Stochastic Timed Automata

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LSV, CNRS & ENS Cachan, France

Based on joint works with Nathalie Bertrand, Thomas Brihaye, Pierre Carlier, Quentin Menet, Christel Baier, ...

Outline

A story that started in 2006...

- My background: timed automata

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  - for measuring system performance (time before failure, ...)
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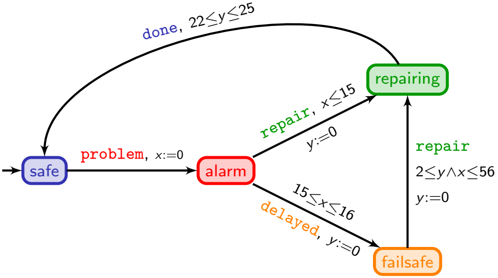
> - FSTTCS'07, LICS'08, QEST'08, QEST'12
> - 72-pages LMCS journal paper
> - Pierre Carlier, joint PhD student between Mons and Cachan, now works on that subject

Outline

The model of timed automata



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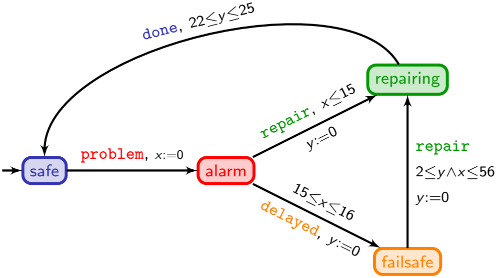


	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe		
x	0		23		0		15.6		15.6	$\cdots$
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\texttt{done}}$	safe
$\cdots$	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

An example: The task graph scheduling problem

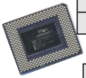
Compute $D\times(C\times(A+B))+(A+B)+(C\times D)$ using two processors:

P_1 (fast):

time	
$+$	2 picoseconds
$\times$	3 picoseconds

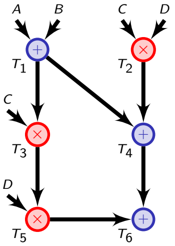
energy	
idle	10 Watt
in use	90 Watts

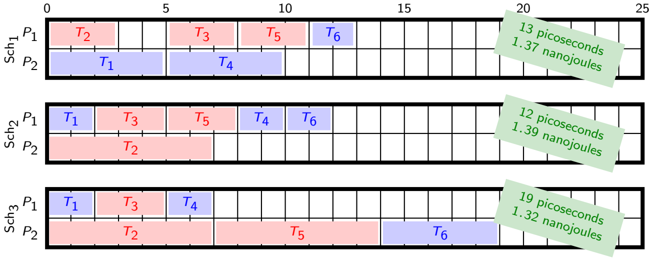
P_2 (slow):



time	
$+$	5 picoseconds
$\times$	7 picoseconds

energy	
idle	20 Watts
in use	30 Watts

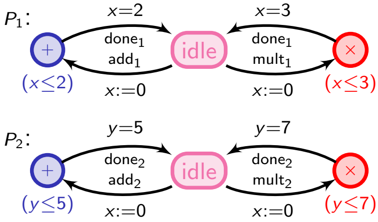




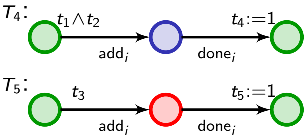
[BFLM10] Bouyer, Fahrenberg, Larsen, Markey. Quantitative Analysis of Real-Time Systems using Priced Timed Automata *(CACM)*.

Modelling the task graph scheduling problem

- Processors

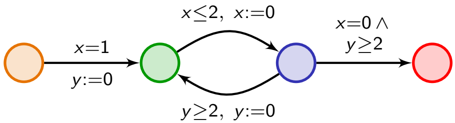


- Tasks



A schedule is a path in the product automaton

Analyzing timed automata



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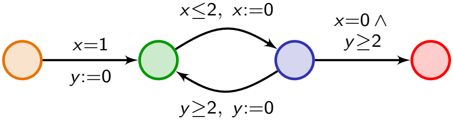

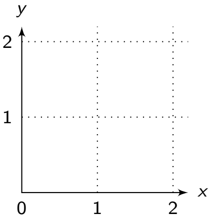

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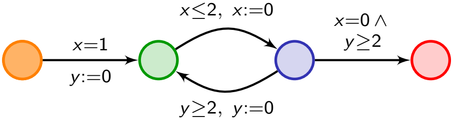

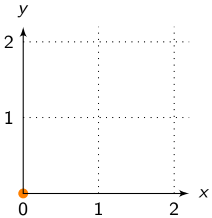

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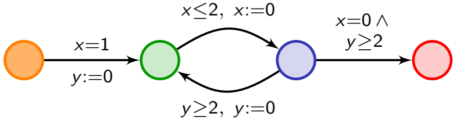

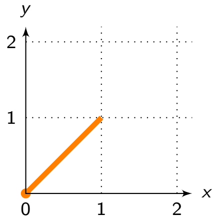

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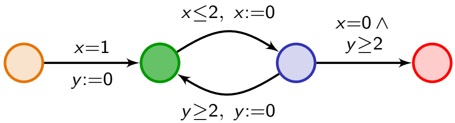

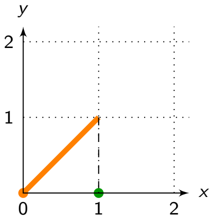

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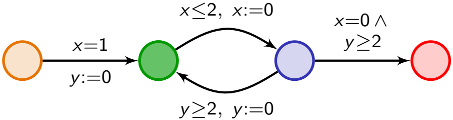

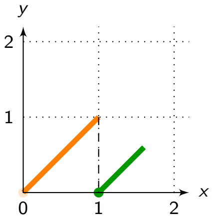

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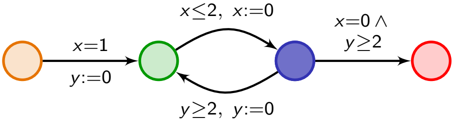

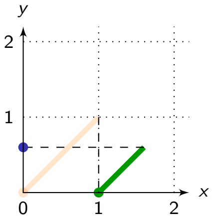

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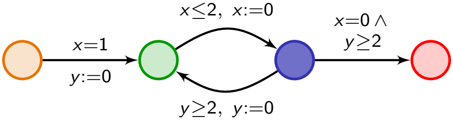

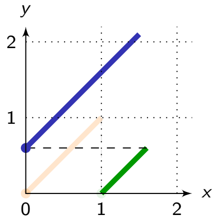

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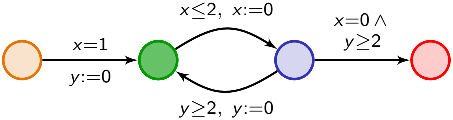

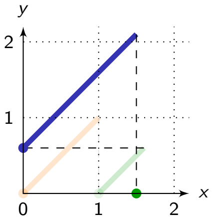

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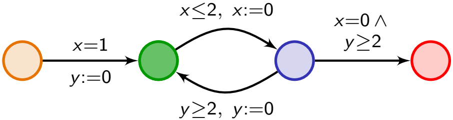

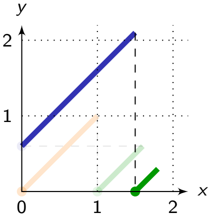

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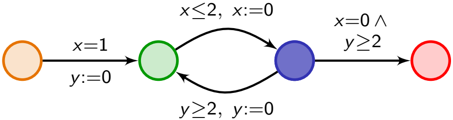

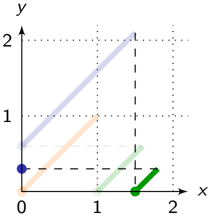

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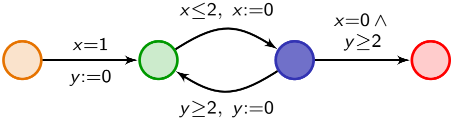

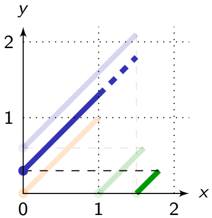

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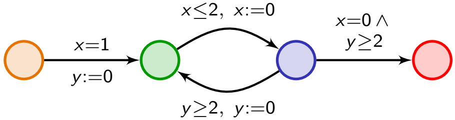

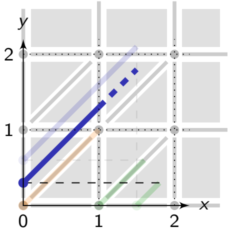

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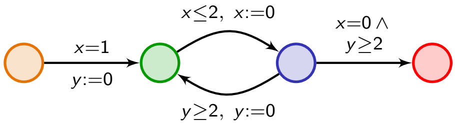

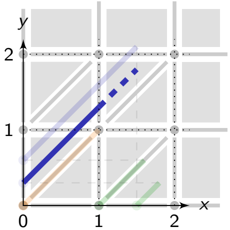

Theorem [AD94]

Reachability in timed automata is decidable (as well as many other important properties). It is PSPACE-complete.

- Technical tool: region abstraction

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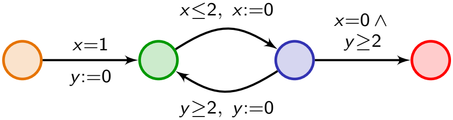

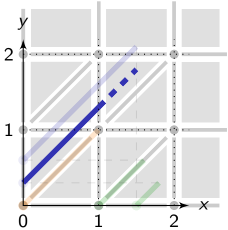

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- Technical tool: region abstraction
- Efficient symbolic technics based on zones, implemented in tools

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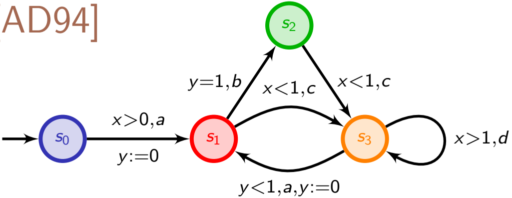
An example [AD94]



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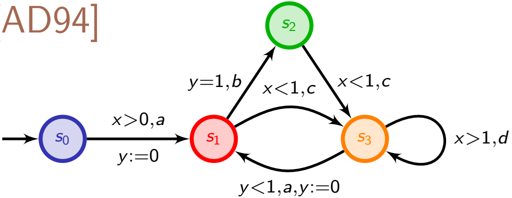

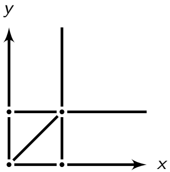

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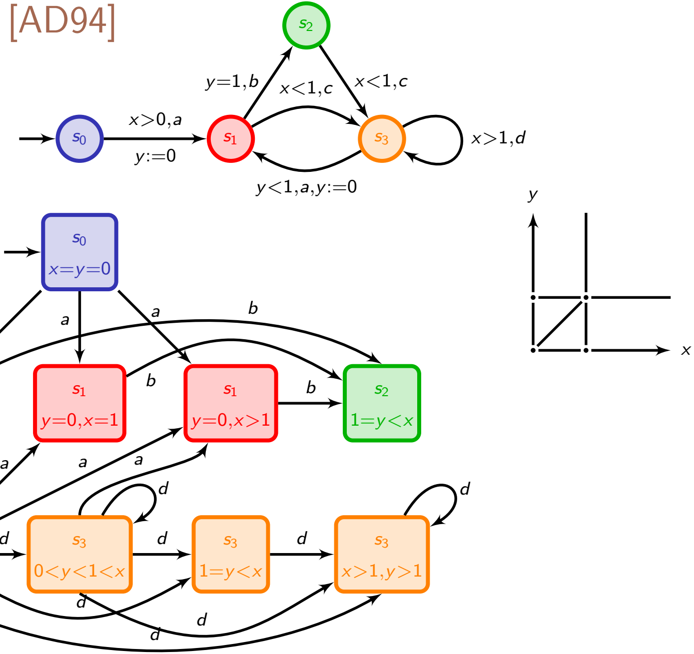
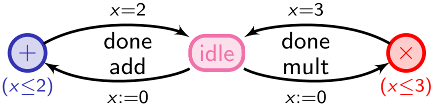


How to model uncertainty over delays?

- Using timed games



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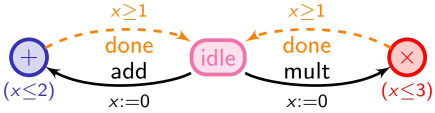

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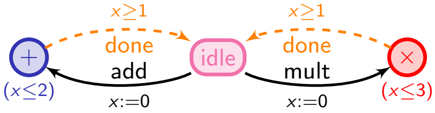

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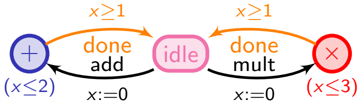

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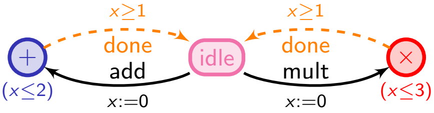

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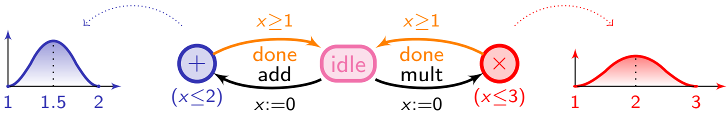

Outline

Existing models?

Models based on timed automata

- Probabilistic timed automata [KNSS99]
  $\rightsquigarrow$ only discrete probabilities over edges
- Continuous probabilistic timed automata [KNSS00]
  $\rightsquigarrow$ resets of clocks are randomized, but only few results

[KNSS99] Kwiatkowska, Norman, Segala, Sproston. Automatic verification of real-time systems with discrete probability distributions *(ARTS'99)*.
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Other related models I was not familiar with in 2006

- Continuous-time Markov chains (CTMCs)
- Generalized semi-Markov processes (GSMPs)
- Process algebras (like Modest) [DK05,BDHK06]

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[KNSS00] Kwiatkowska, Norman, Segala, Sproston. Verifying quantitative properties of continuous probabilistic timed automata *(CONCUR'00)*.
[DK05] D'Argenio, Katoen. Stochastic timed automata, Part I and Part II *(Information and Computation)*.
[BDHK06] Bohnenkamp, D'Argenio, Hermanns, Katoen. MODEST: A compositional modeling formalism for hard and softly timed systems *(IEEE Trans. Software Engineering)*.

Our choice of stochastic timed automata

- Based on the standard timed automata model
  - Model largely adopted for real-time systems
  - Enjoys efficient verification algorithms and corresponding implementations

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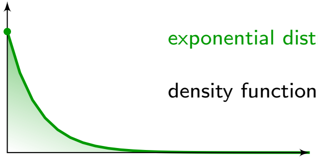
- Based on the standard timed automata model
  - Model largely adopted for real-time systems
  - Enjoys efficient verification algorithms and corresponding implementations
  - We understand it well!
- Randomize delays
- We believe this is an interesting model for systems integrating both:
  - real-time constraints
  - randomized aspects

How can we attach probabilities to delays?

- The example of continuous-time Markov chains



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density function $t \mapsto \begin{cases} \lambda \cdot \exp(-\lambda t) & \text{if } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$

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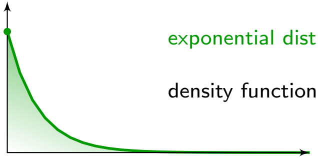

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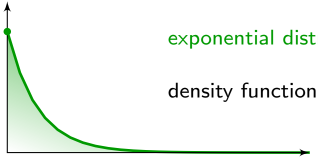

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- But what if bounded intervals?

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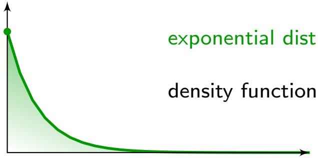
- The example of continuous-time Markov chains



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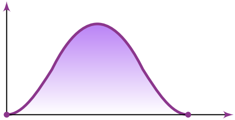
density function $t \mapsto \begin{cases} \lambda \cdot \exp(-\lambda t) & \text{if } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$

$\rightsquigarrow$ this is ok if delays are in $[0, +\infty)$

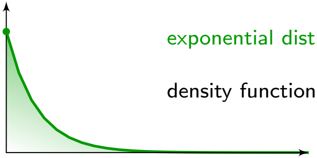
- But what if bounded intervals?



truncated normal distribution

How can we attach probabilities to delays?

- The example of continuous-time Markov chains



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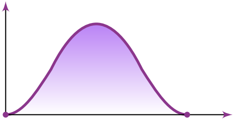

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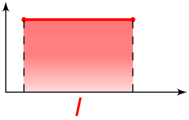
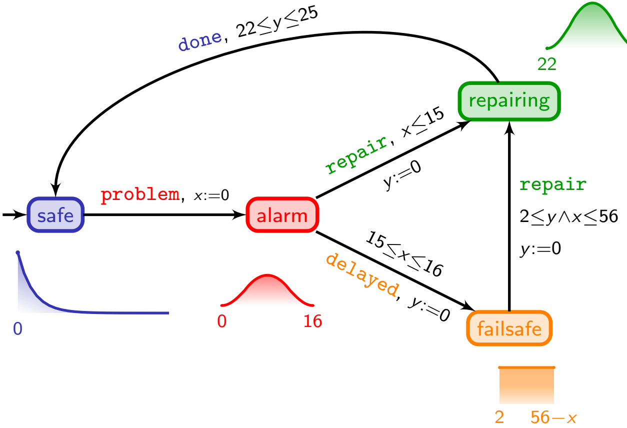


uniform distribution
density function $t \mapsto \begin{cases} \frac{1}{|I|} & \text{if } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$

How does a STA look like?



Formalization of the semantics

- $\pi(s \xrightarrow{e_1} \dots \xrightarrow{e_n})$: symbolic path from s firing edges $e_1, \dots, e_n$

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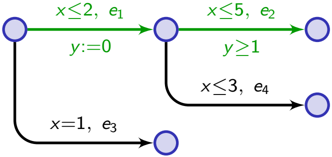

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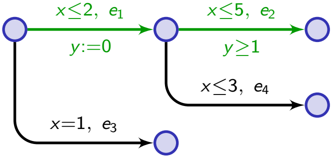

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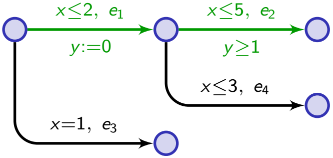

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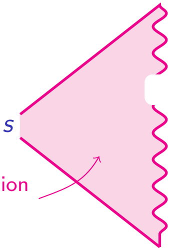

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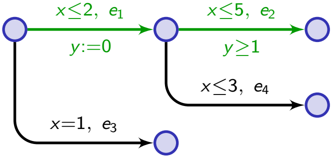

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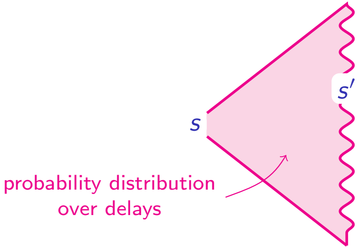

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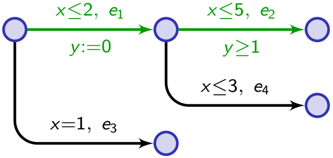

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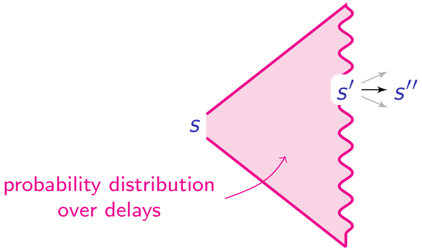

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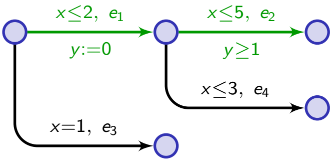

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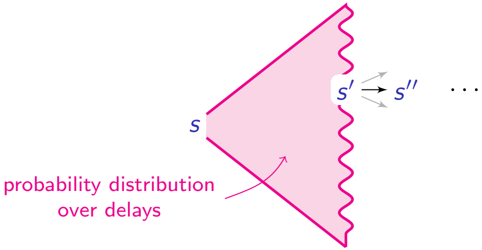

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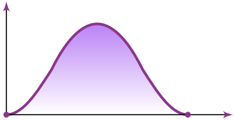

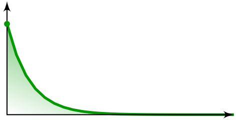

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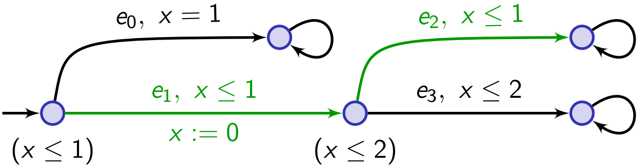
- Example:
  - $\mathsf{Zeno}(s) \;=\; \displaystyle\bigcup_{M \in \mathbb{N}} \bigcap_{n \in \mathbb{N}} \bigcup_{(e_1, \cdots, e_n) \in E^n} \mathsf{Cyl}(\pi_{\sum_i \tau_i \leq M}(s \xrightarrow{e_1} \cdots \xrightarrow{e_n}))$

An example of computation (with uniform distributions)



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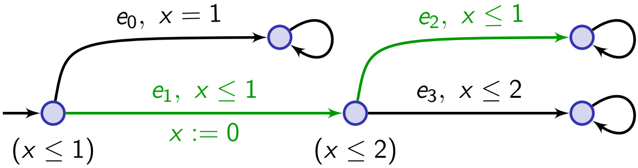

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$$\begin{aligned}
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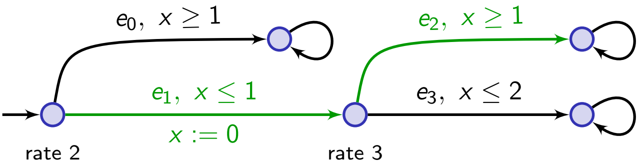
&= \int_0^1 \int_0^1 \left(\frac{\mathbb{P}\big(\pi(s_2)\big)}{2}\mathrm{d}\mu_{s_1}(u)\right)\mathrm{d}\mu_{s_0}(t) \\
&= \int_0^1 \int_0^1 \left(\frac{1}{2}\frac{\mathrm{d}u}{2}\right)\mathrm{d}t \quad = \frac{1}{4}
\end{aligned}$$

An example of computation (with exponential distrib.)



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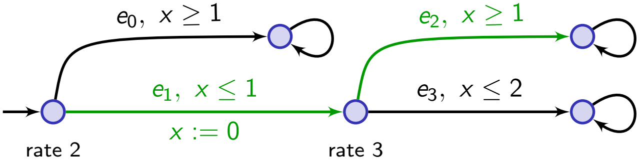

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&= \int_0^1 \left(\int_1^{+\infty} 3\exp(-3u)\mathrm{d}u\right) 2\exp(-2t)\mathrm{d}t \\
&= [-\exp(-2t)]_{t=0}^{1} \cdot [-\exp(-3u)]_{u=1}^{+\infty} \\
&= (1-e^{-2}) \cdot e^{-3} = e^{-3} - e^{-5}
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- Continuous-time Markov chains = STA with a single “useless” clock which is reset on all transitions. The distributions on delays are exponential distributions with a rate per location
- Finite-state generalized semi-Markov processes (residual-lifetime semantics) are STAs (if no fixed-delay events)
- Allows to express richer timing constraints

Outline

Almost-sure model-checking

We are interested in (automatic) model-checking algorithms!

- Qualitative model-checking: decide whether

$$\mathbb{P}\big(\{\varrho \in \mathsf{Runs}(s) \mid \varrho \models \varphi\}\big) = 1$$

  We write $s \models\!\!\!\approx \varphi$ whenever it is the case.
  This is the almost-sure model-checking problem.

- Quantitative model-checking: compute (or approximate) the value

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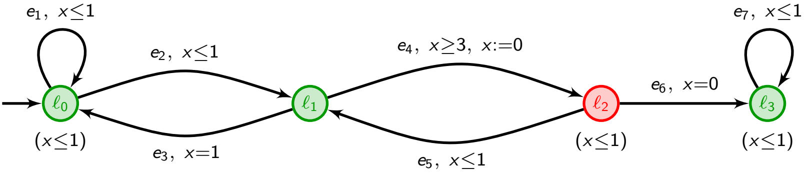
We write $s \models\!\!\!\approx \varphi$ whenever it is the case.
This is the almost-sure model-checking problem.

- Quantitative model-checking: compute (or approximate) the value

$$\mathbb{P}\big(\{\varrho \in \mathsf{Runs}(s) \mid \varrho \models \varphi\}\big)$$

In this talk we focus on
the almost-sure model-checking problem.

An example



$e_1,\ x\leq 1$
$e_2,\ x\leq 1$
$e_3,\ x=1$
$e_4,\ x\geq 3,\ x:=0$
$e_5,\ x\leq 1$
$e_6,\ x=0$
$e_7,\ x\leq 1$
ℓ_0
ℓ_1
ℓ_2
ℓ_3
$(x\leq 1)$
$(x\leq 1)$
$(x\leq 1)$

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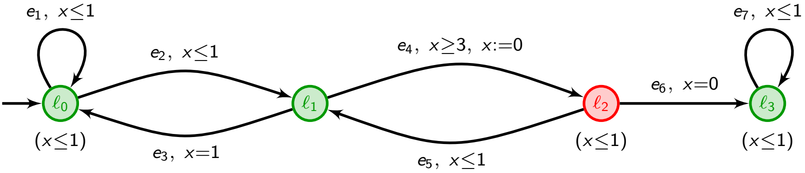

$\mathcal{A} \not\models \mathbf{G}(\text{green} \Rightarrow \mathbf{F}\ \text{red})$

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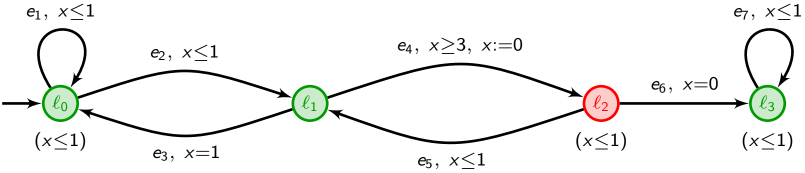

$$\mathcal{A} \not\models \mathbf{G}(\text{green} \Rightarrow \mathbf{F}\,\text{red}) \qquad \text{but} \qquad \mathbb{P}\big(\mathcal{A} \models \mathbf{G}(\text{green} \Rightarrow \mathbf{F}\,\text{red})\big) = 1$$

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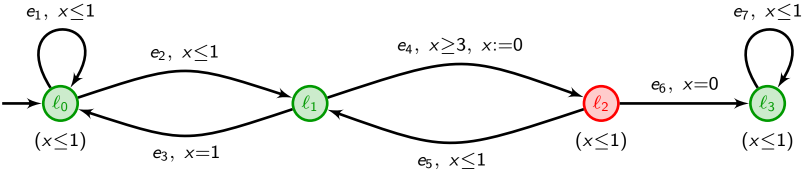
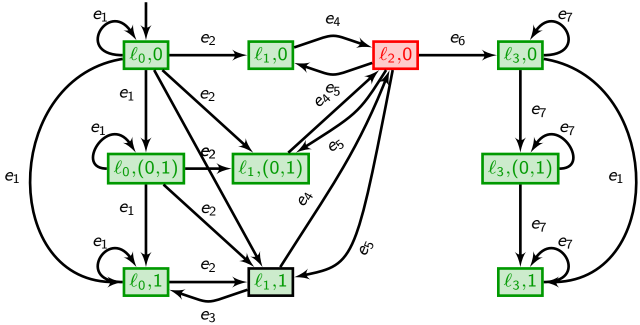


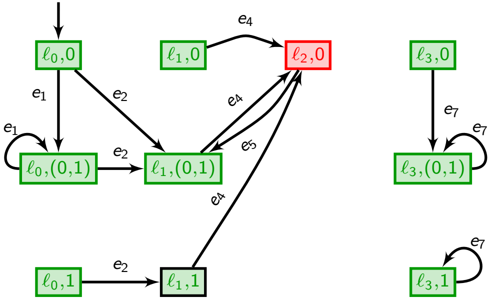
$\mathcal{A} \not\models \mathbf{G}(\text{green} \Rightarrow \mathbf{F}\,\text{red})$ but $\mathbb{P}\big(\mathcal{A} \models \mathbf{G}(\text{green} \Rightarrow \mathbf{F}\,\text{red})\big) = 1$

Indeed, almost surely, paths are of the form $e_1^* e_2 \big(e_4 e_5\big)^\omega$

The classical region automaton



The pruned region automaton



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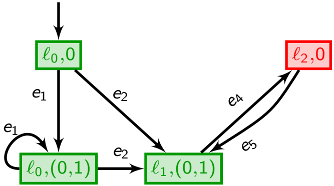

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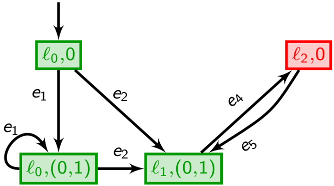

... viewed as a finite Markov chain $MC(\mathcal{A})$

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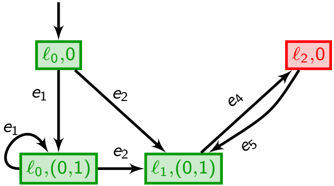

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It holds as well that:

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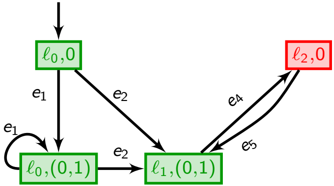

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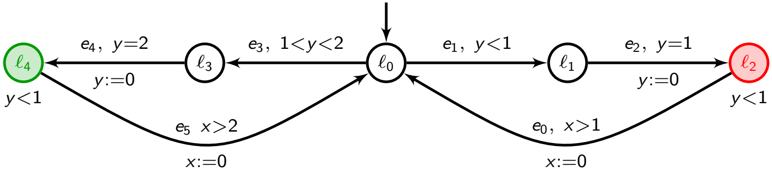
When is that the case that

$$\mathbb{P}\big(\mathcal{A} \models \varphi\big) = 1 \quad \text{iff} \quad \mathbb{P}\big(MC(\mathcal{A}) \models \varphi\big) = 1\ ?$$

A counter-example



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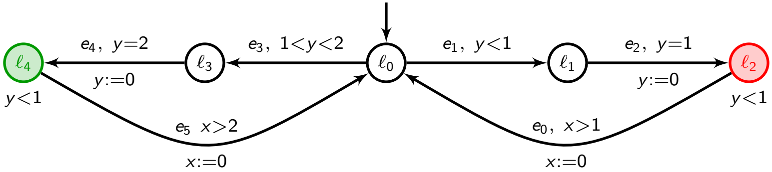

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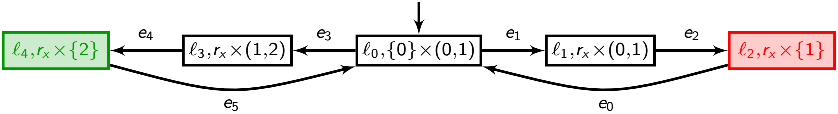

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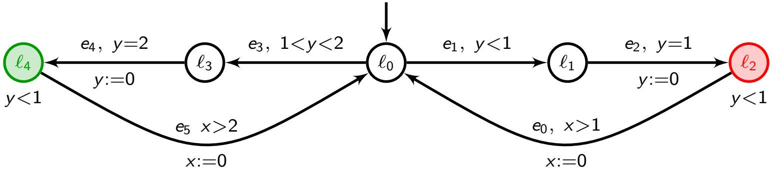

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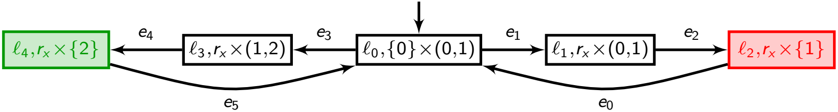

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We clearly have that $\mathbb{P}(MC(\mathcal{A}) \models \varphi) = 1$ BUT $\mathbb{P}(\mathcal{A} \models \varphi) < 1$.

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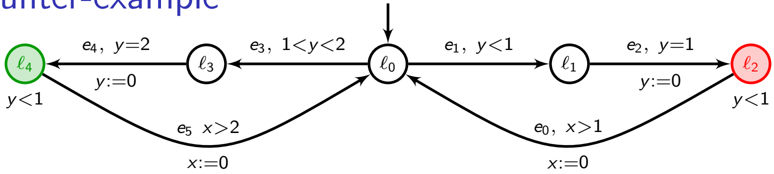

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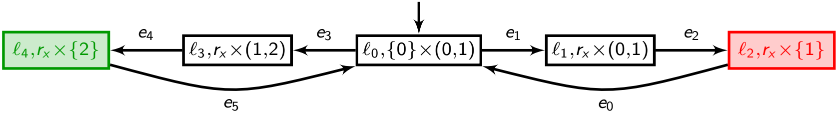

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Let y_n be the value of y at the n^{th} arrival in ℓ_0

$$y_n < 1 \qquad \text{and} \qquad y_n < y_{n+1}$$

Main decidability results [BBB+14]

Theorem

Let $\mathcal{A}$ be a STA and φ a safety property. Then:

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Fairness is a semantic condition:

every edge which is enabled infinitely often is taken infinitely often

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- Proof based on a topology over the set of paths
- Notions of largeness (for proba 1) and meagerness (for proba 0)
- Link between probabilities and topology thanks to the topological games called Banach-Mazur games

[BBB+14] Bertrand, Bouyer, Brihaye, Menet, Baier, Größer, Jurdziński. Stochastic timed automata *(Logical Methods in Computer Science)*.

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[ABM07] Abdulla, Ben Henda, Mayr. Decisive Markov chains *(Logical Methods in Computer Science)*.

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Results [BBB+14]

Theorem

The following classes of STAs are almost-surely fair:

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- (weak-)reactive STAs

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Corollary

The almost-sure model-checking of ω-regular properties in single-clock (resp. reactive) STAs can be decided in NLOGSPACE (resp. PSPACE).
The almost-sure model-checking of of LTL properties in single-clock or reactive STAs can be decided in PSPACE.

[BBB+14] Bertrand, Bouyer, Brihaye, Menet, Baier, Größer, Jurdziński. Stochastic timed automata *(Logical Methods in Computer Science)*.

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- Proof for single-clock STAs:
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  - Technical analysis of single-clock STAs
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- Reactive STAs:
  - There exists $\epsilon > 0$ such that for all s, $\mu_s(]M, +\infty[) > \epsilon$
  - Notion of memoryless region: for every x, either $x = 0$ or $x > M$
  - Borel-Cantelli lemma

Assume $(\mathcal{E}, \mathbb{P})$ is a probabilistic space, and that the measurable events $(E_k)_{k \in \mathbb{N}}$ are independent. If $\sum_{k \in \mathbb{N}} \mathbb{P}(E_k) = +\infty$, then

$$\mathbb{P}\left(\bigcap_{n \in \mathbb{N}} \bigcup_{k \geq n} E_k\right) = 1\,.$$

A note on Zeno behaviours

- The set of Zeno behaviours is measurable:

$$\mathrm{Zeno}(s) \;=\; \bigcup_{M\in\mathbb{N}} \bigcap_{n\in\mathbb{N}} \bigcup_{(e_1,\cdots,e_n)\in E^n} \mathrm{Cyl}(\pi(s \xrightarrow{e_1} \cdots \xrightarrow{e_n}))$$

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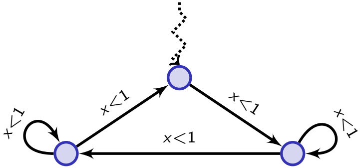

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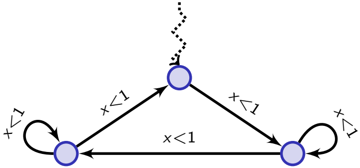

  - an interesting notion of non-Zeno timed automata

- In reactive STAs, Zeno behaviours have probability 0

Outline

Challenge: compositional design of STA

Problematic

componentwise description of systems involving timed constraints and stochastic uncertainties

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    Note: inspiring discussion in [HK09]

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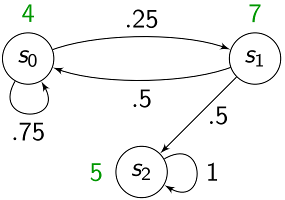
A quick look at CTMCs



A dual representation of CTMCs...

- Exit rates $r(\cdot)$ of states
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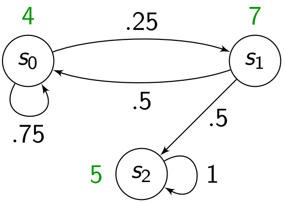

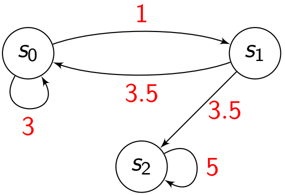

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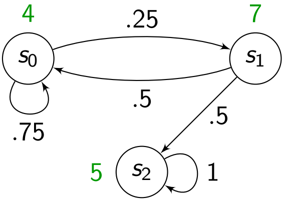

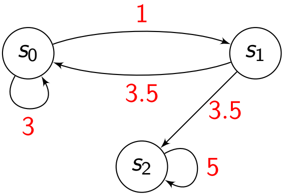

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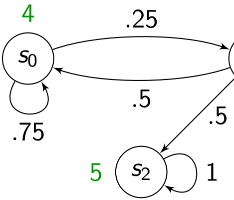

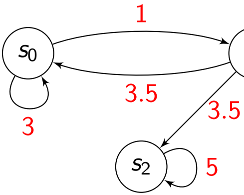

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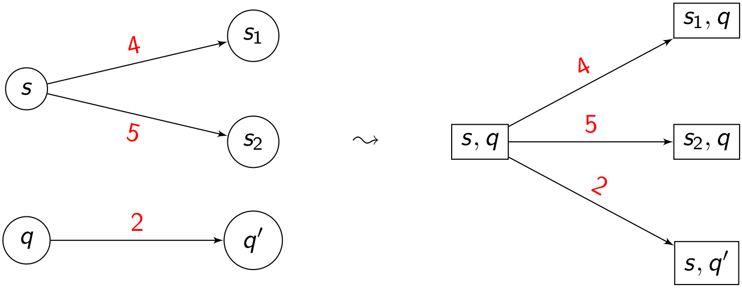
... which allows some computations

- The probability to move from s_0 to s_1 within $[0, t]$ is:

$$\frac{R(s_0, s_1)}{r(s_0)} \cdot (1 - \exp(-r(s_0) \cdot t))$$

Composition of CTMCs made easy



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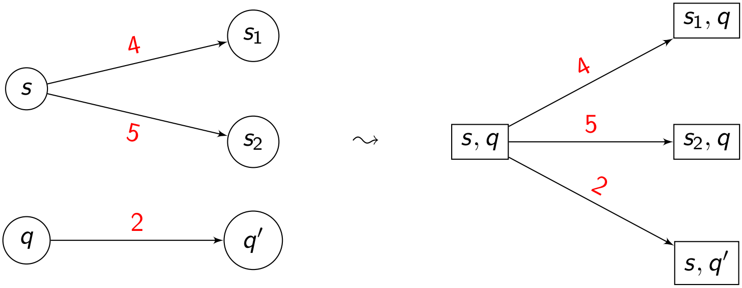

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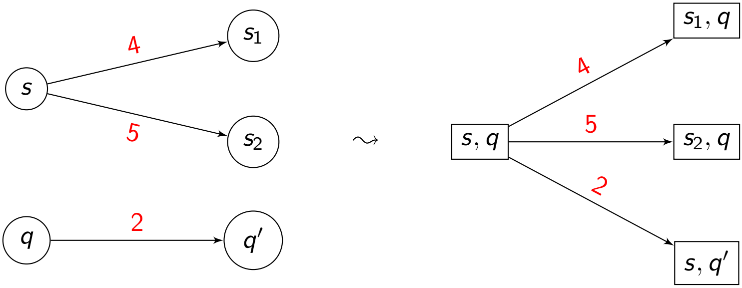

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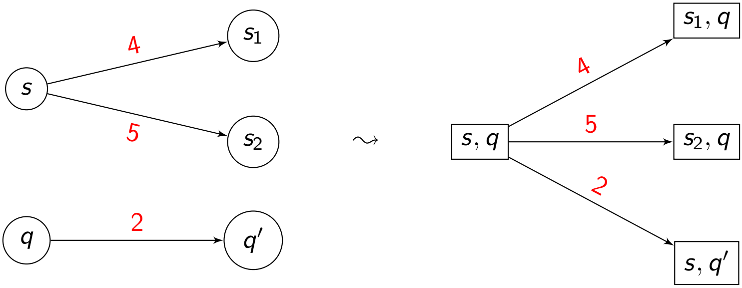
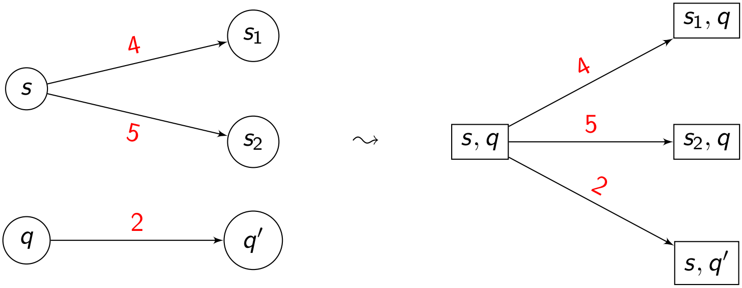


- Race between (s, s_1), (s, s_2) and (q, q')
- If (s, s_i) wins, the system moves to (s_i, q)
- There is a new race between edges from s_i and (q, q') again

Composition of CTMCs made easy



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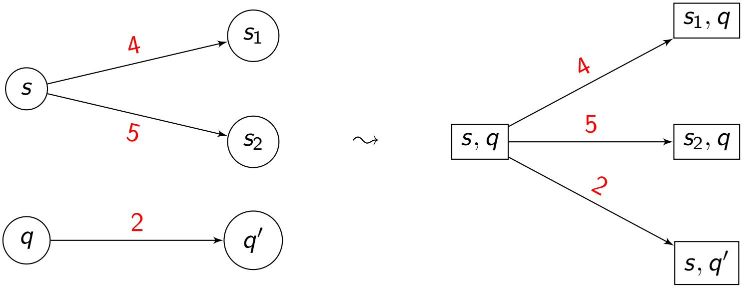

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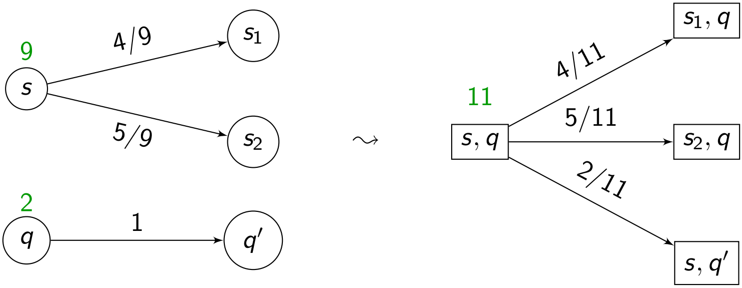

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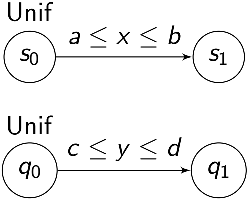
How does that extend to STAs?

Difficulties

- we have to handle guards
- we have to compose more general continuous distributions
  $\rightsquigarrow$ should represent a race between the components
- we want to preserve the structure of the product automaton
- the product should be "interleaving"

An example where everything goes well



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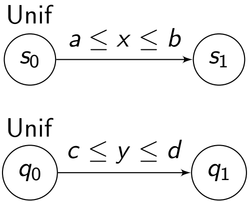

Unif. distrib. over $[a, b]$

- Density function:

$$f(t) = \begin{cases} 0 & \text{if } t < a \text{ or } t > b \\ \frac{1}{b-a} & \text{if } a \leq t \leq b \end{cases}$$

- Cumulative function:

$$\begin{aligned} F(t) &= \int_{d \leq t} f(t)\,\mathrm{d}t \\ &= \begin{cases} 0 & \text{if } t < a \\ \frac{t-a}{b-a} & \text{if } a < t < b \\ 1 & \text{if } t > b \end{cases} \end{aligned}$$

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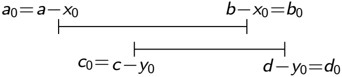
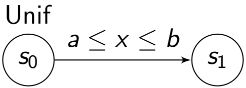
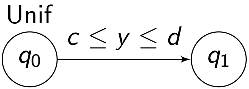
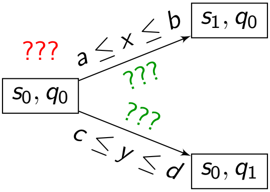
Assume $x_0 \le a$ and $y_0 \le c$ are s.t.









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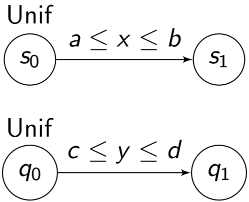

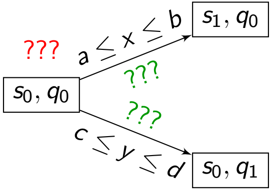

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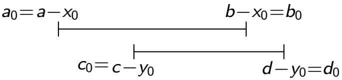

- Distrib. over delays: min. of the two distrib. over delays (race). Its density is:

$$f_{x_0,y_0}(t) = \begin{cases} 0 & \text{if } t < a_0 \text{ or } t > b_0 \\ \frac{1}{b_0 - a_0} & \text{if } a_0 < t < c_0 \\ \frac{d_0 + b_0 - 2t}{(b_0 - a_0)\cdot(d_0 - c_0)} & \text{if } c_0 < t < b_0 \end{cases}$$

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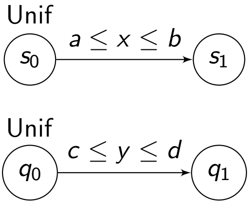

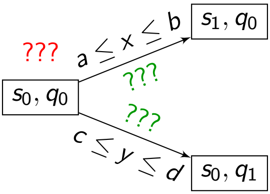

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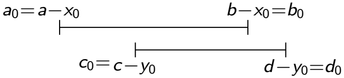

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- Discrete proba. on edges within $[c_0, b_0]$: proba. that the component has won the race

$$\frac{b_0 - t}{d_0 + b_0 - 2t}, \text{ resp. } \frac{d_0 - t}{d_0 + b_0 - 2t}$$

for the bottom, resp. top, edge.

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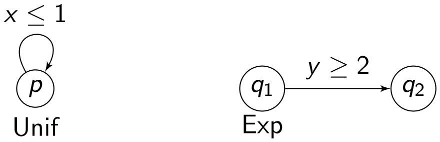

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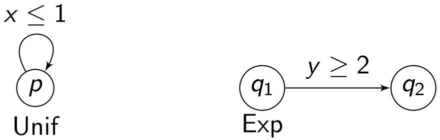

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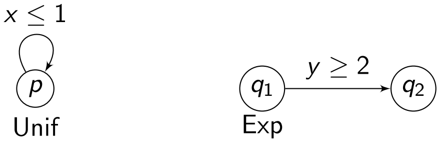

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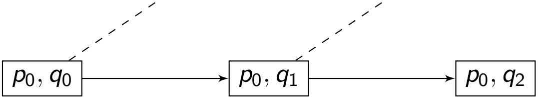

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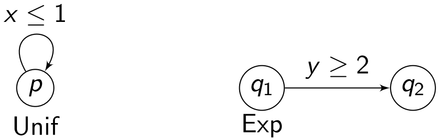

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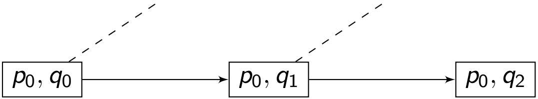

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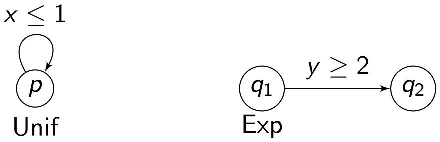

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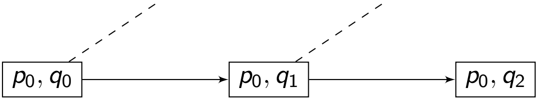

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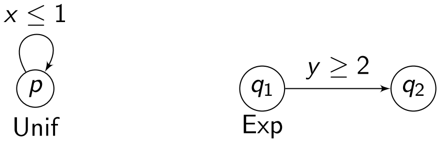

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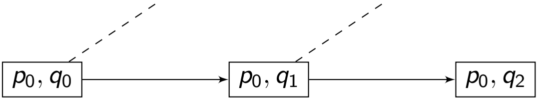

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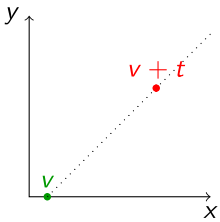

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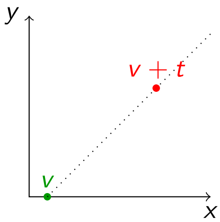

Examples

- $\mathrm{Exp}(\lambda)$ distrib. from v implies $\mathrm{Exp}(\lambda)$ distrib. from $v+t$
- Unif distrib. from v implies Unif distrib. from $v+t$
- Any μ in v can be transferred to some μ' in $v+t$

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Theorem

When STAs are weak-memoryless and almost-surely non-Zeno, the above recipe defines an *internal* parallel composition operator such that:

$$\mathbb{P}_{\mathcal{A}_1 \| \mathcal{A}_2}(\varphi_1 \wedge \varphi_2) = \mathbb{P}_{\mathcal{A}_1}(\varphi_1) \cdot \mathbb{P}_{\mathcal{A}_2}(\varphi_2)$$

Further nice and useful properties

Bisimulation (inspired by [DP03])

An extension of that for CTMCs can be defined: $(\ell, v) \equiv (\ell', v')$ if and only if for every (measurable and reasonable) $\equiv$-closed set C, for every measurable set of delays I,

$$\mathbb{P}\big((\ell, v) \xrightarrow{I,E} C\big) = \mathbb{P}\big((\ell', v') \xrightarrow{I,E} C\big)$$

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The bisimulation is a congruence w.r.t. parallel composition:

$$\mathcal{A}_1 \equiv \mathcal{A}_2 \text{ implies } \mathcal{A}_1 \parallel \mathcal{B} \equiv \mathcal{A}_2 \parallel \mathcal{B}$$

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