The Cost of Punctuality

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   However, it is non-primitive recursive!
4. we propose a tractable though powerful linear-time timed temporal logic which allows punctuality...

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$$\mathsf{MTL} \ni \varphi ::= a \mid \neg a \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \varphi \, \mathbf{U}_I \, \psi \mid \varphi \, \widetilde{\mathbf{U}}_I \, \varphi$$

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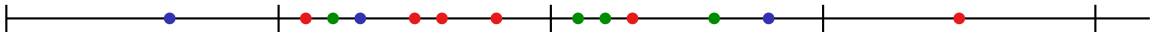

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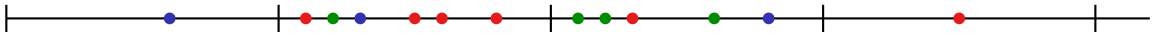

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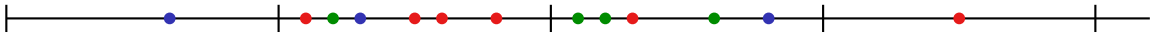

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- $\mathbf{G}_{<2}\Big(\textcolor{blue}{\bullet} \rightarrow \mathbf{F}_{=1}\, \textcolor{red}{\bullet}\Big)$
- $(\textcolor{blue}{\bullet}\, \mathbf{U}_{>3}\, \textcolor{red}{\bullet})\, \mathbf{U}_{\leqslant 1}\, (\mathbf{F}_{>1}\, \textcolor{green}{\bullet})$

Interesting Fragments of MTL

$$\text{MTL} \ni \varphi ::= a \mid \neg a \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \varphi \, \mathbf{U}_I \, \varphi \mid \varphi \, \widetilde{\mathbf{U}}_I \, \varphi$$

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$$\text{LTL} \ni \varphi \ ::= \ a \mid \neg a \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \varphi \, \mathbf{U} \, \varphi \mid \varphi \, \widetilde{\mathbf{U}} \, \varphi$$

$$\text{LTL} \hookrightarrow \text{MTL}$$

[Pnueli77]

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with I non-singular, *i.e.*, with no "punctuality"

$$\text{LTL} \hookrightarrow \text{MITL} \hookrightarrow \text{MTL}$$

[AFH96]

Interesting Fragments of MTL

$$\text{Bounded-MTL} \ni \varphi \ ::= \ a \ | \ \neg a \ | \ \varphi \vee \varphi \ | \ \varphi \wedge \varphi \ | \ \varphi \, \mathbf{U}_I \, \varphi \ | \ \varphi \, \widetilde{\mathbf{U}}_I \, \varphi$$

with I bounded

$$\begin{tikzcd}
\text{Bounded-MTL} \arrow[rr, hook] & & \text{MTL} \\
\text{LTL} \arrow[r, hook] & \text{MITL} \arrow[ur, hook] &
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$$\text{Safety-MTL} \ni \varphi \ ::= \ a \ | \ \neg a \ | \ \varphi \vee \varphi \ | \ \varphi \wedge \varphi \ | \ \varphi \, \mathbf{U}_J \, \varphi \ | \ \varphi \, \widetilde{\mathbf{U}}_I \, \varphi$$

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$$\text{Bounded-MTL} + \text{Invariance} \ \subseteq \text{Safety-MTL}$$

[OW05]

Interesting Fragments of MTL

$$\text{Flat-MTL} \ni \varphi \ ::= \ a \ \mid \ \neg a \ \mid \ \varphi \vee \varphi \ \mid \ \varphi \wedge \varphi \ \mid \ \psi \, \mathbf{U}_I \, \varphi \ \mid \ \varphi \, \widetilde{\mathbf{U}}_I \, \psi$$

with I unbounded $\Rightarrow \psi \in$ LTL

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Bounded-MTL + Invariance $\subseteq$ coFlat-MTL

Some Examples of Formulas

- $\mathbf{G}\left(\text{request} \to \mathbf{F}_{\leqslant 1}(\text{acquire} \wedge \mathbf{F}_{=1}\,\text{release})\right)$ is in coFlat-MTL, but neither in Bounded-MTL, nor in MITL.

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$$\text{Double} = \left(\bullet \rightarrow \mathbf{F}_{=1}(\bullet \wedge \mathbf{X}_{<1}\,\bullet)\right) \wedge \left(\bullet \rightarrow \mathbf{F}_{=1}(\bullet \wedge \mathbf{X}_{<1}\,\bullet)\right)$$

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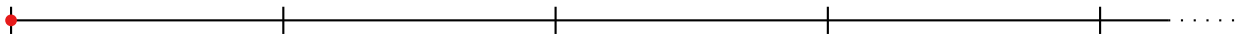

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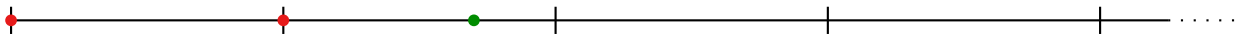

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$\longrightarrow$ enforces in polynomial space a doubly exponential variability

Some Examples of Formulas (cont'd)

- Half $= \mathbf{F}_{=1}\, \mathtt{tt} \vee \mathbf{X}_{\leqslant 1}\, \mathbf{F}_{=1}\, \mathtt{tt}$

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- the formula

  $$\bullet \wedge \text{Double} \wedge \mathbf{G}_{<2^n}\,\text{Double} \wedge \mathbf{G}_{[2^n,2^{n+1})}\,\text{Half} \wedge \mathbf{F}_{=2^{n+1}}\,(\bullet \wedge \mathbf{X}_{=1}\,\mathtt{tt})$$

  hence enforces exact doubling and halfing...

Complexity Results

Over infinite timed words:

	Model Checking	**Satisfiability**
LTL	PSPACE-C. [folklore]	PSPACE-C. [folklore]
MITL	EXPSPACE-C. [AFH96]	EXPSPACE-C. [AFH96]
Bounded-MTL		
Safety-MTL		Decidable [OW06]
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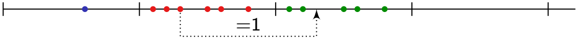

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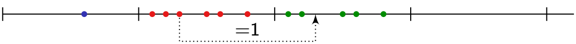

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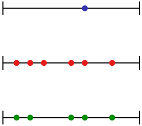

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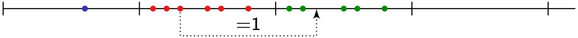

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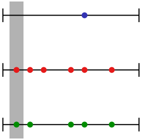

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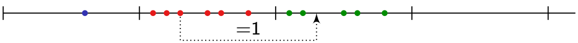

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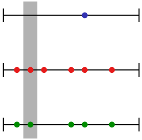

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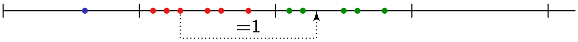

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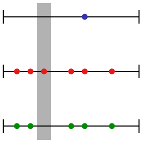

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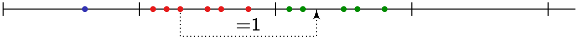

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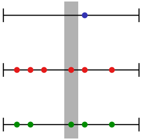

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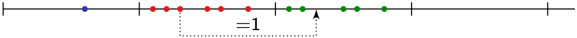

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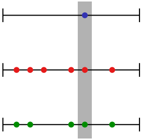

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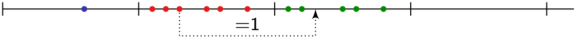

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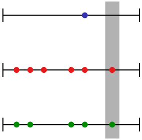
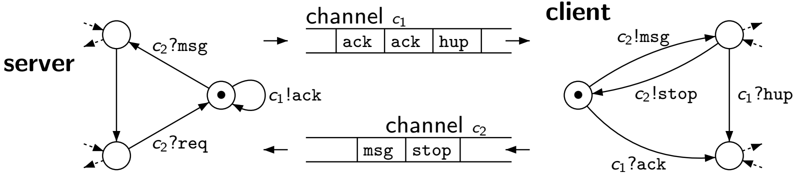


Channel Automata



NB: channels are FIFO...

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We extend channel automata with:

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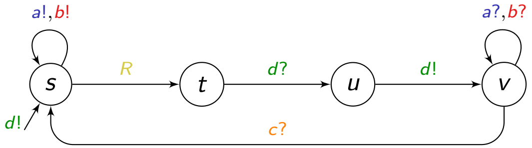

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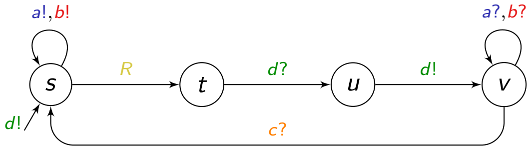

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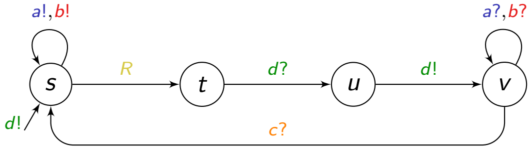
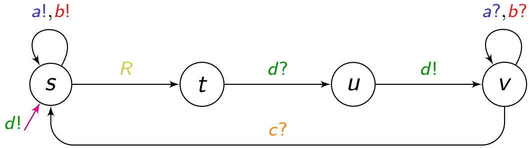
We will be interested in the reachability problem for CAROTs when we bound the number of cycles of the machine

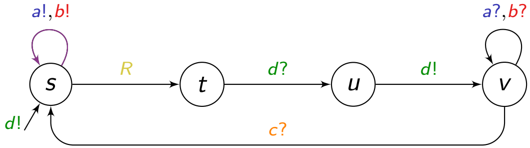


where $R : b \mapsto b \vee c$



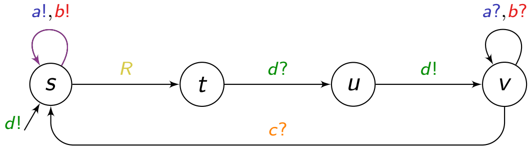
where $R : b \mapsto b \vee c$

d



where $R : b \mapsto b \vee c$

$\underline{b}d$



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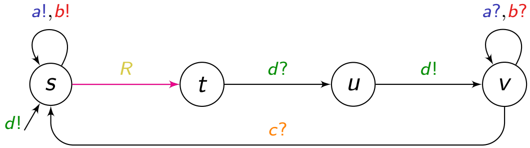
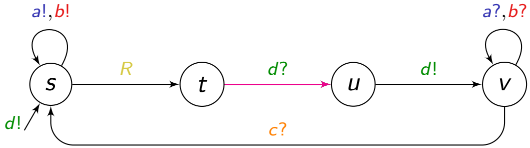
$$b\underline{b}d$$



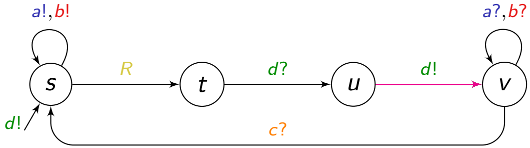
where $R : b \mapsto b \vee c$

$c\underline{b}d$



where $R : b \mapsto b \vee c$

$c\underline{b}$



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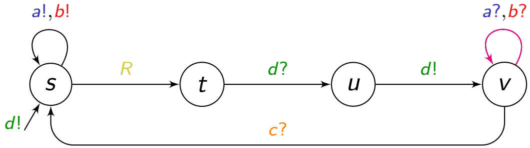
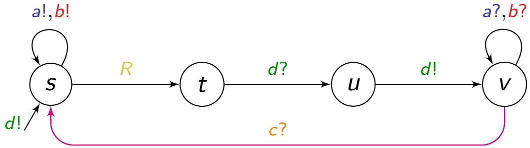
$d\,c\,\underline{b}$



where $R : b \mapsto b \vee c$

dc



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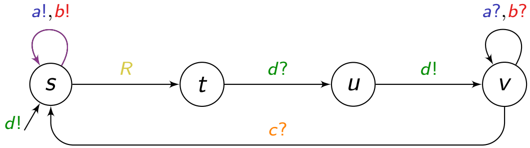

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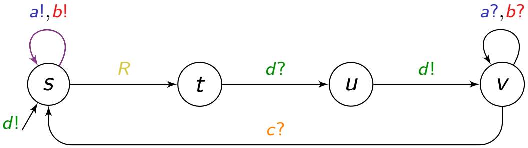

where $R : b \mapsto b \vee c$

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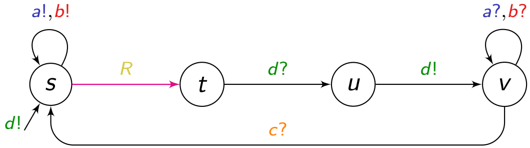

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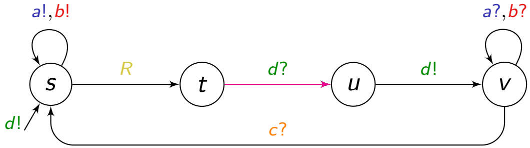

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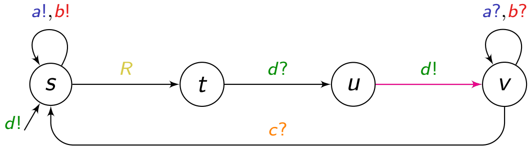

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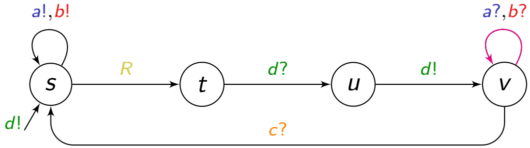

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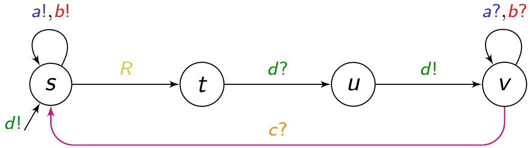

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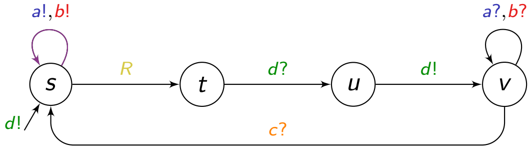

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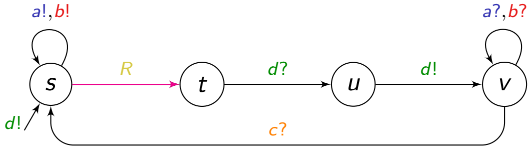

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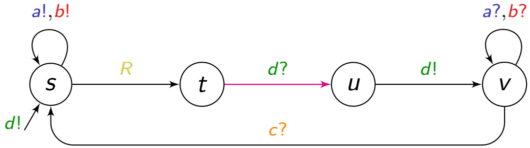

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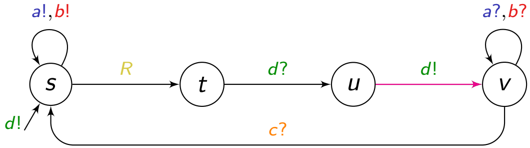

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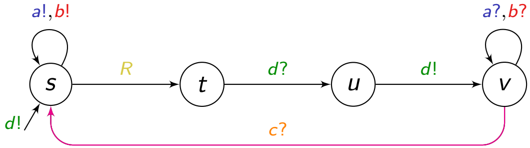

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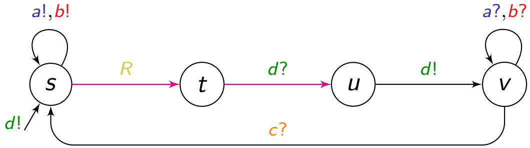

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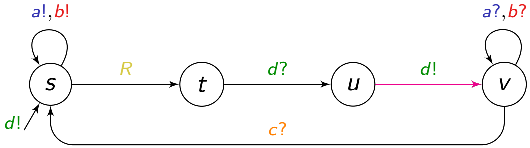

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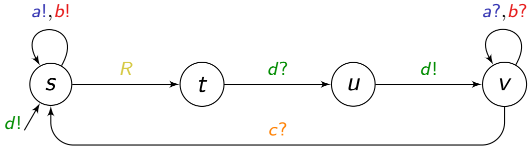
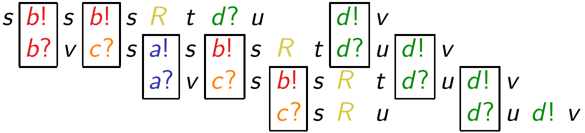


where $R : b \mapsto b \vee c$

Computation table, starting with d on the channel:

$$\begin{array}{ccccccccccccccccccc}
s & b! & s & b! & s & R & t & d? & u & & & d! & v & & & & & & \\
 & b? & v & c? & s & a! & s & b! & s & R & t & d? & u & d! & v & & & & \\
 & & & & & a? & v & c? & s & b! & s & R & t & d? & u & d! & v & & \\
 & & & & & & & & & c? & s & R & u & & & d? & u & d! & v
\end{array}$$

$$
\begin{array}{ccccccccccccccccccc}
s & b! & s & b! & s & R & t & d? & u & & & d! & v & & & & & & \\
 & b? & v & c? & s & a! & s & b! & s & R & t & d? & u & d! & v & & & & \\
 & & & & & a? & v & c? & s & b! & s & R & t & d? & u & d! & v & & \\
 & & & & & & & & & c? & s & R & u & & & d? & u & d! & v
\end{array}
$$



Computation table with sliding window:

s	$b!$	s	$b!$	s	R	t	$d?$	u	u	u	$d!$	v	v	v	v	v	v	v
v	$b?$	v	$c?$	s	$a!$	s	$b!$	s	R	t	$d?$	u	$d!$	v	v	v	v	v
v	v	v	v	v	$a?$	v	$c?$	s	$b!$	s	R	t	$d?$	u	$d!$	v	v	v
v	v	v	v	v	v	v	v	v	$c?$	s	R	u	u	u	$d?$	u	$d!$	v

s b! s b! s R t d? u d! v
b? v c? s a! s b! s R t d? u d! v
a? v c? s b! s R t d? u d! v
c? s R u d? u d! v

Computation table with sliding window:

s	b!	s	b!	s	R	t	d?	u	u	u	d!	v	v	v	v	v	v	v
v	b?	v	c?	s	a!	s	b!	s	R	t	d?	u	d!	v	v	v	v	v
v	v	v	v	v	a?	v	c?	s	b!	s	R	t	d?	u	d!	v	v	v
v	v	v	v	v	v	v	v	v	c?	s	R	u	u	u	d?	u	d!	v

We need to store a window and some extra information for the renaming functions and the occurrence testing.

Theorem

The cycle-bounded reachability problem for CAROTs is solvable in polynomial space in the size of the channel automaton and polynomial space in the value of the cycle bound.

(Can guess and verify a computation table using polynomial space.)

Application to Timed Temporal Logics

- Transform an MTL formula φ into an equivalent one-clock alternating timed automaton $\mathcal{A}_\varphi$ [OW05]

$$\mathbf{G}_{<2}\left(\bullet \rightarrow \mathbf{F}_{=1}\,\bullet\right)$$

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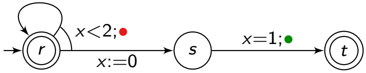
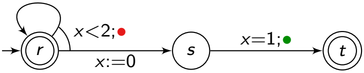
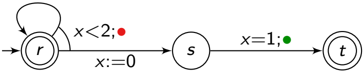
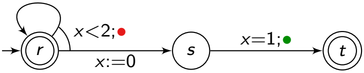



x<2;
x:=0
x=1;
r
s
t



- See a behaviour of this automaton as the content of a FIFO channel



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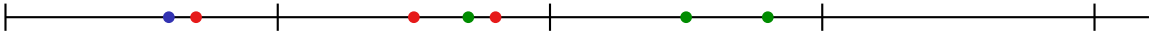

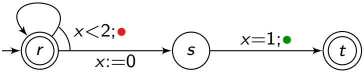

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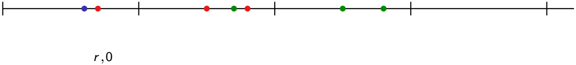

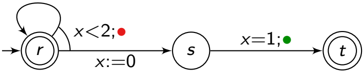

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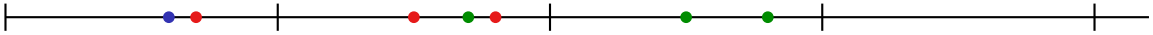
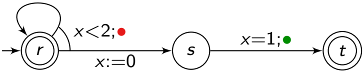


$r,0 \xrightarrow{\bullet} r,0.6$



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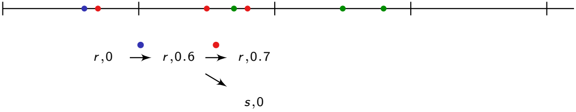

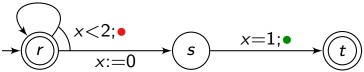

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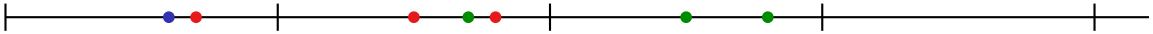

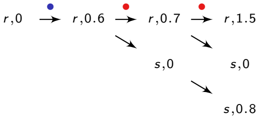

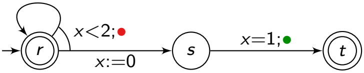

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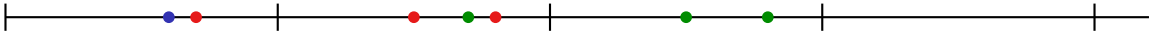

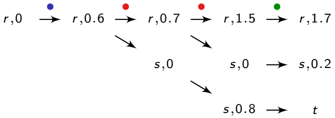

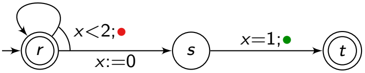

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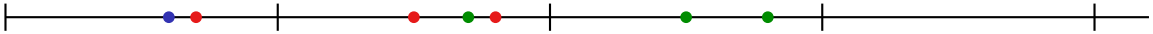

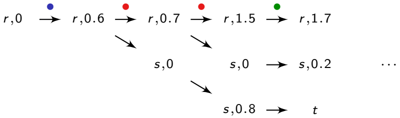

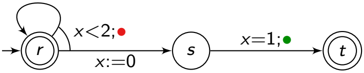

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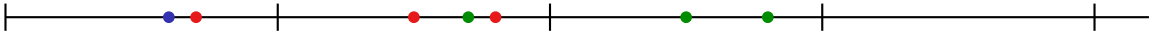

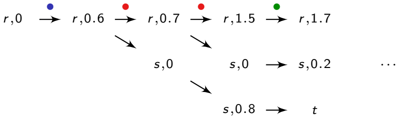

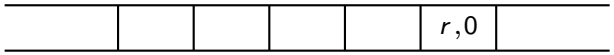

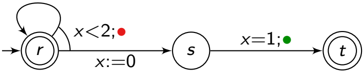

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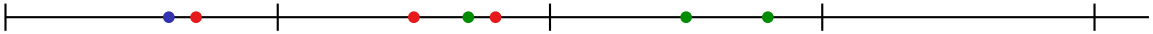

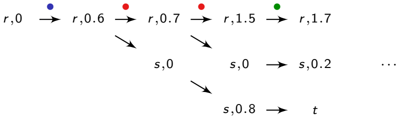

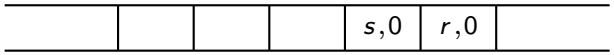

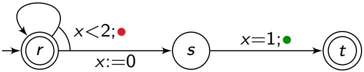

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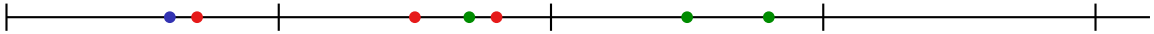

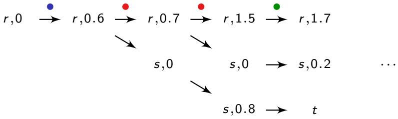

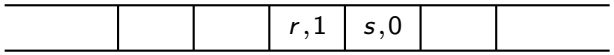

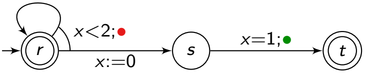

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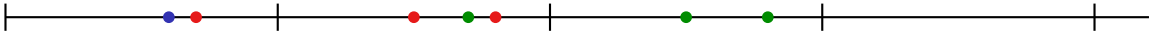
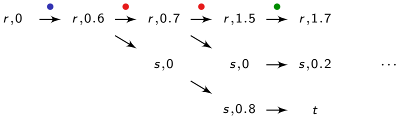
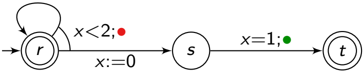




		s,0	r,1	s,0		



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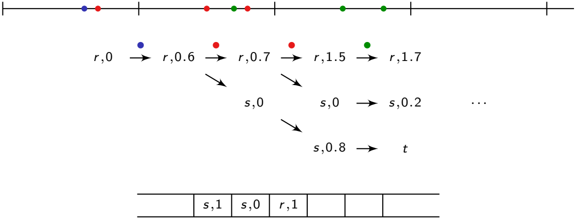

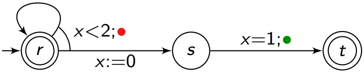

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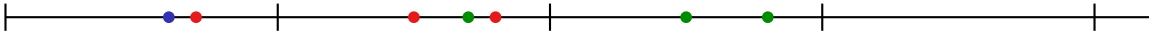

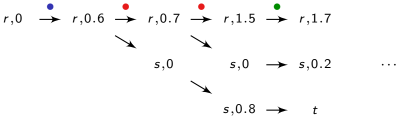

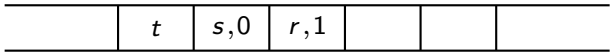

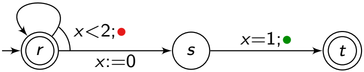

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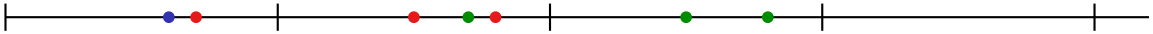
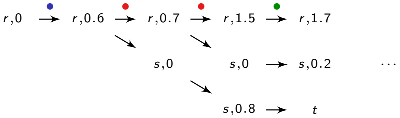
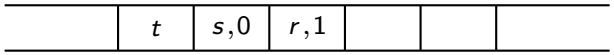
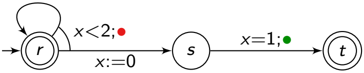






From MTL to CAROTs

Every formula φ can be transformed into a CAROT that "accepts" the models of φ.



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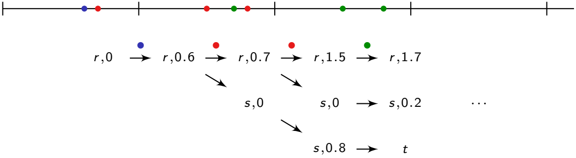

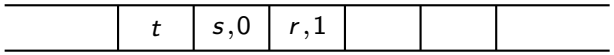

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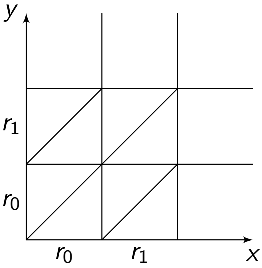
One time unit = one cycle of the CAROT

A Digression on Timed Automata



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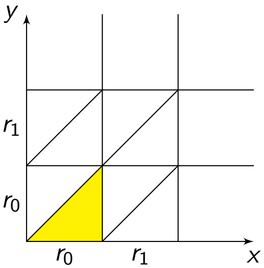

$x, y \in r_0$, $\{y\} < \{x\}$

	(y, r_0)	(x, r_0)	

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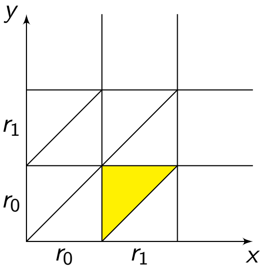

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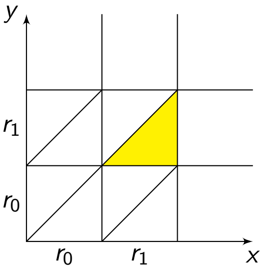

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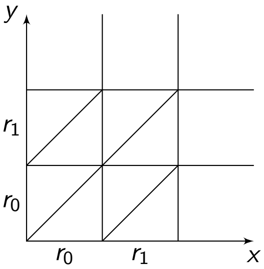

The region graph can be simulated by a channel machine (with a single bounded channel).

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- Model checking and satisfiability checking of Bounded-MTL can be done by a cycle-bounded CAROT, whose number of cycles is exponential (polynomial if constants are encoded in unary). It is hence in EXPSPACE!

The More Involved Case of coFlat-MTL

- We want to bound the number of cycles needed by the CAROT to achieve model-checking of coFlat-MTL, or more simply the satisfiability of Flat-MTL.

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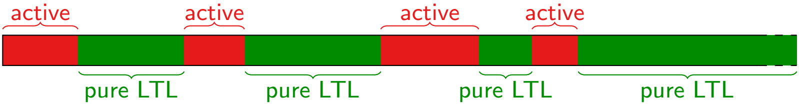

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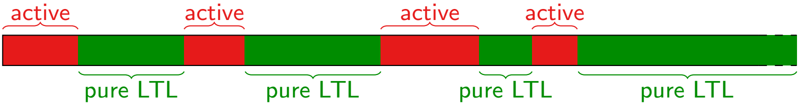

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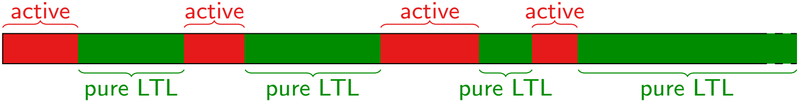

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Theorem

The model checking of coFlat-MTL is in EXPSPACE.

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Encode the halting problem of an EXPSPACE Turing machine:

- generate a doubly exponential number of events in one time unit
- on the next time unit, non-deterministically guess a computation of the EXPSPACE Turing machine
- check it is correct (requires 2^n time units, one for each cell of the machine)
- half, and check that only one event remains

Summary of the Complexity Results

Over infinite timed words:

	Model Checking	**Satisfiability**
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MITL	EXPSPACE-C.	EXPSPACE-C.
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What needs to be done:

- check tractability in practice,
- extend to continuous semantics.