

# Verification of security protocols: from confidentiality to privacy

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- 12 academic departments: mathematics, computer science, chemistry, social sciences, ...
- 13 research laboratories

## Laboratoire Spécification & Vérification

## Verification of critical software and systems

**Goal:** develop the mathematical and algorithmic **foundations** to the development of tools for **automatically** proving correctness and detecting flaws.

**Applications:** computerized systems, databases, **security protocols**



### LSV in figures

- Founded in 1997
- Around 25 permanents + 15 PhD students
- 5 research teams

## Security of Information Systems

- 4 permanents: David Baelde, H. Comon-Lundh, S. Delaune, et J. Goubault-Larrecq.



- 1 engineer + 1 postdoc
- 3 phd students

# Cryptographic protocols everywhere !



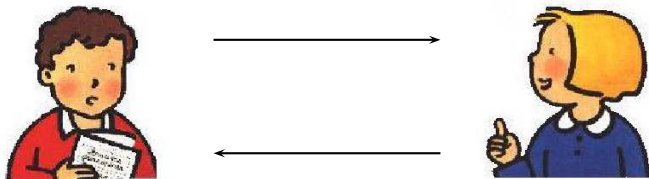
**Goal:** they aim at securing communications over public/insecure networks

# Some security properties

- **Secrecy**: May an intruder learn some secret message between two honest participants?
- **Authentication**: Is the agent **Alice** really talking to **Bob**?
- **Anonymity**: Is an attacker able to learn something about the identity of the participants who are communicating?
- **Non-repudiation**: **Alice** sends a message to **Bob**. **Alice** cannot later deny having sent this message. **Bob** cannot deny having received the message.
- ...

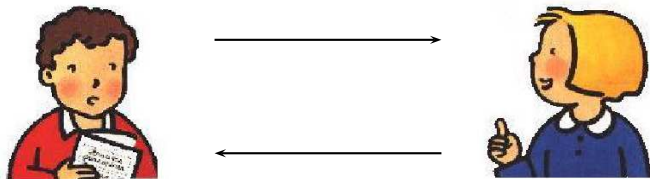
# How does a cryptographic protocol work (or not)?

**Protocol:** small programs explaining how to exchange messages



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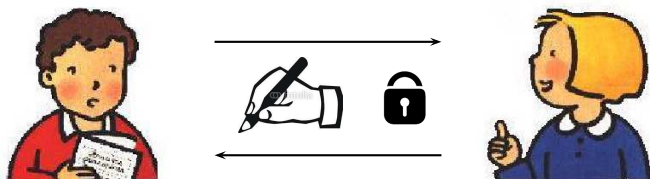
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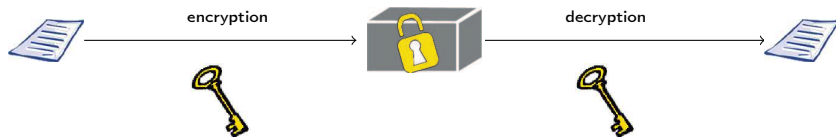
**Cryptographic:** make use of cryptographic primitives

**Examples:** symmetric encryption, asymmetric encryption, signature, hashes, ...



# What is a symmetric encryption scheme?

## Symmetric encryption

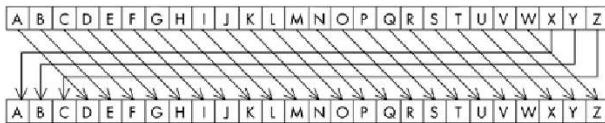


# What is a symmetric encryption scheme?

## Symmetric encryption



**Example:** This might be as simple as shifting each letter by a number of places in the alphabet (e.g. Caesar cipher)



**Today:** DES (1977), AES (2000)

# A famous example

## Enigma machine (1918-1945)

- electro-mechanical rotor cipher machines used by the German to encrypt during World War II
- permutations and substitutions

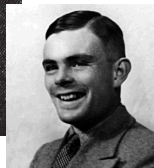
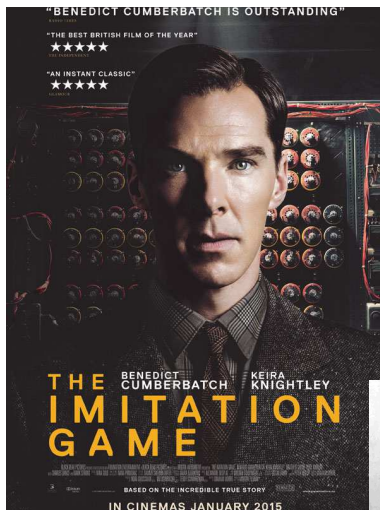


## A bit of history

- 1918: invention of the Enigma machine
- 1940: Battle of the Atlantic during which Alan Turing's Bombe was used to test Enigma settings.

→ Everything about the breaking of the Enigma cipher systems remained secret until the mid-1970s.

# Advertisement



# What is an asymmetric encryption scheme?

## Asymmetric encryption



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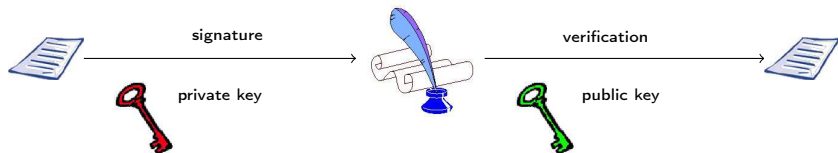
## Examples:

- 1976: first system published by W. Diffie, and M. Hellman,
  - 1977: RSA system published by R. Rivest, A. Shamir, and L. Adleman.
- their security relies on well-known **mathematical problems** (e.g. factorizing large numbers, computing discrete logarithms)

**Today:** those systems are still in use

# What is a signature scheme?

## Signature



## Example:

The RSA cryptosystem (in fact, most public key cryptosystems) can be used as a signature scheme.



# How does a cryptographic protocol work (or not)?

**Example:** A simplified version of the Denning-Sacco protocol (1981)

$A \rightarrow B$  :  $\text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(B))$

$B \rightarrow A$  :  $\text{senc}(s, k)$

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Consider a scenario where  $A$  starts a session with  $C$  who is **dishonest**.

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3.  $B \rightarrow A$  :  $\text{senc}(s, k)$  **Attack !**

# Exercise

We propose to fix the Denning-Sacco protocol as follows:

## Version 1

$$\begin{aligned} A \rightarrow B & : \text{aenc}(\langle A, B, \text{sign}(k, \text{priv}(A)) \rangle, \text{pub}(B)) \\ B \rightarrow A & : \text{senc}(s, k) \end{aligned}$$

## Version 2

$$\begin{aligned} A \rightarrow B & : \text{aenc}(\text{sign}(\langle A, B, k \rangle, \text{priv}(A))), \text{pub}(B)) \\ B \rightarrow A & : \text{senc}(s, k) \end{aligned}$$

Which version would you prefer to use?

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Which version would you prefer to use?      Version 2

→ Version 1 is still vulnerable to the aforementioned attack.

# What about protocols used in real life ?





Serge Humpich case - “ **Yescard** ” (1997)





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**Step 1:** A **logical flaw** in the protocol allows one to copy a card and to use it without knowing the PIN code.

→ not a real problem, there is still a bank account to withdraw







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**Step 1:** A **logical flaw** in the protocol allows one to copy a card and to use it without knowing the PIN code.

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**Step 2:** **breaking encryption** via factorisation of the following (96 digits) number: 213598703592091008239502270499962879705109534182  
6417406442524165008583957746445088405009430865999

→ now, the number that is used is made of **232** digits



Lots of bugs and attacks, with fixes every month

## FREAK attack discovered by Baraghavan et al (Feb. 2015)

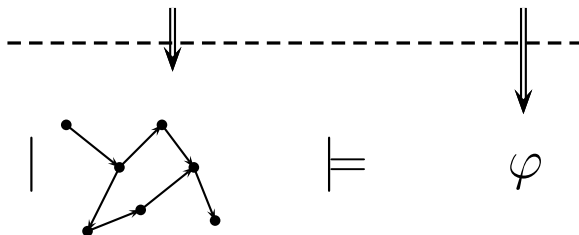
- 1 a logical flaw that allows a **man in the middle attacker** to downgrade connections from 'strong' RSA to 'export-grade' RSA;
- 2 **breaking encryption** via factorisation of such a key can be easily done.

→ 'export-grade' were introduced under the pressure of US governments agencies to ensure that they would be able to decrypt all foreign encrypted communication.

# This talk: formal methods for protocol verification

Does the protocol satisfy a security property?

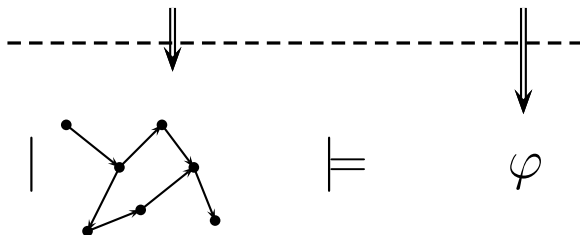
Modelling



# This talk: formal methods for protocol verification

Does the **protocol** *satisfy* a **security property**?

Modelling



## Two main tasks

- 1 Modelling cryptographic protocols and their security properties
- 2 Designing verification algorithms

Modelling messages  
and  
Deciding knowledge  
(in a simple setting)

→ Various models (e.g. [Dolev & Yao, 81]) having some common features

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## The attacker





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## Messages

They are abstracted by **terms**.

## The attacker

- may **read** every message sent on the network,
- may **intercept** and **send** new messages according to its deduction capabilities.  
→ only **symbolic** manipulations on terms.



→ It is important to have a tight modelling of messages

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## Terms

They are built over a **signature**  $\mathcal{F}$ , and an infinite set of **names**  $\mathcal{N}$ .

$$\begin{array}{l} t ::= n \quad \text{name } n \in \mathcal{N} \\ \quad | f(t_1, \dots, t_k) \quad \text{application of symbol } f \in \mathcal{F} \end{array}$$

- Names are used to model **atomic data**  
→ e.g. keys, nonces, agent names, ...
- Function symbols are used to model **cryptographic primitives**  
→ e.g. encryption, signature, ...

# A typical signature

## Standard primitives

$$\mathcal{F} = \{\text{senc}, \text{aenc}, \text{sk}, \text{sign}, \langle \rangle\}$$

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## Going back to the Denning Sacco protocol

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These messages can be modelled as follows:

- 1  $\text{aenc}(\text{sign}(k, \text{sk}(a)), b);$
- 2  $\text{senc}(s, k)$

## Symbolic manipulation on terms

He may **build** new messages following **deduction rules**



### Pairing

$$\frac{x \quad y}{\langle x, y \rangle} \quad \frac{\langle x, y \rangle}{x} \quad \frac{\langle x, y \rangle}{y}$$

### Symmetric encryption

$$\frac{x \quad y}{\text{senc}(x, y)} \quad \frac{\text{senc}(x, y) \quad y}{x}$$

### Asymmetric encryption

$$\frac{x \quad y}{\text{aenc}(x, y)} \quad \frac{\text{aenc}(x, y) \quad \text{sk}(y)}{x}$$

### Signature

$$\frac{x \quad \text{sk}(y)}{\text{sign}(x, \text{sk}(y))} \quad \frac{\text{sign}(x, \text{sk}(y))}{x}$$

We say that  $u$  is **deducible from**  $T$  if there exists a proof tree such that:

- 1 each leaf is labeled by  $v$  with  $v \in T$ ;
- 2 for each node labeled by  $v_0$  and having  $n$  sons labeled by  $v_1, \dots, v_n$ , there exists a deduction rule  $R$  such that

$$\frac{v_1 \quad \dots \quad v_n}{v_0} \quad \text{is an instance of } R$$

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## Exercise - Going back to the Denning Sacco protocol

Let  $T = \{a, b, c, \text{sk}(c), \text{aenc}(\text{sign}(k, \text{sk}(a)), c), \text{senc}(s, k)\}$ .

Is  $s$  deducible from  $T$  ?



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**Answer:** Of course, Yes !

$$\begin{array}{c}
 \text{aenc}(\text{sign}(k, \text{sk}(a)), c) \quad \text{sk}(c) \\
 \hline
 \text{sign}(k, \text{sk}(a)) \\
 \hline
 \text{senc}(s, k) \qquad \qquad \qquad k \\
 \hline
 s
 \end{array}$$

1.  $A \rightarrow C : \text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(C))$
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## Exercise (continued)

Let  $T_0 = \{a, b, c, \text{sk}(c), \text{aenc}(\text{sign}(k, \text{sk}(a)), c)\}$ .

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# Denning Sacco protocol

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Is  $\text{aenc}(\text{sign}(k, \text{sk}(a)), b)$  deducible from  $T_0$ ?

**Answer:** Of course, Yes !

$$\frac{\frac{\text{aenc}(\text{sign}(k, \text{sk}(a)), c) \quad \text{sk}(c)}{\text{sign}(k, \text{sk}(a))} \quad b}{\text{aenc}(\text{sign}(k, \text{sk}(a)), b)}$$

## The deduction problem

**Input:** a finite set of terms  $T$  (the knowledge of the attacker) and a term  $u$  (the secret),

**Output:** Is  $u$  deducible from  $T$ ?

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## Proposition

The deduction problem is decidable in PTIME.

## Algorithm

- 1 **Saturation** of  $T$  with terms in  $St(T \cup \{u\})$  that are deducible in one step;
- 2 if  $u$  is in the saturated set then return **Yes** else return **No**.

**Soundness** If the algorithm returns **Yes** then  $u$  is indeed deducible from  $T$ .  $\longrightarrow$  easy to prove



# Soundness, completeness, and termination

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**Completeness** If the term  $u$  is deducible from  $T$ , then the algorithm returns **Yes**. Otherwise, it returns **No**.  
 $\longrightarrow$  this relies on a **locality property**

## Locality lemma

Let  $T$  and  $u$  be such that  $T \vdash u$ . There exists a proof tree witnessing this fact for which all the nodes are labeled by some  $v$  with  $v \in St(T \cup \{u\})$ .

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Let  $P$  be a proof tree witnessing the fact that  $T \vdash u$  having a **minimal size** (number of nodes). We show by **induction on  $P$**  that:

- if  $P$  ends with root labeled by  $v$  then  $P$  only contains terms in  $St(T \cup \{v\})$ ;

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We first split the deduction rules into **two categories**:

- 1 **composition rules**: encryption, signature, and pairing
- 2 **decomposition rules**: decryption, projections, ...

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- if  $P$  ends with root labeled by  $v$  then  $P$  only contains terms in  $St(T \cup \{v\})$ ;
- if  $P$  ends with a **decomposition rule** then  $P$  only contains terms in  $St(T)$ .

→ this is left as an exercise

# Exercise

Consider the following set of deduction rules:

$$\frac{x \quad sk(y)}{sign(x, sk(y))} \quad \frac{sign(x, sk(y)) \quad vk(y)}{x} \quad \frac{y}{vk(y)}$$

- 1 Give an example showing that these deduction rules are **not local**.
- 2 Extend the notion of subterms to restore the locality property, and show that the deduction problem is decidable.

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## Solution

- 1 Let  $T = \{sign(s, sk(a)); a\}$  and  $u = s$ .
- 2  $St^+(T) = St(T) \cup \{vk(u) \mid sk(u) \in vk(u) \in St(T)\}$ .  
→ the locality proof is left as an exercise

## Exercise

Consider the following set of deduction rules:

$$\frac{x \quad y}{\langle x, y \rangle} \quad \frac{\langle x, y \rangle}{x} \quad \frac{\langle x, y \rangle}{y} \quad \frac{x \quad y}{\text{senc}(x, y)} \quad \frac{\text{senc}(x, y) \quad y}{x}$$

In order to decide whether a term  $u$  is deducible from a set of terms  $T$ , we propose the following algorithm:

- 1 Starting from  $T$ , apply as much as possible the decryption and the projection rules. This leads to a set of terms called  $\text{Decomposition}(T)$ .
- 2 Check whether  $u$  can be obtained by applying the composition rules on top of terms in  $\text{Decomposition}(T)$ .
- 3 In case of success, the algorithm returns **Yes**. Otherwise, it returns **No**.

## Questions

What about termination, soundness, and completeness?

Modelling messages  
and  
Deciding knowledge  
(in a richer setting)



# More cryptographic primitives

We may want to consider a richer term algebra and rely on an **equational theory E** to take into account the properties of the primitives

Exclusive or operator:

$$\begin{array}{lcl} (x \oplus y) \oplus z & = & x \oplus (y \oplus z) \\ x \oplus y & = & y \oplus x \end{array} \qquad \begin{array}{lcl} x \oplus x & = & 0 \\ x \oplus 0 & = & x \end{array}$$

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**Blind signature** (used in evoting protocol)

$$\begin{aligned}\text{check}(\text{sign}(x, y), \text{vk}(y)) &= x \\ \text{unblind}(\text{blind}(y, y), y) &= x \\ \text{unblindsign}(\text{sign}(\text{blind}(x, y), z), y) &= \text{sign}(x, z)\end{aligned}$$

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**Homomorphic encryption:**

$$\begin{aligned}\text{enc}(\langle x, y \rangle, z) &= \langle \text{enc}(x, z), \text{enc}(y, z) \rangle & \text{sdec}(\text{senc}(x, y), y) &= x \\ \text{dec}(\langle x, y \rangle, z) &= \langle \text{dec}(x, z), \text{dec}(y, z) \rangle & \text{proj}_1(\langle x, y \rangle) &= x \\ & & \text{proj}_2(\langle x, y \rangle) &= y\end{aligned}$$

## Going back to the Denning Sacco protocol

$A \rightarrow B$  :  $\text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(B))$

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① symmetric encryption:  $\text{senc}(\cdot, \cdot), \text{sdec}(\cdot, \cdot)$

$\longrightarrow \text{sdec}(\text{senc}(x, y), y) = x$

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② asymmetric encryption:  $\text{aenc}(\cdot, \cdot), \text{adec}(\cdot, \cdot), \text{pk}(\cdot)$

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③ signature:  $\text{sign}(\cdot, \cdot)$ ,  $\text{check}(\cdot, \cdot)$

$$\longrightarrow \text{check}(\text{sign}(x, y), \text{pk}(y)) = x$$

# Deduction in this more general setting

Deduction rules are as follows:

$$\frac{u_1 \quad \cdots \quad u_k}{f(u_1, \dots, u_k)} \quad f \in \mathcal{F} \qquad \frac{u}{u'} \quad u =_{\mathcal{E}} u'$$



# Deduction in this more general setting

Deduction rules are as follows:

$$\frac{u_1 \quad \cdots \quad u_k}{f(u_1, \dots, u_k)} \quad f \in \mathcal{F} \qquad \frac{u}{u'} \quad u =_{\mathbf{E}} u'$$

**Example:** Let  $\mathbf{E} := \text{sdec}(\text{senc}(x, y), y) = x$  and  $T = \{\text{senc}(\text{secret}, k), k\}$ .  
We have that  $T \vdash \text{secret}$ .

$$\frac{\frac{\text{senc}(\text{secret}, k) \quad k}{\text{sdec}(\text{senc}(\text{secret}, k), k)}}{\text{secret}} \quad \text{sdec} \in \mathcal{F} \qquad \text{sdec}(\text{senc}(x, y), y) = x$$

# The deduction problem: is $u$ deducible from $\phi$ ?

We consider a signature  $\mathcal{F}$  and an equational theory  $E$ .

## The deduction problem

**Input** A sequence  $\phi = \{w_1 \triangleright v_1, \dots, w_n \triangleright v_n\}$  of terms and a term  $u$

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## Characterization of deduction

$T \vdash u$  if, and only if, there exists a term  $R$  such that  $R\phi =_E u$ .

$\longrightarrow$  such a term  $R$  is a **recipe** of the term  $u$ .

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- $k$  is deducible from  $\phi$  using  $R_1 = \text{check}(\text{adec}(w_4, w_3), w_1)$ ,
- $s$  is deducible from  $\phi$  using  $R_2 = \text{sdec}(w_5, R_1)$ .

## Proposition

The **deduction problem** is decidable for the equational theory modelling the DS protocol (and actually any subterm convergent equational theory).

## Algorithm:

- 1 **saturation** of  $\phi$  with its deducible **subterm**; we get  $\phi^+$
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- $\phi^+ = \phi \uplus \{w_6 \triangleright \text{sign}(k, ska); w_7 \triangleright \text{pk}(skc); w_8 \triangleright k; w_9 \triangleright s\}.$

## Some other equational theories

### Blind signature

$$\begin{aligned}\text{check}(\text{sign}(x, y), \text{vk}(y)) &= x \\ \text{unblind}(\text{blind}(y, y), y) &= x \\ \text{unblindsign}(\text{sign}(\text{blind}(x, y), z), y) &= \text{sign}(x, z)\end{aligned}$$

Decidability can be shown in a similar fashion **extending the notion of subterm**.

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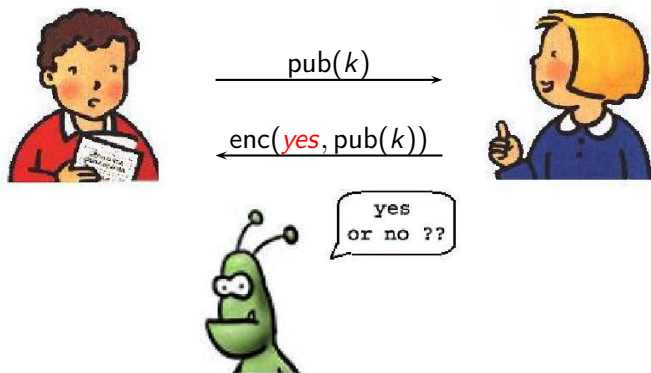
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## Exclusive or

$$\begin{aligned}(x \oplus y) \oplus z &= x \oplus (y \oplus z) & x \oplus x &= 0 \\ x \oplus y &= y \oplus x & x \oplus 0 &= x\end{aligned}$$

The deduction problem can be reduced to the problem of **solving systems of linear equations over  $\mathbb{Z}/2\mathbb{Z}$** .

# Deduction is not always sufficient



→ The intruder **knows** the values **yes** and **no** !

## The real question

Is the intruder able to tell whether Alice sends **yes** or **no**?

## The static equivalence problem

**Input** Two frames  $\phi$  and  $\psi$

$$\phi = \{w_1 \triangleright u_1, \dots, w_\ell \triangleright u_\ell\} \quad \psi = \{w_1 \triangleright v_1, \dots, w_\ell \triangleright v_\ell\}$$

**Output** Can the attacker distinguish the two frames, *i.e.* does there exist a **test**  $R_1 \stackrel{?}{=} R_2$  such that:

$$R_1\phi =_E R_2\phi \text{ but } R_1\psi \neq_E R_2\psi \text{ (or the converse).}$$

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**Example:** Consider the frames:

- $\phi = \{w_1 \triangleright \text{pk}(sks); w_2 \triangleright \text{aenc}(\text{yes}, \text{pk}(sks))\}$ ; and
- $\psi = \{w_1 \triangleright \text{pk}(sks); w_2 \triangleright \text{aenc}(\text{no}, \text{pk}(sks))\}$ .

They are **not** in static equivalence:  $\text{aenc}(\text{yes}, w_1) \stackrel{?}{=} w_2$ .

# Exercise

Consider the equational theories:

- $E_{\text{senc}}$  defined by  $\text{sdec}(\text{senc}(x, y), y) = x$ , and
- $E_{\text{cipher}}$  which extends  $E_{\text{senc}}$  by the equation  $\text{senc}(\text{sdec}(x, y), y) = x$ .

## Questions

Which of the following pairs of frames are statically equivalent ? Whenever applicable give the distinguishing test.

$$\begin{array}{ccc} \{w_1 \triangleright \text{yes}\} & \stackrel{?}{\sim}_{E_{\text{senc}}} & \{w_1 \triangleright \text{no}\} \\ \{w_1 \triangleright \text{senc}(\text{yes}, k)\} & \stackrel{?}{\sim}_{E_{\text{senc}}} & \{w_1 \triangleright \text{senc}(\text{no}, k)\} \\ \{w_1 \triangleright \text{senc}(n, k), w_2 \triangleright k\} & \stackrel{?}{\sim}_{E_{\text{senc}}} & \{w_1 \triangleright \text{senc}(n, k), w_2 \triangleright k'\} \\ \{w_1 \triangleright \text{senc}(n, k), w_2 \triangleright k\} & \stackrel{?}{\sim}_{E_{\text{cipher}}} & \{w_1 \triangleright \text{senc}(n, k), w_2 \triangleright k'\} \end{array}$$



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→ actually, we only need to consider **small tests**

# Example

Consider the frames:

- $\phi = \{w_1 \triangleright \text{aenc}(\langle \text{yes}, r_1 \rangle, \text{pk}(sks)); w_2 \triangleright sks\}$ ; and
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→ **Conclusion:**  $\phi^+$  and  $\psi^+$  are **not** in static equivalence:  $w_4 \stackrel{?}{=} \text{yes}$ .

## Some other equational theories

### Blind signature

$$\begin{aligned}\text{check}(\text{sign}(x, y), \text{vk}(y)) &= x \\ \text{unblind}(\text{blind}(x, y), y) &= x \\ \text{unblindsign}(\text{sign}(\text{blind}(x, y), z), y) &= \text{sign}(x, z)\end{aligned}$$

This can be done in a similar fashion **extending** a bit **the notion of subterm**  
→ again  $\text{sign}(m, k)$  will be considered as a subterm of  $\text{sign}(\text{blind}(m, r), k)$ .

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## Exclusive or

$$\begin{aligned}(x \oplus y) \oplus z &= x \oplus (y \oplus z) & x \oplus x &= 0 \\ x \oplus y &= y \oplus x & x \oplus 0 &= x\end{aligned}$$

The static equivalence problem can be reduced in PTIME to the problem of deciding whether two systems of linear equations have the **same set of solutions** over  $\mathbb{Z}/2\mathbb{Z}$ .

# Existing decidability/complexity results and tools

Theory E	Deduction	Static Equivalence
subterm convergent	PTIME decidable [Abadi & Cortier, TCS'06]	
blind sign., addition, homo. encryption		
ACU	NP-complete	PTIME
Exclusive Or Abelian Group	PTIME	PTIME
ACUNh/AGh	PTIME	decidable
[D., IPL'05; Cortier & D., JAR'12]		

→ A **combination** result for disjoint theories [Cortier & D., JAR'12]

→ **Automatic tools for checking static equivalence**: YAPA M. Baudet (2006); KISS S. Ciobaca (2010); and FAST B. Conchinha (2011)

# Modelling protocols and security properties



## Applied pi calculus

[Abadi & Fournet, 01]

basic programming language with constructs for **concurrency** and **communication**

→ based on the  $\pi$ -calculus [Milner *et al.*, 92], and in some ways similar to the spi-calculus [Abadi & Gordon, 98]

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### Some advantages:

- allows us to model **cryptographic primitives**
- both **reachability** and **equivalence**-based specification of properties

# Protocols as processes - syntax and semantics

Syntax :	$P, Q$	$:=$	0	null process
			$\text{in}(c, x).P$	input
			$\text{out}(c, u).P$	output
			$\text{if } u = v \text{ then } P \text{ else } Q$	conditional
			$P \mid Q$	parallel composition
			$!P$	replication
			$\text{new } n.P$	fresh name generation

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			$\text{in}(c, x).P$	input
			$\text{out}(c, u).P$	output
			$\text{if } u = v \text{ then } P \text{ else } Q$	conditional
			$P \mid Q$	parallel composition
			$!P$	replication
			$\text{new } n.P$	fresh name generation

Semantics  $\rightarrow$ :

Comm	$\text{out}(c, M).P \mid \text{in}(c, x).Q \rightarrow P \mid Q\{M/x\}$
Then	$\text{if } M = N \text{ then } P \text{ else } Q \rightarrow P \text{ when } M =_E N$
Else	$\text{if } M = N \text{ then } P \text{ else } Q \rightarrow Q \text{ when } M \neq_E N$

closed by **structural equivalence** ( $\equiv$ ) and application of **evaluation contexts**.

## Going back to Denning Sacco protocol

$A \rightarrow B$  :  $\text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(B))$

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$P_{DS} = \text{new } sk_a, sk_b. (P_A(sk_a, pk(sk_b)) \mid P_B(sk_b, pk(sk_a)))$



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$\rightarrow$  this simply models a **normal execution** between two **honest** participants

### Confidentiality for process $P$ w.r.t. secret $s$

For **all processes**  $A$  such that  $A \mid P \rightarrow^* Q$ , we have that  $Q$  is not of the form  $C[\text{out}(c, s).Q']$  with  $c$  public.

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### Some difficulties:

- we have to consider **all** the possible executions in presence of an **arbitrary adversary** (modelled as a process)
- we have to consider **realistic** initial configurations
  - replications to model an **unbounded** number of sessions,
  - reveal public keys and private keys to model **dishonest** agents,
  - $P_A/P_B$  may play with other (and perhaps) dishonest agents, ...

# Going back to the Denning Sacco protocol

$A \rightarrow B : \text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(B))$

$B \rightarrow A : \text{senc}(s, k)$

## The aforementioned attack

1.  $A \rightarrow C : \text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(C))$

2.  $C(A) \rightarrow B : \text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(B))$

3.  $B \rightarrow A : \text{senc}(s, k)$

The “minimal” initial configuration to retrieve the attack is:

$\text{new } sk_a. \text{new } sk_b. (\text{out}(c, \text{pk}(sk_b)) \mid P_A(sk_a, \text{pk}(sk_c)) \mid P_B(sk_b, \text{pk}(sk_a)))$

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**Exercise:** Exhibit the process  $A$  (the behaviour of the attacker) that witnesses the aforementioned attack.

## Security properties - authentication

This can be expressed as a **correspondence property**:

*if B finishes a session, thinking he has talked to A then A has also finished a session, thinking she has talked to B (+ possibly agreement on some values).*

Enriched syntax for processes:

$P, Q$	$:=$	$0$	null process
		$\text{in}(c, x).P$	input
		$\dots$	
		$\text{event } p(u_1, \dots, u_n).P$	event

Authentication properties with agreement on some values:

$$\forall x. \text{EndB}(a, b, x) \Rightarrow \text{EndA}(a, b, x)$$



## confidentiality for an unbounded number of sessions

- **undecidable** in general [Even & Goldreich, 83; Durgin *et al*, 99]  
[▶ More details](#)
- some **decidability results** for some restricted fragment, e.g. one variable per protocol's rule [Comon & Cortier, 03]
- **ProVerif**: A tool that does not correspond to any decidability result but works well in practice. [Blanchet, 01]  
[▶ More details](#)

confidentiality for a bounded number of sessions

- a **decidability** result (NP-complete)  
[Rusinowitch & Turuani, 01; Millen & Shmatikov, 01]
- result extended to deal with various cryptographic primitives.

→ various automatic tools, e.g. **AVISPA platform** [Armando *et al.*, 05]  
More details about this tomorrow !

# Challenge

Would you be able to find the attack on the well-known  
Needham-Schroeder protocol?

$$\begin{aligned} A \rightarrow B &: \{A, N_a\}_{\text{pub}(B)} \\ B \rightarrow A &: \{N_a, N_b\}_{\text{pub}(A)} \\ A \rightarrow B &: \{N_b\}_{\text{pub}(B)} \end{aligned}$$



To help you:

<http://www.lsv.ens-cachan.fr/~delaune/VTSA/proverif.pdf>

Questions ?

See you tomorrow !

## Post Correspondence Problem

**Input** A sequence of tiles  $(u_0, v_0) (u_1, v_1) \dots (u_n, v_n)$  with  $u_i, v_i \in \{a, b\}^*$ .

**Output** Does there exist  $k \geq 1$ , and  $1 \leq i_1, \dots, i_k \leq n$  such that

$$u_{i_1} \dots u_{i_k} = v_{i_1} \dots v_{i_k}$$

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Example:

$u_1$	$u_2$	$u_3$	$u_4$	$v_1$	$v_2$	$v_3$	$v_4$
<i>aba</i>	<i>bbb</i>	<i>aab</i>	<i>bb</i>	<i>a</i>	<i>aaa</i>	<i>abab</i>	<i>babba</i>

A solution is **1431**. Indeed, we have that:

$$u_1.u_4.u_3.u_1 = aba.bb.aab.aba = a.babba.abab.a = v_1.v_4.v_3.v_1$$

No solution if we remove the tile  $(u_4, v_4)$ .

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No solution if we remove the tile  $(u_4, v_4)$ .

**Proposition:** The PCP is undecidable.

## Reduction from PCP

We built a protocol that admits an attack ( $s$  is revealed) if, and only if, PCP has a solution.



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We encode words and concatenation using pairs

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**Building words**

- $! \text{in}(\text{senc}(\langle x, y \rangle, k)).\text{out}(\text{senc}(\langle x \cdot u_1, y \cdot v_1 \rangle, k))$
- ...
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**Revealing the secret  $s$ :**  $\text{in}(\text{senc}(\langle z, z \rangle, k)).\text{out}(s)$

ProVerif is a verifier for cryptographic protocols that may **prove** that a protocol is secure or **exhibit attacks**.

- Online demo available at: <http://proverif.rocq.inria.fr/>
- Sources available on Bruno Blanchet's webpage

## Advantages

- fully automatic, and quite efficient
- A rich process algebra: replication, else branches, . . .
- Handles many cryptographic primitives
- Proves various security properties: secrecy, correspondences, equivalences

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## No miracle

Termination is not guaranteed and sometimes the tool is not able to conclude.

→ still, ProVerif works well in practice.

Protocol	Result	ms
Needham-Schroeder shared key	Attack	52
Needham-Schroeder shared key corrected	Secure	109
Denning-Sacco	Attack	6
Denning-Sacco corrected	Secure	7
Otway-Rees	Secure	10
Otway-Rees, variant of Paulson98	Attack	12
Yahalom	Secure	10
Simpler Yahalom	Secure	11
Main mode of Skeme	Secure	23

Pentium III, 1 GHz.