Research internship proposal (M2)

Location : LSV (`http://www.lsv.fr`)

Title : Multiplayer games over graphs with imperfect monitoring

Supervisors :

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Description :

Multiplayer concurrent games over graphs allow to model rich interactions between players. Those games are played as follows. In a state, each player chooses privately and independently an action, defining globally a move (one action per player) ; the next state of the game is then defined as the successor (on the graph) of the current state using that move ; players continue playing from that new state, and form a(n infinite) play. Each player then gets a reward given by a payoff function (one function per player). In particular, objectives of the players may not be contradictory : those games are non-zero-sum games, contrary to two-player games used for controller or reactive synthesis [11, 8].

Using solution concepts borrowed from game theory, one can describe rational behaviours of the players in such a multiplayer situation. One of the most basic solution concepts very much studied by game theorists is that of Nash equilibria [9]. A Nash equilibrium is a strategy profile where no player can improve her payoff by unilaterally changing her strategy. The outcome of a Nash equilibrium can therefore be seen as a rational behaviour of the system. While very much studied by game theoretists (e.g. over (repeated) matrix games), such a concept (and variants thereof) has been only rather recently studied over game graphs. Probably the first works in that direction are [7, 6, 12, 13]. Several series of works have followed. To roughly give an idea of the existing results, pure Nash equilibria always exist in turn-based games for ω-regular objectives [15] but not in concurrent games games ; they can nevertheless be computed [15, 3, 5] for large classes of objectives, though they may not exist in concurrent games, even with simple objectives. The problem becomes harder with mixed Nash equilibria, for which we often cannot decide the existence [14, 4].

Computing Nash equilibria requires to (i) find a good behaviour of the system ; (ii) detect deviations from that behaviour, and identify deviators ; (iii) punish the deviators. Variants of Nash equilibria require slightly different ingredients, but they are mostly of a similar vein.

In (almost) all the works mentioned above, perfect monitoring is assumed. Recently [2], we proposed the notion of *public signal*, and showed that in such a framework, under some further hypotheses about the payoff functions (they should be publicly visible), one can compute Nash equilibria when they exist. This result is based on the construction of an epistemic game abstraction, which is is a two-player turn-based game in which we show that winning strategies of one of the players (Eve) actually correspond to Nash equilibria in the original game. The winning condition for Eve is rather complex, but can be simplified in the case of publicly visible payoff functions. The epistemic game abstraction is inspired by both the epistemic unfolding of [1] used for distributed synthesis, and the suspect game abstraction of [3] used to compute Nash equilibria in concurrent games with hidden actions.

The aim of the internship is to pursue the study of imperfect monitoring in multiplayer games. For instance, one could study whether communication patterns like those in [10] (in the context of matrix games) could be used in games on graphs as well, and yield computability of Nash equilibria. Another possibility would be to extend the approach based on public signal of [2] to more solution concepts (like subgame-perfect equilibria, or robust equilibria, or even rational behaviours described by logical formulas).

This internship will be in the line of the ERC project EQualIS (`http://www.lsv.fr/~bouyer/equalis/`), and can naturally be extended into a PhD thesis.

Références

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