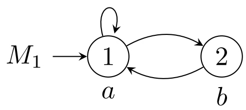
M1 MPRI
Exam on the first part of the Verification module

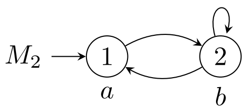
Monday 10th November, 2014

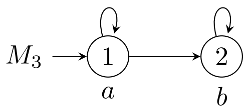
Lecture and exercise notes are allowed. Answers can be written in English or French.

Question 1 *(3 marks)*

For each model and each formula, say if the model satisfies the formula, and give a short explanation.



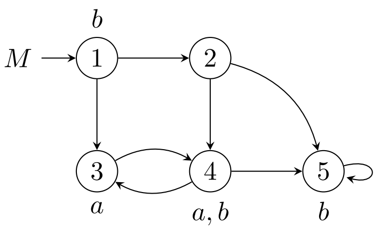




- $\phi_1 = \mathsf{AGF}b$
- $\phi_2 = \mathsf{EGF}b$
- $\phi_3 = \mathsf{AFEG}a$
- $\phi_4 = \mathsf{AGEF}b$

Question 2 *(4 marks)*

For every state subformula of ϕ, give the set of states of M which satisfy it. No explanation is needed.



$\phi = \mathsf{A}\big[(\mathsf{X}b) \vee \mathsf{FA}\big((\mathsf{EFG}a)\mathsf{U}(\mathsf{AG}b)\big)\big]$

Question 3 *(2 marks)*

Are the following formulae equivalent? Give each time a short proof or a counter-example.

- $\mathsf{EF}a$ and $\mathsf{EFEX}a$
- $\mathsf{AGF}a$ and $\mathsf{AGFX}a$
- $a \to \mathsf{EF}a$ and $(\mathsf{E}a) \to (\mathsf{AF}a)$
- $\mathsf{A}((\mathsf{G}a) \to (\mathsf{F}b))$ and $\mathsf{AG}(a \to (\mathsf{F}b))$

Question 4 *(3 marks)*

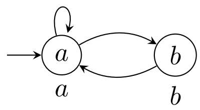
What is the complexity of checking whether two LTL formulae are equivalent? Give a proof.

Question 5 *(2 marks)*

We consider a finite Kripke structure M where the set of atomic propositions is the set of states, and in every state s, $\ell(s) = \{s\}$.

1. Define an LTL formula of size linear in the size of M, that characterizes the set of infinite runs of M.

2. Illustrate your construction on the following Kripke structure.



Question 6 *(6 marks)*

We denote $\mathrm{LTL}^+_{\mathsf{F}}$ the set of LTL formulae with no negation and no other temporal modality than F.

1. *(4 marks)*
   Prove that the existential model-checking problem for $\mathrm{LTL}^+_{\mathsf{F}}$ is in NP.

2. *(2 marks)*
   By reduction from the SAT problem, prove that the existential model-checking problem for $\mathrm{LTL}^+_{\mathsf{F}}$ is NP-hard.