Modelling and verifying e-voting protocols in applied-pi calculus

Stéphanie Delaune and Steve Kremer

LSV, ENS Cachan & CNRS & INRIA Saclay Île-de-France,

Wednesday 1st September
Outline

Lecture 1: Introduction to protocol analysis in applied π
→ today

Lecture 2: Formalisation and verification of security properties

Part I: Privacy-type properties
(based on joint work with M. Ryan)

Part II: Verifiability properties
(based on joint work with M. Ryan and B. Smyth)
→ on Friday
Part I

Formal methods and security protocols
Formal methods for system verification

Does the system satisfy the property?

Modelling

Major successes: formal methods in hardware design, software model-checking of drivers, static analysis of large scale embedded systems, ...

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Formal methods for system verification

Does the system satisfy the property?

Modelling

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2007 Turing award for Computer aided verification

To Clarke, Emerson and Sifakis: For their role in developing Model-Checking into a highly effective verification technology, widely adopted in the hardware and software industries.
Cryptographic protocols everywhere!

Cryptographic protocol: a *distributed* program which uses cryptographic primitives (e.g. encryption, digital signatures, ...) to ensure a *security property* (e.g. confidentiality, authentication, anonymity, ...)
Formal methods for protocol verification

*Does* the protocol *satisfy* a security property?

Modelling
Formal methods for protocol verification

Does the protocol satisfy a security property?

Modelling

protocol is executed in adversarial environment
Formal methods for protocol verification

Does the protocol satisfy a security property?

Modelling

- protocol is executed in adversarial environment
- in this talk: protocols are modelled in the applied pi calculus
Formal methods for protocol verification

Does the protocol satisfy a security property?

Modelling

- protocol is executed in adversarial environment
- in this talk: protocols are modelled in the applied pi calculus
- attackers are any process which can be written in the applied pi calculus
Formal methods for protocol verification

Does the protocol satisfy a security property?

Modelling

- protocol is executed in adversarial environment
- in this talk: protocols are modelled in the applied pi calculus
- attackers are any process which can be written in the applied pi calculus
- partial automation using the verification tool ProVerif
The Needham-Schroeder public key protocol (1978)

- \( A \rightarrow B : \{ A, N_a \}_{\text{pub}(B)} \)
- \( B \rightarrow A : \{ N_a, N_b \}_{\text{pub}(A)} \)
- \( A \rightarrow B : \{ N_b \}_{\text{pub}(B)} \)
The Needham-Schroeder public key protocol (1978)

\[ A \rightarrow B : \{ A, N_a \}_{pub(B)} \]

\[ B \rightarrow A : \{ N_a, N_b \}_{pub(A)} \]

\[ A \rightarrow B : \{ N_b \}_{pub(B)} \]
The Needham-Schroeder public key protocol (1978)

\[
\begin{align*}
A & \rightarrow B : \quad \{A, N_a\}_{\text{pub}(B)} \\
B & \rightarrow A : \quad \{N_a, N_b\}_{\text{pub}(A)} \\
A & \rightarrow B : \quad \{N_b\}_{\text{pub}(B)}
\end{align*}
\]
The Needham-Schroeder public key protocol (1978)

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B & \rightarrow A : \quad \{N_a, N_b\}_{\text{pub}(A)} \\
A & \rightarrow B : \quad \{N_b\}_{\text{pub}(B)}
\end{align*}
\]

\[
\text{INIT} \triangleq \\
in(c, xpkb).\nu na. \\
\text{out}(c, \text{aenc}(\langle pk(ska), na\rangle, xpkb)). \\
in(c, x). \\
\text{if } \text{fst}(\text{aenc}(x, ska)) = na \text{ then} \\
\text{let } xnb = \text{snd}(\text{aenc}(x, ska)) \text{ in} \\
\text{out}(c, \text{aenc}(xnb, xpkb)).0
\]

\[
\text{RESP} \triangleq \\
in(c, y) \\
\text{let } ypka = \text{fst}(\text{aenc}(y, skb)) \text{ in} \\
\text{let } yna = \text{snd}(\text{aenc}(y, skb)) \text{ in} \\
\nu nb.\text{out}(c, \text{aenc}(\langle yna, nb\rangle, ypka)) \\
in(c, z). \\
\text{if } \text{aenc}(z, skb) = nb \text{ then } P
\]

\[
\text{NSPK} \triangleq \nu ska.\text{out}(pk(ska)).!\text{INIT} \quad | \quad \nu skb.\text{out}(pk(skb)).!\text{RESP}
\]
The Needham-Schroeder public key protocol (1978)

\[
\begin{align*}
A &\rightarrow B : \quad \{A, N_a\}_{\text{pub}(B)} \\
B &\rightarrow A : \quad \{N_a, N_b\}_{\text{pub}(A)} \\
A &\rightarrow B : \quad \{N_b\}_{\text{pub}(B)}
\end{align*}
\]

Questions

- Is $N_b$ a shared secret between $A$ and $B$?
- When $B$ receives $\{N_b\}_{\text{pub}(B)}$, does this message really originate from $A$?
The Needham-Schroeder public key protocol (1978)

\[ A \rightarrow B : \quad \{ A, N_a \}_{\text{pub}(B)} \]
\[ B \rightarrow A : \quad \{ N_a, N_b \}_{\text{pub}(A)} \]
\[ A \rightarrow B : \quad \{ N_b \}_{\text{pub}(B)} \]

Questions

- Is \( N_b \) a shared secret between \( A \) and \( B \)?
- When \( B \) receives \( \{ N_b \}_{\text{pub}(B)} \), does this message really originate from \( A \)?

An attack was found 17 years after its publication!
A Man-in-the-middle attack

Agent $A$ \quad Intruder $I$ \quad Agent $B$

\[
\begin{align*}
A & \rightarrow B : \{N_a, A\}_{\text{pub}(B)} \\
B & \rightarrow A : \{N_a, N_b\}_{\text{pub}(A)} \\
A & \rightarrow B : \{N_b\}_{\text{pub}(B)}
\end{align*}
\]
A Man-in-the-middle attack

\[
\{N_a, A\}_{\text{pub}(I)} \Rightarrow
\]

Agent \( A \) \quad Intruder \( I \) \quad Agent \( B \)

- \( A \rightarrow B : \{N_a, A\}_{\text{pub}(B)} \)
- \( B \rightarrow A : \{N_a, N_b\}_{\text{pub}(A)} \)
- \( A \rightarrow B : \{N_b\}_{\text{pub}(B)} \)
A Man-in-the-middle attack

\[
\{N_a, A\}_{\text{pub}(I)} \xrightarrow{\bullet} \{N_a, A\}_{\text{pub}(B)}
\]

Agent \(A\) \hspace{2cm} Intruder \(I\) \hspace{2cm} Agent \(B\)

\[
\begin{align*}
A & \rightarrow B : \{N_a, A\}_{\text{pub}(B)} \\
B & \rightarrow A : \{N_a, N_b\}_{\text{pub}(A)} \\
A & \rightarrow B : \{N_b\}_{\text{pub}(B)}
\end{align*}
\]
A Man-in-the-middle attack

\[ \{N_a, A\}_{\text{pub}(I)} \rightarrow \{N_a, A\}_{\text{pub}(B)} \rightarrow \{N_a, N_b\}_{\text{pub}(A)} \]

Agent A  Intruder I  Agent B

\[ A \rightarrow B : \{N_a, A\}_{\text{pub}(B)} \]
\[ B \rightarrow A : \{N_a, N_b\}_{\text{pub}(A)} \]
\[ A \rightarrow B : \{N_b\}_{\text{pub}(B)} \]
A Man-in-the-middle attack

\[
\begin{align*}
\{ N_a, A \}_\text{pub}(I) & \rightarrow \{ N_a, N_b \}_\text{pub}(A) & \{ N_a, A \}_\text{pub}(B) & \rightarrow \{ N_a, N_b \}_\text{pub}(A)
\end{align*}
\]

Agent A

Intruder I

Agent B

\[
\begin{align*}
A & \rightarrow B : \{ N_a, A \}_\text{pub}(B) \\
\bullet B & \rightarrow A : \{ N_a, N_b \}_\text{pub}(A) \\
A & \rightarrow B : \{ N_b \}_\text{pub}(B)
\end{align*}
\]
A Man-in-the-middle attack

\[
\begin{align*}
\{N_a, A\}_{\text{pub}(I)} & \rightarrow \{N_a, N_b\}_{\text{pub}(A)} \rightarrow \{N_b\}_{\text{pub}(I)} \\
\{N_a, N_b\}_{\text{pub}(A)} & \rightarrow \{N_a, A\}_{\text{pub}(B)} \\
\end{align*}
\]

Agent A

Intruder I

Agent B

\[
\begin{align*}
A & \longrightarrow B : \{N_a, A\}_{\text{pub}(B)} \\
B & \longrightarrow A : \{N_a, N_b\}_{\text{pub}(A)} \\
\bullet A & \longrightarrow B : \{N_b\}_{\text{pub}(B)} \\
\end{align*}
\]
A Man-in-the-middle attack

Agent $A$  

\[
\{N_a, A\}_{\text{pub}(I)} \xrightarrow[]{} \{N_a, N_b\}_{\text{pub}(A)} \xleftarrow[]{} \{N_b\}_{\text{pub}(I)} \xrightarrow[]{} \{N_a, A\}_{\text{pub}(B)} \xleftarrow[]{} \{N_a, N_b\}_{\text{pub}(A)} \xrightarrow[]{} \{N_b\}_{\text{pub}(B)}
\]

Intruder $I$

Agent $B$

\[
A \longrightarrow B : \{N_a, A\}_{\text{pub}(B)} \\
B \longrightarrow A : \{N_a, N_b\}_{\text{pub}(A)} \\
\bullet \ A \longrightarrow B : \{N_b\}_{\text{pub}(B)}
\]
A Man-in-the-middle attack

Agent $A$

Intruder $I$

Agent $B$

Answers

- Is $N_b$ a shared secret between $A$ and $B$?
  - No
A Man-in-the-middle attack

\[
\begin{align*}
\{N_a, A\}_{\text{pub}(l)} & \rightarrow \{N_a, N_b\}_{\text{pub}(A)} \\
\{N_b\}_{\text{pub}(l)} & \rightarrow \{N_a, N_b\}_{\text{pub}(B)} \\
\{N_a, A\}_{\text{pub}(B)} & \rightarrow \{N_b\}_{\text{pub}(A)}
\end{align*}
\]

Agent A  Intruder I  Agent B

**Answers**

- Is \(N_b\) a shared secret between \(A\) and \(B\)?
  \(\rightarrow\) No

- When \(B\) receives \(\{N_b\}_{\text{pub}(B)}\), does this message really originate from \(A\)?
  \(\rightarrow\) No
A Man-in-the-middle attack

\[\{N_a, A\}_{\text{pub}(I)} \rightarrow \{N_a, N_b\}_{\text{pub}(A)} \rightarrow \{N_b\}_{\text{pub}(I)} \rightarrow \{N_a, A\}_{\text{pub}(B)} \rightarrow \{N_a, N_b\}_{\text{pub}(A)} \rightarrow \{N_b\}_{\text{pub}(B)}\]

Agent A \hspace{2cm} Intruder I \hspace{2cm} Agent B

Answers

- Is \(N_b\) a shared secret between \(A\) and \(B\)?
  \[\rightarrow \text{No}\]

- When \(B\) receives \(\{N_b\}_{\text{pub}(B)}\), does this message really originate from \(A\)?
  \[\rightarrow \text{No}\]

Remark: Crypto has not been broken
  \[\rightarrow \text{Attack on the protocol logic}\]
Part II

The applied π calculus
Motivation for using the applied $\pi$-calculus

**Applied pi-calculus:** [Abadi & Fournet, 01]

Basic programming language with constructs for **concurrency** and communication

- based on the $\pi$-calculus [Milner *et al.*, 92]
- in some ways similar to the **spi-calculus** [Abadi & Gordon, 98]
Motivation for using the applied $\pi$-calculus

Applied pi-calculus: [Abadi & Fournet, 01]
basic programming language with constructs for concurrency and communication

- based on the $\pi$-calculus [Milner et al., 92]
- in some ways similar to the spi-calculus [Abadi & Gordon, 98]

Advantages:
- allows us to model less classical cryptographic primitives
- both reachability and equivalence-based specification of properties
- automated proofs using ProVerif tool [Blanchet]
- powerful proof techniques for hand proofs
- successfully used to analyze a variety of security protocols
Modelling messages as terms

First order terms built over a signature $\mathcal{F}$ (finite set of function symbols), an infinite set of names and an infinite set of variables

$$t \ ::= \ term \ | \ x \ variable \ x \ | \ n \ name \ n \ | \ f(t_1, \ldots, t_k) \ application \ of \ symbol \ f \in \mathcal{F}$$

Example: Let $\mathcal{F} = \{\text{enc}(\cdot, \cdot), \text{dec}(\cdot, \cdot), \langle \cdot, \cdot \rangle, \pi_1(\cdot), \pi_2(\cdot)\}$.

$\text{enc}(\langle s_1, a \rangle, k)$ $\text{dec}(\text{enc}(x, y), y)$ $\pi_1(\text{enc}(s, k))$
Modelling messages as terms

First order terms built over a signature $\mathcal{F}$ (finite set of function symbols), an infinite set of names and an infinite set of variables

$$t ::= \text{term} \mid x \quad \text{variable } x \mid n \quad \text{name } n \mid f(t_1, \ldots, t_k) \quad \text{application of symbol } f \in \mathcal{F}$$

Example: Let $\mathcal{F} = \{\text{enc}(\cdot, \cdot), \text{dec}(\cdot, \cdot), \langle \cdot, \cdot \rangle, \pi_1(\cdot), \pi_2(\cdot)\}$.  
$$\text{enc}(\langle s_1, a \rangle, k) \quad \text{dec}(\text{enc}(x, y), y) \quad \pi_1(\text{enc}(s, k))$$

Term algebra is equipped with an equational theory induced by a finite set of equations

Example: Define $E$ by $\text{dec}(\text{enc}(x, y), y) = x$, $\pi_1(\langle x, y \rangle) = x$, $\pi_2(\langle x, y \rangle) = y$  
Then we have that $\pi_1 \text{dec}(\text{enc}(\langle s_1, a \rangle, k)) =_E s$. 
The applied $\pi$-calculus on an example

Syntax:

$$P = \nu s, k. (\text{out}(c_1, \text{enc}(s, k)) | \text{in}(c_1, y).\text{out}(c_2, \text{dec}(y, k))).$$

Special processes: active substitutions $P \mid \{^M/x\}$
The applied $\pi$-calculus on an example

Syntax:

$$P = \nu s, k.(\text{out}(c_1, enc(s, k)) \mid \text{in}(c_1, y).\text{out}(c_2, dec(y, k))).$$

Special processes: active substitutions $P \mid \{^M/x\}$

Semantics:

- **Operational semantics** $\rightarrow$: closed by structural equivalence ($\equiv$) and application of evaluation contexts such that

  - **Comm** $\text{out}(a, x).P \mid \text{in}(a, x).Q \rightarrow P \mid Q$
  - **Then** if $M = M$ then $P$ else $Q \rightarrow P$
  - **Else** if $M = N$ then $P$ else $Q \rightarrow Q$ ($M \neq N$)
The applied $\pi$-calculus on an example

Syntax:

$$ P = \nu s, k. (\text{out}(c_1, \text{enc}(s, k)) \mid \text{in}(c_1, y). \text{out}(c_2, \text{dec}(y, k))). $$

Special processes: active substitutions $P \mid \{^M_x\}$

Semantics:

- **Operational semantics** $\rightarrow$: closed by structural equivalence ($\equiv$) and application of evaluation contexts such that

  Comm: $\text{out}(a, x). P \mid \text{in}(a, x). Q \rightarrow P \mid Q$

  Then: if $M = M$ then $P$ else $Q \rightarrow P$

  Else: if $M = N$ then $P$ else $Q \rightarrow Q$ ($M \neq N$)

Example: $P \rightarrow \nu s, k. \text{out}(c_2, s)$
Labeled operational semantics $\xrightarrow{\alpha}$

Labelled transitions where $\alpha$ is either $\text{in}(c, M)$, $(\nu c').\text{out}(c, c')$ or $(\nu x.)\text{out}(c, x)$

Example:

$$\nu a, \nu k.\text{out}(c, \text{enc}(a, k)).P \xrightarrow{\nu x.\text{out}(c, x)} P | \{\text{enc}(a, k)/x\}$$

Allows processes to communicate with an unspecified environment

Output is done by reference and creates active substitutions
Labeled operational semantics $\xrightarrow{\alpha}$

Labelled transitions where $\alpha$ is either $\text{in}(c, M)$, $(\nu c').\text{out}(c, c')$ or $(\nu x.)\text{out}(c, x)$

Example:

$\nu a, \nu k. \text{out}(c, \text{enc}(a, k)).P \xrightarrow{\nu x.\text{out}(c, x)} P | \{\text{enc}(a, k)/x\}$

Allows processes to communicate with an unspecified environment

Output is done by reference and creates active substitutions

Frames

The frame of a process $\phi(A)$ is build from the process restrictions and active substitutions

Approximation of the process accounting for the static knowledge exposed to the environment
Deducing secrets

Frame
A frame is a process of the form $\nu \tilde{n}.(\{M_1/x_1\} \mid \ldots \mid \{M_n/x_n\})$.

Example

$$P = \nu s, k.(\text{out}(c_2, s) \mid \{\text{enc}(s, k)/x_1\}) \quad \phi(P) = \nu s, k.\{\text{enc}(s, k)/x_1\}$$

Deducibility ($\vdash$)

$\varphi \vdash s$ when

- there exists $M$, such that $M\sigma =_E t$, where $\varphi = \nu \tilde{n}.\sigma$ and $M$ does not use the names $\tilde{n}$
Deducing secrets

Frame
A frame is a process of the form $\nu \tilde{n}.(\{M_1/x_1\} \mid \ldots \mid \{M_n/x_n\})$.

Example

\[ P = \nu s, k. (\text{out}(c_2, s) \mid \{\text{enc}(s, k)/x_1\}) \quad \phi(P) = \nu s, k.\{\text{enc}(s, k)/x_1\} \]

Deducibility ($\vdash$)

$\varphi \vdash s$ when
- there exists $M$, such that $M\sigma =_E t$, where $\varphi = \nu \tilde{n}.\sigma$ and $M$ does not use the names $\tilde{n}$

Example 1: $\nu s.\nu k. (\{\text{enc}(s, k)/x\} \mid \{k/y\}) \vdash s$

as $\text{dec}(x, y)\sigma = \text{dec}(\text{enc}(s, k), k) =_E s$. 

S. Delaune & S. Kremer (LSV)
Deducing secrets

Frame
A frame is a process of the form $\nu \tilde{n}.(\{M_1/x_1\} | \ldots | \{M_n/x_n\})$.

Example

\[ P = \nu s, k.(\text{out}(c_2, s) | \{\text{enc}(s, k)/x_1\}) \quad \phi(P) = \nu s, k.\{\text{enc}(s, k)/x_1\} \]

Deducibility ($\vdash$)

$\varphi \vdash s$ when

- there exists $M$, such that $M\sigma =_E t$, where $\varphi = \nu \tilde{n}.\sigma$ and $M$ does not use the names $\tilde{n}$

Example 2:

$\nu a.\nu k.\{\text{enc}(a, k)/x\} \not\vdash a$
Deducing secrets

Frame
A frame is a process of the form $\nu \tilde{n}.(\{M_1/x_1\} \mid \ldots \mid \{M_n/x_n\})$.

Example

\[ P = \nu s, k.(\text{out}(c_2, s) \mid \{\text{enc}(s, k)/x_1\}) \quad \phi(P) = \nu s, k.\{\text{enc}(s, k)/x_1\} \]

\[ \begin{align*}
\varphi & \vdash s \quad \text{when} \\
& \quad \text{there exists } M, \text{ such that } M\sigma =_E t, \text{ where } \varphi = \nu \tilde{n}.\sigma \text{ and } M \text{ does not use the names } \tilde{n} \\
\end{align*} \]

Deducibility ($\vdash$)

A process $P$ ensures the secret of $s$ if for any $P'$ such that $P \xrightarrow{(\alpha)} *P'$ we have that $\phi(P') \not\vdash s$. 

Static equivalence on frames (≈_s)

φ ≈_s ψ when

- \( dom(φ) = dom(ψ) \) (the frames coincide on unrestricted variables),
- for all terms \( U, V \), \( (U =_E V)φ \) iff \( (U =_E V)ψ \)
Static equivalence on frames (\(\approx_s\))

\(\varphi \approx_s \psi\) when
- \(\text{dom}(\varphi) = \text{dom}(\psi)\) (the frames coincide on unrestricted variables),
- for all terms \(U, V, (U =_E V)\varphi\) iff \((U =_E V)\psi\)

Example 1: \(\nu k.(\{\text{enc}(a,k)/x\} | \{k/y\}) \not\approx_s \nu k.(\{\text{enc}(b,k)/x\} | \{k/y\})\)

because of the test \(\text{dec}(x,y) = a\)
Static equivalence on frames \((\approx_s)\)

\[ \varphi \approx_s \psi \text{ when} \]

1. \(\text{dom}(\varphi) = \text{dom}(\psi)\) (the frames coincide on unrestricted variables),
2. for all terms \(U, V, (U =_E V) \varphi \iff (U =_E V) \psi\)

**Example 1:**

\[ \nu k.(\{^{\text{enc}(a,k)}/x\} \mid \{^k/y\}) \not\approx_s \nu k.(\{^{\text{enc}(b,k)}/x\} \mid \{^k/y\}) \]

because of the test \(\text{dec}(x, y) = a\)

**Example 2:**

\[ \nu k.\{^{\text{enc}(a,k)}/x\} \approx_s \nu k.\{^{\text{enc}(b,k)}/x\} \]
Static equivalence on frames (\(\approx_s\))

\(\varphi \approx_s \psi\) when
- \(\text{dom}(\varphi) = \text{dom}(\psi)\) (the frames coincide on unrestricted variables),
- for all terms \(U, V, (U =_E V)\varphi\) iff \((U =_E V)\psi\)

Example 1: \(\nu k. (\{\text{enc}(a,k)/x\} | \{k/y\}) \not\approx_s \nu k. (\{\text{enc}(b,k)/x\} | \{k/y\})\)

because of the test \(\text{dec}(x,y) = a\)

Example 2: \(\nu k. \{\text{enc}(a,k)/x\} \approx_s \nu k. \{\text{enc}(b,k)/x\}\)

Formalizes the idea that an attacker cannot distinguish two frames
Equivalence of processes

Testing equivalence \((P \approx_t Q)\)

for all closing evaluation contexts \(C[\_]\), we have that:

\[ C[P] \downarrow c \text{ if, and only if, } C[Q] \downarrow c. \]

\[\rightarrow P \downarrow c \text{ when } P \text{ can send a message on the channel } c.\]

Intuition:
An adversary cannot distinguish two processes, even if it can arbitrarily interact with them

Usefull for modelling privacy properties: more on this on Friday in Stéphanie’s talk
Part III

Analysing the protocol by Fujioka, Okamoto, Ohta

[KremerRyan’05]
Anonymous channels
- Implemented using MixNets, Onion Routing, ...

Commitment
- To commit to $m$, I invent a new random $r$ and send you $\text{commit}(m, r)$.
- Later, I'll send you $r$, which you can use to reveal $m$.
- It is binding: one cannot find $r'$, such that the commitment opens correctly to $m'$

Blind signatures
- I want you to sign $m$ but I don’t want you to see its value.
- I send you $\text{blind}(m, r)$. You sign it.
- I use $r$ to extract your signature on $m$. 
Part 1

\[ x = \text{commit}(v, r) \]
\[ e = \text{blind}(x, b) \]

\[ V \rightarrow A : \sigma_V(e) \]
check V is legitimate

\[ A \rightarrow V : \sigma_A(e) \]

\[ \text{unblind}(\sigma_A(e), b) = \sigma_A(x) \]
FOO 92 protocol

[FujiokaOkamotoOhta92]

Part 1

\[ x = \text{commit}(v, r) \]
\[ e = \text{blind}(x, b) \]

\[ \begin{align*}
V & \rightarrow A : \sigma_V(e) \\
& \text{check } V \text{ is legitimate}
\end{align*} \]

\[ A \rightarrow V : \sigma_A(e) \]
\[ \text{unblind}(\sigma_A(e), b) = \sigma_A(x) \]

Voter V

Admin A

Part 2

\[ \begin{align*}
V & \rightarrow C : \sigma_A(x) \\
& \text{enter } (\ell, \sigma_A(x)) \text{ into list}
\end{align*} \]

Voter V

Collector C
FOO 92 protocol  

[FujiokaOkamotoOhta92]

Part 1

\[ x = \text{commit}(v, r) \]
\[ e = \text{blind}(x, b) \]

Voter V

\[ V \rightarrow A : \sigma_V(e) \]
check V is legitimate

A \rightarrow V : \sigma_A(e)

unblind(\sigma_A(e), b) = \sigma_A(x)

Admin A

Part 2

V → C : \sigma_A(x)

enter \((\ell, \sigma_A(x))\) into list

Voter V

Collector C

Part 3

V → C : \ell_i, r

open x using r

publish list \((\ell_i, \sigma_A(x_i))\)

Voter V

Collector C

publish v
Signature

- **commit/2.** commitment
- **open/2.** open commitment
- **sign/2.** digital signature
- **checksign/2.** open digital signature
- **pk/1.** get public key from private key
- **host/1.** get host from public key
- **getpk/1.** get public key from host
- **blind/2.** blinding
- **unblind/2.** undo blinding

Equational theory

- open(commit(m,r),r) = m.
- getpk(host(pubkey)) = pubkey.
- checksign(sign(m,sk),pk(sk)) = m.
- unblind(blind(m,r),r) = m.
- unblind(sign(blind(m,r),sk),r) = sign(m,sk).
Voter process

- ascii version of applied $\pi$-calculus (input to ProVerif tool)
- Hypothesis: All channels are anonymous, unless identification is explicitly given in the message

```
processV =
new blinder; new r;
let blindedcommitedvote=blind(commit(v,r),blinder) in
out(ch,(hostv,sign(blindedcommitedvote,skv)));
in(ch,m2);
let blindedcommitedvote0=checksign(m2,pka) in
if blindedcommitedvote0=blindedcommitedvote then
let signedcommitedvote=unblind(m2,blinder) in
out(ch,signedcommitedvote);
in(ch,(l,=signedcommitedvote));
out(ch,(l,r)).
```
The other processes

- Admin and collector processes similar
- Main process puts everything together:

```plaintext
new ska; new skv;
new privCh;
let pka=pk(ska) in
let hosta = host(pka) in
let pkv=pk(skv) in
let hostv=host(pkv) in
out(ch,pka); out(ch,hosta);
out(ch,pkv); out(ch,hostv);
((out(privCh,pkv); out(privCh,pk(ski))) | (!processV)|(!processA)|(!processC))
```
Blanchet’s ProVerif tool

- Designed and implemented by Bruno Blanchet
  http://www.proverif.ens.fr/
- Input is given in the applied $\pi$-calculus
- Expressive: can model algebraic properties of the crypto, via rewrite rules and equations
- Analyses secrecy/reachability properties of protocols as well as equivalence properties
- Applied $\pi$-calculus is translated into Horn clauses, describing acquisition of knowledge by the attacker
- Unbounded number of sessions
- Sound, but not complete (false attacks are possible)
- Termination not guaranteed
Fairness

Fairness ensures that you cannot obtain exit polls, i.e. early results

Can be modeled as a secrecy property: the vote of a honest voter stays secret until the opening phase

Even a corrupt administrator cannot learn votes: modeled by outputting the admin’s private key

No need for a corrupt collector (collector never uses his private key)

Proofs automated by ProVerif
Fairness using stronger notions of secrecy

Modeling fairness as deducibility may be too weak

Only few possible values for votes make elections particularly vulnerable to offline guessing attacks, aka dictionary attacks

Example: \[ \varphi = \{^\text{enc}(v, pk) / x \} \text{ where } v \in \{0, 1\} \]
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Example: \( \varphi = \{\text{enc}(v, pk)/x\} \) where \( v \in \{0, 1\} \)

Offline guessing attacks can be modelled using static equivalence

\[
\nu \nu. (\varphi | \{v/x\}) \approx_s \nu \nu. (\varphi | \nu v'.\{v'/x\})
\]

Intuition: the attacker cannot distinguish the right guess \( v \) from a wrong guess \( v' \)
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\[ \varphi = \{\text{enc}(v, pk) / x\} \text{ where } v \in \{0, 1\} \]

Offline guessing attacks can be modelled using static equivalence

\[ \nu v.(\varphi | \{v / x\} \approx_s \nu v.(\varphi | \nu v'.\{v' / x\}) \]

Intuition:
the attacker cannot distinguish the right guess \( v \) from a wrong guess \( v' \)

We can verify an even stronger property: strong secrecy [Blanchet’04]

\[ \forall M, N. \quad P\{^M / v\} \approx_o P\{^N / v\} \]
Fairness using stronger notions of secrecy

Modeling fairness as deducibility may be too weak

Only few possible values for votes make elections particularly vulnerable to offline guessing attacks, aka dictionary attacks

Example: \( \varphi = \{^{\text{enc}(\nu;pk)}_x \} \) where \( \nu \in \{0, 1\} \)

Offline guessing attacks can be modelled using static equivalence

\[ \nu \nu. (\varphi | \{^\nu_x \} \approx_s \nu \nu. (\varphi | \nu \nu'. \{^\nu'_x \}) \]

Intuition:
the attacker cannot distinguish the right guess \( \nu \) from a wrong guess \( \nu' \)

We can verify an even stronger property: strong secrecy \[ [\text{Blanchet'04}] \]

\[ \forall M, N. \quad P\{^M_\nu\} \approx_o P\{^N_\nu\} \]

All of these properties have been automatically checked using ProVerif
Only legitimate voters can vote and only once

Do not register intruder and require to vote a challenge vote

Modified collector

[...]
new attack;
if voteV=challengeVote then
  out(ch, attack)
else
  out(ch, voteV).

Proof done by ProVerif

Corrupt administrator: trivial attack found by ProVerif
Outline

Lecture 1: Introduction to protocol analysis in applied pi

→ today

Lecture 2: Formalisation and verification of security properties

Part I: Privacy-type properties
(based on joint work with M. Ryan)

Part II: Verifiability properties
(based on joint work with M. Ryan and B. Smyth)

→ on Friday
Lecture 2: Formalisation and verification of security properties

Part I: Privacy-type properties
(based on joint work with M. Ryan)

Stéphanie Delaune

Part II: Verifiability properties
(based on joint work with M. Ryan and B. Smyth)

Steve Kremer
Privacy-type security properties

Privacy: the fact that a particular voter voted in a particular way is not revealed to anyone

![Vote for me]

Receipt-freeness: a voter cannot prove that she voted in a certain way (this is important to protect voters from coercion)

Coercion-resistance: same as receipt-freeness, but the coercer interacts with the voter during the protocol, (e.g. by preparing messages)
How can we express privacy?

Classically modeled as an \textit{equivalence} between \textit{two slightly different processes} $P_1$ and $P_2$.

In applied pi calculus, such an equivalence can be:

1. Testing equivalence $(P_1 \approx_t P_2)$
2. Observational equivalence $(P_1 \approx_o P_2)$
Testing equivalence \((P \approx_t Q)\)

for all closing evaluation contexts \(C[\_]\), we have that:

\[ C[P] \Downarrow c \text{ if, and only if, } C[Q] \Downarrow c. \]

\[ \rightarrow \quad P \Downarrow c \text{ when } P \text{ can send a message on the channel } c. \]
Testing equivalence ($P \approx_t Q$)

for all closing evaluation contexts $C[_]$, we have that:

$$C[P] \Downarrow c \text{ if, and only if, } C[Q] \Downarrow c.$$ 

$\rightarrow$ $P \Downarrow c$ when $P$ can send a message on the channel $c$.

Example 1: $\text{out}(a, s) \not\approx_t \text{out}(a, s')$
Testing equivalence \((P \approx_t Q)\)

for all closing evaluation contexts \(C[\_]\), we have that:

\[ C[P] \downarrow c \text{ if, and only if, } C[Q] \downarrow c. \]

\[ \rightarrow \quad P \downarrow c \text{ when } P \text{ can send a message on the channel } c. \]

Example 1:

\[ \text{out}(a, s) \not\approx_t \text{out}(a, s') \]

\[ \rightarrow \quad C[\_] = \text{in}(a, x).\text{if } x = s \text{ then out}(c, \text{ok}) | \_ \]
Testing equivalence \((P \approx_t Q)\)

for all closing evaluation contexts \(C[\_]\), we have that:

\[C[P] \Downarrow c \text{ if, and only if, } C[Q] \Downarrow c.\]

\[\rightarrow \quad P \Downarrow c \text{ when } P \text{ can send a message on the channel } c.\]

Example 2:

\[\nu s. \text{out}(a, \text{enc}(s, k)).\text{out}(a, \text{enc}(s, k')) \approx_t \nu s, s'. \text{out}(a, \text{enc}(s, k)).\text{out}(a, \text{enc}(s', k'))\]
Testing equivalence

Testing equivalence \( (P \approx_t Q) \)

for all closing evaluation contexts \( C[\_] \), we have that:

\[ C[P] \Downarrow c \text{ if, and only if, } C[Q] \Downarrow c. \]

\[ \rightarrow P \Downarrow c \text{ when } P \text{ can send a message on the channel } c. \]

Example 2:

\[ \nu s. \text{out}(a, \text{enc}(s, k)).\text{out}(a, \text{enc}(s, k')) \]

\[ \not\approx_t \]

\[ \nu s, s'. \text{out}(a, \text{enc}(s, k)).\text{out}(a, \text{enc}(s', k')) \]

\[ \rightarrow C[\_] = \text{in}(a, x).\text{in}(a, y).\text{if } (\text{dec}(x, k) = \text{dec}(y, k')) \text{ then } \text{out}(c, ok) \mid _\]
Testing equivalence

Testing equivalence \( (P \approx_t Q) \)

for all closing evaluation contexts \( C[\_] \), we have that:

\[ C[P] \Downarrow c \text{ if, and only if, } C[Q] \Downarrow c. \]

\[ \rightarrow \quad P \Downarrow c \text{ when } P \text{ can send a message on the channel } c. \]

Example 3: \( \nu s.\text{out}(a, s) \approx_t \nu s.\nu k.\text{out}(a, \text{enc}(s, k)) \)
Observational equivalence

Observational equivalence ($\approx_o$)

The largest symmetric relation $\mathcal{R}$ on processes such that $P \mathcal{R} Q$ implies

1. if $P \Downarrow c$, then $Q \Downarrow c$,
2. if $P \rightarrow^* P'$, then $Q \rightarrow^* Q'$ and $P' \mathcal{R} Q'$ for some $Q'$,

S. Delaune & S. Kremer (LSV)
Observational equivalence ($\approx_o$)

The largest symmetric relation $\mathcal{R}$ on processes such that $P \mathcal{R} Q$ implies

1. if $P \downarrow c$, then $Q \downarrow c$,
2. if $P \rightarrow^* P'$, then $Q \rightarrow^* Q'$ and $P' \mathcal{R} Q'$ for some $Q'$,

Lemma

We have that: $P \approx_o Q \implies P \approx_t Q$
May testing vs observational equivalence

In general, testing equivalence does not imply observational equivalence.

Example:

**Process $P$**

\[ \text{out}(c, a). (\text{out}(c, b_1) + \text{out}(c, b_2)) \]

**Process $Q$**

\[ \text{out}(c, a). \text{out}(c, b_1) + \text{out}(c, a). \text{out}(c, b_2) \]
May testing vs observational equivalence

In general, testing equivalence does not imply observational equivalence.

Example:

Process $P$
\[ \text{out}(c, a).\left(\text{out}(c, b_1) + \text{out}(c, b_2)\right) \]

Process $Q$
\[ \text{out}(c, a).\text{out}(c, b_1) + \text{out}(c, a).\text{out}(c, b_2) \]

\[ \approx_t = \approx_o \? \]

On determinate processes, the two notions coincide.
Outline

Formalising Privacy
Formalisation of privacy

Classically modeled as equivalences between two slightly different processes $P_1$ and $P_2$, but

- changing the identity
  \[ S[V_A\{^a/v\}] \approx S[V_B\{^a/v\}] \]
  does not work, as identities are revealed
- changing the vote
  \[ S[V_A\{^a/v\}] \approx S[V_A\{^b/v\}] \]
  does not work, as the votes are revealed at the end
Formalisation of privacy

Classically modeled as equivalences between two slightly different processes $P_1$ and $P_2$, but

- changing the **identity**
  
  $$S[V_A\{^a/_v\}] \approx S[V_B\{^a/_v\}]$$

  does not work, as **identities are revealed**

- changing the **vote**
  
  $$S[V_A\{^a/_v\}] \approx S[V_A\{^b/_v\}]$$

  does not work, as the **votes are revealed** at the end

**Solution**

Consider 2 honest voters and **swap** their votes.
Formal Definition of privacy

Definition (S. Kremer & M. Ryan, 2005)

A voting protocol respects privacy if

\[ S[V_A\{a/v\} \mid V_B\{b/v\}] \approx S[V_A\{b/v\} \mid V_B\{a/v\}] \]
Formal Definition of privacy

Definition (S. Kremer & M. Ryan, 2005)

A voting protocol respects privacy if

\[ S[V_A^{a/v} \mid V_B^{b/v}] \approx S[V_A^{b/v} \mid V_B^{a/v}] \]

Some remarks

- **robust** in case of an unanimous scrutin
- **flexible** w.r.t. authorities required to be honest

**Limitation:** This definition does not say anything about the privacy of a voter who wants to nullify her vote.
Example 1

Voter process

\[ V = \text{out}(ch, \{v\}_{pub(s)}) \]

What about privacy?

\[ V_A\{^a/v\} \mid V_B\{^b/v\} \approx V_A\{^b/v\} \mid V_B\{^a/v\} \]
Example 1

Voter process

\[ V = \text{out}(ch, \{v\}_{\text{pub}(S)}) \]

What about privacy?

\[ V_A\{^a/v\} \mid V_B\{^b/v\} \approx V_A\{^b/v\} \mid V_B\{^a/v\} \]

i.e.

\[ \text{out}(ch, \{^a\}_{\text{pub}(S)}) \mid \text{out}(ch, \{^b\}_{\text{pub}(S)}) \approx \text{out}(ch, \{^b\}_{\text{pub}(S)}) \mid \text{out}(ch, \{^a\}_{\text{pub}(S)}) \]
Example 1

Voter process

\[ V = \text{out}(ch, \{v\}_{\text{pub}(S)}) \]

What about privacy?

\[ V_A\{^a/v\} \mid V_B\{^b/v\} \approx V_A\{^b/v\} \mid V_B\{^a/v\} \]

i.e.

\[ \text{out}(ch, \{^a\}_{\text{pub}(S)}) \mid \text{out}(ch, \{^b\}_{\text{pub}(S)}) \approx \text{out}(ch, \{^b\}_{\text{pub}(S)}) \mid \text{out}(ch, \{^a\}_{\text{pub}(S)}) \]

\[ \rightarrow \text{The equivalence holds.} \]

Some remarks:

- \(ch\) is assumed to be an anonymous channel;
- the server is not assumed to be honest.
Example 2

Voter process

\[ V(Id) = \text{out}(ch, \langle Id, \{v\}_{\text{pub}(S)} \rangle) \]

What about privacy (for someone who does not know \text{priv}(S))?

\[ V_A\{^a/v\} \mid V_B\{^b/v\} \approx V_A\{^b/v\} \mid V_B\{^a/v\} \]
Example 2

Voter process

\[ V(Id) = \text{out}(ch, \langle Id, \{v\}_{\text{pub}(S)} \rangle) \]

What about privacy (for someone who does not know priv(S))? 

\[ V_A\{a/v\} | V_B\{b/v\} \approx \quad V_A\{b/v\} | V_B\{a/v\} \]

i.e.

\[ \text{out}(ch, \langle A, \{a\}_{\text{pub}(S)} \rangle) | \text{out}(ch, \langle B, \{b\}_{\text{pub}(S)} \rangle) \]

\[ \approx \]

\[ \text{out}(ch, \langle A, \{b\}_{\text{pub}(S)} \rangle) | \text{out}(ch, \langle B, \{a\}_{\text{pub}(S)} \rangle) \]
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\[ V(Id) = \text{out}(ch, \langle Id, \{v\}_{\text{pub}(S)} \rangle) \]

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\[ V_A \{a/v\} | V_B \{b/v\} \approx V_A \{b/v\} | V_B \{a/v\} \]

i.e.

\[ \text{out}(ch, \langle A, \{a\}_{\text{pub}(S)} \rangle) | \text{out}(ch, \langle B, \{b\}_{\text{pub}(S)} \rangle) \approx \]

\[ \text{out}(ch, \langle A, \{b\}_{\text{pub}(S)} \rangle) | \text{out}(ch, \langle B, \{a\}_{\text{pub}(S)} \rangle) \]

\[ \rightarrow \text{The equivalence does not hold (with deterministic encryption).} \]

\[ \rightarrow \text{The equivalence holds with probabilistic encryption.} \]
Example: Fujioka et al. protocol (1992)

First Phase:
the voter gets a “token” from the administrator.

1. \( V \rightarrow A : V, \text{sign}(\text{blind}(\text{commit}(\text{vote}, r), b), V) \)
2. \( A \rightarrow V : \text{sign}(\text{blind}(\text{commit}(\text{vote}, r), b), A) \)

Voting phase:

3. \( V \rightarrow C : \text{commit}(\text{vote}, r), \text{sign}(\text{commit}(\text{vote}, r), A) \)
4. \( C \rightarrow : l, \text{commit}(\text{vote}, r), \text{sign}(\text{commit}(\text{vote}, r), A) \)

Counting phase:

5. \( V \rightarrow C : l, r \)
6. \( C \) publishes the outcome of the vote

What about privacy?

\[
V_A^{a/v} \mid V_B^{b/v} \approx V_A^{b/v} \mid V_B^{a/v}
\]
Example: Fujioka et al. protocol (1992)

Process synchronisation: the protocol is divided into 3 phases
→ synchronisation is crucial for privacy to hold.
Example: Fujioka et al. protocol (1992)

Process synchronisation: the protocol is divided into 3 phases
→ synchronisation is crucial for privacy to hold.

Authorities: privacy holds even if the authorities are corrupted
- we do not require any private keys to be secret;
- we have just to ensure that both voters use the same public key for
  the administrator.
Formalising Receipt-Freeness
Receipt-freeness: Leaking secrets to the coencer

To model receipt-freeness we need to specify that a coerced voter cooperates with the coencer by leaking secrets on a channel \( ch \).

We denote by \( V^{ch} \) the process built from the process \( V \) as follows:

- \( 0^{ch} \equiv 0 \),
- \( (P \mid Q)^{ch} \equiv P^{ch} \mid Q^{ch} \),
- \( (\nu n.P)^{ch} \equiv \nu n.\text{out}(ch, n).P^{ch} \),
- \( (\text{in}(u, x).P)^{ch} \equiv \text{in}(u, x).\text{out}(ch, x).P^{ch} \),
- \( (\text{out}(u, M).P)^{ch} \equiv \text{out}(u, M).P^{ch} \),
- \ldots

We denote by \( V \setminus \text{out}(ch, \cdot) \equiv \nu ch.(V \mid !\text{in}(ch, x)) \).
Definition (S. Delaune, S. Kremer & M. Ryan, 2006)

A voting protocol is receipt-free if there exists a process $V'$, satisfying

1. $V' \setminus \text{out}(\text{cht}, \cdot) \approx V_A^{\{a/v\}},$
2. $S[V_A^{\{c/v\}} \cdot \text{cht} | V_B^{\{a/v\}}] \approx S[V' | V_B^{\{c/v\}}].$
Receipt-freeness

Definition (S. Delaune, S. Kremer & M. Ryan, 2006)

A voting protocol is receipt-free if there exists a process $V'$, satisfying

- $V'\out(chc;\cdot) \approx V_A\{a/v\}$,
- $S[V_A\{c/v\}^{chc} | V_B\{a/v\}] \approx S[V' | V_B\{c/v\}]$.

Limitations:

- This definition does not take into account randomization and forced-abstention attacks.
Example 1

**Voter process**

\[ V = \text{out}(ch, \{v\}_{\text{pub}(s)}) \]

What about receipt-freeness?

\( i.e. \) Does there exist \( V' \) such that

1. \( V' \setminus \text{out}(ch, \cdot) \approx V_A^{\{a/v\}} \),
2. \( V_A^{\{c/v\}} \cdot \text{ chc } | \ V_B^{\{a/v\}} \approx V' | V_B^{\{c/v\}} \).
Example 1

Voter process

\[ V = \text{out}(ch, \{v\}_{\text{pub}(s)}) \]

What about receipt-freeness?

\textit{i.e.} Does there exist \( V' \) such that

1. \( V' \setminus \text{out}(ch, \cdot) \cong V_A\{a/v\}, \)
2. \( V_A\{c/v\}^{\text{ chc}} \mid V_B\{a/v\} \cong V' \mid V_B\{c/v\}. \)

The process \( V_A\{a/v\} \) satisfies the two requirements.

1. \( V_A\{a/v\} \setminus \text{out}(ch, \cdot) \cong V_A\{a/v\}, \)
2. \( V_A\{c/v\}^{\text{ chc}} \mid V_B\{a/v\} \cong V_A\{a/v\} \mid V_B\{c/v\}. \)

\[ \rightarrow \text{Receipt-freeness holds.} \]
Voter process

\[ V(Id) = \nu_r . \text{out}(ch, \langle Id, \{ v \}^r_{\text{pub}(S)} \rangle) \]

What about receipt-freeness?
Example 2 (with probabilistic encryption)

Voter process

\[ V(Id) = \nu r \cdot \text{out}(ch, \langle Id, \{v\}_\text{pub}(S) \rangle) \]

What about receipt-freeness?

\[ \implies \text{Receipt-freeness does not hold: } r \text{ can be used as a receipt.} \]

We have that:

\[ V_A\{c/v\}^{chc} = \nu r \cdot \text{out}(chc, r) \cdot \text{out}(ch, \langle A, \{c\}_\text{pub}(S) \rangle). \]
Example: Fujioka et al. protocol (1992)

First Phase:
the voter gets a “token” from the administrator.

1. \( V \rightarrow A \) : \( V, \text{sign}(\text{blind}(\text{commit}(\text{vote}, r), b), V) \)
2. \( A \rightarrow V \) : \( \text{sign}(\text{blind}(\text{commit}(\text{vote}, r), b), A) \)

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4. \( C \rightarrow \) : \( l, \text{commit}(\text{vote}, r), \text{sign}(\text{commit}(\text{vote}, r), A) \)

Counting phase:

5. \( V \rightarrow C \) : \( l, r \)
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What about receipt-freeness?
Example: Fujioka et al. protocol (1992)

This protocol is not receipt-free and it was not designed with receipt-freeness in mind.

→ the blinding factor $b_A$, the commitment key $r_A$, and the private key of the voter can be used as a receipt
Example: Fujioka et al. protocol (1992)

This protocol is not receipt-free and it was not designed with receipt-freeness in mind.

→ the blinding factor $b_A$, the commitment key $r_A$, and the private key of the voter can be used as a receipt

How can we ensure receipt-freeness?

1. reencryption mechanism
2. trapdoor commitment scheme
   → not always sufficient to ensure coercion-resistance
Receipt-freeness implies privacy

**Proposition**

If a voting protocol is receipt-free then it also respects privacy (for the same context $S$).
Receipt-freeness implies privacy

**Proposition**

If a voting protocol is **receipt-free** then it also respects **privacy** (for the same context $S$).

**Proof.** By hypothesis, there exists a process $V'$, such that

1. $V^{\text{out}(chc, \cdot)} \approx V_A^{a/v}$, and
2. $S[V_A^{c/v}^{chc} | V_B^{a/v}] \approx S[V' | V_B^{c/v}]$. 


Receipt-freeness implies privacy

Proposition

If a voting protocol is receipt-free then it also respects privacy (for the same context S).

Proof. By hypothesis, there exists a process $V'$, such that

- $V' \setminus \text{out}(chc, \cdot) \approx V_A\{a / v\}$, and
- $S[V_A\{c / v\}^{chc} \mid V_B\{a / v\}] \approx S[V' \mid V_B\{c / v\}]$.

Apply the evaluation context $\nu chc.( \_ \mid \text{!in}(chc, x))$ on both sides:

$S[V_A\{c / v\}^{chc} \mid V_B\{a / v\}]\setminus \text{out}(chc, \cdot) \approx S[V' \mid V_B\{c / v\}]\setminus \text{out}(chc, \cdot)$
Receipt-freeness implies privacy

**Proposition**

If a voting protocol is receipt-free then it also respects privacy (for the same context $S$).

**Proof.** By hypothesis, there exists a process $V'$, such that

- $V \setminus \text{out}(chc,\cdot) \approx V_A^{\{a/v\}}$, and
- $S[V_A^{\{c/v\}}^{chc} \mid V_B^{\{a/v\}}] \approx S[V' \mid V_B^{\{c/v\}}].$

Apply the evaluation context $\nu chc.(\_ \mid \text{in}(chc,x))$ on both sides:

$$S[V_A^{\{c/v\}}^{chc} \mid V_B^{\{a/v\}}] \setminus \text{out}(chc,\cdot) \approx S[V' \mid V_B^{\{c/v\}}] \setminus \text{out}(chc,\cdot)$$

Then, we show that we can push $\setminus \text{out}(chc,\cdot)$ inside:

$$S[(V_A^{\{c/v\}}^{chc}) \setminus \text{out}(chc,\cdot) \mid V_B^{\{a/v\}}] \approx S[V' \setminus \text{out}(chc,\cdot) \mid V_B^{\{c/v\}}]$$
Receipt-freeness implies privacy

**Proposition**

If a voting protocol is receipt-free then it also respects privacy (for the same context $S$).

**Proof.** By hypothesis, there exists a process $V'$, such that

- $V \setminus \text{out}(chc, \_ \_ ) \approx V_A\{^{a} / v\}$, and
- $S[V_A\{^c / v\}^{chc} \mid V_B\{^{a} / v\}] \approx S[V' \mid V_B\{^{c} / v\}]$.

Apply the evaluation context $\nu chc.( _ | \! \text{in}(chc, x))$ on both sides:

$$S[V_A\{^c / v\}^{chc} \mid V_B\{^{a} / v\}]\setminus \text{out}(chc, \_ \_ ) \approx S[V' \mid V_B\{^{c} / v\}]\setminus \text{out}(chc, \_ \_ )$$

Then, we show that we can push $\setminus \text{out}(chc, \_ \_ )$ inside:

$$S[\nu chc.( V_A\{^c / v\}^{chc}) \setminus \text{out}(chc, \_ \_ ) \mid V_B\{^{a} / v\}] \approx S[V'\setminus \text{out}(chc, \_ \_ ) \mid V_B\{^{c} / v\}]$$

Thus Privacy holds: $S[V_A\{^c / v\} \mid V_B\{^{a} / v\}] \approx S[V_A\{^{a} / v\} \mid V_B\{^{c} / v\}]$.
Outline

Formalising Coercion Resistance
Coercion-resistance (1)

Leaking secrets to the coencer $V^{c_1,c_2}$:
- the coencer will receive the message from the coerced voter $V$ on $c_2$;
- the coencer will give some prepared messages on $c_1$. 
Coercion-resistance (1)

Leaking secrets to the coercer $V^{c_1,c_2}$:

- the coercer will receive the message from the coerced voter $V$ on $c_2$;
- the coercer will give some prepared messages on $c_1$.

First approximation

There exists $V'$ such that

$$S[V_A\{?/v\}^{c_1,c_2} \mid V_B\{a/v\}] \approx S[V' \mid V_B\{c/v\}]$$
Coercion-resistance (1)

Leaking secrets to the coercer $V^{c_1,c_2}$:
- the coencer will receive the message from the coerced voter $V$ on $c_2$;
- the coencer will give some prepared messages on $c_1$.

First approximation

There exists $V'$ such that

$$S[V_A\{?/v\}^{c_1,c_2} | V_B\{a/v\}] \approx S[V' | V_B\{c/v\}] .$$

Problems:
- This assumes that the coencer will vote $c$.
- If the coencer votes $c' \neq c$, then the equivalence will not hold.
Coercion-resistance (2)

First approximation

There exists \( V' \) such that

\[
S[V_A\{?/v\}^{c_1,c_2} \mid V_B\{^a/v\}] \approx S[V' \mid V_B\{^c/v\}].
\]

To get rid of this problem, two possible solutions:

- add some conditions to ensure that the coercer will vote \( c \).
  → our approach (with Steve Kremer and Mark D. Ryan)
    (CSFW’06, Journal of Computer Security’09)

- allow the voter \( V_B \) to adapt his choice to counterbalance the vote
done by the coerced voter.
  → approach followed by Backes et al. (CSF’08). To achieve this,
they rely on an extractor process.
Verification of equivalence-based properties
How can we establish privacy in e-voting protocols?

→ we have to establish equivalence properties between processes

### Main difficulties

- quantification over all contexts,
- some specific features (anonymous channel, synchronisation phase, bulletin board)
- quite complexe cryptographic primitives
  → e.g. blind signatures, reencryption mechanism, …
How can we establish privacy in e-voting protocols?

we have to establish equivalence properties between processes

Main difficulties

- quantification over all contexts,
- some specific features (anonymous channel, synchronisation phase, bulletin board)
- quite complex cryptographic primitives
  e.g. blind signatures, reencryption mechanism, ...

Manual proofs are quite error-prone.

Existing automated tools designed for secrecy and authentication are not well-suited for verifying e-voting protocols.
Intuitively, static equivalence formalizes the idea that an attacker cannot distinguish two sequences of messages.
Static equivalence on frames - passive attacker

\[ \rightarrow \text{Intuitively, static equivalence formalizes the idea that an attacker cannot distinguish two sequences of messages} \]

Example: \( E = \{ \text{dec(enc}(x, y), y) = x \} \)

\[ \phi_1 = \text{yes, no, } k, \{\text{yes}\}_k \text{ and } \phi_2 = \text{yes, no } k, \{\text{no}\}_k \]

\[ \rightarrow \text{not statically equivalent, choose } M = \text{dec}(x_4, x_3) \text{ and } N = x_1 \]
Results on static equivalence

Decidability results:

- for the class of subterm convergent equational theories;
- for many theories involving an AC operator
  \[\rightarrow \text{ e.g. XOR, Abelian group, }\ldots\]
- for specific theories used in e-voting, e.g. blind signatures, trapdoor bit commitment, re-encryption, \ldots
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Combination result:
- for disjoint equational theories
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- for the class of subterm convergent equational theories;
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Combination result:

- for disjoint equational theories

Existing tools:

- YAPA - Yet Another Protocol Analyser
  http://www.lsv.ens-cachan.fr/~baudet/yapa/
- KiSs - Knowledge In Security protocolS
  http://www.lsv.ens-cachan.fr/~ciobaca/kiss/
The ProVerif tool (B. Blanchet)

http://www.proverif.ens.fr/

Input: processes written in applied pi calculus

Characteristics
- unbounded number of sessions
- primitives given by an equational theory
- security properties: (strong) secrecy, correspondence properties, equivalence properties
- sound but not complete
  \[\rightarrow\] sometimes, the tool reports some false attacks

Limitation
ProVerif tries to establish diff-equivalence (too strong).
Going beyond the ProVerif tool

Let \( P(x_1, x_2) = \text{out}(x_1); \text{synch}; \text{out}(x_2). \)

\[
P(a, b) \mid P(b, a) \approx P(a, a) \mid P(b, b).
\]

\[\rightarrow \text{ProVerif fails to establish this equivalence.}\]

To overcome this limitation \((\text{Joint with M. Ryan and B. Smyth}):\)

- we propose a \textit{transformation} to conclude in more cases;
- Then, using ProVerif on the resulting processes, we propose the \textit{first automated} proof of privacy for the FOO protocol.
Going beyond the ProVerif tool

Let $P(x_1, x_2) = \text{out}(x_1); \text{synch}; \text{out}(x_2)$.

$$P(a, b) \mid P(b, a) \approx P(a, a) \mid P(b, b).$$

$\rightarrow$ ProVerif fails to establish this equivalence.

To overcome this limitation (Joint with M. Ryan and B. Smyth):

- we propose a transformation to conclude in more cases;
- Then, using ProVerif on the resulting processes, we propose the first automated proof of privacy for the FOO protocol.

Still some limitations:

- some primitives can not be handled, e.g. reencryption, trapdoor bit commitment, ...
- unable in general to establish receipt-freeness properties.
Another approach – constraint solving

→ bounded number of sessions (i.e. processes without replication)
Another approach – constraint solving

\[
\rightarrow \text{bounded number of sessions (i.e. processes without replication)}
\]

**Step 1:** reduction to the problem of checking **symbolic equivalence** between constraint systems.

\[
\rightarrow \text{for simple processes} \quad \text{joint work with V. Cortier}
\]

\[
\rightarrow \text{for general processes} \quad \text{joint work with S. Kremer and M. Ryan}
\]

- this reduction is sound but not complete.
Another approach – constraint solving

→ bounded number of sessions (*i.e.* processes without replication)

**Step 1**: reduction to the problem of checking *symbolic equivalence* between constraint systems.

→ for simple processes

→ for general processes

joint work with V. Cortier

joint work with S. Kremer and M. Ryan

- this reduction is sound but not complete.

**Step 2**: decision procedures for *symbolic equivalence*

→ several procedures already exist for subterm convergent theories

→ we propose another one (for a specific set of primitives) together with an *efficient implementation* (ADECS tool)

joint work with V. Cheval and H. Comon-Lundh

http://www.lsv.ens-cachan.fr/~cheval/
Conclusion for privacy-type properties

Formalising properties in applied pi

- **Nice definitions.** The quantification on $V'$ in the receipt-freeness property should not be a problem.
- These definitions can be reused to model similar properties in other applications, *e.g.* privacy in Vehicular Ad-hoc NETwork, privacy-type properties in e-auction, ... .

Verification in applied pi (of equivalence-based properties)

- still an active research area;
- existing results and procedure are quite limited;
- **Challenge:** a verification tool that performs automated proofs of privacy-type properties in e-voting protocols.
Lecture 2: Formalisation and verification of security properties

Part II: Verifiability properties
(based on joint work with M. Ryan and B. Smyth)

Steve Kremer
Formalising Verifiability
Election verifiability

verifiability
verifiability
auditability
Election verifiability

end-to-end \{ verifiability auditability \}
Election verifiability

end-to-end : verifiability, auditability

- Election results can be fully verified by voters/observers
- The software provided by election authorities does not need to be trusted
- The software used to perform the verification can be sourced independently
Election verifiability

**Individual verifiability**
A voter can check her own vote is included in the tally.

**Universal verifiability**
Anyone can check that the declared outcome corresponds to the tally.

**Eligibility verifiability**
Anyone can check that only eligible votes are included in the declared outcome.

Remarks
- Verifiability $\neq$ correctness
- What system components need to be trusted in order to carry out these checks?
Election verifiability

We suppose that the protocol involves

- Voter credentials (typically, a public part and a private part for each voter)
- A bulletin board, on which are placed entries corresponding to voter’s outputs.

A protocol satisfies *election verifiability* if there are tests $\phi^{IV}$, $\phi^{UV}$ and $\phi^{EV}$ satisfying certain acceptability conditions.
Formalizing voting processes

Voting process specification: \( \langle V, A \rangle \) where
- \( V \) plain process without replication (the voter)
- \( A \) a closed evaluation context s.t. \( fv(V) = \{v\} \) (the admins)

Voting process

\[
VP_n(s_1, \ldots, s_n) = A[V^{s_1/v} | \cdots | V^{s_n/v}]
\]

models \( n \) voters casting votes for \( s_1, \ldots, s_n \)
Formalizing voting processes

Voting process specification: \( \langle V, A \rangle \) where

- \( V \): plain process without replication (the voter)
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Voting process

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\]

models \( n \) voters casting votes for \( s_1, \ldots, s_n \)

Voting on Satan’s computer

- Extend attacker model to software and hardware, i.e. \( V, A \) only represent the trusted parts of the protocol
- Ideally this is only the interaction between the voter and the terminal!
- In practice some parts need to be added, motivated by auditing parts, distributed authorities, …
Augmented voting process

We add to the applied pi calculus a $\text{rec}(r, t)$ construct: adds a special entry $\{t / r\}$ to frame not accessible to the attacker

the process $R(P)$ is like $P$ but replaces
- $\nu n$ by $\nu n.\text{rec}(r, n)$ for some fresh $r$;
- $\text{in}(c, x)$ by $\nu n.\text{rec}(r, x)$ for some fresh $r$.

Augmented voting process

$$\text{VP}_n^+(s_1, \ldots, s_n) = A[V_1^+ | \cdots | V_n^+]$$

where $V_i^+ = R(V)\{s_i / \nu\}\{r_i / r | r \in rv R(V)\}$
Example: a “raising hands” protocol

Idea: Voter simply outputs her signed vote.

Admin: generates and distributes keys via a private channel \( d \)

\[
A \triangleq \nu d. \nu skA. ( \langle \nu skv. \text{out}(d, skv). \text{out}(c, \text{sign}(skA, pk(skv))) \rangle \\
\{ \text{pk}(skA)/x_{pkA} \mid _- \})
\]

Voter: received private key and outputs signed vote

\[
V \triangleq \text{in}(d, x_{skv}). \text{out}(c, \langle \text{pk}(x_{skv}), \text{sign}(x_{skv}, v) \rangle)
\]
Verifiability tests

We require the existence of tests

\[ \phi^{IV}(v, w, \tilde{x}, y, \tilde{r}) \quad \phi^{UV}(\tilde{v}, \tilde{x}, \tilde{y}, \tilde{z}) \quad \phi^{EV}(\tilde{w}, \tilde{x}, \tilde{y}, \tilde{z}) \]

where

- \( v \) refers to the vote, \( \tilde{v} \) to the declared outcome
- \( w \) refers to the public cred., \( \tilde{w} \) to all voters’ public cred.
- \( \tilde{x} \) expected to refer to global election values
- \( y \) expected to refer to the voter’s ballot on the BB, \( \tilde{y} \) to all voters ballots
- \( \tilde{r} \) refer to the voter’s private data
- \( \tilde{z} \) expected to refer to outputs generated for UV and EV
A voting specification \( \langle V, A \rangle \) satisfies individual and universal verifiability if

\[ \exists \phi^{IV}, \phi^{UV} \text{ s.t.} \]

### Soundness.

\[ \forall C, B \text{ s.t. } C[VP_n^+(s_1, \ldots, s_n)] \xrightarrow{(\alpha)}^* B, \phi(B) \equiv \nu \tilde{n}.\sigma: \]

\[ \forall i, j. \quad \phi^{IV} \{s_i/v, \tilde{r}_i/\tilde{r}\}\sigma \land \phi^{IV} \{s_j/v, \tilde{r}_j/\tilde{r}\}\sigma \Rightarrow i = j \quad (1) \]

\[ \phi^{UV}\sigma \land \phi^{UV} \{\tilde{v}'/\tilde{v}\}\sigma \Rightarrow \tilde{v}\sigma \simeq \tilde{v}'\sigma \quad (2) \]

\[ \land_{1 \leq i \leq n} \phi^{IV} \{s_i/v, \tilde{r}_i/\tilde{r}, y_i/y\}\sigma \land \phi^{UV}\sigma \Rightarrow \tilde{s} \simeq \tilde{v}\sigma \quad (3) \]

### Effectiveness.

\[ \exists C, B \text{ s.t. } C[VP_n^+(s_1, \ldots, s_n)] \xrightarrow{(\alpha)}^* B, \phi(B) \equiv \nu \tilde{n}.\sigma: \]

\[ \land_{1 \leq i \leq n} \phi^{IV} \{s_i/v, \tilde{r}_i/\tilde{r}, y_i/y\}\{y_i/y\}\sigma \land \phi^{UV}\sigma \quad (4) \]
A voting specification $\langle V, A \rangle$ satisfies individual and universal verifiability if $\exists \phi^IV, \phi^UV$ s.t.

**Soundness.** $\forall C, B$ s.t. $C[VP_n^+(s_1, \ldots, s_n)] \xrightarrow{(\alpha)}^* B$, $\phi(B) \equiv \nu \tilde{n}.\sigma$:

$$\forall i, j. \quad \phi^IV \{s_i / v, \tilde{r}_i / \bar{r}\} \sigma \land \phi^IV \{s_j / v, \tilde{r}_j / \bar{r}\} \sigma \Rightarrow i = j \quad (1)$$

$$\phi^UV \sigma \land \phi^UV \{\tilde{v}' / \bar{v}\} \sigma \Rightarrow \tilde{v} \sigma \simeq \tilde{v}' \sigma \quad (2)$$

$$\land_{1 \leq i \leq n} \phi^IV \{s_i / v, \tilde{r}_i / \bar{r}, y_i / y\} \sigma \land \phi^UV \sigma \Rightarrow \tilde{s} \simeq \tilde{v} \sigma \quad (3)$$

**Effectiveness.** $\exists C, B$ s.t. $C[VP_n^+(s_1, \ldots, s_n)] \xrightarrow{(\alpha)}^* B$, $\phi(B) \equiv \nu \tilde{n}.\sigma$:

$$\land_{1 \leq i \leq n} \phi^IV \{s_i / v, \tilde{r}_i / \bar{r}, y_i / y\} \{y_i / y\} \sigma \land \phi^UV \sigma \quad (4)$$

**Intuition:** a same BB entry $y$ cannot validate $\phi^IV$ for two different voters
A voting specification $\langle V, A \rangle$ satisfies **individual and universal verifiability** if $\exists \phi^{IV}, \phi^{UV}$ s.t.

**Soundness.** $\forall C, B$ s.t. $C[VP_n^+(s_1, \ldots, s_n)] \xrightarrow{(\alpha)}* B$, $\phi(B) \equiv \nu \tilde{n}.\sigma$:

1. $\forall i, j. \quad \phi^{IV} \{s_i / v, \tilde{r}_i / \tilde{r}\} \sigma \land \phi^{IV} \{s_j / v, \tilde{r}_j / \tilde{r}\} \sigma \Rightarrow i = j$ \hspace{1cm} (1)
2. $\phi^{UV} \sigma \land \phi^{UV} \{\tilde{v}' / \tilde{v}\} \sigma \Rightarrow \tilde{v} \sigma \simeq \tilde{v}' \sigma$ \hspace{1cm} (2)
3. $\land_{1 \leq i \leq n} \phi^{IV} \{s_i / v, \tilde{r}_i / \tilde{r}, y_i / y\} \sigma \land \phi^{UV} \sigma \Rightarrow \tilde{s} \simeq \tilde{v} \sigma$ \hspace{1cm} (3)

**Effectiveness.** $\exists C, B$ s.t. $C[VP_n^+(s_1, \ldots, s_n)] \xrightarrow{(\alpha)}* B$, $\phi(B) \equiv \nu \tilde{n}.\sigma$:

$\land_{1 \leq i \leq n} \phi^{IV} \{s_i / v, \tilde{r}_i / \tilde{r}, y_i / y\} \{y_i / y\} \sigma \land \phi^{UV} \sigma$ \hspace{1cm} (4)

**Intuition:** for a same election $\phi^{UV}$ can only validate one outcome
Individual and universal verifiability

A voting specification \( \langle V, A \rangle \) satisfies **individual and universal verifiability** if \( \exists \phi^{IV}, \phi^{UV} \) s.t.

**Soundness.** \( \forall C, B \) s.t. \( C[VP_n^+(s_1, \ldots, s_n)] \xrightarrow{(\alpha)} B, \phi(B) \equiv \nu \tilde{n}.\sigma: \)

\[
\forall i, j. \quad \phi^{IV}\{s_i/v, \tilde{r}_i/\tilde{r}\}\sigma \land \phi^{IV}\{s_j/v, \tilde{r}_j/\tilde{r}\}\sigma \Rightarrow i = j \tag{1}
\]

\[
\phi^{UV}\sigma \land \phi^{UV}\{\tilde{v'/}\tilde{v}\}\sigma \Rightarrow \tilde{v}\sigma \simeq \tilde{v'}\sigma \tag{2}
\]

\[
\bigwedge_{1 \leq i \leq n} \phi^{IV}\{s_i/v, \tilde{r}_i/\tilde{r}, y_i/y\}\sigma \land \phi^{UV}\sigma \Rightarrow \tilde{s} \simeq \tilde{v}\sigma \tag{3}
\]

**Effectiveness.** \( \exists C, B \) s.t. \( C[VP_n^+(s_1, \ldots, s_n)] \xrightarrow{(\alpha)} B, \phi(B) \equiv \nu \tilde{n}.\sigma: \)

\[
\bigwedge_{1 \leq i \leq n} \phi^{IV}\{s_i/v, \tilde{r}_i/\tilde{r}, y_i/y\}\{y_i/y\}\sigma \land \phi^{UV}\sigma \tag{4}
\]

**Intuition:** if \( \phi^{IV} \)'s hold on votes \( s_1, \ldots, s_n \) then \( \phi^{UV} \) can only validate this particular outcome
Individual and universal verifiability

A voting specification $\langle V, A \rangle$ satisfies individual and universal verifiability if $\exists \phi^{IV}, \phi^{UV}$ s.t.

**Soundness.** $\forall C, B$ s.t. $C[VP^n_+(s_1, \ldots, s_n)] \xrightarrow{(\alpha)} B$, $\phi(B) \equiv \nu \tilde{n}.\sigma$:

1. $\forall i, j. \phi^{IV} \{s_i / v, \tilde{r}_i / \tilde{r}\} \sigma \land \phi^{IV} \{s_j / v, \tilde{r}_j / \tilde{r}\} \sigma \Rightarrow i = j$ (1)
2. $\phi^{UV} \sigma \land \phi^{UV} \{\tilde{v}' / \tilde{v}\} \sigma \Rightarrow \tilde{v} \sigma \simeq \tilde{v}' \sigma$ (2)
3. $\bigwedge_{1 \leq i \leq n} \phi^{IV} \{s_i / v, \tilde{r}_i / \tilde{r}, y_i / y\} \sigma \land \phi^{UV} \sigma \Rightarrow \tilde{s} \simeq \tilde{v} \sigma$ (3)

**Effectiveness.** $\exists C, B$ s.t. $C[VP^n_+(s_1, \ldots, s_n)] \xrightarrow{(\alpha)} B$, $\phi(B) \equiv \nu \tilde{n}.\sigma$:

4. $\bigwedge_{1 \leq i \leq n} \phi^{IV} \{s_i / v, \tilde{r}_i / \tilde{r}, y_i / y\} \{y_i / y\} \sigma \land \phi^{UV} \sigma$ (4)

Avoids vacuous tests where $\phi^{IV}, \phi^{UV}$ are false
Example: “raising hands” verifiability

The expected BB entry should be

$$\langle pk(skv), \text{sign}(skv, v) \rangle$$

Define the tests

$$\phi^{IV} \triangleq y =_E \langle pk(r_{skv}), \text{sign}(r_{skv}, v) \rangle \quad \phi^{UV} \triangleq \bigwedge_{1 \leq i \leq n} \text{getmsg}(\pi_2(y_i)) =_E v_i$$
Example: “raising hands” verifiability

The expected BB entry should be

\[ \langle \text{pk}(skv), \text{sign}(skv, v) \rangle \]

Define the tests

\[ \phi^IV \triangleq y = E \langle \text{pk}(r_{skv}), \text{sign}(r_{skv}, v) \rangle \quad \phi^UV \triangleq \bigwedge_{1 \leq i \leq n} \text{getmsg}(\pi_2(y_i)) = E v_i \]

Easy proof that individual and universal verifiability hold:

1. Suppose that \( \phi^IV_i \sigma \) and \( \phi^IV_j \sigma \) hold, i.e.,

\[ y\sigma = E \langle \text{pk}(r_{skv_i}\sigma), \text{sign}(r_{skv_i}\sigma, s_i) \rangle \quad y\sigma = E \langle \text{pk}(r_{skv_j}\sigma), \text{sign}(r_{skv_j}\sigma, s_j) \rangle \]

Hence, \( r_{skv_i}\sigma = E r_{skv_j}\sigma \). From the voting process spec. for every reachable \( \sigma \), \( i \neq j \) implies that \( r_{skv_i}\sigma \not= E r_{skv_j}\sigma \).

2,3 Immediate.

4) Holds for \( C = \_ \).
Example: FOO

What are the minimal parts of the protocol to be trusted?

The voting process specification

\[ V_{\text{foo}} \doteq \nu \text{rnd}.\text{out}(c, \nu).\text{out}(c, \text{rnd}) \quad \text{and} \quad A_{\text{foo}}[\_] \doteq \_ \]

where \( \text{rnd} \) is intended to be the randomness used for the commitment

The augmented voting process

\[ \text{VP}_n^+(s_1, \ldots, s_n) \doteq \nu \text{rnd}.\text{rec}(r_1, \text{rnd}).\text{out}(c, s_1).\text{out}(c, \text{rnd}) \mid \ldots \mid \nu \text{rnd}.\text{rec}(r_n, \text{rnd}).\text{out}(c, s_n).\text{out}(c, \text{rnd}) \]
Example: FOO

What are the minimal parts of the protocol to be trusted?

The voting process specification

\[ V_{\text{foo}} \triangleq \nu rnd.\text{out}(c, \nu).\text{out}(c, rnd) \quad \text{and} \quad A_{\text{foo}}[\_] \triangleq \_ \]

where \( rnd \) is intended to be the randomness used for the commitment

The augmented voting process

\[
VP_n^+(s_1, \ldots, s_n) = \nu \text{rnd.rec}(r_1, \text{rand}).\text{out}(c, s_1).\text{out}(c, \text{rand}) | \ldots | \nu \text{rnd.rec}(r_n, \text{rand}).\text{out}(c, s_n).\text{out}(c, \text{rand})
\]

Remark: Other properties need different trust assumptions!
Example: FOO

The expected BB entry should be

$$\langle r, \text{commit}(r, v) \rangle$$

Define the tests

$$\phi^{IV} \models y =_E \langle r, \text{commit}(r, v) \rangle \quad \phi^{UV} \models \bigwedge_{1 \leq i \leq n} v_i =_E \text{open}(\pi_1(y), \pi_2(y))$$

Theorem

$$\langle V_{foo}, A_{foo} \rangle$$ satisfies individual and universal verifiability.
A voting specification $\langle V, A \rangle$ satisfies election verifiability if
\[ \exists \phi^{IV}, \phi^{UV}, \phi^{EV} \text{ s.t. additionally} \]

Let \( X = f_v(\phi^{EV}) \setminus \text{dom} \text{VP}_n^+(s_1, \ldots, s_n) \)

**Soundness.** \( \forall C, B \text{ s.t. } C[\text{VP}_n^+(s_1, \ldots, s_n)] \xrightarrow{\alpha}^* B, \phi(B) \equiv \nu \tilde{n}.\sigma: \)

\[
\phi^{EV} \sigma \land \phi^{EV} \{x'/x \mid x \in X \setminus \tilde{y}\} \sigma \Rightarrow \tilde{w} \sigma \simeq \tilde{w}' \sigma \quad (5)
\]
\[
\bigwedge_{1 \leq i \leq n} \phi^{IV}_i \sigma \land \phi^{EV} \{\tilde{w}'/\tilde{w}\} \sigma \Rightarrow \tilde{w} \sigma \simeq \tilde{w}' \sigma \quad (6)
\]
\[
\phi^{EV} \sigma \land \phi^{EV} \{x'/x \mid x \in X \setminus \tilde{w}\} \sigma \Rightarrow \tilde{y} \sigma \simeq \tilde{y}' \sigma \quad (7)
\]

**Effectiveness.** \( \exists C, B \text{ s.t. } C[\text{VP}_n^+(s_1, \ldots, s_n)] \xrightarrow{\alpha}^* B, \phi(B) \equiv \nu \tilde{n}.\sigma: \)

\[
\bigwedge_{1 \leq i \leq n} \phi^{IV} \{s_i/v, \tilde{r}_i /\tilde{r}, y_i/y\} \{y_i/y\} \sigma \land \phi^{UV} \sigma \land \phi^{EV} \sigma \quad (8)
\]
Election verifiability

A voting specification $\langle V, A \rangle$ satisfies election verifiability if 
$\exists \phi^{IV}, \phi^{UV}, \phi^{EV}$ s.t. additionally

Let $X = \text{fv}(\phi^{EV}) \setminus \text{dom} \text{VP}_n^+(s_1, \ldots, s_n)$

**Soundness.** $\forall C, B$ s.t. $C[\text{VP}_n^+(s_1, \ldots, s_n)] \xrightarrow{(\alpha)}^* B$, $\phi(B) \equiv \nu \tilde{n}.\sigma$:

\[
\phi^{EV} \sigma \land \phi^{EV} \{x'/x \mid x \in X \setminus \tilde{y}\} \sigma \Rightarrow \tilde{w} \sigma \simeq \tilde{w}' \sigma \quad \text{(5)}
\]

\[
\bigwedge_{1 \leq i \leq n} \phi^{IV}_i \sigma \land \phi^{EV} \{\tilde{w}' / \tilde{w}\} \sigma \Rightarrow \tilde{w} \sigma \simeq \tilde{w}' \sigma \quad \text{(6)}
\]

\[
\phi^{EV} \sigma \land \phi^{EV} \{x'/x \mid x \in X \setminus \tilde{w}\} \sigma \Rightarrow \tilde{y} \sigma \simeq \tilde{y}' \sigma \quad \text{(7)}
\]

**Effectiveness.** $\exists C, B$ s.t. $C[\text{VP}_n^+(s_1, \ldots, s_n)] \xrightarrow{(\alpha)}^* B$, $\phi(B) \equiv \nu \tilde{n}.\sigma$:

\[
\bigwedge_{1 \leq i \leq n} \phi^{IV} \{s_i / v, \tilde{r}_i / \tilde{r}, y_i / y\} \{y_i / y\} \sigma \land \phi^{UV} \sigma \land \phi^{EV} \sigma \quad \text{(8)}
\]

**Intuition:** given ballots $\tilde{y} \sigma$, provided by the environment, $\phi^{EV}$ succeeds for a unique list of public credentials
Election verifiability

A voting specification $\langle V, A \rangle$ satisfies election verifiability if 
$\exists \phi^{IV}, \phi^{UV}, \phi^{EV}$ s.t. additionally 

Let $X = fV(\phi^{EV}) \setminus \text{dom}\ VP_n^+(s_1, \ldots, s_n)$

Soundness. $\forall C, B$ s.t. $C[VP_n^+(s_1, \ldots, s_n)] \xrightarrow{(\alpha)_\ast} B, \phi(B) \equiv \nu \tilde{n} \cdot \sigma$: 

\begin{align*}
\phi^{EV} & \sigma \land \phi^{EV} \{x' / x | x \in X \setminus \tilde{y}\} \sigma \Rightarrow \tilde{w} \sigma \simeq \tilde{w}' \sigma \quad (5) \\
\bigwedge_{1 \leq i \leq n} \phi_i^{IV} & \sigma \land \phi^{EV} \{\tilde{w}' / \tilde{w}\} \sigma \Rightarrow \tilde{w} \sigma \simeq \tilde{w}' \sigma \quad (6) \\
\phi^{EV} & \sigma \land \phi^{EV} \{x' / x | x \in X \setminus \tilde{w}\} \sigma \Rightarrow \tilde{y} \sigma \simeq \tilde{y}' \sigma \quad (7)
\end{align*}

Effectiveness. $\exists C, B$ s.t. $C[VP_n^+(s_1, \ldots, s_n)] \xrightarrow{(\alpha)_\ast} B, \phi(B) \equiv \nu \tilde{n} \cdot \sigma$: 

\begin{align*}
\bigwedge_{1 \leq i \leq n} \phi_i^{IV} & \{s_i / v, \tilde{r}_i / \tilde{r}, y_i / y\} \{y_i / y\} \sigma \land \phi^{UV} \sigma \land \phi^{EV} \sigma \quad (8)
\end{align*}

Intuition: if BB contains the ballots of voters with public cred. $\tilde{w} \sigma$ then $\phi^{EV}$ only holds on these credentials
Election verifiability

A voting specification \( \langle V, A \rangle \) satisfies election verifiability if
\[ \exists \phi^{IV}, \phi^{UV}, \phi^{EV} \text{ s.t. additionally} \]

Let \( X = fV(\phi^{EV}) \setminus \text{dom}\text{VP}_n^+(s_1, \ldots, s_n) \)

**Soundness.** \( \forall C, B \text{ s.t. } C[\text{VP}_n^+(s_1, \ldots, s_n)] \xrightarrow{(\alpha)}^* B, \phi(B) \equiv \nu\tilde{n}.\sigma: \)

\[
\begin{align*}
\phi^{EV} \sigma \land \phi^{EV} \{x' / x \mid x \in X \setminus \tilde{y}\} \sigma &\Rightarrow \tilde{w}\sigma \simeq \tilde{w}'\sigma \quad (5) \\
\bigwedge_{1 \leq i \leq n} \phi^{IV}_i \sigma \land \phi^{EV} \{\tilde{w}' / \tilde{w}\} \sigma &\Rightarrow \tilde{w}\sigma \simeq \tilde{w}'\sigma \quad (6) \\
\phi^{EV} \sigma \land \phi^{EV} \{x' / x \mid x \in X \setminus \tilde{w}\} \sigma &\Rightarrow \tilde{y}\sigma \simeq \tilde{y}'\sigma \quad (7)
\end{align*}
\]

**Effectiveness.** \( \exists C, B \text{ s.t. } C[\text{VP}_n^+(s_1, \ldots, s_n)] \xrightarrow{(\alpha)}^* B, \phi(B) \equiv \nu\tilde{n}.\sigma: \)

\[
\bigwedge_{1 \leq i \leq n} \phi^{IV} \{s_i / v, \tilde{r}_i / \tilde{r}, y_i / y\}\{y_i / y\} \sigma \land \phi^{UV} \sigma \land \phi^{EV} \sigma \quad (8)
\]

**Intuition:** given a set of credentials \(\tilde{w}\), only one set of BB entries \(\tilde{y}\) are accepted by \(\phi^{EV}\)
Election verifiability

A voting specification \( \langle V, A \rangle \) satisfies election verifiability if 
\[ \exists \phi^{IV}, \phi^{UV}, \phi^{EV} \text{ s.t. additionally} \]

Let \( X = f_v(\phi^{EV}) \setminus \text{dom}\ VP^+_n(s_1, \ldots, s_n) \)

Soundness. \( \forall C, B \text{ s.t. } C[VP^+_n(s_1, \ldots, s_n)] \xrightarrow{(\alpha)}^* B, \phi(B) \equiv \nu \tilde{n}.\sigma: \)

\[
\phi^{EV} \sigma \land \phi^{EV} \{x'/x \mid x \in X \setminus \tilde{y}\} \sigma \Rightarrow \tilde{w}\sigma \simeq \tilde{w}'\sigma \quad (5)
\]

\[
\bigwedge_{1 \leq i \leq n} \phi^{IV}_i \sigma \land \phi^{EV} \{\tilde{w}'/\tilde{w}\} \sigma \Rightarrow \tilde{w}\sigma \simeq \tilde{w}'\sigma \quad (6)
\]

\[
\phi^{EV} \sigma \land \phi^{EV} \{x'/x \mid x \in X \setminus \tilde{w}\} \sigma \Rightarrow \tilde{y}\sigma \simeq \tilde{y}'\sigma \quad (7)
\]

Effectiveness. \( \exists C, B \text{ s.t. } C[VP^+_n(s_1, \ldots, s_n)] \xrightarrow{(\alpha)}^* B, \phi(B) \equiv \nu \tilde{n}.\sigma: \)

\[
\bigwedge_{1 \leq i \leq n} \phi^{IV}_i \{s_i/\nu, \tilde{r}_i/\tilde{r}, \tilde{y}_i/y\}\{y_i/y\} \sigma \land \phi^{UV} \sigma \land \phi^{EV} \sigma \quad (8)
\]

Avoids vacuous tests where \( \phi^{IV}, \phi^{UV}, \phi^{EV} \) are false
Election verifiability may ensure the needed transparency for electronic voting to be acceptable.

Formal definition of election verifiability as tests with acceptability conditions (generally rather easy to prove)

We have analysed

- **FOO**: individual and universal verifiable, but not election verifiability
- **Helios 2.0**: individual and universal verifiable, but not election verifiability
- **JCJ/Civitas**: verifies election verifiability

Allows for each of the protocols to identify the trust assumptions

Detailed analysis available in [Kremer, Ryan, Smyth, ESORICS 2010]
http://www.bensmyth.com/publications/10tech/CSR-10-06.pdf