Model-Checking Timed Temporal Logics

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Based on joint works with Fabrice Chevalier, Nicolas Markey,
Joël Ouaknine and James Worrell

Outline

Model-checking

system:

property:

$\Rightarrow$

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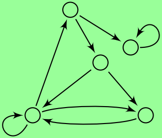

$\mathbf{G}\,(\texttt{request} \rightarrow \mathbf{F}\,\texttt{grant})$

Model-checking

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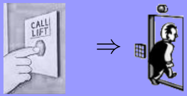
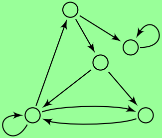
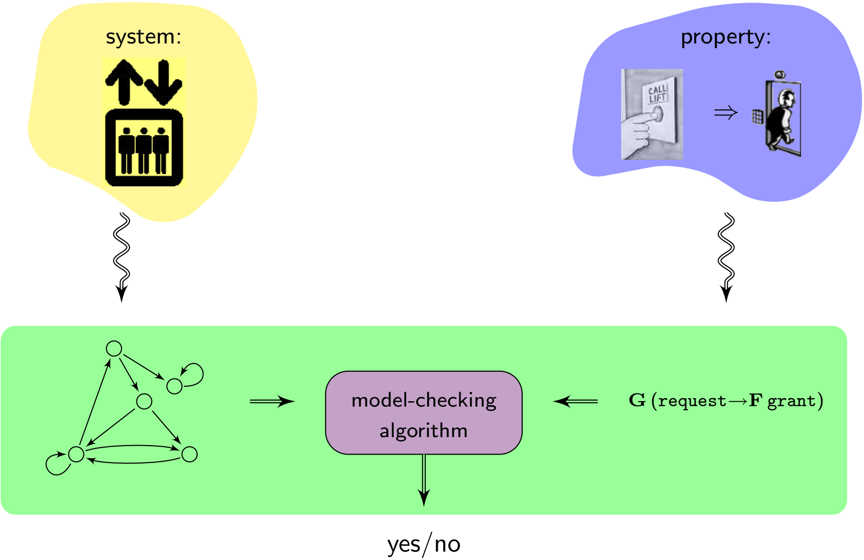




model-checking algorithm

$\mathbf{G}\,(\mathtt{request} \rightarrow \mathbf{F}\,\mathtt{grant})$

Model-checking



The untimed (linear-time) framework

Linear-time temporal logic [Pnu77]

$$\mathsf{LTL} \ni \varphi ::= p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \neg\varphi \mid \mathbf{X}\,\varphi \mid \varphi\,\mathbf{U}\,\varphi$$

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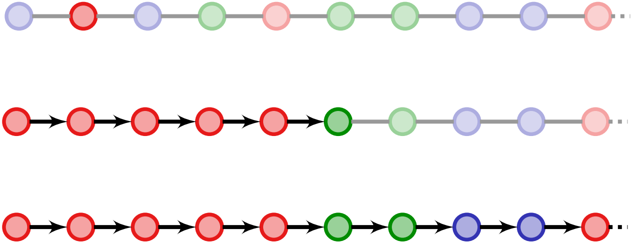

$\models \quad \mathbf{X}\,\textcolor{red}{\bullet}$

$\models \quad \textcolor{red}{\bullet}\,\mathbf{U}\,\textcolor{green}{\bullet}$

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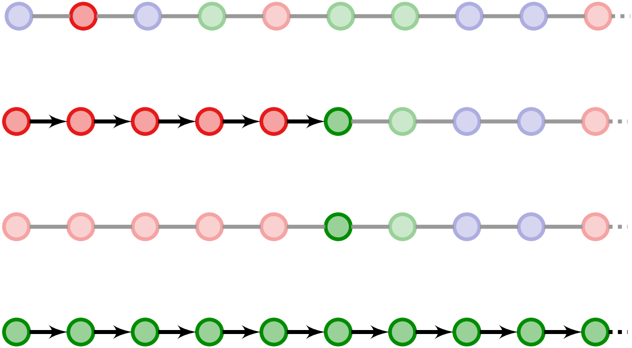

$$\models \quad \mathbf{X}\,\bullet$$

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$$\models \quad \mathbf{G}\,\bullet \;\equiv\; \neg\mathbf{F}\,\neg\bullet$$

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- response property:

$$\mathbf{G}\,({\color{red}\bullet} \to \mathbf{F}\,{\color{green}\bullet})$$

- liveness property:

$$\mathbf{G}\,\mathbf{F}\,{\color{green}\bullet}$$

- safety property:

$$\mathbf{G}\,\neg{\color{red}\bullet}$$

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- safety property:

$$\mathbf{G}\,\neg\bullet$$

- a more complex property:

$$(\bullet \wedge (\mathbf{F}\,\bullet \vee \mathbf{G}\,\bullet))\,\mathbf{U}\,\bullet$$

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Adding timing requirements

- Need for timed models
  - the behaviour of most systems depends on time;
  - faithful modelling has to take time into account.

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- Need for timed specification languages
  - the behaviour of most systems depends on time;
  - untimed specifications are not sufficient
    (for instance, bounded response timed, etc...)

☞ TCTL, MTL, TPTL, timed μ-calculus...

Outline

Metric Temporal Logic (MTL)

[Koy90]

$$\mathsf{MTL} \ni \varphi \ ::= \ a \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \varphi \, \mathbf{U}_I \, \varphi$$

where I is an interval with integral bounds.

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  - this distinction is fundamental

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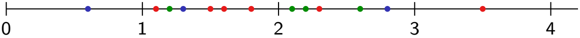
The pointwise semantics

MTL formulas are interpreted over timed words:



$$(\bullet, .6)(\bullet, 1.1)(\bullet, 1.2)(\bullet, 1.3)\ldots$$

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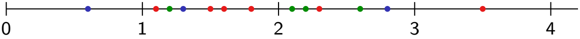

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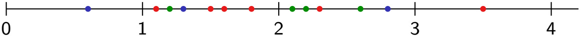

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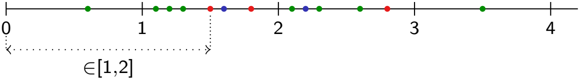

$\models \bullet \ \mathbf{U}_{[1,2]} \bullet$

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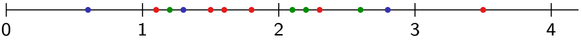

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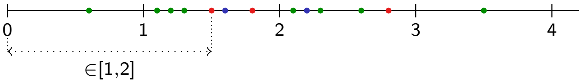

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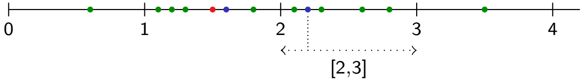

$\not\models \mathbf{G}_{[2,3]} \bullet$

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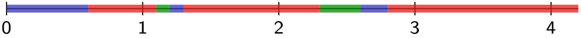
MTL formulas are interpreted over (finitely variable) signals:



$t \in [0,.6] \mapsto \blacksquare$ $\qquad$ $t \in (.6,1.1) \mapsto \blacksquare$ $\qquad$ $t \in [1.1,1.2) \mapsto \blacksquare$

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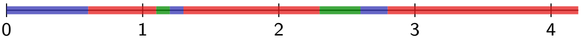

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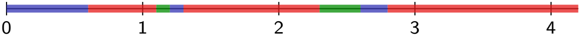

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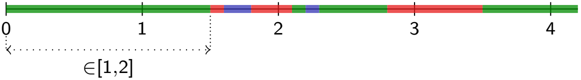

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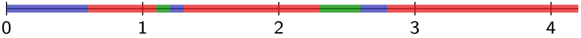

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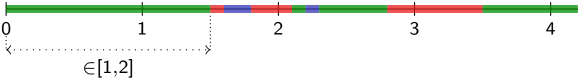

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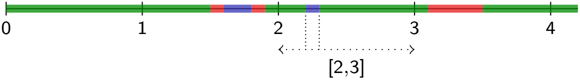

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Some examples

- "Every problem is followed within 56 time units by an alarm"

$$\mathbf{G}\,(\texttt{problem} \rightarrow \mathbf{F}_{\leqslant 56}\,\texttt{alarm})$$

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- "Every problem is followed within 56 time units by an alarm"

$$\mathbf{G}\,(\texttt{problem} \rightarrow \mathbf{F}_{\leqslant 56}\,\texttt{alarm})$$

- "Each time there is a problem, it is either repaired within the next 15 time units, or an alarm rings during 3 time units 12 time units later"

$$\mathbf{G}\,(\texttt{problem} \rightarrow (\mathbf{F}_{\leqslant 15}\,\texttt{repair} \vee \mathbf{G}_{[12,15)}\,\texttt{alarm}))$$

Some further extensions

- Timed Propositional Temporal Logic (TPTL) [AH89]

TPTL = LTL + clock variables + clock constraints

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Theorem

LTL+Past is as expressive as LTL [Kam68,GPSS80].

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- This is true in the pointwise semantics.
- This is wrong in the continuous semantics!

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The TPTL formula

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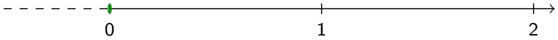

$$\mathbf{G}\ \bullet \rightarrow \left\{ \right.$$

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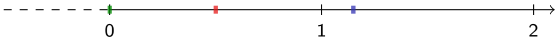

$$\mathbf{G}\ \bullet\ \rightarrow \left\{ \begin{array}{l} \mathbf{F}_{\leqslant 1}\ \bullet\ \wedge\ \mathbf{F}_{[1,2]}\ \bullet \\ \\ \\ \end{array} \right.$$

The TPTL formula

$$\mathbf{G}\left({\color{green}\bullet} \rightarrow x.\mathbf{F}\left({\color{red}\bullet} \wedge \mathbf{F}\left({\color{blue}\bullet} \wedge x \leqslant 2\right)\right)\right)$$

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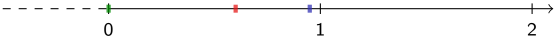

$$\mathbf{G}\ {\color{green}\bullet} \ \rightarrow \ \left\{ \begin{array}{l} \quad \mathbf{F}_{\leqslant 1}\ {\color{red}\bullet} \ \wedge \ \mathbf{F}_{[1,2]}\ {\color{blue}\bullet} \\ \vee \\ \quad \mathbf{F}_{\leqslant 1}\left({\color{red}\bullet} \ \wedge \ \mathbf{F}_{\leqslant 1}\,{\color{blue}\bullet}\right) \\ \\ \end{array} \right.$$

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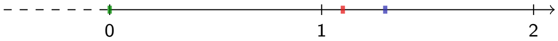
can be expressed in MTL in the continuous semantics



$$\mathbf{G}\ \textcolor{green}{\bullet}\ \rightarrow \left\{ \begin{array}{ll} & \mathbf{F}_{\leqslant 1}\ \textcolor{red}{\bullet}\ \wedge\ \mathbf{F}_{[1,2]}\ \textcolor{blue}{\bullet} \\ \vee & \\ & \mathbf{F}_{\leqslant 1}\,(\ \textcolor{red}{\bullet}\ \wedge\ \mathbf{F}_{\leqslant 1}\,\textcolor{blue}{\bullet}\ ) \\ \vee & \\ & \end{array} \right.$$

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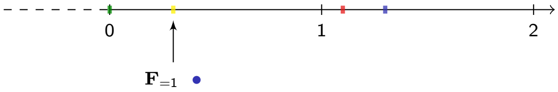

$$\mathbf{G}\ \bullet\ \rightarrow \begin{cases} \quad \mathbf{F}_{\leqslant 1}\ \bullet\ \wedge\ \mathbf{F}_{[1,2]}\ \bullet \\ \vee \\ \quad \mathbf{F}_{\leqslant 1}\,(\ \bullet\ \wedge\ \mathbf{F}_{\leqslant 1}\,\bullet\ ) \\ \vee \\ \end{cases}$$

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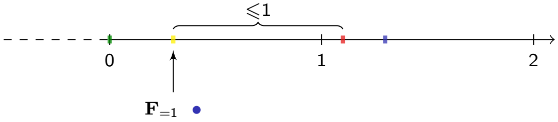

$$\mathbf{G}\ {\color{green}\bullet} \ \rightarrow \ \left\{ \begin{array}{ll} & \mathbf{F}_{\leqslant 1}\ {\color{red}\bullet}\ \wedge\ \mathbf{F}_{[1,2]}\ {\color{blue}\bullet} \\ \vee & \\ & \mathbf{F}_{\leqslant 1}\,(\ {\color{red}\bullet}\ \wedge\ \mathbf{F}_{\leqslant 1}\,{\color{blue}\bullet}\ ) \\ \vee & \\ & \end{array} \right.$$

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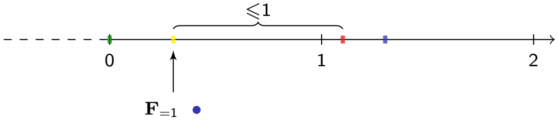

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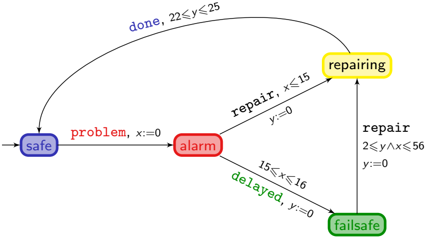
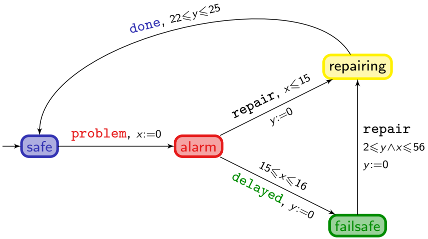
Outline

Timed automata

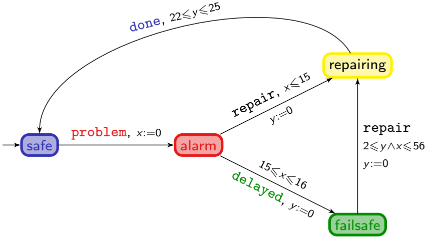


Timed automata



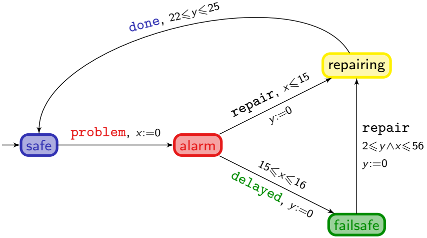
	safe
x	0
y	0

Timed automata



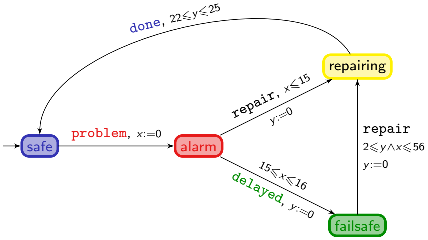
	safe	$\xrightarrow{23}$	safe
x	0		23
y	0		23

Timed automata



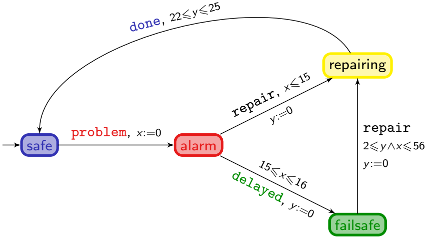
	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm
x	0		23		0
y	0		23		23

Timed automata



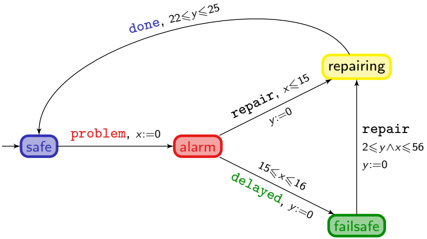
	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm
x	0		23		0		15.6
y	0		23		23		38.6

Timed automata



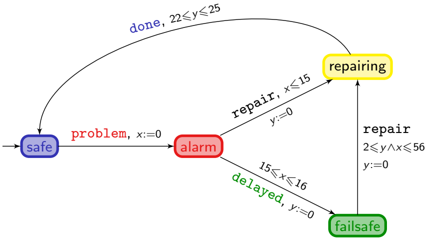
	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe
x	0		23		0		15.6		15.6
y	0		23		23		38.6		0

Timed automata



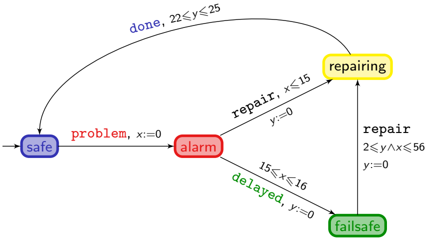
	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\text{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\text{delayed}}$	failsafe	$\xrightarrow{2.3}$	failsafe
x	0		23		0		15.6		15.6		17.9
y	0		23		23		38.6		0		2.3

Timed automata



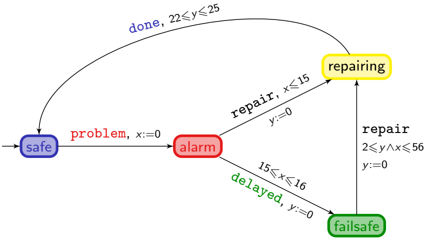
	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	reparation
x	0		23		0		15.6		15.6		17.9		17.9
y	0		23		23		38.6		0		2.3		0

Timed automata



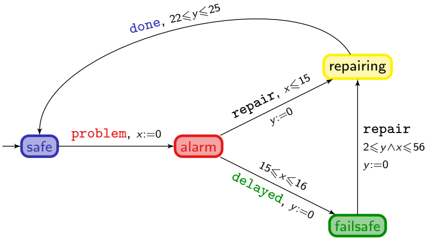
	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	reparation	$\xrightarrow{22.1}$	reparation
x	0		23		0		15.6		15.6		17.9		17.9		40
y	0		23		23		38.6		0		2.3		0		22.1

Timed automata



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	reparation	$\xrightarrow{22.1}$	reparation	$\xrightarrow{\texttt{done}}$	safe
x	0		23		0		15.6		15.6		17.9		17.9		40		40
y	0		23		23		38.6		0		2.3		0		22.1		22.1

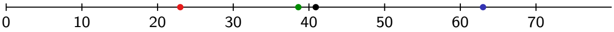
Timed automata



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	reparation	$\xrightarrow{22.1}$	reparation	$\xrightarrow{\texttt{done}}$	safe
x	0		23		0		15.6		15.6		17.9		17.9		40		40
y	0		23		23		38.6		0		2.3		0		22.1		22.1

Can be viewed:

- as the timed word (`problem`,23)(`delayed`,38.6)(`repair`,40.9)(`done`,63)



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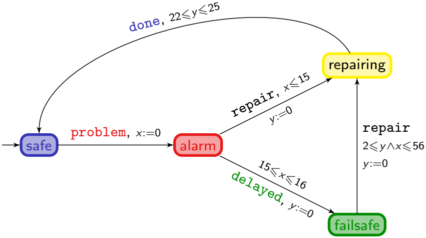

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	reparation	$\xrightarrow{22.1}$	reparation	$\xrightarrow{\texttt{done}}$	safe
x	0		23		0		15.6		15.6		17.9		17.9		40		40
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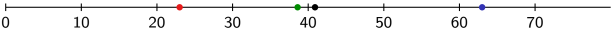
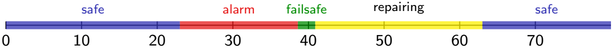


- as the signal



Basic result on timed automata

Theorem

The reachability problem is decidable (and PSPACE-complete) for timed automata [AD94].

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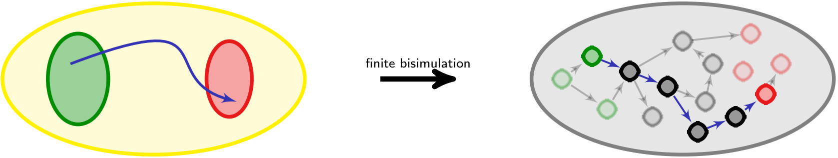

timed automaton

large (but finite) automaton
(region automaton)

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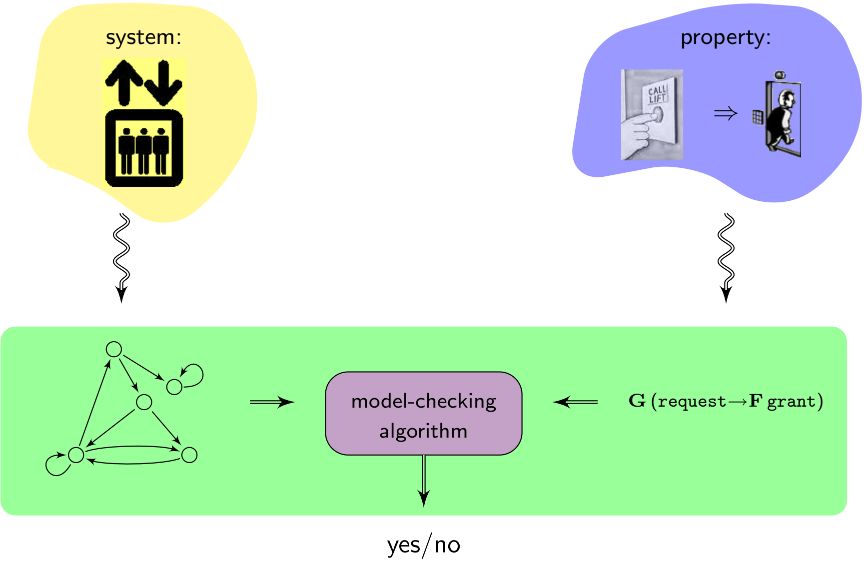
Outline

Back to the model-checking problem



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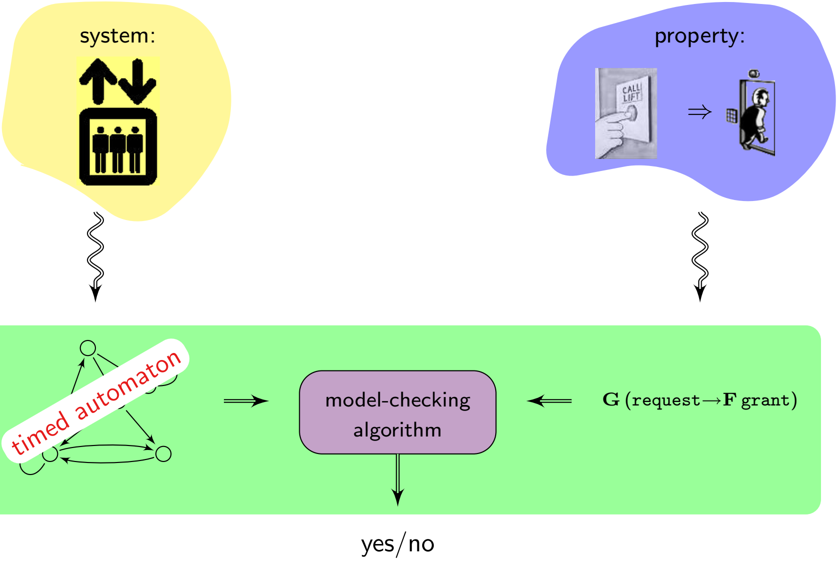

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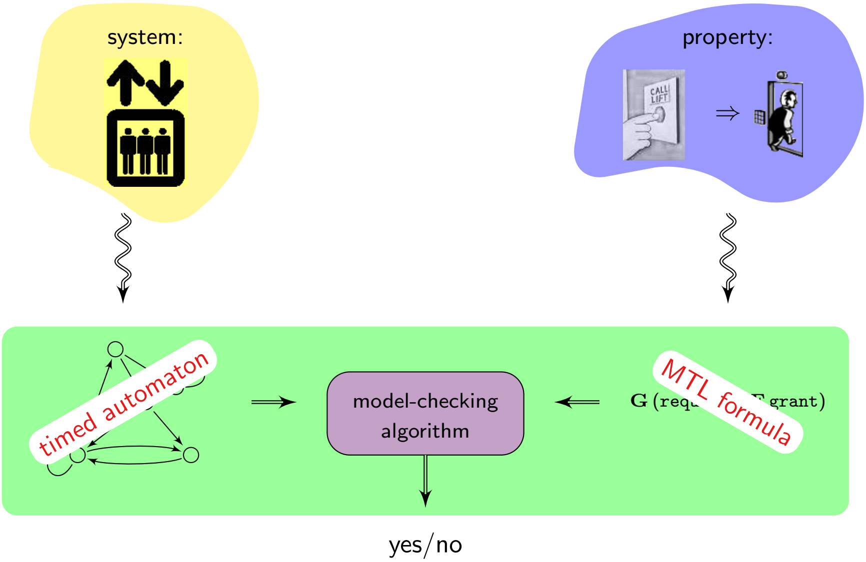

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Over finite runs, the model-checking problem is:

	pointwise sem.	continuous sem.
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From LTL to alternating automata

LTL formulas can be turned into linear alternating (Büchi) automata

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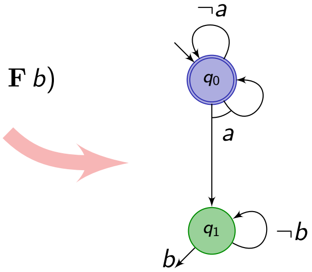

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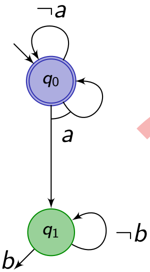

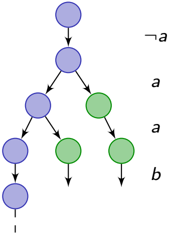

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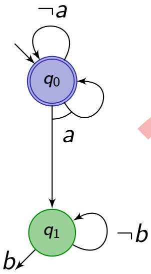

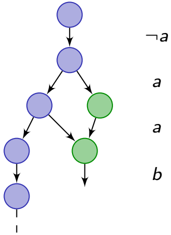

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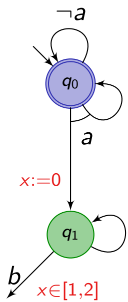

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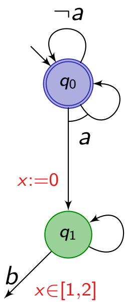
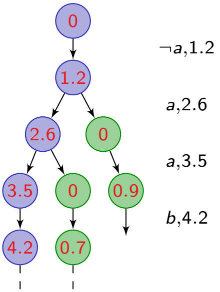
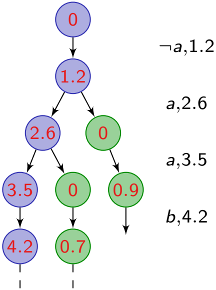




An abstract transition system



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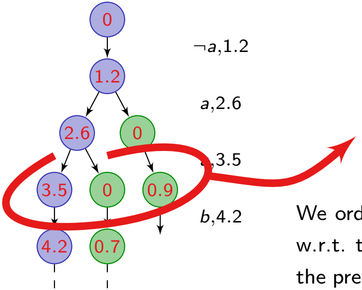

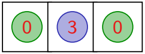

We order elements in a slice of the tree w.r.t. their fractional part, and we forget the precise values of the fractional parts.

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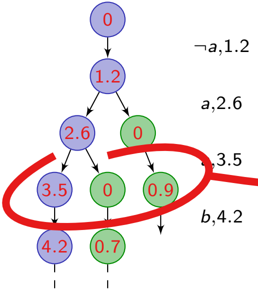

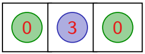

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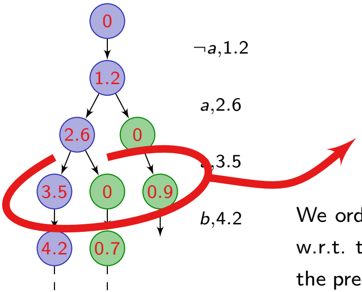

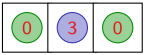

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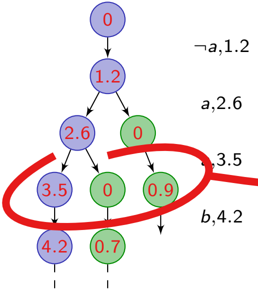

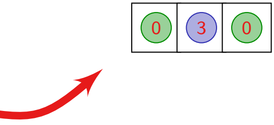

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- ☞ it is (time-abstract) bisimilar to the transition system of the alternating timed automata
- ☞ there is a well quasi-order on the set of abstract configurations (subword relation):

$$\text{higman} \sqsubseteq \text{highmountain}$$

Summary

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Over finite runs, the model-checking problem is:

	pointwise sem.	continuous sem.
MTL	undecidable [OW06]*	undecidable [AFH96]
MTL+Past	undecidable	undecidable
TPTL	undecidable [AH94]	undecidable [AH94]

* by reduction of the recurrence problem for channel machines

[MH84] Miyano, Hayashi. Alternating finite automata on ω-words (TCS, 1984).
[OW06] Ouaknine, Worrell. On metric temporal logic and faulty Turing machines (FoSSaCS'06).

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☞ something more clever needs to be done

[HR04] Hirshfeld, Rabinovich. Logics for real time: decidability and complexity (Fundamenta Informaticae, 2004).

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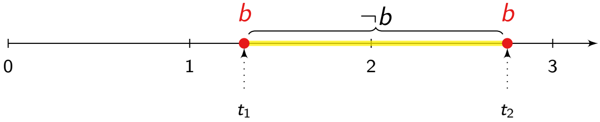

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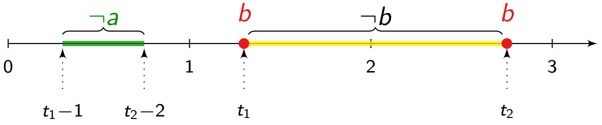

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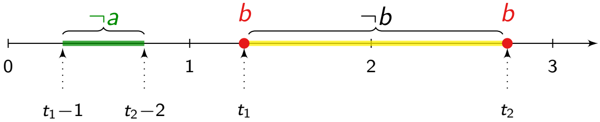

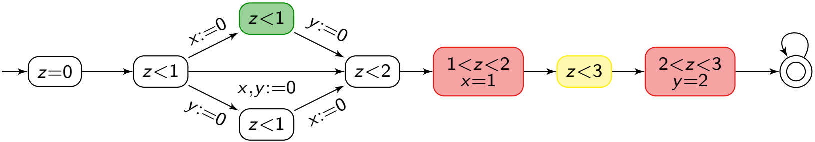

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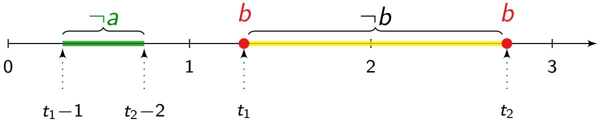

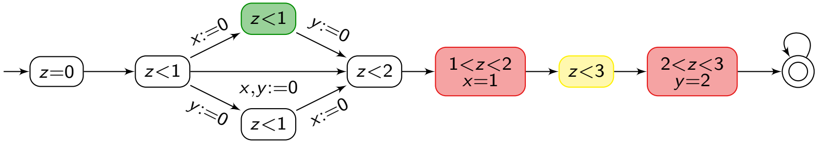

☞ This idea can be extended to any formula in MITL

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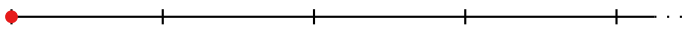

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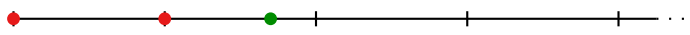

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- A Bounded-MTL formula may define a non timed-regular language:

$$\mathbf{G}_{\leqslant 1}\,(\textcolor{red}{\bullet} \rightarrow \mathbf{F}_{=1}\,\textcolor{green}{\bullet}) \wedge \mathbf{G}_{\leqslant 1}\,\textcolor{red}{\bullet} \wedge \mathbf{G}_{(1,2]}\,\textcolor{green}{\bullet}$$

defines the context-free language $\{\textcolor{red}{\bullet}^n \textcolor{green}{\bullet}^m \mid n \leqslant m\}$.

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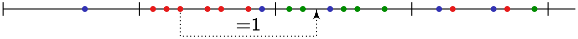

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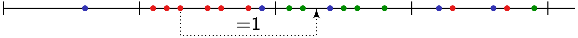

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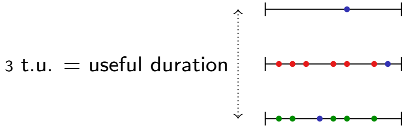

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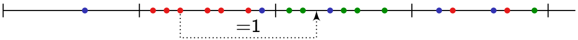

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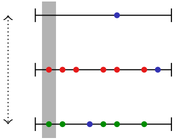

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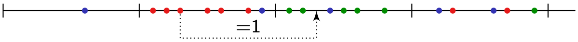

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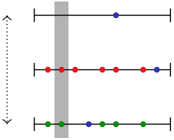

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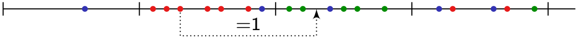

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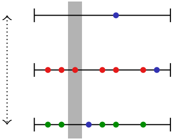

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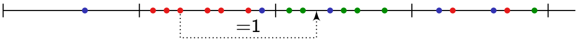

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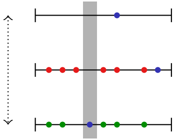

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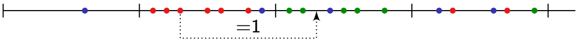

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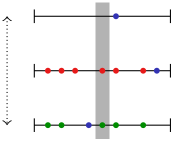

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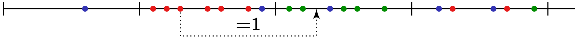

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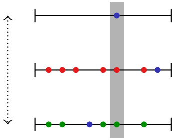

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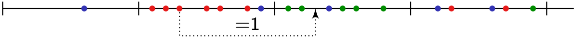

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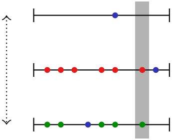

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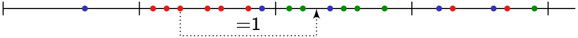

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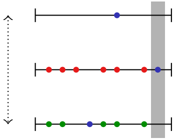

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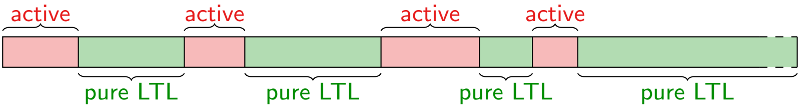

where - the number of active fragments is at most exponential
- the total duration of active fragments is at most exponential

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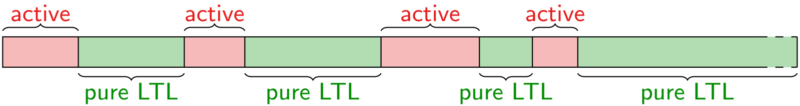

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- active fragment = cycle-bounded computation in a channel machine
- pure LTL part = finite automaton computation

Outline

Conclusion

- Recent advances have raised a new interest for linear-time timed temporal logics
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  - Some rather ‘efficient’ subclasses
    - non-punctual formulas
    - structurally (co-)flat formulas

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Conclusion

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  - Some rather 'efficient' subclasses
    - non-punctual formulas
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- A recent result: coFlat-$\mathsf{MTL}_{\mathsf{MITL}}$ unifies coFlat-MTL and MITL, and is EXPSPACE-complete [BMOW08]!
- No real data structures do exist for these logics.

[BMOW08] Bouyer, Markey, Ouaknine, Worrell. On Expressiveness and Complexity in Real-time Model Checking. Submitted.