A Probabilistic Semantics for Timed Automata

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- Timed automata, an idealized mathematical model for real-time systems

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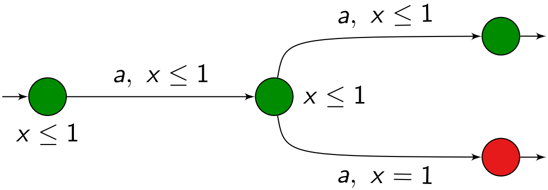
Aim: Use probabilities to "relax" the semantics of timed automata

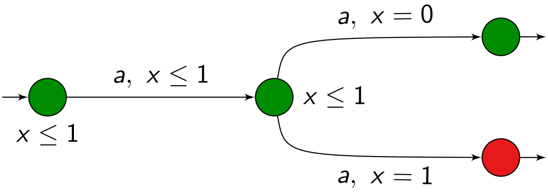
Initial example



Intuition: from the initial state,

this automaton *almost-surely* satisfies "$\mathbf{G}$ green"

A maybe less intuitive example



Does it *almost-surely* satisfy "$\mathbf{F}$ red"?

Our proposition

- $\pi(s \xrightarrow{e_1} \dots \xrightarrow{e_n})$: symbolic path from s firing edges $e_1, \dots, e_n$

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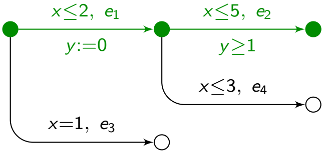

$$\pi(s_0 \xrightarrow{e_1}\xrightarrow{e_2}) = \{s_0 \xrightarrow{\tau_1, e_1} s_1 \xrightarrow{\tau_2, e_2} s_2 \mid \tau_1 \leq 2,\ \tau_1 + \tau_2 \leq 5,\ \tau_2 \geq 1\}$$

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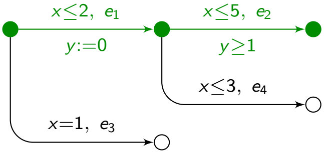

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- Idea:

From state s_0:

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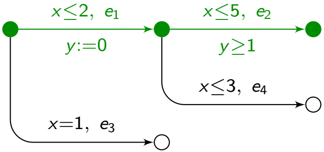

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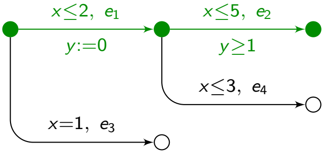

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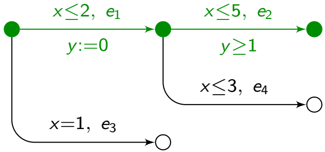

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- Idea:

From state s_0:

- randomly choose a delay
- then randomly select an edge
- then continue

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symbolic path: $\pi(s \xrightarrow{e_1} \cdots \xrightarrow{e_n}) = \{s \xrightarrow{\tau_1, e_1} s_1 \cdots \xrightarrow{\tau_n, e_n} s_n\}$

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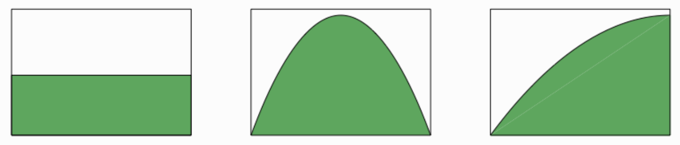

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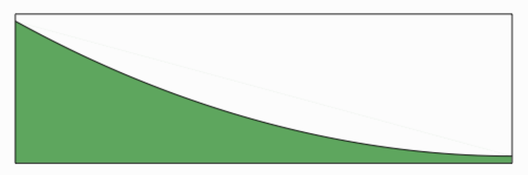

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  $$\pi_{\mathcal{C}}\big(s \xrightarrow{e_1} \cdots \xrightarrow{e_n}\big) = \{s \xrightarrow{\tau_1, e_1} s_1 \cdots \xrightarrow{\tau_n, e_n} s_n \mid (\tau_1, \cdots, \tau_n) \models \mathcal{C}\}$$

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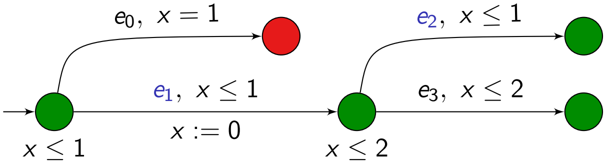
- Example:
  - $\mathsf{Zeno}(s) = \bigcup_{M \in \mathbb{N}} \bigcap_{n \in \mathbb{N}} \bigcup_{(e_1, \cdots, e_n) \in E^n} \mathsf{Cyl}(\pi_{\Sigma_i \tau_i \leq M}(s \xrightarrow{e_1} \cdots \xrightarrow{e_n}))$

An example of computation (with uniform distributions)



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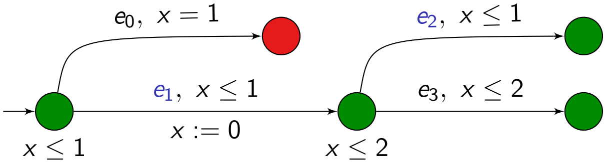

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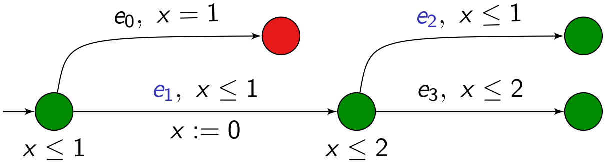

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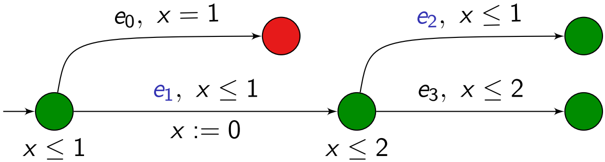

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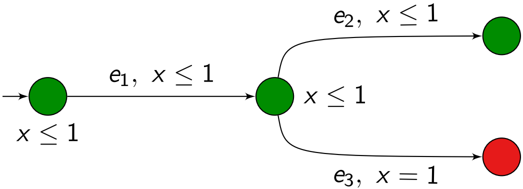
$$= \int_0^1 \int_0^1 \left(\frac{1}{2}\frac{\mathrm{d}u}{2}\right)\mathrm{d}t \quad = \frac{1}{4}$$

Back to the first example



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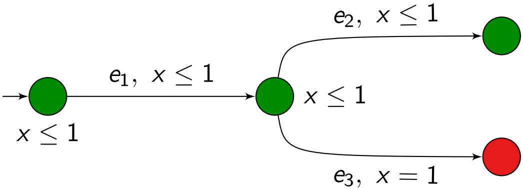

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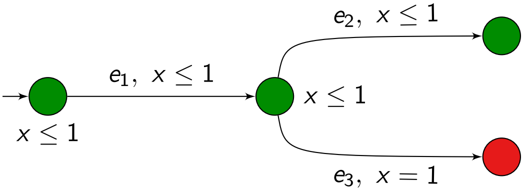

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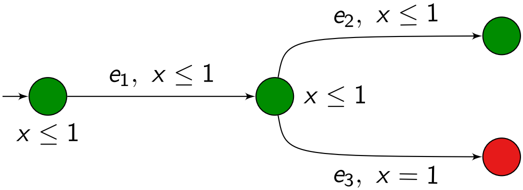

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- $\mathbb{P}\Big(\mathbf{G}\ \text{green}\Big) = 1$

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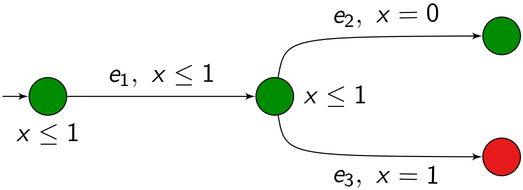

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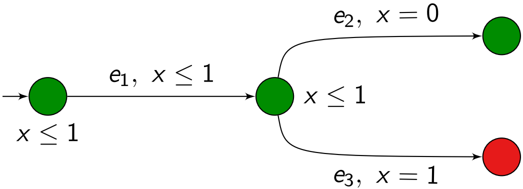

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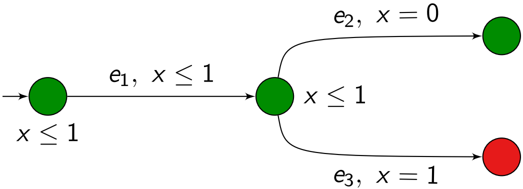

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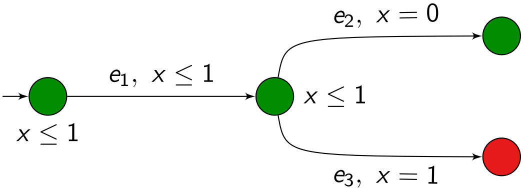

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Almost-sure model-checking

If φ is an LTL formula,

$$s \mathrel{|\!\!\approx} \varphi \quad \stackrel{\text{def}}{\Leftrightarrow} \quad \mathbb{P}\Big(\{\varrho \in \mathsf{Runs}(s) \mid \varrho \models \varphi\}\Big) = 1$$

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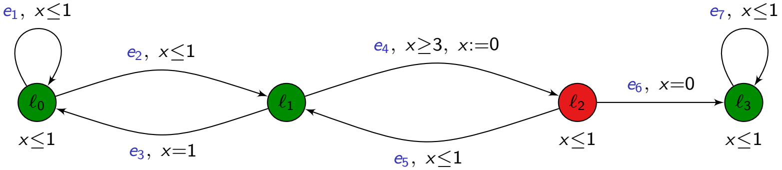
We want to decide the almost-sure model-checking...
(This is a qualitative question)

An example



e_1, $x \leq 1$
e_2, $x \leq 1$
e_4, $x \geq 3$, $x := 0$
e_6, $x = 0$
e_7, $x \leq 1$
ℓ_0
ℓ_1
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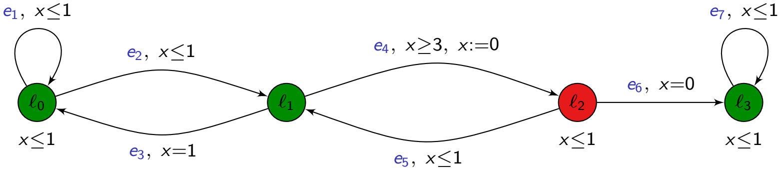

$\mathcal{A} \not\models$ **G**(green $\Rightarrow$ **F** red)

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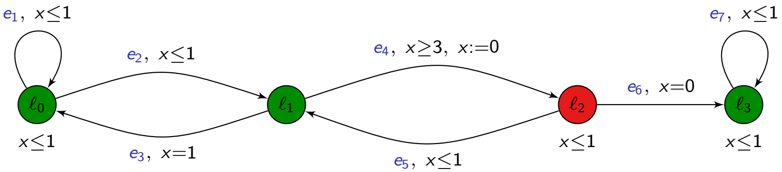

$\mathcal{A} \not\models \mathbf{G}(\text{green} \Rightarrow \mathbf{F}\ \text{red})$ but $\mathcal{A} \mathrel{|}\!\approx \mathbf{G}(\text{green} \Rightarrow \mathbf{F}\ \text{red})$

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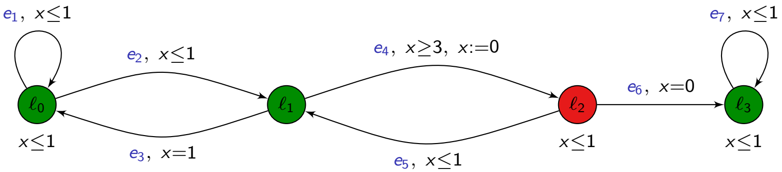

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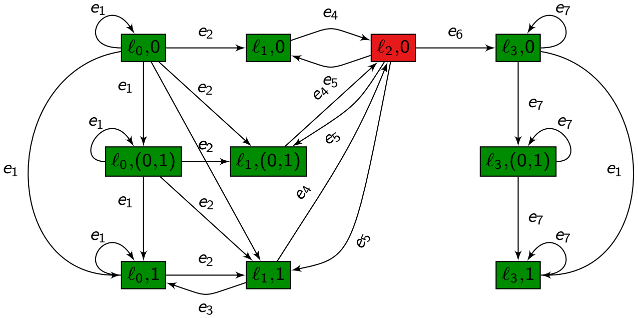
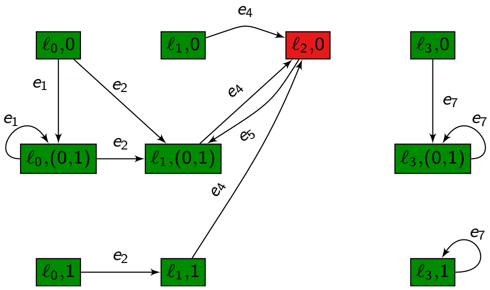
Indeed, almost surely, paths are of the form $e_1^* e_2 \left(e_4 e_5\right)^\omega$

The classical region automaton



The pruned region automaton



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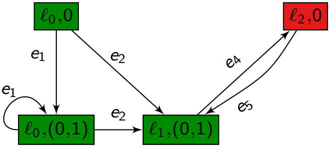

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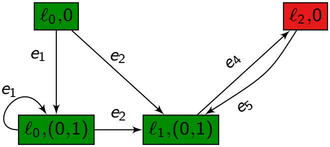

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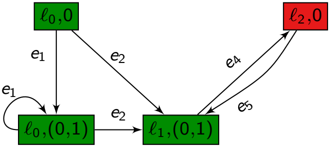

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Theorem

For single-clock timed automata,

$$\mathcal{A} \models\!\!\approx \varphi \quad \text{iff} \quad \mathbb{P}(MC(\mathcal{A}) \models \varphi) = 1$$

Result

Theorem

For single-clock timed automata, the almost-sure model-checking

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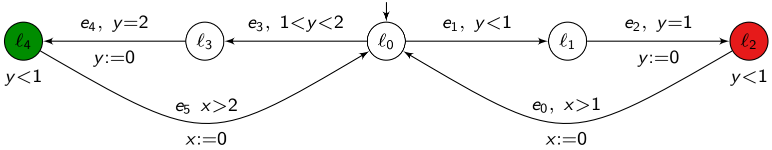
  - link between probabilities and topology thanks to the topological games called Banach-Mazur games

An example with two clocks



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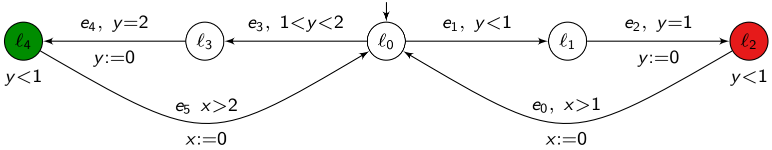

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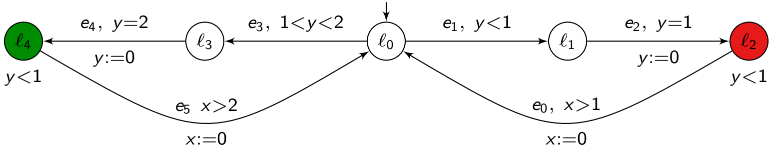

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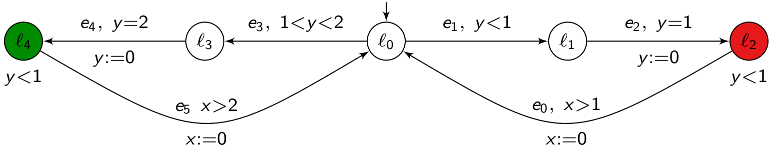

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- There is a *strange* convergence phenomenon: along an execution, if $\delta_i > 0$ is the delay in location ℓ_4, then we have that $\sum_i \delta_i \leq 1$

A note on Zeno behaviours

- The set of Zeno behaviours is measurable:

$$\mathsf{Zeno}(s) = \bigcup_{M \in \mathbb{N}} \bigcap_{n \in \mathbb{N}} \bigcup_{(e_1, \cdots, e_n) \in E^n} \mathsf{Cyl}(\pi(s \xrightarrow{e_1} \cdots \xrightarrow{e_n}))$$

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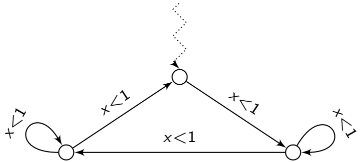

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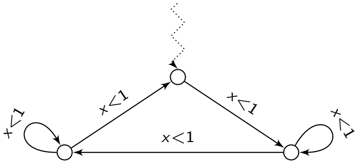

  - an interesting notion of non-Zeno timed automata

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- Strong relation with robustness
  - robust timed automata [GHJ97,HR00]
  - robust model-checking
    [Puri98,DDR04,DDMR04,ALM05,BMR06,BMR08]
    cf Pierre-Alain Reynier's talk tomorrow

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Further works

- efficient zone-based algorithm
- apply to relevant examples
- add non-determinism (*à la* MDP)
- handle several clocks
- timed properties