Zero-Knowledge Proofs of Knowledge

Stéphanie Delaune

September 6, 2013

Proofs of knowledge

Proof of knowledge are often used to

- prove one's identity (*e.g.* authentication protocol)
- prove one's belonging to a group
- prove that one has done something correctly (*e.g.* mix net)

Example

- Alice knows the product of two prime numbers, (*e.g.* $p_1 \times p_2$),
- Alice knows also the pair (p_1, p_2).

Now, assume that Bob knows only the product $p_1 \times p_2$

- He is not able to retieve the pair (p_1, p_2) of Alice
  $\longrightarrow$ factorisation in prime numbers is a very hard problem
- If Alice gives him p_1 and p_2 he is convinced that she knows the result.

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Two kinds of proofs of knowledge

First Solution: (*e.g.* password mechanism)

- the verifier learns (or even already knows) the password,
- an eavesdropper learns the password

Second Solution: zero-knowledge proof

- an eavesdropper will not learn the solution,
- the verifier will not learn the solution.

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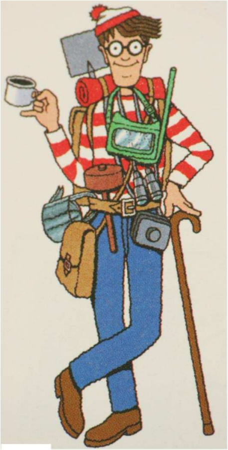
> *"Zero-knowledge proofs are fascinating and extremely useful constructs. They are both convincing and yet yield nothing beyond the validity of the assertion being proved."* O. Goldreich

Example: Where is Charlie?



Goal:

1. find the reporter Charlie in a big picture,
2. convince the verifier (me) that you have the solution without revealing it (neither to me, nor to the others).

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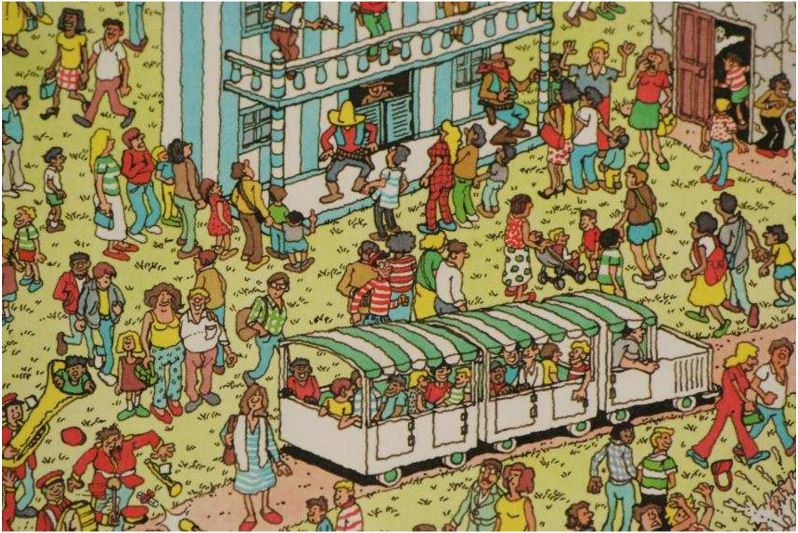
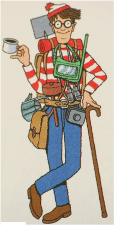


Example: Where is Charlie?

How can you prove that you know where is Charlie without saying nothing about where he is?



Solutions:

Example: Where is Charlie?

How can you prove that you know where is Charlie **without** saying nothing about where he is?

Solutions:

1. get a copy of the picture, cut out Charlie and show it to me.
2. put a big mask with a window having the shape of Charlie and show me Charlie through the window.

What is it ?

Zero-knowledge proofs are proofs that are both convincing and yet yield nothing beyond the validity of the assertion being proved.

$\longrightarrow$ introduced 20 years ago by Goldwasser, Micali and Rackoff [1985]

- Completeness: if the statement is true, the honest verifier will be convinced of this fact by an honest prover.
- Soundness: if the statement is false, no cheating prover can convince the honest verifier that it is true.
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The definitions given above seem to be contradictory.

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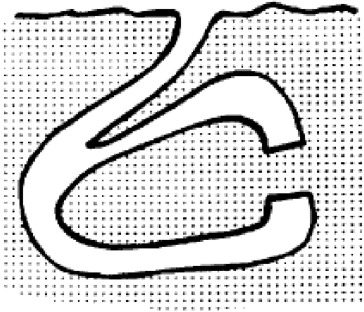
Example: The strange cave of Ali Baba



- a cave shaped like a circle, with entrance on one side and the magic door blocking the opposite side
- the door can be opened by saying some magic words "....".

Goal:

Ali Baba wants to convince me that he knows the secret without revealing it.

How can Ali Baba proceed?

Example: The strange cave of Ali Baba

Ali Baba wants to convince me that he knows the magic words.

Ali Baba hides inside the cave

I ask him to exit on the right side or on the left side
$\longrightarrow$ I choose

Ali Baba exits from the side I just asked.

… and we repeat this procedure several times.

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I can be convinced that Ali Baba knows the magic words.

Why?

- If Ali Baba does not know the magic word, then he can only return by the same path. Since, I randomly choose the path, he has 50% chance of guessing correctly.
- By repeating this trick many times, say 20 times, his chance of succesfully anticipating all my requests becomes very small.

Moreover,

- I learn nothing about the magic word beyond the fact this word allows Ali Baba to open the magic door, and
- I am not able to prove to someone else that I know the magic words.

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The End
– of Ali Baba story –

To know the magic word:

How to explain Zero-Knowledge Protocols to Your Children.
Jean-Jacques Quisquater and Louis Guillou.

Graph 3-coloring

Definition (3-coloring)

A 3-coloring of a graph is an assignment of colors in $\{\bullet, \bullet, \bullet\}$ to vertices such that no pair of adjacent vertices are assigned to the same color.

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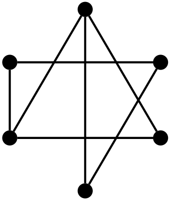

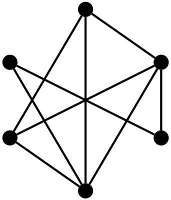

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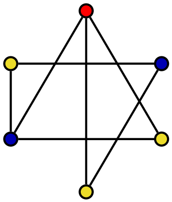

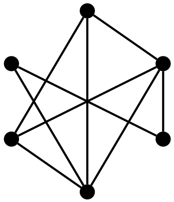

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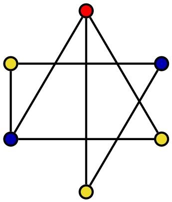
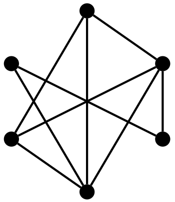
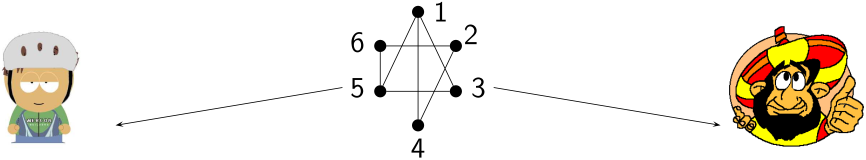




3-coloring problem

Given graph G, the problems of deciding if the graph G is 3-colorable is a very hard problem. It is also very hard to find a 3-coloring of a large graph.

Protocol based on the 3-coloring problem



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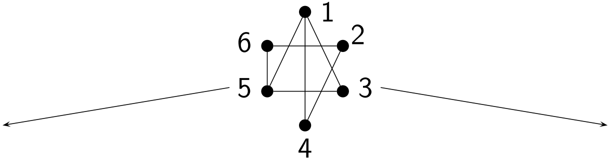

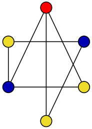

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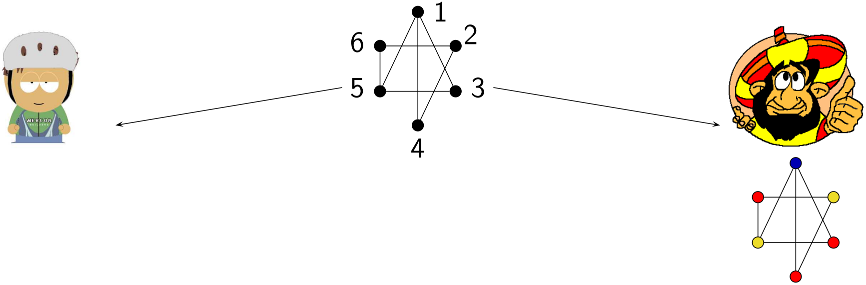

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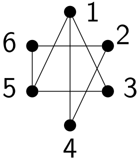

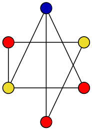

$\{\bullet\}_{k_1}, \{\bullet\}_{k_2}, \{\bullet\}_{k_3}, \{\bullet\}_{k_4}, \{\bullet\}_{k_5}, \{\bullet\}_{k_6}$

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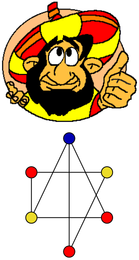

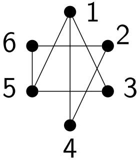

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choose an edge

$(1, 4)$

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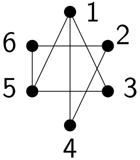

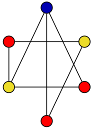

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k_1 and k_4

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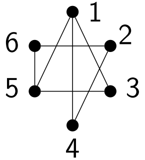

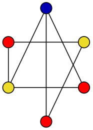

$\{{\color{blue}\bullet}\}_{k_1}, \{{\color{olive}\bullet}\}_{k_2}, \{{\color{red}\bullet}\}_{k_3}, \{{\color{red}\bullet}\}_{k_4}, \{{\color{olive}\bullet}\}_{k_5}, \{{\color{red}\bullet}\}_{k_6}$ ←

choose an edge

$(1, 4)$ →

send keys

k_1 and k_4 ←

decrypt $\{{\color{blue}\bullet}\}_{k_1}$ with k_1
decrypt $\{{\color{red}\bullet}\}_{k_4}$ with k_4

accept since ${\color{blue}\bullet} \neq {\color{red}\bullet}$

... repeat the procedure several times

Discussion on the protocol

- Completeness: if the statement is true, the honest verifier will be convinced of this fact by an honest prover.

  $\longrightarrow$ if Ali Baba knows the 3-coloring of the graph, then the verifier will accept his proof.

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Applications

- Credit card payment
  $\longrightarrow$ to prove that you know the secret code without revealing it
- prove your identity
- prove that you belongs to a group without revealing who you are
  $\longrightarrow$ to ensure privacy
- to enforce honest behavior in mix net (e.g. e-voting protocols)
- to convince someone that you have solved a Sudoku puzzle without revealing the solution.

Conclusion and Further Reading

Zero-knowledge proofs are fascinating due to their seemingly contradictory definitions. Nevertheless, such kind of proofs really exist.

It turns out that in an Internet-like setting, where multiple protocols may be executed concurrently, building zero-knowledge proofs is more challenging.

Bibiliography

1. *How to explain Zero-Knowledge Protocols to Your Children.* Jean-Jacques Quisquater and Louis Guillou.
2. *Zero-Knowledge twenty years after its invention.* Oded Goldreich.
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