In this session we will find the set of critical pairs, given a rewrite system. Remember that the rewrite system is given to us as a list of pairs of terms. The desired output is also a list of pairs of terms, which are the critical pairs of the system.

∀ rules For each pair of rules \((l_1, r_1)\) and \((l_2, r_2)\),

Rename Rename the variables of the two rules such that they are disjoint

∀ subterms For each context \(C\) and non variable term \(l_1'\) such that \(l_1 = C[l_1']\),
If \(S = mgu(l_1', l_2)\),
Add \((S(r_1, S(C[r_2])))\) to the list of critical pairs.

∀ rules also includes the case with \(l_1 = l_2\) and \(r_1 = r_2\). Notice that in ∀subterms, we need to remember the context while taking the non variable subterms of \(l_1\). A context can be a function of type \((\text{term} \to \text{term})\).