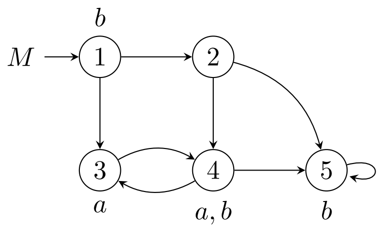
M1 MPRI
Exam on the first part of the Verification module

Tuesday 17th November, 2015

Lecture and exercise notes are allowed. Answers can be written in English or French.

Question 1 *(4 points)*

For every state subformula of ϕ, give the set of states of M which satisfy it. No explanation is needed.



$\phi = \mathsf{A}\big[(\mathsf{G}b)\ \mathsf{U}\ \mathsf{E}\big((\mathsf{EX}b) \wedge \mathsf{FAX}a\big)\big]$

Question 2 *(2 points)*

Are the following formulae equivalent? Give each time a short proof or a counter-example.

- $\mathsf{E}((\mathsf{X}b)\ \mathsf{U}\ (\neg b))$ and $\mathsf{EG}(b \wedge \mathsf{AX}\neg b)$
- $\mathsf{EGAF}a$ and $\mathsf{AF}a$
- $(\mathsf{X}a)\ \mathsf{U}\ b$ and $\mathsf{X}(a\ \mathsf{U}\ (a \wedge b))$
- $\mathsf{G}(a\ \mathsf{U}\ \mathsf{G}b)$ and $(\mathsf{G}a)\ \mathsf{U}\ (\mathsf{G}b)$

Question 3 *(2 points)*

Give two models (finite Kripke structures) which satisfy the same LTL formulae but not the same CTL formulae. Give a proof.

Question 4 *(3 points)*

What is the complexity of deciding whether two models satisfy the same LTL formulae? Give a proof.

Question 5 *(2 points)*

Propose a Büchi automaton whose language is the language of the LTL formula $(\mathsf{F}(a \wedge \mathsf{X}b)) \wedge (\mathsf{XG}\neg \mathsf{a})$. No explanation is needed. Your Büchi automaton does not need to be the one obtained using the construction presented in the course.

Question 6 *(8 points)*

A CTL* formula ϕ is a *behaviorally state formula* if, for every model M and for every state s, $M, s \models \mathsf{E}\phi$ iff $M, s \models \mathsf{A}\phi$.

1. Is the following formula a behaviorally state formula? Give an explanation.

$$(\mathsf{F}a) \wedge (\mathsf{AXG}\neg\mathsf{a}) \wedge (\mathsf{EX}b)$$

2. Is the following formula a behaviorally state formula? Give an explanation.

$$(\mathsf{F}a) \wedge (\mathsf{XG}\neg\mathsf{a})$$

3. Prove that every formula of the form $\mathsf{E}\phi$ or $\mathsf{A}\phi$ is a behaviorally state formula.

4. Prove that the problem of deciding if a CTL* formula is not a behaviorally state formula can be reduced to the problem of satisfiability of a CTL* formula.

5. Given a CTL* formula ϕ, a model M and a state s such that $M, s \models \mathsf{E}\neg\phi$, show that ϕ is satisfiable iff $\mathsf{X}\phi$ is not a behaviorally state formula.