On the verification and control of timed systems

Patricia Bouyer-Decitre

LSV, CNRS & ENS Cachan, France

Two main parts:

1. An introduction to timed systems (this morning)
2. Modelling resources in timed systems (this afternoon)

An introduction to timed systems

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Outline

Time!

Context: verification of critical systems

Time

- naturally appears in real systems (for ex. protocols, embedded systems)
- appears in properties (for ex. bounded response time)

"Will the airbag oben within 5ms after the car crashes?"

$\leadsto$ Need of models and specification languages integrating timing aspects

Adding timing informations

- **Untimed case:** sequence of observable events
  a: send message b: receive message

$$a\ b\ a\ b\ a\ b\ a\ b\ a\ b\ \cdots = (a\ b)^\omega$$

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$$(a, d_1)\ (b, d_2)\ (a, d_3)\ (b, d_4)\ (a, d_5)\ (b, d_6)\ \cdots$$

  d_1: date at which the first a occurs
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  - **Discrete-time semantics:** dates are *e.g.* taken in $\mathbb{N}$
    Ex: $(a, 1)(b, 3)(c, 4)(a, 6)$

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  - **Discrete-time semantics:** dates are *e.g.* taken in $\mathbb{N}$
    Ex: $(a, 1)(b, 3)(c, 4)(a, 6)$
  - **Dense-time semantics:** dates are *e.g.* taken in $\mathbb{Q}_+$, or in $\mathbb{R}_+$
    Ex: $(a, 1.28).(b, 3.1).(c, 3.98)(a, 6.13)$

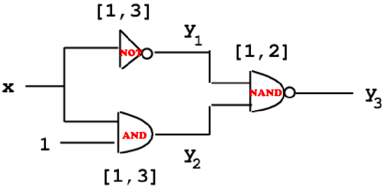
A case for dense-time

Time domain: discrete (*e.g.* $\mathbb{N}$) or dense (*e.g.* $\mathbb{Q}_+$ or $\mathbb{R}_+$)

- A compositionality problem with discrete time
- Dense-time is a more general model than discrete time
- But, can we not always discretize?

A digital circuit [Alu91]

Discussion in the context of reachability problems for asynchronous digital circuits [BS91]



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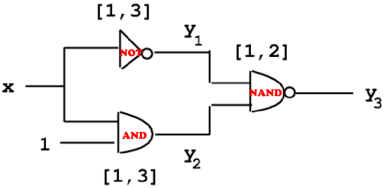

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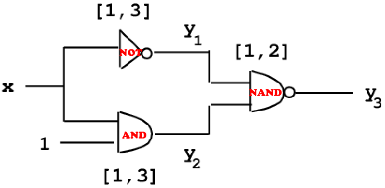

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The input x changes to 1. The corresponding stable state is y=[011]

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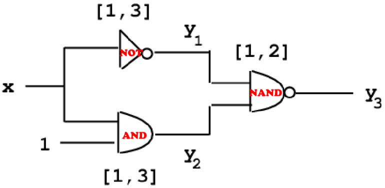

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However, many possible behaviours, e.g.

$$[101] \xrightarrow[1.2]{y_2} [111] \xrightarrow[2.5]{y_3} [110] \xrightarrow[2.8]{y_1} [010] \xrightarrow[4.5]{y_3} [011]$$

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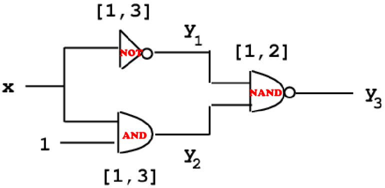

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Reachable configurations: $\{[101],\ [111],\ [110],\ [010],\ [011],\ [001]\}$

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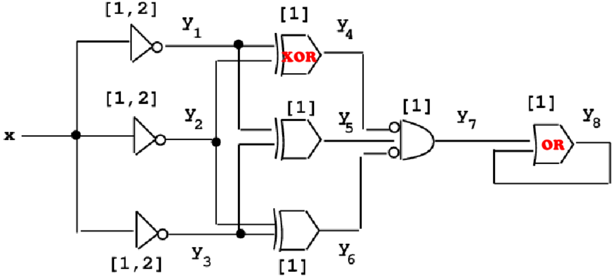

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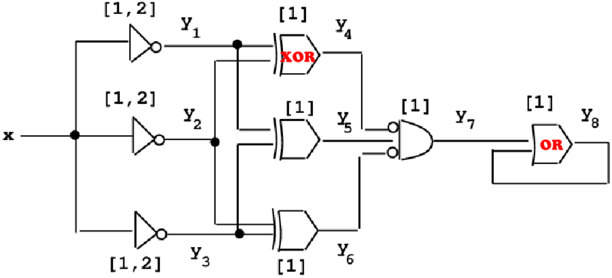

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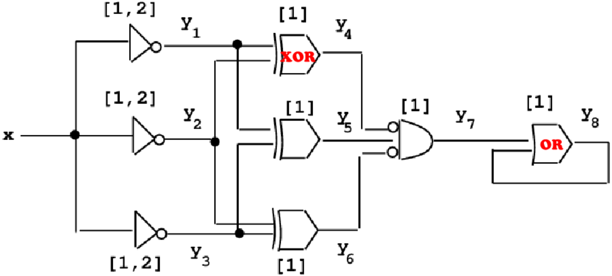

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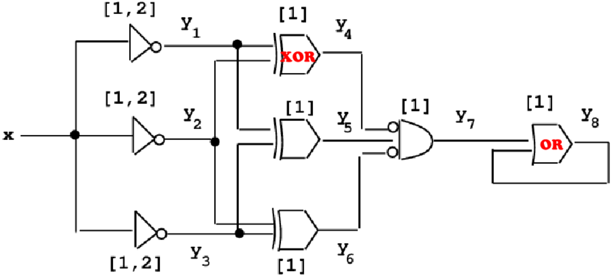

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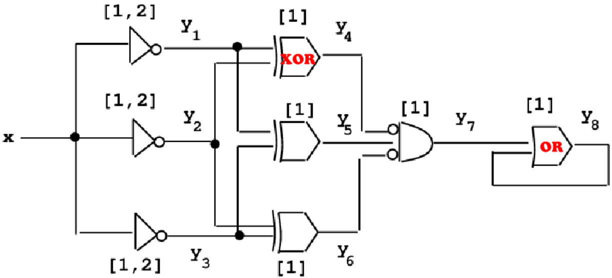

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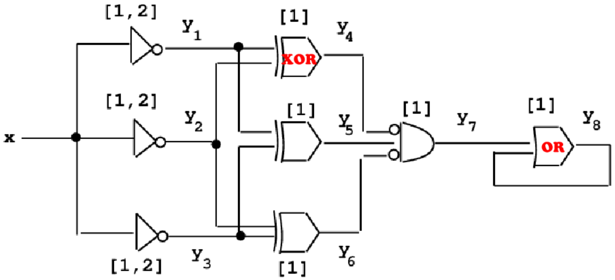

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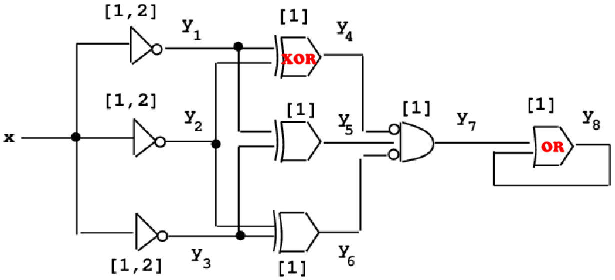

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Theorem [BS91]

For every $k \geq 1$, there exists a digital circuit such that the reachability set of states in dense-time is strictly larger than the one in discrete time (with granularity $\frac{1}{k}$).

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Further counter-example

There exist systems for which no granularity exists. (see later)

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Hence, we better consider a dense-time domain!

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Outline

A plethora ot models...

- ... for real-time systems:
    - timed circuits,
    - time(d) Petri nets,
    - timed process algebra,
    - timed automata,
    - ...

- ... and for real-time properties:
    - timed observers,
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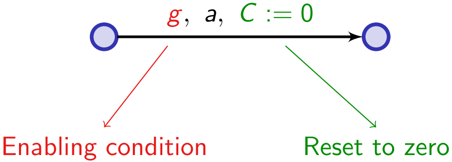
Timed automata [AD90]

- A finite control structure + variables (clocks)
- A transition is of the form:



- An enabling condition (or guard) is:

$$g \quad ::= \quad x \sim c \;\mid\; x - y \sim c \;\mid\; g \wedge g$$

where $\sim \in \{<, \leq, =, \geq, >\}$

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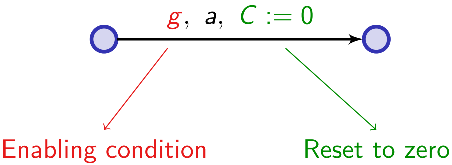

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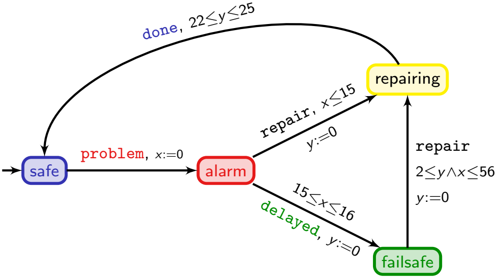
An example of a timed automaton



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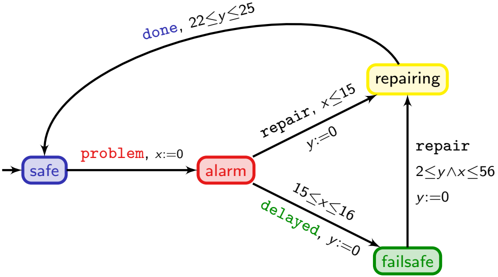

	safe
x	0
y	0

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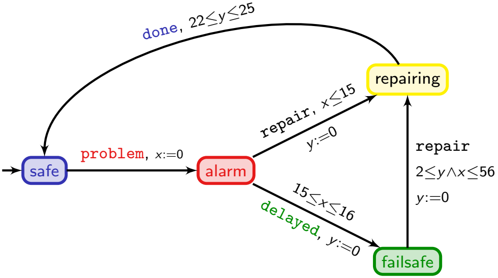

	safe	$\xrightarrow{23}$	safe
x	0		23
y	0		23

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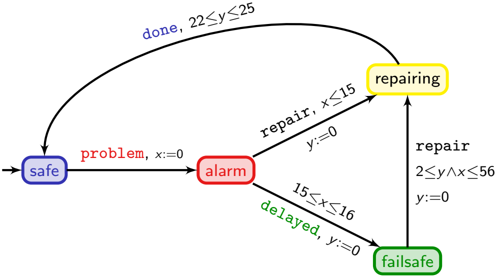

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm
x	0		23		0
y	0		23		23

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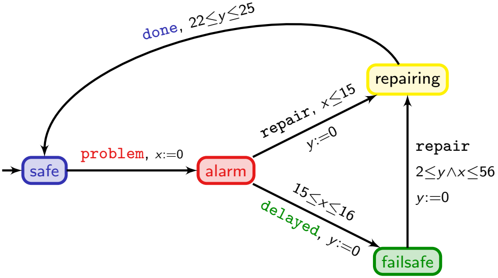

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm
x	0		23		0		15.6
y	0		23		23		38.6

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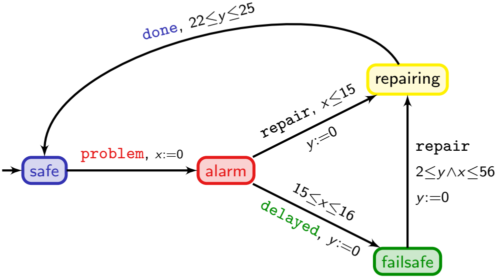

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	···
y	0		23		23		38.6		0	

	failsafe
···	15.6
	0

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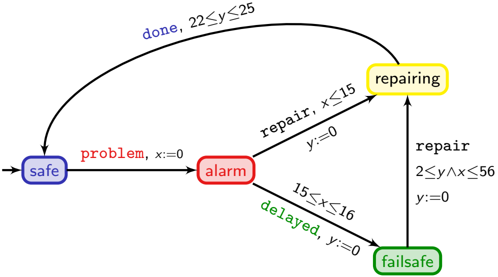

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe
⋯	15.6		17.9
	0		2.3

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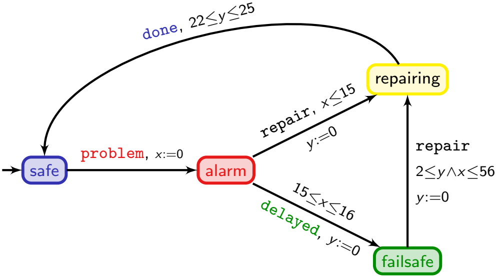

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing
⋯	15.6		17.9		17.9
	0		2.3		0

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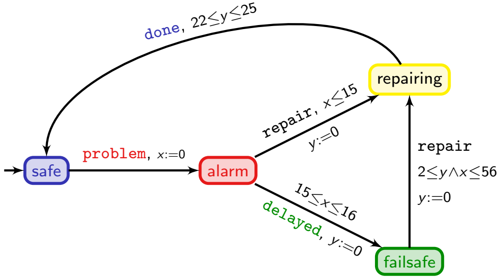
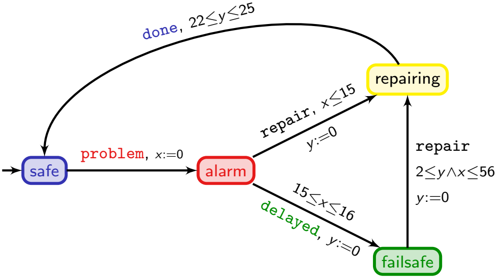


	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	···
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing
···	15.6		17.9		17.9		40
	0		2.3		0		22.1

An example of a timed automaton



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x	0		23		0		15.6		15.6	⋯
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	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\texttt{done}}$	safe
⋯	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

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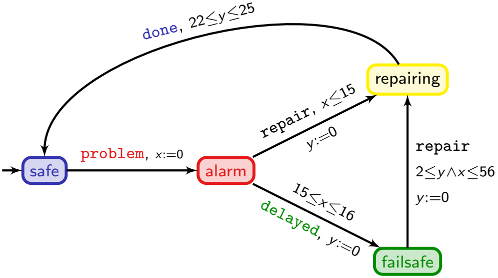

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⋯	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

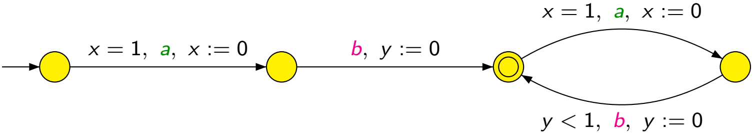
This run read the timed word
$(\texttt{problem}, 23)(\texttt{delayed}, 38.6)(\texttt{repair}, 40.9), (\texttt{done}, 63)$.

Timed automata semantics

- $\mathcal{A} = (\Sigma, L, X, \longrightarrow)$ is a TA
- **Configurations:** $(\ell, v) \in L \times T^X$ where T is the time domain

  v is called the (clock) valuation
- **Timed transition system:**
  - **action transition**: $(\ell, v) \xrightarrow{a} (\ell', v')$ if $\exists \ell \xrightarrow{g,a,r} \ell' \in \mathcal{A}$ s.t. $\begin{cases} v \models g \\ v' = v[r \leftarrow 0] \end{cases}$
  - **delay transition**: $(\ell, v) \xrightarrow{\delta(d)} (\ell, v + d)$ if $d \in T$

Discrete *vs* dense-time semantics



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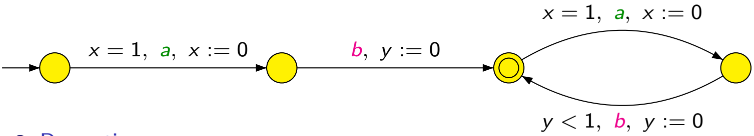
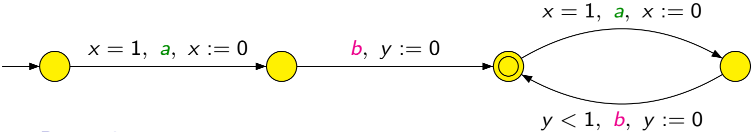


- Dense-time:
  $L_{dense} = \{((ab)^\omega, \tau) \mid \forall i,\ \tau_{2i-1} = i \text{ and } \tau_{2i} - \tau_{2i-1} > \tau_{2i+2} - \tau_{2i+1}\}$

Discrete *vs* dense-time semantics



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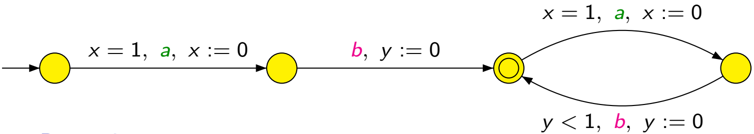

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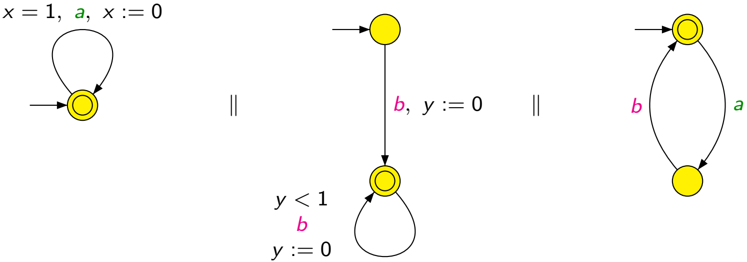
However, it does result from the following parallel composition:



Classical verification problems

- reachability of a control state
- $\mathcal{S} \sim \mathcal{S}'$: bisimulation, etc...
- $L(\mathcal{S}) \subseteq L(\mathcal{S}')$: language inclusion
- $\mathcal{S} \models \varphi$ for some formula φ: model-checking
- $\mathcal{S} \parallel A_T$ + reachability: testing automata
- ...

Classical temporal logics

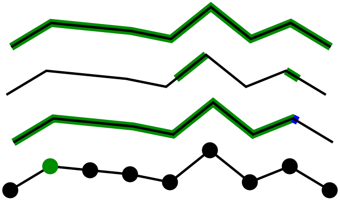
Path formulas:

$\mathbf{G}\,\varphi$ "Always"

$\mathbf{F}\,\varphi$ "Eventually"

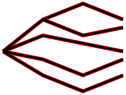
$\varphi\,\mathbf{U}\,\varphi'$ "Until"

$\mathbf{X}\,\varphi$ "Next"

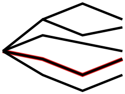


State formulas:

$\mathbf{A}\psi$



$\mathbf{E}\psi$



$\rightsquigarrow$ LTL: Linear Temporal Logic [Pnu77],
CTL: Computation Tree Logic [EC82]

[Pnu77] Pnueli. The temporal logic of programs *(FoCS'77)*.
[EC82] Emerson, Clarke. Using branching time temporal logic to synthesize synchronization skeletons *(Science of Computer Programming 1982)*.

Adding time to temporal logics

Classical temporal logics allow us to express that

"any problem is followed by an alarm"

[ACD90] Alur, Courcoubetis, Dill. Model-checking for real-time systems *(LICS'90)*.
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$\rightsquigarrow$ TCTL: Timed CTL **[ACD90,ACD93,HNSY94]**

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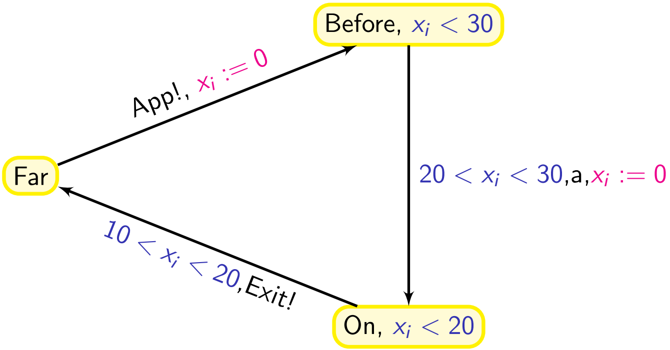
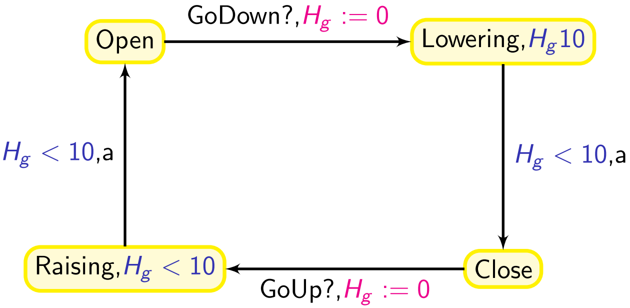
The train crossing example (1)

Train$_i$ with $i = 1, 2, \ldots$



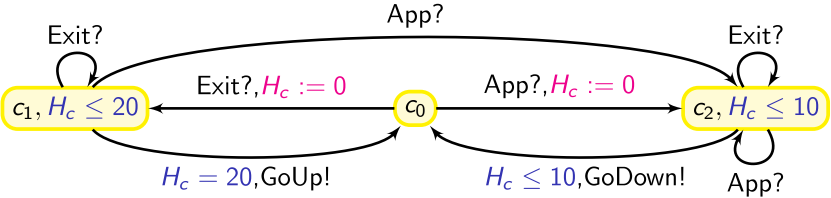
The train crossing example (2)

The gate:



The train crossing example (3)

The controller:



The train crossing example (4)

We use the synchronization function f:

Train_1	Train_2	Gate	Controller	
App!	.	.	App?	App
.	App!	.	App?	App
Exit!	.	.	Exit?	Exit
.	Exit!	.	Exit?	Exit
a	.	.	.	a
.	a	.	.	a
.	.	a	.	a
.	.	GoUp?	GoUp!	GoUp
.	.	GoDown?	GoDown!	GoDown

to define the parallel composition $(\text{Train}_1 \parallel \text{Train}_2 \parallel \text{Gate} \parallel \text{Controller})$

NB: the parallel composition does not add expressive power!

The train crossing example (5)

Some properties one could check:

- Is the gate closed when a train crosses the road?

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$$\neg\mathbf{EF}\,(\text{gate.Close} \wedge \mathbf{E}(\text{gate.Close}\ \ \mathbf{U}_{>5\ \text{min}}\ \ \neg\text{gate.Close}))$$

Another example: A Fischer protocol

A mutual exclusion protocol with a shared variable *id* [AL94].

[AL94] Abadí, Lamport. An old-fashioned recipe for real time *(ACM Transactions on Programming Languages and Systems)*.

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Process i:

a : await ($id = 0$);
b : set id to i;
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$\leadsto$ a max. delay k_1 between a and b
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$\rightsquigarrow$ See the demo with the tool Uppaal
(can be downloaded freely on http://www.uppaal.com/)

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Outline

Verification

Emptiness problem: is the language accepted by a timed automaton empty?

- basic reachability/safety properties (final states)
- basic liveness properties (ω-regular conditions)

[AD90] Alur, Dill. Automata for modeling real-time systems *(ICALP'90)*.
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Emptiness problem: is the language accepted by a timed automaton empty?

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The emptiness problem for timed automata is decidable and PSPACE-complete.

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Method: construct a finite abstraction

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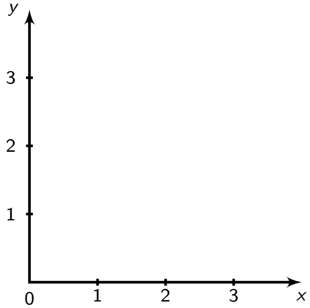
The region abstraction



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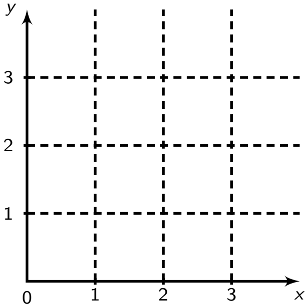
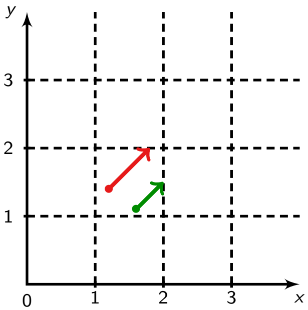


- "compatibility" between regions and constraints

The region abstraction



- "compatibility" between regions and constraints
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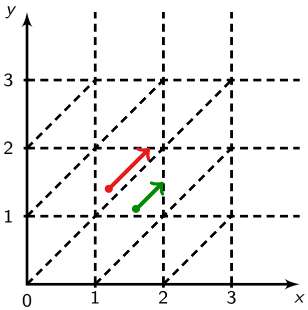

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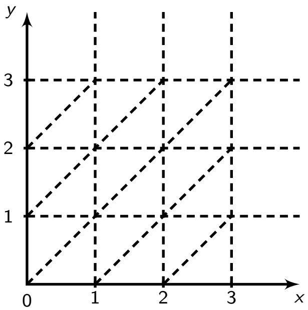

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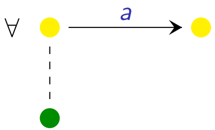
$\rightsquigarrow$ an equivalence of finite index
a time-abstract bisimulation

Time-abstract bisimulation



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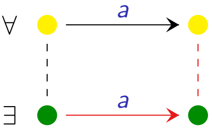

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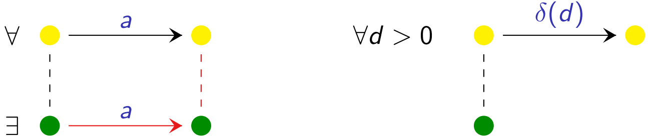

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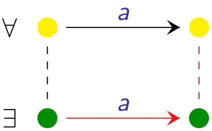

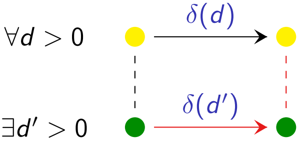

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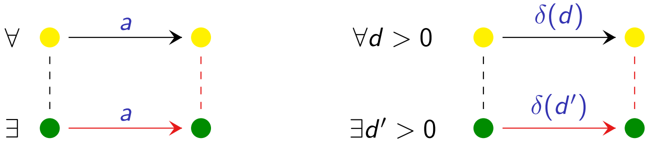
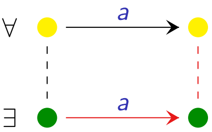
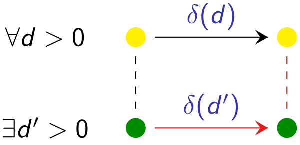


$$(\ell_0, v_0) \xrightarrow{a_1, t_1} (\ell_1, v_1) \xrightarrow{a_2, t_2} (\ell_2, v_2) \xrightarrow{a_3, t_3} \dots$$

Time-abstract bisimulation

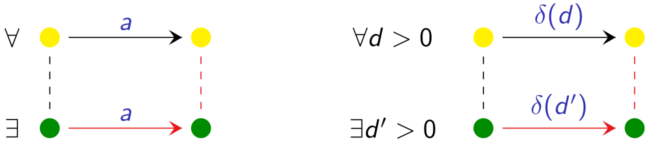




$$\begin{tikzcd}
(\ell_0, v_0) \arrow[r, "a_1, t_1"] \arrow[d] & (\ell_1, v_1) \arrow[r, "a_2, t_2"] \arrow[d] & (\ell_2, v_2) \arrow[r, "a_3, t_3"] \arrow[d] & \ldots \\
(\ell_0, R_0) \arrow[r, "a_1"] & (\ell_1, R_1) \arrow[r, "a_2"] & (\ell_2, R_2) \arrow[r, "a_3"] & \ldots
\end{tikzcd}$$

with $v_i \in R_i$ for all i.

Time-abstract bisimulation



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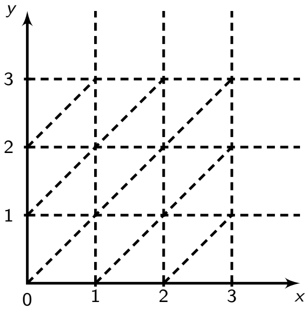

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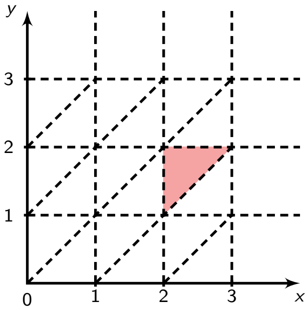
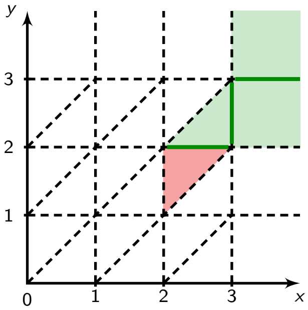


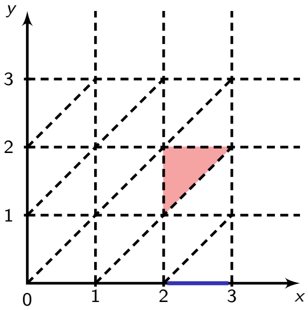
$$\begin{cases} 2 < x < 3 \\ 1 < y < 2 \\ \{x\} < \{y\} \end{cases}$$

The region abstraction



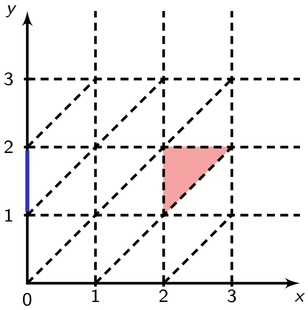
time successors

The region abstraction



reset of clock y

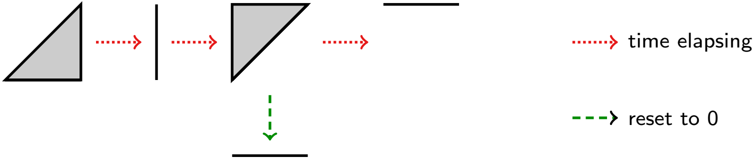
The region abstraction



reset of clock x

The region graph

A finite graph representing time elapsing and reset of clocks:



Region automaton $\equiv$ finite bisimulation quotient

timed automaton $\otimes$ region graph

Region automaton $\equiv$ finite bisimulation quotient

timed automaton $\bigotimes$ region graph

$\ell \xrightarrow{g,a,C:=0} \ell'$ is transformed into:

$(\ell, R) \xrightarrow{a} (\ell', R')$ if there exists $R'' \in \mathsf{Succ}_t^*(R)$ s.t.

- $R'' \subseteq g$
- $[C \leftarrow 0]R'' \subseteq R'$

Region automaton $\equiv$ finite bisimulation quotient

timed automaton $\bigotimes$ region graph

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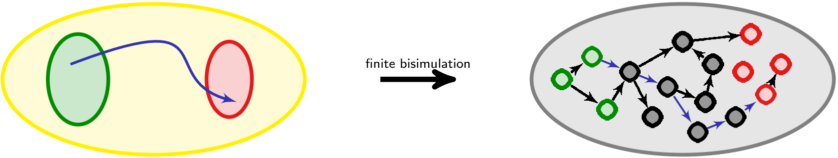
$\mathcal{L}$(reg. aut.) = UNTIME($\mathcal{L}$(timed aut.))

where $\mathrm{UNTIME}((a_1, t_1)(a_2, t_2)\dots) = a_1 a_2 \dots$

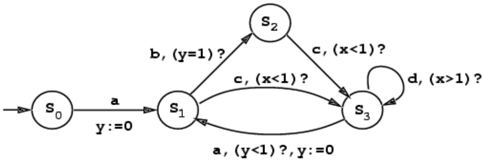


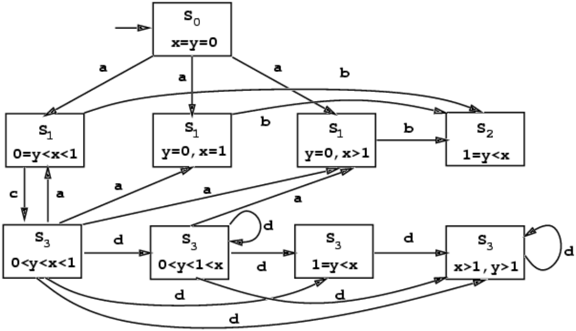
timed automaton

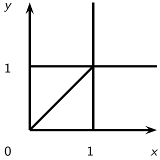
large (but finite) automaton
(region automaton)

$$\mathcal{L}(\textbf{reg. aut.}) = \textbf{UNTIME}(\mathcal{L}(\textbf{timed aut.}))$$

An example [AD94]







PSPACE membership

The size of the region graph is in $\mathcal{O}(|X|!.2^{|X|})$

- **One configuration:** a discrete location + a region

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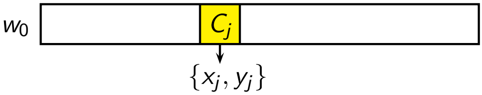
$\rightsquigarrow$ in NPSPACE, thus in PSPACE

PSPACE-hardness

$$\left.\begin{array}{c}\mathcal{M}\ \text{LBTM}\\ w_0 \in \{a,b\}^*\end{array}\right\} \rightsquigarrow \begin{array}{l}A_{\mathcal{M},w_0}\ \text{s.t.}\ \mathcal{M}\ \text{accepts}\ w_0\ \text{iff the final state}\\ \text{of}\ A_{\mathcal{M},w_0}\ \text{is reachable}\end{array}$$



C_j contains an "a" if $x_j = y_j$
C_j contains a "b" if $x_j < y_j$

(these conditions are invariant by time elapsing)

LBTM: linearly bounded Turing machine (a witness for PSPACE-complete problems)

PSPACE-hardness (cont.)

If $q \xrightarrow{\alpha,\alpha',\delta} q'$ is a transition of $\mathcal{M}$, then for each position i of the tape, we have a transition

$$(q,i) \xrightarrow{g,r:=0} (q',i')$$

where:

- g is $x_i = y_i$ (resp. $x_i < y_i$) if $\alpha = a$ (resp. $\alpha = b$)
- $r = \{x_i, y_i\}$ (resp. $r = \{x_i\}$) if $\alpha' = a$ (resp. $\alpha' = b$)
- $i' = i + 1$ (resp. $i' = i - 1$) if δ is right and $i < n$ (resp. left)

Enforcing time elapsing: on each transition, add the condition $t = 1$ and clock t is reset.

Initialization: init $\xrightarrow{t=1, r_0:=0} (q_0, 1)$ where $r_0 = \{x_i \mid w_0[i] = b\} \cup \{t\}$

Termination: $(q_f, i) \longrightarrow$ end

The case of single-clock timed automata

Exercise [LMS04]

Think of the special case of single-clock timed automata. Can we do better than PSPACE?

[LMS04] Laroussinie, Markey, Schnoebelen. Model checking timed automata with one or two clocks *(CONCUR'04)*.

Consequence of region automata construction

Region automata:

correct finite (and exponential) abstraction for checking reachability/Büchi-like properties.

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correct finite (and exponential) abstraction for checking reachability/Büchi-like properties.

However...

everything can not be reduced to finite automata...

A model not far from undecidability

Some bad news...

- Language universality is undecidable [AD90]
- Language inclusion is undecidable [AD90]
- Complementability is undecidable [Tri03,Fin06]
- ...

[Tri03] Tripakis. Folk theorems on the determinization and minimization of timed automata *(FORMATS'03)*.
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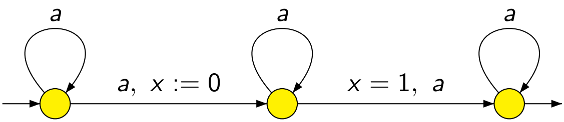

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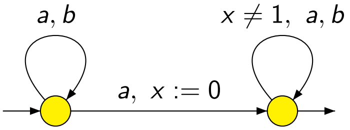

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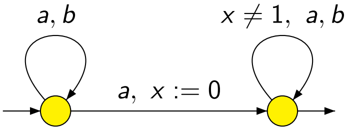

UNTIME $\left(\overline{L} \cap \{(a^* b^*, \tau) \mid \text{all } a's \text{ happen before 1 and no two } a's \text{ simultaneously}\}\right)$ is not regular **(exercise!)**

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The two-counter machine

Definition

A two-counter machine is a finite set of instructions over two counters (c and d):

- Incrementation:
  (p): $c := c + 1$; goto (q)
- Decrementation:
  (p): if $c > 0$ then $c := c - 1$; goto (q) else goto (r)

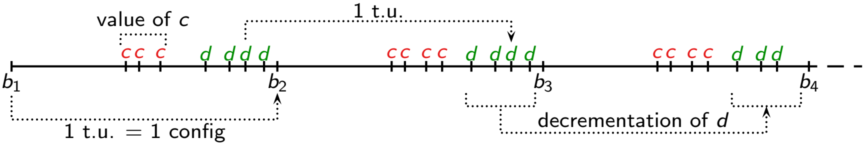
Theorem [Minsky 67]

The halting problem for two counter machines is undecidable.

Undecidability of universality

Theorem [AD90]

Universality of timed automata is undecidable.

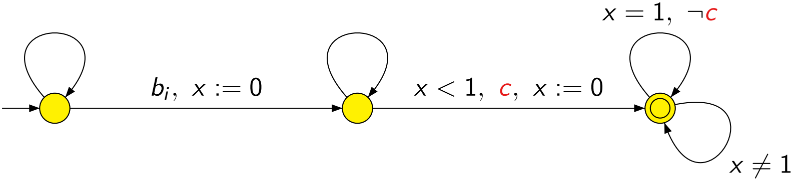


- one configuration is encoded in one time unit
- number of c's: value of counter c
- number of d's: value of counter d
- one time unit between two corresponding c's (resp. d's)

$\rightsquigarrow$ **We encode "non-behaviours" of a two-counter machine**

Example

Module to check that if instruction i does not decrease counter c, then all actions c appearing less than 1 t.u. after b_i has to be followed by an other c 1 t.u. later.



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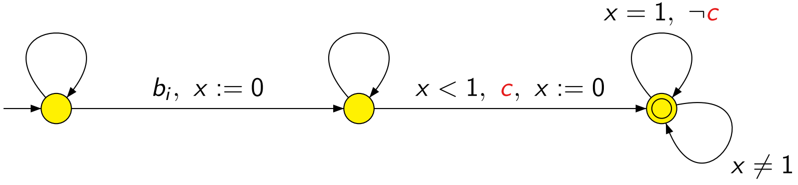

The union of all small modules is not universal
iff
The two-counter machine has a recurring computation

Partial conclusion

- This idea of a finite bisimulation quotient has been applied to many "timed" or "hybrid" systems:
  - various extensions of timed automata
    [Bérard,Diekert,Gastin,Petit 1998] [Choffrut,Goldwurm 2000]
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  - $\cdots$
- Note however that it might be hard to prove there is a finite bisimulation quotient!

Outline

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A general model: hybrid systems

[Hen96]

What is a hybrid system?

a discrete control (the mode of the system)
$+$ a continuous evolution within a mode (given by variables)

[Hen96] Henzinger. The theory of hybrid automata *(LICS'96)*.

A general model: hybrid systems

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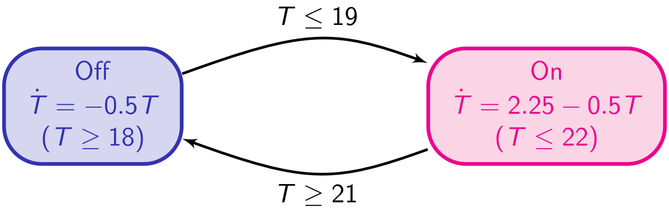
$+$ a continuous evolution within a mode (given by variables)

Example (The thermostat)

A simple thermostat, where T (the temperature) depends on the time:



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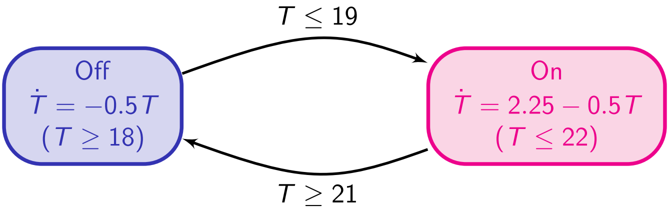

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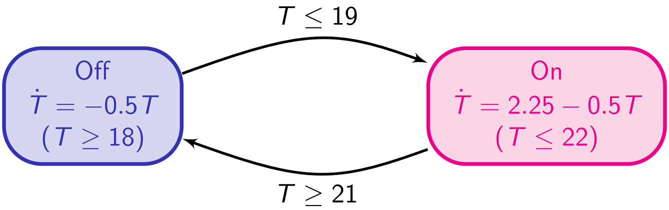

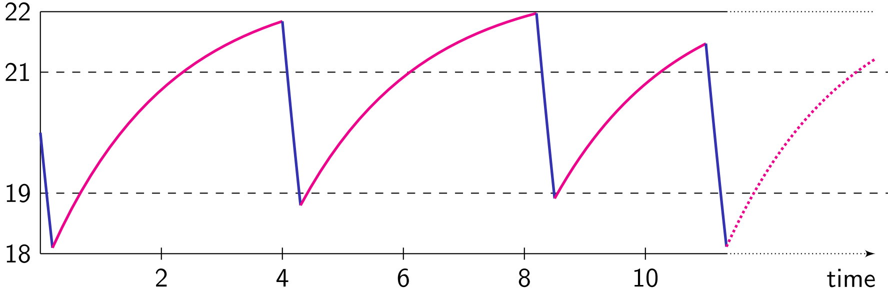

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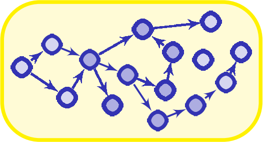

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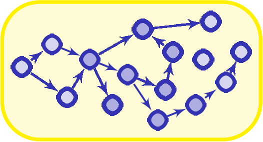

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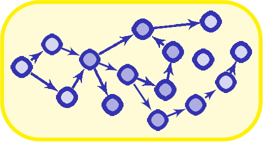

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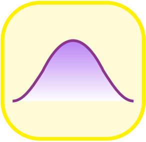

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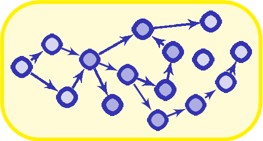

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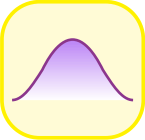

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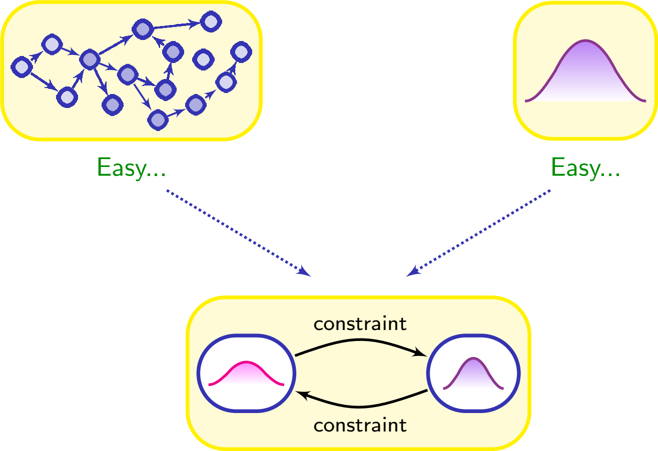
Ok... but?



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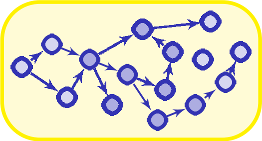

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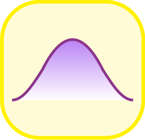

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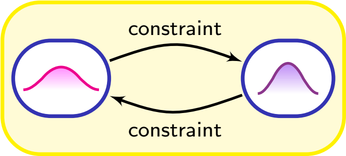

Hard!

What about decidability?

$\rightsquigarrow$ almost everything is undecidable

Negative results [HKPV95]

- The class of hybrid systems with clocks and only one variable having possibly two slopes $k_1 \neq k_2$ is undecidable.
- The class of *stopwatch* automata is undecidable.

[HKPV95] Henzinger, Kopke, Puri, Varaiya. What's decidable about hybrid automata? *(STOC'95)*.

Outline

Role of diagonal constraints

$$\boxed{x - y \sim c \quad \text{and} \quad x \sim c}$$

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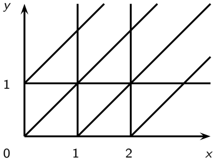
- **Decidability:** yes, using the region abstraction



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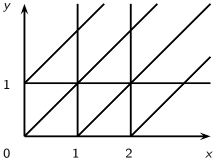
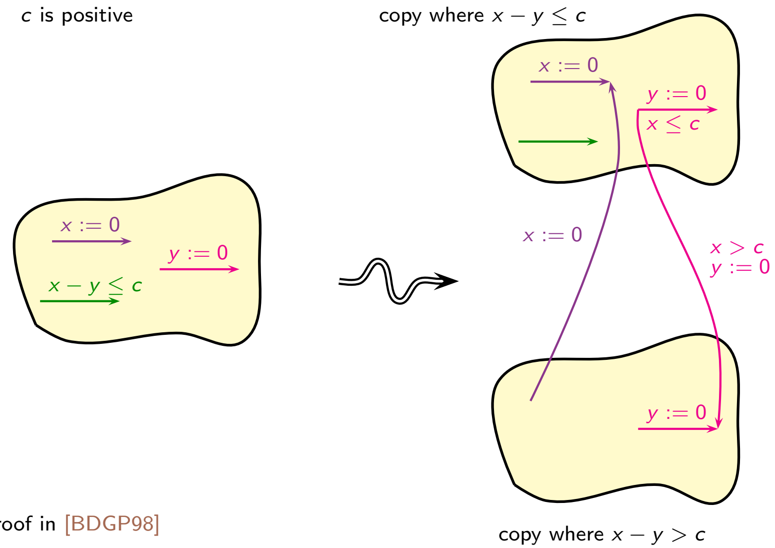


- **Expressiveness:** no additional expressive power

Role of diagonal constraints (cont.)



$\rightsquigarrow$ proof in [BDGP98]

[BDGP98] Bérard, Diekert, Gastin, Petit. Characterization of the expressive power of silent transitions in timed automata *(Fundamenta Informaticae)*.

Role of diagonal constraints (cont.)

Exercise [BC05]

Consider, for every positive integer n, the timed language:

$$\mathcal{L}_n = \{(a, t_1) \dots (a, t_{2^n}) \mid 0 < t_1 < \dots < t_{2^n} < 1\}$$

1. Construct a timed automaton with diagonal constraints which recognizes $\mathcal{L}_n$. What is the size of this automaton?
2. Idem without diagonal constraints. Can you do better?
3. Conclude.

[BC05] Bouyer, Chevalier. On conciseness of extensions of timed automata *(Journal of Automata, Languages and Combinatorics)*.

Adding silent actions

$$\xrightarrow{g,\ \varepsilon,\ C := 0}$$

[BDGP98]

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  (actions have no influence on region automaton construction)

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[BDGP98]

- **Decidability:** yes

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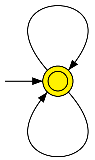
- **Expressiveness:** strictly more expressive!



Adding additive constraints

$\boxed{x + y \sim c \quad \text{and} \quad x \sim c}$ **[BD00]**

[BD00] Bérard, Dufourd. Timed automata and additive clock constraints *(Information Proecessing Letters)*.

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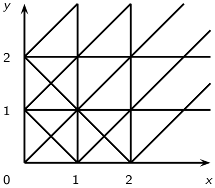

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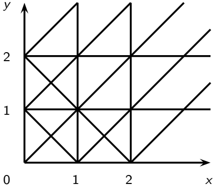
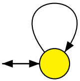


  - for four clocks (or more), undecidable!

- **Expressiveness:** more expressive! (even using two clocks)

$$\{(a^n, t_1 \dots t_n) \mid n \geq 1 \text{ and } t_i = 1 - \frac{1}{2^i}\}$$



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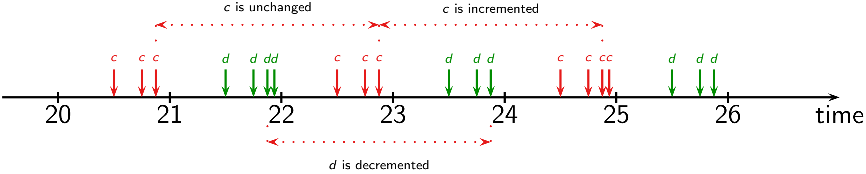
Undecidability proof



$\rightsquigarrow$ simulation of
- decrementation of a counter
- incrementation of a counter

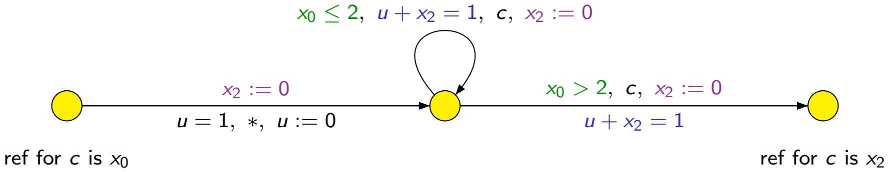
We will use 4 clocks:

- u, "tic" clock (each time unit)
- x_0, x_1, x_2: reference clocks for the two counters

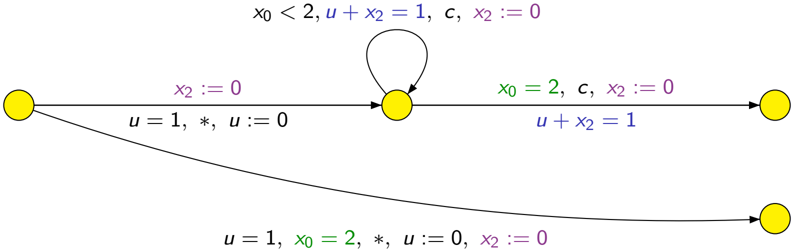
"x_i reference for c" $\equiv$ "the last time x_i has been reset is the last time action c has been performed"

Undecidability proof (cont.)

- **Incrementation of counter c:**

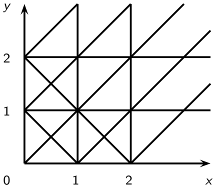


- **Decrementation of counter c:**



Adding constraints of the form $x + y \sim c$

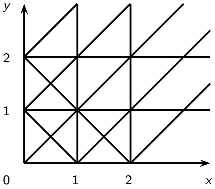
- Two clocks: decidable using the abstraction



- Four clocks (or more): undecidable!

Adding constraints of the form $x + y \sim c$

- Two clocks: decidable using the abstraction



- **Three clocks: open question**

- Four clocks (or more): undecidable!

Adding new operations on clocks

Several types of updates: $x := y + c$, $x :< c$, $x :> c$, etc...

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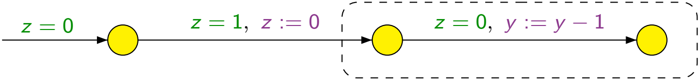
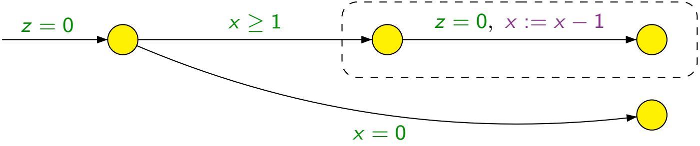
- Only decrementation also leads to undecidability

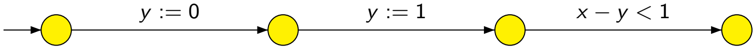
  - **Incrementation of counter x**



  - **Decrementation of counter x**



Decidability



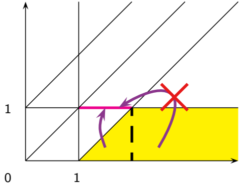


image by $y := 1$

$\rightsquigarrow$ the bisimulation property is not met

The classical region automaton construction is not correct.

Decidability (cont.)

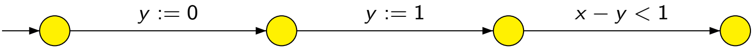
$$\begin{array}{lcl} \mathcal{A} & \rightsquigarrow & \text{Diophantine linear inequations system} \\ & \rightsquigarrow & \text{is there a solution?} \\ & \rightsquigarrow & \text{if yes, belongs to a decidable class} \end{array}$$

Examples:

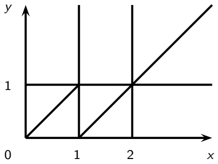
- constraint $x \sim c$ $\quad c \leq \max_x$
- constraint $x - y \sim c$ $\quad c \leq \max_{x,y}$
- update $x :\sim y + c$ $\quad \max_x \leq \max_y + c$
  and for each clock z, $\max_{x,z} \geq \max_{y,z} + c$, $\max_{z,x} \geq \max_{z,y} - c$
- update $x :< c$ $\quad c \leq \max_x$
  and for each clock z, $\max_z \geq c + \max_{z,x}$

The constants $(\max_x)$ and $(\max_{x,y})$ define a set of regions.

Decidability (cont.)



$$\begin{cases} \max_y \geq 0 \\ \max_x \geq 0 + \max_{x,y} \\ \max_y \geq 1 \\ \max_x \geq 1 + \max_{x,y} \\ \max_{x,y} \geq 1 \end{cases} \quad \text{implies} \quad \begin{cases} \max_x = 2 \\ \max_y = 1 \\ \max_{x,y} = 1 \\ \max_{y,x} = -1 \end{cases}$$

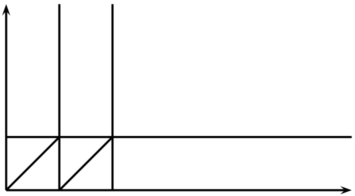


The bisimulation property is met.

What's wrong when undecidable?

Decrementation $x := x - 1$

$$\max_x \leq \max_x - 1$$



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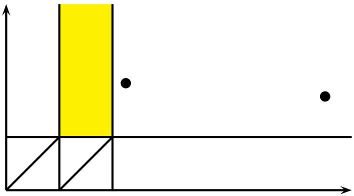

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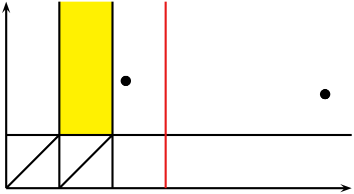

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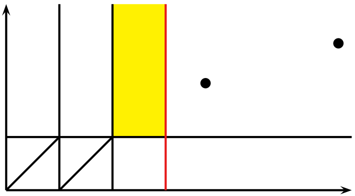

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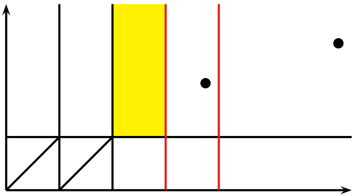

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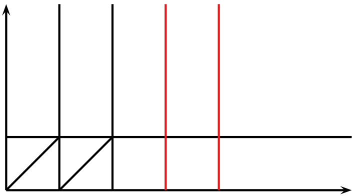

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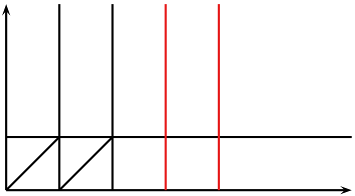

etc...

Decidability (cont.)

	Diagonal-free constraints	**General constraints**
$x := c,\ x := y$	PSPACE-complete	PSPACE-complete
$x := x + 1$		Undecidable
$x := y + c$		
$x := x - 1$	Undecidable	
$x :< c$	PSPACE-complete	PSPACE-complete
$x :> c$		Undecidable
$x :\sim y + c$		
$y + c <: x :< y + d$		
$y + c <: x :< z + d$	Undecidable	

[BDFP00]

[BDFP00] Bouyer, Dufourd, Fleury, Petit. Updatable timed automata *(Theoretical Computer Science)*.

Outline

The example of alternating timed automata

Alternating timed automata $\equiv$ ATA

[LW05,OW05]

[LW05] Lasota, Walukiewicz. Alternating timed automata *(FoSSaCS'05)*.
[OW05] Ouaknine, Worrell. On the decidability of Metric Temporal Logic *(LICS'05)*.

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Example

"No two a's are separated by 1 unit of time"

$$\begin{cases} \ell_0, a, \mathit{true} & \mapsto \quad \ell_0 \wedge (x := 0, \ell_1) \\ \ell_1, a, x \neq 1 & \mapsto \quad \ell_1 \\ \ell_1, a, x = 1 & \mapsto \quad \ell_2 \\ \ell_2, a, \mathit{true} & \mapsto \quad \ell_2 \end{cases}$$

$$\begin{cases} \ell_0 \text{ initial state} \\ \ell_0, \ell_1 \text{ final states} \\ \ell_2 \text{ losing state} \end{cases}$$

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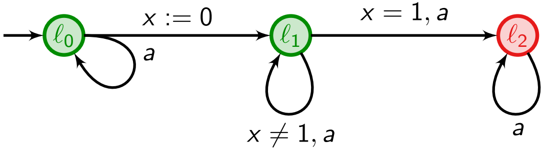

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Theorem

- Emptiness of ATA is undecidable.
- Emptiness of one-clock ATA is decidable, but non-primitive recursive.
- Emptiness for Büchi properties of one-clock ATA is undecidable.
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Lower bound: simulation of a lossy channel system... [Sch02]

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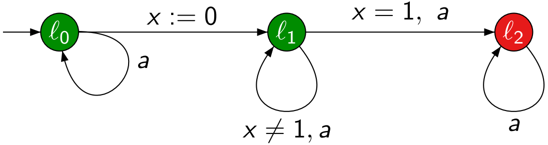

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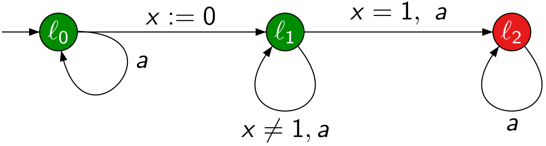
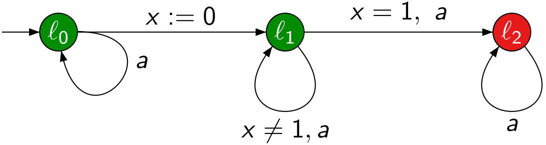


Execution over timed word $(a, .3)(a, .8)(a, 1.4)(a, 1.8)(a, 2)$

Example



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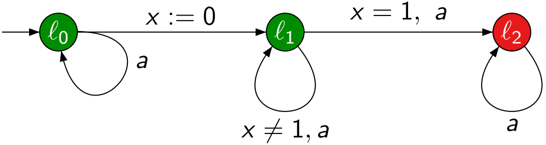
Example



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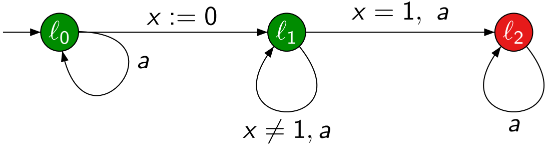
$$\{\ (\ell_0, 0)\ \}$$
$$\downarrow$$
$$\{\ (\ell_0, .3)\ ,\ (\ell_1, 0)\ \}$$

Example



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$$\downarrow$$
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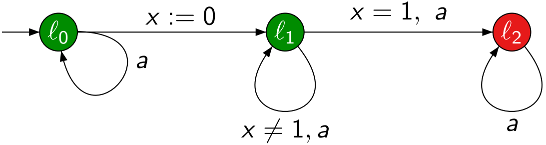

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$$\{(\ell_0, 0)\}$$
$$\downarrow$$
$$\{(\ell_0, .3), (\ell_1, 0)\}$$
$$\downarrow$$
$$\{(\ell_0, .8), (\ell_1, 0), (\ell_1, .5)\}$$
$$\downarrow$$
$$\{(\ell_0, 1.4), (\ell_1, 0), (\ell_1, .6), (\ell_1, 1.1)\}$$

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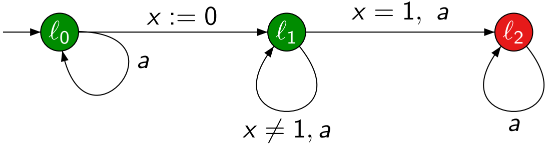

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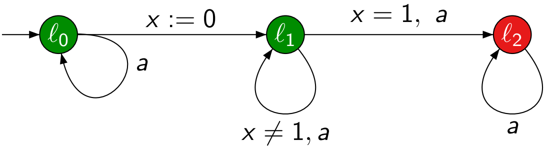
Example



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$$\downarrow$$
$$\{(\ell_0, 2), (\ell_1, 0), (\ell_1, .2), (\ell_1, .6), (\ell_2, 1.2), (\ell_1, 1.7)\}$$

An abstraction

A configuration $=$ a finite set of pairs (ℓ, x)

$$(\ell, 0) \quad (\ell, 0.3) \quad (\ell, 1.2) \quad (\ell, 2.3) \quad (\ell', 0.4) \quad (\ell', 1) \quad (\ell', 0.8)$$

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$\{(\ell, 0), (\ell', 1)\}$

0.0

An abstraction

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$\{(\ell, 0), (\ell', 1)\} \quad \{(\ell, 1)\}$

0.0 0.2

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0.0 0.2 0.3

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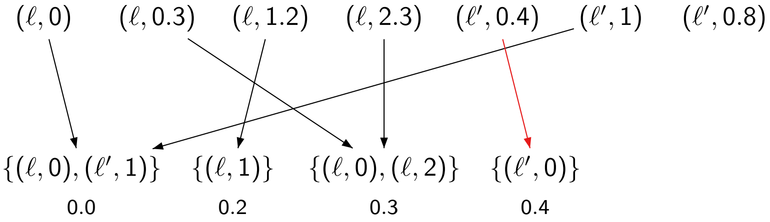

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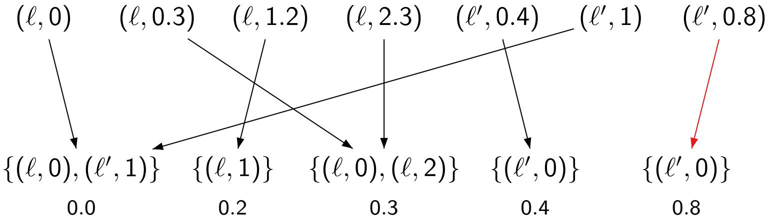

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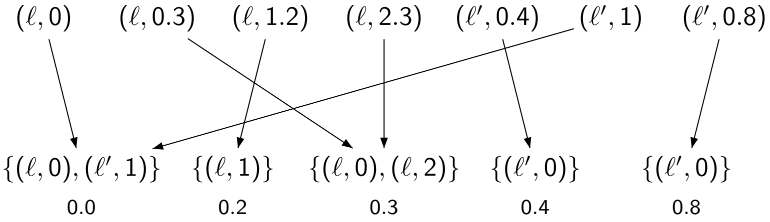

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$(\ell, 0)$ $\quad (\ell, 0.3)$ $\quad (\ell, 1.2)$ $\quad (\ell, 2.3)$ $\quad (\ell', 0.4)$ $\quad (\ell', 1)$ $\quad (\ell', 0.8)$

$\{(\ell, 0), (\ell', 1)\}$	$\{(\ell, 1)\}$	$\{(\ell, 0), (\ell, 2)\}$	$\{(\ell', 0)\}$	$\{(\ell', 0)\}$
0.0	0.2	0.3	0.4	0.8

Abstracted into: $\{(\ell, 0), (\ell', 1)\} \cdot \{(\ell, 1)\} \cdot \{(\ell, 0), (\ell, 2)\} \cdot \{(\ell', 0)\} \cdot \{(\ell', 0)\}$

Abstract transition system

$$\{(\ell, 0), (\ell', 1)\} \cdot \{(\ell, 1)\} \cdot \{(\ell, 0), (\ell, 2)\} \cdot \{(\ell', 0)\} \cdot \{(\ell', 0)\}$$

Abstract transition system

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Time successors:

Abstract transition system

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Abstract transition system

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Time successors:

$$\{(\ell',1)\} \cdot \{(\ell,0),(\ell',1)\} \cdot \{(\ell,1)\} \cdot \{(\ell,0),(\ell,2)\} \cdot \{(\ell',0)\}$$

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$$\{(\ell,2)\} \cdot \{(\ell,1),(\ell,3)\} \cdot \{(\ell',1)\} \cdot \{(\ell',1)\} \cdot \{(\ell,0),(\ell',1)\}$$

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Transition $\ell \xrightarrow{x>2,x:=0} \ell''$:

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$$\{(\ell'',0)\}\cdot\{(\ell,1),(\ell',2)\}\cdot\{(\ell,1)\}\cdot\{(\ell',1)\}\cdot\{(\ell',1)\}$$

What can we do with that abstract transition system?

Correctness?

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The previous abstraction is (almost) a time-abstract bisimulation.

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  $$(\gamma_1 \sqsubseteq \gamma_1' \text{ and } \gamma_1' \rightsquigarrow \gamma_2') \Rightarrow (\gamma_1 \rightsquigarrow^* \gamma_2 \text{ and } \gamma_2 \sqsubseteq \gamma_2')$$

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A recipe:

$$(\text{Higman's lemma} + \text{Koenig's lemma}) \Rightarrow \text{termination}$$

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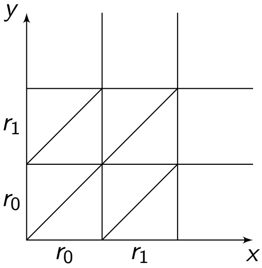
Alternative

The abstract transition system can be simulated by a kind of FIFO channel machine.

A digression on timed automata



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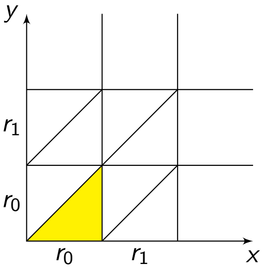

$x, y \in r_0$, $\{y\} < \{x\}$

$\boxed{(y, r_0)} \cdot \boxed{(x, r_0)}$

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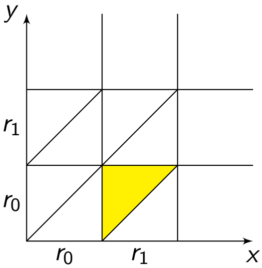

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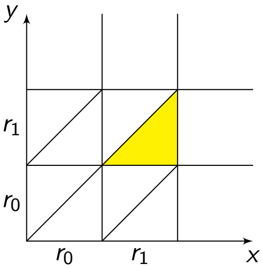

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$(y, r_1) \cdot (x, r_1)$

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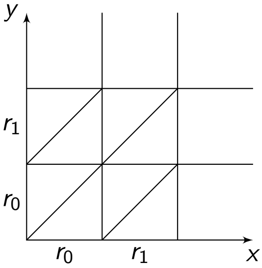

The classical region automaton can be simulated by a channel machine (with a single bounded channel).

Partial conclusion

Similar technics apply to:

- networks of single-clock timed automata **[Abdulla,Jonsson 1998]**

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Outline

What about the practice?

- the region automaton is never computed
- instead, symbolic computations are performed

What do we need?

- Need of a symbolic representation:

Finite representation of infinite sets of configurations

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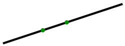

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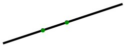

  - set of words aa, $aaaa$, $aaaaaa$...
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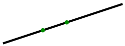

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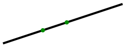

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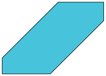

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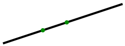

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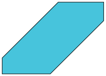

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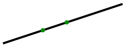

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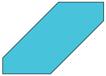


  - BDDs, DBMs (see later), CDDs, etc...

- Need of abstractions, heuristics, etc...

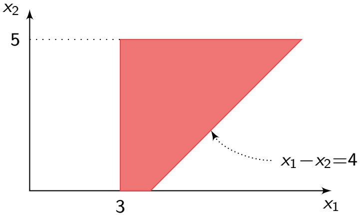
An example of computation with HyTech

```
command:  /usr/local/bin/hytech gas_burner
=================================================================
HyTech:  symbolic model checker for embedded systems
Version 1.04f (last modified 1/24/02) from v1.04a of 12/6/96
For more info:
    email:  hytech@eecs.berkeley.edu
    http://www.eecs.berkeley.edu/~tah/HyTech
Warning:  Input has changed from version 1.00(a).  Use -i for more info
=================================================================

Backward computation
Number of iterations required for reachability:  6
System satisfies non-leaking duration property

Location:  not_leaking
x >= 0 & t >= 3 & y <= 20t & y >= 0
| x + 20t >= y + 11 & y <= 20t + 19 & t >= 2 & x >= 0 & y >= 0
| y >= 0 & t >= 1 & x + 20t >= y + 22 & y <= 20t + 8 & x >= 0
| y >= 0 & x + 20t >= y + 33 & 20t >= y + 3 & x >= 0
Location:  leaking
19x + y <= 20t + 19 & y >= x + 59 & x <= 1 & x >= 0
| t >= x + 2 & x <= 1 & y >= 0 & 19x + y <= 20t + 19 & x >= 0
| t >= x + 1 & x <= 1 & y >= 0 & 19x + y <= 20t + 8 & x >= 0
| 20t >= 19x + y + 3 & y >= 0 & x <= 1 & x >= 0

=================================================================
Max memory used = 0 pages = 0 bytes = 0.00 MB
Time spent = 0.02u + 0.00s = 0.02 sec total

=================================================================
```

Zones: A symbolic representation for timed systems

Example of a zone and its DBM representation

$$Z \;=\; (x_1 \geq 3) \;\wedge\; (x_2 \leq 5) \;\wedge\; (x_1 - x_2 \leq 4)$$



$$\begin{array}{c c} & \begin{array}{ccc} x_0 & x_1 & x_2 \end{array} \\ \begin{array}{c} x_0 \\ x_1 \\ x_2 \end{array} & \begin{pmatrix} \infty & -3 & \infty \\ \infty & \infty & 4 \\ 5 & \infty & \infty \end{pmatrix} \end{array}$$

DBM: Difference Bound Matrice [BM83,Dill89]

[BM83] Berthomieu, Menasche. An enumerative approach for analyzing time Petri nets *World Comupter Congress*.
[Dill89] Dill. Timing assumptions and verification of finite-state concurrent systems *(Automatic Verification Methods for Finite State Systems)*.

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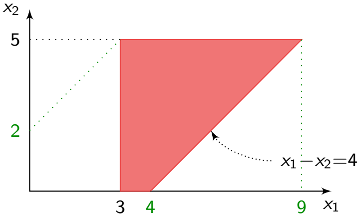

$$\begin{array}{c c} & \begin{array}{ccc} x_0 & x_1 & x_2 \end{array} \\ \begin{array}{c} x_0 \\ x_1 \\ x_2 \end{array} & \begin{pmatrix} 0 & -3 & 0 \\ 9 & 0 & 4 \\ 5 & 2 & 0 \end{pmatrix} \end{array}$$

"normal form"

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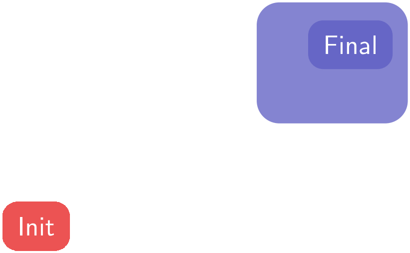

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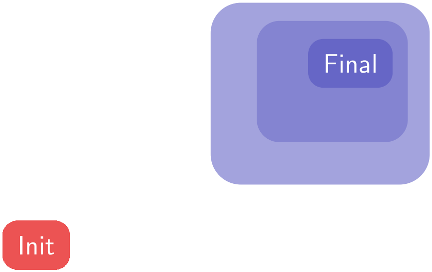

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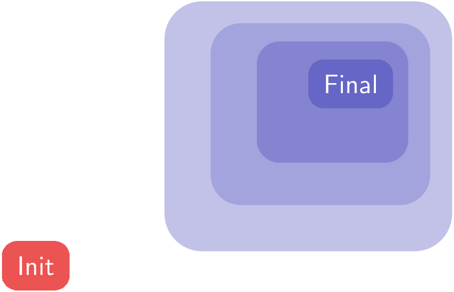

Backward computation

Note on the backward analysis of TA

$$\ell \xrightarrow{g,\ a,\ C := 0} \ell'$$

$$\overleftarrow{[C \leftarrow 0]^{-1}(Z \cap (C = 0)) \cap g} \qquad\qquad Z$$

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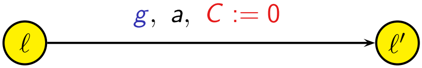
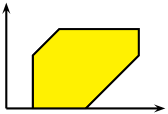


$\overleftarrow{[C \leftarrow 0]^{-1}(Z \cap (C = 0)) \cap g}$

Z



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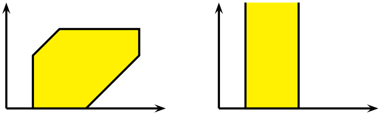

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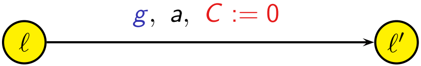

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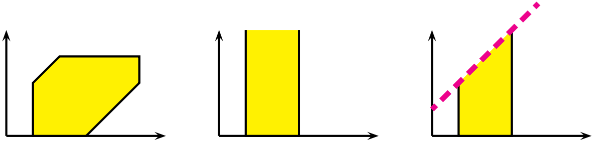

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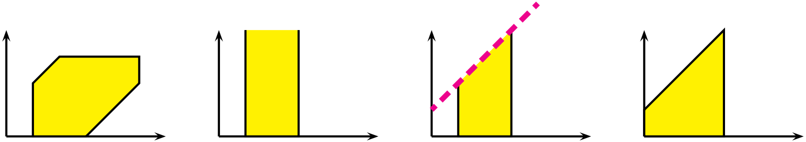

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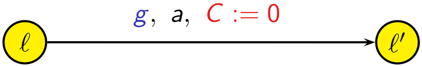

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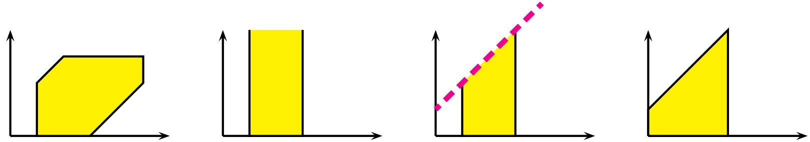

Z $\qquad [C \leftarrow 0]^{-1}(Z \cap (C = 0))$ $\qquad \overleftarrow{[C \leftarrow 0]^{-1}(Z \cap (C = 0)) \cap g}$

☺ the backward computation always terminates!

☺☺ ... and it is correct!!!

Note on the backward analysis (cont.)

If $\mathcal{A}$ is a timed automaton, we construct its corresponding set of regions.

Because of the bisimulation property, we get that:

"Every set of valuations which is computed along the backward computation is a finite union of regions"

Note on the backward analysis (cont.)

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Let R be a region. Assume:

- $v \in \overleftarrow{R}$ (for ex. $v + t \in R$)
- $v' \equiv_{\text{reg.}} v$

There exists t' s.t. $v' + t' \equiv_{\text{reg.}} v + t$, which implies that $v' + t' \in R$ and thus $v' \in \overleftarrow{R}$.

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Because of the bisimulation property, we get that:

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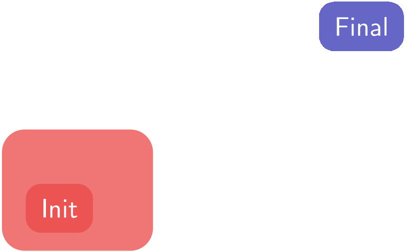
But, the backward computation is not so nice, when also dealing with integer variables...

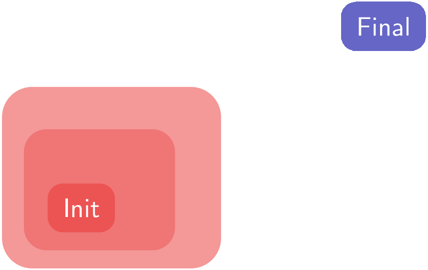
$$i := j.k + \ell.m$$

Forward computation

Forward computation



Forward computation



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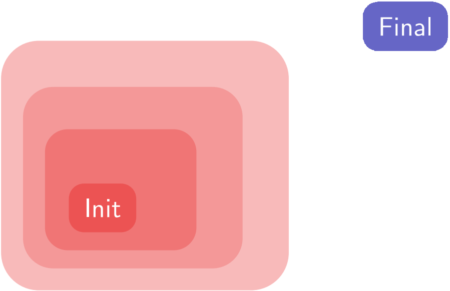
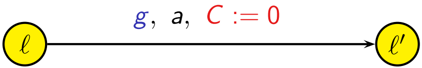


Forward computation

Forward analysis of timed automata



zones $\qquad Z \qquad\qquad [C \leftarrow 0](\overrightarrow{Z} \cap g)$

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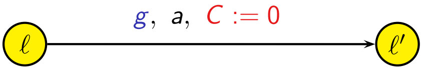

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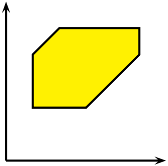

Z

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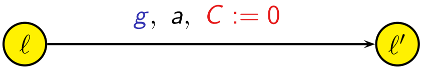

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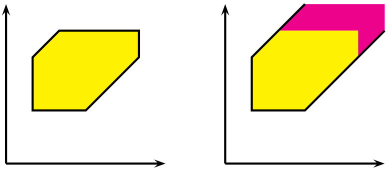

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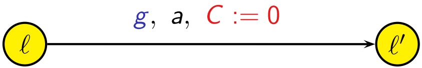

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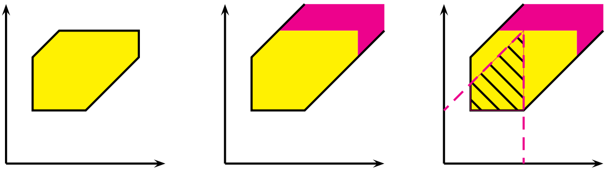

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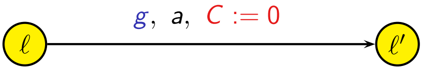

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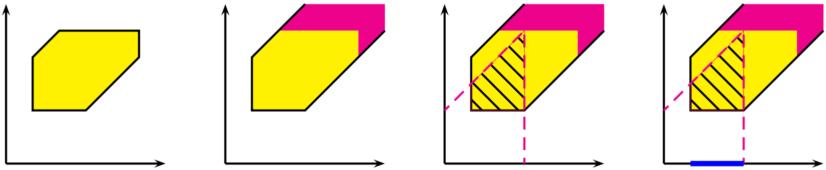

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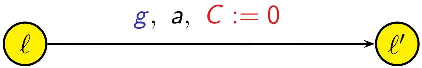

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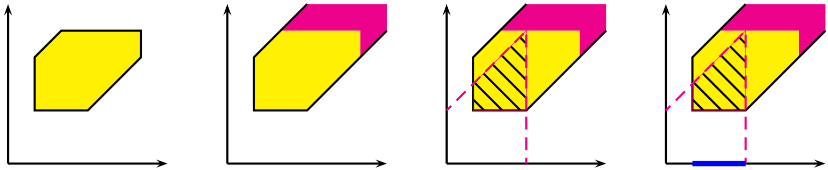

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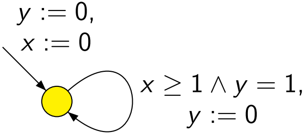
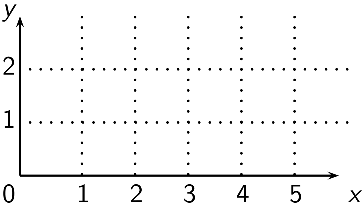
☹ the forward computation may not terminate...

Non termination of the forward analysis





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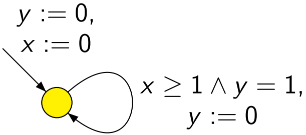

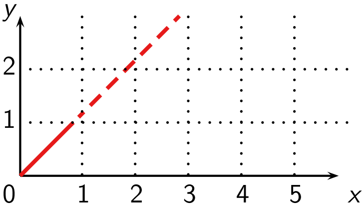

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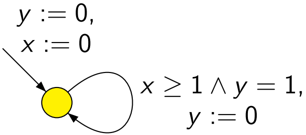

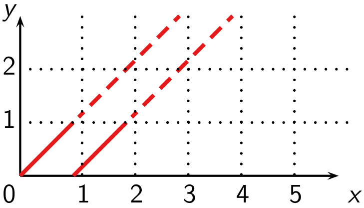

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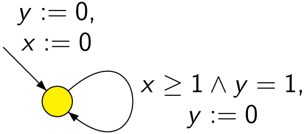

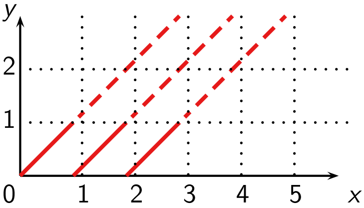

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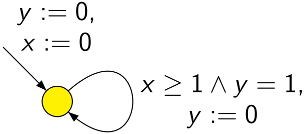

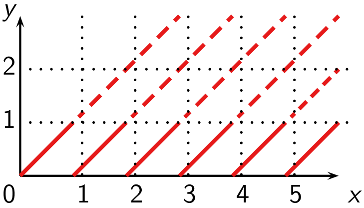

$\rightsquigarrow$ an infinite number of steps...

"Solutions" to this problem

(f.ex. in [Larsen,Pettersson,Yi 1997] or in [Daws,Tripakis 1998])

- **inclusion checking**: if $Z \subseteq Z'$ and Z' already considered, then we don't need to consider Z

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- **activity**: eliminate redundant clocks **[Daws,Yovine 1996]**

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$$q \xrightarrow{g,a,C:=0} q' \quad \text{implies} \quad \mathrm{Act}(q) = \mathrm{clocks}(g) \cup (\mathrm{Act}(q') \setminus C)$$

...

"Solutions" to this problem (cont.)

- **convex-hull approximation**: if Z and Z' are computed then we overapproximate using "$Z \sqcup Z'$".

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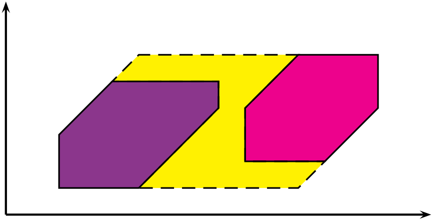

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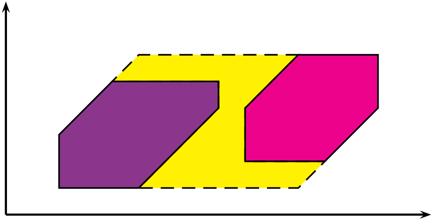
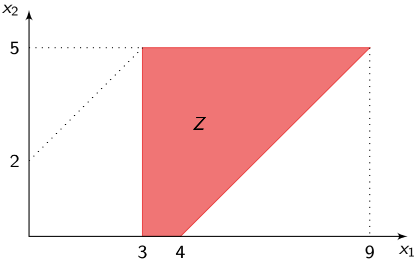


- **extrapolation**, an abstraction operator on zones

An abstraction: the extrapolation operator

$\mathsf{Approx}_2(Z)$: "the smallest zone containing Z that is defined only with constants no more than 2"



$$\begin{pmatrix} 0 & -3 & 0 \\ 9 & 0 & 4 \\ 5 & 2 & 0 \end{pmatrix}$$

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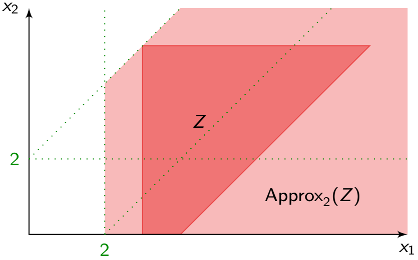

$$\begin{pmatrix} 0 & -3 & 0 \\ 9 & 0 & 4 \\ 5 & 2 & 0 \end{pmatrix} \xRightarrow{\mathsf{Approx}_2} \begin{pmatrix} 0 & -2 & 0 \\ \infty & 0 & \infty \\ \infty & 2 & 0 \end{pmatrix}$$

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Classical algorithm, focus on correctness

Challenge

Choose a **good** constant for the extrapolation so that the forward computation is correct. Classical algorithm, the choice goes to the maximal constant.

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The classical algorithm is correct for diagonal-free timed automata.

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This theorem does not extend to timed automata using diagonal clock constraints... [Bou03,Bou04]

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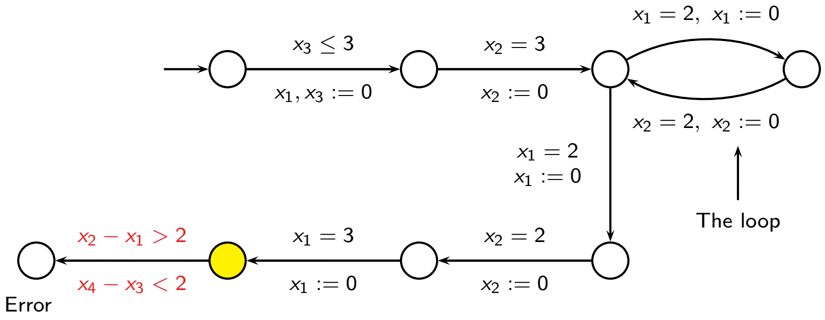
A problematic automaton



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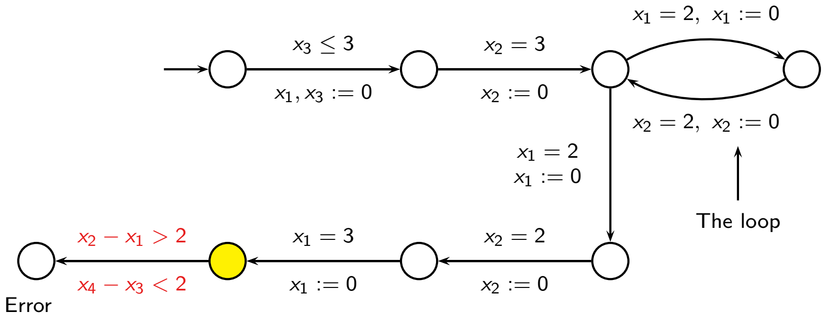

$$\begin{cases} v(x_1) = 0 \\ v(x_2) = d \\ v(x_3) = 2\alpha + 5 \\ v(x_4) = 2\alpha + 5 + d \end{cases}$$

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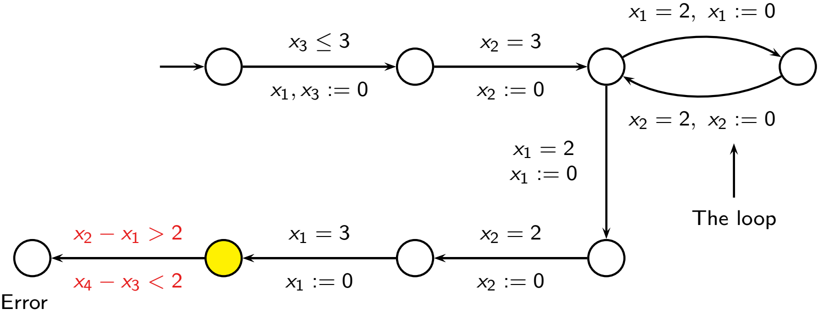

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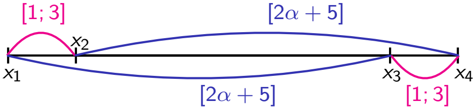

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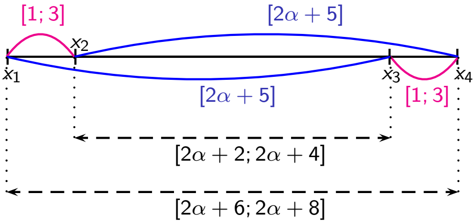

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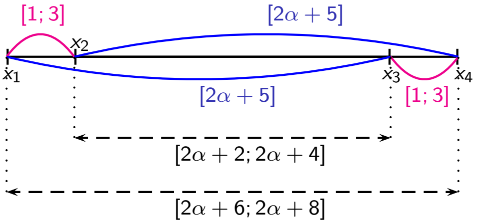

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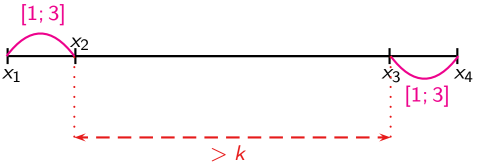

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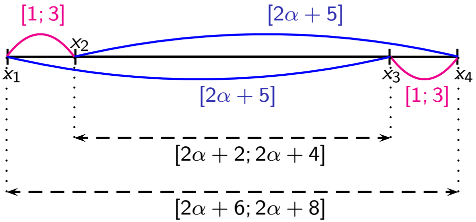

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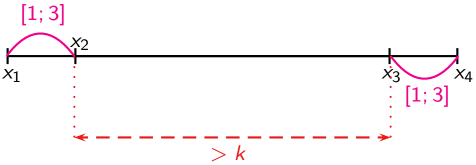

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Hence, any choice of constant is erroneous!

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For the previous automaton,

no abstraction operator can satisfy all these criteria!

Why that?

Assume there is a “nice” operator Abs.

The set $\{M \text{ DBM representing a zone } \mathrm{Abs}(Z)\}$ is finite.

$\rightsquigarrow$ k the max. constant defining one of the previous DBMs

We get that, for every zone Z,

$$Z \subseteq \mathrm{Extra}_k(Z) \subseteq \mathrm{Abs}(Z)$$

Problem!

Open questions:
- which conditions can be made weaker?
- find a clever termination criterium?
- use an other data structure than zones/DBMs?

Improving the classical algorithm

- the extrapolation operator can be made coarser:
  - local extrapolation constants [BBFL03];
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$\rightsquigarrow$ the tool Uppaal is under development since 1995...

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- Several technics for proving decidability of real-time systems
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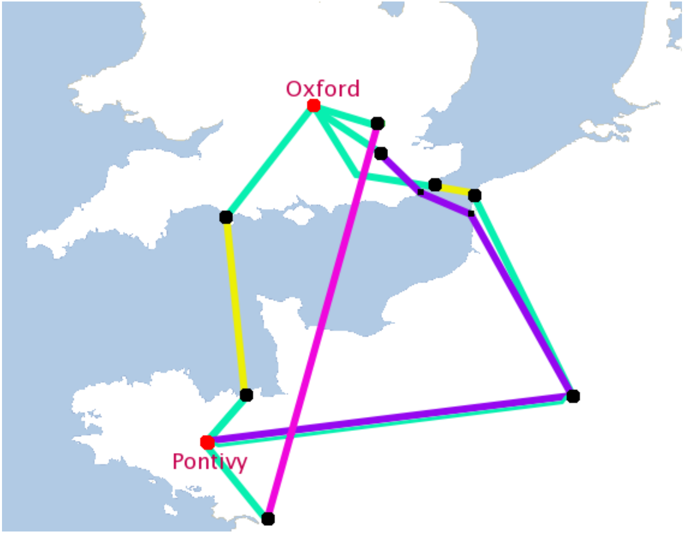
Modelling and analyzing resources in timed systems

Patricia Bouyer-Decitre

LSV, CNRS & ENS Cachan, France

Outline

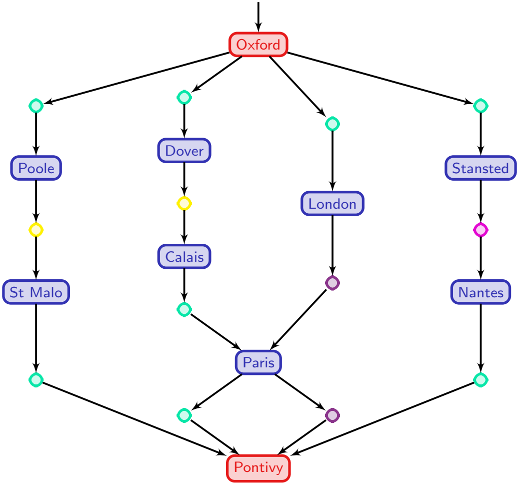
A starting example



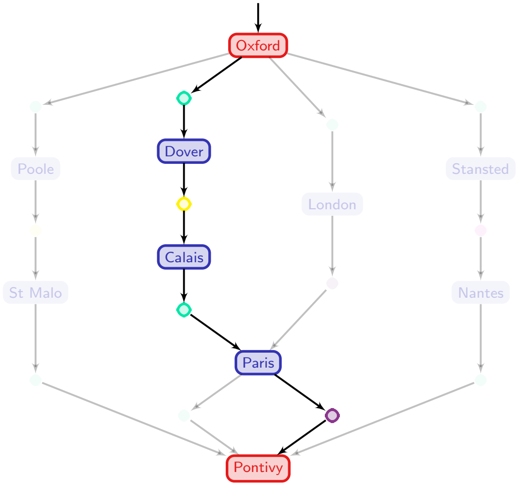
Natural questions

- **Can I reach Pontivy from Oxford?**
- **What is the minimal time to reach Pontivy from Oxford?**
- **What is the minimal fuel consumption to reach Pontivy from Oxford?**
- **What if there is an unexpected event?**
- **Can I use my computer all the way?**

A first model of the system

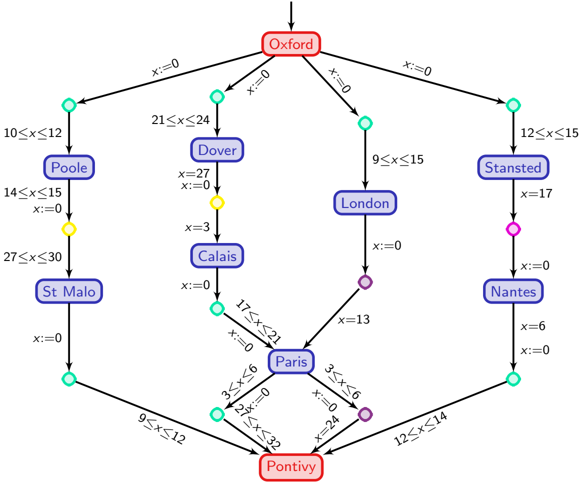


Can I reach Pontivy from Oxford?

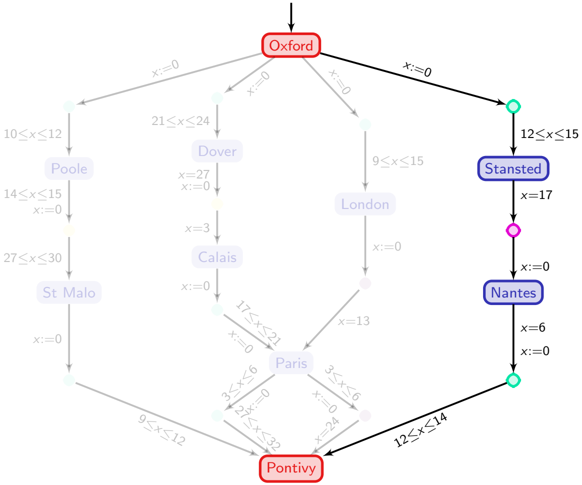


This is a reachability question in a finite graph: Yes, I can!

A second model of the system

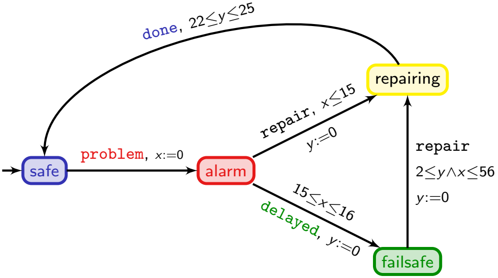


How long will that take?

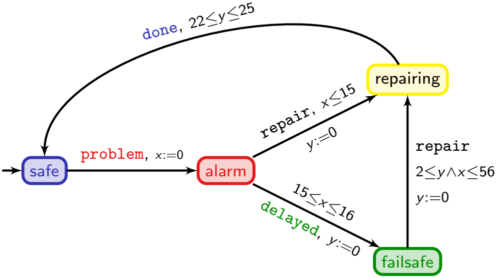


It is a reachability (and optimization) question
in a timed automaton: at least 350mn = 5h50mn!

An example of a timed automaton

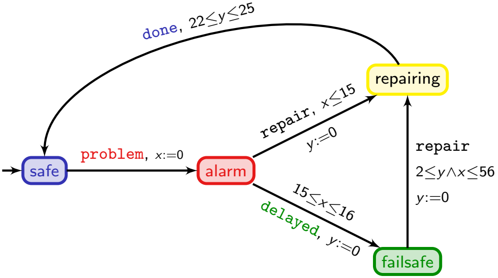


An example of a timed automaton



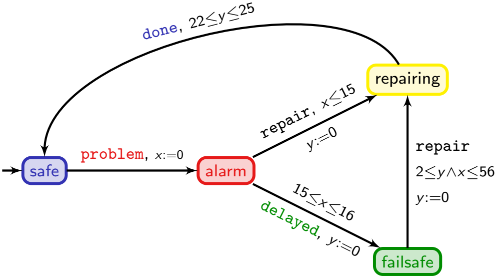
	safe
x	0
y	0

An example of a timed automaton



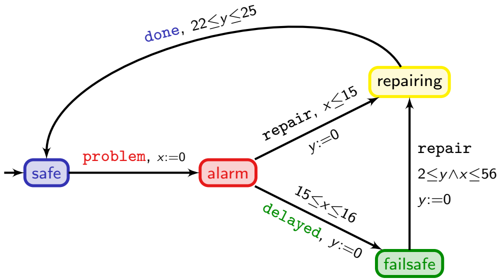
	safe	$\xrightarrow{23}$	safe
x	0		23
y	0		23

An example of a timed automaton



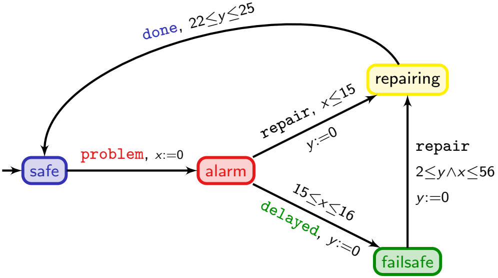
	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm
x	0		23		0
y	0		23		23

An example of a timed automaton



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm
x	0		23		0		15.6
y	0		23		23		38.6

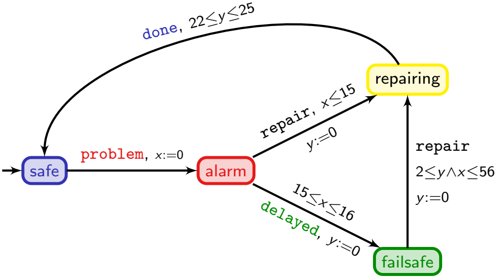
An example of a timed automaton



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	$\cdots$
y	0		23		23		38.6		0	

	failsafe
$\cdots$	15.6
	0

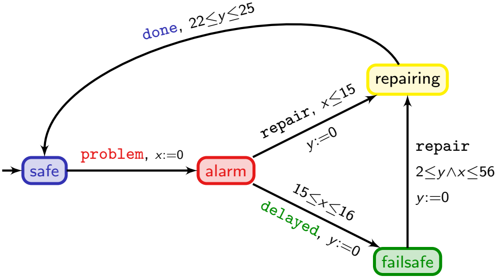
An example of a timed automaton



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe
⋯	15.6		17.9
	0		2.3

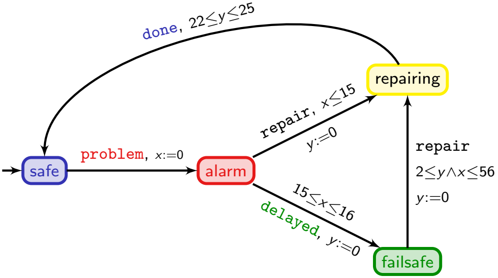
An example of a timed automaton



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	···
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing
···	15.6		17.9		17.9
	0		2.3		0

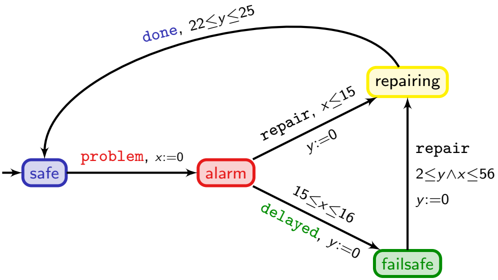
An example of a timed automaton



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	$\cdots$
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing
$\cdots$	15.6		17.9		17.9		40
	0		2.3		0		22.1

An example of a timed automaton



$$\begin{array}{cccccccccccc}
 & \text{safe} & \xrightarrow{23} & \text{safe} & \xrightarrow{\texttt{problem}} & \text{alarm} & \xrightarrow{15.6} & \text{alarm} & \xrightarrow{\texttt{delayed}} & \text{failsafe} & \\
\text{x} & 0 & & 23 & & 0 & & 15.6 & & 15.6 & \cdots \\
\text{y} & 0 & & 23 & & 23 & & 38.6 & & 0 &
\end{array}$$

$$\begin{array}{cccccccccccc}
 & \text{failsafe} & \xrightarrow{2.3} & \text{failsafe} & \xrightarrow{\texttt{repair}} & \text{repairing} & \xrightarrow{22.1} & \text{repairing} & \xrightarrow{\texttt{done}} & \text{safe} \\
\cdots & 15.6 & & 17.9 & & 17.9 & & 40 & & 40 \\
 & 0 & & 2.3 & & 0 & & 22.1 & & 22.1
\end{array}$$

Timed automata

Theorem [AD90,CY92]

The (time-optimal) reachability problem is decidable (and PSPACE-complete) for timed automata.

[AD90] Alur, Dill. Automata for modeling real-time systems *(ICALP'90)*.
[CY92] Courcoubetis, Yannakakis. Minimum and maximum delay problems in real-time systems *(Formal Methods in System Design)*.

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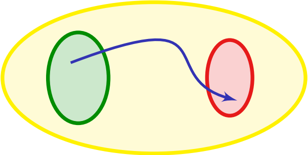
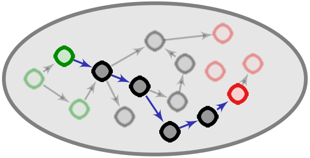


timed automaton

finite bisimulation



large (but finite) automaton
(region automaton)

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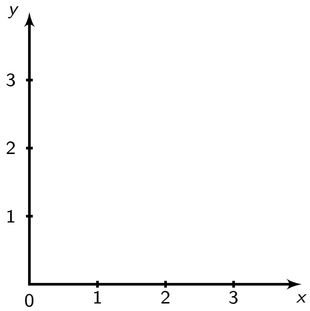
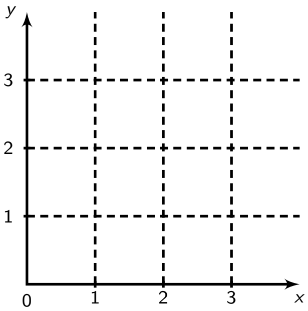
[CY92] Courcoubetis, Yannakakis. Minimum and maximum delay problems in real-time systems *(Formal Methods in System Design)*.

The region abstraction

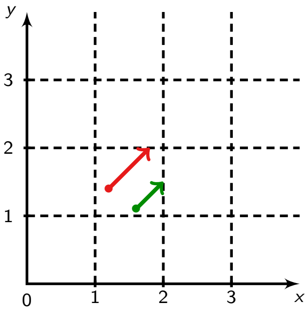


The region abstraction



- "compatibility" between regions and constraints

The region abstraction



- "compatibility" between regions and constraints
- "compatibility" between regions and time elapsing

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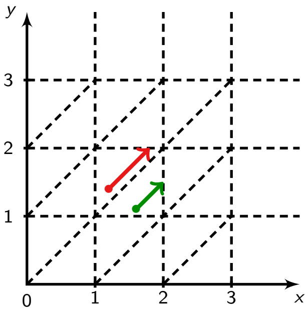

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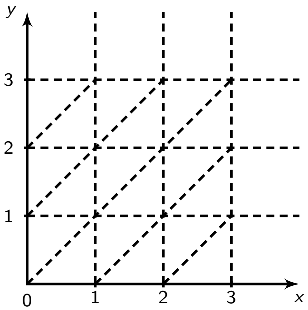

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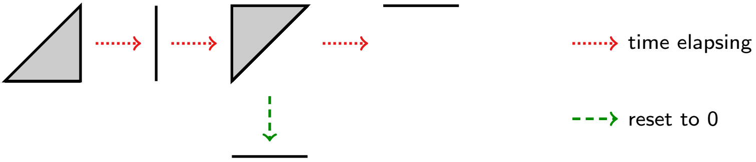
- "compatibility" between regions and time elapsing

$\rightsquigarrow$ an equivalence of finite index
a time-abstract bisimulation

The region abstraction



Outline

1. Introduction
2. Modelling and optimizing resources in timed systems
3. Managing resources
4. Conclusion

Modelling resources in timed systems

- System resources might be relevant and even crucial information

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  - energy consumption,
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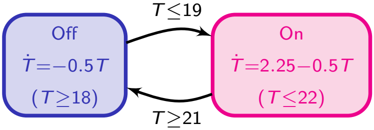
The thermostat example



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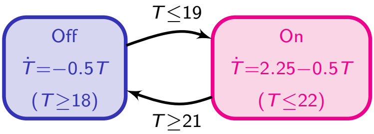

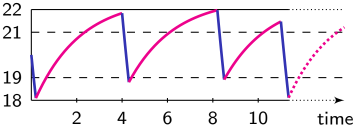

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Theorem [HKPV95]

The reachability problem is undecidable in hybrid automata.

[HKPV95] Henzinger, Kopke, Puri, Varaiya. What's decidable wbout hybrid automata? *(SToC'95)*.

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  - ...

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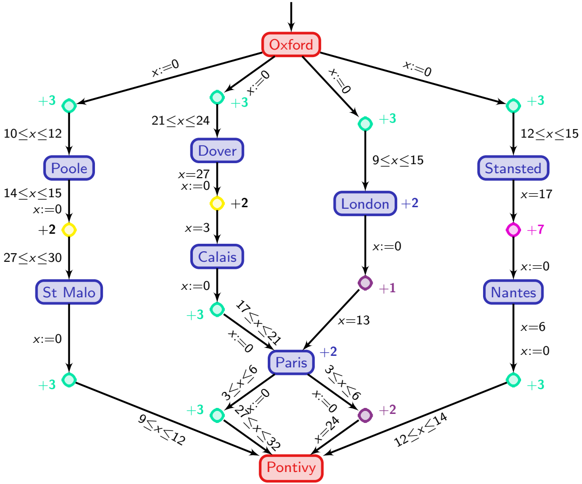
Theorem [HKPV95]

The reachability problem is undecidable in hybrid automata.

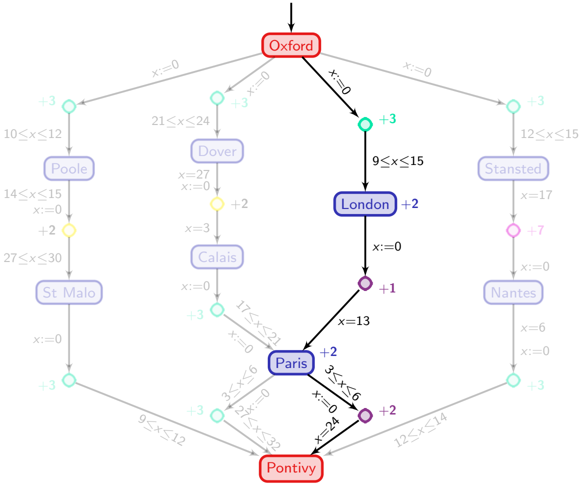
- An alternative: weighted/priced timed automata [ALP01,BFH+01]
  $\rightsquigarrow$ hybrid variables do not constrain the system
  hybrid variables are observer variables

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
[BFH+01] Behrmann, Fehnker, Hune, Larsen, Pettersson, Romijn, Vaandrager. Minimum-cost reachability in priced timed automata *(HSCC'01)*.

A third model of the system

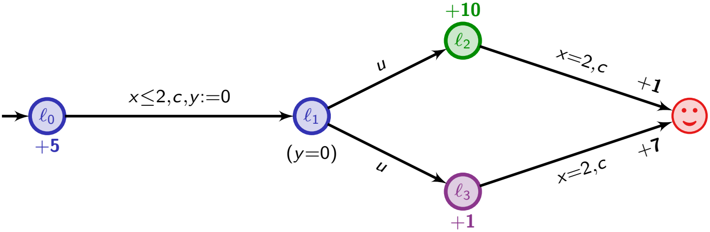


How much fuel will I use?



It is a quantitative (optimization) problem
in a priced timed automaton: at least 68 anti-planet units!

Weighted/priced timed automata [ALP01,BFH+01]



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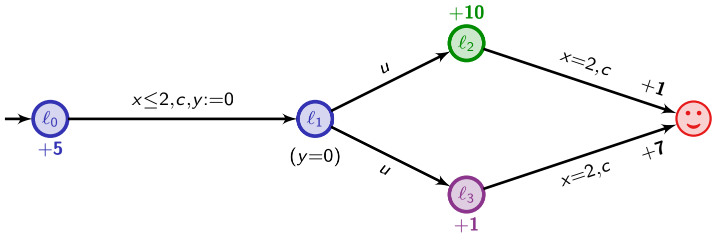

	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

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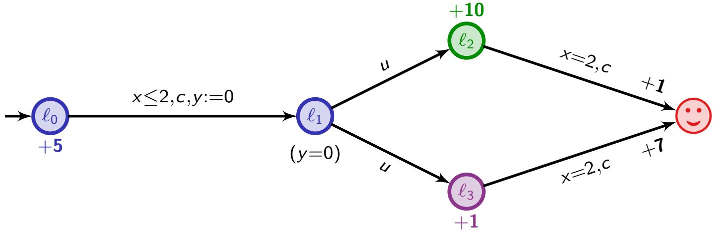

$$
\begin{array}{ccccccccccccc}
 & \ell_0 & \xrightarrow{1.3} & \ell_0 & \xrightarrow{c} & \ell_1 & \xrightarrow{u} & \ell_3 & \xrightarrow{0.7} & \ell_3 & \xrightarrow{c} & ☺ \\
x & 0 & & 1.3 & & 1.3 & & 1.3 & & 2 & & \\
y & 0 & & 1.3 & & 0 & & 0 & & 0.7 & &
\end{array}
$$

cost :

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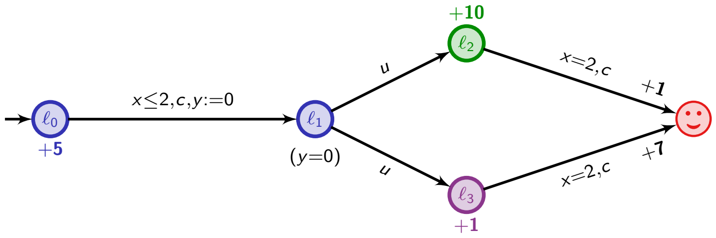

	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : 6.5

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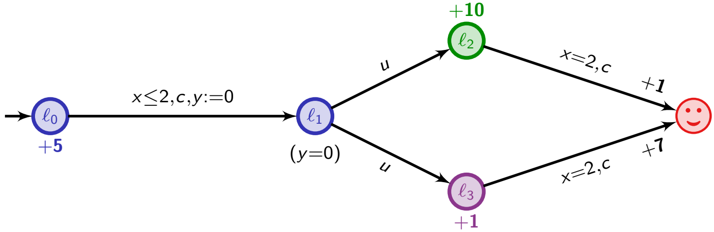

$$
\begin{array}{cccccccccccc}
 & \ell_0 & \xrightarrow{1.3} & \ell_0 & \xrightarrow{c} & \ell_1 & \xrightarrow{u} & \ell_3 & \xrightarrow{0.7} & \ell_3 & \xrightarrow{c} & ☺ \\
x & 0 & & 1.3 & & 1.3 & & 1.3 & & 2 & & \\
y & 0 & & 1.3 & & 0 & & 0 & & 0.7 & &
\end{array}
$$

cost : $\quad 6.5 \quad + \quad 0$

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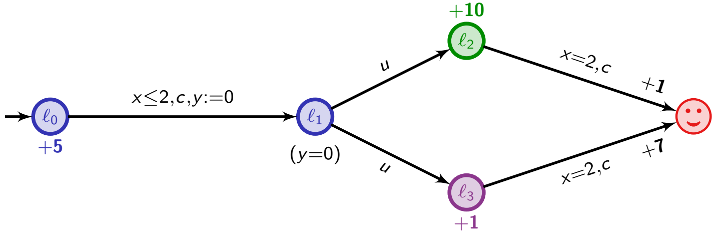
Weighted/priced timed automata [ALP01,BFH+01]



	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		
cost :		6.5	+	0	+	0					

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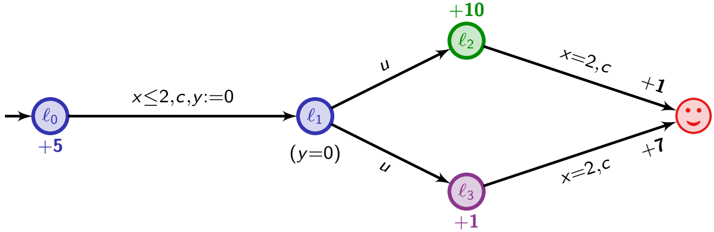
Weighted/priced timed automata [ALP01,BFH+01]



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x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : $6.5 + 0 + 0 + 0.7$

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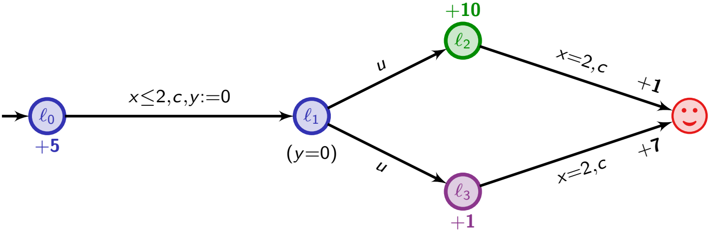

	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		
cost :		6.5	+	0	+	0	+	0.7	+	7	

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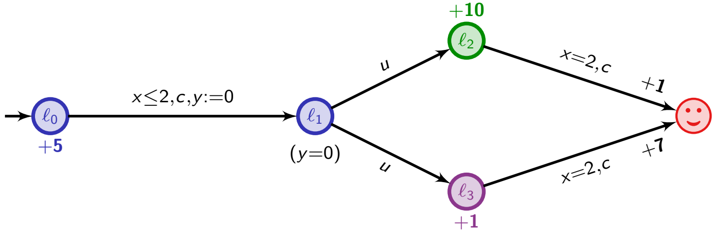
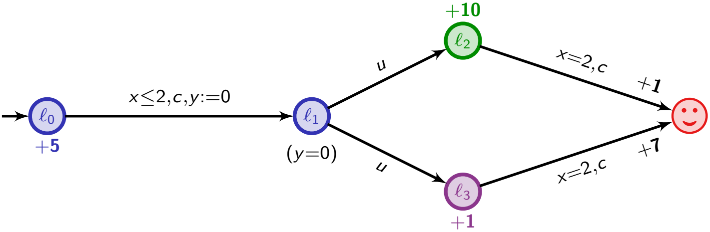


	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺		
x	0		1.3		1.3		1.3		2				
y	0		1.3		0		0		0.7				
cost :		6.5	+	0	+	0	+	0.7	+	7		=	14.2

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
[BFH+01] Behrmann, Fehnker, Hune, Larsen, Pettersson, Romijn, Vaandrager. Minimum-cost reachability in priced timed automata *(HSCC'01)*.

Weighted/priced timed automata [ALP01,BFH+01]



Question: what is the optimal cost for reaching  ?

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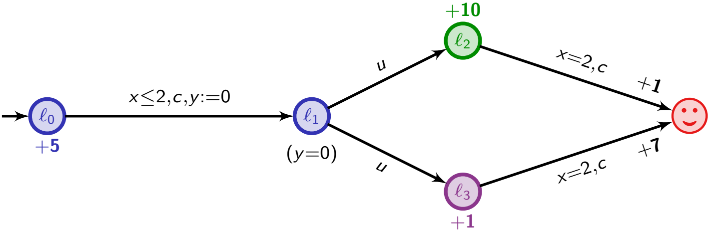

Question: what is the optimal cost for reaching 🙂?

$$5t + 10(2 - t) + 1$$

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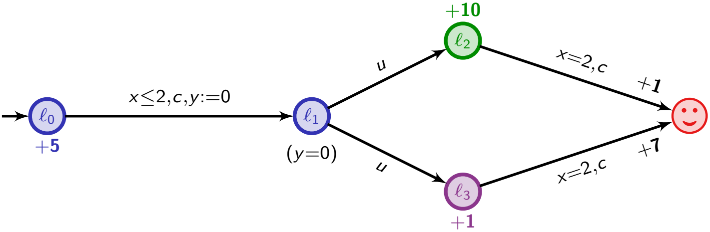

Question: what is the optimal cost for reaching 🙂 ?

$$5t + 10(2-t) + 1\ ,\ 5t + (2-t) + 7$$

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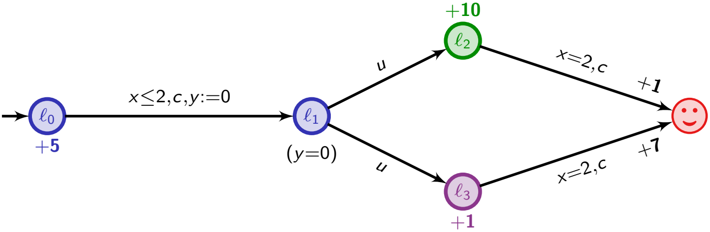

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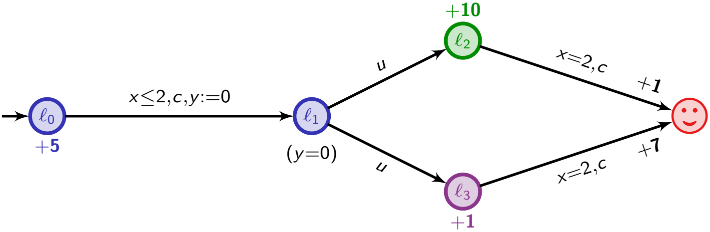

Question: what is the optimal cost for reaching 🙂?

$$\inf_{0 \leq t \leq 2} \min\left(5t + 10(2-t) + 1 \, , \, 5t + (2-t) + 7 \right) = 9$$

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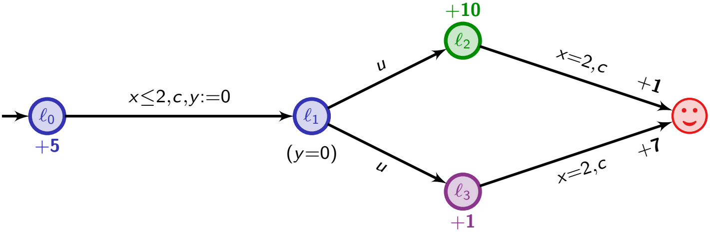

Question: what is the optimal cost for reaching 🙂?

$$\inf_{0 \leq t \leq 2} \min\left(5t + 10(2-t) + 1 \;,\; 5t + (2-t) + 7 \right) = 9$$

$\rightsquigarrow$ *strategy:* leave immediately ℓ_0, go to ℓ_3, and wait there 2 t.u.

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
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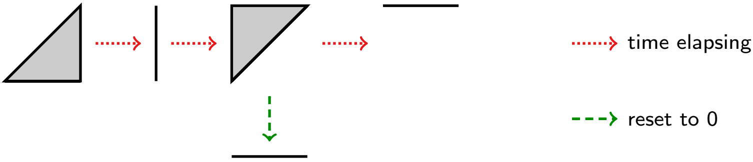
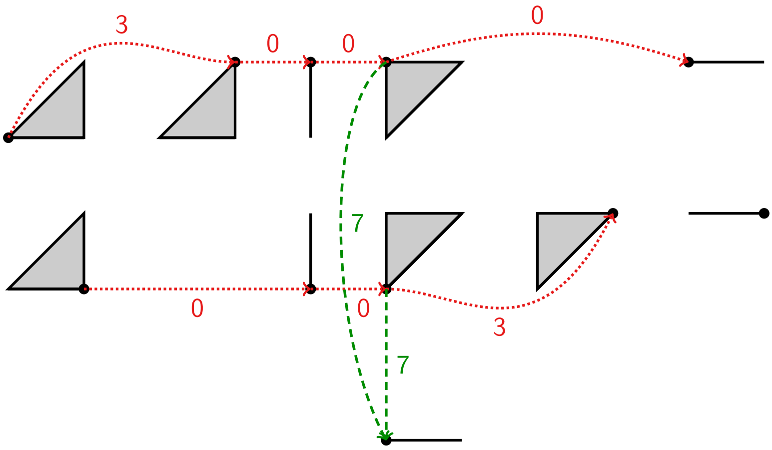
The region abstraction is not fine enough



The corner-point abstraction



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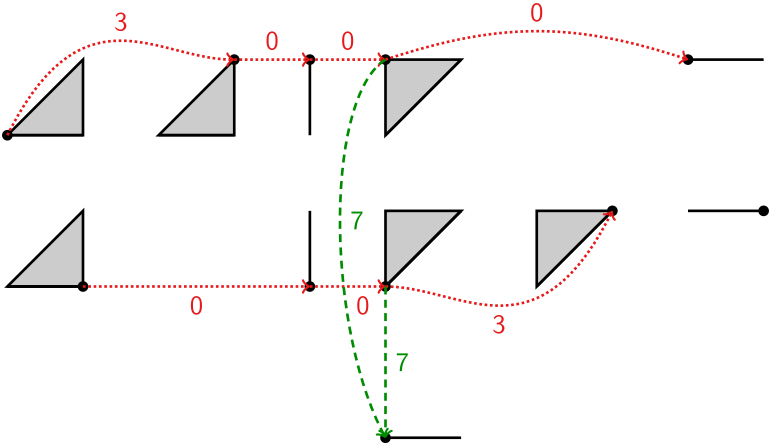

We can somehow discretize the behaviours...

From timed to discrete behaviours

Optimal reachability as a linear programming problem

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$\circ \xrightarrow{t_1} \circ \xrightarrow{t_2} \circ \xrightarrow{t_3} \circ \xrightarrow{t_4} \circ \xrightarrow{t_5} \circ \quad \cdots$$

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$\circ \xrightarrow{t_1} \circ \xrightarrow[x\leq 2]{t_2} \circ \xrightarrow{t_3} \circ \xrightarrow{t_4} \circ \xrightarrow{t_5} \circ \quad \cdots \qquad \left\{ \begin{array}{l} t_1+t_2\leq 2 \\ \\ \end{array} \right.$$

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$\circ \xrightarrow[y:=0]{t_1} \circ \xrightarrow[x\leq 2]{t_2} \circ \xrightarrow{t_3} \circ \xrightarrow[y\geq 5]{t_4} \circ \xrightarrow{t_5} \circ \quad \cdots \qquad \begin{cases} t_1+t_2\leq 2 \\ t_2+t_3+t_4\geq 5 \end{cases}$$

From timed to discrete behaviours

Optimal reachability as a linear programming problem

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Lemma

Let Z be a bounded zone and f be a function

$$f : (t_1, \ldots, t_n) \mapsto \sum_{i=1}^{n} c_i t_i + c$$

well-defined on $\overline{Z}$. Then $inf_Z f$ is obtained on the border of $\overline{Z}$ with integer coordinates.

From timed to discrete behaviours

Optimal reachability as a linear programming problem

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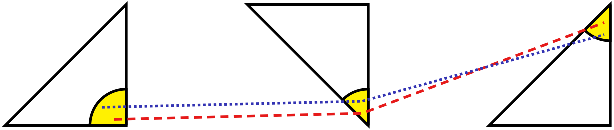
$\rightsquigarrow$ for every finite path π in $\mathcal{A}$, there exists a path Π in $\mathcal{A}_{\text{cp}}$ such that

$$\text{cost}(\Pi) \leq \text{cost}(\pi)$$

[Π is a "corner-point projection" of π]

From discrete to timed behaviours

Approximation of abstract paths:



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From discrete to timed behaviours

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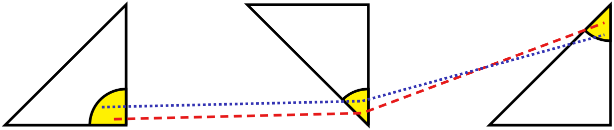
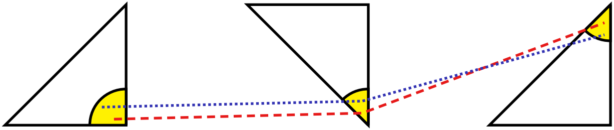


For any path Π of $\mathcal{A}_{\mathsf{cp}}$, for any $\varepsilon > 0$,

From discrete to timed behaviours

Approximation of abstract paths:



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$$\|\Pi - \pi_\varepsilon\|_\infty < \varepsilon$$

From discrete to timed behaviours

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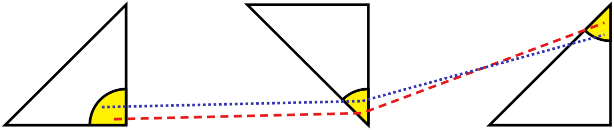

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$$\|\Pi - \pi_\varepsilon\|_\infty < \varepsilon \Rightarrow |\mathrm{cost}(\Pi) - \mathrm{cost}(\pi_\varepsilon)| < \eta$$

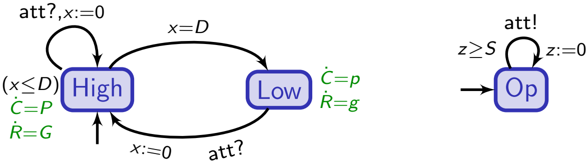
Optimal-cost reachability

Theorem [ALP01,BFH+01,BBBR07]

The optimal-cost reachability problem is decidable (and PSPACE-complete) in (priced) timed automata.

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
[BFH+01] Behrmann, Fehnker, Hune, Larsen, Pettersson, Romijn, Vaandrager. Minimum-cost reachability in priced timed automata *(HSCC'01)*.
[BBBR07] Bouyer, Brihaye, Bruyère, Raskin. On the optimal reachability problem *(Formal Methods in System Design)*.

Going further 1: mean-cost optimization



[BBL08] Bouyer, Brinksma, Larsen. Optimal infinite scheduling for multi-priced timed automata *(Formal Methods in System Designs)*.

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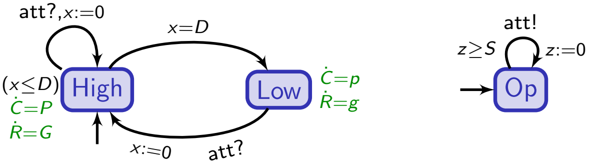

$\rightsquigarrow$ compute optimal infinite schedules that minimize

$$\text{mean-cost}(\pi) = \limsup_{n \to +\infty} \frac{\text{cost}(\pi_n)}{\text{reward}(\pi_n)}$$

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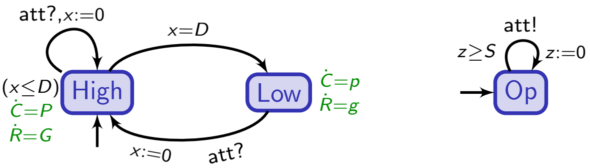

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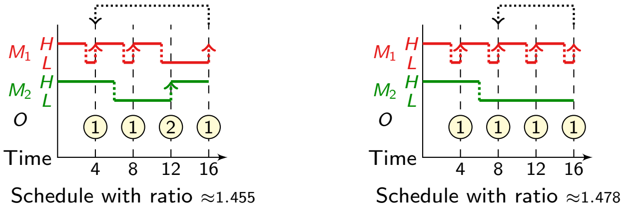

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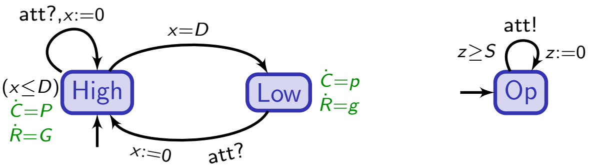

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Theorem [BBL08]

The mean-cost optimization problem is decidable (and PSPACE-complete) for priced timed automata.

$\rightsquigarrow$ the corner-point abstraction can be used

[BBL08] Bouyer, Brinksma, Larsen. Optimal infinite scheduling for multi-priced timed automata *(Formal Methods in System Designs)*.

From timed to discrete behaviours

- **Finite behaviours:** based on the following property

Lemma

Let Z be a bounded zone and f be a function

$$f : (t_1, \ldots, t_n) \mapsto \frac{\sum_{i=1}^{n} c_i t_i + c}{\sum_{i=1}^{n} r_i t_i + r}$$

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mean-cost$(\Pi) \leq$ mean-cost(π)

From timed to discrete behaviours

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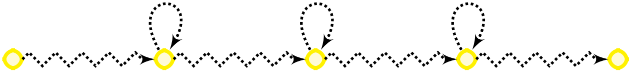
- **Infinite behaviours:** decompose each sufficiently long projection into cycles:



The (acyclic) linear part will be negligible!

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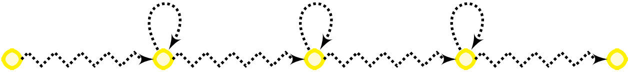
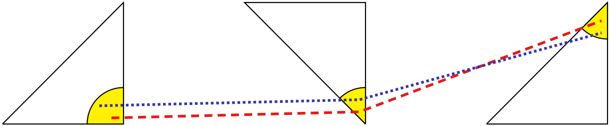


The (acyclic) linear part will be negligible!

$\rightsquigarrow$ the optimal cycle of $\mathcal{A}_{\mathsf{cp}}$ is better than any infinite path of $\mathcal{A}$!

From discrete to timed behaviours

Approximation of abstract paths:



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From discrete to timed behaviours

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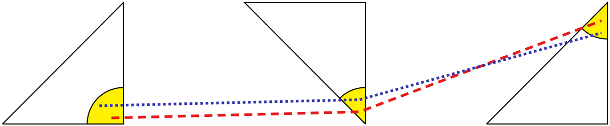
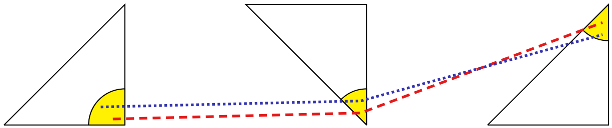


For any path Π of $\mathcal{A}_{\mathsf{cp}}$, for any $\varepsilon > 0$,

From discrete to timed behaviours

Approximation of abstract paths:



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From discrete to timed behaviours

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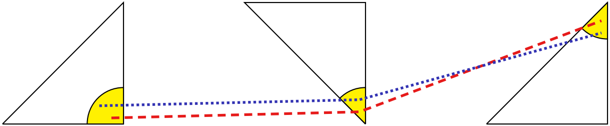

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Going further 2: concavely-priced cost functions

$\rightsquigarrow$ A general abstract framework for quantitative timed systems

Theorem [JT08]

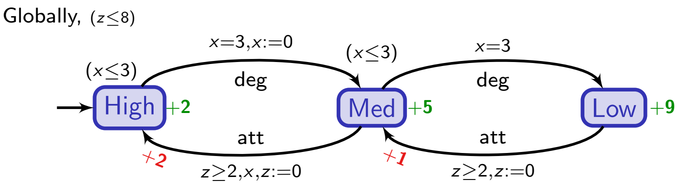
Optimal cost in concavely-priced timed automata is computable, if we restrict to quasi-concave price functions. For the following cost functions, the (decision) problem is even PSPACE-complete:

- optimal-time and optimal-cost reachability;
- optimal discrete discounted cost;
- optimal average-time and average-cost;
- optimal mean-cost.

$\rightsquigarrow$ a slight extension of corner-point abstraction can be used

[JT08] Judziński, Trivedi. Concavely-priced timed automata *(FORMATS'08)*.

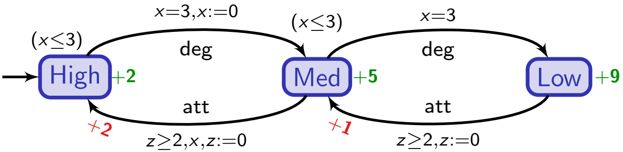
Going further 3: discounted-time cost optimization



[FL08] Fahrenberg, Larsen. Discount-optimal infinite runs in priced timed automata *(INFINITY'08)*.

Going further 3: discounted-time cost optimization

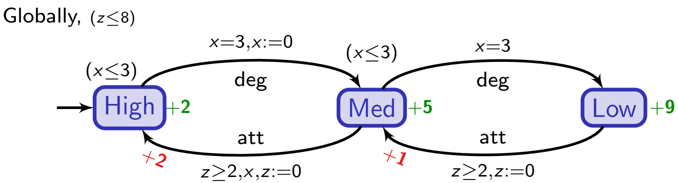
Globally, $(z \leq 8)$



$\rightsquigarrow$ compute optimal infinite schedules that minimize
discounted cost over time

[FL08] Fahrenberg, Larsen. Discount-optimal infinite runs in priced timed automata *(INFINITY'08)*.

Going further 3: discounted-time cost optimization



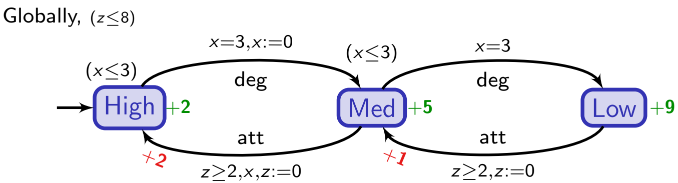
$\rightsquigarrow$ compute optimal infinite schedules that minimize

$$\text{discounted-cost}_{\lambda}(\pi) = \sum_{n \geq 0} \lambda^{T_n} \int_{t=0}^{\tau_{n+1}} \lambda^t \text{cost}(\ell_n)\, \mathrm{d}t + \lambda^{T_{n+1}} \text{cost}(\ell_n \xrightarrow{a_{n+1}} \ell_{n+1})$$

if $\pi = (\ell_0, v_0) \xrightarrow{\tau_1, a_1} (\ell_1, v_1) \xrightarrow{\tau_2, a_2} \cdots$ and $T_n = \sum_{i \leq n} \tau_i$

[FL08] Fahrenberg, Larsen. Discount-optimal infinite runs in priced timed automata *(INFINITY'08)*.

Going further 3: discounted-time cost optimization



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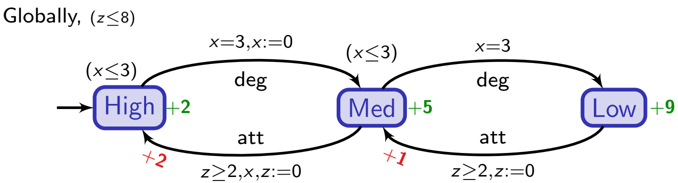
Going further 3: discounted-time cost optimization



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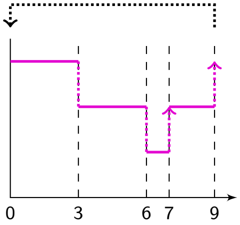
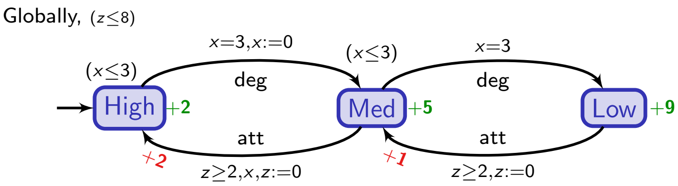


if $\lambda = e^{-1}$, the discounted cost of that infinite schedule is ≈ 2.16

[FL08] Fahrenberg, Larsen. Discount-optimal infinite runs in priced timed automata *(INFINITY'08)*.

Going further 3: discounted-time cost optimization



$\leadsto$ compute optimal infinite schedules that minimize discounted cost over time

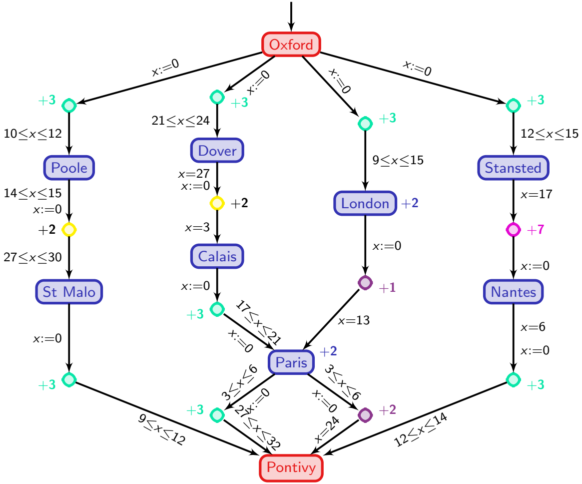
Theorem [FL08]

The optimal discounted cost is computable in EXPTIME in priced timed automata.

$\leadsto$ the corner-point abstraction can be used

[FL08] Fahrenberg, Larsen. Discount-optimal infinite runs in priced timed automata *(INFINITY'08)*.

A fourth model of the system
What if there is an unexpected event?



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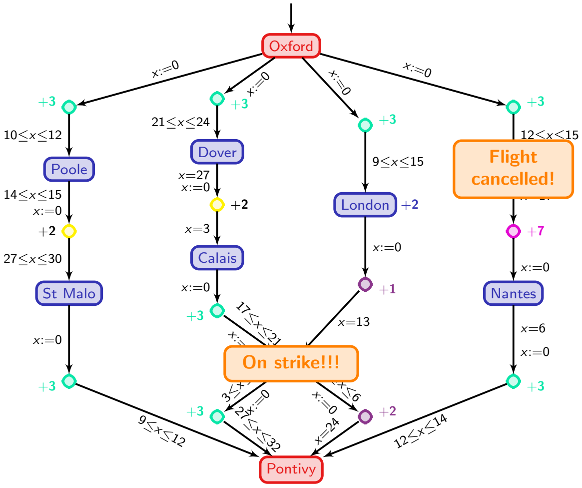
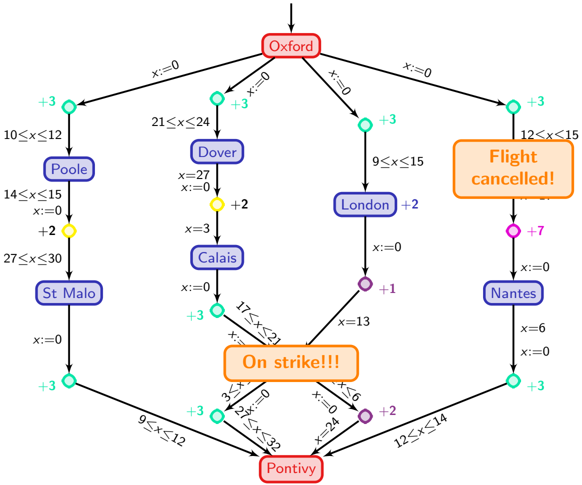
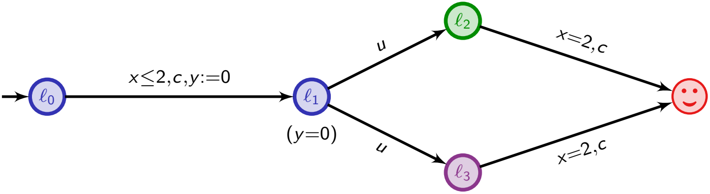


A fourth model of the system
What if there is an unexpected event?



$\rightsquigarrow$ modelled as timed games

A simple example of timed game



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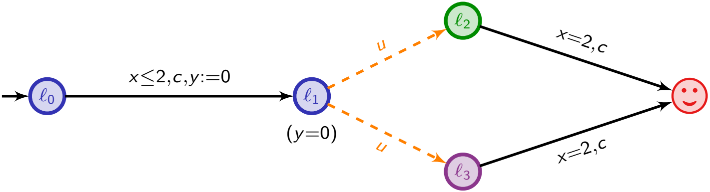
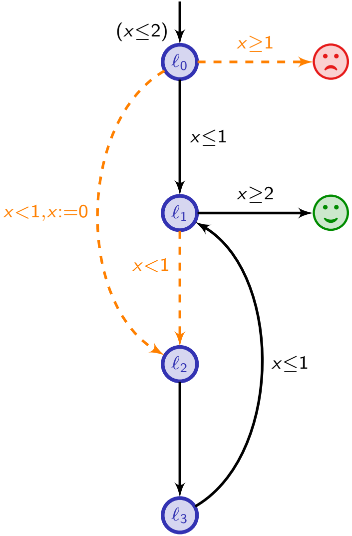


Another example



Decidability of timed games

Theorem [AMPS98,HK99]

Safety and reachability control in timed automata are decidable and EXPTIME-complete.

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Optimal-time reachability timed games are decidable and EXPTIME-complete.

[AM99] Asarin, Maler. As soon as possible: time optimal control for timed automata *(HSCC'99)*.
[BHPR07] Brihaye, Henzinger, Prabhu, Raskin. Minimum-time reachability in timed games *(ICALP'07)*.
[JT07] Jurdzinński, Trivedi. Reachability-time games on timed automata *(ICALP'07)*.

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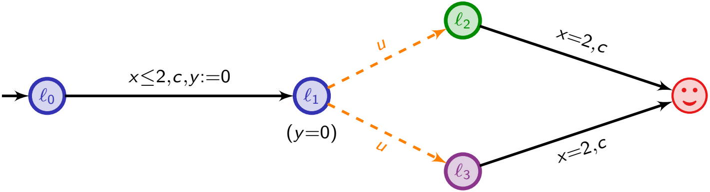
$\rightsquigarrow$ let's play with Uppaal Tiga! [BCD+07]

[BCD+07] Behrmann, Cougnard, David, Fleury, Larsen, Lime. Uppaal-Tiga: Time for playing games! *(CAV'07).*

Back to the simple example



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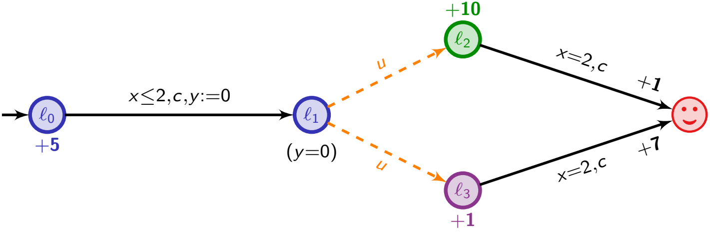

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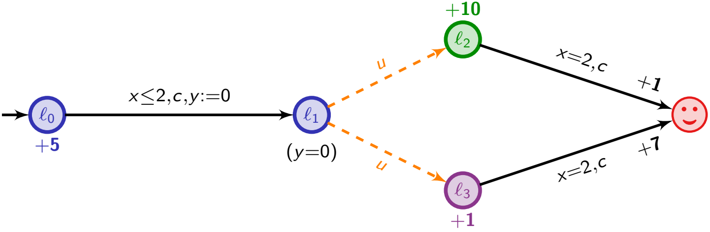

Question: what is the optimal cost we can ensure while reaching 🙂?

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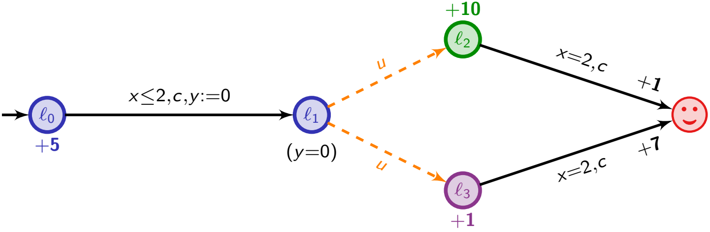

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$5t + 10(2 - t) + 1$

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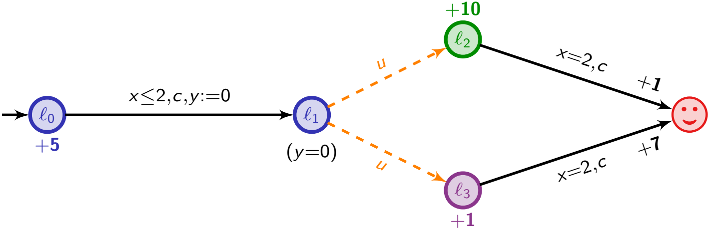

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$$5t + 10(2-t) + 1 \ , \ 5t + (2-t) + 7$$

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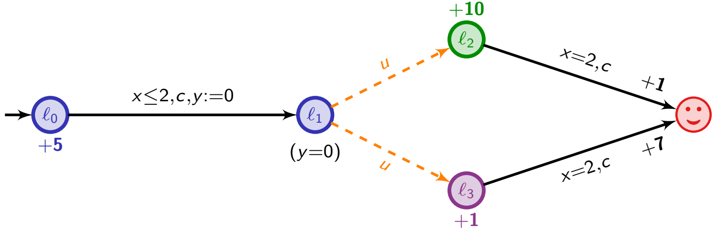

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$$\max\,(\;5t + 10(2-t) + 1\;,\;5t + (2-t) + 7\;)$$

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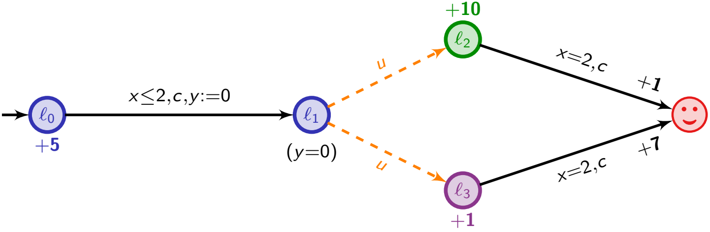

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$$\inf_{0 \leq t \leq 2} \max \left(5t + 10(2-t) + 1 \,,\ 5t + (2-t) + 7 \right) = 14 + \frac{1}{3}$$

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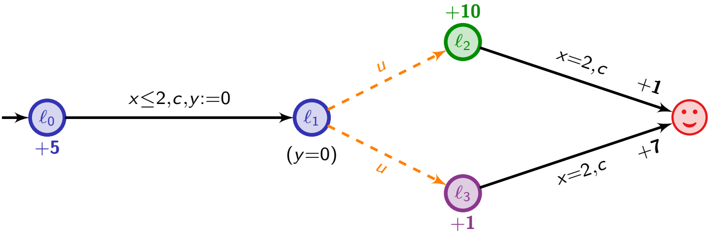

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$\rightsquigarrow$ *strategy:* wait in ℓ_0, and when $t = \frac{4}{3}$, go to ℓ_1

Optimal reachability in priced timed games

This topic has been fairly hot these last couple of years...

e.g. [LMM02,ABM04,BCFL04]

[LMM02] La Torre, Mukhopadhyay, Murano. Optimal-reachability and control for acyclic weighted timed automata *(TCS@02)*.
[ABM04] Alur, Bernardsky, Madhusudan. Optimal reachability in weighted timed games *(ICALP'04)*.
[BCFL04] Bouyer, Cassez, Fleury, Larsen. Optimal strategies in priced timed game automata *(FSTTCS'04)*.

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Optimal timed games are undecidable, as soon as automata have three clocks or more.

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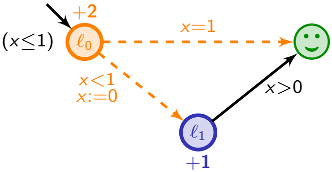

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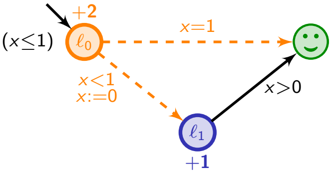

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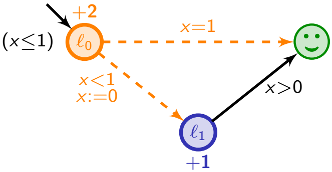

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- Rather involved proof of correctness for a simple algorithm.

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Given two clocks x and y, we can check whether $y = 2x$.

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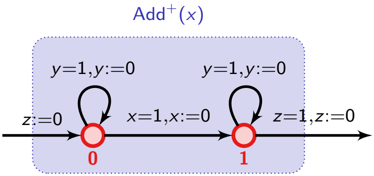
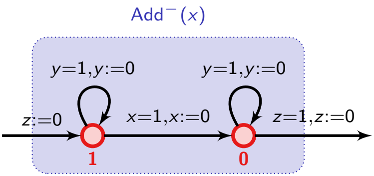


The cost is increased by x_0



The cost is increased by $1-x_0$

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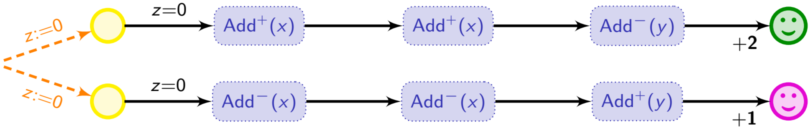

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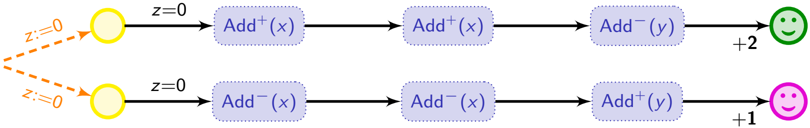

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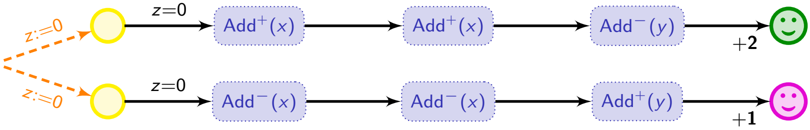

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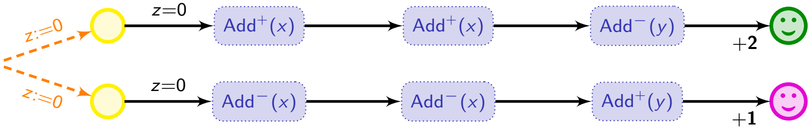

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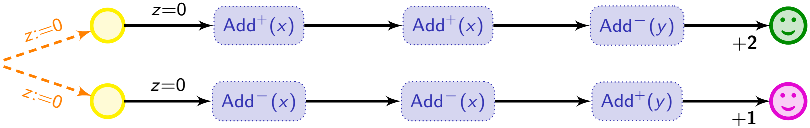

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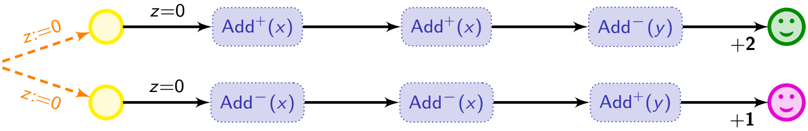

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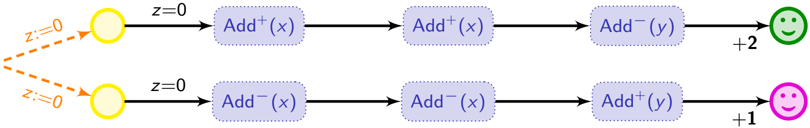

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  if $y_0 = 2x_0$, in both branches, cost $= 3$
- Player 1 has a winning strategy with cost ≤ 3 iff $y_0 = 2x_0$

The negative side: why is that hard?

Player 1 will simulate a two-counter machine:

- each instruction is encoded as a module;
- the values c_1 and c_2 of the counters are encoded by the values of two clocks:
$$x = \frac{1}{2^{c_1}} \qquad \text{and} \qquad y = \frac{1}{3^{c_2}}$$
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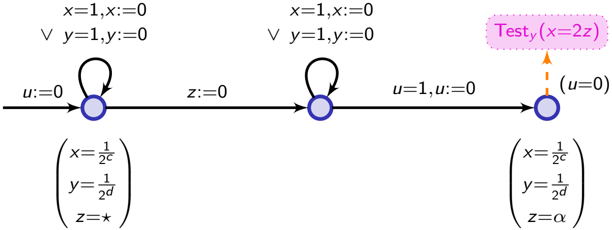
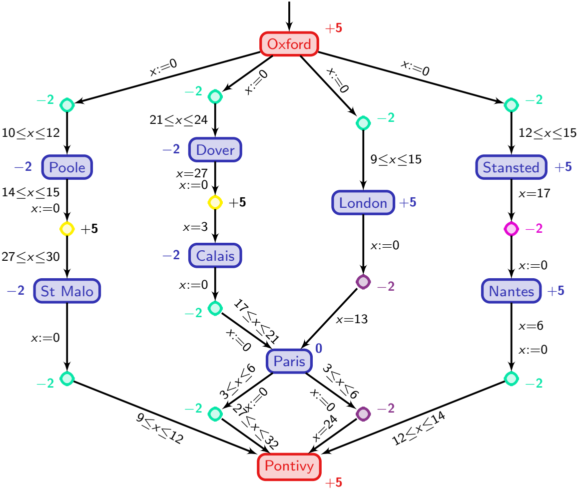
Globally, $(x \leq 1, y \leq 1, u \leq 1)$

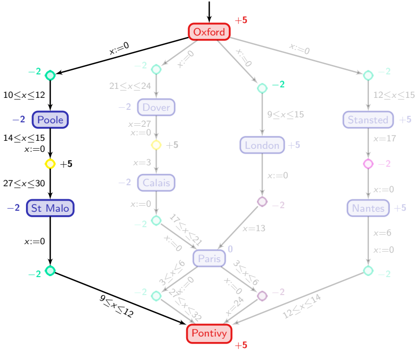


Outline

A fifth model of the system



Can I work with my computer all the way?



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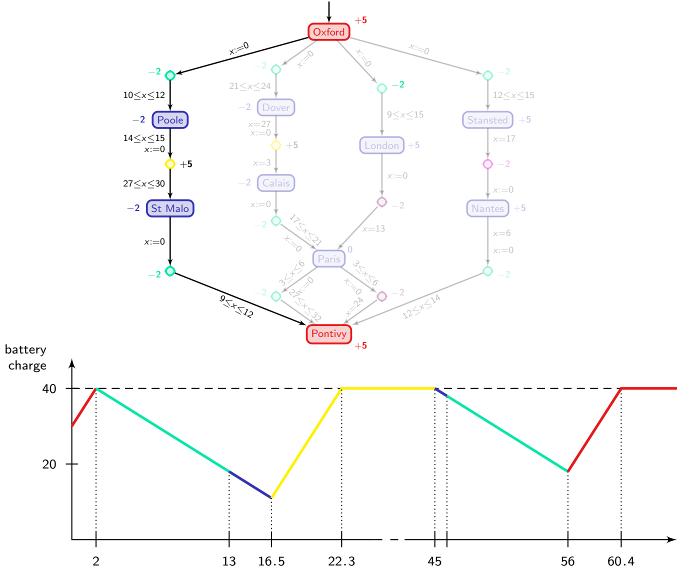
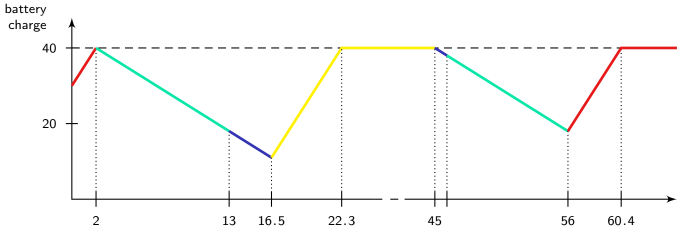


Can I work with my computer all the way?

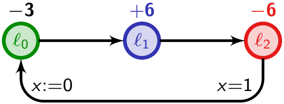
Energy is not only consumed, but can be regained.

$\rightsquigarrow$ the aim is to continuously satisfy some energy constraints.



An example of resource management

Globally $(x \leq 1)$



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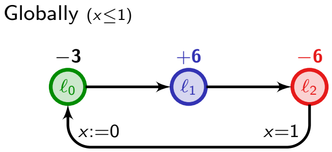

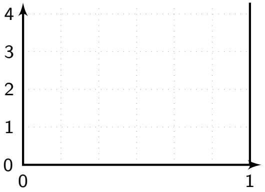

- Lower-bound problem: can we stay above 0?

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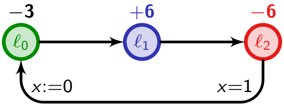

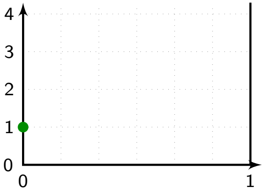

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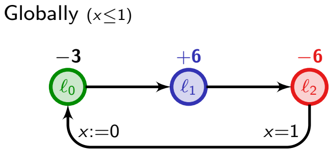

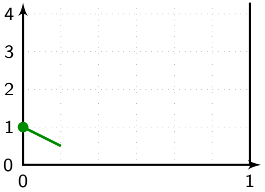

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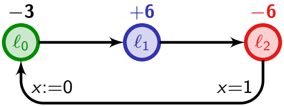

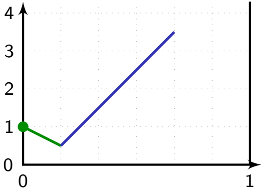

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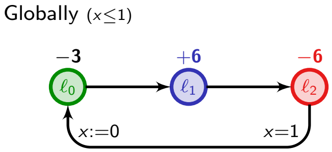

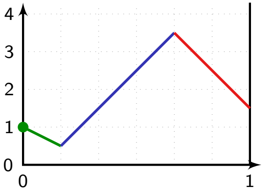

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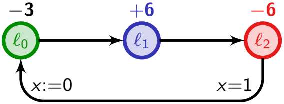

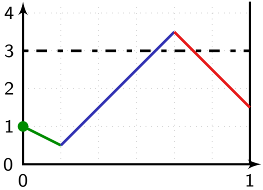

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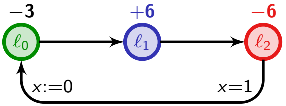
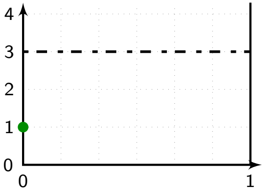
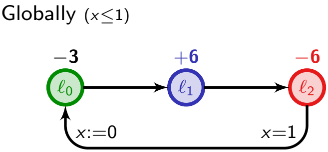
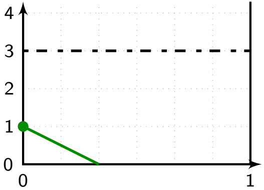




- Lower-bound problem
- Lower-upper-bound problem: can we stay within bounds?

An example of resource management





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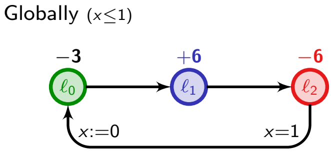

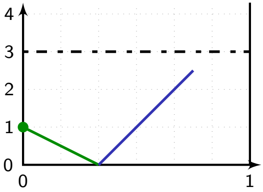

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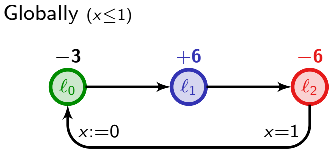

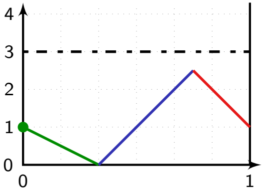

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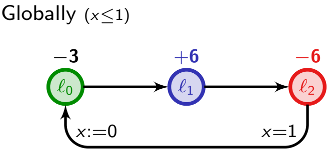

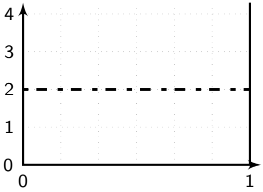

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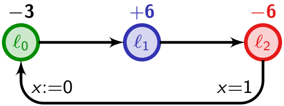

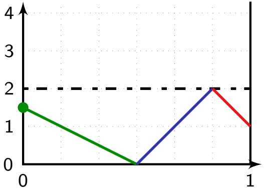

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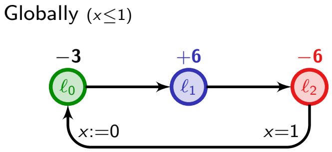

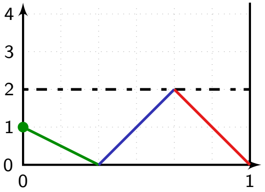

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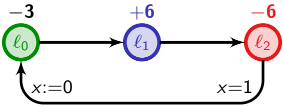

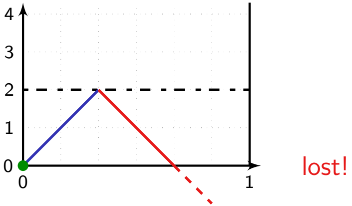

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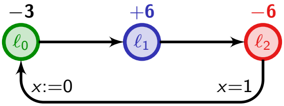

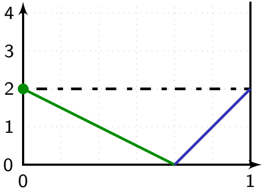

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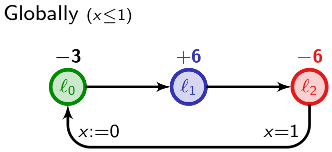

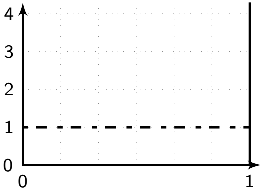

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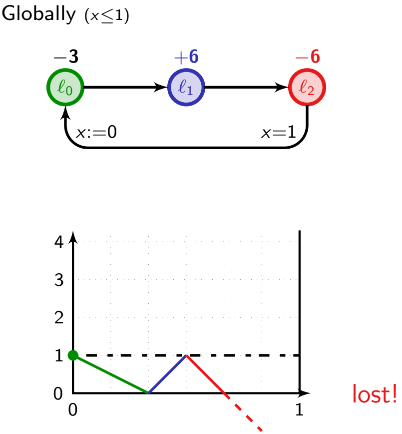

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An example of resource management

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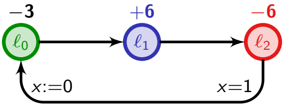
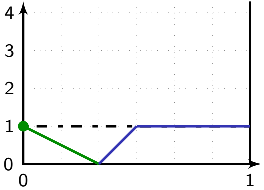
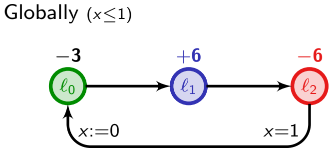
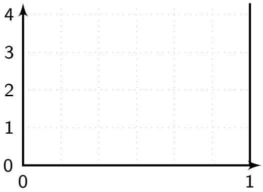




- Lower-bound problem
- Lower-upper-bound problem
- Lower-weak-upper-bound problem: can we "weakly" stay within bounds?

An example of resource management





- Lower–bound problem $\rightsquigarrow$ **L**
- Lower-upper-bound problem $\rightsquigarrow$ **L+U**
- Lower-weak-upper-bound problem $\rightsquigarrow$ **L+W**

Only partial results so far [BFLMS08]

0 clock!	**exist. problem**	**univ. problem**	**games**
L	$\in$ PTIME	$\in$ PTIME	$\in$ UP $\cap$ co-UP PTIME-hard
L+W	$\in$ PTIME	$\in$ PTIME	$\in$ NP $\cap$ co-NP PTIME-hard
L+U	$\in$ PSPACE NP-hard	$\in$ PTIME	EXPTIME-c.

[BFLMS08] Bouyer, Fahrenberg, Larsen, Markey, Srba. Infinite runs in weighted timed automata with energy constraints *(FORMATS'08)*.

Only partial results so far [BFLMS08]

1 clock	exist. problem	univ. problem	games
L	$\in$ PTIME	$\in$ PTIME	?
L+W	$\in$ PTIME	$\in$ PTIME	?
L+U	?	?	undecidable

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Only partial results so far [BFLMS08]

n clocks	exist. problem	univ. problem	games
L	?	?	?
L+W	?	?	?
L+U	?	?	undecidable

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Relation with mean-payoff games

Definition

Mean-payoff games: in a weighted game graph, does there exists a strategy s.t. the mean-cost of any play is nonnegative?

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- from **L**-games to mean-payoff games: transform the game as follows:

Single-clock $\mathbf{L}+\mathbf{U}$-games

Theorem

The single-clock $\mathbf{L}+\mathbf{U}$-games are undecidable.

Single-clock **L**+**U**-games

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We encode the behaviour of a two-counter machine:

- each instruction is encoded as a module;
- the values c_1 and c_2 of the counters are encoded by the energy level

$$e = 5 - \frac{1}{2^{c_1} \cdot 3^{c_2}}$$

when entering the corresponding module.

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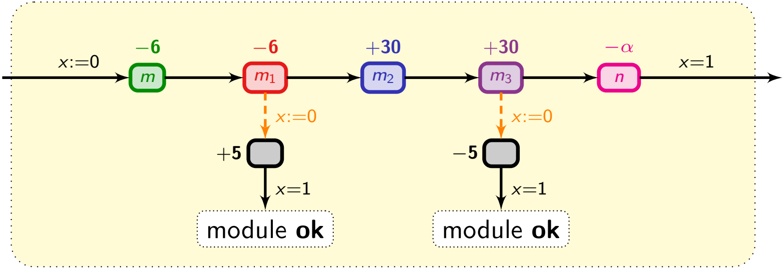
$\rightsquigarrow$ We present a generic construction for incrementing/decrementing the counters.

Generic module for incrementing/decrementing



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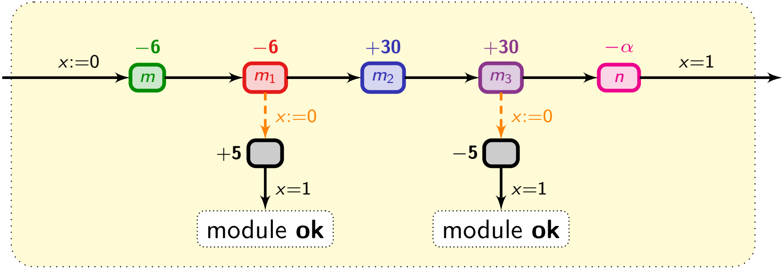

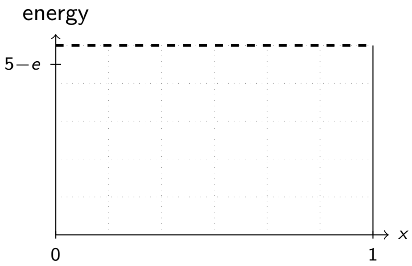

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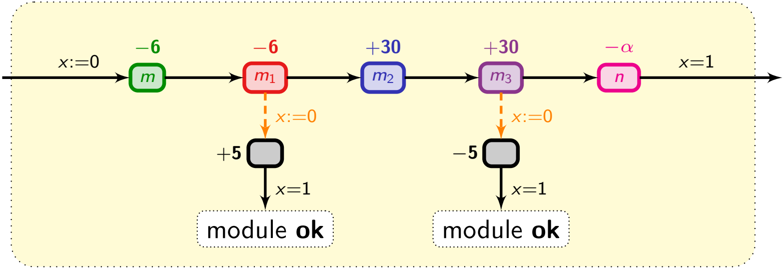

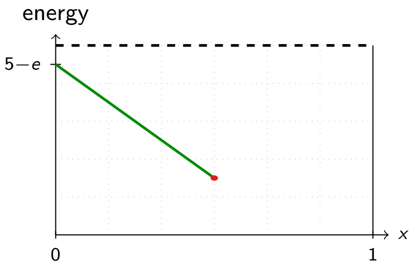

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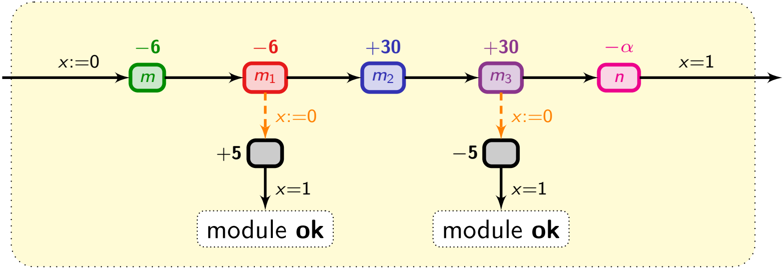

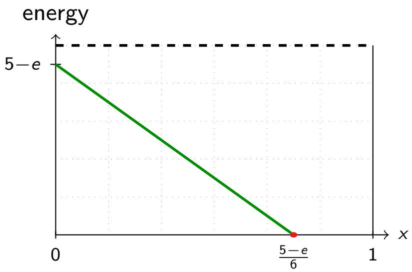

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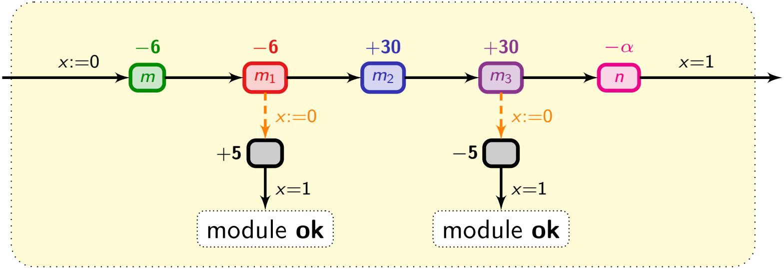

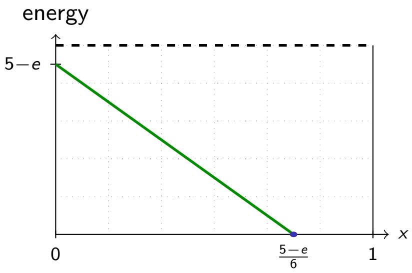

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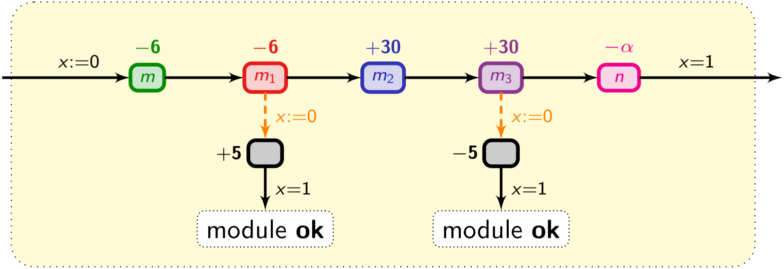

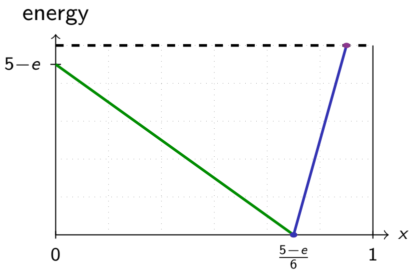

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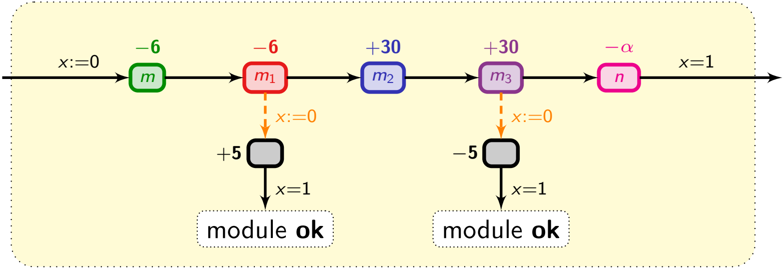

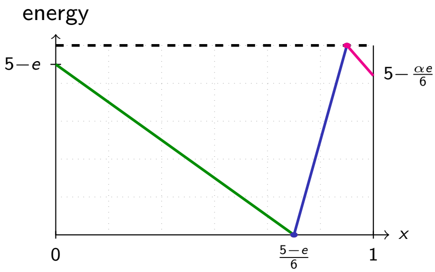

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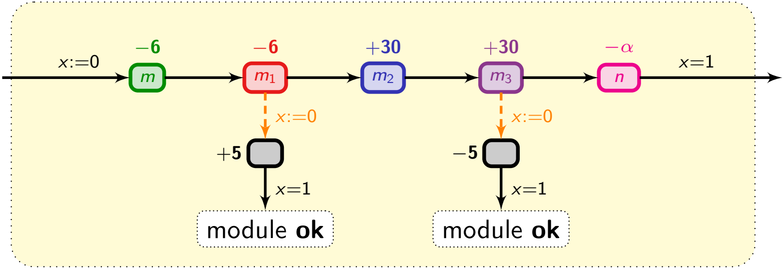
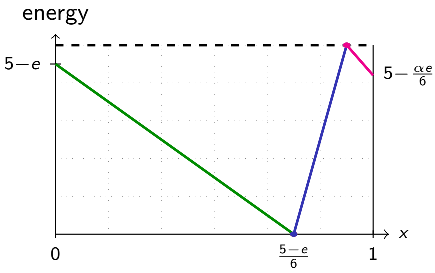




- $\alpha{=}3$: increment c_1
- $\alpha{=}2$: increment c_2
- $\alpha{=}12$: decrement c_1
- $\alpha{=}18$: decrement c_2

Outline

Some applications

Tools

- Uppaal (timed automata)
- Uppaal Cora (priced timed automata)
- Uppaal Tiga (timed games)

Case studies

- A lacquer production scheduling problem [BBHM05]
- Task graph scheduling problems [AKM03]
- An oil pump control problem [CJL+09]

[BBHM05] Behrmann, Brinksma, Hendriks, Mader. Scheduling lacquer production by reachability analysis - A case study *(IFAC'05)*.
[AKM03] Abdeddaïm, Kerbaa, Maler. Task graph scheduling using timed automata *(IPDPS'03)*.
[CJL+09] Cassez, Jessen, Larsen, Raskin, Reynier. Automatic synthesis of robust and optimal controllers - An industrial case study *(HSCC'09)*.

Task graph scheduling problems

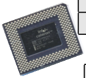
Compute $D \times (C \times (A+B)) + (A+B) + (C \times D)$ using two processors:

P_1 (fast):

time	
$+$	2 picoseconds
$\times$	3 picoseconds

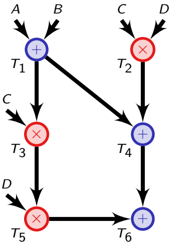
energy	
idle	10 Watt
in use	90 Watts

P_2 (slow):



time	
$+$	5 picoseconds
$\times$	7 picoseconds

energy	
idle	20 Watts
in use	30 Watts



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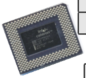

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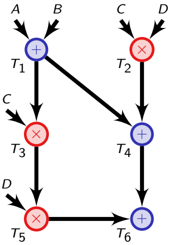

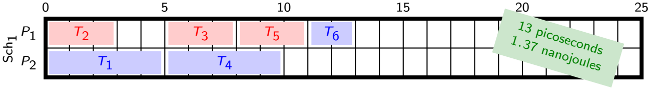

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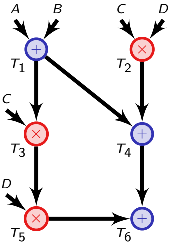

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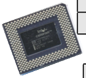

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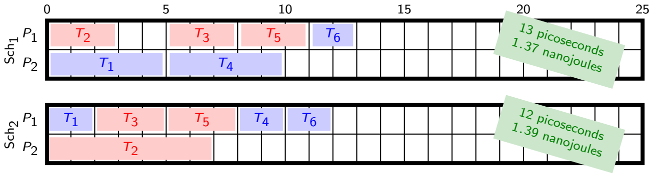

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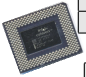

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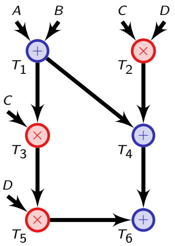

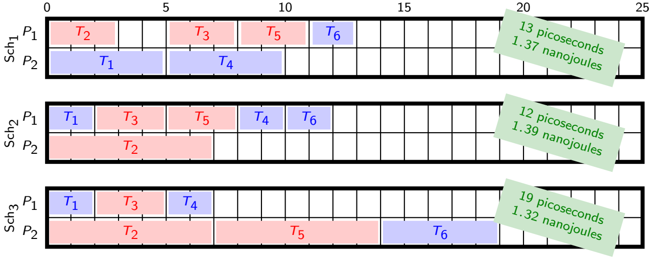

Modelling the task graph scheduling problem

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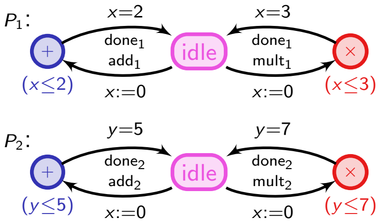
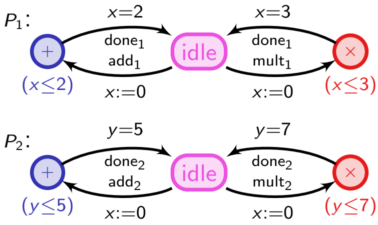
- Processors

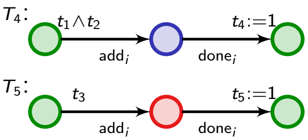


Modelling the task graph scheduling problem

- Processors



- Tasks



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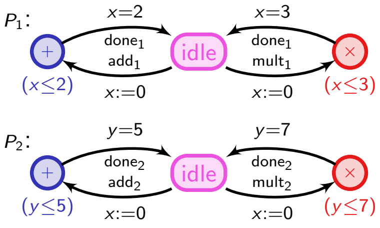
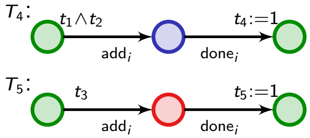
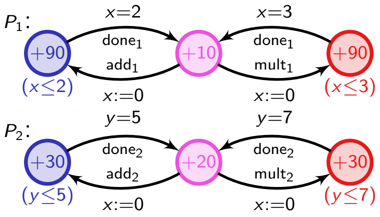


- Tasks



- Modelling energy



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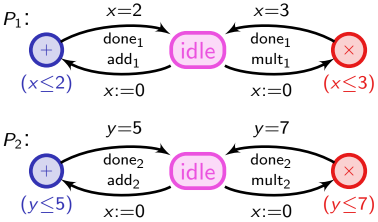
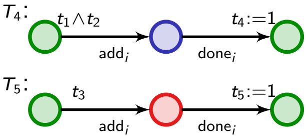


- Tasks



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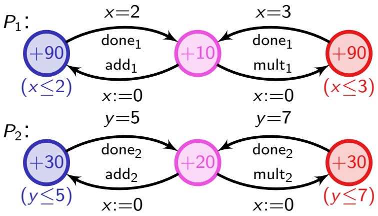
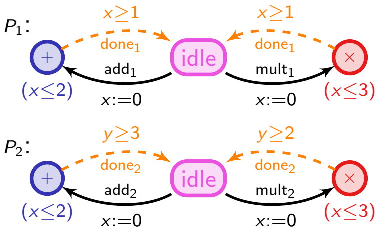


- Modelling uncertainty



Conclusion

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  ... and not all of them have been answered!

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- Current and further work:
  - further cost functions (e.g. exponential)
  - computation of approximate optimal values
  - further investigation of safe games + several cost variables?
  - discounted-time optimal games
  - link between discounted-time games and mean-cost games?
  - computation of equilibria
  - ...