Robustness in Timed Automata

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Based on joint works with Nicolas Markey, Pierre-Alain Reynier and Ocan Sankur

Acknowledgment to Nicolas and Ocan for slides

ImpRo EQualIS

Outline

1. Introduction

2. Robust model-checking and implementation
   - Parameterized enlarged semantics
   - Automatic generation of an implementation
   - Implementation by shrinking

3. Robust realisability
   - Excess semantics
   - Conservative semantics

4. Conclusion

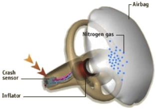
Time-dependent systems

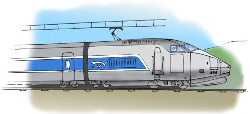
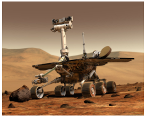
- We are interested in timed systems

Time-dependent systems

- We are interested in timed systems

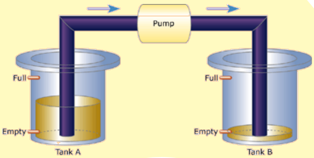




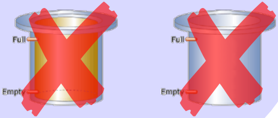
Model-checking and control

system:



[http://www.embedded.com]

property:



Model-checking and control

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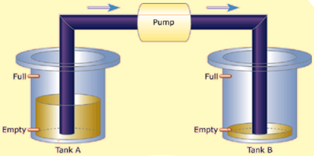

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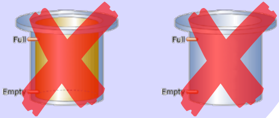
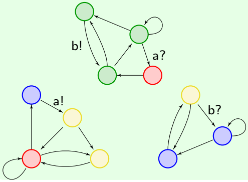
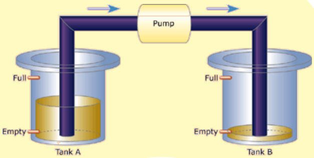




$\mathbf{AG}(\neg \texttt{B.overfull} \wedge \neg \texttt{B.dried\_up})$

Model-checking and control

system:



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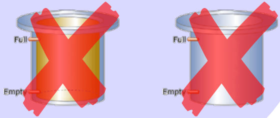
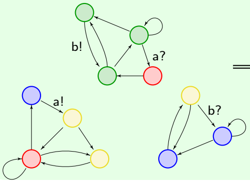


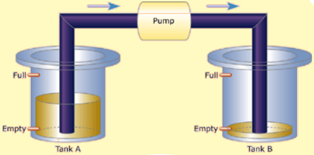


algorithm

AG($\neg$`B.overfull` $\wedge$ $\neg$`B.dried_up`)

Model-checking and control

system:



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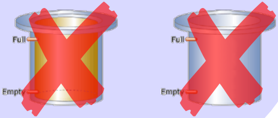
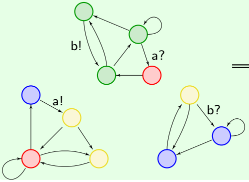




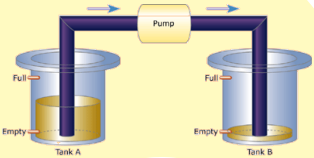
model-checking algorithm

$\mathbf{AG}(\neg\mathtt{B.overfull}$
$\wedge\ \neg\mathtt{B.dried\_up})$

yes/no

Model-checking and control

system:



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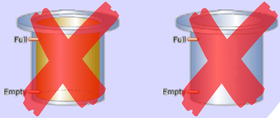
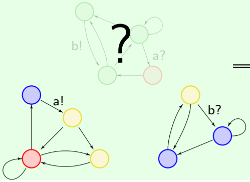
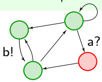




control/synthesis algorithm

AG($\neg$`B.overfull`

$\wedge$ $\neg$`B.dried_up`)



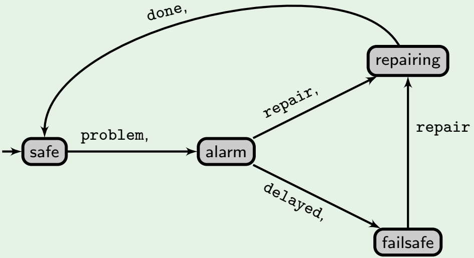
Reasoning about real-time systems

The model of timed automata [AD94]

A timed automaton is made of

- a finite automaton-based structure

Example



[AD9

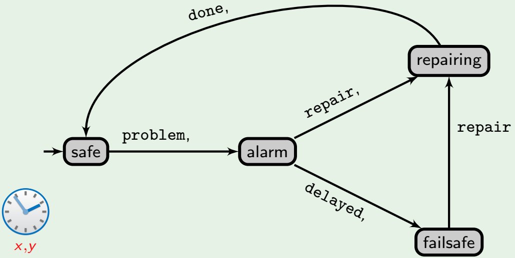
Reasoning about real-time systems

The model of timed automata [AD94]

A timed automaton is made of

- a finite automaton-based structure
- a set of clocks

Example



[AD94] Alur, Dill: A Theory of Timed Automata. Theor. Comp. Science, 1994.

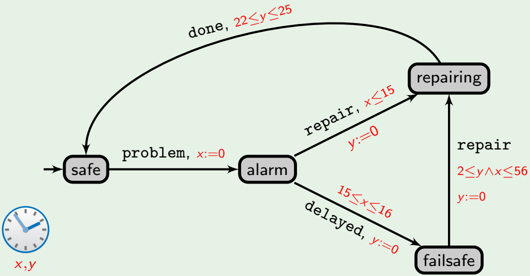
Reasoning about real-time systems

The model of timed automata [AD94]

A timed automaton is made of

- a finite automaton-based structure
- a set of clocks
- timing constraints on transitions

Example



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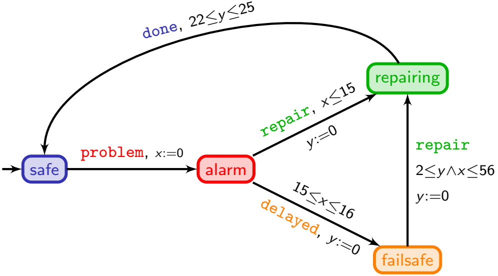
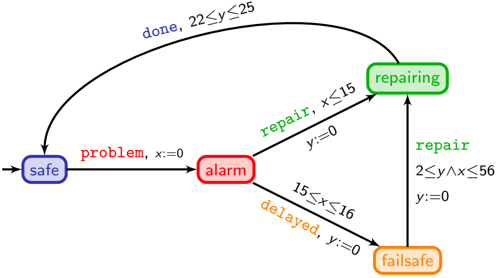
Example – cont'd

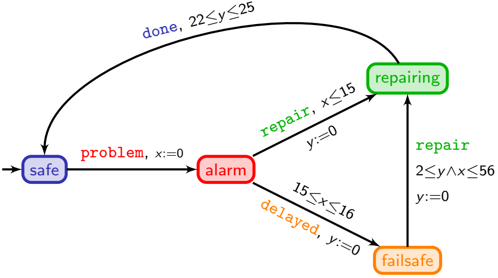


Example – cont'd



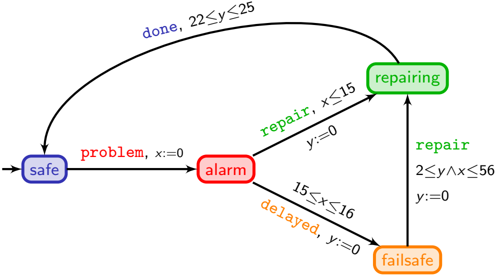
	safe
x	0
y	0

Example – cont'd



	safe	$\xrightarrow{23}$	safe
x	0		23
y	0		23

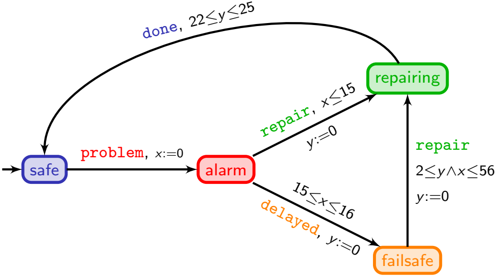
Example – cont'd



$$\text{safe} \xrightarrow{23} \text{safe} \xrightarrow{\texttt{problem}} \text{alarm}$$

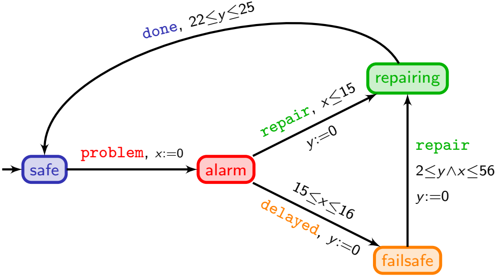
x	0	23	0
y	0	23	23

Example – cont'd



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm
x	0		23		0		15.6
y	0		23		23		38.6

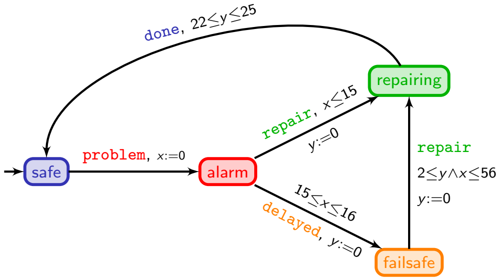
Example – cont'd



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	···
y	0		23		23		38.6		0	

	failsafe
···	15.6
	0

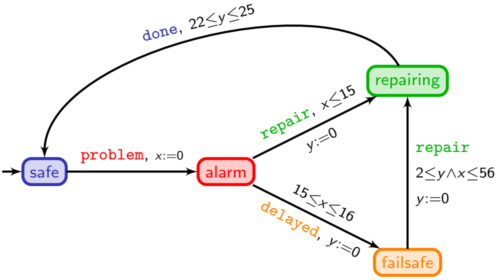
Example – cont'd



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	$\cdots$
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe
$\cdots$	15.6		17.9
	0		2.3

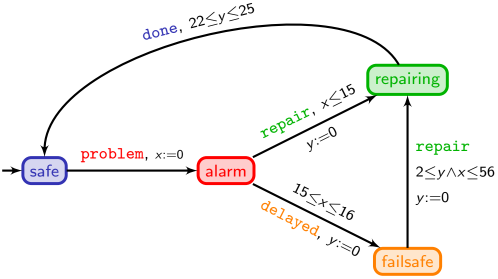
Example – cont'd



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	···
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing
···	15.6		17.9		17.9
	0		2.3		0

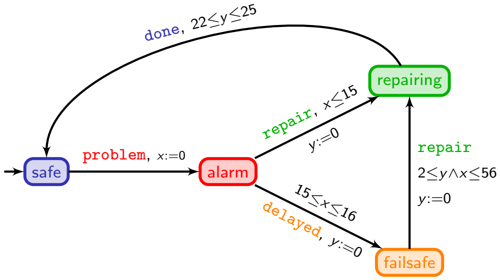
Example – cont'd



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing
⋯	15.6		17.9		17.9		40
	0		2.3		0		22.1

Example – cont'd



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\texttt{done}}$	safe
⋯	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

Discrete-time semantics

...because computers are digital!

[Alur91] Techniques for automatic verification of real-time systems. *PhD thesis*, 1991.

Discrete-time semantics

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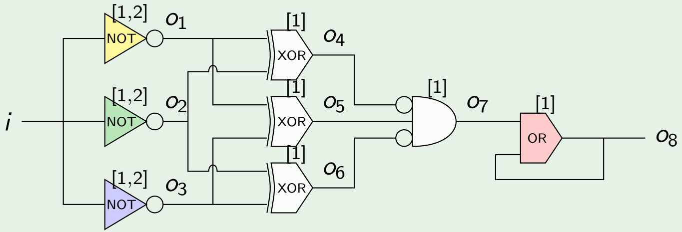
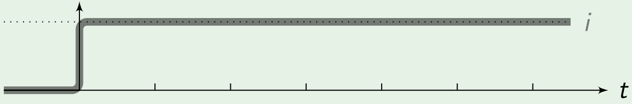
Example [Alur91]



- under discrete-time, the output is always 0:



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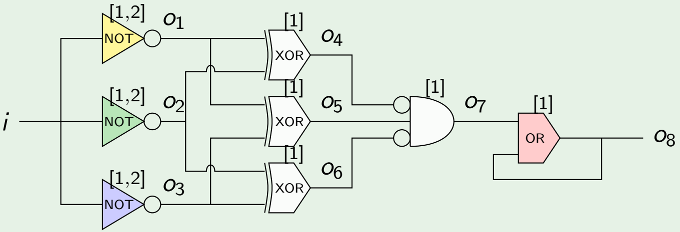

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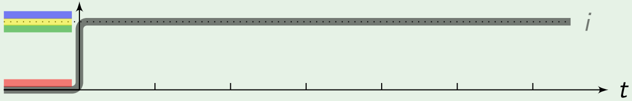

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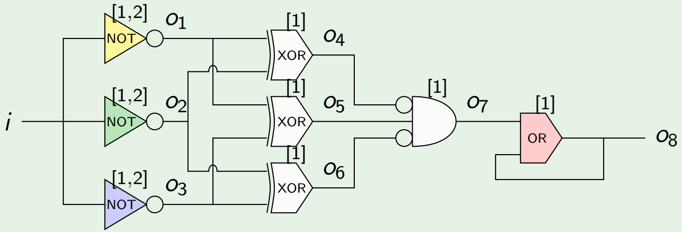

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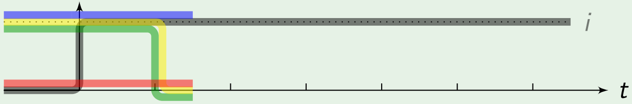

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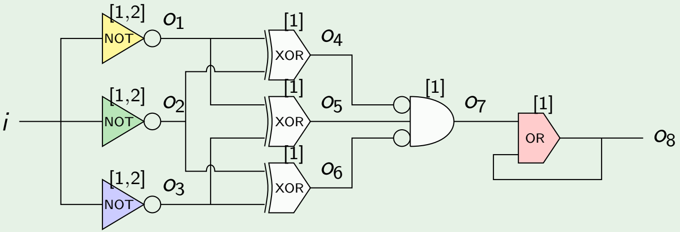

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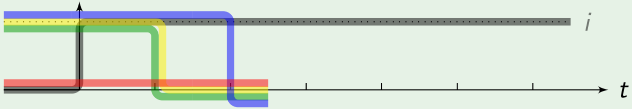

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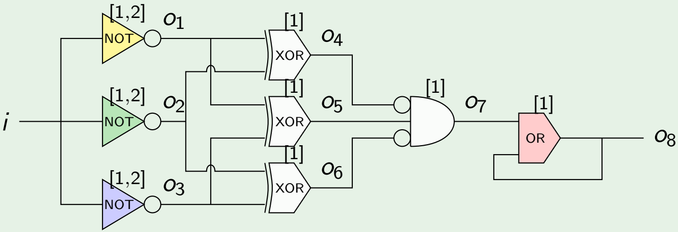

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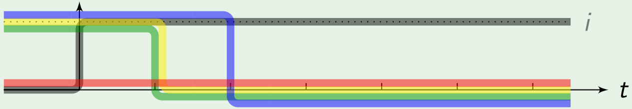

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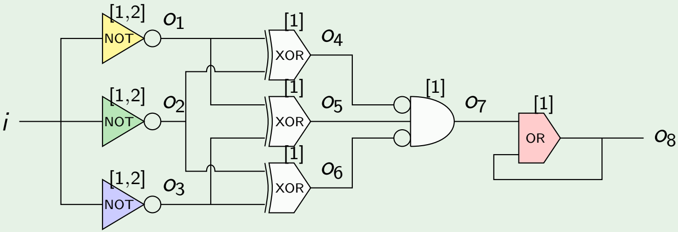
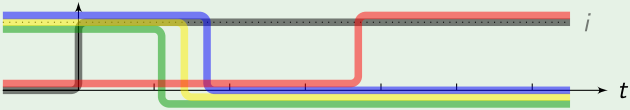


- under continuous-time, the output can be 1:



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Continuous-time semantics

...real-time models for real-time systems!

Continuous-time semantics

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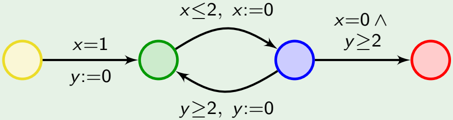
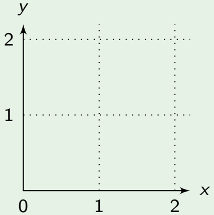
Example





Continuous-time semantics

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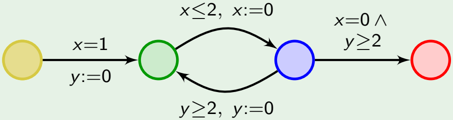
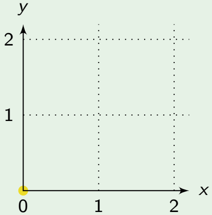
Example





Continuous-time semantics

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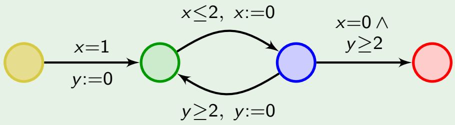
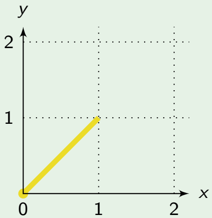
Example





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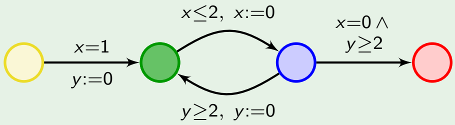

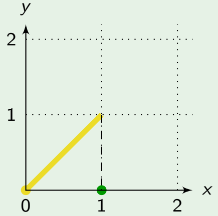

Continuous-time semantics

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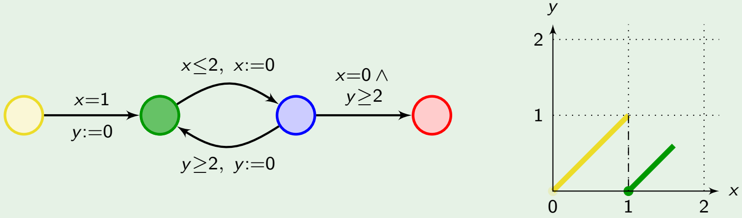

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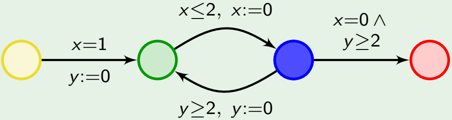

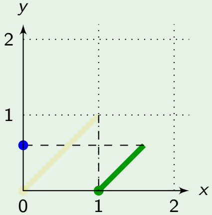

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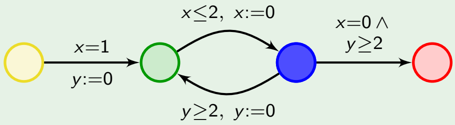

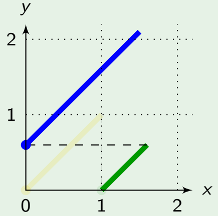

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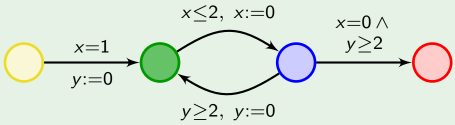

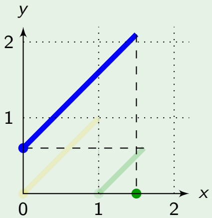

Continuous-time semantics

...real-time models for real-time systems!

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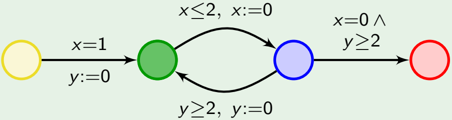

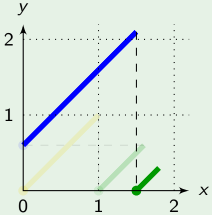

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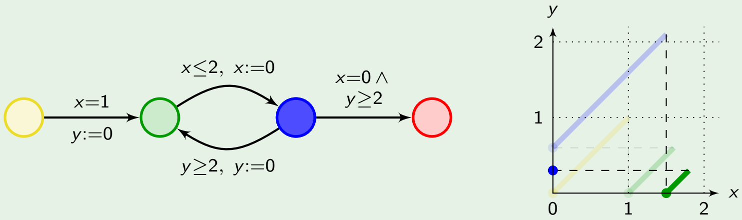

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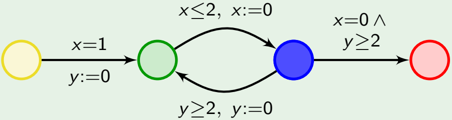

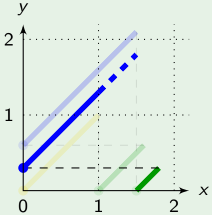

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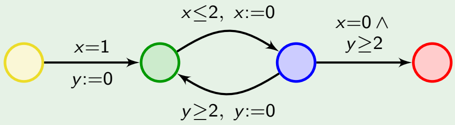

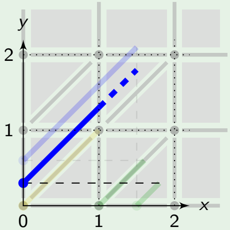

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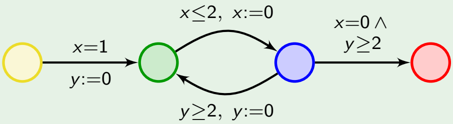

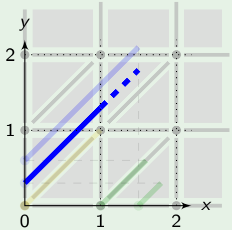

Theorem [AD94]

Reachability in timed automata is decidable (as well as many other important properties). It is PSPACE-complete.

- Technical tool: region abstraction

Continuous-time semantics

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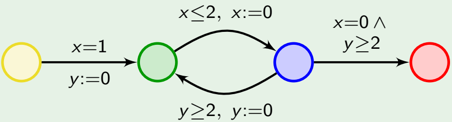
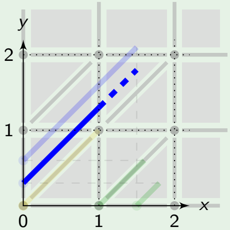
Example





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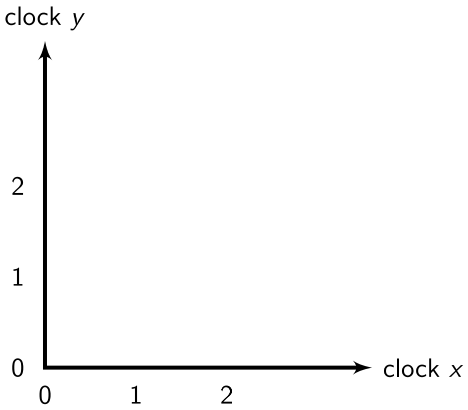
Reachability in timed automata is decidable (as well as many other important properties). It is PSPACE-complete.

- Technical tool: region abstraction
- Efficient symbolic technics based on zones, implemented in tools

Technical tool: Region abstraction



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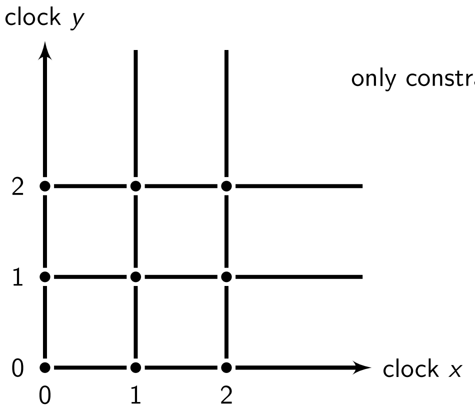

only constraints: $x \sim c$ with $c \in \{0, 1, 2\}$
$y \sim c$ with $c \in \{0, 1, 2\}$

- "compatibility" between regions and constraints

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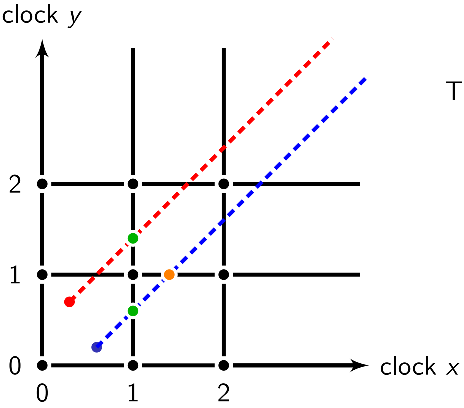

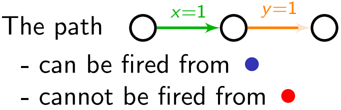

- "compatibility" between regions and constraints
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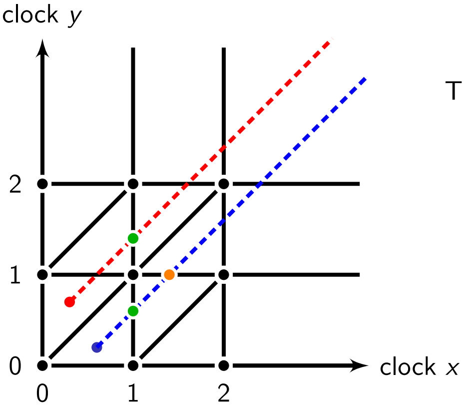

The path 

- can be fired from ●
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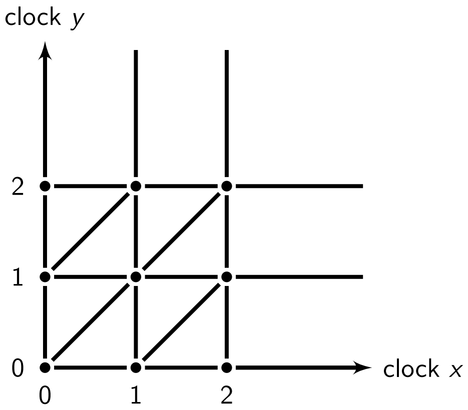

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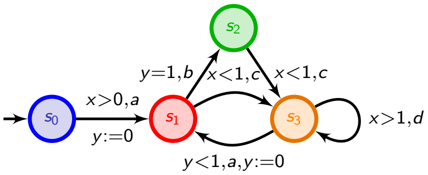
$\rightsquigarrow$ an equivalence of finite index

Technical tool: Region abstraction – An example [AD94]



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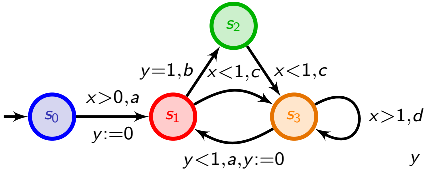

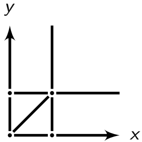

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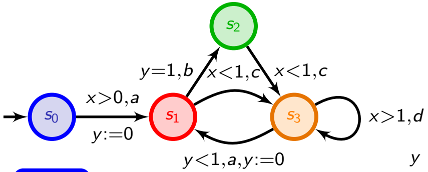

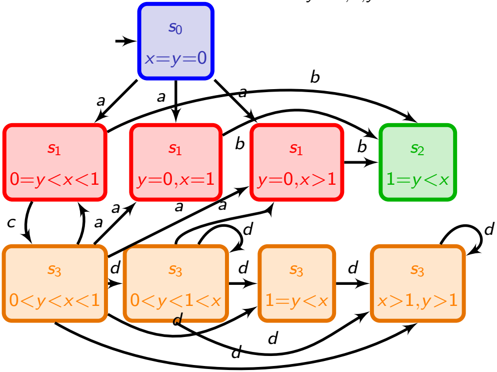

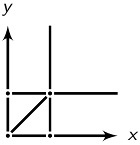

Technical tool: Zones and DBMs

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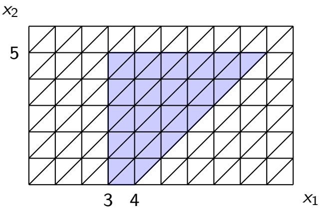
Zone: $(x_1 \geq 3) \wedge (x_2 \leq 5) \wedge (x_1 - x_2 \leq 4)$



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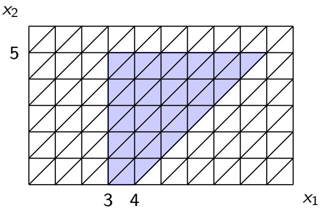

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$$|||$$

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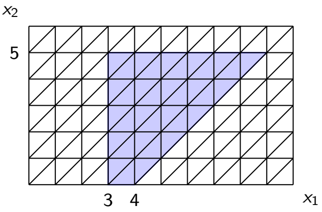
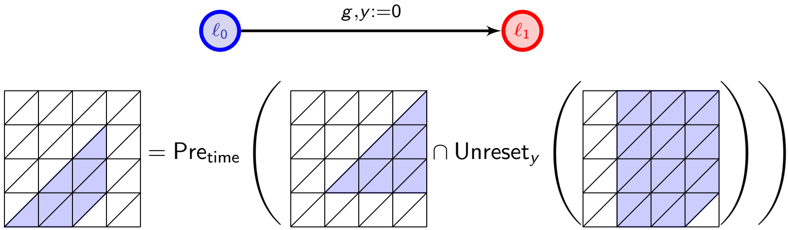


Technical tool: Zones and DBMs

They can be used to compute sets of states in timed automata



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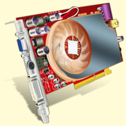
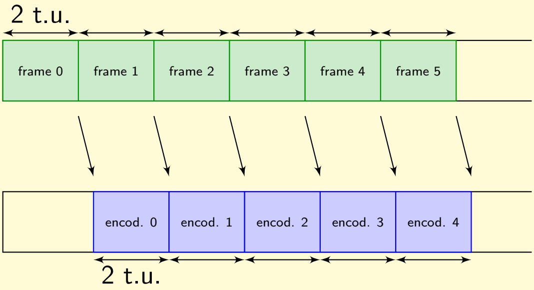
Important questions

- Is the real system correct when it is proven correct on the model?
- Does actual work transfer to real-world systems? To what extent?

Example 1: Imprecision on clock values

Frame capture [ACS10]





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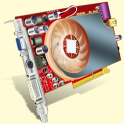
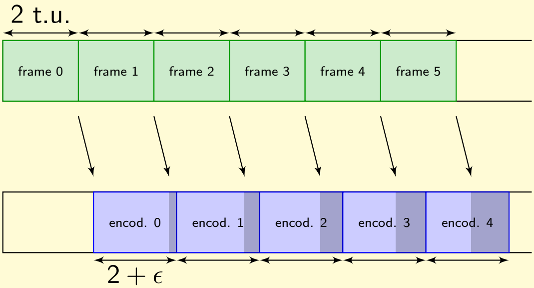


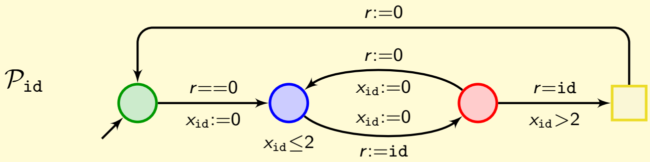


$\rightsquigarrow$ A frame will eventually be skipped

[ACS10] Abdellatif, Combaz, Sifakis. Model-based implementation of real-time applications. *Int. Conf. Embedded Software*, ACM 2010.

Example 2: Strict timing constraints

Mutual exclusion protocol [KLL+97]



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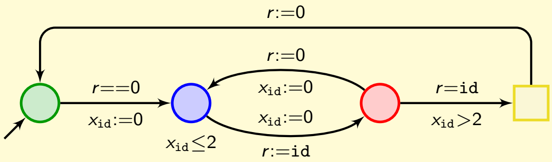
Mutual exclusion protocol [KLL+97]

$\mathcal{P}_{\mathtt{id}}$



- When $\mathcal{P}_1$ and $\mathcal{P}_2$ run in parallel (sharing variable r), the state where both of them are in □ is not reachable.

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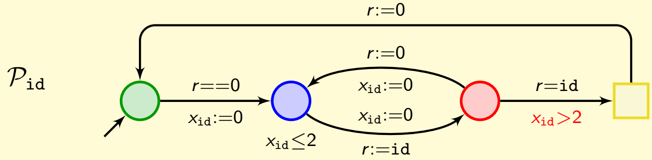

- When $\mathcal{P}_1$ and $\mathcal{P}_2$ run in parallel (sharing variable r), the state where both of them are in □ is not reachable.
- This property is lost when $x_{\mathtt{id}} > 2$ is replaced with $x_{\mathtt{id}} \geq 2$.

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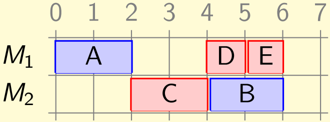
Example 3: Scheduling and timing anomaly

- Scheduling analysis with timed automata [AAM06]
- **Goal:** analyze a *work-conserving* scheduling policy on given scenarios (no machine is idle if a task is waiting for execution)

Example of a scenario



with the dependency constraints: $A \to B$ and $C \to D, E$.

1. A, D, E must be scheduled on machine M_1
2. B, C must be scheduled on machine M_2
3. C starts no sooner than 2 time units

[AAM06] Abdeddaim, Asarin, Maler. Scheduling with timed automata. *Theor. Comp. Science*, 2006.

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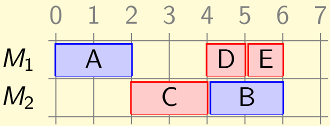

$\rightsquigarrow$ Schedulable in 6 time units

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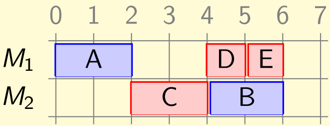

⇝ Schedulable in 6 time units

- Unexpectedly, the duration of A drops to 1.999

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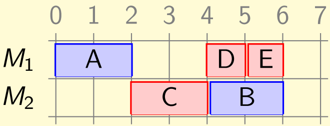

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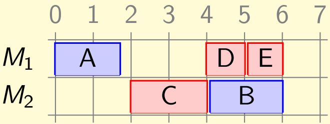

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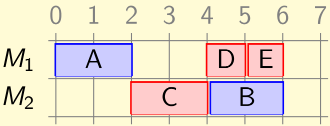

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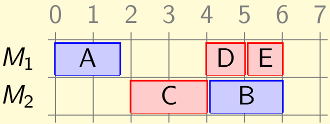

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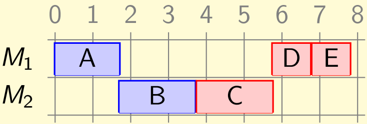

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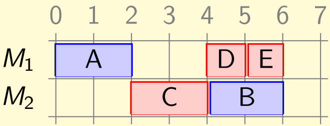

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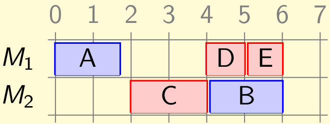

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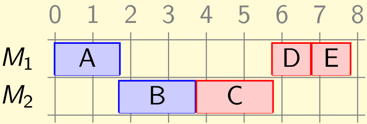

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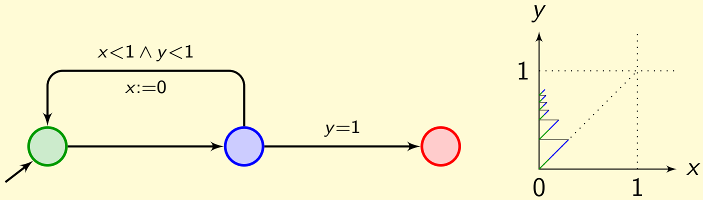
$\rightsquigarrow$ Standard analysis does not capture this **timing anomaly**

Example 4: Zeno behaviours



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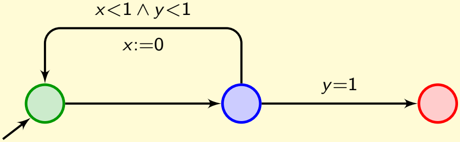

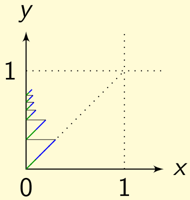

- Those are easy to detect and can be handled; [HS11]

[HS11] Herbreteau, Srivathsan. Coarse abstractions make Zeno behaviours difficult to detect, *Logic. Meth. Comp. Science*, 2011.

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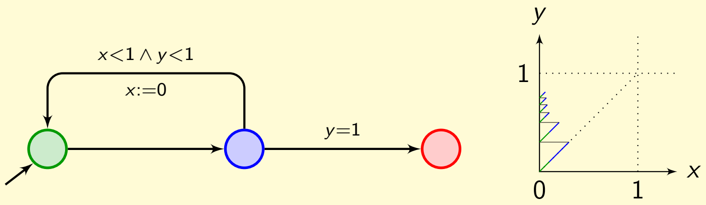

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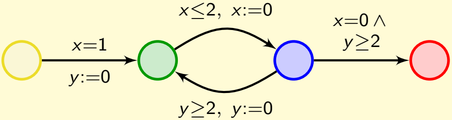
- They are easy to remove by construction.

[HS11] Herbreteau, Srivathsan. Coarse abstractions make Zeno behaviours difficult to detect, *Logic. Meth. Comp. Science*, 2011.

Example 5: More complex convergence phenomena



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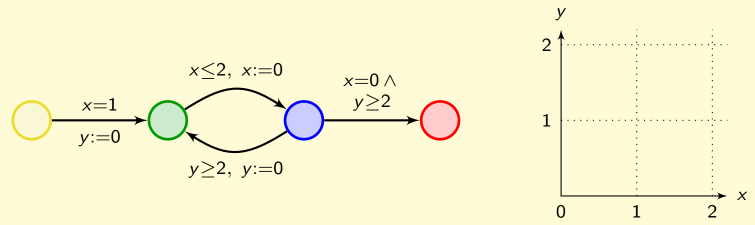

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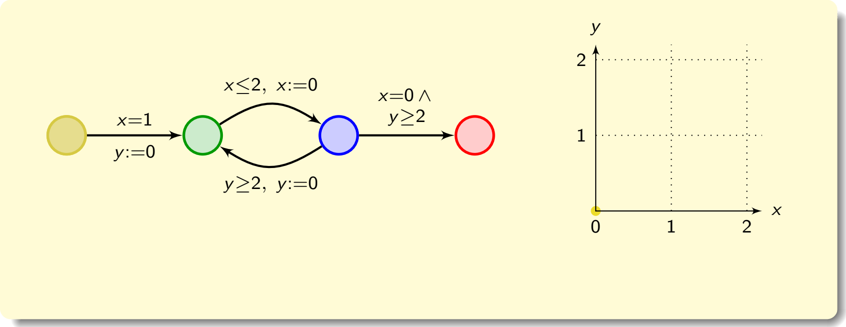

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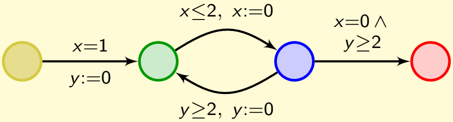

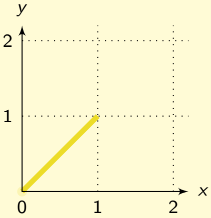

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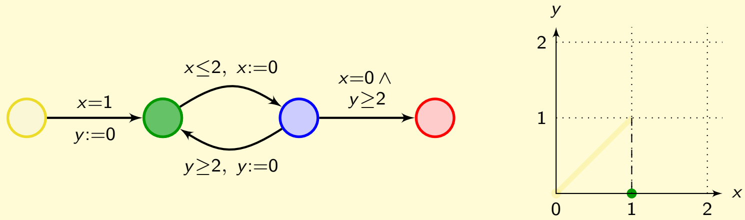

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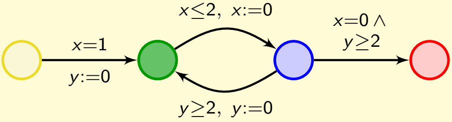

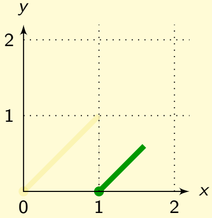

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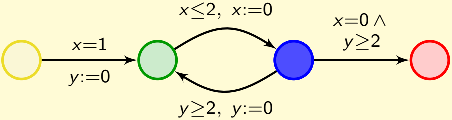

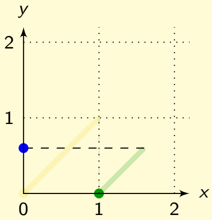

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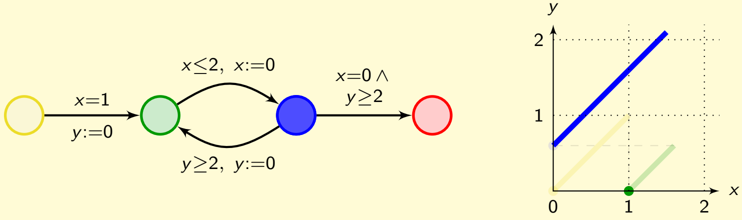

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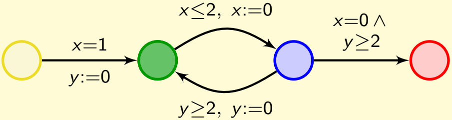

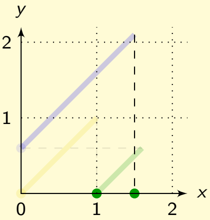

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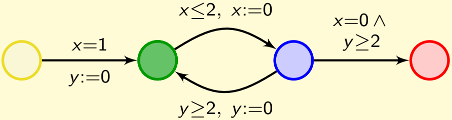

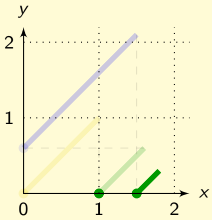

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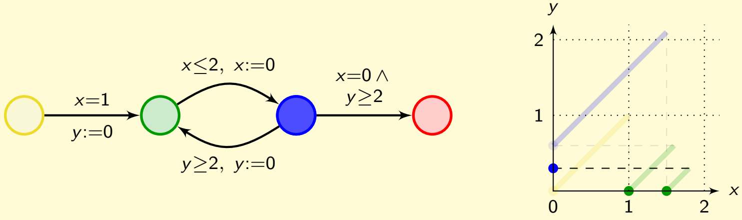

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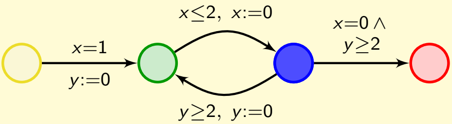

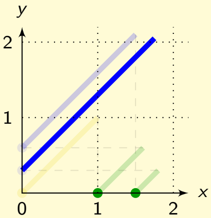

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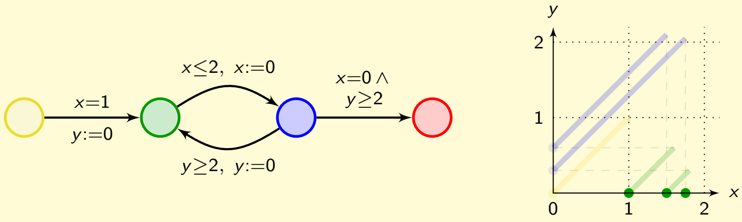

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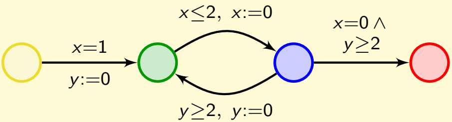

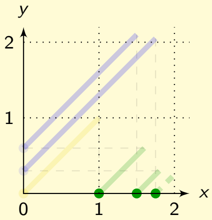

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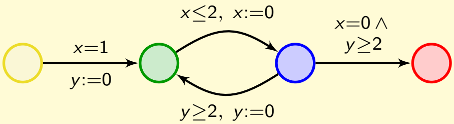

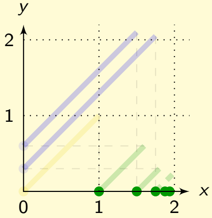

$\rightsquigarrow$ Value of clock x when hitting ◯ is converging,
even though global time diverges

The goal

Add robustness to the theory of timed automata

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Rest of the talk

We present a couple of frameworks that have been developed recently in this context. We focus on perturbations on time measurements and jitter.

Outline

1. Introduction

2. Robust model-checking and implementation
   - Parameterized enlarged semantics
   - Automatic generation of an implementation
   - Implementation by shrinking

3. Robust realisability
   - Excess semantics
   - Conservative semantics

4. Conclusion

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We describe two such frameworks:

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2. or we build $\mathcal{A}$ and implement $\mathcal{B}$, and we prove:

$$\begin{aligned} \text{correctness of } \mathcal{A} \quad &\Rightarrow \quad \text{“robust” correctness of } \mathcal{B} \\ &\Rightarrow \quad \text{correctness of } \mathcal{B}_{\texttt{real}} \end{aligned}$$

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Parameterized enlarged semantics for timed automata

A transition can be taken at any time in $[t - \delta; t + \delta]$

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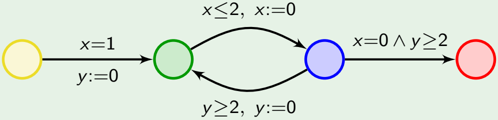
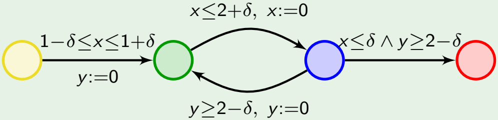
Example

Given a parameter δ,



is transformed into



Parameterized enlarged semantics – Discussion

What is the relevance of this semantics?

- This is a worst-case approach
- This captures approximate behaviours of the system
- One can define program semantics such that for every $\epsilon > 0$:

$$\mathcal{A} \subseteq \mathtt{program}_\epsilon(\mathcal{A}) \subseteq \mathcal{A}_{f(\epsilon)}$$

ϵ: parameters of the semantics

[DDR04] De Wulf, Doyen, Raskin. Almost ASAP semantics: From timed models to timed implementations *HSCC*, 2004.
[SBM11] Sankur, Bouyer, Markey. Shrinking Timed Automata. *FSTTCS*, 2011.

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- Verify $\mathcal{A}_\delta$ (better if δ is a parameter)
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$\rightsquigarrow$ This is good for designing systems
with simple timing constraints (e.g. equalities).

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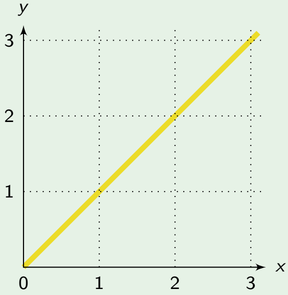

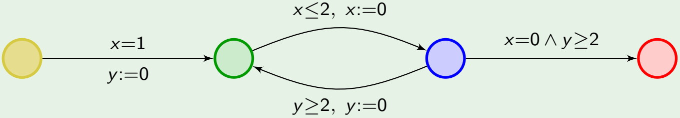

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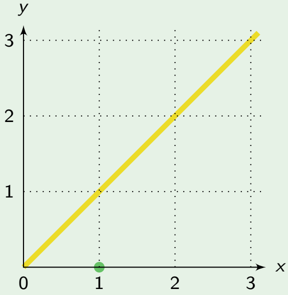

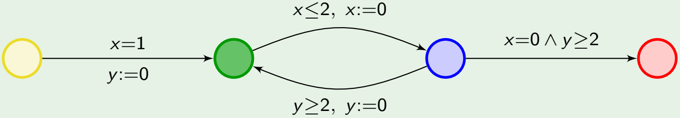

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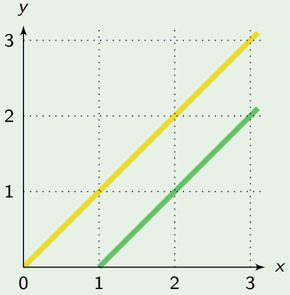

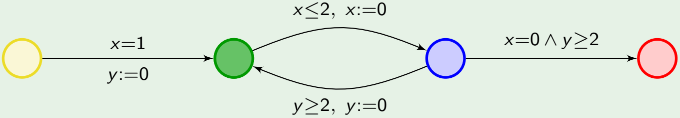

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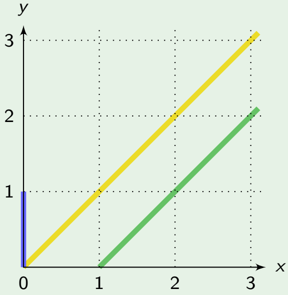

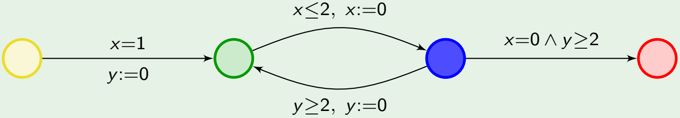

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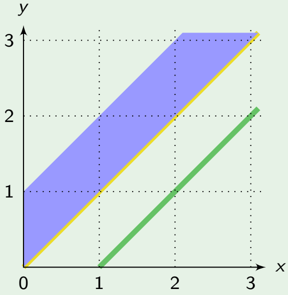

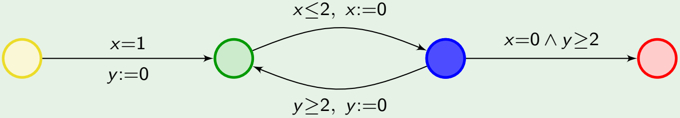

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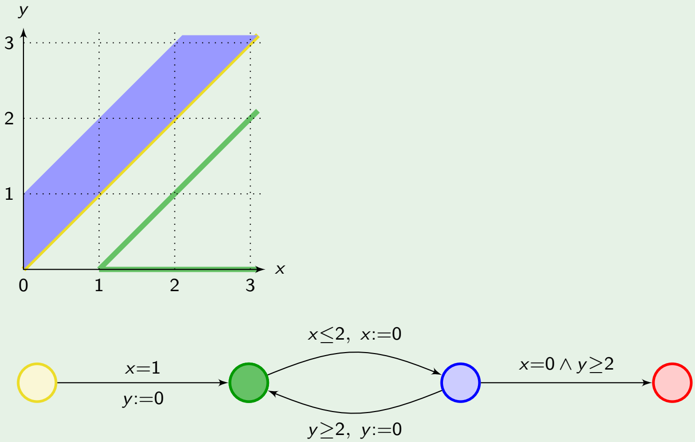

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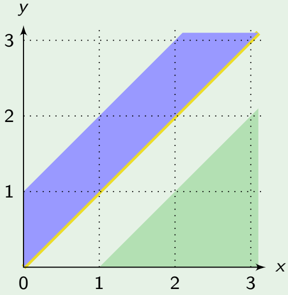

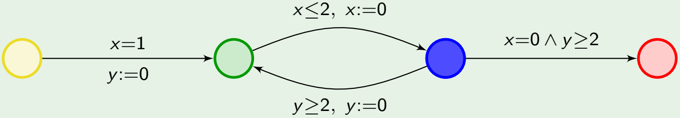

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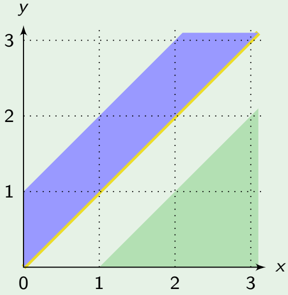

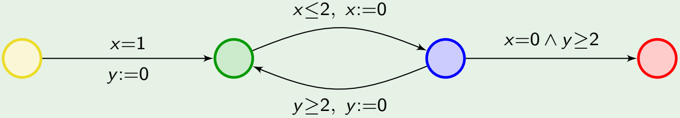

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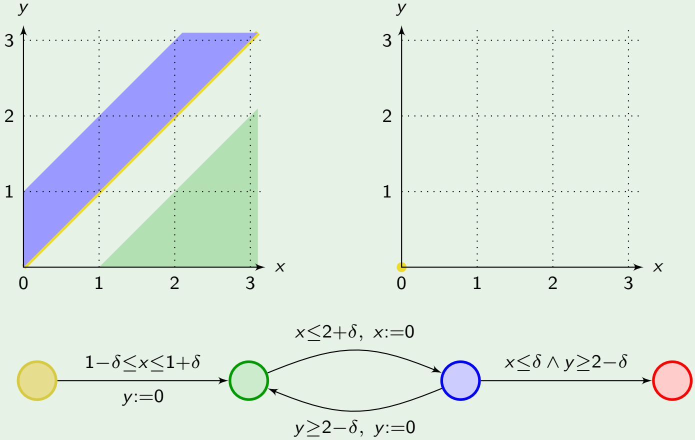

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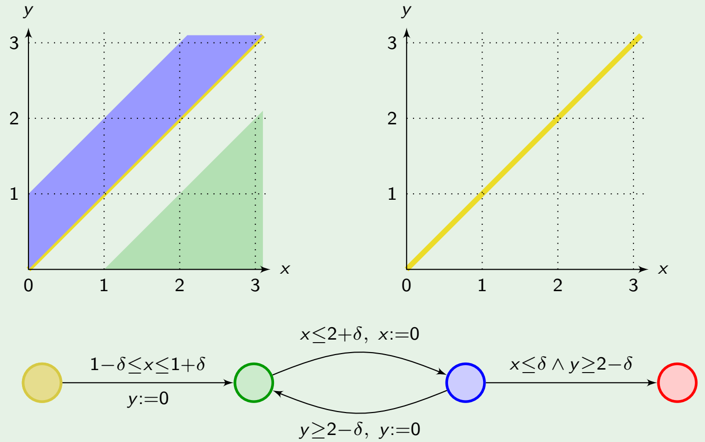

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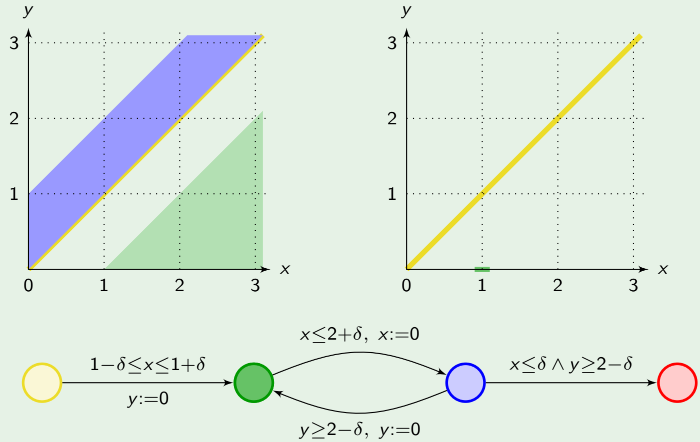

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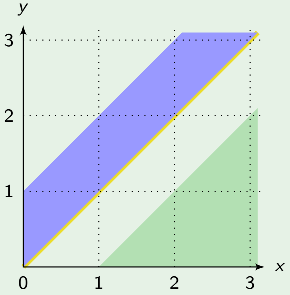

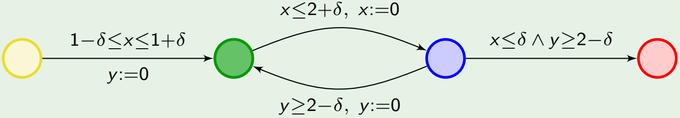

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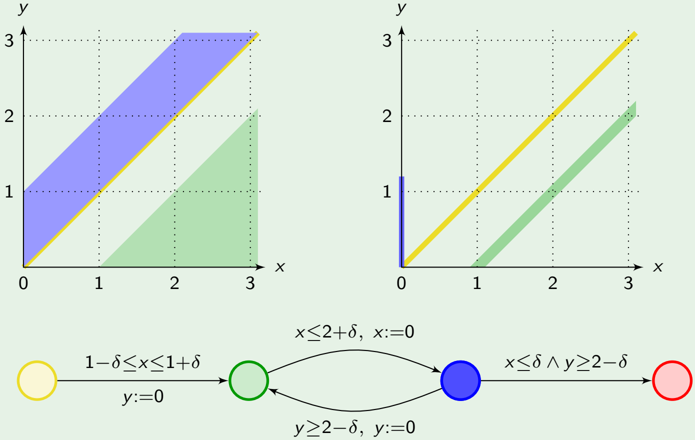

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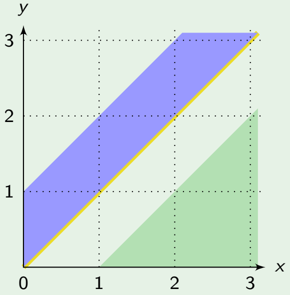

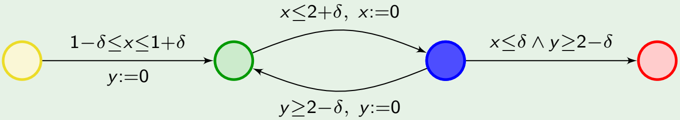

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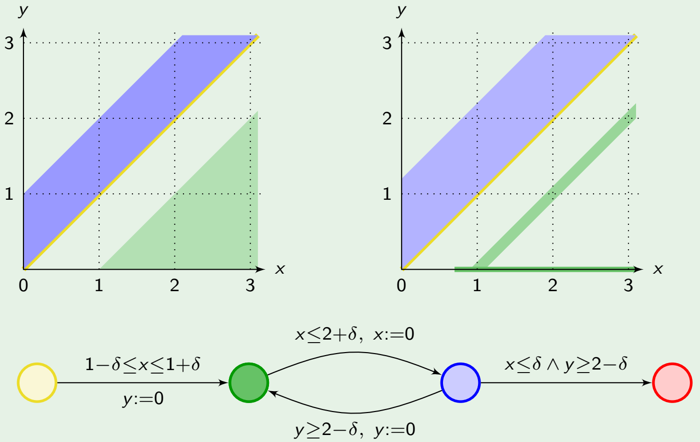

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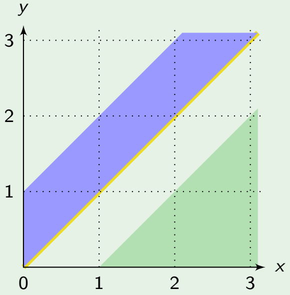

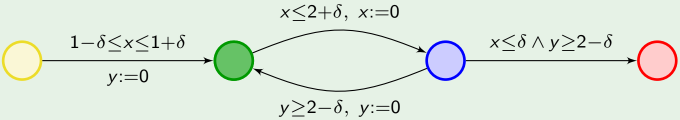

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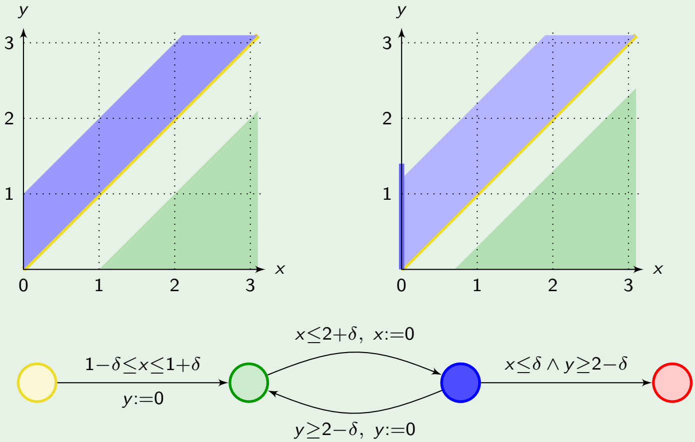

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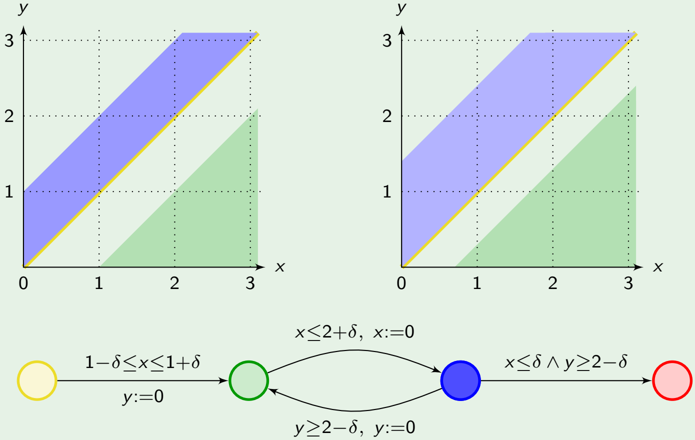

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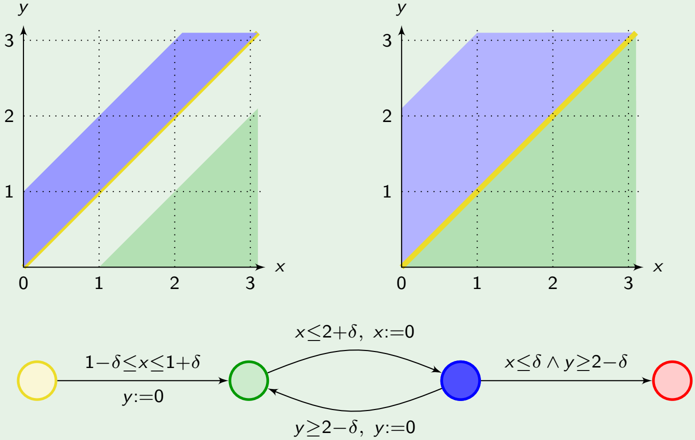

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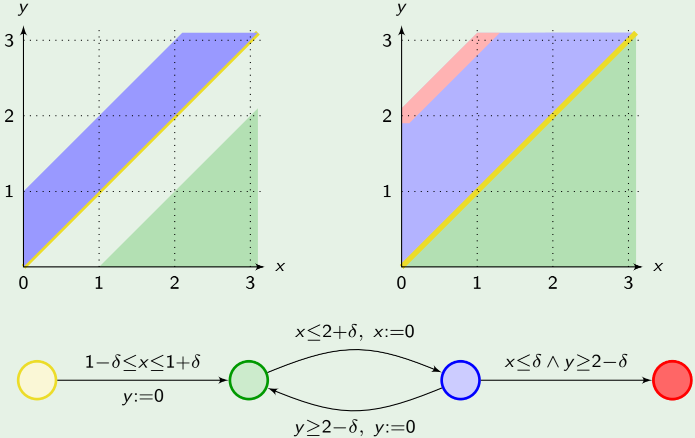

Parameterized enlarged semantics – Algorithmics

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The (parameterized) robust model-checking problem

It asks whether there is some $\delta_0 > 0$ such that for every $0 \leq \delta \leq \delta_0$, $\mathcal{A}_\delta \models \varphi$.

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Theorem

Robust model-checking of reachability, Büchi, LTL, CoflatMTL properties is decidable. Complexities are those of standard non robust model-checking problems.

[Puri00] Puri. Dynamical properties of timed automata. *Disc. Event Dyn. Syst., 2000.*
[DDMR08] De Wulf, Doyen, Markey, Raskin. Robust safety of timed automata. *FMSD*, 2008.
[BMR06] Bouyer, Markey, Reynier. Robust model-checking of timed automata. *LATIN*, 2006.
[BMR08] Bouyer, Markey, Reynier. Robust analysis of timed automata via channel machines. *FoSSaCS*, 2008.

Outline

1. Introduction

2. Robust model-checking and implementation
   - Parameterized enlarged semantics
   - Automatic generation of an implementation
   - Implementation by shrinking

3. Robust realisability
   - Excess semantics
   - Conservative semantics

4. Conclusion

Automatic generation of an implementation

The (approx.) implementation synthesis problem

Given $\mathcal{A}$, build $\mathcal{A}'$ such that:

- $\mathcal{A}'$ 'identical' (e.g. bisimilar) to $\mathcal{A}$
- $\mathcal{A}'$ is 'robust' (that is, good enough for implementation)

[BLM+11] Bouyer, Larsen, Markey, Sankur, Thrane. Timed automata can always be made implementable. *CONCUR*, 2011.

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Theorem

All timed automata are approximately implementable!
(for approx. bisimulation)

- Technical tool: region construction

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Automatic generation of an implementation

Methodology

- Design and verify $\mathcal{A}$
- Implement $\mathcal{A}'$ (automatically generated)

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- ☺ Separates design and implementation
- ☹ $\mathcal{A}'$ is much bigger than $\mathcal{A}$

[BLM$^+$11] Bouyer, Larsen, Markey, Sankur, Thrane. Timed automata can always be made implementable. *CONCUR*, 2011.

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Parameterized shrunk semantics for timed automata

A constraint $[a, b]$ **is shrunk to** $[a + k\delta; b - h\delta]$

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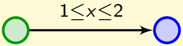
Why should we do that?

Abstract model



Real-world model

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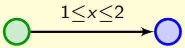

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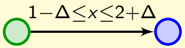

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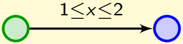

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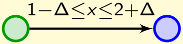

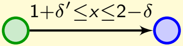

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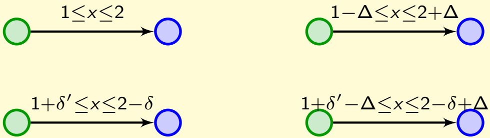

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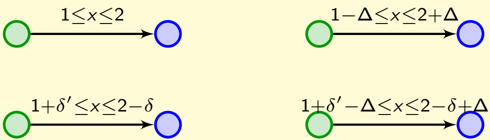

It is fine as soon as $[1 + \delta' - \Delta; 2 - \delta + \Delta] \subseteq [1; 2]$,
which is the case when $\delta, \delta' \geq \Delta$.

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Parameterized shrunk semantics for timed automata

A constraint $[a, b]$ **is shrunk to** $[a + k\delta; b - h\delta]$

Summary of the approach

$\rightsquigarrow$ Shrink the clock constraints in the model, to prevent additional behaviours in the implementation

- If $\mathcal{B} = \mathcal{A}_{-\mathbf{k}\delta}$, then

$$\mathcal{B} \subseteq \texttt{program}_\epsilon(\mathcal{B}) \subseteq \mathcal{B}_{f(\epsilon)} = \mathcal{A}_{-\mathbf{k}\delta+f(\epsilon)} \subseteq \mathcal{A}$$

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Parameterized shrunk semantics – Discussion

What is the relevance of that approach?

Anticipate imprecisions to prevent additional behaviours in the real-world

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$\leadsto$ This is good for designing systems with strong/hard timing constraints

⚠ Problem

Make sure that no important behaviours are lost in $\mathcal{A}_{-\mathbf{k}\delta}$!!

Parameterized shrunk semantics – Algorithmics

The (parameterized) shrinkability problem

Find parameters $\mathbf{k}$ and δ such that:

- $\mathcal{A} \sqsubseteq_{\mathtt{t.a.}} \mathcal{A}_{-\mathbf{k}\delta}$ (or $\mathcal{F} \sqsubseteq_{\mathtt{t.a.}} \mathcal{A}_{-\mathbf{k}\delta}$ for some finite automaton $\mathcal{F}$)
  [shrinkability w.r.t. untimed simulation]
- $\mathcal{A}_{-\mathbf{k}\delta}$ is non-blocking whenever $\mathcal{A}$ is non-blocking
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Theorem

Parameterized shrinkability can be decided (in exponential time).

- Challenge: take care of the accumulation of perturbations
- Technical tools: parameterized shrunk DBM, max-plus equations
- Tool Shrinktech developed by Ocan Sankur [San13]

  `http://www.lsv.ens-cachan.fr/Software/shrinktech/`

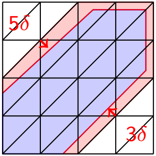
[San13] Sankur. Shrinktech: A tool for the robustness analysis of timed automata. *CAV*, 2013.

Technical tools: Shrunk DBMs and max-plus equations

Shrunk DBMs with parameter δ...

... are used to represent sets of states of shrunk automata:

DBM: $x - y \leq \alpha$
Shrunk DBM: $x - y \leq \alpha - 5\delta$



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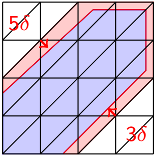

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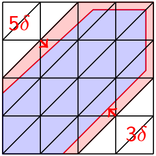

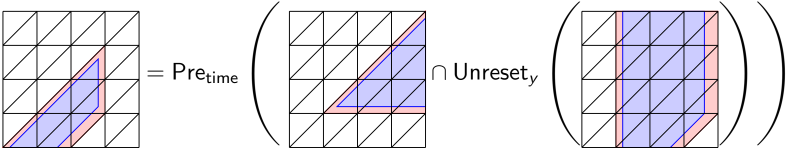

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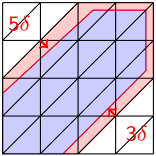
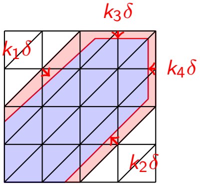
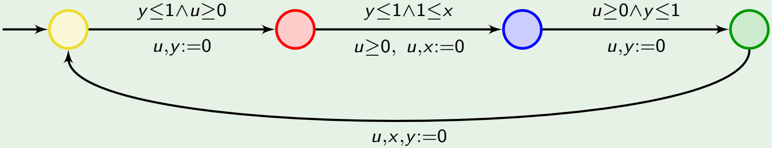


... to max-plus equations



$$\rightsquigarrow \quad \begin{aligned} k_3 &= \max(k_1 + k_2, k_3) \\ k_2 &= \max(k_2, k_1) + k_3 \end{aligned}$$

Example


$y{\leq}1{\wedge}u{\geq}0$
$u,y{:=}0$
$y{\leq}1{\wedge}1{\leq}x$
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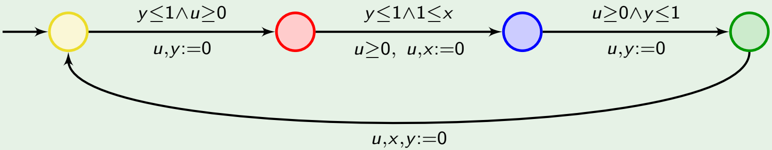
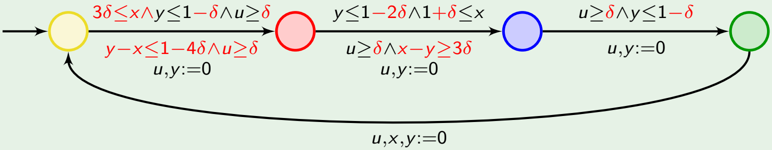


The largest shrunk automaton which is correct w.r.t. untimed simulation and non-blockingness is:



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Here, a strategy in a timed automaton is a way to resolve (time and action) non-determinism

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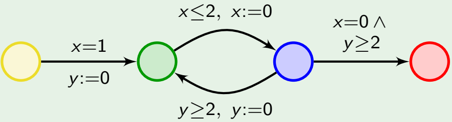

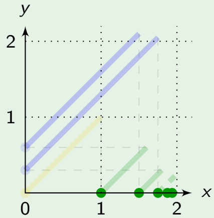

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Robust realisability

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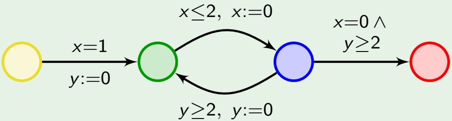

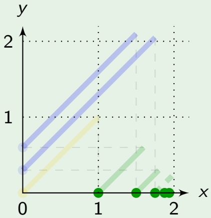

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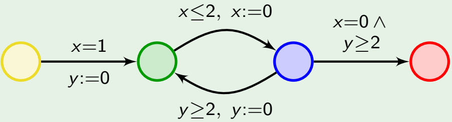

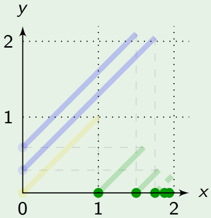

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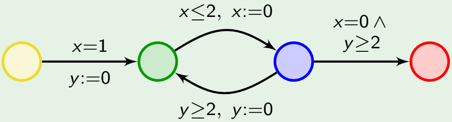

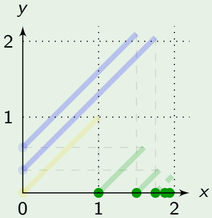

Strategy: in location with value x, delay $\frac{2-x}{2}$

- This strategy requires infinite precision
- In practice, when x is close to 2, no additional delay is supported: the run is theoretically infinite, but it is actually blocking
- And that is unavoidable

Robust realisability

Here, a strategy in a timed automaton is a way to resolve (time and action) non-determinism

Idea of robust realisability

Synthesize strategies that realise some property, even under perturbations: strategies should adapt to previous imprecisions imprecisions

$\rightsquigarrow$ develop a theory of robust strategies that tolerate errors/imprecisions and avoid convergence

Game semantics of a timed automaton

Game semantics $\mathcal{G}_\delta(\mathcal{A})$ of timed automaton $\mathcal{A}$...

... between Controller and Perturbator:

- from (ℓ, v), Controller suggests a delay $d \geq \delta$ and a next edge $e = (\ell \xrightarrow{g,Y} \ell')$ that is available after delay d
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A δ-robust strategy for Controller is then a strategy that satisfies the expected property, whatever plays Perturbator.

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The excess game semantics

Constraints may not be satisfied after the perturbation: that is, only $v + d$ should satisfy g

[BMS12] Bouyer, Markey, Sankur. Robust reachability in timed automata: A game-based approach. *ICALP*, 2012.

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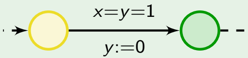

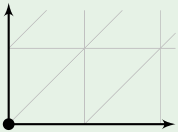

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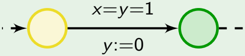

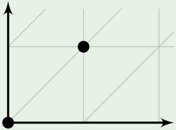

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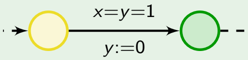

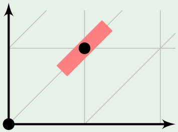

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Constraints may not be satisfied after the perturbation: that is, only $v + d$ should satisfy g

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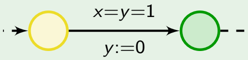

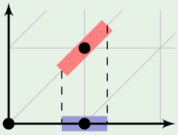

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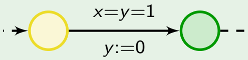

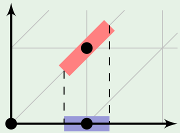

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The excess game semantics – Algorithmics

The (parameterized) synthesis problem

Synthesize $\delta > 0$ and a δ-robust strategy that achieves a given goal.

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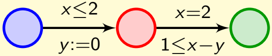

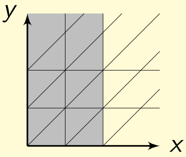

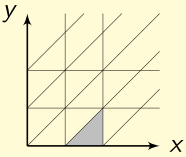

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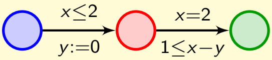

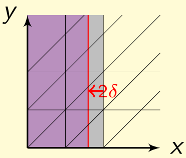

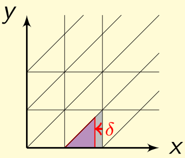

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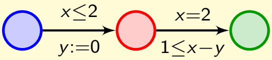
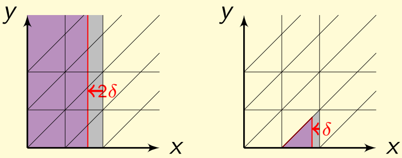
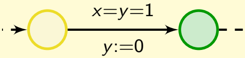
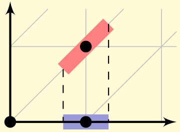




2. New regions become reachable





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Theorem

The parameterized synthesis problem for reachability properties is decidable and EXPTIME-complete. Furthermore, uniform winning strategies (w.r.t. δ) can be computed.

- Technical tool: a region-based refined game abstraction
- ☺ Extends to two-player games (i.e. to real control problems)
- ☹ Only valid for reachability properties

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Constraints have to be satisfied after the perturbation: that is, $v + d + \epsilon$ should satisfy g for every $\epsilon \in [-\delta; +\delta]$

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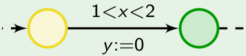

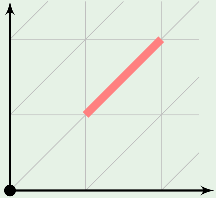

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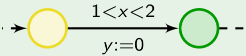

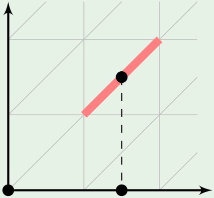

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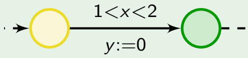

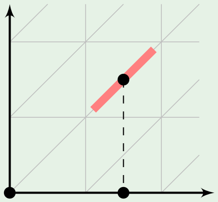

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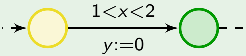

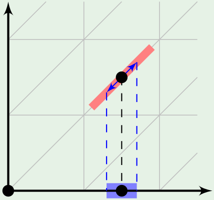

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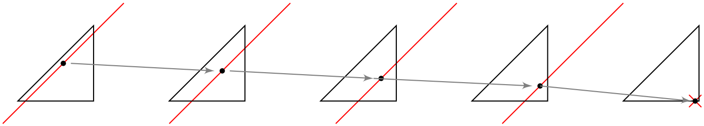

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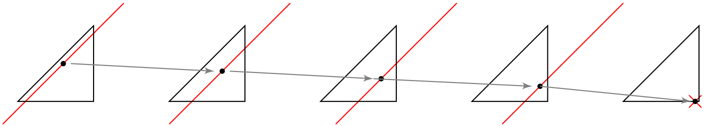

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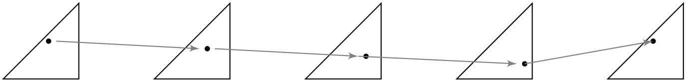

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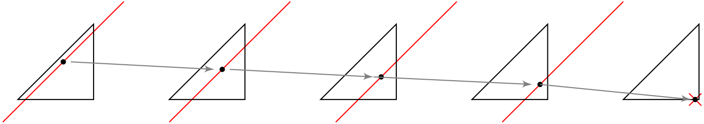

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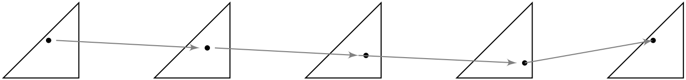

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Tools for solving the synthesis problem

- Orbit graphs, forgetful cycles [AB11]
- Forgetful (that is, strongly connected) orbit graph $\Leftrightarrow$ no convergence phenomena
  $\rightsquigarrow$ strong relation with thick automata.

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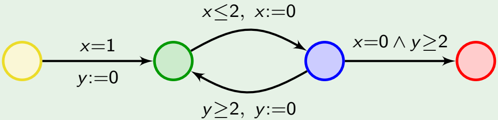

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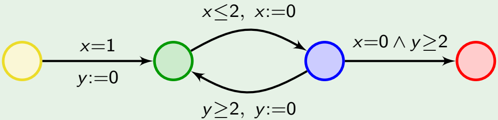
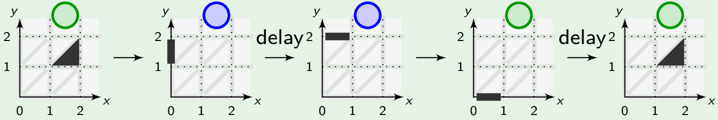


A region cycle:



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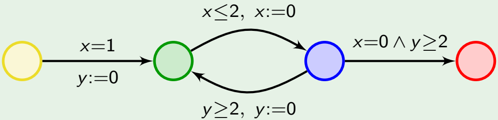

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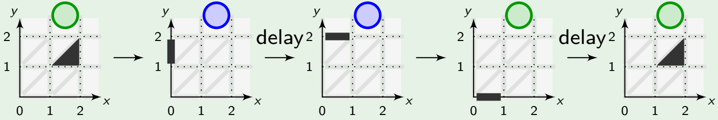
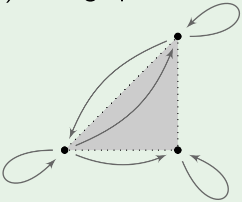


The corresponding (folded) orbit graph:



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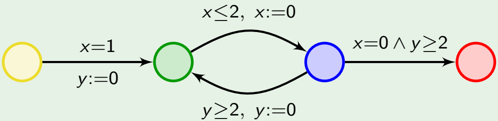
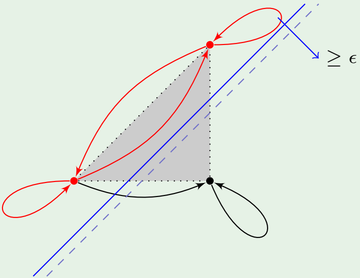


The cycle is not forgetful (that is, not strongly connected), Perturbator can enforce convergence:



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- This list of possible approaches is not exhaustive:
  - tube acceptance [GHJ97]
  - sampling approach [KP05,BLM+11]
  - probabilistic approach [BBB+08,BBJM12]
  - ...