Verification of security protocols via constraint solving

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Cryptographic protocols

Cryptographic protocols

- small programs designed to secure communication
- use cryptographic primitives (*e.g.* encryption, hash function, . . .)

Security properties

Secrecy: May an intruder learn some secret message between two honest participants ?

Authentication: Is the agent Alice really talking to Bob ?

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Authentication: Is the agent Alice really talking to Bob ?

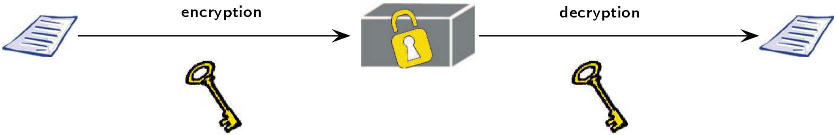
Privacy: Alice participate to an election. May a participant learn something about the vote of Alice ?

Receipt-Freeness: Alice participate to an election. Does Alice gain any information (a receipt) which can be used to prove to a coercer that she voted in a certain way ?

Fairness: ...

Cryptographic primitives

Symmetric encryption



Cryptographic primitives

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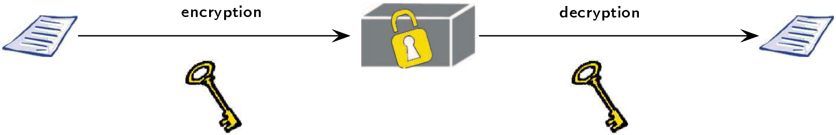
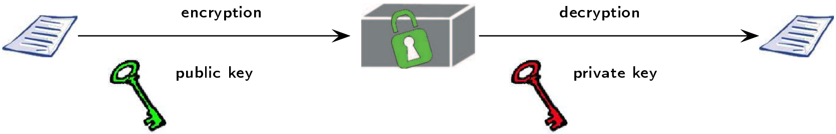


Asymmetric encryption



Verification of cryptographic protocols

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How cryptographic protocols can be attacked?

Breaking encryption

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Logical attack

Logical attack – What is it?

transfer 100 euros on merchant's bank account ⟶

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transfer 100 euros on
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$\longrightarrow$

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transfer 100 euros on
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$\longrightarrow$

$\vdots$

transfer 100 euros on
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$\longrightarrow$

Credit Card Payment Protocol

Example: credit card payment

- The client Cl puts his credit card C in the terminal T.
- The merchant enters the amount M of the sale.
- The terminal authenticates the credit card.
- The client enters his PIN.
  If $M \geq$ €100, then in 20% of cases,
  - The terminal contacts the bank B.
  - The banks gives its authorisation.

More details

the Bank B , the Client Cl, the Credit Card C and the Terminal T

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- a private signature key – $\mathrm{priv}(B)$
- a public key to verify a signature – $\mathrm{pub}(B)$
- a secret key shared with the credit card – K_{CB}

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Credit Card

- some *Data*: name of the cardholder, expiry date ...
- a signature of the *Data* – $\mathsf{sign}(Data, \mathsf{priv}(B))$
- a secret key shared with the bank – K_{CB}

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Terminal

- the public key of the bank – $\text{pub}(B)$

Payment protocol

the terminal T reads the credit card C:

$$1. \quad C \quad \rightarrow \quad T: \quad Data, \mathsf{sign}(Data, \mathsf{priv}(B))$$

Payment protocol

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the terminal T asks the code:

$$\begin{array}{llcll} 2. & T & \rightarrow & Cl : & code? \\ 3. & Cl & \rightarrow & C : & 1234 \\ 4. & C & \rightarrow & T : & ok \end{array}$$

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$$\begin{array}{lllll} 2. & T & \rightarrow & CI: & code? \\ 3. & CI & \rightarrow & C: & 1234 \\ 4. & C & \rightarrow & T: & ok \end{array}$$

the terminal T requests authorisation the bank B:

$$\begin{array}{lllll} 5. & T & \rightarrow & B: & auth? \\ 6. & B & \rightarrow & T: & 4528965874123 \\ 7. & T & \rightarrow & C: & 4528965874123 \\ 8. & C & \rightarrow & T: & \mathsf{enc}(4528965874123, K_{CB}) \\ 9. & T & \rightarrow & B: & \mathsf{enc}(4528965874123, K_{CB}) \\ 10. & B & \rightarrow & T: & ok \end{array}$$

Attacks on the credit card

Security was initially ensured by:

- the cards were difficult to reproduce,
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Security was initially ensured by:

- the cards were difficult to reproduce,
- the protocol and the keys were secret.

But there are some flaws:

- cryptographic flaw: keys of 320 bits are too small,
- logical flaw: no link between the secret code and the authentication of the card;
- fake cards can be easily build.

⟶ "YesCard" built by Serge Humpich (1997).

YesCard: How does it work?

Logical Flaw:

$$
\begin{array}{llll}
1.C & \rightarrow T & : \text{Data}, \text{sign}(\text{Data}, \text{priv}(B)) \\
2.T & \rightarrow CI & : code? \\
3.CI & \rightarrow C & : 1234 \\
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$$

YesCard: How does it work?

Logical Flaw:

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$\longrightarrow$ Note that there is someone to debit.

YesCard: How does it work?

Logical Flaw:

$$\begin{array}{llll} 1.C & \to T & : \text{Data}, \text{sign}(\text{Data}, \text{priv}(B)) \\ 2.T & \to CI & : code? \\ 3.CI & \to C' & : 0000 \\ 4.C' & \to T & : ok \end{array}$$

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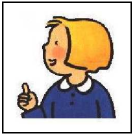
YesCard (by Serge Humpich)

$$\begin{array}{llll} 1.C' & \to T & : \text{XXX}, \text{sign}(\text{XXX}, \text{priv}(B)) \\ 2.T & \to CI & : code? \\ 3.CI & \to C' & : 0000 \\ 4.C' & \to T & : ok \end{array}$$

Needham-Schroeder's protocol

Needham-Schroeder's Protocol (1978)



- $$\begin{array}{rcrr} A & \rightarrow & B: & \{A, N_a\}_{\mathsf{pub}(B)} \\ B & \rightarrow & A: & \{N_a, N_b\}_{\mathsf{pub}(A)} \\ A & \rightarrow & B: & \{N_b\}_{\mathsf{pub}(B)} \end{array}$$

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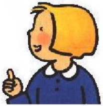

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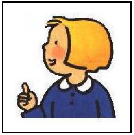

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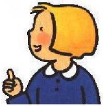

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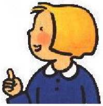

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- Is N_b secret between A and B ?
- When B receives $\{N_b\}_{\mathsf{pub}(B)}$, does this message really comes from A ?

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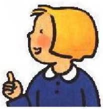

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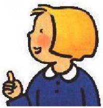
Attack

An attack was found 17 years after its publication! [Lowe 96]

Example: Man in the Middle Attack



Agent A

Intrus I

Agent B

Attack

- involving 2 sessions in parallel,
- an honest agent has to initiate a session with I.

$$
\begin{aligned}
A \rightarrow B &: \{A, N_a\}_{\mathsf{pub}(B)} \\
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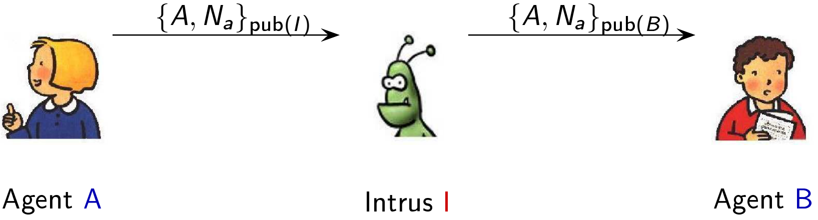

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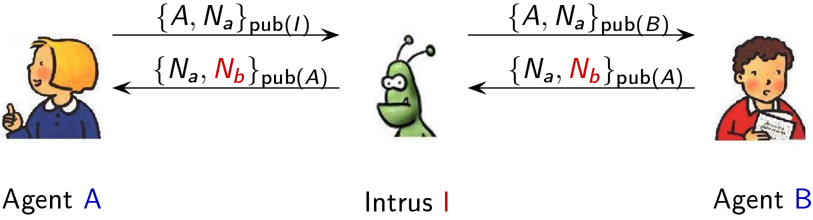

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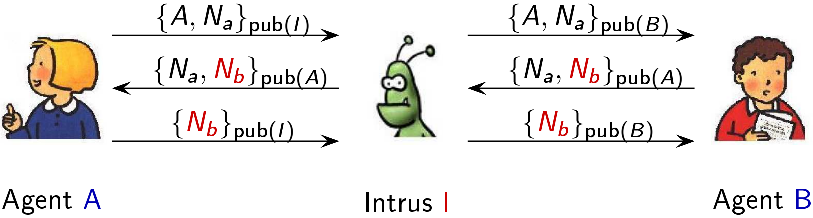

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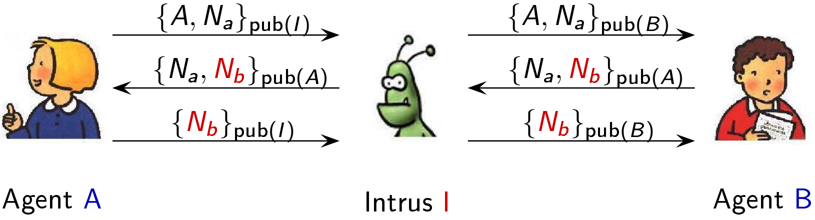

Attack

- the intruder knows N_b,
- When B finishes his session (apparently with A), A has never talked with B.

$$
\begin{array}{lll}
A \rightarrow B & : & \{A, N_a\}_{pub(B)} \\
B \rightarrow A & : & \{N_a, N_b\}_{pub(A)} \\
A \rightarrow B & : & \{N_b\}_{pub(B)}
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Logical attacks - How to detect them?

Symbolic approach

- messages are represented by terms rather than bit-strings
  $\hookrightarrow \{m\}_k$ encryption of the message m with key k,
  $\hookrightarrow \langle m_1, m_2 \rangle$ pairing of messages m_1 and m_2, ...
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Relevance of the approach

- numerous attacks have already been obtained,
- allows us to perform automatic verification, *e.g.* AVISPA, Proverif, ...
- soundness results already exist, *e.g.* [Micciancio & Warinschi'04]

Outline of the talk

1. Introduction

2. How to deal with trace properties (e.g. secrecy, authentication, ...)?

3. How to deal with equivalence based properties (e.g. privacy, ...)?

4. Conclusion

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Deduction capabilities of the attacker

Composition rules

$$\frac{T \vdash u \quad T \vdash v}{T \vdash \langle u, v \rangle} \qquad \frac{T \vdash u \quad T \vdash v}{T \vdash \mathsf{f}(u, v)} \ \textit{with } \mathsf{f} \in \{\mathsf{enc}, \mathsf{enca}, \mathsf{sign}\}$$

Decomposition rules

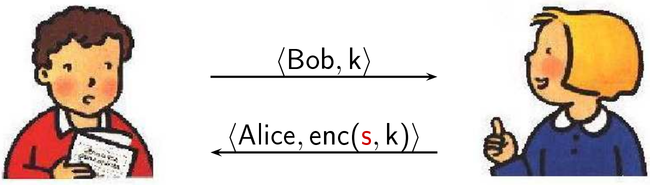
$$\frac{}{T \vdash u}\ u \in T \qquad \frac{T \vdash \langle u, v \rangle}{T \vdash u} \qquad \frac{T \vdash \langle u, v \rangle}{T \vdash v} \qquad \frac{T \vdash \mathsf{enc}(u, v) \quad T \vdash v}{T \vdash u}$$

$$\frac{T \vdash \mathsf{enca}(u, \mathsf{pub}(v)) \quad T \vdash \mathsf{priv}(v)}{T \vdash u} \qquad \frac{T \vdash \mathsf{sign}(u, \mathsf{priv}(v))}{T \vdash u}\ \textit{(optional)}$$

Deducibility relation

A term u is deducible from a set of terms T, denoted by $T \vdash u$, if there exists a prooftree witnessing this fact.

A simple protocol



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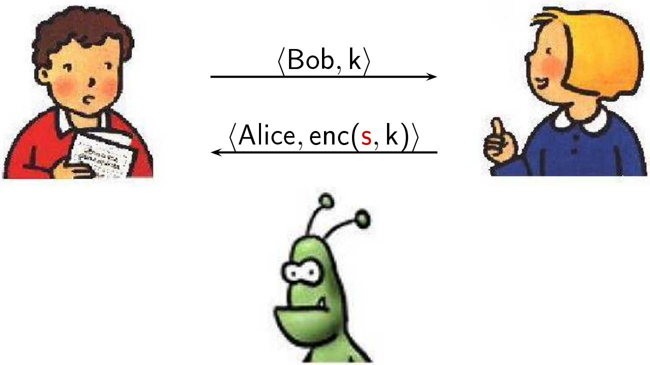


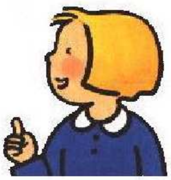
Question?

Can the attacker learn the secret s?

A simple protocol

$$\xrightarrow{\langle \mathsf{Bob}, \mathsf{k} \rangle}$$

$$\xleftarrow{\langle \mathsf{Alice}, \mathsf{enc}(\mathsf{s}, \mathsf{k}) \rangle}$$



Answer: Of course, Yes!

$$\frac{\dfrac{\langle \mathsf{Alice}, \mathsf{enc}(\mathsf{s}, \mathsf{k}) \rangle}{\mathsf{enc}(\mathsf{s}, \mathsf{k})} \qquad \dfrac{\langle \mathsf{Bob}, \mathsf{k} \rangle}{\mathsf{k}}}{\mathsf{s}}$$

Deducibility problem - Some existing results

$\longrightarrow$ depends on the deduction capabilities of the intruder

Dolev-Yao intruder

The deducibility problem is decidable in polynomial time.

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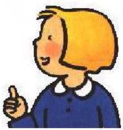
Taking into account algebraic properties of the cryptographic primitives (*e.g.* RSA encrytpion)

$$\mathsf{E} := \left\{ \begin{array}{rcl} \mathsf{dec}(\mathsf{enc}(x, \mathsf{pub}(y)), \mathsf{priv}(y)) & = & x \\ \mathsf{enc}(\mathsf{dec}(x, \mathsf{priv}(y)), \mathsf{pub}(y)) & = & x \end{array} \right.$$

$$\frac{T \vdash m \quad T \vdash k}{T \vdash \mathsf{f}(m, k)} \ \mathsf{f} \in \{\mathsf{dec}, \mathsf{enc}\} \qquad \frac{T \vdash m_1}{T \vdash m_2} \ m_1 =_{\mathsf{E}} m_2$$

Needham Schroeder protocol:



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Protocol – Example: Needham Schroeder protocol (1978)

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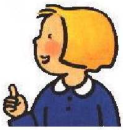

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A protocol is a finite set of roles:

Exemple:

role $\Pi(1)$ corresponding to the 1st participant played by a talking to b:

$$\begin{array}{l} \mathsf{init} \xrightarrow{N} \mathsf{enca}(\langle N, a\rangle, \mathsf{pub}(b)) \\ \mathsf{enca}(\langle N, x\rangle, \mathsf{pub}(a)) \rightarrow \mathsf{enca}(x, \mathsf{pub}(b)). \end{array}$$

Trace properties in presence of an active attacker

Insecurity problem (bounded number of sessions)

Let $\mathcal{I}$ be an inference system modelling the attacker.

INPUT: a finite set $R_1, \ldots, R_m$ of instances of roles,
a finite set T_0 of terms (initial intruder knowledge),
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OUTPUT: Does there exist an interleaving of $R_1, \ldots, R_m$ runnable from T_0 w.r.t. $\mathcal{I}$ at the end of which

- the intruder knowledge is T, and
- s is deducible from T in $\mathcal{I}$?

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Security properties (trace properties): *e.g.* secrecy, some kinds of authentication properties, ...

Secrecy via constraint system

Constraint systems are used to specify secrecy preservation under a particular, finite scenario.

Scenario

$$
\begin{array}{rcl}
\mathrm{rcv}(u_1) & \xrightarrow{N_1} & \mathrm{snd}(v_1) \\
\mathrm{rcv}(u_2) & \xrightarrow{N_2} & \mathrm{snd}(v_2) \\
 & \ldots & \\
\mathrm{rcv}(u_n) & \xrightarrow{N_n} & \mathrm{snd}(v_n)
\end{array}
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Constraint System

$$\mathcal{C} = \left\{ \begin{array}{l} T_0 \Vdash u_1 \\ T_0, v_1 \Vdash u_2 \\ \quad \ldots \\ T_0, v_1, .., v_n \Vdash s \end{array} \right.$$

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Constraint System

$$\mathcal{C} = \begin{cases} T_0 \Vdash u_1 \\ T_0, v_1 \Vdash u_2 \\ \quad \ldots \\ T_0, v_1, .., v_n \Vdash s \end{cases}$$

Solution of a constraint system

A substitution σ such that

for every $T \Vdash u \in \mathcal{C}$, $u\sigma$ *is deducible from* $T\sigma$.

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Well-formed constraint system

- monotonicity: intruder never forgets information
- origination: a variable first appear in a right hand side.

Running example: Needham-Schroeder's protocol

$R_A(a, I)$ and $R_B(b)$ (running in parallel)

$$\begin{array}{rcl} \text{init} & \rightarrow & \{a, n_a\}_{\mathrm{pub}(I)} \\ \{n_a, x_{n_b}\}_{\mathrm{pub}(a)} & \rightarrow & \{x_{n_b}\}_{\mathrm{pub}(I)} \\ & & \\ \{y_a, y_{n_a}\}_{\mathrm{pub}(b)} & \rightarrow & \{y_{n_a}, n_b\}_{\mathrm{pub}(y_a)} \end{array}$$

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Constraints System

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Constraints System

$$T_0,\ \{a, n_a\}_{\mathrm{pub}(I)}$$

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Constraints System

$$T_0,\ \{a, n_a\}_{\mathrm{pub}(I)} \ \Vdash\ \{y_a, y_{n_a}\}_{\mathrm{pub}(b)}$$

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 & & & \\
\mathbf{2} & \{y_a, y_{n_a}\}_{\text{pub}(b)} & \rightarrow & \{y_{n_a}, n_b\}_{\text{pub}(y_a)}
\end{array}
$$

Constraints System

$$
\begin{array}{l}
T_0, \ \{a, n_a\}_{\text{pub}(I)} \ \Vdash \ \{y_a, y_{n_a}\}_{\text{pub}(b)} \\
T_0, \ \{a, n_a\}_{\text{pub}(I)}, \ \{y_{n_a}, n_b\}_{\text{pub}(y_a)}
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Constraints System

$$
\begin{array}{l}
T_0,\ \{a, n_a\}_{\mathsf{pub}(I)} \Vdash \{y_a, y_{n_a}\}_{\mathsf{pub}(b)} \\
T_0,\ \{a, n_a\}_{\mathsf{pub}(I)},\ \{y_{n_a}, n_b\}_{\mathsf{pub}(y_a)} \Vdash \{n_a, x_{n_b}\}_{\mathsf{pub}(a)}
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Constraints System

$$\begin{array}{l} T_0,\ \{a, n_a\}_{\text{pub}(I)} \Vdash \{y_a, y_{n_a}\}_{\text{pub}(b)} \\ T_0,\ \{a, n_a\}_{\text{pub}(I)},\ \{y_{n_a}, n_b\}_{\text{pub}(y_a)} \Vdash \{n_a, x_{n_b}\}_{\text{pub}(a)} \\ T_0,\ \{a, n_a\}_{\text{pub}(I)},\ \{y_{n_a}, n_b\}_{\text{pub}(y_a)},\ \{x_{n_b}\}_{\text{pub}(I)} \end{array}$$

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T_0,\ \{a, n_a\}_{\mathsf{pub}(I)} \Vdash \{y_a, y_{n_a}\}_{\mathsf{pub}(b)} \\
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\end{array}
$$

Solution $\quad \sigma = \{y_a \mapsto \quad , \quad y_{n_a} \mapsto \quad , \quad x_{n_b} \mapsto \quad \}$

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Constraints System

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$$T_0,\ \{a, n_a\}_{\mathrm{pub}(I)},\ \{y_{n_a}, n_b\}_{\mathrm{pub}(y_a)},\ \{x_{n_b}\}_{\mathrm{pub}(I)} \Vdash n_b$$

Solution $\quad \sigma = \{y_a \mapsto a,\ \ y_{n_a} \mapsto n_a,\ \ x_{n_b} \mapsto \quad \}$

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\end{array}$$

Constraints System

$$\begin{array}{l}
T_0,\ \{a, n_a\}_{\text{pub}(I)} \Vdash \{y_a, y_{n_a}\}_{\text{pub}(b)} \\
T_0,\ \{a, n_a\}_{\text{pub}(I)},\ \{y_{n_a}, n_b\}_{\text{pub}(y_a)} \Vdash \{n_a, x_{n_b}\}_{\text{pub}(a)} \\
T_0,\ \{a, n_a\}_{\text{pub}(I)},\ \{y_{n_a}, n_b\}_{\text{pub}(y_a)},\ \{x_{n_b}\}_{\text{pub}(I)} \Vdash n_b
\end{array}$$

Solution $\sigma = \{y_a \mapsto a,\ \ y_{n_a} \mapsto n_a,\ \ x_{n_b} \mapsto n_b\}$

Some existing results

Many theoretical results for different intruder models

- to take into account algebraic properties of cryptographic primitives (exclusive or, cipher block chaining, ...)
- to take into account the fact that some data are poorly-chosen (*e.g.* passwords)

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Few generic results

- procedure to solve constraint systems for a class of intruder
  $\hookrightarrow$ *e.g.* any intruder who can be described by a subterm convergent rewiting system
- combination result for disjoint intruder models.

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- combination result for disjoint intruder models.

Some tools

- AVISPA tool (Atse, OFMC)

Outline of the talk

Motivation: Electronic voting

Advantages:

- Convenient,
- Efficient facilities for tallying votes.

Drawbacks:

- Risk of large-scale and undetectable fraud,
- Such protocols are extremely error-prone.

> *"A 15-year-old in a garage could manufacture smart cards and sell them on the Internet that would allow for multiple votes"* Avi Rubin

Possible issue: formal methods

abstract analysis of the protocol against formally-stated properties

Expected properties

Privacy: the fact that a particular voter voted in a particular way is not revealed to anyone

Receipt-freeness: a voter cannot prove that she voted in a certain way (this is important to protect voters from coercion)

Coercion-resistance: same as receipt-freeness, but the coercer interacts with the voter during the protocol, e.g. by preparing messages

How to model such security properties?

Formalisation of Privacy
$\hookrightarrow$ consider 2 honest voters and swap their votes

Privacy

A voting protocol respects privacy if

$$S[V_A\{^a/_v\} \mid V_B\{^b/_v\}] \approx S[V_A\{^b/_v\} \mid V_B\{^a/_v\}].$$

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Formalisation of Receipt-freeness and Coercion-resistance in term of equivalence.

Constraint solving

In terms of constraint system, the main ingredient to decide $\approx$:

$\mathcal{C}_1 \sim \mathcal{C}_2$: equivalence of (well-formed) constraint systems

What does it mean?

1. this does not mean that $\mathcal{C}_1$ and $\mathcal{C}_2$ have the same set of (first-order) solutions.
2. Given a solution σ, let $\Lambda_\sigma = \{\lambda_\sigma^1, \ldots, \lambda_\sigma^k\}$ be the witnesses of the fact that σ is a solution of

$$\mathcal{C} := \begin{cases} T_1 \Vdash u_1 \\ \vdots \\ T_\ell \Vdash u_\ell \end{cases}$$

$$\mathcal{C}_1 \sim \mathcal{C}_2 \quad \text{iff} \quad \{\Lambda_\sigma \mid \sigma \in Sol(\mathcal{C}_1)\} = \{\Lambda_\sigma \mid \sigma \in Sol(\mathcal{C}_2)\}.$$

Existing results

A lot of results in the passive case

- to take into account algebraic properties (exclusive or, . . .)
- combination result for disjoint equational theories,
- YAPA tool that works for subterm convergent theories and more

Active case: very few results

- decision procedure for subterm convergent theories (not implemented)
- ProVerif tool

Ongoing work

Motivation: verification of privacy type proeprties in e-voting protocols

Passive case:
$\longrightarrow$ to deal with more complex cryptographic primitives, those that are frequently used in e-voting protocols

- blind signature (already done in the passive case)
- trapdoor bit commitment
- reencryption mechanism

Active case:
design a procedure to decide equivalence of constraint systems in presence of blind signature.

$\longrightarrow$ this will allow us to decide privacy in e-voting protocols, e.g. protocol due to Fujioka, Okamoto and Ohta.

Conclusion

Verification via constraint solving
$\longrightarrow$ a useful approach to verify security protocols

- can be adapted to other cryptographic primitives;
- useful for trace properties but also equivalence based properties;
- can be adapted to deal with regular constraints, e.g. $u \in L$;
- limits: only a bounded number of sessions

Conclusion

Verification via constraint solving
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Questions?