Trace equivalence via constraint solving

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Cryptographic protocols

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Cryptographic protocols

- small programs designed to secure communication (*e.g.* secrecy, authentication, anonymity, ...)
- use cryptographic primitives (*e.g.* encryption, signature, ...)

The network is unsecure!

Communications take place over a public network like the Internet.

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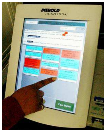

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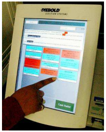
It becomes more and more important to protect our privacy.



Example: electronic passport

$\longrightarrow$ studied in [Arapinis, Chothia, Ritter & Ryan,10]

An electronic passport is a passport with an RFID tag embedded in it.

The RFID tag stores:

- the information printed on your passport,
- a JPEG copy of your picture.

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The Basic Access Control (BAC) protocol is a key establishment protocol that has been designed to also ensure unlinkability.

ISO/IEC standard 15408

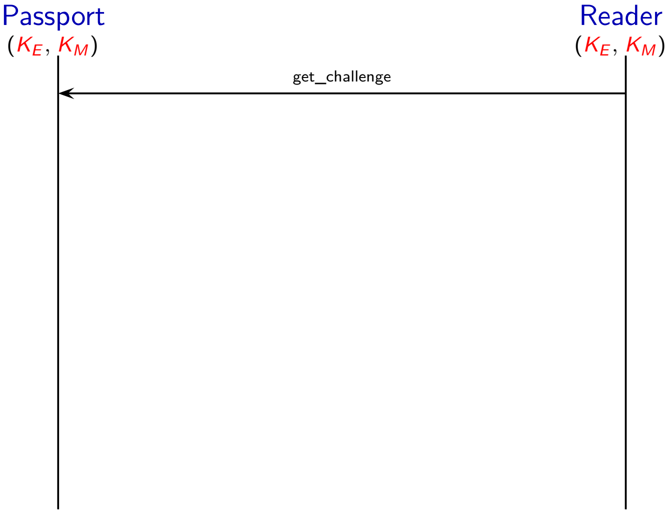
Unlinkability aims to ensure *that a user may make multiple uses of a service or resource without others being able to link these uses together.*

The electronic passport protocol

Passport
(K_E, K_M)

Reader
(K_E, K_M)

The electronic passport protocol



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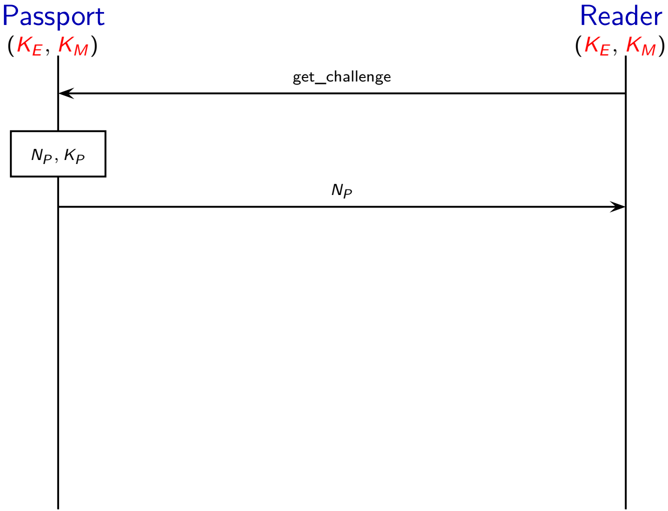

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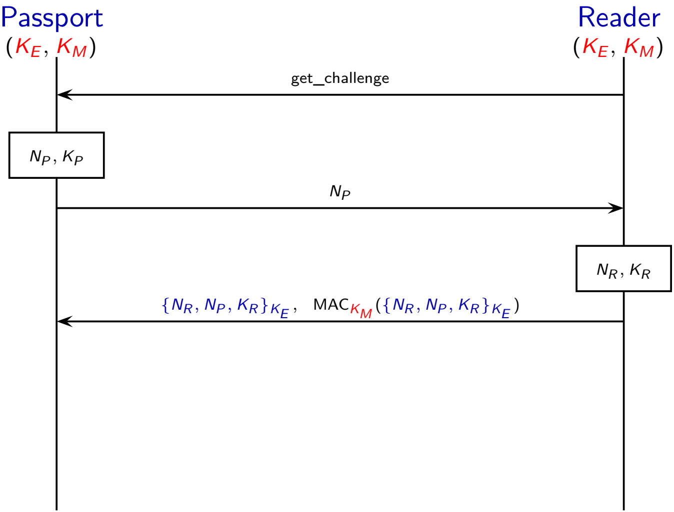

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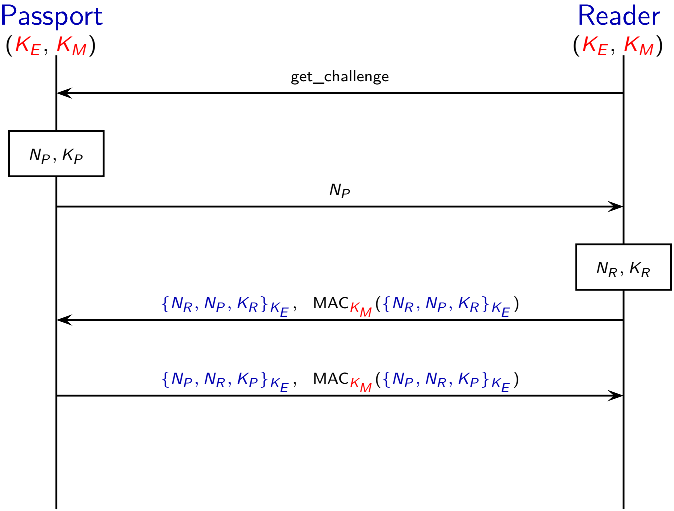

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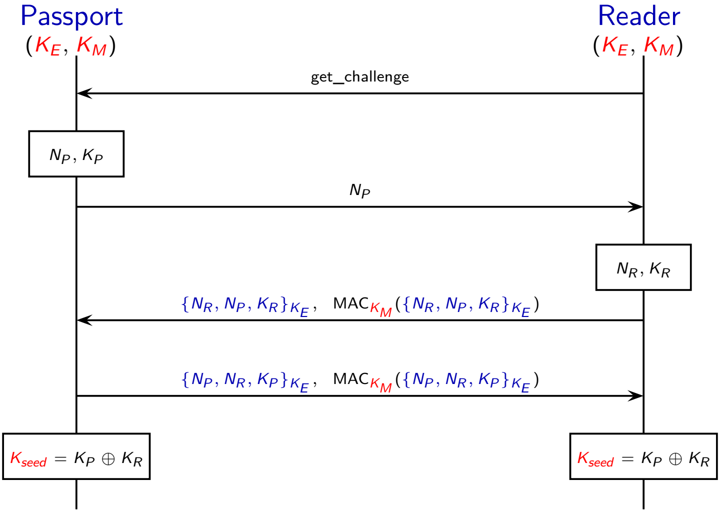

Privacy-type security properties

Equivalence based properties

"An observer cannot observe any difference between P and Q"

$\longrightarrow$ unlinkability, anonymity, privacy related properties in e-voting, strong secrecy, ...

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$$P \mid P \mid !Reader \overset{?}{\approx} P \mid P' \mid !Reader$$

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What is the attacker able to distinguish?

- $\mathtt{new}\ k;\ \mathsf{out}(\{0\}_k) \approx \mathtt{new}\ k; \mathsf{out}(\{1\}_k)$

  $\longrightarrow$ *We assume a Dolev-Yao attacker and perfect cryptography*

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What is the attacker able to distinguish?

- $\texttt{if } \phi \texttt{ then } P \texttt{ else } Q \approx \texttt{if } \neg\phi \texttt{ then } Q \texttt{ else } P$

  $\longrightarrow$ *He can not observe the result of a test*

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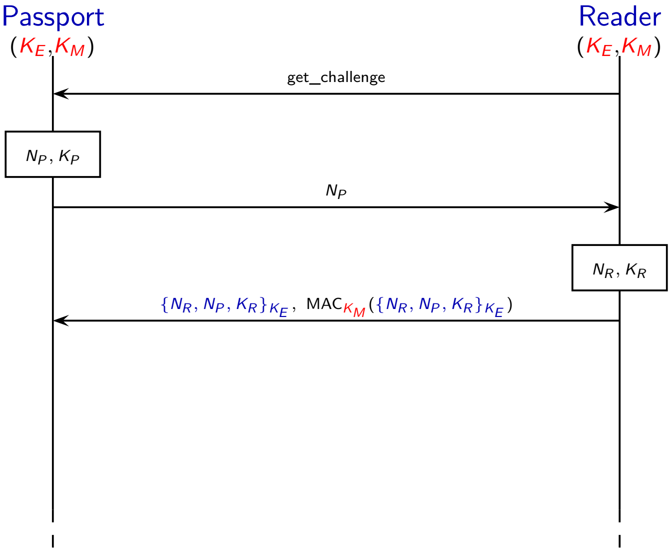
- $\mathsf{out}(a); (\mathsf{out}(b) + \mathsf{out}(c)) \approx \mathsf{out}(a); \mathsf{out}(b) + \mathsf{out}(b); \mathsf{out}(c)$

  $\longrightarrow$ *We consider trace equivalence (also called may-testing)*

French electronic passport

$\longrightarrow$ the passport must reply to all received messages.



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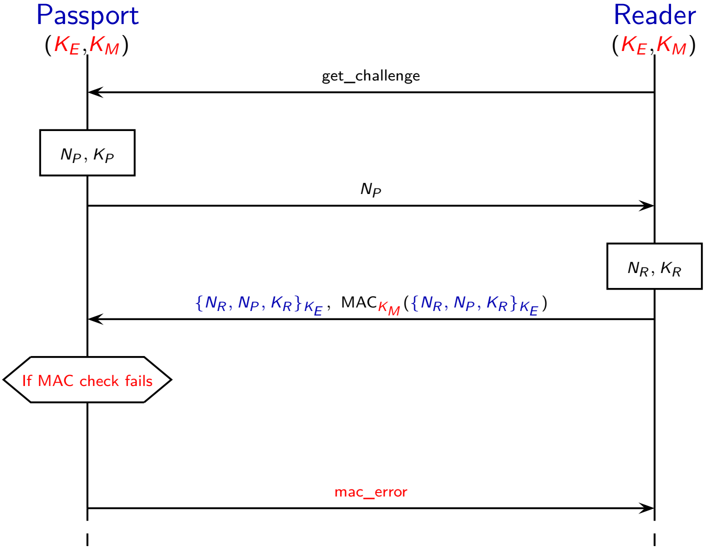

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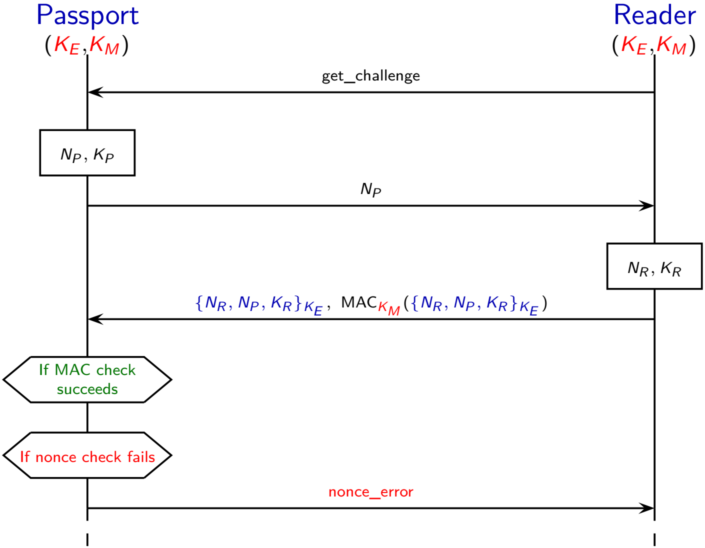

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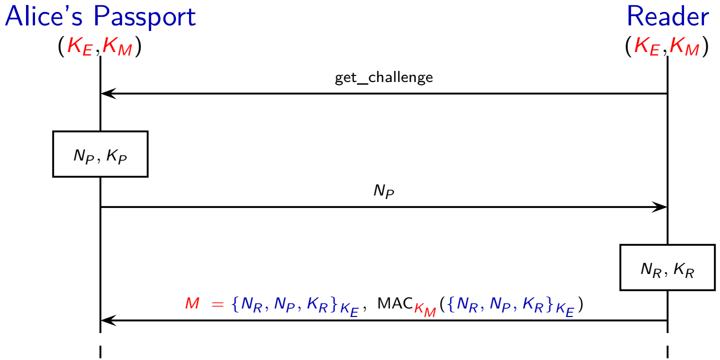
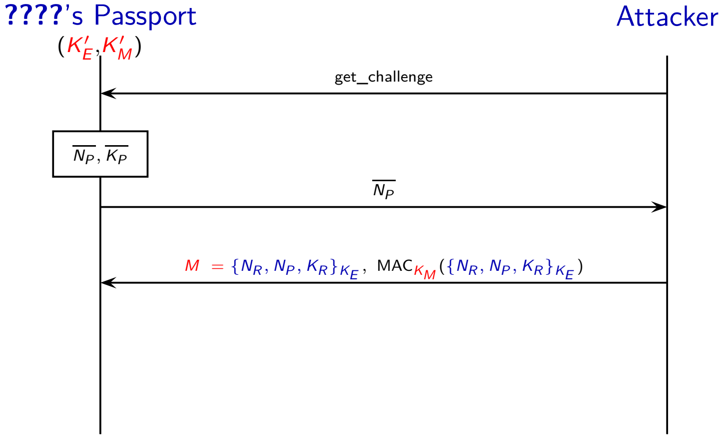
Part 1 of the attack. The attacker eavesdropes on Alice using her passport and records message M.



Part 2 of the attack.

The attacker replays the message M and checks the error code he receives.



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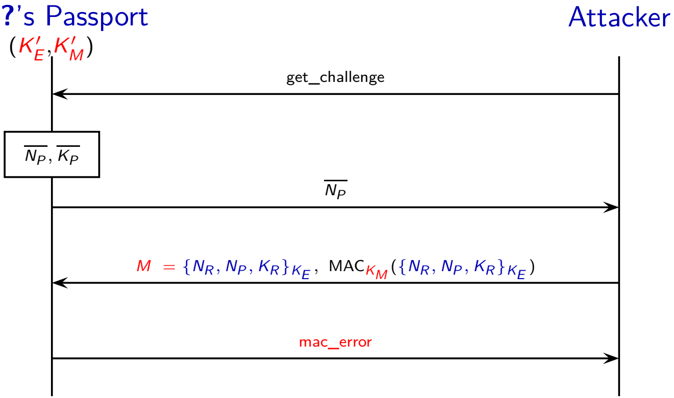

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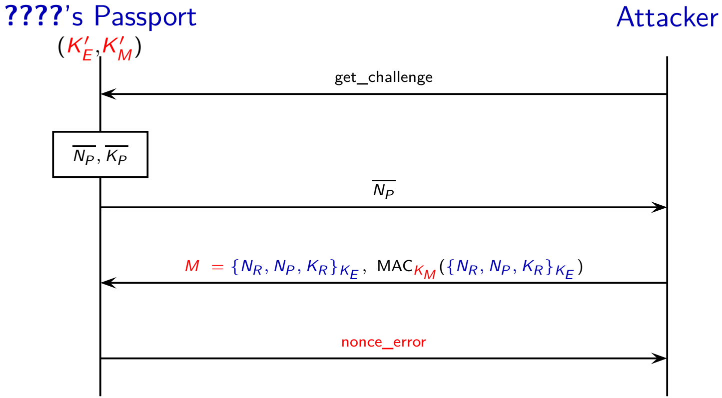

$\Longrightarrow$ MAC check succeeded $\Longrightarrow$ $K'_M = K_M$ $\Longrightarrow$ **????** is Alice

State of the art in a nutshell

for analysing equivalence-based security properties

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Bounded number of sessions
e.g. [Baudet, 05], [Dawson & Tiu, 10], [Chevalier & Rusinowitch, 10], ...

$\longrightarrow$ this allows us to decide trace equivalence between simple processes with trivial else branches. [Cortier & Delaune, 09]

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ProVerif tool [Blanchet, 01] `http://www.proverif.ens.fr/`

- + unbounded number of sessions; various cryptographic primitives;
- − termination is not guaranteed; diff-equivalence (too strong)

$\longrightarrow$ ProSwapper extension [Smyth, 10]

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$\longrightarrow$ ProSwapper extension [Smyth, 10]

$\longrightarrow$ None of these results is able to analyse the e-passport protocol.

Our contribution

Main result

A procedure for deciding trace equivalence for a large class of processes.

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- + non-trivial else branches, private channels, and non-deterministic choice;
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Some applications:

- unlinkability in RFID protocols (*e.g.* e-passport protocol);
- anonymity/privacy (*e.g.* private authentication protocols [Abadi & Fournet, 04]).

Outline

1. Introduction

2. From trace equivalence to symbolic equivalence

3. Deciding symbolic equivalence using constraint solving techniques

4. Conclusion

Outline

From processes to constraint systems

Passport P - (K_E, K_M)

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\begin{array}{l}
\mathrm{in}(= \mathit{get\_challenge}); \mathtt{new}\ N_P; \mathtt{new}\ K_P;\\
\mathrm{in}(\langle z_E, z_M\rangle);\\
\mathtt{if}\ z_M = \mathrm{MAC}_{K_M}(z_E)\\
\quad \mathtt{then}\ \mathtt{let}\ (x_R, x'_P, y_R) = \mathrm{dec}(z_E, K_E)\ \mathit{in}\\
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Once an interleaving of symbolic actions has been fixed (*e.g.* in; in; out), we generate the associated constraint systems:

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$$\longrightarrow \mathcal{C}_{\mathrm{nonce}}$$

Once an interleaving of symbolic actions has been fixed (*e.g.* in; in; out), we generate the associated constraint systems:

$$\mathcal{C}_{\mathrm{mac}};\ \mathcal{C}_{\mathrm{nonce}};$$

From processes to constraint systems

Passport P - (K_E, K_M)

$$
\begin{array}{l}
\mathrm{in}(= \mathit{get\_challenge}); \mathtt{new}\ N_P; \mathtt{new}\ K_P;\\
\mathrm{in}(\langle z_E, z_M\rangle);\\
\mathtt{if}\ z_M = \mathrm{MAC}_{K_M}(z_E)\\
\quad \mathtt{then}\ \mathtt{let}\ (x_R, x'_P, y_R) = \mathrm{dec}(z_E, K_E)\ \mathit{in}\\
\qquad\quad \mathtt{if}\ N_P = x'_P\\
\qquad\qquad \mathtt{then}\ \mathtt{let}\ m = \{\langle N_P, x_R, K_P\rangle\}_{K_E}\ \mathit{in}\\
\qquad\qquad\qquad \mathrm{out}(\langle m, \mathrm{MAC}_{K_M}(m)\rangle)\\
\qquad\qquad \mathtt{else}\ \mathrm{out}(\mathit{nonce\_error})\\
\quad \mathtt{else}\ \mathrm{out}(\mathit{mac\_error})
\end{array}
$$

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Trace equivalence via constraint solving

To check whether $P \approx P'$, we have to check whether

$\Sigma \approx_s \Sigma'$ for all sequence of symbolic actions (*e.g.* in;in;out).

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Symbolic equivalence $\Sigma \approx_s \Sigma'$

- for all $\mathcal{C} \in \Sigma$ for all $(\sigma, \theta) \in \mathsf{Sol}(\mathcal{C})$, there exists $\mathcal{C}' \in \Sigma'$ such that:
$$(\sigma', \theta) \in \mathsf{Sol}(\mathcal{C}') \quad \text{and} \quad \Phi\sigma \sim \Phi'\sigma' \text{ (static equivalence).}$$
- and conversely

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Going back to the E-passport example

Among others, we have to check whether

$$\{\mathcal{C}_{\mathsf{mac}};\ \mathcal{C}_{\mathsf{nonce}};\ldots\} \overset{?}{\approx}_s \{\mathcal{C}'_{\mathsf{mac}};\mathcal{C}'_{\mathsf{nonce}};\ldots\}$$

where $\mathcal{C}'_{\mathsf{mac}}$, $\mathcal{C}'_{\mathsf{nonce}}$, ... are the counterparts of $\mathcal{C}_{\mathsf{mac}}$, $\mathcal{C}_{\mathsf{nonce}}$, ... in which K_E and K_M have been replaced by K'_E and K'_M.

French passport (1/2)

$$\{\mathcal{C}_{\mathsf{mac}};\ \mathcal{C}_{\mathsf{nonce}};\ \ldots\} \overset{?}{\approx}_s \{\mathcal{C}'_{\mathsf{mac}};\mathcal{C}'_{\mathsf{nonce}};\ldots\}$$

when T_0 contains $\langle\{\overline{N_R},\overline{N_P},\overline{K_R}\}_{K_E},\ \mathsf{MAC}_{K_M}(\{\overline{N_R},\overline{N_P},\overline{K_R}\}_{K_E})\rangle$

$\longrightarrow$ the answer should be no

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$$\mathcal{C}_{\mathsf{nonce}} = \left\{ \begin{array}{c} T_0 \overset{?}{\vdash} get\_challenge \\ T_0 \overset{?}{\vdash} \langle z_E, z_M\rangle \\ z_M \overset{?}{=} \mathsf{MAC}_{K_M}(z_E) \\ \langle x_R, x'_P, y_R\rangle = \mathsf{dec}(z_E, K_E) \\ N_P \overset{?}{\neq} x'_P \\ \hline \Phi = T_0; nonce\_error \end{array} \right.$$

$\longrightarrow$ A solution for $\mathcal{C}_{\mathsf{nonce}}$ consists of replaying the message in T_0.

French passport (2/2)

If the attacker performed this replay, what will happen in the other side?

$$\{\mathcal{C}'_{\mathsf{mac}};\ \mathcal{C}'_{\mathsf{nonce}};\ \ldots\}$$

$\longrightarrow$ this computation does not lead to a solution for constraint system that contains $z_M = \mathsf{MAC}_{K'_M}(z_E)$.

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$\longrightarrow$ this computation leads to a solution for $\mathcal{C}'_{\mathsf{mac}}$ but the resulting sequence of messages Φ and Φ' are not in static equivalence.

Related work on symbolic equivalence

Several works have already been done:

- for subterm convergent equational theories:

  [Baudet,05]; [Chevalier & Rusinowitch,10]

  $\longrightarrow$ does not lead to a practical algorithm

- for a fixed set of cryptographic primitives:

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Two main limitations

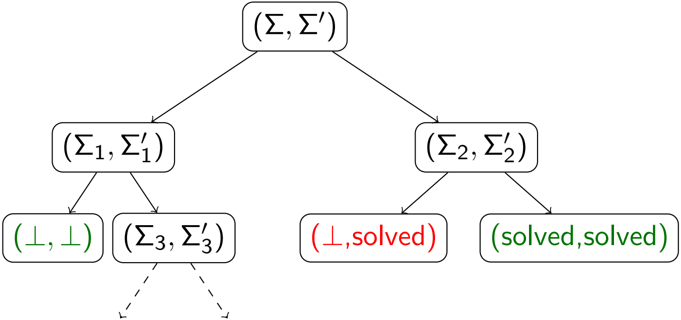
- positive constraint systems only;
- symbolic equivalence between two constraint systems (and not sets of constraint systems)

Outline

1. Introduction
2. From trace equivalence to symbolic equivalence
3. Deciding symbolic equivalence using constraint solving techniques
4. Conclusion

Our procedure in a nutshell

Main idea of our procedure: We rewrite pairs of sets of constraint systems until a trivial failure or a trivial success is found.



Our simplification rules

We propose a finite set of simplification rules that transform a constraint system into two constraint system.

Example: the Cons simplification rule

$$\mathcal{C} = \begin{cases} \dots \\ T \overset{?}{\vdash} \mathsf{enc}(u_1, u_2) \\ \dots \end{cases}$$

$$\mathcal{C}_1 = \begin{cases} \dots \\ T \overset{?}{\vdash} u_1;\ T \overset{?}{\vdash} u_2 \\ \dots \end{cases} \qquad \mathcal{C}_2 = \begin{cases} \dots \\ T \overset{?}{\vdash} \mathsf{enc}(u_1, u_2) \\ \text{using a recipe } \zeta \text{ such} \\ \text{that } \mathsf{Top}(\zeta) \neq \mathsf{enc} \\ \dots \end{cases}$$

$\longrightarrow$ We have also an Axiom rule and a Dest rule.

Step 1: reaching a constraint system in pre-solved form

$\longrightarrow$ the CONS, DEST, and AXIOM rules allow us to reach a pre-solved form, *i.e.* a system of the form

$$\mathcal{C} = \begin{cases} T_1 \overset{?}{\vdash} x_1 \\ T_2 \overset{?}{\vdash} x_2 \\ \ldots \\ T_n \overset{?}{\vdash} x_n \end{cases} + \text{ some disequalities}$$

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$\longrightarrow$ this is sufficient to decide satisfiability but not sufficient to decide symbolic equivalence. Why?

$$\mathcal{C} = \begin{cases} T_0 \overset{?}{\vdash} x \\ T_0 \overset{?}{\vdash} y \end{cases} + \quad y \overset{?}{\neq} \mathsf{enc}(x, x) \qquad\qquad \mathcal{C}' = \begin{cases} T'_0 \overset{?}{\vdash} x' \\ T'_0 \overset{?}{\vdash} y' \end{cases}$$

$\mathcal{C}$ and $\mathcal{C}'$ are in pre-solved form but they are not in symbolic equivalence.

Step 2: dealing with disequations

For these we have some specific rules to:

- simplify the disequations; and
- "match" the disequations of each constraint system.

$$\mathcal{C} = \left\{ \begin{array}{l} T_0 \overset{?}{\vdash} x \\ T_0 \overset{?}{\vdash} y \end{array} \right. \quad + \quad y \overset{?}{\neq} \mathsf{enc}(x, x) \qquad\qquad \mathcal{C}' = \left\{ \begin{array}{l} T_0' \overset{?}{\vdash} x' \\ T_0' \overset{?}{\vdash} y' \end{array} \right.$$

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Apply a rule to split each constraint system into two constraint systems:

$$\begin{array}{ccc} & \boxed{(\mathcal{C}; \mathcal{C}')} & \\ \swarrow & & \searrow \\ \boxed{(\perp;\ \mathcal{C}' + y' = \mathsf{enc}(x', x'))} & & \boxed{(\mathcal{C};\ \mathcal{C}' + y' \neq \mathsf{enc}(x', x'))} \end{array}$$

Step 3: dealing with static equivalence

$\longrightarrow$ The two resulting sequences of messages have to be indistinguishable.

$$\mathcal{C} = \begin{cases} a;\ \mathrm{pub}(b) \overset{?}{\vdash} x \\ \Phi = a;\ \mathrm{pub}(b);\ \{x\}_{\mathrm{pub}(b)} \end{cases} \quad \mathcal{C}' = \begin{cases} a';\ \mathrm{pub}(b'); \overset{?}{\vdash} x' \\ \Phi' = a';\ \mathrm{pub}(b');\ \{x'\}_{\mathrm{pub}(c')} \end{cases}$$

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Applying $\textsc{Ded-subterm}$ on $(\mathcal{C}, \mathcal{C}')$ will generate $(\mathcal{C}_1; \mathcal{C}'_1)$ (on one branch):

$$\begin{cases} \dots \\ a; \mathrm{pub}(b), \{x\}_{\mathrm{pub}(b)} \overset{?}{\vdash} x \\ a; \mathrm{pub}(b); \{x\}_{\mathrm{pub}(b)} \overset{?}{\vdash} \mathrm{pub}(b) \end{cases} \qquad \begin{cases} \dots \\ a'; \mathrm{pub}(b'), \{x'\}_{\mathrm{pub}(c')} \overset{?}{\vdash} x' \\ a'; \mathrm{pub}(b'); \{x'\}_{\mathrm{pub}(c')} \overset{?}{\vdash} \mathrm{pub}(c') \end{cases}$$

Main results on our algorithm

Termination

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Soundness/Completeness

Let (Σ_0, Σ_0') be pair of sets of constraint systems, and consider a binary tree obtained by applying our simplification rule following a strategy $\mathcal{S}$.

1. soundness: If all leaves of the tree are labeled with $(\perp, \perp)$ or $(\mathit{solved}, \mathit{solved})$, then $\Sigma_0 \approx_s \Sigma_0'$.
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Theorem

Given two sets Σ_0, Σ_0' of constraint systems, it is decidable whether

$$\Sigma_0 \approx_s \Sigma_0'$$

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Main result

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To go further

1. an efficient implementation
   $\longrightarrow$ it seems necessary to come with some optimisations to reduce the search space (*e.g.* the number of interleavings)
2. more primitives
   $\longrightarrow$ this will allow us to analyse a larger class of protocols (*e.g* e-voting protocols, ...)
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VIP project Jan. 2012 - Dec 2015.

$\longrightarrow$ A postdoc position and a PhD position are available.