The Cost of Punctuality

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Motivation

**Context:** verification of timed systems towards linear-time timed temporal logics
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1. linear-time timed temporal logics: interesting for specifying properties of systems, but we cannot verify them! [AH93]
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→ **punctuality is undecidable!**
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2. MITL, a palliative to these negative results \[\text{[AFH96]}\]
   (MITL: disallows punctual constraints)
   \[\rightarrow \text{punctuality is undecidable!}\]

3. Safety-MTL: a decidable logic which partly allows punctuality \[\text{[OW0\{5,6\}}\]
   However, it is **non-primitive recursive**!
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4. we propose a tractable though powerful linear-time timed temporal logic which allows punctuality...
Metric Temporal Logic

MTL: Metric Temporal Logic

\[ \text{MTL } \exists \varphi ::= a \mid \neg a \mid \varphi \lor \varphi \mid \varphi \land \varphi \mid \varphi \mathbf{U}_I \psi \mid \varphi \mathbf{\tilde{U}}_I \varphi \]

where \( I \) is an interval with integral bounds
Metric Temporal Logic

\textbf{MTL}: Metric Temporal Logic

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\text{MTL } \varrho ::= a \mid \neg a \mid \varphi \lor \varphi \mid \varphi \land \varphi \mid \varphi \mathcal{U}_I \psi \mid \varphi \mathcal{\overline{U}}_I \varphi
\]

where \( I \) is an interval with integral bounds

We interpret \textbf{MTL} formulas over timed words (this is the so-called point-based semantics):
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We use classical shorthands, like \( F \), \( G \), \( X \), etc...
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- \( G <_2 (\bullet \rightarrow F =_1 \bullet) \)
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We interpret **MTL** formulas over timed words (this is the so-called point-based semantics):

![Timed Word Example](image)

We use classical shorthands, like \( \mathbf{F} \), \( \mathbf{G} \), \( \mathbf{X} \), etc...

- \( \mathbf{G}_{<2}(\bullet \to \mathbf{F}_{=1}\bullet) \)
- \( (\bullet \mathbf{U}_{>3}\bullet) \mathbf{U}_{\leq 1} (\mathbf{F}_{>1}\bullet) \)
Interesting Fragments of MTL

\[ \text{MTL } \exists \varphi \ ::= \ a \mid \neg a \mid \varphi \lor \varphi \mid \varphi \land \varphi \mid \varphi \mathcal{U} \varphi \mid \varphi \tilde{\mathcal{U}} \varphi \]
Interesting Fragments of MTL

\[ \text{LTL } \exists \phi ::= a \mid \neg a \mid \phi \lor \phi \mid \phi \land \phi \mid \phi \mathbf{U} \phi \mid \phi \tilde{\mathbf{U}} \phi \]

[PNueli77]
Interesting Fragments of MTL

\[
\text{MITL } \exists \varphi ::= a \mid \neg a \mid \varphi \lor \varphi \mid \varphi \land \varphi \mid \varphi \Uparrow I \varphi \mid \varphi \Tilde{\Uparrow} I \varphi
\]

with \( I \) non-singular, \emph{i.e.}, with no “punctuality”

[AFH96]
Interesting Fragments of MTL

Bounded-MTL $\exists \varphi ::= a \mid \neg a \mid \varphi \lor \varphi \mid \varphi \land \varphi \mid \varphi \mathsf{U} / \varphi \mid \varphi \mathsf{\tilde{U}} / \varphi$

with / bounded
Interesting Fragments of MTL

\[
\text{Safety-MTL} \ni \varphi \ ::= \ a \mid \neg a \mid \varphi \lor \varphi \mid \varphi \land \varphi \mid \varphi \mathbf{U}_J \varphi \mid \varphi \tilde{\mathbf{U}}_I \varphi
\]

with \( J \) bounded

\[
\text{Bounded-MTL} \quad \text{Safety-MTL} \quad \text{MTL} \quad \text{MITL} \quad \text{LTL}
\]

\[
\text{Bounded-MTL} + \text{Invariance} \subseteq \text{Safety-MTL}
\]

[OW05]
Interesting Fragments of MTL

Flat-MTL \ni \varphi ::= a \mid \neg a \mid \varphi \lor \varphi \mid \varphi \land \varphi \mid \psi U_I \varphi \mid \varphi \tilde{U}_I \psi

with I unbounded \Rightarrow \psi \in LTL
Interesting Fragments of MTL

\[ \text{coFlat-MTL} \ni \varphi ::= a \mid \neg a \mid \varphi \lor \varphi \mid \varphi \land \varphi \mid \varphi \mathbf{U}_I \psi \mid \psi \mathbf{\tilde{U}}_I \varphi \]

with \( I \) unbounded \( \Rightarrow \psi \in \text{LTL} \)

\[ \text{Bounded-MTL} \rightarrow \text{Safety-MTL} \]
\[ \text{coFlat-MTL} \leftarrow \text{MTL} \]
\[ \text{LTL} \rightarrow \text{MITL} \]

\[ \text{Bounded-MTL} + \text{Invariance} \subseteq \text{coFlat-MTL} \]
Some Examples of Formulas

- $G\left(\text{request} \rightarrow F_{\leq 1} (\text{acquire} \land F_{\geq 1} \text{release})\right)$ is in $\text{coFlat-MTL}$, but neither in $\text{Bounded-MTL}$, nor in $\text{MITL}$. 
Some Examples of Formulas

- $G\left(request \rightarrow F_{\leq 1} \text{(acquire} \land F_{= 1} \text{release)}\right)$ is in coFlat-MTL, but neither in Bounded-MTL, nor in MITL.

- $\varphi_n = \bullet \land \text{Double} \land G_{< 2^n} \text{Double}$ where

  
  
  Double = \left(\bullet \rightarrow F_{= 1} (\bullet \land X_{< 1} \bullet)\right) \land \left(\bullet \rightarrow F_{= 1} (\bullet \land X_{< 1} \bullet)\right)

  

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\[05/21\]
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$\rightarrow$ enforces in polynomial space a doubly exponential variability
Some Examples of Formulas (cont’d)

- Half $= F_{=1} \top \top \lor X_{\leq 1} F_{=1} \top \top$ → may eliminate one over two actions
Some Examples of Formulas (cont’d)

- Half = $F_{=1} \ U X_{\leq 1} F_{=1} \ U$
  
  may eliminate one over two actions

- the formula

  $\bullet \ W Double \ W G_{<2^n} Double \ W G_{[2^n,2^{n+1})} \ Half \ W F_{=2^{n+1}} \ (\bullet \ W X_{=1} \ U)$

  hence enforces exact doubling and halving…
## Complexity Results

Over infinite timed words:

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<tr>
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<td></td>
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An Example

Assume one wants to verify formula

$$G_{<2}(\bullet \rightarrow F_{=1} \bullet)$$
An Example

Assume one wants to verify formula

\[ G <_2 (\bullet \rightarrow F = 1 \bullet) \]
An Example

Assume one wants to verify formula

\[ G <_2 \left( \bullet \rightarrow F =_1 \bullet \right) \]

Offline, we stack all time units and use a sliding window:
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Assume one wants to verify formula

\[ G_{<2} \left( \bullet \rightarrow F_{=1} \bullet \right) \]

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\[ G_{<2}(\cdot \rightarrow F=1 \cdot) \]

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$$G_{<2}\left( \cdot \rightarrow F_{=1} \cdot \right)$$

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Offline, we stack all time units and use a sliding window:
Channel Automata

NB: channels are FIFO...
Extended Channel Automata

We extend channel automata with:

- renaming (a letter can be replaced non-det. by another one);
- occurrence testing (check whether a letter appears on a channel).

→ CAROT
Extended Channel Automata

We extend channel automata with:

- **renaming** (a letter can be replaced non-det. by another one);
- **occurrence testing** (check whether a letter appears on a channel).

\[ \rightarrow \text{CAROT} \]

where \( R \) non-deterministically rename \( b \) to either \( b \) or \( c \).
Extended Channel Automata

We extend channel automata with:

- renaming (a letter can be replaced non-det. by another one);
- occurrence testing (check whether a letter appears on a channel).

→ CAROT

We will be interested in the reachability problem for CAROTs when we bound the number of cycles of the machine.
where \( R : b \mapsto b \lor c \)
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$dcb$
where \( R : b \mapsto b \lor c \)
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where $R : b \mapsto b \lor c$

$$c \underline{a} d$$
where $R : b \mapsto b \lor c$
where $R : b \mapsto b \lor c$
where $R : b \mapsto b \lor c$

$d\!c$
where $R : b \mapsto b \lor c$
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Computation table, starting with $d$ on the channel:
Computation table with sliding window:

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<table>
<thead>
<tr>
<th>s</th>
<th>b!</th>
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<th>b!</th>
<th>s</th>
<th>R</th>
<th>t</th>
<th>d?</th>
<th>u</th>
<th>d!</th>
<th>v</th>
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<tbody>
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```
We need to store a window and some extra information for the renaming functions and the occurrence testing.
Theorem

The cycle-bounded reachability problem for CAROTs is solvable in polynomial space in the size of the channel automaton and polynomial space in the value of the cycle bound.

(Can guess and verify a computation table using polynomial space.)
Application to Timed Temporal Logics

- Transform an MTL formula $\varphi$ into an equivalent one-clock alternating timed automaton $A_{\varphi}$

$$G_{<2} \left( \bullet \rightarrow F_{=1} \bullet \right)$$
Transform an MTL formula $\varphi$ into an equivalent one-clock alternating timed automaton $A_\varphi$.

$$
G_{<2} \left( \text{red} \rightarrow F_{=1} \text{green} \right)
$$
\[ r \xleftarrow{x < 2; \bullet} s \xrightarrow{x = 1; \bullet} t \]

\[ x := 0 \]
See a behaviour of this automaton as the content of a FIFO channel
See a behaviour of this automaton as the content of a FIFO channel:

$r$ $\xrightarrow{x<2; \bullet} s$ $\xrightarrow{x=0}s$ $\xrightarrow{x=1; \bullet} t$
See a behaviour of this automaton as the content of a FIFO channel

$r,0$
See a behaviour of this automaton as the content of a FIFO channel.

\( r,0 \rightarrow r,0.6 \)
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See a behaviour of this automaton as the content of a FIFO channel

\[ r \xrightarrow{x < 2;} s \xrightarrow{x := 0} t \]

\[ r,0 \rightarrow r,0.6 \rightarrow r,0.7 \rightarrow r,1.5 \rightarrow r,1.7 \]

\[ s,0 \rightarrow s,0.2 \rightarrow s,0.8 \rightarrow t \]
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\[ \begin{array}{c}
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From MTL to CAROTs
Every formula $\varphi$ can be transformed into a CAROT that “accepts” the models of $\varphi$. 
▶ See a behaviour of this automaton as the content of a FIFO channel

```
  r,0  →  r,0.6  →  r,0.7  →  r,1.5  →  r,1.7
  \   \     \     \     \     \     \\
  s,0  s,0  →  s,0.2  \    \    \    \    \    ...
  s,0.8 →  t
```

From MTL to CAROTs
Every formula $\varphi$ can be transformed into a CAROT that “accepts” the models of $\varphi$.

One time unit = one cycle of the CAROT
A Digression on Timed Automata

\[ Y \]
\[ r_1 \]
\[ r_0 \]
\[ r_0 \quad r_1 \quad X \]
A Digression on Timed Automata

\[ x, y \in r_0, \{y\} < \{x\} \]

\[(y, r_0), (x, r_0)\]
A Digression on Timed Automata

\[ x \in r_1, y \in r_0, \{x\} < \{y\} \]

\[(x, r_1) \mid (y, r_0)\]
A Digression on Timed Automata

\[ x, y \in r_1, \{y\} < \{x\} \]

\[ (y, r_1) \parallel (x, r_1) \]
The region graph can be simulated by a channel machine (with a single bounded channel).
The “Simple” Case of Bounded-MTL

- Bounded-MTL enjoys a small-model property:
The “Simple” Case of Bounded-MTL

- Bounded-MTL enjoys a small-model property:
  Every satisfiable formula has a model with relevant prefix of size at most exponential in the size of the formula (e.g., the sum of all constants appearing in the formula).
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- Model checking and satisfiability checking of Bounded-MTL can be done by a cycle-bounded CAROT, whose number of cycles is exponential (polynomial if constants are encoded in unary). It is hence in EXPSPACE!
The More Involved Case of coFlat-MTL

- We want to bound the number of cycles needed by the CAROT to achieve model-checking of coFlat-MTL, or more simply the satisfiability of Flat-MTL.

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\text{Flat-MTL} \ni \varphi ::= a \mid \neg a \mid \varphi \lor \varphi \mid \varphi \land \varphi \mid \psi \mathcal{U}_I \varphi \mid \varphi \mathcal{U}_I \psi
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**Theorem**

The model checking of coFlat-MTL is in EXPSPACE.
Hardness

Theorem
The satisfiability problem for Bounded-MTL is EXPSPACE-Hard.
Hardness

Theorem

The satisfiability problem for Bounded-MTL is EXPSPACE-Hard.

Encode the halting problem of an EXPSPACE Turing machine:

- generate a doubly exponential number of events in one time unit
- on the next time unit, non-deterministically guess a computation of the EXPSPACE Turing machine
- check it is correct (requires $2^n$ time units, one for each cell of the machine)
- half, and check that only one event remains
Summary of the Complexity Results

Over infinite timed words:

<table>
<thead>
<tr>
<th></th>
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</tr>
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<tbody>
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- contains punctual constraints,
- contains invariance,
- is tractable in theory.
Conclusion

In this work, we have exhibited a subclass of MTL which:

- contains punctual constraints,
- contains invariance,
- is tractable in theory.

What needs to be done:

- check tractability in practice,
- extend to continuous semantics.