning Sacco protocol a good key exchange protocol? No!

Description of a possible attack:

\[ \text{aenc}(\text{sign}(k_{AC}, \text{priv}(A)), \text{pub}(C)) \]
Example: Denning Sacco protocol (1981)

\[ \text{aenc} \left( \text{sign}(k_{AB}, \text{priv}(A)), \text{pub}(B) \right) \]

Is the Denning Sacco protocol a good key exchange protocol? **No!**

Description of a possible attack:

\[ \text{aenc} \left( \text{sign}(k_{AC}, \text{priv}(A)), \text{pub}(C) \right) \]

\[ \text{sign}(k_{AC}, \text{priv}(A)) \]

\[ k_{AC} \]

\[ \text{aenc} \left( \text{sign}(k_{AC}, \text{priv}(A)), \text{pub}(B) \right) \]

S. Delaune (LSV)
We propose to fix the Denning-Sacco protocol as follows:

Version 1

\[ A \rightarrow B \ : \ aenc(\langle A, B, \text{sign}(k, \text{priv}(A))\rangle, \text{pub}(B)) \]

Version 2

\[ A \rightarrow B \ : \ aenc(\text{sign}(\langle A, B, k\rangle, \text{priv}(A)), \text{pub}(B)) \]

Which version would you prefer to use?
We propose to fix the Denning-Sacco protocol as follows:

**Version 1**

\[ A \rightarrow B : \ aenc(\langle A, B, \text{sign}(k, \text{priv}(A))\rangle, \text{pub}(B)) \]

**Version 2**

\[ A \rightarrow B : \ aenc(\text{sign}(\langle A, B, k\rangle, \text{priv}(A))\rangle, \text{pub}(B)) \]

Which version would you prefer to use? Version 2

\[ \rightarrow \] Version 1 is still vulnerable to the aforementioned attack.
What about protocols used in real life?
Serge Humpich case - “Yescard” (1997)
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Step 1: A logical flaw in the protocol allows one to copy a card and to use it without knowing the PIN code.

→ not a real problem, there is still a bank account to withdraw
Credit Card payment protocol

Serge Humpich case - “Yescard” (1997)

Step 1: A logical flaw in the protocol allows one to copy a card and to use it without knowing the PIN code.

→ not a real problem, there is still a bank account to withdraw

Step 2: breaking encryption via factorisation of the following (96 digits) number:

213598703592091008239502270499962879705109534182
6417406442524165008583957746445088405009430865999

→ now, the number that is used is made of 232 digits
HTTPS connections

Lots of bugs and attacks, with fixes every month

FREAK attack discovered by Baraghavan et al (Feb. 2015)

1. A logical flaw that allows a man in the middle attacker to downgrade connections from 'strong' RSA to 'export-grade' RSA;
2. Breaking encryption via factorisation of such a key can be easily done.

→ 'export-grade' were introduced under the pressure of US governments agencies to ensure that they would be able to decrypt all foreign encrypted communication.
This is a passport with an **RFID tag** embedded in it.

The **RFID tag** stores:

- the information printed on your passport,
- a JPEG copy of your picture.
Electronic passport

This is a passport with an RFID tag embedded in it.

The RFID tag stores:
- the information printed on your passport,
- a JPEG copy of your picture.

The Basic Access Control (BAC) protocol is a key establishment protocol that has been designed to also ensure unlinkability.

ISO/IEC standard 15408

Unlinkability aims to ensure that a user may make multiple uses of a service or resource without others being able to link these uses together.
BAC protocol

Passport
$(K_E, K_M)$

Reader
$(K_E, K_M)$
BAC protocol

Passport $(K_E, K_M)$

Reader $(K_E, K_M)$

get_challenge
BAC protocol

Passport
\((K_E, K_M)\)

Reader
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\(N_P, K_P\)

\(N_P\)
BAC protocol

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$(K_E, K_M)$

Reader
$(K_E, K_M)$

get_challenge

$N_P, K_P$

$N_P$


$N_R, K_R$
BAC protocol

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$(K_E, K_M)$

Reader
$(K_E, K_M)$

$N_P, K_P$

$N_P$

$N_R, K_R$


get_challenge
BAC protocol

Passport
\((K_E, K_M)\)

Reader
\((K_E, K_M)\)

\[\text{get\_challenge} \]

\(N_P, K_P\)

\(N_P\)


\(K_{seed} = f(K_P, K_R)\)

\(K_{seed} = f(K_P, K_R)\)
Does the protocol satisfy a security property?
Does the protocol satisfy a security property?

Modelling

\[ \text{Modelling} \rightarrow \text{Graph} \rightarrow \models \phi \]

E-passport application

What about unlinkability of the ePassport holders?
This talk: formal methods for protocol verification

Outline of the this talk

1. Modelling cryptographic protocols and their security properties
2. Designing verification algorithms

Does the protocol satisfy a security property?
Part I

Modelling cryptographic protocols and their security properties
Two major families of models ...

... with some advantages and some drawbacks.

Computational model

- + messages are bitstring, a general and powerful adversary
- - manual proofs, tedious and error-prone

Symbolic model

- - abstract model, e.g. messages are terms
- + automatic proofs
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Symbolic model

– abstract model, e.g. messages are terms
+ automatic proofs

Some results allowed to make a link between these two very different models.

→ Abadi & Rogaway 2000
Protocols as processes

Applied pi calculus

basic programming language with constructs for concurrency and communication

→ based on the π-calculus [Milner et al., 92] ...

\[
P, Q := 0 \\
in(c, x).P \\
out(c, u).P \\
\text{if } u = v \text{ then } P \text{ else } Q \\
P \mid Q \\
!P \\
\text{new } n.P
\]

null process
input
output
conditional
parallel composition
replication
fresh name generation
Protocols as processes

Applied pi calculus [Abadi & Fournet, 01]

basic programming language with constructs for concurrency and communication

→ based on the $\pi$-calculus [Milner et al., 92] ...

\[
P, Q := \begin{align*}
0 & \quad \text{null process} \\
in(c, x).P & \quad \text{input} \\
out(c, u).P & \quad \text{output} \\
\text{if } u = v \text{ then } P \text{ else } Q & \quad \text{conditional} \\
P | Q & \quad \text{parallel composition} \\
!P & \quad \text{replication} \\
\text{new } n.P & \quad \text{fresh name generation}
\end{align*}
\]

... but messages that are exchanged are not necessarily atomic!
Terms are built over a set of names $\mathcal{N}$, and a signature $\mathcal{F}$.

\[
t ::= n \quad \text{name } n \\
| f(t_1, \ldots, t_k) \quad \text{application of symbol } f \in \mathcal{F}
\]
Messages as terms

Terms are built over a set of names $\mathcal{N}$, and a signature $\mathcal{F}$.

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 t ::= \begin{array}{ll}
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 f(t_1, \ldots, t_k) & \text{application of symbol } f \in \mathcal{F}
\end{array}
\]

Example: representation of $\{a, n\}_k$

- Names: $n, k, a$
- Constructors: senc, pair,
Messages as terms

Terms are built over a set of names $\mathcal{N}$, and a signature $\mathcal{F}$.

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**Example:** representation of $\{a, n\}_k$

- **Names:** $n, k, a$
- **constructors:** $\text{senc}, \text{pair}$,
- **destructors:** $\text{sdec}, \text{proj}_1, \text{proj}_2$.

The term algebra is equipped with an equational theory $\mathcal{E}$.

\[
\begin{align*}
\text{sdec}(\text{senc}(x, y), y) &= x \\
\text{proj}_1(\text{pair}(x, y)) &= x \\
\text{proj}_2(\text{pair}(x, y)) &= y
\end{align*}
\]

**Example:** $\text{sdec}(\text{senc}(s, k), k) =_\mathcal{E} s$. 
Semantics →:

Comm \quad \text{out}(c, u).P \parallel \text{in}(c, x).Q \rightarrow P \parallel Q\{u/x\}

Then \quad \text{if } u = v \text{ then } P \text{ else } Q \rightarrow P \text{ when } u \equiv_E v

Else \quad \text{if } u = v \text{ then } P \text{ else } Q \rightarrow Q \text{ when } u \not\equiv_E v
Semantics →:

- **Comm** \( \text{out}(c, u).P \mid \text{in}(c, x).Q \rightarrow P \mid Q\{u/x\} \)
- **Then** if \( u = v \) then \( P \) else \( Q \rightarrow P \) when \( u \equiv E v \)
- **Else** if \( u = v \) then \( P \) else \( Q \rightarrow Q \) when \( u \neq E v \)

Closed by

- **Structural equivalence** \( (\equiv) \):
  \[
  P \mid Q \equiv Q \mid P, \quad P \mid 0 \equiv P, \quad \ldots
  \]

- **Application of evaluation contexts**:
  \[
  \frac{P \rightarrow P'}{\text{new} n. \ P \rightarrow \text{new} n. \ P'} \quad \frac{P \rightarrow P'}{P \mid Q \rightarrow P' \mid Q}
  \]
Going back to the Denning Sacco protocol (1/2)

\[
\begin{align*}
A \rightarrow B & : \ \text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(B)) \\
B \rightarrow A & : \ \text{senc}(s, k)
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What function symbols and equations do we need to model this protocol?
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What function symbols and equations do we need to model this protocol?

1. **Symmetric encryption**: \( \text{senc}(\cdot, \cdot), \text{sdec}(\cdot, \cdot) \)

\[ \rightarrow \text{sdec(\text{senc}(x, y), y) = x} \]
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2. **asymmetric encryption**: \(\text{aenc}(. , .)\), \(\text{adec}(. , .)\), \(\text{pk}(. )\)
   \[ \rightarrow \text{adec}(\text{aenc}(x, \text{pk}(y)), y) = x \]
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3. **signature**: \( \text{ok}, \text{sign}(\cdot, \cdot), \text{check}(\cdot, \cdot), \text{getmsg}(\cdot) \)
   
   \[ \rightarrow \text{check}(\text{sign}(x, y), \text{pk}(y)) = \text{ok} \]
   
   \[ \rightarrow \text{getmsg}(\text{sign}(x, y)) = x \]
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The two terms involved in a normal execution are:

\( \text{aenc}(\text{sign}(k, ska), \text{pk}(skb)), \text{ and } \text{senc}(s, k) \)
Going back to the Denning Sacco protocol (2/2)

\[ A \rightarrow B : \text{aenc(}\text{sign}(k, \text{priv}(A)), \text{pub}(B)) \]

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Alice and Bob as processes:

\[ P_A(sk_a, pk_b) = \text{new } k. \text{out}(c, \text{aenc}(\text{sign}(k, sk_a), pk_b)).\text{in}(c, x_a). \ldots \]
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One possible scenario:

\[ P_{DS} = \text{new } \text{sk}_a, \text{sk}_b.(P_A(\text{sk}_a, \text{pk}(\text{sk}_b)) \mid P_B(\text{sk}_b, \text{pk}(\text{sk}_a)) \]
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\[ \rightarrow \text{new } sk_a, sk_b, k. (\text{in}(c, x_a). \ldots \]
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Going back to the Denning Sacco protocol (2/2)

\[ A \to B : \text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(B)) \]

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Alice and Bob as processes:

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\[ \rightarrow \text{new } sk_a, sk_b, k.( \text{in}(c, x_a). \ldots \]

\[ \text{new s}.\text{out}(c, \text{senc}(s, \text{getmsg(\text{adec(\text{aenc(sign}(k, sk_a), pk_b), sk_b))}))))) \]

\[ \rightarrow \text{this simply models a normal execution between two honest participants} \]
Confidentiality for process $P$ w.r.t. secret $s$

For all processes $A$ such that $A | P \rightarrow^* Q$, we have that $Q$ is not of the form $C[\text{out}(c, s).Q']$ with $c$ public.
Confidentiality for process $P$ w.r.t. secret $s$

For all processes $A$ such that $A \parallel P \rightarrow^* Q$, we have that $Q$ is not of the form $C[\text{out}(c, s).Q']$ with $c$ public.

Some difficulties:

- we have to consider all the possible executions in presence of an arbitrary adversary (modelled as a process)
- we have to consider realistic initial configurations
  - an unbounded number of agents,
  - replications to model an unbounded number of sessions,
  - reveal public keys and private keys to model dishonest agents,
  - honest agents may initiate a session with a dishonest agent, ...
Going back to the Denning Sacco protocol

\[ A \rightarrow B : \text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(B)) \]
\[ B \rightarrow A : \text{senc}(s, k) \]

The aforementioned attack

1. \( A \rightarrow C : \text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(C)) \)
2. \( C(A) \rightarrow B : \text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(B)) \)
3. \( B \rightarrow A : \text{senc}(s, k) \)

The “minimal” initial configuration to retrieve the attack is:

\[ P_{DS} = \text{new } sk_a, sk_b. (P_A(sk_a, pk(sk_c)) \mid P_B(sk_b, pk(sk_a) \mid \text{out}(c, pk(skb)))) \]
Going back to the Denning Sacco protocol

\[ A \to B : \text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(B)) \]
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The aforementioned attack

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The “minimal” initial configuration to retrieve the attack is:

\[ P_{DS} = \text{new } sk_a, sk_b.(P_A(sk_a, pk(sk_c)) | P_B(sk_b, pk(sk_a) | \text{out}(c, pk(skb)))) \]

Exercise: Exhibit the process \( A \) (the behaviour of the attacker) that witnesses the aforementioned attack, i.e. such that:

\[ A \mid P_{DS} \to^* C[\text{out}(c, s).Q'] \]
Privacy-type properties are modelled as equivalence-based properties.

Testing equivalence between $P$ and $Q$, denoted $P \approx Q$

for all processes $A$, we have that:

$$(A \mid P) \Downarrow_c \text{ if, and only if, } (A \mid Q) \Downarrow_c$$

where $R \Downarrow_c$ means that $R$ can evolve and emits on public channel $c$. 
Privacy-type properties are modelled as equivalence-based properties.

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**Exercise 1:**

$\text{out}(a, \text{yes}) \approx \text{out}(a, \text{no})$
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Exercise 1:

$$\text{out}(a, \text{yes}) \not\approx \text{out}(a, \text{no})$$

$$\rightarrow A = \text{in}(a, x).\text{if } x = \text{yes} \text{ then } \text{out}(c, \text{ok})$$
Privacy-type properties are modelled as equivalence-based properties.

Testing equivalence between $P$ and $Q$, denoted $P \approx Q$

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Exercise 2: $k$ and $k'$ are known to the attacker:

$$\text{new } s.\text{out}(a, \text{senc}(s, k)).\text{out}(a, \text{senc}(s, k')) \approx \text{new } s, s'.\text{out}(a, \text{senc}(s, k)).\text{out}(a, \text{senc}(s', k'))$$
Privacy-type properties are modelled as equivalence-based properties.

**Testing equivalence between** $P$ **and** $Q$, denoted $P \approx Q$

For all processes $A$, we have that:

$$(A | P) \downarrow_c \text{ if, and only if, } (A | Q) \downarrow_c$$

where $R \downarrow_c$ means that $R$ can evolve and emits on public channel $c$.

**Exercise 2:** $k$ and $k'$ are known to the attacker

\[
\begin{align*}
\text{new } s. & \text{out}(a, \text{senc}(s, k)).\text{out}(a, \text{senc}(s, k')) \neq \\
\text{new } s, & \text{s'}.\text{out}(a, \text{senc}(s, k)).\text{out}(a, \text{senc}(s', k'))
\end{align*}
\]

$$A = \text{in}(a, x).\text{in}(a, y).\text{if } (\text{sdec}(x, k) = \text{sdec}(y, k')) \text{ then out}(c, \text{ok})$$
Privacy-type properties are modelled as **equivalence-based properties**

**Testing equivalence between** $P$ and $Q$, denoted $P \equiv Q$

for all processes $A$, we have that:

$$(A \mid P) \Downarrow_c \text{ if, and only if, } (A \mid Q) \Downarrow_c$$

where $R \Downarrow_c$ means that $R$ can evolve and emits on public channel $c$.

**Exercise 3:** Are the two following processes in testing equivalence?

$$\text{new } s.\text{out}(a, s) \equiv \text{new } s.\text{new } k.\text{out}(a, \text{senc}(s, k))$$
Some privacy-type properties

Unlinkability

\[ \text{!new } \text{ke}. \text{new } \text{km}. (\text{!P}_{\text{BAC}} \mid \text{!R}_{\text{BAC}}) \approx \text{!new } \text{ke}. \text{new } \text{km}. (\text{P}_{\text{BAC}} \mid \text{!R}_{\text{BAC}}) \]

\[ \uparrow \]

many sessions
for each passport

only one session
for each passport

[Arapinis et al, 2010]
Some privacy-type properties

Unlinkability

\[
\text{old } \text{key}. \text{new } \text{key}. (!P_{BAC} \mid !R_{BAC}) \approx \text{old } \text{key}. \text{new } \text{key}. (P_{BAC} \mid !R_{BAC})
\]

\[
\uparrow
\]

many sessions
for each passport

only one session
for each passport

[Vrapinis et al, 2010]

Vote privacy

\[
S[V_A(\text{yes}) \mid V_B(\text{no})] \approx S[V_A(\text{no}) \mid V_B(\text{yes})]
\]

\[
\uparrow
\]

A votes yes
B votes no

A votes no
B votes yes

[Kremer and Ryan, 2005]
Designing verification algorithms
(from confidentiality to privacy)
State of the art in a nutshell

for analysing confidentiality properties

Unbounded number of sessions

- undecidable in general [Even & Goldreich, 83; Durgin et al, 99]
- decidable for restricted classes [Lowe, 99; Rammanujam & Suresh, 03]

→ ProVerif: A tool that does not correspond to any decidability result but works well in practice. [Blanchet, 01]
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- **undecidable** in general  
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→ **ProVerif**: A tool that does not correspond to any decidability result but works well in practice.  
  [Blanchet, 01]

Bounded number of sessions

- a **decidability** result (NP-complete)  
  [Rusinowitch & Turuani, 01; Millen & Shmatikov, 01]
- result extended to deal with various cryptographic primitives.

→ various automatic tools, e.g. **AVISPA platform**  
  [Armando et al., 05]
The deduction problem: is \( u \) deducible from \( T \)?

We consider a signature \( \mathcal{F} \) and an equational theory \( E \).

The deduction problem

<table>
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| output  | Can the attacker learn \( s \) from \( \phi \), i.e. does there exist a term (called recipe) \( R \) built using public symbols and \( w_1, \ldots, w_n \) such that \( R\phi \models_E s \). |
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Exercise: Let $\phi = \{ w_1 \triangleright v_1, \ldots, w_n \triangleright v_n \}$.

1. Is $k$ deducible from $\phi$?
2. What about $s$?
The deduction problem: is $u$ deducible from $T$?

We consider a signature $\mathcal{F}$ and an equational theory $E$.

The deduction problem

| **input** | A sequence $\phi$ of ground terms (i.e. messages) and a term $s$ (the secret) $\phi = \{w_1 \triangleright v_1, \ldots, w_n \triangleright v_n\}$ |
| **output** | Can the attacker learn $s$ from $\phi$, i.e. does there exist a term (called recipe) $R$ built using public symbols and $w_1, \ldots, w_n$ such that $R\phi =_E s$. |

Exercise: Let $\phi = \{w_1 \triangleright pk(ska); w_2 \triangleright pk(skb); w_3 \triangleright skc; w_4 \triangleright aenc(sign(k, ska), pk(skc)); w_5 \triangleright senc(s, k)\}$.

1. Is $k$ deducible from $\phi$? Yes, using $R_1 = \text{getmsg}(\text{adec}(w_4, w_3))$
2. What about $s$?
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### The deduction problem

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Exercise: Let \( \phi = \{w_1 \triangleright \text{pk}(ska); \ w_2 \triangleright \text{pk}(skb); \ w_3 \triangleright \text{skc}; \ w_4 \triangleright \text{aenc} (\text{sign}(k, ska), \text{pk}(skc)); \ w_5 \triangleright \text{senc}(s, k)\}. \)

1. Is \( k \) deducible from \( \phi \)? Yes, using \( R_1 = \text{getmsg} (\text{adec}(w_4, w_3)) \)
2. What about \( s \)? Yes, using \( R_2 = \text{sdec}(w_5, R_1) \).
The deduction problem

**Proposition**

The **deduction problem** is decidable in PTIME for the equational theory modelling the DS protocol (and for many others)

**Algorithm**

1. saturation of $\phi$ with its deducible subterms in one-step: $\phi^+$
2. does there exist $R$ such that $R\phi^+ = s$  \((\text{syntaxic equality})\)
The deduction problem

Proposition

The **deduction problem** is decidable in PTIME for the equational theory modelling the DS protocol (and for many others)

Algorithm

1. saturation of $\phi$ with its deducible subterms in one-step: $\phi^+$
2. does there exist $R$ such that $R\phi^+ = s$ (syntactic equality)

Going back to the previous example:

- $\phi = \{w_1 \triangleright pk(ska); w_2 \triangleright pk(skb); w_3 \triangleright skc; w_4 \triangleright aenc(sign(k, ska), pk(skc)); w_5 \triangleright senc(s, k)\}.$

- $\phi^+ = \phi \cup \{w_6 \triangleright sign(k, ska); w_7 \triangleright k; w_8 \triangleright s\}$.
Soundness If the algorithm returns Yes then $u$ is indeed deducible from $\phi$. \quad \rightarrow \text{easy to prove}
Soundness If the algorithm returns Yes then $u$ is indeed deducible from $\phi$. 

Termination The set of subterms is finite and polynomial, and one-step deducibility can be checked in polynomial time.

--- easy to prove for the deduction rules under study
Soundness, completeness, and termination

**Soundness** If the algorithm returns Yes then $u$ is indeed deducible from $\phi$. \hspace{1cm} \rightarrow \hspace{1cm} \text{easy to prove}

**Termination** The set of subterms is finite and polynomial, and one-step deducibility can be checked in polynomial time. \hspace{1cm} \rightarrow \hspace{1cm} \text{easy to prove for the deduction rules under study}

**Completeness** If $u$ is deducible from $\phi$, then the algorithm returns Yes.
Soundness, completeness, and termination

**Soundness** If the algorithm returns *Yes* then $u$ is indeed deducible from $\phi$. \(\rightarrow\) easy to prove

**Termination** The set of subterms is finite and polynomial, and one-step deducibility can be checked in polynomial time. \(\rightarrow\) easy to prove for the deduction rules under study

**Completeness** If $u$ is deducible from $\phi$, then the algorithm returns *Yes*. \(\rightarrow\) this relies on a locality property

**Locality lemma**

Let $\phi$ be a frame and $u$ be a deducible subterm of $\phi$. There exists a recipe $R$ witnessing this fact which satisfies the locality property: for any $R'$ subterm of $R$, we have that $R'\phi \downarrow$ is a subterm of $\phi$. 
Caution!

One should never underestimate the attacker!

The attacker can listen to the communication but also:

- intercept the messages that are sent by the participants,
- build new messages according to his deduction capabilities, and
- send messages on the communication network.

→ this is the co-called active attacker
Confidentiality using the constraint solving approach

→ active attacker, only for a bounded number of sessions
Confidentiality using the constraint solving approach

→ active attacker, only for a bounded number of sessions

Two main steps:

1. A symbolic exploration of all the possible traces
   
The infinite number of possible traces (i.e. experiment) are represented by a finite set of constraint systems
   
   → this set can be huge (exponential on the number of sessions) ...
   
   but some optimizations are used to reduce this number

2. A decision procedure for deciding whether a constraint system has a solution or not.

   → this algorithm works quite well
Step 1: confidentiality via constraint solving

We consider a finite sequence of actions:

\[ \text{in}(u_1); \text{out}(v_1); \text{in}(u_2); \ldots \text{out}(v_n) \]

We build the following constraint system:

\[
C = \left\{ \begin{array}{l}
T_0 ? u_1 \\
T_0, v_1 ? u_2 \\
\ldots \\
T_0, v_1, \ldots, v_n ? s
\end{array} \right. 
\]

\( u_i \) and \( v_i \) may contain variables.
Step 1: confidentiality via constraint solving

We consider a finite sequence of actions:

\[ \text{in}(u_1); \text{out}(v_1); \text{in}(u_2); \ldots \text{out}(v_n) \]

We build the following constraint system:

\[ C = \begin{cases} 
T_0 \vdash u_1 \\
T_0, v_1 \vdash u_2 \\
\ldots \\
T_0, v_1, \ldots, v_n \vdash s 
\end{cases} \]

**Solution of a constraint system \( C \)**

A substitution \( \sigma \) such that: for every \( T \vdash u \in C \), \( u\sigma \) is deducible from \( T\sigma \).
Going back to the Denning Sacco protocol

\[ A \to B : \ aenc(\text{sign}(k, \text{priv}(A)), \text{pub}(B)) \]
\[ B \to A : \ senc(s, k) \]

One possible interleaving:

\[
\text{out}(aenc(\text{sign}(k, ska), \text{pk}(skc)))
\]
\[
\text{in}(aenc(\text{sign}(x, ska), \text{pk}(skb))); \text{out}(senc(s, x))
\]
Going back to the Denning Sacco protocol

\[ A \to B : \text{aenc(sign}(k, \text{priv}(A)), \text{pub}(B)) \]
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One possible interleaving:

\[ \text{out(aenc(sign}(k, \text{ska}), \text{pk}(\text{skc})))} \]
\[ \text{in(aenc(sign}(x, \text{ska}), \text{pk}(\text{skb}))); \text{out(senc}(s, x)) \]

The associated constraint system is:

\[ T_0; \text{aenc(sign}(k, \text{ska}), \text{pk}(\text{skc})) \vdash \text{aenc(sign}(x, \text{ska}), \text{pk}(\text{skb})) \]
\[ T_0; \text{aenc(sign}(k, \text{ska}), \text{pk}(\text{skc})); \text{senc}(s, x) \vdash s \]

with \( T_0 = \{ \text{pk}(\text{ska}), \text{pk}(\text{skb}); \text{skc} \}. \)
Going back to the Denning Sacco protocol

\[ \begin{align*}
A \to B & : \ aenc(\text{sign}(k, \text{priv}(A)), \text{pub}(B)) \\
B \to A & : \ senc(s, k)
\end{align*} \]

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\text{out}(aenc(\text{sign}(k, ska), \text{pk}(skc))) \\
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\end{align*} \]

with \( T_0 = \{ \text{pk}(ska), \text{pk}(skb); skc \} \).

Question: Does \( C \) admit a solution?
Going back to the Denning Sacco protocol

\[ A \to B : \text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(B)) \]
\[ B \to A : \text{senc}(s, k) \]

One possible interleaving:

\[ \text{out}(\text{aenc}(\text{sign}(k, ska), \text{pk}(skc))) \]
\[ \text{in}(\text{aenc}(\text{sign}(x, ska), \text{pk}(skb))); \text{out}(\text{senc}(s, x)) \]

The associated constraint system is:

\[ T_0; \text{aenc}(\text{sign}(k, ska), \text{pk}(skc)) \vdash \text{aenc}(\text{sign}(x, ska), \text{pk}(skb)) \]
\[ T_0; \text{aenc}(\text{sign}(k, ska), \text{pk}(skc)); \text{senc}(s, x) \vdash s \]

with \( T_0 = \{\text{pk}(ska), \text{pk}(skb); skc\} \).

Question: Does \( C \) admit a solution? Yes: \( x \to k \).
The general case: is the constraint system $C$ satisfiable?

**Main idea**: simplify them until reaching $\bot$ or solved forms

**Constraint system in solved form**

$$C = \begin{cases} T_0 \vdash x_0 \\ T_0 \cup T_1 \vdash ? x_1 \\ \vdots \\ T_0 \cup T_1 \ldots \cup T_n \vdash ? x_n \end{cases}$$

**Question**

Is there a solution to such a system?
The general case: is the constraint system $\mathcal{C}$ satisfiable?

Main idea: simplify them until reaching $\bot$ or solved forms

Constraint system in solved form

$$\mathcal{C} = \begin{cases}
T_0 \vdash x_0 \\
T_0 \cup T_1 \vdash x_1 \\
\vdots \\
T_0 \cup T_1 \ldots \cup T_n \vdash x_n
\end{cases}$$

Question

Is there a solution to such a system?

Of course, yes! Choose $u_0 \in T_0$, and consider the substitution:

$$\sigma = \{x_0 \mapsto u_0, \ldots, x_n \mapsto u_0\}$$
Step 2: simplification rules

→ these rules deal with pairs and symmetric encryption only

\[ \begin{align*}
R_{ax} : & \quad C \land T \vdash u \iff C \quad \text{if } u \text{ is deducible from} \\
& \quad T \cup \{x \mid T' \vdash x \in C, T' \subsetneq T\}
\end{align*} \]

\[ \begin{align*}
R_{unif} : & \quad C \land T \vdash u \iff C\sigma \land T\sigma \vdash u\sigma \quad \text{if } \sigma = \text{mgu}(t_1, t_2) \text{ where } t_1, t_2 \in st(T) \cup \{u\}
\end{align*} \]

\[ \begin{align*}
R_{fail} : & \quad C \land T \vdash u \iff \bot \quad \text{if } \text{vars}(T \cup \{u\}) = \emptyset \text{ and } T \not\vdash u
\end{align*} \]

\[ \begin{align*}
R_f : & \quad C \land T \vdash f(u_1, u_2) \iff C \land T \vdash u_1 \land T \vdash u_2 \quad f \in \{\langle\rangle, \text{senc}\}
\end{align*} \]
Applying rule $R_f$

\[ R_f : \quad C \land T \vdash f(u_1, u_2) \rightsquigarrow C \land T \vdash u_1 \land T \vdash u_2 \]

Example:

\[ T_0; aenc(sign(k, ska), pk(skb)) \vdash aenc(sign(x, ska), pk(skb)) \]
Applying rule $R_f$

$$R_f : \quad C \land T \vdash f(u_1, u_2) \rightsquigarrow C \land T \vdash u_1 \land T \vdash u_2$$

Example:

$$T_0; \ aenc(\text{sign}(k, ska), \text{pk}(skc)) \vdash aenc(\text{sign}(x, ska), \text{pk}(skb))$$

$$\rightsquigarrow \begin{cases} 
T_0; \ aenc(\text{sign}(k, ska), \text{pk}(skc)) \vdash \text{sign}(x, ska) \\
T_0; \ aenc(\text{sign}(k, ska), \text{pk}(skc)) \vdash \text{pk}(skb) 
\end{cases}$$
Applying rule $R_{\text{unif}}$

$R_{\text{unif}} : \quad C \land T \vdash u \xRightarrow{\sigma} C\sigma \land T\sigma \vdash u\sigma$

if $\sigma = \text{mgu}(t_1, t_2)$ where $t_1, t_2 \in \text{st}(T) \cup \{u\}$

Example:

\[
\begin{aligned}
T_0; \text{aenc}(\text{sign}(k, ska), pk(skc)) & \vdash \text{sign}(x, ska) \\
T_0; \text{aenc}(\text{sign}(k, ska), pk(skc)) & \vdash \text{pk}(skb)
\end{aligned}
\]
Applying rule $R_{\text{unif}}$

$$R_{\text{unif}} : \quad C \land T \vdash u \leadsto_{\sigma} C\sigma \land T\sigma \vdash u\sigma$$

if $\sigma = \text{mgu}(t_1, t_2)$ where $t_1, t_2 \in st(T) \cup \{u\}$

Example:

$$\begin{align*}
T_0; \ aenc(\text{sign}(k, ska), \ pk(skc)) & \vdash \ \text{sign}(x, ska) \\
T_0; \ aenc(\text{sign}(k, ska), \ pk(skc)) & \vdash \ \text{pk}(skb) \\
\end{align*}$$

$$\leadsto \begin{align*}
T_0; \ aenc(\text{sign}(k, ska), \ pk(skc)) & \vdash \ \text{sign}(k, ska) \\
T_0; \ aenc(\text{sign}(k, ska), \ pk(skc)) & \vdash \ \text{pk}(skb) \\
\end{align*}$$
Applying rule $R_{ax}$

$$
R_{ax} : \quad C \land T \vdash u \quad \Rightarrow \quad C \quad \text{if } u \text{ deducible from } T \cup \{x \mid T' \vdash x \in C, T' \subset T\}
$$

Example: (assuming that $skc$ and $pk(skb)$ are in $T_0$)

$$
\left\{ \begin{array}{l}
T_0; \ aenc(sign(k, ska), pk(skc)) \vdash sign(k, ska) \\
T_0; \ aenc(sign(k, ska), pk(skc)) \vdash pk(skb)
\end{array} \right.
$$
Applying rule $R_{ax}$

$$R_{ax} : \quad C \land T \vdash u \implies C \quad \text{if } u \text{ deducible from}$$
$$\quad T \cup \{x \mid T' \vdash x \in C, T' \subset T\}$$

Example: (assuming that $skc$ and $pk(skb)$ are in $T_0$)

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\begin{array}{l}
T_0; \ aenc(\text{sign}(k, ska), pk(skc)) \vdash \text{sign}(k, ska) \\
T_0; \ aenc(\text{sign}(k, ska), pk(skc)) \vdash pk(skb)
\end{array}
\right\}$$

$$\implies \left\{ 
\begin{array}{l}
T_0; \ aenc(\text{sign}(k, ska), pk(skc)) \vdash \text{sign}(k, ska)
\end{array}
\right\}$$
Applying rule $R_{ax}$

\[ R_{ax} : \ C \land T \vdash u \rightsquigarrow C \ \text{if} \ u \ \text{deducible from} \]
\[ T \cup \{x \mid T' \vdash x \in C, T' \subset T\} \]

Example: (assuming that $skc$ and $pk(skb)$ are in $T_0$)

\[
\begin{cases}
T_0; \ aenc(sign(k, ska), pk(skc)) \vdash sign(k, ska) \\
T_0; \ aenc(sign(k, ska), pk(skc)) \vdash pk(skb)
\end{cases}
\]

\[
\rightsquigarrow \begin{cases}
T_0; \ aenc(sign(k, ska), pk(skc)) \vdash sign(k, ska)
\end{cases}
\]

\[
\rightsquigarrow \emptyset \ (\text{empty constraint system})
\]
Results on the simplification rules

\[ R_{ax} : \quad C \land T \vdash u \quad \leadsto \quad C \quad \text{if } u \text{ is deducible from} \]
\[ T \cup \{x \mid T' \vdash x \in C, T' \subset T\} \]

\[ R_{unif} : \quad C \land T \vdash u \quad \leadsto \sigma \quad C\sigma \land T\sigma \vdash u\sigma \]
\[ \text{if } \sigma = \text{mgu}(t_1, t_2) \text{ where } t_1, t_2 \in st(T) \cup \{u\} \]

\[ R_{fail} : \quad C \land T \vdash u \quad \leadsto \bot \quad \text{if } \text{vars}(T \cup \{u\}) = \emptyset \text{ and } T \not\vdash u \]

\[ R_f : \quad C \land T \vdash f(u_1, u_2) \quad \leadsto \quad C \land T \vdash u_1 \land T \vdash u_2 \quad f \in \{\langle\rangle, \text{senc}\} \]

Given a (well-formed) constraint system \( C \):

**Soundness**

If \( C \leadsto^*_\sigma C' \) and \( \theta \) solution of \( C' \) then \( \sigma\theta \) is a solution of \( C \).

\[ \longrightarrow \text{ easy to show} \]
Results on the simplification rules

\[ \text{R}_{ax} : \quad C \land T \vdash u \quad \rightsquigarrow \quad C \quad \text{if } u \text{ is deducible from} \]
\[ T \cup \{ x \mid T' \vdash x \in C, T' \subset T \} \]

\[ \text{R}_{unif} : \quad C \land T \vdash u \quad \rightsquigarrow_{\sigma} \quad C\sigma \land T\sigma \vdash u\sigma \]
\[ \text{if } \sigma = \text{mgu}(t_1, t_2) \text{ where } t_1, t_2 \in st(T) \cup \{ u \} \]

\[ \text{R}_{fail} : \quad C \land T \vdash u \quad \rightsquigarrow \quad \bot \quad \text{if } \text{vars}(T \cup \{ u \}) = \emptyset \text{ and } T \not\vdash u \]

\[ \text{R}_f : \quad C \land T \vdash f(u_1, u_2) \quad \rightsquigarrow \quad C \land T \vdash u_1 \land T \vdash u_2 \quad f \in \{ \langle \rangle, \text{senc} \} \]

Given a (well-formed) constraint system \( C \):

Exercise: Termination

There is no infinite chain \( C \rightsquigarrow_{\sigma_1} C_1 \ldots \rightsquigarrow_{\sigma_n} C_n \).
Results on the simplification rules

\[ R_{ax} : \quad C \land T \vdash u \leadsto C \text{ if } u \text{ is deducible from } \]
\[ T \cup \{ x \mid T' \vdash x \in C, T' \subset T \} \]

\[ R_{unif} : \quad C \land T \vdash u \leadsto_{\sigma} C\sigma \land T\sigma \vdash u\sigma \text{ if } \sigma = \text{mgu}(t_1, t_2) \text{ where } t_1, t_2 \in st(T) \cup \{u\} \]

\[ R_{fail} : \quad C \land T \vdash u \leadsto \bot \text{ if } \text{vars}(T \cup \{u\}) = \emptyset \text{ and } T \not\vdash u \]

\[ R_{f} : \quad C \land T \vdash f(u_1, u_2) \leadsto C \land T \vdash u_1 \land T \vdash u_2 \text{ if } f \in \{\langle\rangle, \text{senc}\} \]

Given a (well-formed) constraint system \( C \):

Exercise: Termination

There is no infinite chain \( C \leadsto_{\sigma_1} C_1 \ldots \leadsto_{\sigma_n} C_n \).

\[ \longrightarrow \text{ using the lexicographic order (number of var, size of rhs)} \]
Results on the simplification rules

- **R\textsubscript{ax}**: $\mathcal{C} \land T \vdash u \leadsto \mathcal{C}$ if $u$ is deducible from $T \cup \{x \mid T' \vdash x \in \mathcal{C}, T' \subset T\}$

- **R\textsubscript{unif}**: $\mathcal{C} \land T \vdash u \leadsto_{\sigma} \mathcal{C}\sigma \land T\sigma \vdash u\sigma$
  
  if $\sigma = \text{mgu}(t_1, t_2)$ where $t_1, t_2 \in \text{st}(T) \cup \{u\}$

- **R\textsubscript{fail}**: $\mathcal{C} \land T \vdash u \leadsto \bot$
  
  if $\text{vars}(T \cup \{u\}) = \emptyset$ and $T \nvdash u$

- **R\textsubscript{f}**: $\mathcal{C} \land T \vdash f(u_1, u_2) \leadsto \mathcal{C} \land T \vdash u_1 \land T \vdash u_2 \ f \in \{\langle\rangle, \text{senc}\}$

Given a (well-formed) constraint system $\mathcal{C}$:

**Completeness**

If $\theta$ is a solution of $\mathcal{C}$ then there exists $\mathcal{C}'$ and $\theta'$ such that $\mathcal{C} \leadsto^{*}_{\sigma} \mathcal{C}'$, $\theta'$ is a solution of $\mathcal{C}'$, and $\theta = \sigma\theta'$.

→ more involved to show
Step 2: procedure for solving a constraint system

Main idea of the procedure:

\[
C = \begin{cases}
T_0, \quad u_1 \\
T_0, v_1, \quad u_2 \\
T_0, v_1, \ldots, v_n, \quad s
\end{cases}
\]

\[\rightarrow\] this gives us a symbolic representation of all the solutions.
Main result

Theorem

Deciding confidentiality for a bounded number of sessions is decidable for classical primitives (actually in co-NP).

Exercise: NP-hardness can be shown by encoding 3-SAT
Main result

**Theorem**

Deciding confidentiality for a bounded number of sessions is decidable for classical primitives (actually in co-NP).

**Exercise:** NP-hardness can be shown by encoding 3-SAT

**Some extensions that already exist:**

1. disequality tests (protocol with else branches)
2. more primitives: asymmetric encryption, blind signature, exclusive-or, ...
This approach has been implemented in the Avantssar Platform.

http://www.avantssar.eu

→ Typically concludes within few seconds over the flawed protocols of the Clark/Jacob library.
Designing verification algorithms
(from confidentiality to privacy)
Deduction is not always sufficient

→ The intruder knows the values yes and no!

The real question

Is the intruder able to tell whether Alice sends yes or no?
The ground case: are $\phi$ and $\psi$ in static equivalence?

The static equivalence problem

**Input** Two frames $\phi$ and $\psi$

\[
\phi = \{ w_1 \triangleright u_1, \ldots, w_\ell \triangleright u_\ell \} \quad \psi = \{ w_1 \triangleright v_1, \ldots, w_\ell \triangleright v_\ell \}
\]

**Output** Can the attacker distinguish the two frames, i.e. does there exist a test $R_1 \not\equiv R_2$ such that:

\[
R_1 \phi =_E R_2 \phi \text{ but } R_1 \psi \not\equiv E R_2 \psi \text{ (or the converse).}
\]
The ground case: are $\phi$ and $\psi$ in static equivalence?

The static equivalence problem

**input** Two frames $\phi$ and $\psi$

$$\phi = \{ w_1 \triangleright u_1, \ldots, w_\ell \triangleright u_\ell \} \quad \psi = \{ w_1 \triangleright v_1, \ldots, w_\ell \triangleright v_\ell \}$$

**output** Can the attacker distinguish the two frames, i.e. does there exist a test $R_1 \not\equiv R_2$ such that:

$$R_1\phi \equiv E_R_2\phi \text{ but } R_1\psi \not\equiv E_R_2\psi \text{ (or the converse).}$$

Example: Consider the frames:

- $\phi = \{ w_1 \triangleright \text{aenc}(\langle \text{yes}, r_1 \rangle, \text{pk}(\text{sks})); \ w_2 \triangleright \text{sks} \};$ and
- $\psi = \{ w_1 \triangleright \text{aenc}(\langle \text{no}, r_2 \rangle, \text{pk}(\text{sks})); \ w_2 \triangleright \text{sks} \}.$

They are not in static equivalence: $\text{proj}_1(\text{adec}(w_1, w_2)) \not\equiv \text{yes}.$
Consider the equational theories:

- $E_{\text{senc}}$ defined by $sdec(senc(x, y), y) = x$, and
- $E_{\text{cipher}}$ which extends $E_{\text{senc}}$ by the equation $senc(sdec(x, y), y) = x$.

Questions

Which of the following pairs of frames are statically equivalent? Whenever applicable give the distinguishing test.

\[
\begin{align*}
\{ w_1 \triangleright yes \} & \sim_{E_{\text{senc}}} \{ w_1 \triangleright no \} \\
\{ w_1 \triangleright senc(yes, k) \} & \sim_{E_{\text{senc}}} \{ w_1 \triangleright senc(no, k) \} \\
\{ w_1 \triangleright senc(n, k), w_2 \triangleright k \} & \sim_{E_{\text{senc}}} \{ w_1 \triangleright senc(n, k), w_2 \triangleright k' \} \\
\{ w_1 \triangleright senc(n, k), w_2 \triangleright k \} & \sim_{E_{\text{cipher}}} \{ w_1 \triangleright senc(n, k), w_2 \triangleright k' \}
\end{align*}
\]

$k, k', n$ are a priori unknown to the attacker.
Consider the equational theories:

- $E_{\text{senc}}$ defined by $s\text{dec}(s\text{enc}(x, y), y) = x$, and
- $E_{\text{cipher}}$ which extends $E_{\text{senc}}$ by the equation $s\text{enc}(s\text{dec}(x, y), y) = x$.

Questions

Which of the following pairs of frames are statically equivalent? Whenever applicable give the distinguishing test.

$$
\{w_1 \triangleright \text{yes}\} \overset{?}{\sim}_{E_{\text{senc}}} \{w_1 \triangleright \text{no}\} \quad X
$$

$$
\{w_1 \triangleright s\text{enc}(\text{yes}, k)\} \overset{?}{\sim}_{E_{\text{senc}}} \{w_1 \triangleright s\text{enc}(\text{no}, k)\}
$$

$$
\{w_1 \triangleright s\text{enc}(n, k), w_2 \triangleright k\} \overset{?}{\sim}_{E_{\text{senc}}} \{w_1 \triangleright s\text{enc}(n, k), w_2 \triangleright k'\}
$$

$$
\{w_1 \triangleright s\text{enc}(n, k), w_2 \triangleright k\} \overset{?}{\sim}_{E_{\text{cipher}}} \{w_1 \triangleright s\text{enc}(n, k), w_2 \triangleright k'\}
$$

$k, k'$, and $n$ are a priori unknown to the attacker.
Exercise

Consider the equational theories:

- $E_{senc}$ defined by $\text{sdec}(\text{senc}(x, y), y) = x$, and
- $E_{cipher}$ which extends $E_{senc}$ by the equation $\text{senc}(\text{sdec}(x, y), y) = x$.

Questions

Which of the following pairs of frames are statically equivalent? Whenever applicable give the distinguishing test.

$$
\begin{align*}
\{ w_1 \triangleright yes \} \sim_{E_{senc}} \{ w_1 \triangleright no \} & \quad \text{X} \\
\{ w_1 \triangleright \text{senc}(yes, k) \} \sim_{E_{senc}} \{ w_1 \triangleright \text{senc}(no, k) \} & \quad \checkmark \\
\{ w_1 \triangleright \text{senc}(n, k), w_2 \triangleright k \} \sim_{E_{senc}} \{ w_1 \triangleright \text{senc}(n, k), w_2 \triangleright k' \} & \\
\{ w_1 \triangleright \text{senc}(n, k), w_2 \triangleright k \} \sim_{E_{cipher}} \{ w_1 \triangleright \text{senc}(n, k), w_2 \triangleright k' \} & 
\end{align*}
$$

$k, k', \text{ and } n$ are a priori unknown to the attacker.
Exercise

Consider the equational theories:

- \( E_{senc} \) defined by \( sdec(senc(x, y), y) = x \), and
- \( E_{cipher} \) which extends \( E_{senc} \) by the equation \( senc(sdec(x, y), y) = x \).

Questions

Which of the following pairs of frames are statically equivalent? Whenever applicable give the distinguishing test.

\[
\begin{align*}
\{ w_1 \triangleright yes \} & \not\sim_{E_{senc}} \{ w_1 \triangleright no \} & \checkmark \\
\{ w_1 \triangleright senc(yes, k) \} & \not\sim_{E_{senc}} \{ w_1 \triangleright senc(no, k) \} \\
\{ w_1 \triangleright senc(n, k), w_2 \triangleright k \} & \not\sim_{E_{senc}} \{ w_1 \triangleright senc(n, k), w_2 \triangleright k' \} \\
\{ w_1 \triangleright senc(n, k), w_2 \triangleright k \} & \not\sim_{E_{cipher}} \{ w_1 \triangleright senc(n, k), w_2 \triangleright k' \}
\end{align*}
\]

\( k, k', \) and \( n \) are a priori unknown to the attacker.
Exercise

Consider the equational theories:

- $E_{\text{senc}}$ defined by $\text{sdec}(\text{senc}(x, y), y) = x$, and
- $E_{\text{cipher}}$ which extends $E_{\text{senc}}$ by the equation $\text{senc}(\text{sdec}(x, y), y) = x$.

Questions

Which of the following pairs of frames are statically equivalent? Whenever applicable give the distinguishing test.

\[
\begin{align*}
\{ w_1 \triangleright \text{yes} \} & \sim_{E_{\text{senc}}} \{ w_1 \triangleright \text{no} \} \quad \text{X} \\
\{ w_1 \triangleright \text{senc}(\text{yes}, k) \} & \sim_{E_{\text{senc}}} \{ w_1 \triangleright \text{senc}(\text{no}, k) \} \quad \checkmark \\
\{ w_1 \triangleright \text{senc}(n, k), w_2 \triangleright k \} & \sim_{E_{\text{senc}}} \{ w_1 \triangleright \text{senc}(n, k), w_2 \triangleright k' \} \quad \text{X} \\
\{ w_1 \triangleright \text{senc}(n, k), w_2 \triangleright k \} & \sim_{E_{\text{cipher}}} \{ w_1 \triangleright \text{senc}(n, k), w_2 \triangleright k' \} \quad \checkmark
\end{align*}
\]

$k, k', n$ are a priori unknown to the attacker.
The static equivalence problem

Proposition

The **static equivalence problem** is decidable in PTIME for the theory modelling the DS protocol (and for many others)
The static equivalence problem

Proposition

The static equivalence problem is decidable in PTIME for the theory modelling the DS protocol (and for many others)

Algorithm

1. saturation of $\phi/\psi$ with their deducible subterms $\phi^+/\psi^+$
2. does there exist a test $R_1 \equiv R_2$ such that $R_1\phi^+ = R_2\phi^+$ whereas $R_1\psi^+ \neq R_2\psi^+$ (again syntaxic equality) ?
   → Actually, we only need to consider small tests
Going back to our previous example

Example

- $\phi = \{ w_1 \triangleright aenc(\langle yes, r_1 \rangle, pk(sks)); \ w_2 \triangleright sks \}; \ \text{and}$
- $\psi = \{ w_1 \triangleright aenc(\langle no, r_2 \rangle, pk(sks)); \ w_2 \triangleright sks \}.$

They are not in static equivalence: $\text{proj}_1(\text{adec}(w_1, w_2)) \not= \text{yes}.$
Example

\[ \phi = \{ w_1 \triangleright aenc(\langle yes, r_1 \rangle, pk(sks)); \ w_2 \triangleright sks \}; \ \text{and} \]
\[ \psi = \{ w_1 \triangleright aenc(\langle no, r_2 \rangle, pk(sks)); \ w_2 \triangleright sks \}. \]

They are not in static equivalence: \( \text{proj}_1(\text{adec}(w_1, w_2)) \neq yes. \)

Applying the algorithm

\[ \phi^+ = \phi \uplus \{ \]
\[ \psi^+ = \psi \uplus \{ \]
\[ , \ \text{and} \]
\[ . \]
Going back to our previous example

Example

\[ \phi = \{ w_1 \triangleright aenc(\langle yes, r_1 \rangle, pk(sks)); w_2 \triangleright sks \}; \text{ and} \]
\[ \psi = \{ w_1 \triangleright aenc(\langle no, r_2 \rangle, pk(sks)); w_2 \triangleright sks \}. \]

They are not in static equivalence: proj\(_1\)(adec(w_1, w_2)) \neq yes.

Applying the algorithm

\[ \phi^+ = \phi \cup \{ w_3 \triangleright \langle yes, r_1 \rangle; \}
\[ \psi^+ = \psi \cup \{ w_3 \triangleright \langle no, r_2 \rangle; \} \text{, and} \]
Going back to our previous example

Example

\[ \phi = \{ w_1 \triangleright aenc(\langle yes, r_1 \rangle, pk(sks)); w_2 \triangleright sks \}; \text{ and} \]

\[ \psi = \{ w_1 \triangleright aenc(\langle no, r_2 \rangle, pk(sks)); w_2 \triangleright sks \}. \]

They are not in static equivalence: \( \text{proj}_1(\text{adec}(w_1, w_2)) \neq yes \).

Applying the algorithm

\[ \phi^+ = \phi \uplus \{ w_3 \triangleright \langle yes, r_1 \rangle; w_4 \triangleright yes \}; \text{ and} \]

\[ \psi^+ = \psi \uplus \{ w_3 \triangleright \langle no, r_2 \rangle; w_4 \triangleright no \}. \]
Going back to our previous example

Example

\[ \phi = \{ w_1 \triangleright \text{aenc}(\langle \text{yes}, r_1 \rangle, \text{pk}(\text{sks})); \ w_2 \triangleright \text{sks} \}; \text{ and} \]

\[ \psi = \{ w_1 \triangleright \text{aenc}(\langle \text{no}, r_2 \rangle, \text{pk}(\text{sks})); \ w_2 \triangleright \text{sks} \}. \]

They are not in static equivalence: \( \text{proj}_1(\text{adec}(w_1, w_2)) \not= \text{yes} \).

Applying the algorithm

\[ \phi^+ = \phi \uplus \{ w_3 \triangleright \langle \text{yes}, r_1 \rangle; \ w_4 \triangleright \text{yes}; \ w_5 \triangleright r_1 \}, \text{ and} \]

\[ \psi^+ = \psi \uplus \{ w_3 \triangleright \langle \text{no}, r_2 \rangle; \ w_4 \triangleright \text{no}; \ w_5 \triangleright r_2 \}. \]
Going back to our previous example

Example

\[ \phi = \{ w_1 \triangleright \text{aenc}(\langle \text{yes}, r_1 \rangle, \text{pk}(sks)); \ w_2 \triangleright sks \}; \ \text{and} \]

\[ \psi = \{ w_1 \triangleright \text{aenc}(\langle \text{no}, r_2 \rangle, \text{pk}(sks)); \ w_2 \triangleright sks \}. \]

They are not in static equivalence: \( \text{proj}_1(\text{adec}(w_1, w_2)) \neq \text{yes} \).

Applying the algorithm

\[ \phi^+ = \phi \uplus \{ w_3 \triangleright \langle \text{yes}, r_1 \rangle; \ w_4 \triangleright \text{yes}; \ w_5 \triangleright r_1 \}, \ \text{and} \]

\[ \psi^+ = \psi \uplus \{ w_3 \triangleright \langle \text{no}, r_2 \rangle; \ w_4 \triangleright \text{no}; \ w_5 \triangleright r_2 \}. \]

\[ \rightarrow \phi^+ \ \text{and} \ \psi^+ \ \text{are not in static equivalence:} \ w_4 \neq \text{yes}. \]
State of the art in a nutshell (active attacker)

for analysing privacy properties

Unbounded number of sessions

- **undecidable** in general (and even under quite severe restriction)
- decidable for restricted classes [Chrétien PhD thesis, 16]

→ ProVerif checks diff-equivalence (too strong) [Blanchet et al, 05]
State of the art in a nutshell

for analysing privacy properties

Unbounded number of sessions

- **undecidable** in general (and even under quite severe restriction)
- decidable for **restricted** classes

\[ \rightarrow \text{ProVerif checks diff-equivalence (too strong)} \]

[Chrétien PhD thesis, 16]

Bounded number of sessions

- several decision procedures under various restrictions
  - e.g. [Baudet, 05], [Dawson & Tiu, 10], [Chevalier & Rusinowitch, 10],
  [Chadha et al., 12], [Cheval PhD thesis, 12].
One “recent” contribution

A procedure for deciding testing equivalence for a large class of processes for a bounded number of sessions.

→ PhD thesis of V. Cheval, 2012
Main result

A procedure for deciding testing equivalence for a large class of processes for a bounded number of sessions.

Class of processes:

- non-trivial else branches, private channels, and non-deterministic choice;
- a fixed set of cryptographic primitives (signature, encryption, hash function, mac).

→ PhD thesis of V. Cheval, 2012
Privacy using the constraint solving approach

What about unlinkability of the ePassport holders?

\[ P_{\text{BAC}}(K_E, K_M) \cong P_{\text{BAC}}(K'_E, K'_M) \]

Two main steps:

1. A symbolic exploration of all the possible traces
   The infinite number of possible traces (i.e. experiment) are represented by a finite set of constraint systems
   → this set can be huge (exponential on the number of sessions)!

2. A decision procedure for deciding (symbolic) equivalence between sets of constraint systems
   → this algorithm works quite well
the passport must reply to all received messages.
the passport must reply to all received messages.
the passport must reply to all received messages.

Passport \((K_E, K_M)\)

\(N_P, K_P\)

Reader \((K_E, K_M)\)

\(N_P\)


If MAC check succeeds

If nonce check fails

nonce_error
Step 1: from processes to constraint systems

Passport $P(K_E, K_M)$

\begin{verbatim}
in(= get_challenge); new $N_P$; new $K_P$;
out($N_P$); in($\langle z_E, z_M \rangle$);
if $z_M = \text{mac}_{K_M}(z_E)$ then
  if $N_P = \text{proj}_1(\text{proj}_2(s\text{dec}(z_E, K_E)))$ then
    out($\langle m, \text{mac}_{K_M}(m) \rangle$)
  else
    out($\text{nonce_error}$)
else
  out($\text{mac_error}$)
\end{verbatim}

where
\[
m = \{\langle N_P, \text{proj}_1(s\text{dec}(z_E, K_E)), K_P \rangle\}_{K_E}
\]

Once an interleaving of symbolic actions has been fixed (e.g. in; in; out), we generate the associated constraint systems:
Step 1: from processes to constraint systems

Passport $P(K_E, K_M)$

\[
\begin{align*}
\text{in}(= \text{get\_challenge}); & \text{new } N_P; \text{new } K_P; \\
\text{out}(N_P); & \text{in}(\langle z_E, z_M \rangle); \\
\text{if } z_M = \text{mac}_{K_M}(z_E) \text{ then } & \\
\quad \text{if } N_P = \text{proj}_1(\text{proj}_2(\text{sdec}(z_E, K_E))) \text{ then } \\
\quad & \text{out}(\langle m, \text{mac}_{K_M}(m) \rangle) \\
\text{else } & \text{out}(\text{nonce\_error}) \\
\text{else } & \text{out}(\text{mac\_error})
\end{align*}
\]

where

\[m = \{\langle N_P, \text{proj}_1(\text{sdec}(z_E, K_E)), K_P \rangle\}_{K_E}\]

Once an interleaving of symbolic actions has been fixed (e.g. in; in; out), we generate the associated constraint systems:

\[T_0 \vdash \text{get\_challenge}\]

\[\Phi = T_0;\]
Step 1: from processes to constraint systems

Passport \( P(K_E, K_M) \)

\[
\begin{align*}
\text{in}(= \text{get\_challenge}); \text{new } N_P; \text{new } K_P; \\
\text{out}(N_P); \text{in}([z_E, z_M]); \\
\text{if } z_M = \text{mac}_{K_M}(z_E) \text{ then} \\
\quad \text{if } N_P = \text{proj}_1(\text{proj}_2(\text{sdec}(z_E, K_E))) \text{ then} \\
\qquad \text{out}([m, \text{mac}_{K_M}(m)]) \\
\quad \text{else out}(\text{nonce\_error}) \\
\text{else out}(\text{mac\_error})
\end{align*}
\]

where
\[
m = \{[N_P, \text{proj}_1(\text{sdec}(z_E, K_E)), K_P]\}_{K_E}
\]

Once an interleaving of symbolic actions has been fixed (e.g. in; in; out), we generate the associated constraint systems:

\[ T_0 \vdash \text{get\_challenge} \]

\[ \Phi = T_0; N_P; \]
Step 1: from processes to constraint systems

**Passport** \( P(K_E, K_M) \)

\[
\begin{align*}
\text{in}(=\text{get\_challenge}); & \text{new } N_P; \text{new } K_P; \\
\text{out}(N_P); \text{in}(\langle z_E, z_M \rangle); \\
\text{if } z_M = \text{mac}_{K_M}(z_E) \text{ then} & \\
\quad \text{if } N_P = \text{proj}_1(\text{proj}_2(\text{sdec}(z_E, K_E))) \text{ then} & \\
\qquad \text{out}(\langle m, \text{mac}_{K_M}(m) \rangle) & \\
\text{else } \text{out}(\text{nonce\_error}) & \\
\text{else } \text{out}(\text{mac\_error}) & \\
\end{align*}
\]

where

\[
m = \{\langle N_P, \text{proj}_1(\text{sdec}(z_E, K_E)), K_P \rangle\}_{K_E}
\]

Once an interleaving of symbolic actions has been fixed (e.g. in; in; out), we generate the associated constraint systems:

\[
T_0 \vdash \text{get\_challenge} \\
T_0, N_P \vdash \langle z_E, z_M \rangle
\]

\[
\Phi = T_0; N_P;
\]
Step 1: from processes to constraint systems

Passport $P(K_E, K_M)$

\[
\begin{align*}
\text{in(= get\_challenge); new } N_P; \text{ new } K_P; \\
\text{out}(N_P); \text{in}(\langle z_E, z_M \rangle); \\
\text{if } z_M = \text{mac}_{K_M}(z_E) \text{ then} \\
\quad \text{if } N_P = \text{proj}_1(\text{proj}_2(\text{sdec}(z_E, K_E))) \text{ then} \\
\quad\quad \text{out}(\langle m, \text{mac}_{K_M}(m) \rangle) \\
\quad \text{else out}(\text{nonce\_error}) \\
\text{else out}(\text{mac\_error})
\end{align*}
\]

where

\[
m = \{ \langle N_P, \text{proj}_1(\text{sdec}(z_E, K_E)), K_P \rangle \}_{K_E}
\]

Once an interleaving of symbolic actions has been fixed (e.g. in; in; out), we generate the associated constraint systems:

\[
\begin{align*}
T_0 \vdash \text{get\_challenge} \\
T_0, N_P \vdash \langle z_E, z_M \rangle \\
z_M \neq \text{mac}_{K_M}(z_E)
\end{align*}
\]

\[\Phi = T_0; N_P; \text{mac\_error}\]
Step 1: from processes to constraint systems

Passport $P(K_E, K_M)$

\[
\begin{align*}
\text{in}(= \text{get\_challenge}); & \text{new } N_P; \text{new } K_P; & \\
\text{out}(N_P); & \text{in}(\langle z_E, z_M \rangle); & \\
\text{if } z_M = \text{mac}_{K_M}(z_E) \text{ then } & \text{if } N_P = \text{proj}_1(\text{proj}_2(\text{sdec}(z_E, K_E))) \text{ then } & \\
& \text{out}(\langle m, \text{mac}_{K_M}(m) \rangle) & \\
\text{else out}(\text{nonce\_error}) & & \\
\text{else out}(\text{mac\_error}) & & \\
\end{align*}
\]

where

\[m = \{\langle N_P, \text{proj}_1(\text{sdec}(z_E, K_E)), K_P \rangle\}_K_E\]

Once an interleaving of symbolic actions has been fixed (e.g. in; in; out), we generate the associated constraint systems:

\[\mathcal{C}_{\text{mac}};\]

\[\Phi = T_0; N_P; \text{mac\_error} \rightarrow \mathcal{C}_{\text{mac}}\]
Step 1: from processes to constraint systems

**Passport** $P(K_E, K_M)$

\[
in(= \text{get}_\text{-challenge}); \text{new } N_P; \text{new } K_P; \\
\text{out}(N_P); \text{in}(\langle z_E, z_M \rangle); \\
\text{if } z_M = \text{mac}_{K_M}(z_E) \text{ then} \\
\quad \text{if } N_P = \text{proj}_1(\text{proj}_2(\text{sdec}(z_E, K_E))) \text{ then} \\
\quad \quad \text{out}(\langle m, \text{mac}_{K_M}(m) \rangle) \\
\quad \text{else out}(\text{nonce}_\text{-error}) \\
\text{else out}(\text{mac}_\text{-error})
\]

where
\[
m = \{\langle N_P, \text{proj}_1(\text{sdec}(z_E, K_E)), K_P \rangle\}_{K_E}
\]

Once an interleaving of symbolic actions has been fixed (*e.g.* in; in; out), we generate the associated constraint systems:

$$\mathcal{C}_{\text{mac}};$$
Step 1: from processes to constraint systems

Passport \( P(K_E, K_M) \)

\[
\begin{align*}
\text{in} &= \text{get\_challenge}; \text{new } N_P; \text{new } K_P; \\
\text{out} &= (N_P); \text{in} \langle z_E, z_M \rangle; \\
\text{if } z_M &= \text{mac}_{K_M}(z_E) \text{ then} \\
& \quad \text{if } N_P = \text{proj}_1(\text{proj}_2(\text{sdec}(z_E, K_E))) \text{ then} \\
& \quad \quad \text{out} \langle m, \text{mac}_{K_M}(m) \rangle \\
& \quad \text{else out} (\text{nonce\_error}) \\
\text{else out} &= (\text{mac\_error})
\end{align*}
\]

\[\Phi = T_0; N_P;\]

Once an interleaving of symbolic actions has been fixed (e.g. in; in; out), we generate the associated constraint systems:

\[C_{\text{mac}};\]
Step 1: from processes to constraint systems

**Passport $P(K_E, K_M)$**

\[
\text{in}(= \text{get\_challenge}); \text{new } N_P; \text{new } K_P; \\
\text{out}(N_P); \text{in}(\langle z_E, z_M \rangle); \\
\text{if } z_M = \text{mac}_{K_M}(z_E) \text{ then} \\
\quad \text{if } N_P = \text{proj}_1(\text{proj}_2(\text{sdec}(z_E, K_E))) \text{ then} \\
\quad \quad \text{out}(\langle m, \text{mac}_{K_M}(m) \rangle) \\
\quad \text{else out(\text{nonce\_error})} \\
\text{else out(\text{mac\_error})}
\]

where
\[
m = \{\langle N_P, \text{proj}_1(\text{sdec}(z_E, K_E)), K_P \rangle\}_{K_E}
\]

Once an interleaving of symbolic actions has been fixed (*e.g.* in; in; out), we generate the associated constraint systems:

\[
\mathcal{C}_{\text{mac}};
\]

\[
T_0 \vdash \text{get\_challenge} \\
T_0, N_P \vdash \langle z_E, z_M \rangle
\]

\[
\Phi = T_0; N_P;
\]
Step 1: from processes to constraint systems

Passport \( P(K_E, K_M) \)

\[
\begin{align*}
\text{in}(= \text{get\_challenge}); \text{new } N_P; \text{new } K_P; \\
\text{out}(N_P); \text{in}(\langle z_E, z_M \rangle); \\
\text{if } z_M &= \text{mac}_{K_M}(z_E) \text{ then} \\
& \quad \text{if } N_P = \text{proj}_1(\text{proj}_2(\text{sdec}(z_E, K_E))) \text{ then} \\
& \quad \quad \text{out}(\langle m, \text{mac}_{K_M}(m) \rangle) \\
& \quad \text{else out(\text{nonce\_error})} \\
\text{else out(\text{mac\_error})}
\end{align*}
\]

where
\[
m = \{\langle N_P, \text{proj}_1(\text{sdec}(z_E, K_E)), K_P \rangle\}_{K_E}
\]

Once an interleaving of symbolic actions has been fixed (e.g. in; in; out), we generate the associated constraint systems:

\[
\mathcal{C}_{\text{mac}};
\]

\[
T_0 \vdash \text{get\_challenge} \\
T_0, N_P \vdash \langle z_E, z_M \rangle \\
? z_M = \text{mac}_{K_M}(z_E) \\
N_P \not= \text{proj}_1(\text{proj}_2(\text{sdec}(z_E, K_E))) \\
\Phi = T_0; N_P; \text{nonce\_error}
\]
Step 1: from processes to constraint systems

Passport $P(K_E, K_M)$

\[
\begin{align*}
\text{in}(= \text{get\_challenge}); & \text{new } N_P; \text{new } K_P; \\
\text{out}(N_P); & \text{in}(\langle z_E, z_M \rangle); \\
\text{if } z_M = \text{mac}_{K_M}(z_E) \text{ then } & \text{if } N_P = \text{proj}_1(\text{proj}_2(\text{sdec}(z_E, K_E))) \text{ then } \\
& \text{out}(\langle m, \text{mac}_{K_M}(m) \rangle) \\
& \text{else out}(\text{nonce\_error}) \\
\text{else out}(\text{mac\_error})
\end{align*}
\]

where

\[
m = \{\langle N_P, \text{proj}_1(\text{sdec}(z_E, K_E)), K_P \rangle\}_{K_E}
\]

Once an interleaving of symbolic actions has been fixed (e.g. in; in; out), we generate the associated constraint systems:

\[
C_{\text{mac}}; C_{\text{nonce}};
\]

\[
\begin{align*}
T_0 \vdash \text{get\_challenge} \\
T_0, N_P \vdash \langle z_E, z_M \rangle \\
z_M ? \vdash \text{mac}_{K_M}(z_E) \\
N_P \not\vdash \text{proj}_1(\text{proj}_2(\text{sdec}(z_E, K_E))) \\
\Phi = T_0; N_P; \text{nonce\_error} \\
\rightarrow C_{\text{nonce}}
\end{align*}
\]
Step 1: from processes to constraint systems

Passport $P(K_E, K_M)$

\[
\text{in}(= \text{get\_challenge}); \text{new } N_P; \text{new } K_P;
\text{out}(N_P); \text{in}((z_E, z_M));
\text{if } z_M = \text{mac}_{K_M}(z_E) \text{ then }
\text{if } N_P = \text{proj}_1(\text{proj}_2(\text{sdec}(z_E, K_E))) \text{ then }
\text{out}(⟨m, \text{mac}_{K_M}(m)⟩)
\text{else out}(\text{nonce\_error})
\text{else out}(\text{mac\_error})
\]

where
\[
m = \{⟨N_P, \text{proj}_1(\text{sdec}(z_E, K_E)), K_P⟩\}_{K_E}
\]

Once an interleaving of symbolic actions has been fixed (e.g. in; in; out), we generate the associated constraint systems:

\[C_{\text{mac}}; C_{\text{nonce}}; \ldots\]
Step 2: symbolic equivalence

To check whether $P \approx P'$, we have to check whether

$$\Sigma \approx_s \Sigma'$$

for all sequence of symbolic actions (e.g. $\text{in;}\text{in;}\text{out}$).
Step 2: symbolic equivalence

To check whether $P \approx P'$, we have to check whether

\[ \Sigma \approx_s \Sigma' \] for all sequence of symbolic actions (e.g. `in;in;out`).

**Symbolic equivalence $\Sigma \approx_s \Sigma'$**

- for all $C \in \Sigma$ for all $(\sigma, \theta) \in \text{Sol}(C)$, there exists $C' \in \Sigma'$ such that:
  
  \[ (\sigma', \theta) \in \text{Sol}(C') \quad \text{and} \quad \Phi \sigma \sim \Phi' \sigma' \] (static equivalence).

- and conversely
Step 2: symbolic equivalence

To check whether $P \approx P'$, we have to check whether

$$\Sigma \approx_{s} \Sigma'$$

for all sequence of symbolic actions (e.g. in;in;out).

Symbolic equivalence $\Sigma \approx_{s} \Sigma'$

- for all $C \in \Sigma$ for all $(\sigma, \theta) \in \text{Sol}(C)$, there exists $C' \in \Sigma'$ such that:
  $$(\sigma', \theta) \in \text{Sol}(C')$$ and $$\Phi \sigma \sim \Phi' \sigma'$$ (static equivalence).

- and conversely

Going back to the E-passport example

$$P_{BAC}(K_E, K_M) \approx ? P_{BAC}(K'_E, K'_M)$$

Among others, we have to check: $$\{C_{\text{mac}}; C_{\text{nonce}}; \ldots\} \approx_{s} \{C'_{\text{mac}}; C'_{\text{nonce}}; \ldots\}$$

where $C'_{\text{mac}}, C'_{\text{nonce}}, \ldots$ are the counterparts of $C_{\text{mac}}, C_{\text{nonce}}, \ldots$ in which $K_E/K_M$ are replaced by $K'_E/K'_M$. 

\{C_{\text{mac}}; C_{\text{nonce}}; \ldots\} \approx_s \{C'_{\text{mac}}; C'_{\text{nonce}}; \ldots\}

when \( T_0 \) contains \( \omega_0 > \langle \{N^0_R, N^0_P, K^0_R\}K_E, \text{mac}_{K_M}(\{N^0_R, N^0_P, K^0_R\}K_E) \rangle \)
\{C_{\text{mac}}; C_{\text{nonce}}; \ldots\} \approx_s \{C'_{\text{mac}}; C'_{\text{nonce}}; \ldots\}

when \ T_0\ contains \ w_0 \triangleright \langle \{N_0^R, N_0^P, K_0^R\} \mathit{K_E}, \ \text{mac}_K_M(\{N_0^R, N_0^P, K_0^R\} \mathit{K_E}) \rangle

\begin{align*}
\mathcal{C}_{\text{nonce}} = \begin{cases}
T_0 \triangleright \text{get\_challenge} \\
T_0 \triangleright \langle z_E, z_M \rangle \\
z_M = \text{mac}_K_M(z_E) \\
N_P \neq \text{proj}_1(\text{proj}_2(\text{sdec}(z_E, K_E)))
\end{cases}
\end{align*}

\[ \Phi = T_0; \text{nonce\_error} \]

\[ \rightarrow \text{A solution for } \mathcal{C}_{\text{nonce}} \text{ is:} \]

\[ \sigma = \{ z_E \mapsto \{N_0^R, N_0^P, K_0^R\} \mathit{K_E}, \ z_M \mapsto \text{mac}_K_M(\{N_0^R, N_0^P, K_0^R\} \mathit{K_E}) \} \]

with \( \theta = \{ X_1 \mapsto \text{get\_challenge}, \ X_2 \mapsto w_0 \} \).
\{C_{\text{mac}}; C_{\text{nonce}}; \ldots\} \approx_s \{C'_{\text{mac}}; C'_{\text{nonce}}; \ldots\}

when \ T_0 \text{ contains } w_0 \triangleright \langle \{N_R^0, N_P^0, K_R^0\}_{K_E}, \text{mac}_{K_M}(\{N_R^0, N_P^0, K_R^0\}_{K_E}) \rangle

Is \ \theta = \{X_1 \mapsto \text{get\_challenge}, \ X_2 \mapsto w_0\} \text{ also a solution on the other side?}
\{C_{\text{mac}}; C_{\text{nonce}}; \ldots\} \approx_s \{C'_{\text{mac}}; C'_{\text{nonce}}; \ldots\}

when \( T_0 \) contains \( w_0 \triangleright \langle \{N^0_R, N^0_P, K^0_R\}K_E, \text{mac}_{K_M}(\{N^0_R, N^0_P, K^0_R\}K_E) \rangle \)

Is \( \theta = \{X_1 \mapsto \text{get\_challenge}, X_2 \mapsto w_0\} \) also a solution on the other side?

What about the constraint system \( C'_{\text{nonce}} \)?

\[
C'_{\text{nonce}} = \begin{cases}
T_0 \vdash \text{get\_challenge} \\
T_0, N_P \vdash \langle z_E, z_M \rangle \\
z_M \vdash \text{mac}_{K'_M}(z_E) \\
N_P \neq \text{proj}_1(\text{proj}_2(\text{sdec}(z_E, K'_E)))
\end{cases}
\]

\( \Phi = T_0; N_P; \text{nonce\_error} \)
\{C_{\text{mac}}; C_{\text{nonce}}; \ldots\} \approx_s \{C'_{\text{mac}}; C'_{\text{nonce}}; \ldots\}

when $T_0$ contains $w_0 \triangleright \langle \{N^0_R, N^0_P, K^0_R\} K_E, \text{mac}_{K_M}(\{N^0_R, N^0_P, K^0_R\} K_E) \rangle$

Is $\theta = \{X_1 \mapsto \text{get\_challenge}, X_2 \mapsto w_0\}$ also a solution on the other side?

What about the constraint system $C'_{\text{nonce}}$?

\[
\begin{align*}
C'_{\text{nonce}} &= \left\{ \begin{array}{c}
T_0 \vdash \text{get\_challenge} \\
T_0, N_P \vdash \langle z_E, z_M \rangle \\
z_M \vdash \text{mac}_{K'_M}(z_E) \\
N_P \neq \text{proj}_1(\text{proj}_2(\text{sdec}(z_E, K'_E))))
\end{array} \right. \\
\Phi &= T_0; N_P; \text{nonce\_error}
\end{align*}
\]

$\rightarrow \theta$ is not a solution!
\{C_{\text{mac}}; C_{\text{nonce}}; \ldots\} \cong_s \{C'_{\text{mac}}; C'_{\text{nonce}}; \ldots\}

when $T_0$ contains $w_0 \triangleright \langle \{N_R^0, N_P^0, K_R^0\} \rangle_{K_E}$, $\text{mac}_{K_M}(\{N_R^0, N_P^0, K_R^0\} \rangle_{K_E})$

Is $\theta = \{X_1 \mapsto \text{get\textunderscore challenge}, X_2 \mapsto w_0\}$ also a solution on the other side?

What about the constraint system $C'_{\text{mac}}$?

\[C'_{\text{mac}} = \begin{cases} 
T_0 \vdash \text{get\textunderscore challenge} \\
T_0 \vdash \langle z_E, z_M \rangle \\
z_M \neq \text{mac}_{K'_M}(z_E) \\
\Phi' = T_0; N_P; \text{mac\textunderscore error}
\end{cases}\]
\{C_{\text{mac}}; \ C_{\text{nonce}}; \ldots\} \approx_s \{C'_{\text{mac}}; C'_{\text{nonce}}; \ldots\}

when \( T_0 \) contains \( w_0 \triangleright \langle \{N_R^0, N_P^0, K_R^0\} K_E, \ \text{mac}_{K_M}(\{N_R^0, N_P^0, K_R^0\} K_E) \rangle \)

Is \( \theta = \{X_1 \mapsto \text{get}_\text{-challenge}, \ X_2 \mapsto w_0\} \) also a solution on the other side?

What about the constraint system \( C'_{\text{mac}} \)?

\[
C'_{\text{mac}} = \begin{cases}
T_0 \vdash \text{get}_\text{-challenge} \\
T_0 \vdash \langle z_E, z_M \rangle \\
z_M \neq \text{mac}_{K_M'}(z_E)
\end{cases}
\]

\[\Phi' = T_0; N_P; \ \text{mac}_\text{-error}\]

\( \rightarrow \theta \) is a solution ...

but the resulting sequence of messages are not in static equivalence.

\( T_0; N_P; \ \text{nonce}_\text{-error} \not\sim T_0; N_P, \ \text{mac}_\text{-error} \)
An attack on the French passport  [Chothia & Smirnov, 10]

**Attack against unlinkability**

An attacker can track a French passport, provided he has once witnessed a successful authentication.
An attack on the French passport [Chothia & Smirnov, 10]

Attack against unlinkability

An attacker can track a French passport, provided he has once witnessed a successful authentication.

Part 1 of the attack. The attacker eavesdrops on Alice using her passport and records message $M$.

Alice’s Passport

$(K_E, K_M)$

$N_P, K_P$

Reader

$(K_E, K_M)$

$N_R, K_R$

$N_P$

An attack on the French passport [Chothia & Smirnov, 10]

Part 2 of the attack.
The attacker replays the message $M$ and checks the error code he receives.

$$????'s\ \text{Passport} \quad (K'_E, K'_M)$$

Attacker

get\_challenge

$N'_P, K'_P$

$N'_P$

$M = \{N_R, N_P, K_R\}_K_E, \ MAC_{K_M}(\{N_R, N_P, K_R\}_K_E)$
Part 2 of the attack.
The attacker replays the message $M$ and checks the error code he receives.

$\text{???'s Passport}$

$(K_E', K_M')$

$\text{Attacker}$

$\text{get\_challenge}$

$N_P'$

$M = \{N_R, N_P, K_R\}_K E, \ MAC_{K_M}(\{N_R, N_P, K_R\}_K E)$

$\text{mac\_error}$

$\Rightarrow \text{MAC check failed} \quad \Rightarrow \quad K_M' \neq K_M \quad \Rightarrow \quad ???? \text{ is not Alice}$
An attack on the French passport [Chothia & Smirnov, 10]

Part 2 of the attack.
The attacker replays the message $M$ and checks the error code he receives.

???'s Passport $(K'_E, K'_M)$

Attacker

get_challenge

$N'_P$, $K'_P$


nonce_error

$\Rightarrow$ MAC check succeeded $\Rightarrow K'_M = K_M \Rightarrow$ ???' is Alice
Main idea: We rewrite pairs \((\Sigma, \Sigma')\) of sets of constraint systems (extended to keep track of some information) until a trivial failure or a trivial success is found.
Results on the simplification rules

Termination
Applying blindly the simplification rules does not terminate but there is a particular strategy $S$ that allows us to ensure termination.

Soundness/Completeness
Let $(\Sigma_0, \Sigma'_0)$ be pair of sets of constraint systems, and consider a binary tree obtained by applying our simplification rule following a strategy $S$.

1. **soundness**: If all leaves of the tree are labeled with $(\bot, \bot)$ or $(\text{solved}, \text{solved})$, then $\Sigma_0 \approx_s \Sigma'_0$.

2. **completeness**: if $\Sigma_0 \approx_s \Sigma'_0$, then all leaves of the tree are labeled with $(\bot, \bot)$ or $(\text{solved}, \text{solved})$. 
APTE- Algorithm for Proving Trace Equivalence

http://projects.lsv.ens-cachan.fr/APTE (Ocaml - 12 KLocs)

→ developed by Vincent Cheval  [Cheval, TACAS’14]
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→ but a limited practical impact because it scales badly
Main objective

to develop POR techniques that are suitable for analysing security protocols (especially testing equivalence)
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to develop POR techniques that are suitable for analysing security protocols (especially testing equivalence)

Example: \( \text{in}(c_1, x_1) \cdot \text{out}(c_1, \text{ok}) | \text{in}(c_2, x_2) \cdot \text{out}(c_2, \text{ok}) \)

We propose two optimizations:

1. **Compression**: we impose a simple strategy on the exploration of the available actions (roughly outputs are performed first and using a fixed arbitrary order)
2. **Reduction**: we avoid exploring some redundant traces taking into account the data that are exchanged
Practical impact of our optimizations (in APTE)

Toy example

Denning Sacco protocol

→ Each optimisation brings an exponential speedup.
Practical impact of our optimizations (in APTE)

→ Each optimisation brings an exponential speedup.

<table>
<thead>
<tr>
<th>Protocol</th>
<th>reference</th>
<th>with POR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yahalom (3-party)</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Needham Schroeder (3-party)</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>Private Authentication (2-party)</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>E-Passport PA (2-party)</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>Denning-Sacco (3-party)</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Wide Mouthed Frog (3-party)</td>
<td>6</td>
<td>13</td>
</tr>
</tbody>
</table>

Maximum number of parallel processes verifiable in 20 hours.

→ Our optimisations make Apte much more useful in practice for investigating interesting scenarios.
Limitations of this approach

1. the algebraic properties of the primitives are abstracted away
   → no guarantee if the protocol relies on an encryption that satisfies some additional properties (e.g. RSA, ElGamal)

2. only the specification is analysed and not the implementation
   → most of the passports are actually linkable by a careful analysis of time or message length.

   http://www.loria.fr/~glondu/epassport/attaque-tailles.html

3. not all scenario are checked
   → no guarantee if the protocol is used one more time!
To sum up

Cryptographic protocols are:
  - **difficult** to design and analyse;
  - particularly vulnerable to **logical attacks**.

Strong primitives are necessary . . .

. . . but this is not sufficient!
To sum up

Cryptographic protocols are:

- difficult to design and analyse;
- particularly vulnerable to logical attacks.

It is important to ensure that the protocols we are using every day work properly.

We now have automatic and powerful verification tools to analyse:

- classical security goals, e.g. secrecy and authentication;
- relatively small protocols;
- protocols that rely on standard cryptographic primitives.
Conclusion

A need of formal methods in verification of security protocols. Regarding confidentiality (or authentication), powerful tool support that are nowdays used by industrials and security agencies.

It remains a lot to do for analysing privacy-type properties:

- formal definitions of some subtle security properties
  \(\rightarrow\) receipt-freeness, coercion-resistance in e-voting

- algorithms (and tools!) for checking automatically trace equivalence for various cryptographic primitives;
  \(\rightarrow\) homomorphic encryption used in e-voting, exclusive-or used in RFID protocols

- more composition results
  \(\rightarrow\) Could we derive some security guarantees of the whole e-passport application from the analysis performed on each subprotocol?