When are timed automata determinizable?

Christel Baier [1] Nathalie Bertrand [2] Patricia Bouyer [3] Thomas Brihaye [4]

[1] Technische Universität Dresden – Germany
[2] IRISA – INRIA Rennes – France
[3] LSV – CNRS & ENS Cachan – France
[4] Université de Mons – Belgium

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Outline

Verification and formal languages

Real systems $\mathit{Syst} \rightsquigarrow \mathcal{A}_{\mathit{Syst}}$

Specification $\mathit{Spec} \rightsquigarrow \mathcal{A}_{\varphi}$

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- The universality problem: $\Sigma^* \subseteq \mathcal{L}(\mathcal{A})$

Verification and formal languages (finite automata)

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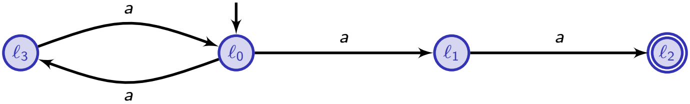
Every finite automaton is determinizable
into an exponential size finite automaton.

Determinizing finite automata (on finite words)

Example: $\mathcal{L}(\mathcal{A}) = (aa)^* aa$



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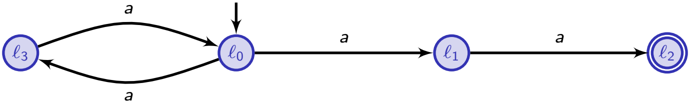
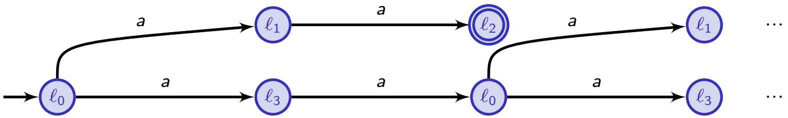


Unfolding $\mathcal{A}$



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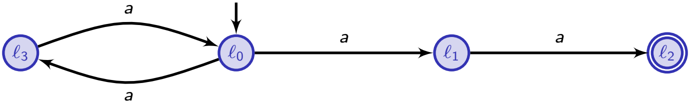

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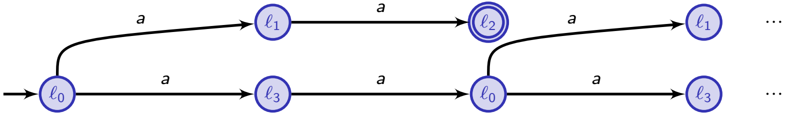
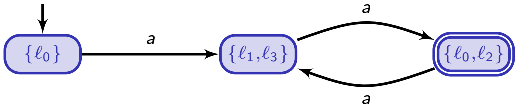
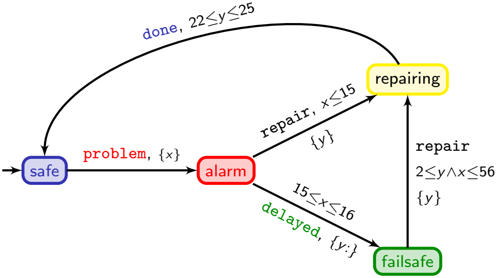


A deterministic version of $\mathcal{A}$



Outline

What is a timed automaton?



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	···
y	0		23		23		38.6		0	

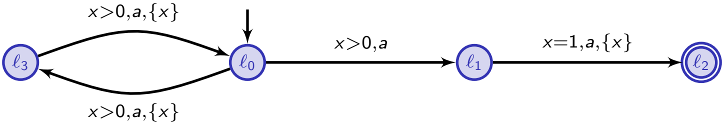
	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\texttt{done}}$	safe
···	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

$\rightsquigarrow$ It reads the timed word $(\texttt{problem}, 23)(\texttt{delayed}, 38.6)(\texttt{repair}, 40.9)(\texttt{done}, 63)$

Timed languages accepted by timed automata

Example

Let $\mathcal{A}$ be the following timed automaton:



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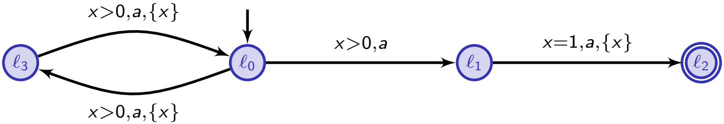

$$\mathcal{L}(\mathcal{A}) = \{(a, t_1)(a, t_2)\cdots(a, t_{2n}) \mid n \geq 1,\ 0 < t_1 < t_2 < \cdots < t_{2n-1} \text{ and } t_{2n} - t_{2n-2} = 1\}$$

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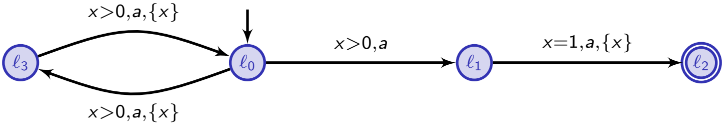

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The timed word $w = (a,0.2)(a,0.5)(a,1.2)(a,1.5)$ is in $\mathcal{L}(\mathcal{A})$.

Results on timed automata [AD90,AD94]

Emptiness problem

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$\rightsquigarrow$ prevents using timed automata as a specification language

Timed automata and determinism

Deterministic timed automaton

A timed automaton $\mathcal{A}$ is deterministic whenever for every timed word w, there is at most one initial run (starting from $(\ell_0, 0)$) which reads u.

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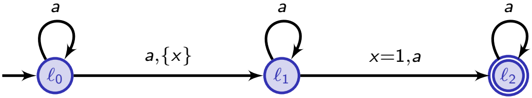
There exist timed automata that are not determinizable [AD90]



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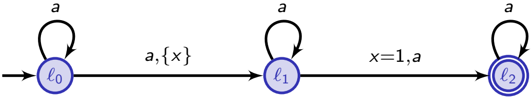

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Theorem [Tri03,Fin06]

We cannot decide whether a timed automaton can be determinized.

Event-clock timed automata [AFH94]

Event-clock timed automata

An event-clock timed automaton is a timed automaton that contains only event-recording clocks: for every letter $a \in \Sigma$, there is a clock x_a, which is reset at every occurrence of an a.

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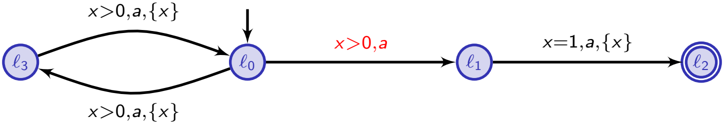

$\mathcal{A}$ is not an event-clock timed automaton

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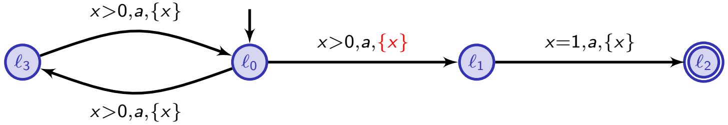

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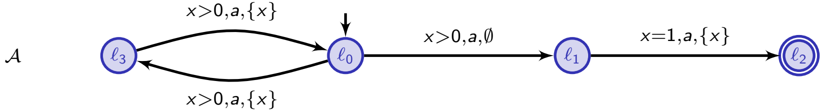
Theorem

- Event-clock timed automata are determinizable.
- Checking universality (and language inclusion) is PSPACE-complete for event-clock timed automata.

Outline

1. General framework
2. Timed automata
3. Towards a determinization procedure for timed automata...
   - Unfolding
   - Region equivalence
   - Symbolic determinization
   - Reducing the number of clocks
   - Reducing the number of locations
4. When can we apply the procedure?
5. Conclusion

Unfolding



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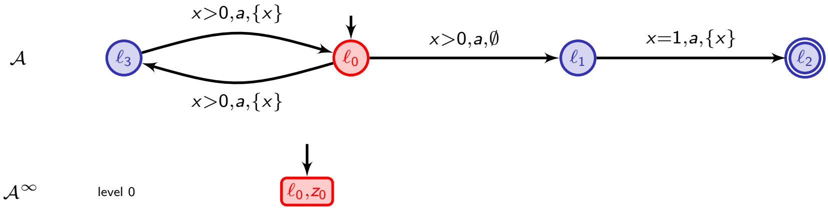

$\mathcal{A}$
$x>0, a, \{x\}$
$x>0, a, \{x\}$
$x>0, a, \emptyset$
$x=1, a, \{x\}$
ℓ_3
ℓ_0
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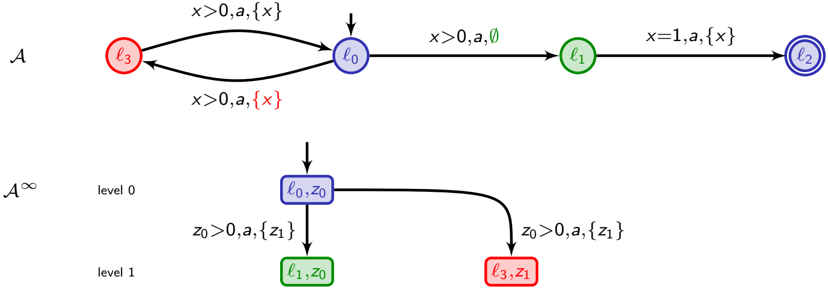
ℓ_2
$\mathcal{A}^\infty$
level 0
ℓ_0, z_0

Unfolding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level 0
ℓ_0,z_0
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level 1
ℓ_1,z_0
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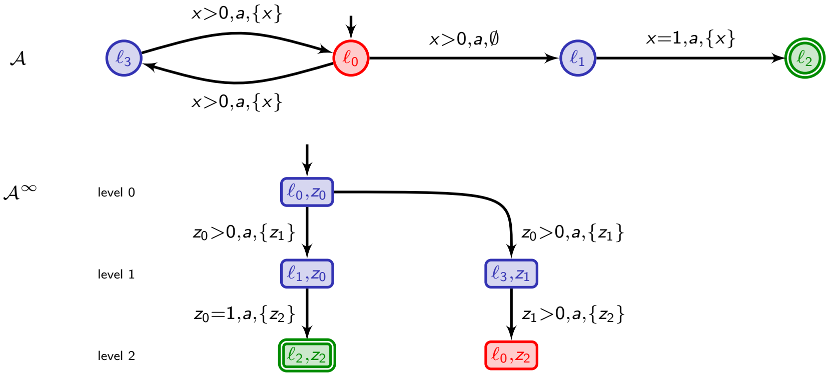
Unfolding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ℓ_0
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ℓ_2
$\mathcal{A}^\infty$
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level 1
level 2
ℓ_0, z_0
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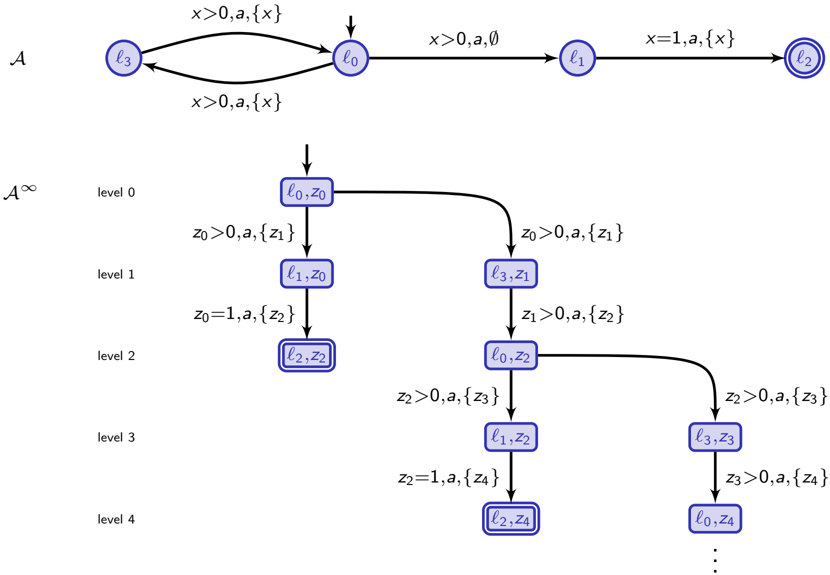
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⋮

Properties of the unfolding

Advantage of the unfolding: "input-determinacy"

Given a finite timed word w, there is a unique valuation v_w such that every initial run reading w ends in a configuration (n, v_w) with $level(n) = |w|$.

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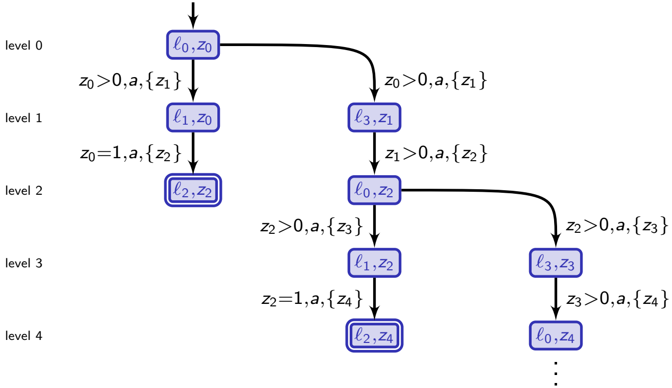

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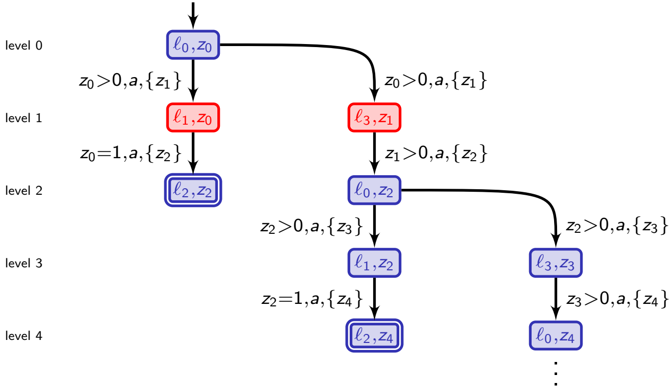

$w = (a, 0.2)$ $\quad\rightsquigarrow\quad$ $v_w = (\underset{z_0}{0.2}, \underset{z_1}{0})$

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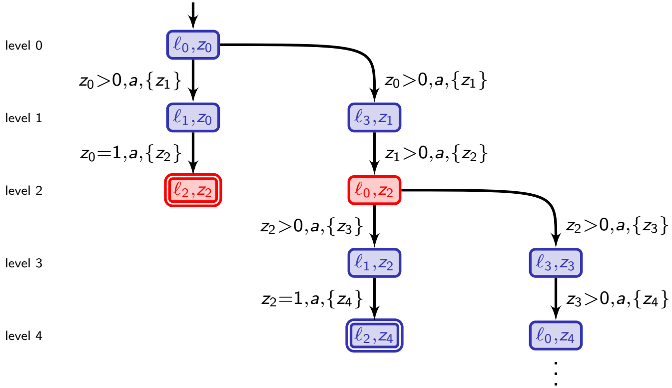

$w = (a, 0.2)(a, 0.5) \quad \rightsquigarrow \quad v_w = (\underset{z_0}{0.5}, \underset{z_1}{0.3}, \underset{z_2}{0})$

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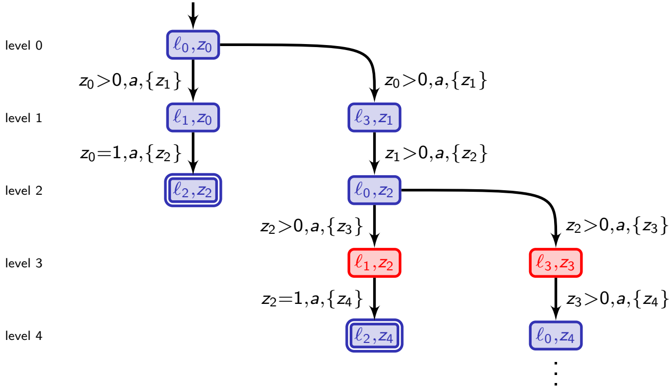

$$w = (a, 0.2)(a, 0.5)(a, 1.2) \qquad \rightsquigarrow \quad v_w = (\underset{z_0}{1.2}, \underset{z_1}{1}, \underset{z_2}{0.7}, \underset{z_3}{0})$$

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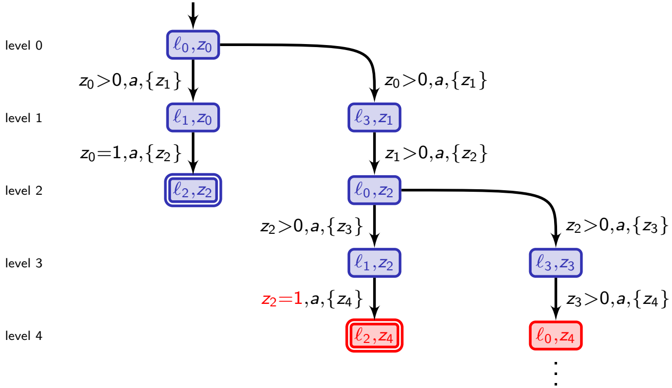

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Drawbacks of the unfolding

- $\mathcal{A}^\infty$ has infinitely many locations.
- $\mathcal{A}^\infty$ has infinitely many clocks.
- $\mathcal{A}^\infty$ is not deterministic.

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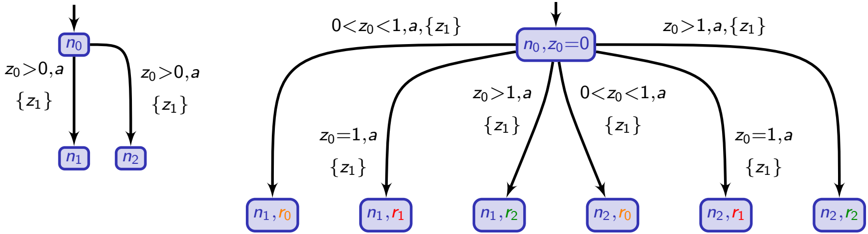
Lemma

$\mathcal{A}$ and $\mathcal{A}^\infty$ are strongly timed bisimilar.
In particular $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}^\infty)$.

Region equivalence on $\mathcal{A}^\infty$

The standard region equivalence naturally extends to $\mathcal{A}^\infty$,

at level i we only consider region over $\{z_1, \ldots, z_i\}$.



where $r_0 = (0 = z_1 < z_0 < 1)$, $r_1 = (0 = z_1 < z_0 = 1)$, $r_2 = (0 = z_1 < 1 < z_0)$.

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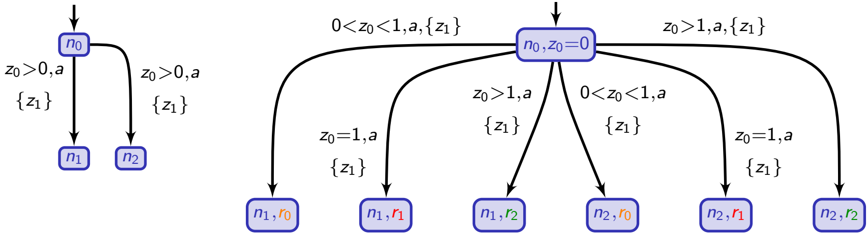

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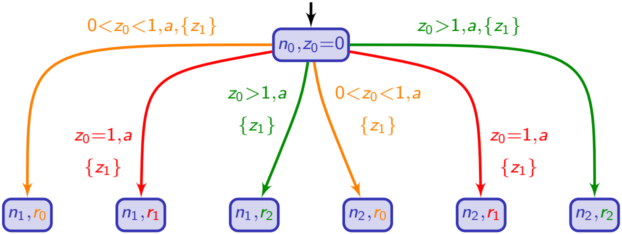
Lemma

$\mathcal{A}^\infty$ and $R(\mathcal{A}^\infty)$ are strongly timed bisimilar.
In particular, $\mathcal{L}(\mathcal{A}^\infty) = \mathcal{L}(R(\mathcal{A}^\infty))$.

Symbolic determinization of $R(\mathcal{A}^\infty)$



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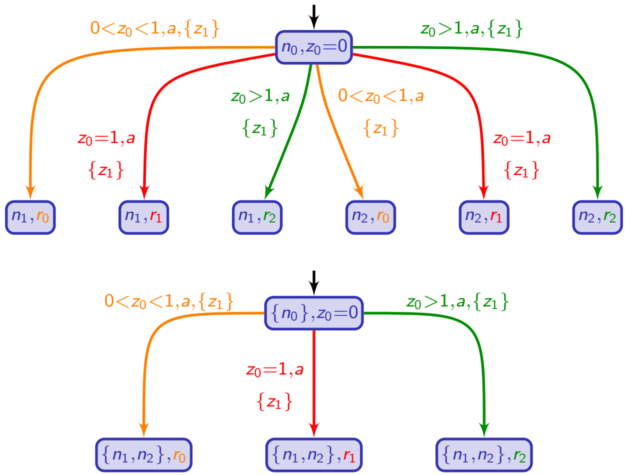

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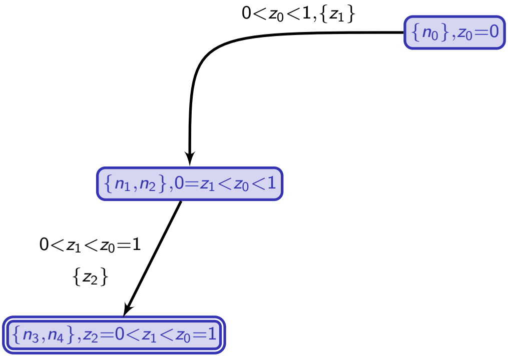
Lemma

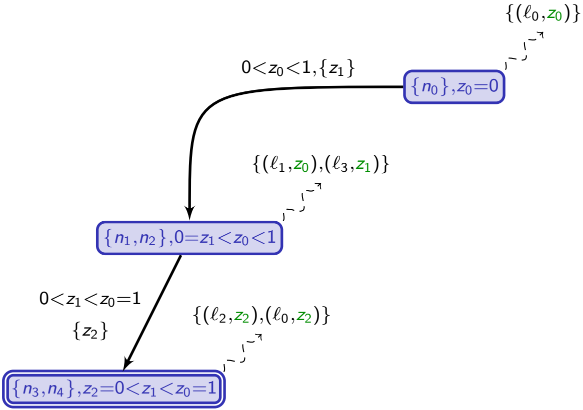
$\mathcal{L}(\mathcal{A}) = \mathcal{L}(\text{SymbDet}(R(\mathcal{A}^\infty)))$.

Notion of active clocks



Notion of active clocks

Remember where nodes come from!



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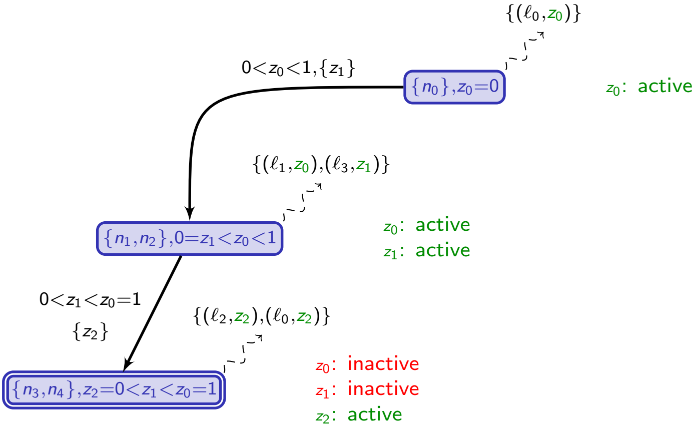

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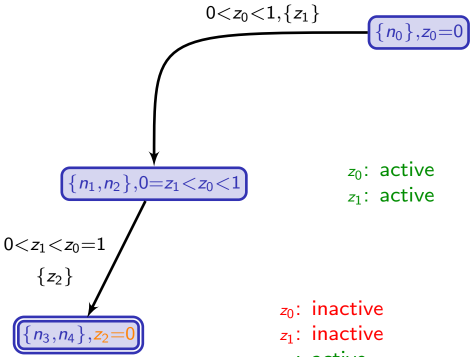
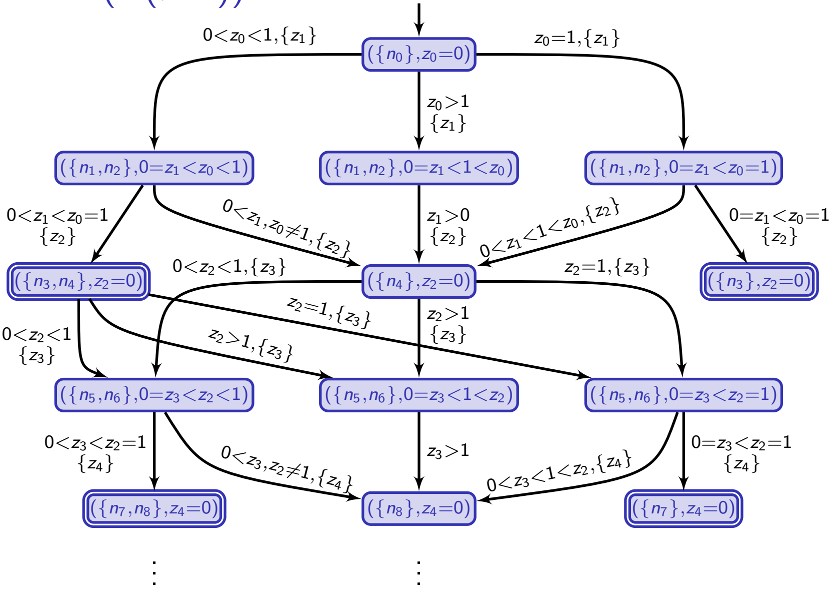


z_0: active

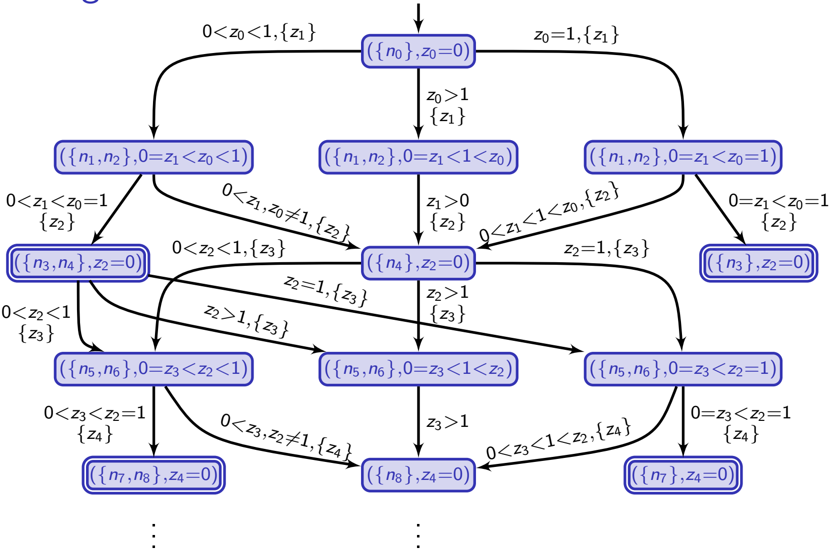
z_0: active
z_1: active

z_0: inactive
z_1: inactive
z_2: active

SymbDet($R(\mathcal{A}^\infty)$)



Reducing the number of clocks



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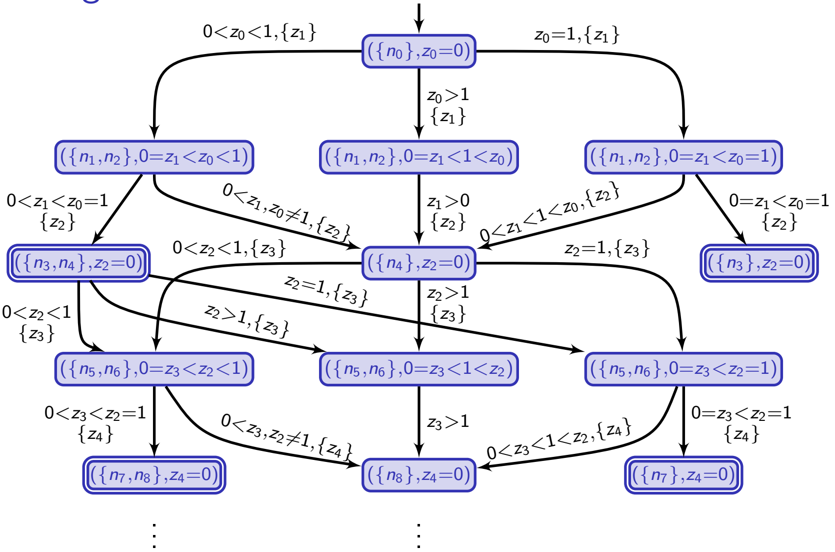

Two clocks are sufficient to get full timing information! $\rightsquigarrow$ $\begin{cases} z_{2i} & \mapsto x_1 \\ z_{2i+1} & \mapsto x_2 \end{cases}$

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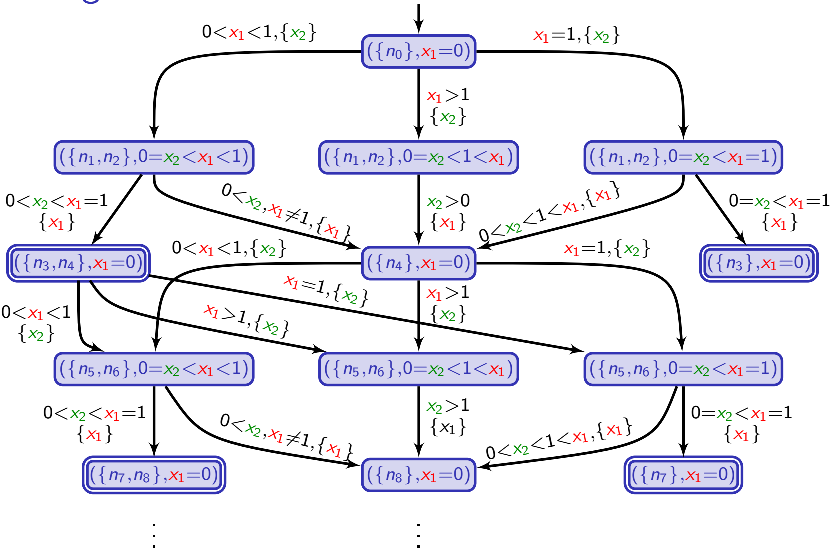

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Given $\gamma \in \mathbb{N}$, we say that SymbDet($R(\mathcal{A}^\infty)$) is γ-clock-bounded if in every node, the number of active clocks is bounded by γ.

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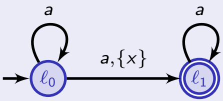

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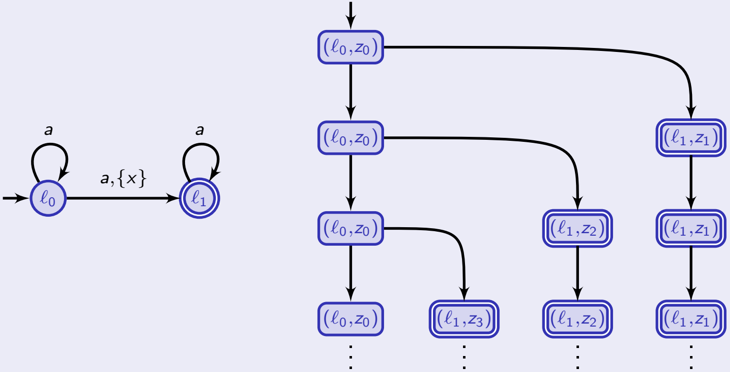

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In case $\mathsf{SymbDet}(R(\mathcal{A}^\infty))$ is γ-clock-bounded, we construct using a deterministic policy $\Gamma_\gamma(\mathsf{SymbDet}(R(\mathcal{A}^\infty)))$, an equivalent timed system with clocks $\{x_1, \ldots, x_\gamma\}$.

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Advantages of Γ_γ(SymbDet($R(\mathcal{A}^\infty)$))

- Γ_γ(SymbDet($R(\mathcal{A}^\infty)$)) is deterministic!
- Γ_γ(SymbDet($R(\mathcal{A}^\infty)$)) has finitely many clocks.

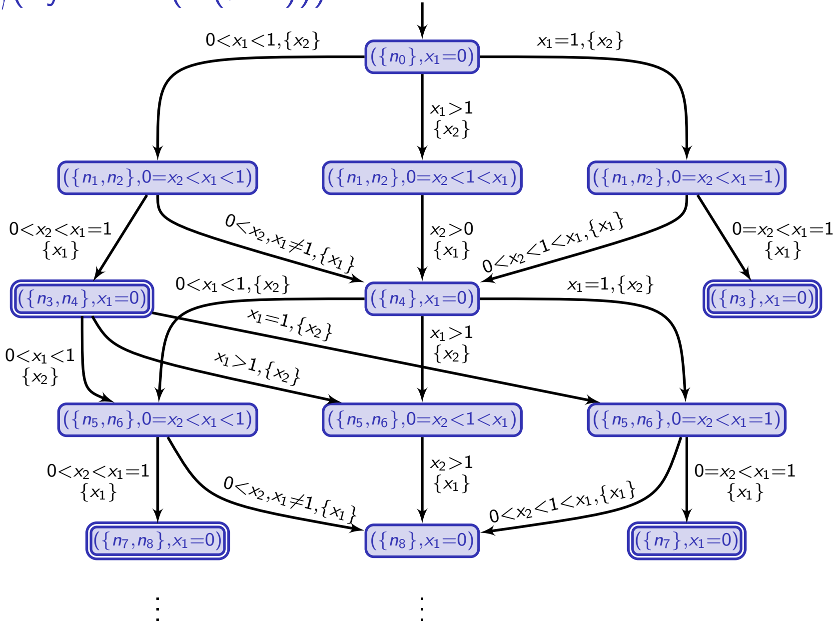
Drawback of Γ_γ(SymbDet($R(\mathcal{A}^\infty)$))

Γ_γ(SymbDet($R(\mathcal{A}^\infty)$)) has infinitely many locations.

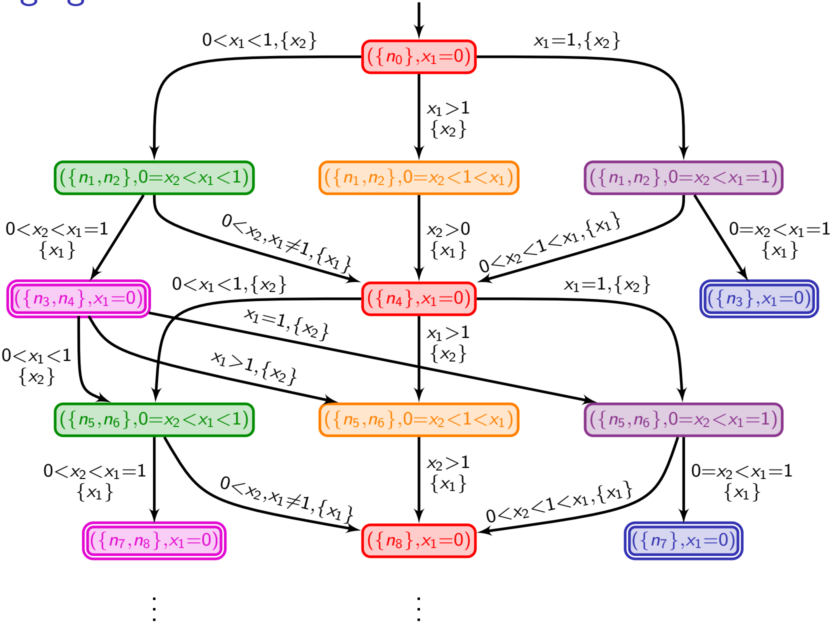
Lemma

$\mathcal{L}(\mathcal{A}) = \mathcal{L}(\Gamma_\gamma(\text{SymbDet}(R(\mathcal{A}^\infty))))$.

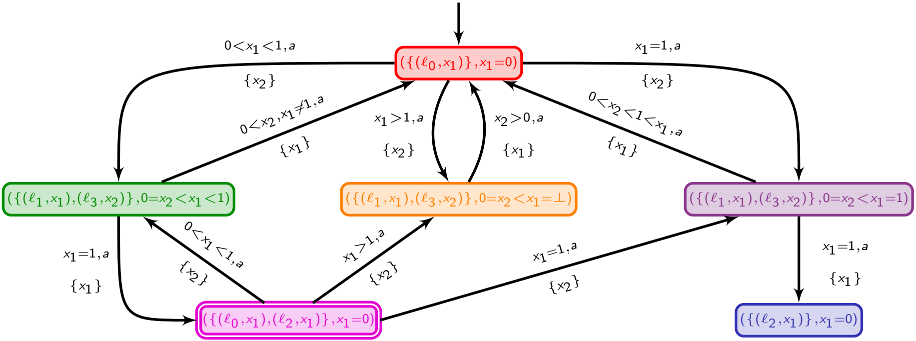
$\Gamma_\gamma(\mathsf{SymbDet}(R(\mathcal{A}^\infty)))$



Merging “identical” nodes



A deterministic timed automaton equivalent to $\mathcal{A}$



Properties of the location reduction

In case $\mathsf{SymbDet}(R(\mathcal{A}^\infty))$ is γ-clock-bounded, we define $\mathcal{B}_{\mathcal{A},\gamma}$ obtained by merging the nodes of $\Gamma_\gamma(\mathsf{SymbDet}(R(\mathcal{A}^\infty)))$ with "the same labels".

Theorem

In case $\mathsf{SymbDet}(R(\mathcal{A}^\infty))$ is γ-clock-bounded, $\mathcal{B}_{\mathcal{A},\gamma}$ is a deterministic timed automaton such that $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{B}_{\mathcal{A},\gamma})$.

Outline

When can we apply our procedure?

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The p-assumption

Given $p \in \mathbb{N}$, $\mathcal{A}$ satisfies the *p-assumption* if for every $n \geq p$, for every run

$$\varrho = (\ell_0, v_0) \xrightarrow{\tau_1, a_1} (\ell_1, v_1) \ldots \xrightarrow{\tau_n, a_n} (\ell_n, v_n)$$

for every clock $x \in X$, either x is reset along ϱ or $v_n(x) > M$.

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Classes to which the procedure applies

- Event-clock timed automata (with $\gamma = |\Sigma|$)
- Strongly non-Zeno timed automata (since they satisfy the p-assumption)
- timed automata with integer resets [SPKM08]

Hardness issues

We can prove EXPSPACE-hardness of:

- the universality problem for timed automata satisfying the p-assumption and for timed automata with integer resets;
- the inclusion problem for strongly non-Zeno timed automata.

The results

Summary of the complexity results

	size of the det. TA	universality problem	inclusion problem
TA_p	doubly exp.	EXPSPACE-compl.	EXPSPACE-compl.
SnZTA	doubly exp.	trivial	EXPSPACE-compl.
ECTA [AFH94]	exp.	PSPACE-compl.	PSPACE-compl.
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Remark

In case $\mathcal{A}$ has one clock, $\mathrm{SymbDet}(R(\mathcal{A}^\infty))$ allows to recover the decidability of the universality problem in one-clock TA [OW04].

Outline

Conclusion

What we have done

- We have described a procedure to determinize timed automata...
- ... which terminates for several subclasses of timed automata
  - event-clock timed automata
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- We recover known results, but also describe new determinizable classes of timed automata.
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What we will do now

- We want to see whether other determinizable classes (open timed automata) could fit our framework.
- We will extend to infinite timed words (with a Safra-like construction mixed with our procedure?)