Security via constraint solving

Stéphanie Delaune

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Cryptographic protocols

Cryptographic protocols

- small programs designed to secure communication
- use cryptographic primitives (*e.g.* encryption, hash function, . . .)

Goals

- Secrecy: May an intruder learn some secret message between two honest participants ?
- Authentication: Is the agent Alice really talking to Bob ?

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- Secrecy: May an intruder learn some secret message between two honest participants ?
- Authentication: Is the agent Alice really talking to Bob ?
- Fairness: Alice and Bob want to sign a contract. Alice initiates the protocol. May Bob obtain some advantage ?
- Privacy: Alice participate to an election. May a participant learn something about the vote of Alice ?
- Receipt-Freeness: Alice participate to an election. Does Alice gain any information (a receipt) which can be used to prove to a coercer that she voted in a certain way ?
- ...

Needham-Schroeder's Protocol (1978)

$$\begin{array}{rcrl} A & \rightarrow & B: & \{A, N_a\}_{\text{pub}(B)} \\ B & \rightarrow & A: & \{N_a, N_b\}_{\text{pub}(A)} \\ A & \rightarrow & B: & \{N_b\}_{\text{pub}(B)} \end{array}$$

Needham-Schroeder's Protocol (1978)

$$A \to B : \{A, N_a\}_{\mathsf{pub}(B)}$$

- $$B \to A : \{N_a, N_b\}_{\mathsf{pub}(A)}$$

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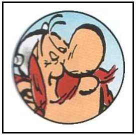

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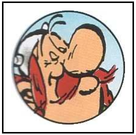

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Questions

- Is N_b secret between A and B ?
- When B receives $\{N_b\}_{\mathrm{pub}(B)}$, does this message really comes from A ?

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Attack

An attack was found 17 years after its publication! [Lowe 96]

How cryptographic protocols can be attacked?

Breaking encryption

Verification of cryptographic protocols

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Breaking encryption

Logical attack

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Logical attacks

- can be mounted even assuming perfect cryptography,
  ↪ replay attack, man-in-the middle attack, . . .
- are numerous, see SPORE, Security Protocols Open REpository
  ↪ `http://www.lsv.ens-cachan.fr/spore/`
- subtle and hard to detect by “eyeballing” the protocol

Example: Man in the Middle Attack

Agent A

Intrus I

Agent B

Attack

- involving 2 sessions in parallel,
- an honest agent has to initiate a session with I.

$$\begin{array}{lcl} A \rightarrow B & : & \{A, N_a\}_{\mathsf{pub}(B)} \\ B \rightarrow A & : & \{N_a, N_b\}_{\mathsf{pub}(A)} \\ A \rightarrow B & : & \{N_b\}_{\mathsf{pub}(B)} \end{array}$$

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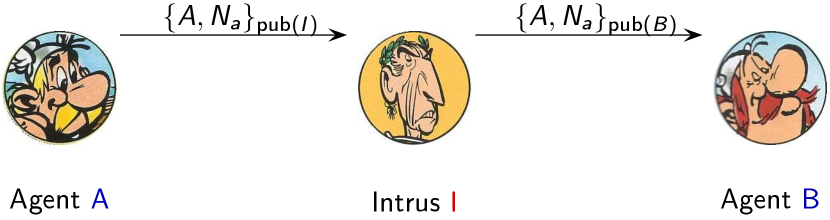

$$
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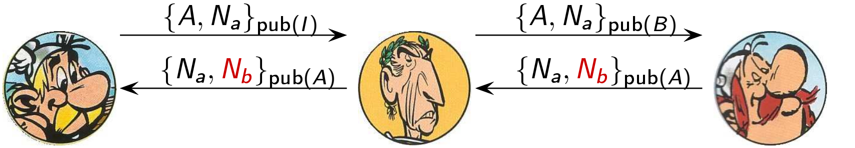

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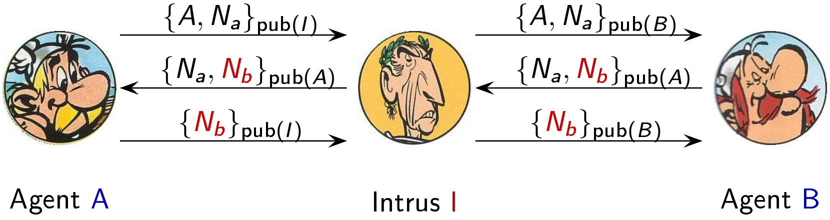

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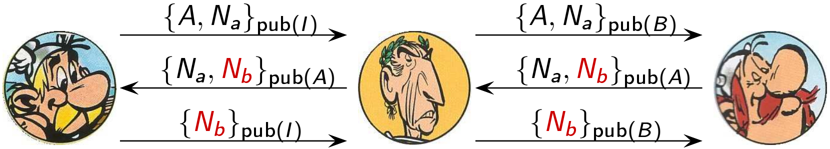

Agent A | Intrus I | Agent B

Attack

- the intruder knows N_b,
- When B finishes his session (apparently with A), A has never talked with B.

$$A \rightarrow B \quad : \{A, N_a\}_{pub(B)}$$
$$B \rightarrow A \quad : \{N_a, N_b\}_{pub(A)}$$
$$A \rightarrow B \quad : \{N_b\}_{pub(B)}$$

Logical attacks - How to detect them?

Symbolic approach

- messages are represented by terms rather than bit-strings
  $\hookrightarrow$ $\{m\}_k$ encryption of the message m with key k,
  $\hookrightarrow$ $\langle m_1, m_2 \rangle$ pairing of messages m_1 and m_2, ...
- attacker controls the network and can perform specific actions

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Relevance of the approach

- numerous attacks have already been obtained,
- allows us to perform automatic verification, *e.g.* AVISPA, Proverif, ...
- soundness results already exist, *e.g.* [Micciancio & Warinschi'04]

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Presence of an attacker ...

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Security problem for an unbounded number of sessions is undecidable.

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Security problem for a fixed number of sessions is decidable.

Outline of the talk

1. Introduction
2. How to deal with trace properties? (e.g. secrecy, authentication)
3. Work in progress: Equivalence based security properties (e.g. anonymity)

Outline of the talk

Dolev-Yao Intruder Model

m_1, m_2 and A are messages (terms)
T a finite set of messages (intruder's knowledge)

Ax. (A) $$\frac{}{T \vdash m_1} \quad m_1 \in T$$

Pair (P) $$\frac{T \vdash m_1 \quad T \vdash m_2}{T \vdash \langle m_1, m_2 \rangle}$$

Enc. (E) $$\frac{T \vdash m_1 \quad T \vdash \mathsf{pub}(A)}{T \vdash \{m_1\}_{\mathsf{pub}(A)}}$$

Proj. (Prj$_2$) $$\frac{T \vdash \langle m_1, m_2 \rangle}{T \vdash m_2}$$

Dec. (D) $$\frac{T \vdash \{m_1\}_{\mathsf{pub}(A)} \quad T \vdash \mathsf{priv}(A)}{T \vdash m_1}$$

Proj. (Prj$_1$) $$\frac{T \vdash \langle m_1, m_2 \rangle}{T \vdash m_1}$$

Deducibility problem

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OUTPUT: Does there exist a proof of $T \vdash s$?

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$$\dfrac{\dfrac{s_1 \in T}{T \vdash s_1}\,(A) \qquad \dfrac{\dfrac{\{s_2\}_k \in T}{T \vdash \{s_2\}_k}\,(A) \quad \dfrac{k \in T}{T \vdash k}\,(A)}{T \vdash s_2}\,(D)}{T \vdash \langle s_1, s_2 \rangle}\,(P)$$

Deducibility problem - Some existing results

$\longrightarrow$ depends on the deduction capabilities of the intruder

Dolev-Yao intruder

The deducibility problem is decidable in polynomial time.

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Prefix Intruder (*e.g.* Cipher Block Chaining)

$$\frac{T \vdash \{\langle m_1, m_2 \rangle\}_{\mathsf{pub}(A)}}{T \vdash \{m_1\}_{\mathsf{pub}(A)}}$$

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Taking into account algebraic properties of the cryptographic primitives (*e.g.* RSA encrytpion)

$$\mathsf{E} := \begin{cases} \mathsf{dec}(\mathsf{enc}(x, \mathsf{pub}(y)), \mathsf{priv}(y)) & = & x \\ \mathsf{enc}(\mathsf{dec}(x, \mathsf{priv}(y)), \mathsf{pub}(y)) & = & x \end{cases}$$

$$\frac{T \vdash m \quad T \vdash k}{T \vdash \mathsf{f}(m, k)} \; \mathsf{f} \in \{\mathsf{dec}, \mathsf{enc}\} \qquad \frac{T \vdash m_1}{T \vdash m_2} \; m_1 =_{\mathsf{E}} m_2$$

In presence of an active attacker

Insecurity problem (bounded number of sessions)

Let $\mathcal{I}$ be an inference system modelling the attacker.

INPUT: a finite set $R_1, \ldots, R_m$ of instances of roles,
a finite set T_0 of terms (initial intruder knowledge),
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- the intruder knowledge is T, and
- s is deducible from T in $\mathcal{I}$?

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Security properties (trace properties): *e.g.* secrecy, some kinds of authentication properties, . . .

Running example: Needham-Schroeder's protocol

$$
\begin{array}{lcl}
A \rightarrow B & : & \{A, N_a\}_{\mathsf{pub}(B)} \\
B \rightarrow A & : & \{N_a, N_b\}_{\mathsf{pub}(A)} \\
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\end{array}
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Roles composing the protocol

$$
\begin{array}{lcl}
R_A(x_a, x_b) & : & \nu n_a.\ \mathsf{out}(\{x_a, n_a\}_{\mathsf{pub}(x_b)}); \\
& & \quad \mathsf{in}(\{n_a, x_{n_b}\}_{\mathsf{pub}(x_a)}); \mathsf{out}(\{x_{n_b}\}_{\mathsf{pub}(x_b)}) \\
& & \\
R_B(y_b) & : & \nu n_b.\ \mathsf{in}(\{y_a, y_{n_a}\}_{\mathsf{pub}(y_b)}); \mathsf{out}(\{y_{n_a}, n_b\}_{\mathsf{pub}(y_a)})
\end{array}
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To retrieve the well-known man-in-the-middle attack, we consider

- $R_A(a, I)$ and $R_B(b)$ (running in parallel).
- $T_0 = \{a, b, I, \mathrm{pub}(a), \mathrm{pub}(b), \mathrm{pub}(I), \mathrm{priv}(I)\}$
- Is n_b deducible by the intruder?

Insecurity problem via constraint solving

Protocol rules	Constraint System

$$\begin{array}{c} \mathsf{in}(u_1);\ \mathsf{out}(v_1) \\ \mathsf{in}(u_2);\ \mathsf{out}(v_2) \\ \ldots \\ \mathsf{in}(u_n);\ \mathsf{out}(v_n) \end{array}$$

$$\mathcal{C} = \left\{ \begin{array}{l} T_0 \Vdash u_1 \\ T_0, v_1 \Vdash u_2 \\ \quad \ldots \\ T_0, v_1, .., v_n \Vdash s \end{array} \right.$$

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$$\begin{array}{c} \mathrm{in}(u_1);\ \mathrm{out}(v_1) \\ \mathrm{in}(u_2);\ \mathrm{out}(v_2) \\ \ldots \\ \mathrm{in}(u_n);\ \mathrm{out}(v_n) \end{array}$$

Constraint System

$$\mathcal{C} = \begin{cases} T_0 \Vdash u_1 \\ T_0, v_1 \Vdash u_2 \\ \quad \ldots \\ T_0, v_1, .., v_n \Vdash s \end{cases}$$

Solution of a constraint system in $\mathcal{I}$

A substitution σ such that

for every $T \Vdash u \in \mathcal{C}$, $u\sigma$ is deducible from $T\sigma$ in $\mathcal{I}$.

Running example: Needham-Schroeder's protocols

$R_A(a, I)$ and $R_B(b)$ (running in parallel)

$$\begin{array}{rcl} & & \mathrm{out}(\{a, n_a\}_{\mathrm{pub}(I)}) \\ \mathrm{in}(\{n_a, x_{n_b}\}_{\mathrm{pub}(a)}) & ; & \mathrm{out}(\{x_{n_b}\}_{\mathrm{pub}(I)}) \\ \\ \mathrm{in}(\{y_a, y_{n_a}\}_{\mathrm{pub}(b)}) & ; & \mathrm{out}(\{y_{n_a}, n_b\}_{\mathrm{pub}(y_a)}) \end{array}$$

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1			$\mathrm{out}(\{a, n_a\}_{\mathrm{pub}(I)})$
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2	$\mathrm{in}(\{y_a, y_{n_a}\}_{\mathrm{pub}(b)})$	;	$\mathrm{out}(\{y_{n_a}, n_b\}_{\mathrm{pub}(y_a)})$

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Constraints System

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Constraints System

$$T_0, \ \{a, n_a\}_{\mathsf{pub}(I)}$$

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Constraints System

$$T_0,\ \{a, n_a\}_{\mathrm{pub}(I)} \Vdash \{y_a, y_{n_a}\}_{\mathrm{pub}(b)}$$

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Constraints System

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Solution $\quad \sigma = \{y_a \mapsto \quad,\ y_{n_a} \mapsto \quad,\ x_{n_b} \mapsto \quad\}$

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2	$\text{in}(\{y_a, y_{n_a}\}_{\text{pub}(b)})$	;	$\text{out}(\{y_{n_a}, n_b\}_{\text{pub}(y_a)})$

Constraints System

$$T_0,\ \{a, n_a\}_{\text{pub}(I)} \Vdash \{y_a, y_{n_a}\}_{\text{pub}(b)}$$
$$T_0,\ \{a, n_a\}_{\text{pub}(I)},\ \{y_{n_a}, n_b\}_{\text{pub}(y_a)} \Vdash \{n_a, x_{n_b}\}_{\text{pub}(a)}$$
$$T_0,\ \{a, n_a\}_{\text{pub}(I)},\ \{y_{n_a}, n_b\}_{\text{pub}(y_a)},\ \{x_{n_b}\}_{\text{pub}(I)} \Vdash n_b$$

Solution $\sigma = \{y_a \mapsto a,\ \ y_{n_a} \mapsto n_a,\ \ x_{n_b} \mapsto \quad \}$

Running example: Needham-Schroeder's protocols

$R_A(a, I)$ and $R_B(b)$ (running in parallel)

1			$\mathrm{out}(\{a, n_a\}_{\mathrm{pub}(I)})$
3	$\mathrm{in}(\{n_a, x_{n_b}\}_{\mathrm{pub}(a)})$	;	$\mathrm{out}(\{x_{n_b}\}_{\mathrm{pub}(I)})$
2	$\mathrm{in}(\{y_a, y_{n_a}\}_{\mathrm{pub}(b)})$	;	$\mathrm{out}(\{y_{n_a}, n_b\}_{\mathrm{pub}(y_a)})$

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Existing results

Many theoretical results for different intruder models

- to take into account algebraic properties of cryptographic primitives (exclusive or, cipher block chaining, ...)
- to take into account the fact that some data are poorly-chosen (*e.g.* passwords)

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- procedure to solve constraint systems for a class of intruder
  $\hookrightarrow$ *e.g.* any intruder who can be described by a subterm convergent rewiting system
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Some tools

- AVISPA tool (Atse, OFMC)

Outline of the talk

1. Introduction
2. How to deal with trace properties? (e.g. secrecy, authentication)
3. Work in progress: Equivalence based security properties (e.g. anonymity)

Motivation: Electronic voting

Advantages:

- Convenient,
- Efficient facilities for tallying votes.

Drawbacks:

- Risk of large-scale and undetectable fraud,
- Such protocols are extremely error-prone.

> *"A 15-year-old in a garage could manufacture smart cards and sell them on the Internet that would allow for multiple votes"* Avi Rubin

Possible issue: formal methods

abstract analysis of the protocol against formally-stated properties

Expected properties

Privacy: the fact that a particular voter voted in a particular way is not revealed to anyone

Receipt-freeness: a voter cannot prove that she voted in a certain way (this is important to protect voters from coercion)

Coercion-resistance: same as receipt-freeness, but the coercer interacts with the voter during the protocol, e.g. by preparing messages

How to model such security properties?

[Kremer & Ryan, 2005] – Formalisation of Privacy
$\hookrightarrow$ consider 2 honest voters and swap their votes

Privacy

A voting protocol respects privacy if

$$S[V_A\{^a/_v\} \mid V_B\{^b/_v\}] \approx S[V_A\{^b/_v\} \mid V_B\{^a/_v\}].$$

[Delaune, Kremer & Ryan, 2006]
Formalisation of Receipt-freeness and Coercion-resistance in term of equivalence.

Some examples

$$S[V_A\{^a/_v\} \mid V_B\{^b/_v\}] \approx S[V_A\{^b/_v\} \mid V_B\{^a/_v\}]$$

Naive vote protocol (version 1)

$$V \rightarrow S : \{v\}_{\text{pub}(S)}$$

What about privacy?

Some examples

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Naive vote protocol (version 2)

$$V \rightarrow S : Id, \{v\}_{\mathsf{pub}(S)}$$

What about privacy?

Some examples

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What about privacy?

- deterministic encryption: NOT OK
- probabilistic encryption: OK

More formally

Labeled bisimilarity ($\approx_\ell$)

The largest symmetric relation $\mathcal{R}$ on processes, such that $A \mathcal{R} B$ implies

1. $\phi(A) \approx_s \phi(B)$ (depends on E),
2. if $A \rightarrow A'$, then $B \rightarrow^* B'$ and $A' \mathcal{R} B'$ for some B',
3. if $A \xrightarrow{\alpha} A'$, then $B \rightarrow^* \xrightarrow{\alpha} \rightarrow^* B'$ and $A' \mathcal{R} B'$ for some B'.

This relation is in genral undecidable. Why?

- unfolding tree is infinite in depth
- unfolding tree is infinititely branching (because of inputs)
- equational theories may be complex

Tool: Proverif

$\longrightarrow$ Obviously, the procedure is not complete. Proverif is not able to conclude for privacy even for naive voting protocols (version 1)

Work in Progress

Our Goal:

to do better than Proverif in the context of a bounded number of sessions

- Infinite depth:
  $\hookrightarrow$ we restrict to consider processes without replication (finite processes),
- Infinite branching:
  $\hookrightarrow$ we define a notion of symbolic processes and symbolic bisimulation

Concrete	$in(x).out(\{x\}_k) \xrightarrow{in(m_1)} out(\{m_1\}_k)$
Symbolic	$(in(x).out(\{x\}_k);\, \mathcal{C}) \xrightarrow{in(x)} (out(\{x\}_k);\, \mathcal{C} \cup \phi(P) \Vdash x)$

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Symbolic $\quad (in(x).out(\{x\}_k);\ \mathcal{C}) \xrightarrow{in(x)} (out(\{x\}_k);\ \mathcal{C} \cup \phi(P) \Vdash x)$

Then, we plan:

- to design a procedure to solve our constaint systems for a class of equational theory as larger as possible
- to implement a tool