On the Model Checking of Timed and Weighted Temporal Logics

Patricia Bouyer

LSV, CNRS & ENS Cachan, France

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Outline

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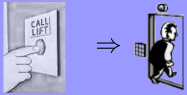
Model-checking

system:

property:



Model-checking

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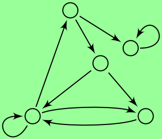
property:

$\Rightarrow$



$\mathbf{G}\,(\texttt{request} \rightarrow \mathbf{F}\,\texttt{grant})$

Model-checking

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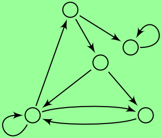
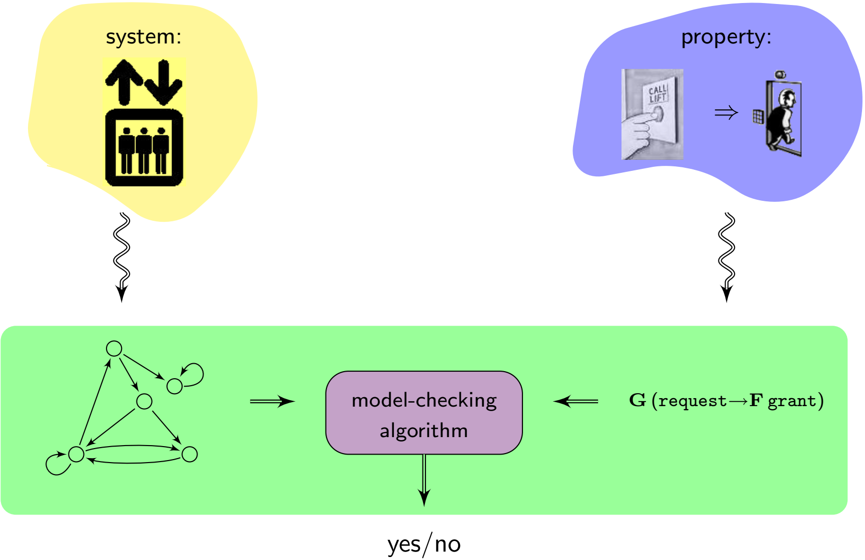


model-checking algorithm

$\mathbf{G}\,(\mathtt{request} \rightarrow \mathbf{F}\,\mathtt{grant})$

Model-checking



The untimed (linear-time) framework

Linear-time temporal logic [Pnu77]

$$\text{LTL} \ni \varphi ::= p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \neg\varphi \mid \mathbf{X}\,\varphi \mid \varphi\,\mathbf{U}\,\varphi$$

[Pnu77] Pnueli. The temporal logic of programs (FOCS'77).

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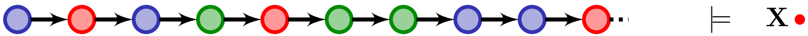

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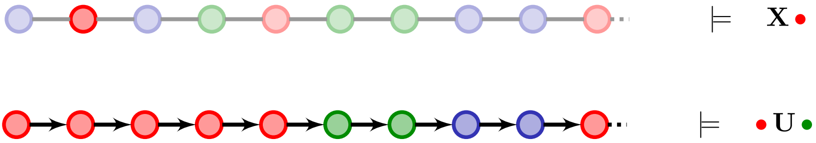

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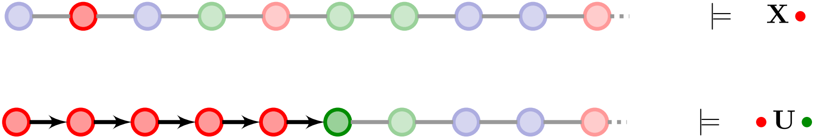

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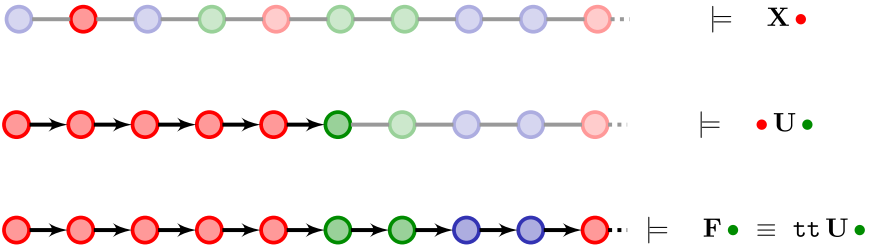

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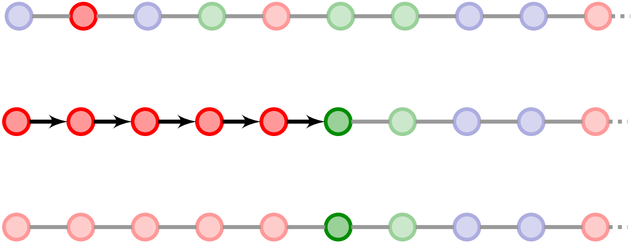

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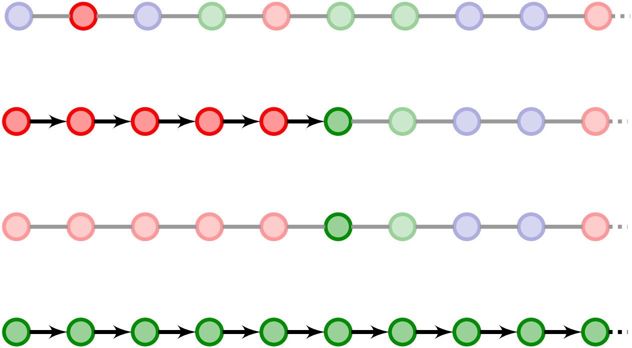

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- response property:

$$\mathbf{G}\,({\color{red}\bullet} \rightarrow \mathbf{F}\,{\color{green}\bullet})$$

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- safety property:

$$\mathbf{G}\,\neg\textcolor{red}{\bullet}$$

- a more complex property:

$$(\textcolor{green}{\bullet} \wedge (\mathbf{F}\,\textcolor{blue}{\bullet} \vee \mathbf{G}\,\textcolor{orange}{\bullet}))\,\mathbf{U}\,\textcolor{red}{\bullet}$$

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Adding timing requirements

- Need for timed models
  - the behaviour of most systems depends on time;
  - faithful modelling has to take time into account.

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- Need for timed specification languages
  - the behaviour of most systems depends on time;
  - untimed specifications are not sufficient (for instance, bounded response timed, etc...)

  ☞ TCTL, MTL, TPTL, timed μ-calculus...

Outline

Metric Temporal Logic (MTL)

[Koy90]

$$\mathsf{MTL} \ni \varphi ::= a \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \varphi \, \mathbf{U}_I \, \varphi$$

where I is an interval with integral bounds.

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- Can be interpreted over finite or infinite behaviours
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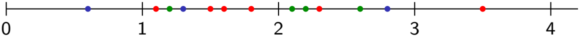
The pointwise semantics

MTL formulas are interpreted over timed words:



$$(\bullet, .6)(\bullet, 1.1)(\bullet, 1.2)(\bullet, 1.3)\ldots$$

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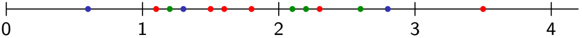

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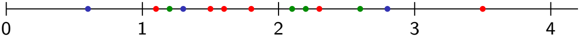

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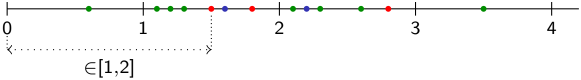

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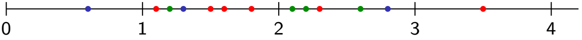

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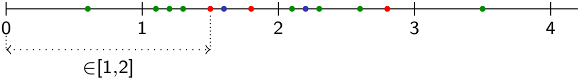

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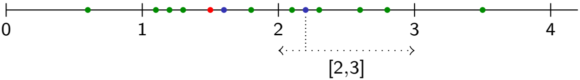

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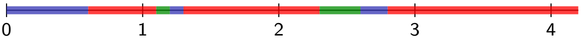
MTL formulas are interpreted over (finitely variable) signals:



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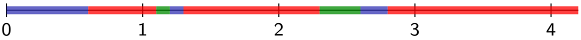

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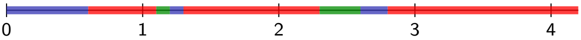

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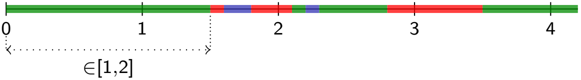

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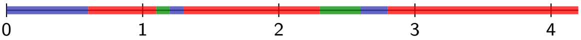

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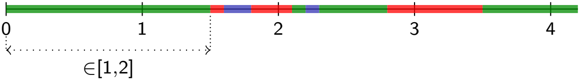

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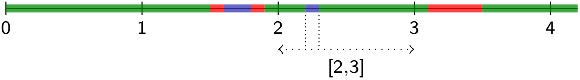

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Some examples

- "Every problem is followed within 56 time units by an alarm"

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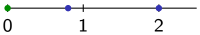

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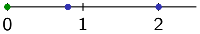

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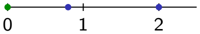

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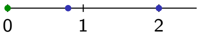

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  - in the pointwise semantics, $\mathbf{F}_{=2}\,\bullet \not\equiv \mathbf{F}_{=1}\,\mathbf{F}_{=1}\,\bullet$
  - in the continuous semantics, $\mathbf{F}_{=2}\,\bullet \equiv \mathbf{F}_{=1}\,\mathbf{F}_{=1}\,\bullet$

Some further extensions

- Timed Propositional Temporal Logic (TPTL) [AH89]

  TPTL = LTL + clock variables + clock constraints

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A note on the expressiveness

Theorem

LTL+Past is as expressive as LTL [Kam68,GPSS80].

[Kam68] Kamp. Tense logic and the theory of linear order (PhD Thesis UCLA 1968).
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A note on the expressiveness

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$$\mathbf{G}\,(\bullet \rightarrow x.\mathbf{F}\,(\bullet \wedge \mathbf{F}\,(\bullet \wedge x \leqslant 2)))$$

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- This is wrong in the continuous semantics!

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The TPTL formula

$$\mathbf{G}\left({\color{green}\bullet} \rightarrow x.\mathbf{F}\left({\color{red}\bullet} \wedge \mathbf{F}\left({\color{blue}\bullet} \wedge x \leqslant 2\right)\right)\right)$$

can be expressed in MTL in the continuous semantics

The TPTL formula

$$\mathbf{G}\left(\textcolor{green}{\bullet} \rightarrow x.\mathbf{F}\left(\textcolor{red}{\bullet} \wedge \mathbf{F}\left(\textcolor{blue}{\bullet} \wedge x \leqslant 2\right)\right)\right)$$

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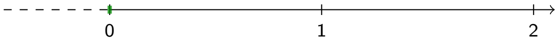

$$\mathbf{G}\ \textcolor{green}{\bullet} \rightarrow \left\{ \right.$$

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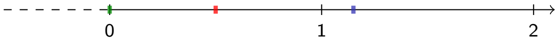

$$\mathbf{G}\ {\color{green}\bullet} \ \rightarrow \begin{cases} \mathbf{F}_{\leqslant 1}\ {\color{red}\bullet} \ \wedge \ \mathbf{F}_{[1,2]}\ {\color{blue}\bullet} \\ \\ \\ \end{cases}$$

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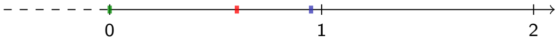

$$\mathbf{G}\ {\color{green}\bullet} \rightarrow \begin{cases} \quad \mathbf{F}_{\leqslant 1}\ {\color{red}\bullet} \ \wedge\ \mathbf{F}_{[1,2]}\ {\color{blue}\bullet} \\ \vee \\ \quad \mathbf{F}_{\leqslant 1}\left({\color{red}\bullet} \ \wedge\ \mathbf{F}_{\leqslant 1}{\color{blue}\bullet}\right) \\ \end{cases}$$

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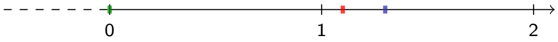

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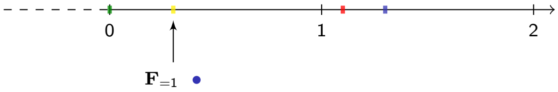

$$\mathbf{G}\ {\color{green}\bullet} \ \rightarrow \left\{ \begin{array}{ll} & \mathbf{F}_{\leqslant 1}\ {\color{red}\bullet} \ \wedge\ \mathbf{F}_{[1,2]}\ {\color{blue}\bullet} \\ \vee & \\ & \mathbf{F}_{\leqslant 1}\,(\ {\color{red}\bullet} \ \wedge\ \mathbf{F}_{\leqslant 1}\,{\color{blue}\bullet}\ ) \\ \vee & \\ & \end{array} \right.$$

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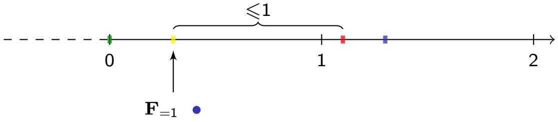

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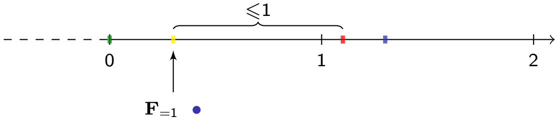

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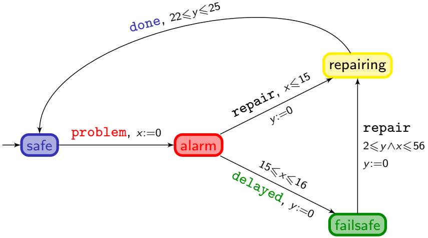
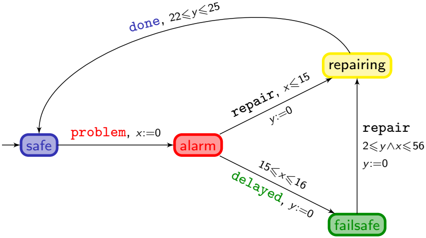
Outline

Timed automata

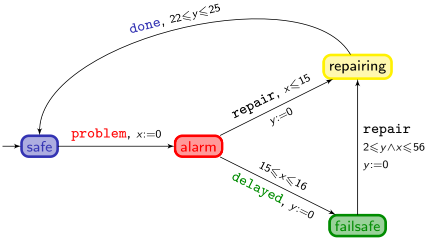


Timed automata



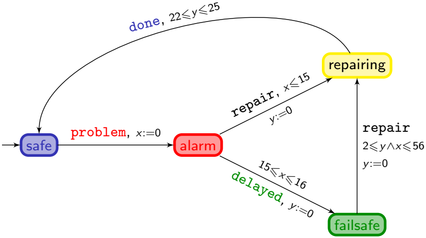
	safe
x	0
y	0

Timed automata



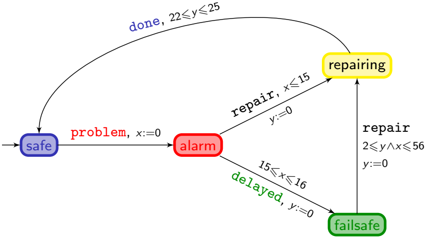
	safe	$\xrightarrow{23}$	safe
x	0		23
y	0		23

Timed automata



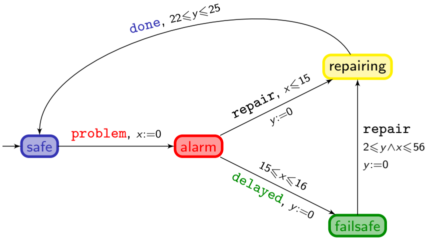
	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\text{problem}}$	alarm
x	0		23		0
y	0		23		23

Timed automata



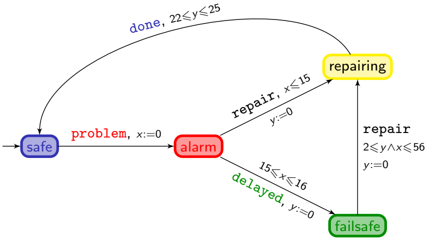
	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\text{problem}}$	alarm	$\xrightarrow{15.6}$	alarm
x	0		23		0		15.6
y	0		23		23		38.6

Timed automata



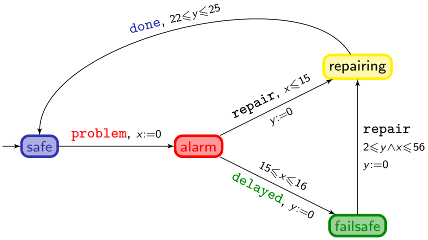
	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe
x	0		23		0		15.6		15.6
y	0		23		23		38.6		0

Timed automata



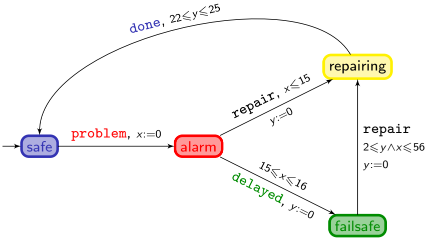
	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	$\xrightarrow{2.3}$	failsafe
x	0		23		0		15.6		15.6		17.9
y	0		23		23		38.6		0		2.3

Timed automata



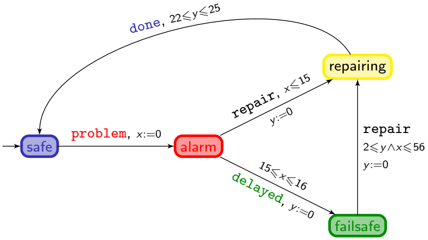
	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\text{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\text{delayed}}$	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\text{repair}}$	reparation
x	0		23		0		15.6		15.6		17.9		17.9
y	0		23		23		38.6		0		2.3		0

Timed automata



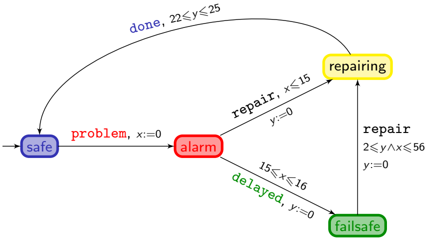
	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\text{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\text{delayed}}$	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\text{repair}}$	reparation	$\xrightarrow{22.1}$	reparation
x	0		23		0		15.6		15.6		17.9		17.9		40
y	0		23		23		38.6		0		2.3		0		22.1

Timed automata



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	reparation	$\xrightarrow{22.1}$	reparation	$\xrightarrow{\texttt{done}}$ safe
x	0		23		0		15.6		15.6		17.9		17.9		40	40
y	0		23		23		38.6		0		2.3		0		22.1	22.1

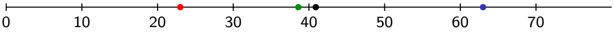
Timed automata



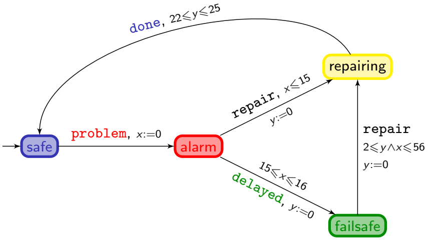
	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\text{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\text{delayed}}$	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\text{repair}}$	reparation	$\xrightarrow{22.1}$	reparation	$\xrightarrow{\text{done}}$	safe
x	0		23		0		15.6		15.6		17.9		17.9		40		40
y	0		23		23		38.6		0		2.3		0		22.1		22.1

Can be viewed:

- as the timed word (problem,23)(delayed,38.6)(repair,40.9)(done,63)



Timed automata



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\text{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\text{delayed}}$	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\text{repair}}$	reparation	$\xrightarrow{22.1}$	reparation	$\xrightarrow{\text{done}}$	safe
x	0		23		0		15.6		15.6		17.9		17.9		40		40
y	0		23		23		38.6		0		2.3		0		22.1		22.1

Can be viewed:

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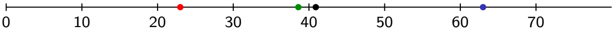
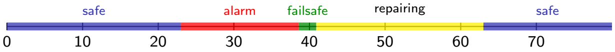


- as the signal



Basic result on timed automata

Theorem

The reachability problem is decidable (and PSPACE-complete) for timed automata [AD94].

[AD94] Alur, Dill. A theory of timed automata (TCS, 1994).

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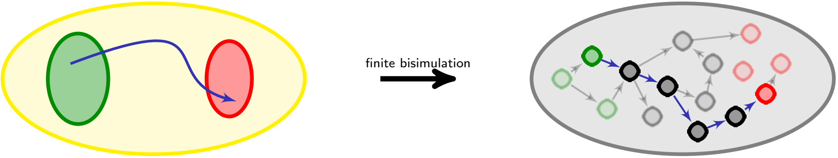

timed automaton

large (but finite) automaton
(region automaton)

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Basic result on timed automata

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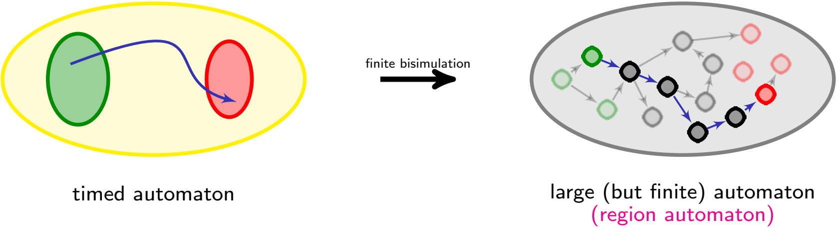


☞ It can be extended to model-check TCTL [ACD93].

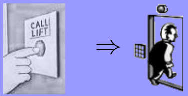
[AD94] Alur, Dill. A theory of timed automata (TCS, 1994).
[ACD93] Alur, Courcoubetis, Dill. Model-checking in dense real-time (I&C, 1993).

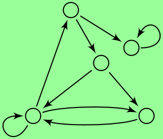
Outline

Back to the model-checking problem

system:

property:

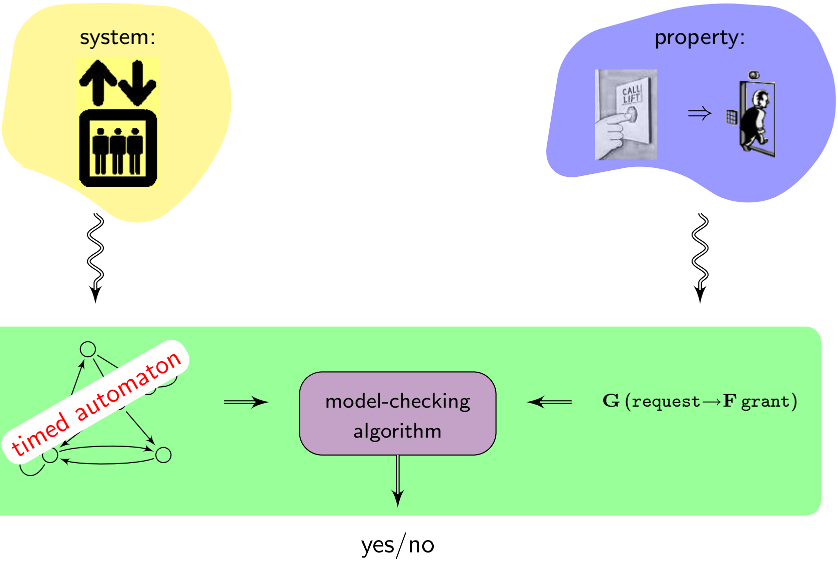




$\Longrightarrow$ model-checking algorithm $\Longleftarrow$ $\mathbf{G}\,(\mathtt{request} \rightarrow \mathbf{F}\,\mathtt{grant})$

yes/no

Back to the model-checking problem

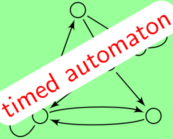


Back to the model-checking problem

system:

property:

$\Rightarrow$



$\Longrightarrow$ model-checking algorithm $\Longleftarrow$ **G** (requ … **F** grant)

MTL formula

yes/no

Results

Theorem

Over finite runs, the model-checking problem is:

	pointwise sem.	continuous sem.
MTL	decidable, NPR [OW05]	undecidable [AFH96]
MTL+Past	undecidable	undecidable
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- Model-checking linear-time timed temporal logics is hard!

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  (reminder: model-checking TCTL is PSPACE-complete)

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  (reminder: model-checking TCTL is PSPACE-complete)

☞ we will explain this high complexity, following [Che07]

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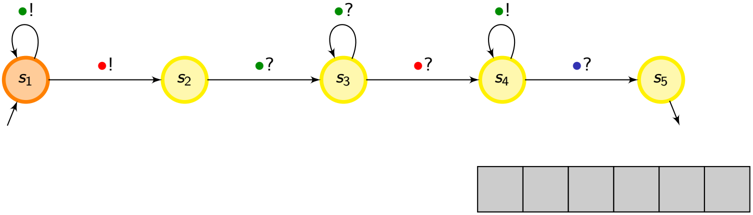
[AFH96] Alur, Feder, Henzinger. The benefits of relaxing punctuality (Journal of the ACM, 1996).
[AH94] Alur, Henzinger. A really temporal logic (Journal of the ACM, 1994).
[Che07] Chevalier. Logiques pour les systèmes temporisés : contrôle et expressivité (PhD Thesis ENS Cachan, June 2007).

A short visit to channel machines (1)

A channel machine $=$ a finite automaton $+$ a FIFO channel



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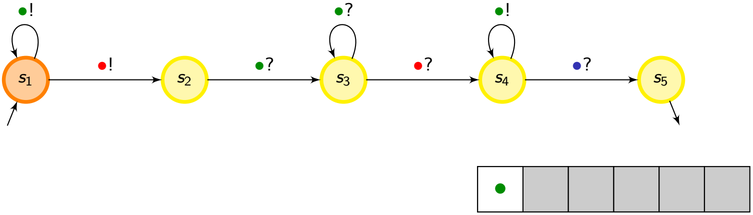

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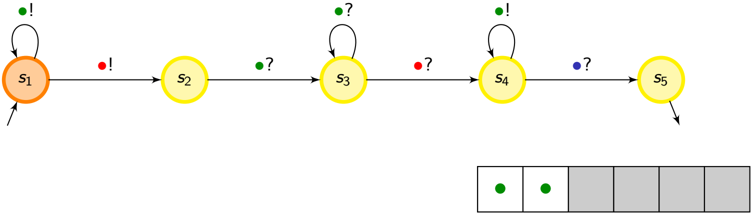

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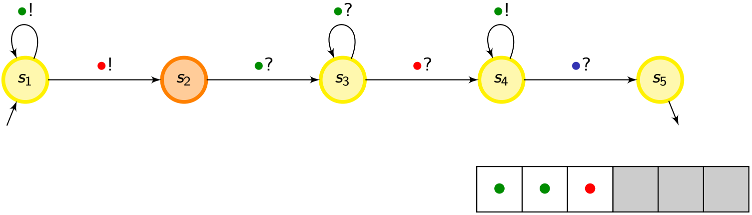

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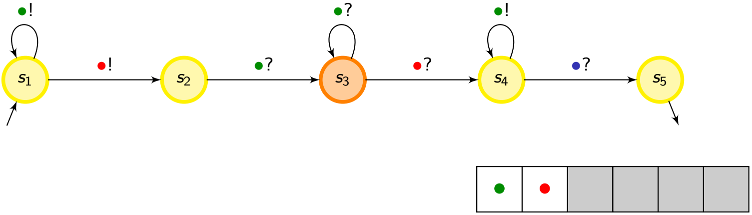

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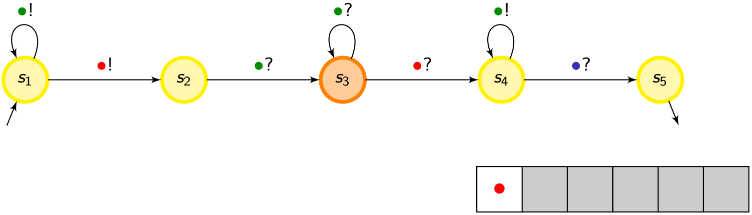

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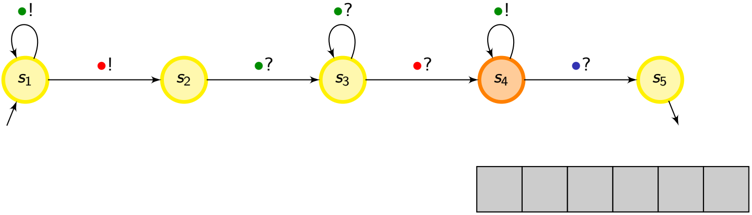

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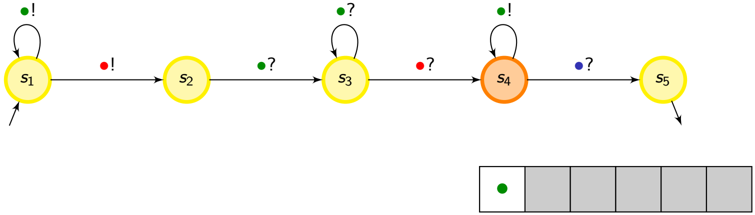

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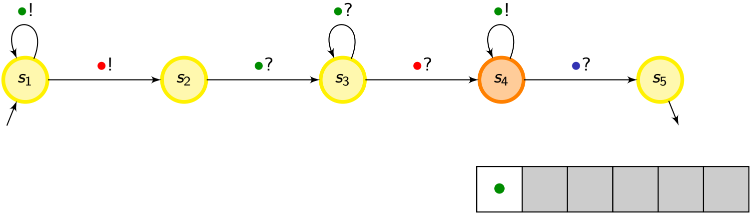

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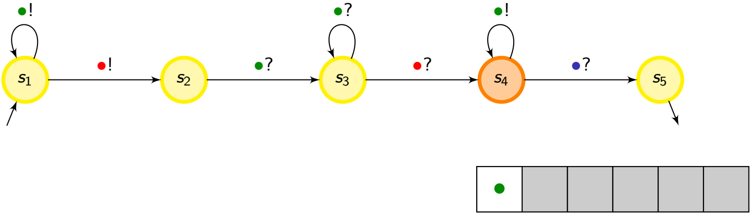

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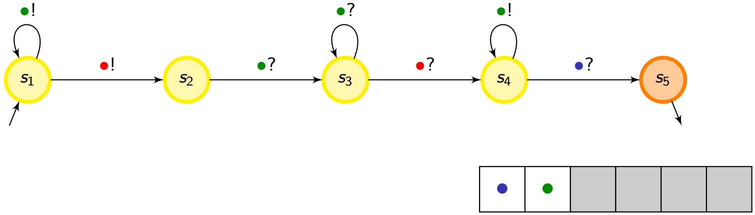

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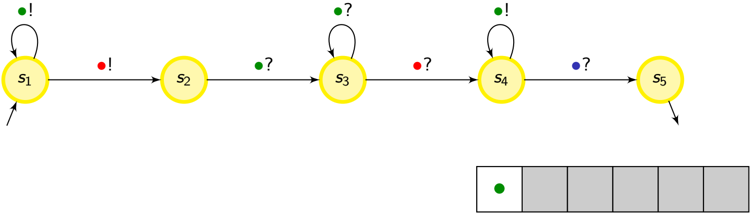

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A short visit to channel machines (2)

Halting problem: is there an execution ending in a halting state?

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Proposition

- The halting problem is undecidable for channel machines [BZ83].
- The halting problem is decidable but NPR for channel machines with insertion errors [Sch02].

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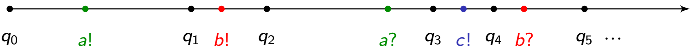
Channel machines and timed words

We encode an execution of a channel machine as a timed word:

$$(q_0, \varepsilon) \xrightarrow{a!} (q_1, a) \xrightarrow{b!} (q_2, ab) \xrightarrow{a?} (q_3, b) \xrightarrow{c!} (q_4, bc) \xrightarrow{b?} (q_5, c) \cdots$$



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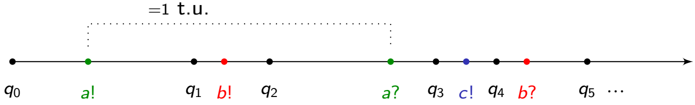

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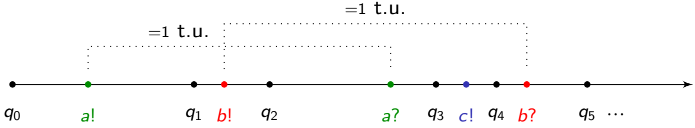

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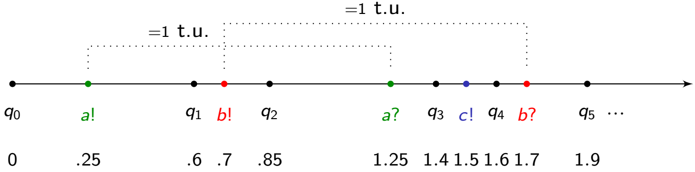

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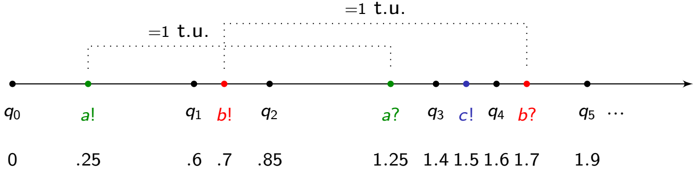

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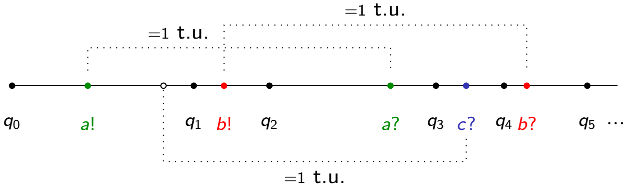

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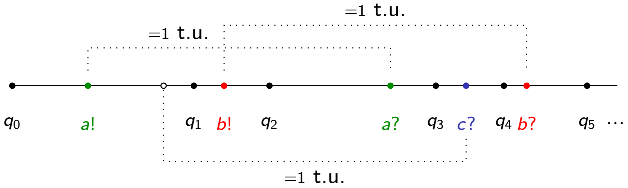

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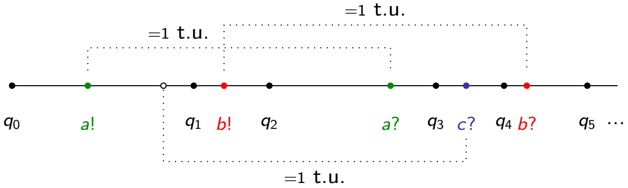

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☞ model-checking MTL is NPR

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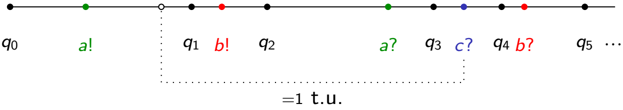

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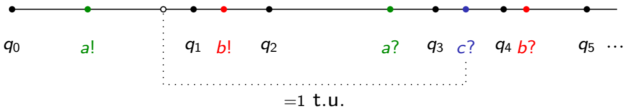

  - this formula is in TPTL (pointwise sem.), not in MTL

What we have proved so far

Theorem

Over finite runs, the model-checking problem is:

	pointwise sem.	continuous sem.
MTL	NPR [OW07]	undecidable [AFH96]
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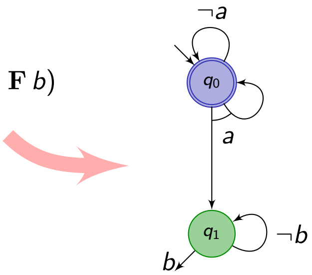

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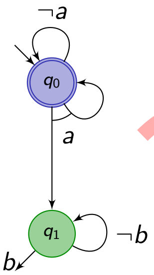

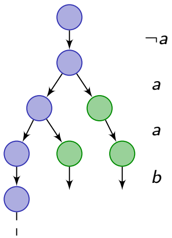

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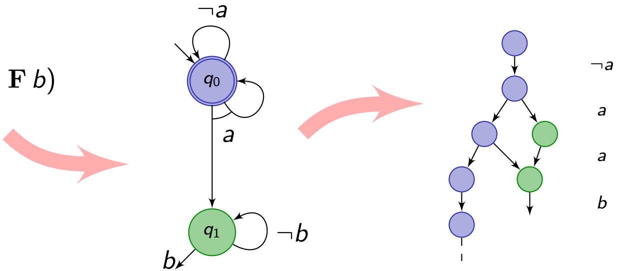

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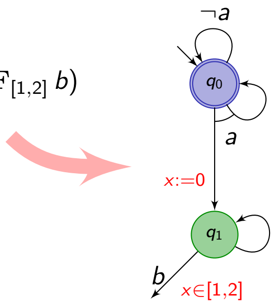

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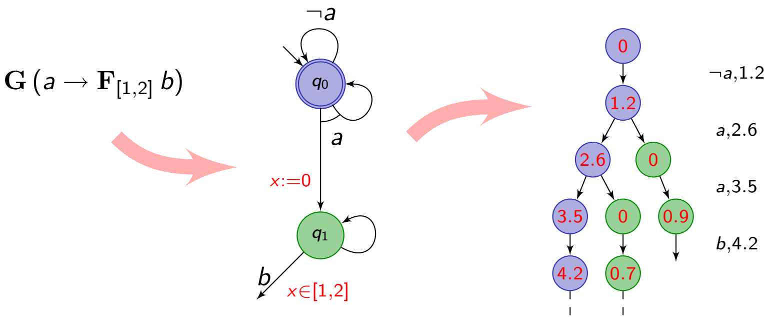
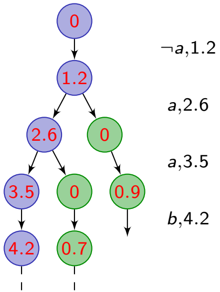


An abstract transition system



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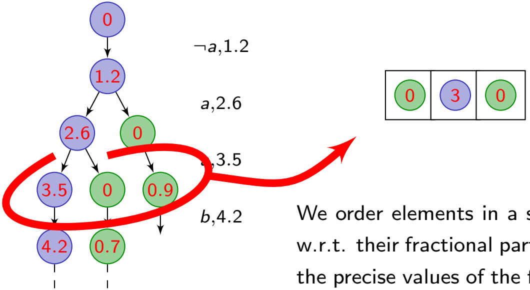

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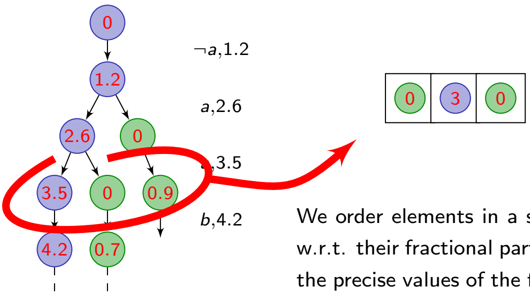

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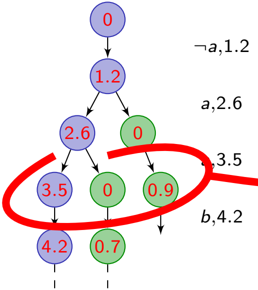

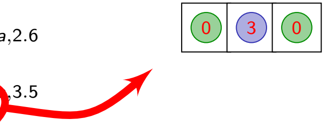

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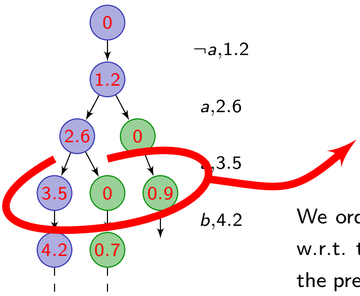

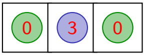

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- ☞ there is a well quasi-order on the set of abstract configurations (subword relation):

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Summary

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What about infinite behaviours?

- the previous algorithm cannot be lifted to the infinite behaviours framework

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Outline

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Metric Interval Temporal Logic (MITL): [AFH96]

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- Examples:
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The fragment without punctuality

- The undecidability/NPR proofs heavily rely on punctual constraints.

  Claim: "Any logic strong enough to express the property $\mathbf{G}\,(\color{red}\bullet \color{black}\rightarrow \mathbf{F}_{=1}\,\color{green}\bullet\color{black})$ is undecidable"

- What if we forbid punctual constraints in MTL?

Metric Interval Temporal Logic (MITL): [AFH96]

$$\mathsf{MITL} \ni \varphi \ ::= \ a \ | \ \neg\varphi \ | \ \varphi \vee \varphi \ | \ \varphi \wedge \varphi \ | \ \varphi\,\mathbf{U}_{I}\,\varphi$$

with I a non-punctual interval

- Examples:
  - $\mathbf{G}\,(\color{red}\bullet \color{black}\rightarrow \mathbf{F}_{=1}\,\color{green}\bullet\color{black})$ is not in MITL
  - $\mathbf{G}\,(\color{red}\bullet \color{black}\rightarrow \mathbf{F}_{[1,2]}\,\color{green}\bullet\color{black})$ is in MITL

[AFH96] Alur, Feder, Henzinger. The benefits of relaxing punctuality (Journal of the ACM, 1996).

Model-checking MITL is “easy”

Theorem

The model-checking problem for MITL is EXPSPACE-complete [AFH96].
If constants are encoded in unary, it is even PSPACE-complete [HR04].

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☞ something more clever needs to be done

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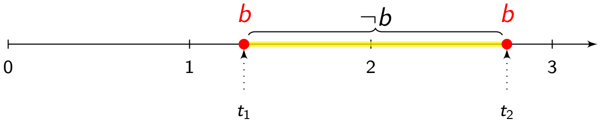

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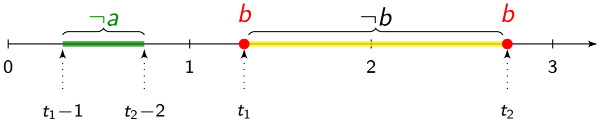

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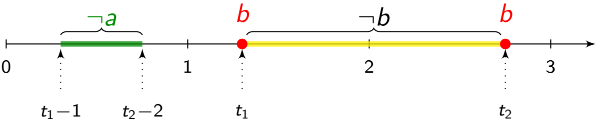

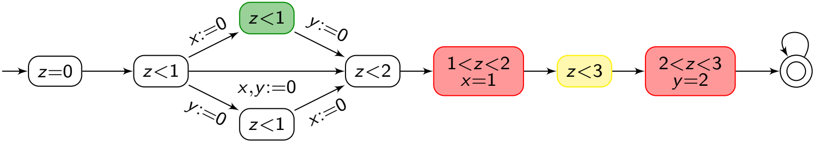

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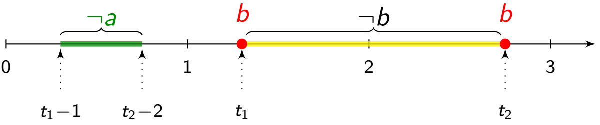

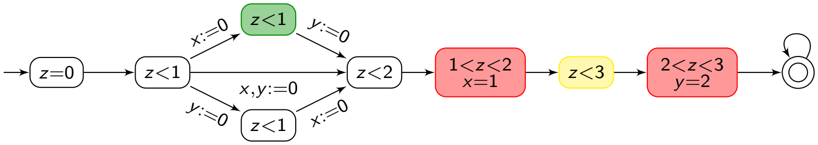

☞ This idea can be extended to any formula in MITL

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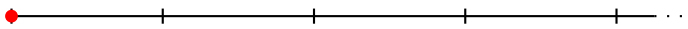

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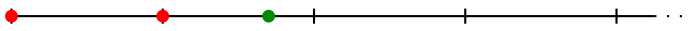

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- A Bounded-MTL formula may define a non timed-regular language:

$$\mathbf{G}_{\leqslant 1}\,({\color{red}\bullet} \rightarrow \mathbf{F}_{=1}\,{\color{green}\bullet}) \wedge \mathbf{G}_{\leqslant 1}\,{\color{red}\bullet} \wedge \mathbf{G}_{(1,2]}\,{\color{green}\bullet}$$

defines the context-free language $\{{\color{red}\bullet}^n {\color{green}\bullet}^m \mid n \leqslant m\}$.

Algorithm for Bounded-MTL

Assume one wants to verify the formula

$$\mathbf{G}_{<2}\Big(\textcolor{red}{\bullet} \longrightarrow \mathbf{F}_{=1}\,\textcolor{green}{\bullet}\Big)$$

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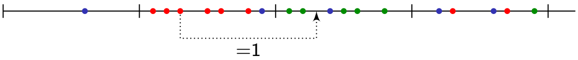

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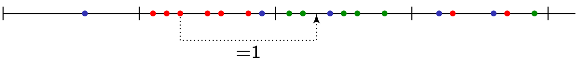
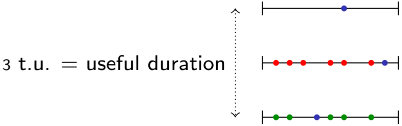


Offline, we stack all ‘relevant’ time units and use a sliding window:



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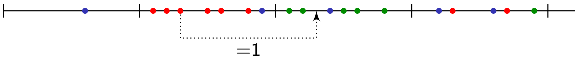

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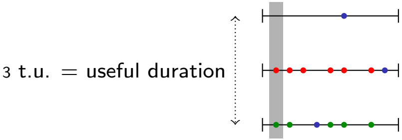

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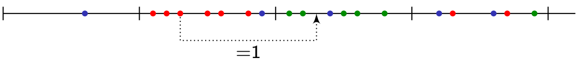

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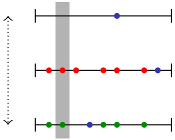
3 t.u. = useful duration



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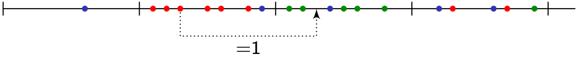

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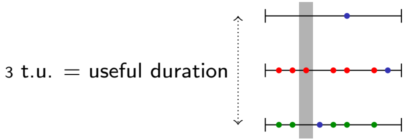

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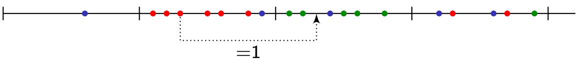

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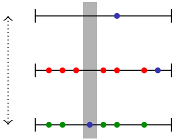

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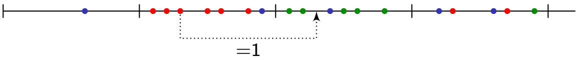

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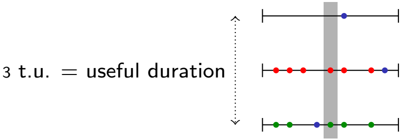

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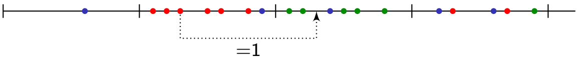

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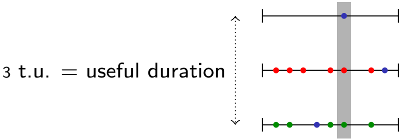

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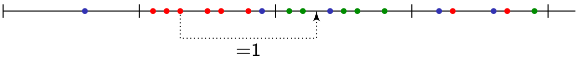

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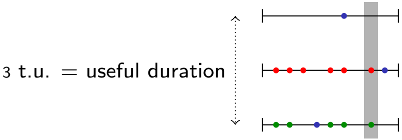

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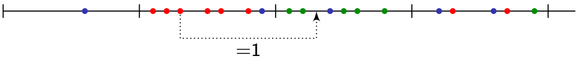

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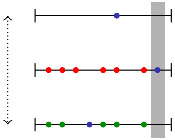

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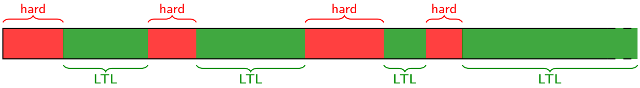

where - the number of hard fragments is at most exponential
- the total duration of hard fragments is at most exponential

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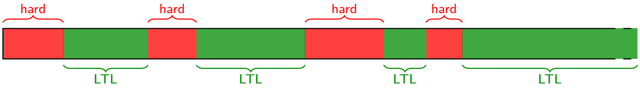

where - the number of hard fragments is at most exponential
- the total duration of hard fragments is at most exponential

- hard fragment = sliding window algorithm
- LTL fragment = finite automaton computation

The ultimate fragment?

- MITL and coFlat-$\mathsf{MTL}_{\mathsf{LTL}}$ have ‘low’ complexities for pretty different reasons...

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$$\text{coFlat-MTL}_{\mathrm{MITL}} \ni \varphi \ ::= \ a \mid \neg a \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \varphi\, \mathbf{U}_I\, \psi \mid \psi\, \widetilde{\mathbf{U}}_I\, \varphi$$

where I unbounded $\Rightarrow \psi \in$ MITL.

[BMOW08] Bouyer, Markey, Ouaknine, Worrell. On Expressiveness and Complexity in Real-time Model Checking (ICALP'08).

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- Examples:
  - $\mathbf{F}\,\mathbf{G}_{\leqslant 1} \bullet$ is in coFlat-MTL_{MITL}
  - $\mathbf{F}\,\mathbf{G}_{=1} \bullet$ is not in coFlat-MTL_{MITL}
  - coFlat-MTL_{MITL} generalizes both logics MITL and coFlat-MTL_{LTL}

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Model-checking coFlat-$\mathsf{MTL}_{\mathsf{MITL}}$

Theorem

The model-checking problem for coFlat-$\mathsf{MTL}_{\mathsf{MITL}}$ is EXPSPACE-complete [BMOW08], regardless of the encoding of constants in unary or in binary.

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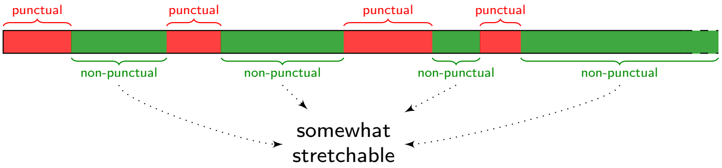

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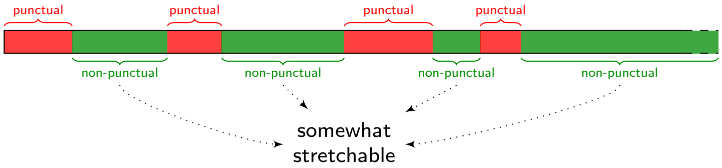
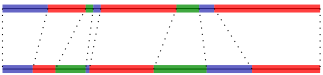


Stretchable signal:



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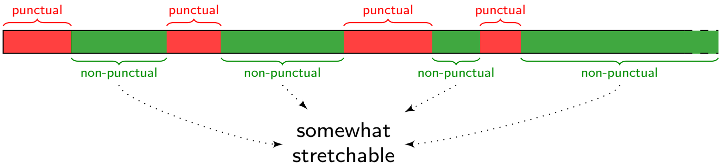

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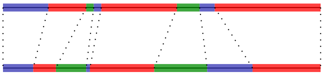

- Any model of an LTL formula is stretchable.

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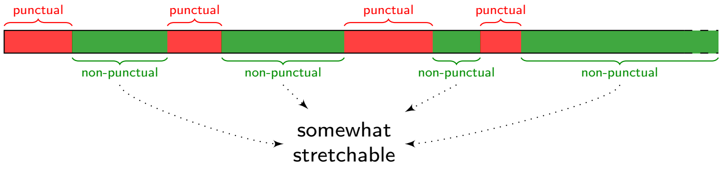

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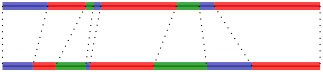

- Any model of an LTL formula is stretchable.
- Any model of an MITL formula is *somewhat* stretchable.

- non-punctual part: somewhat stretchable
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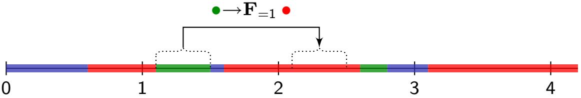

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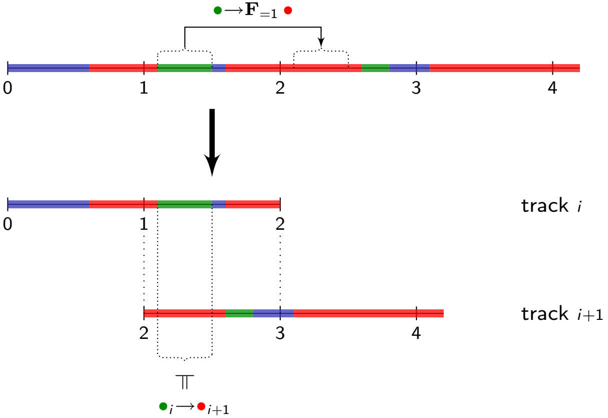
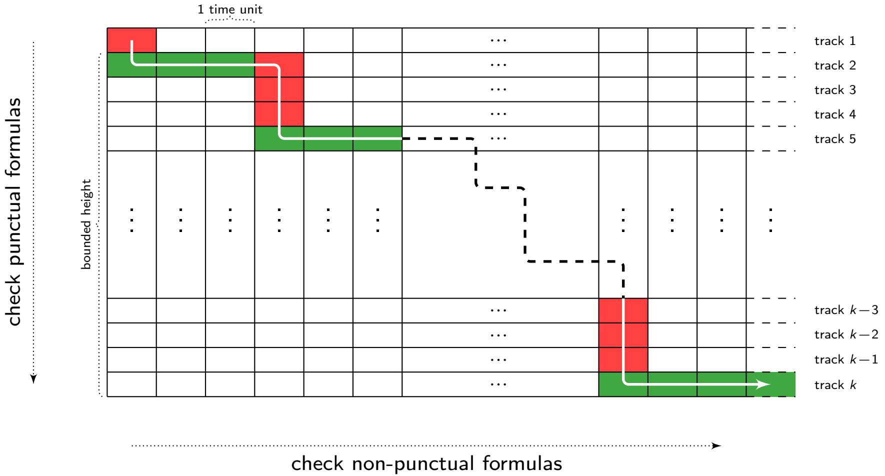


A tableau satisfiability problem



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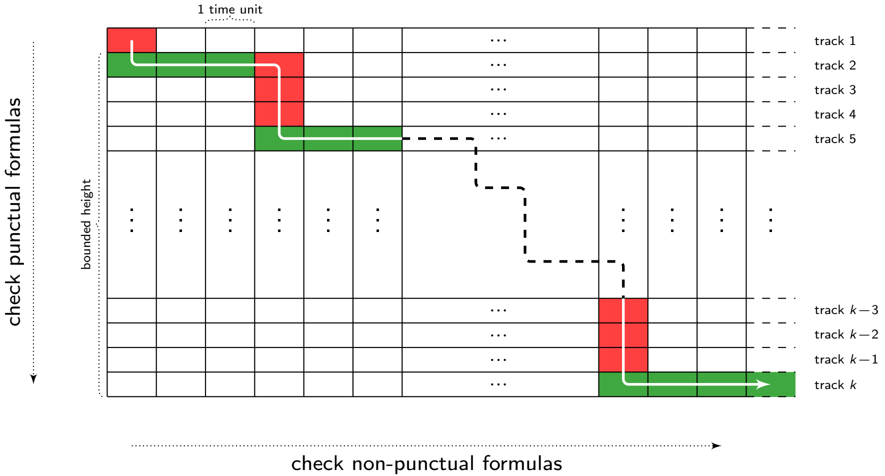
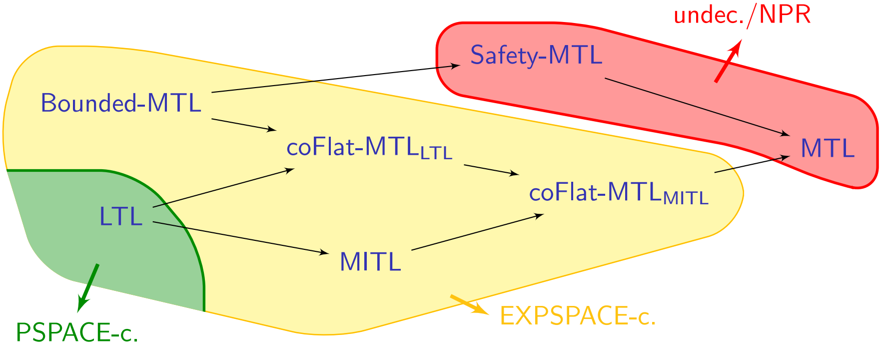


☞ transform into sat. prob. for LTL+Past over $\mathbb{R}_+$ (PSPACE: [Rey04])

[Rey04] Reynolds. The complexity of the temporal logic over the reals (Subm.'04).

The end of the quest for tractable fragments of MTL?



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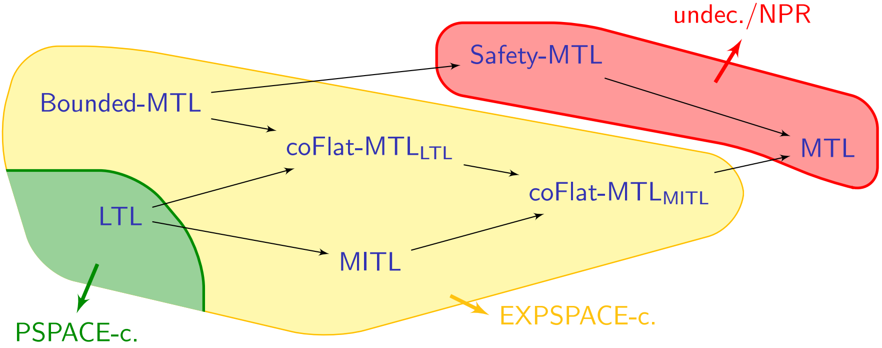

+ add Positive-MTL [PW09]

[PW09] Parys, Walukiewicz. Weak Alternating Timed Automata *(ICALP'09)*.

Outline

Conclusion

- Recent advances have raised a new interest for linear-time timed temporal logics
    - Not everything is undecidable
    - Some rather ‘efficient’ subclasses
        - non-punctual formulas
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Conclusion

- Recent advances have raised a new interest for linear-time timed temporal logics
  - Not everything is undecidable
  - Some rather 'efficient' subclasses
    - non-punctual formulas
    - structurally (co-)flat formulas
- No real data structures do exist for these logics.

Going further with quantities...

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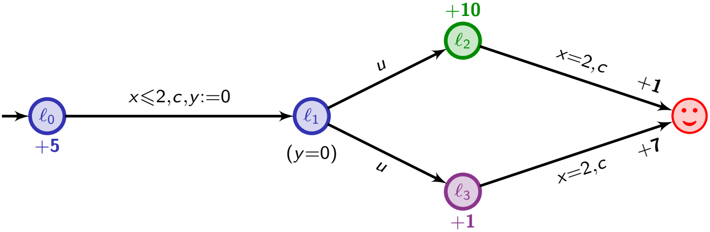
  ... but this is a very active field of research!

Weighted/priced timed automata [ALP01,BFH+01]



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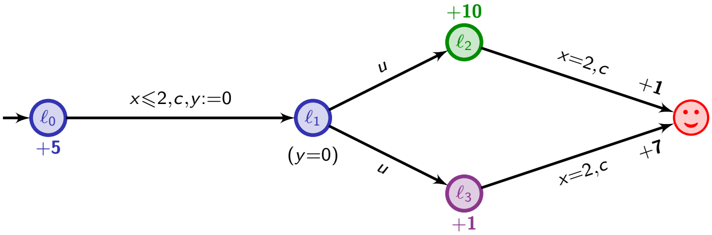

	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
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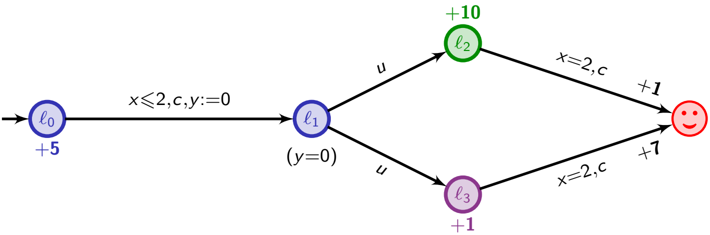

$$\begin{array}{ccccccccccccc} & \ell_0 & \xrightarrow{1.3} & \ell_0 & \xrightarrow{c} & \ell_1 & \xrightarrow{u} & \ell_3 & \xrightarrow{0.7} & \ell_3 & \xrightarrow{c} & ☺ \\ x & 0 & & 1.3 & & 1.3 & & 1.3 & & 2 & & \\ y & 0 & & 1.3 & & 0 & & 0 & & 0.7 & & \end{array}$$

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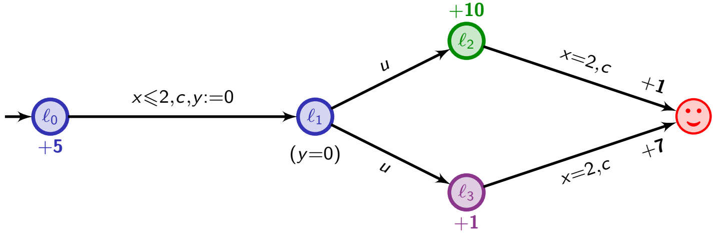

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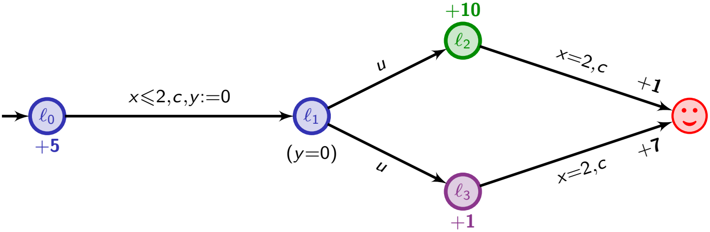

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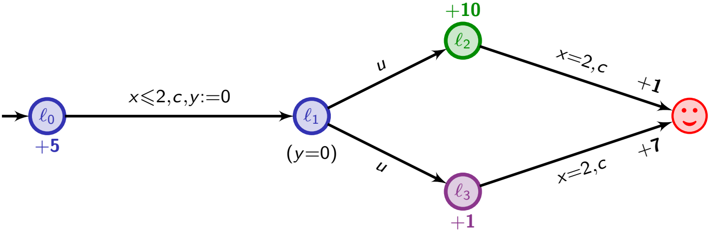

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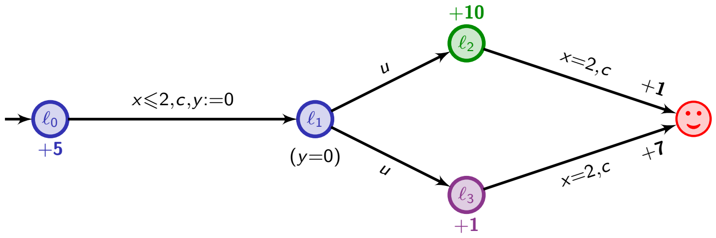

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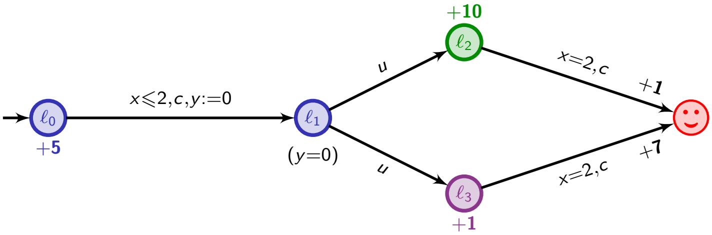

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cost :		6.5	+	0	+	0	+	0.7	+	7	

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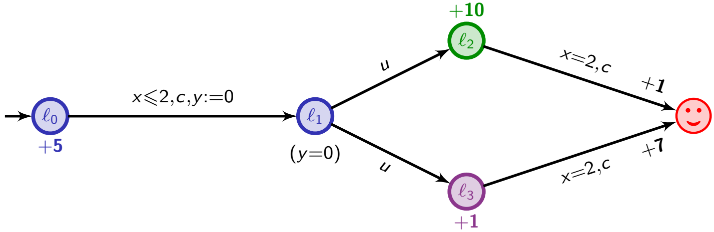

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cost : $6.5 + 0 + 0 + 0.7 + 7 = 14.2$

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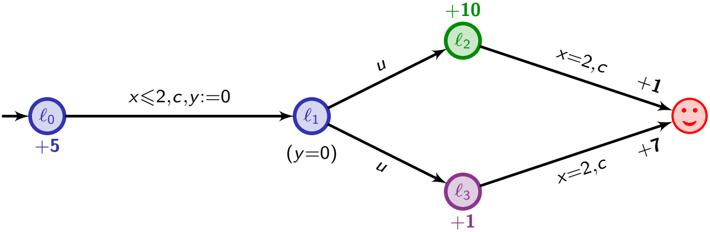

Question: what is the optimal cost for reaching 🙂 ?

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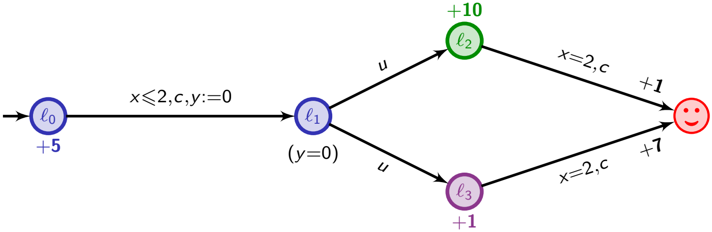

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$$5t + 10(2 - t) + 1$$

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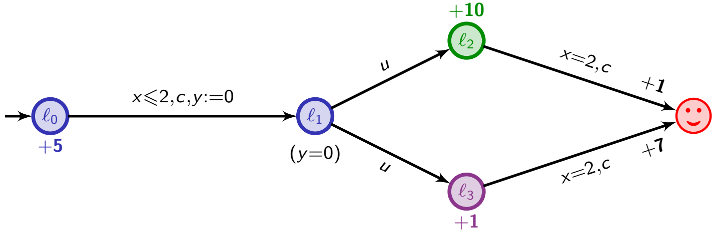

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$$5t + 10(2 - t) + 1 \;,\; 5t + (2 - t) + 7$$

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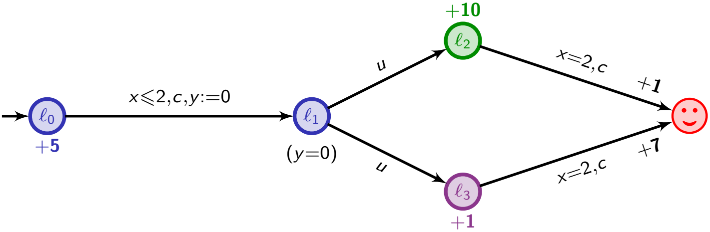

Question: what is the optimal cost for reaching ?

$$\min\left(\ 5t+10(2-t)+1\ ,\ 5t+(2-t)+7\ \right)$$

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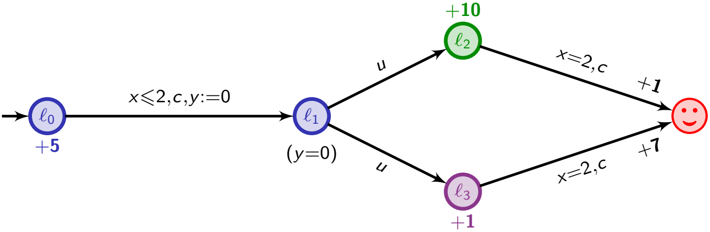

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$$\inf_{0\leqslant t\leqslant 2} \min\left(5t+10(2-t)+1\ ,\ 5t+(2-t)+7 \right) = 9$$

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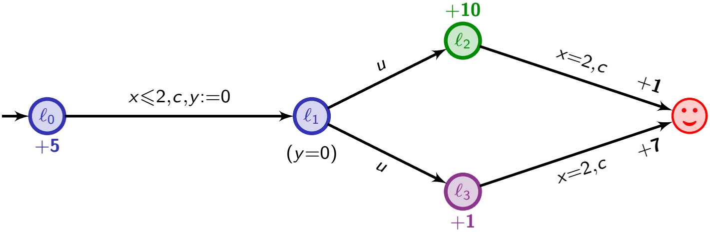

Question: what is the optimal cost for reaching 🙂 ?

$$\inf_{0 \leqslant t \leqslant 2} \min \left(5t + 10(2-t) + 1 \, , \, 5t + (2-t) + 7 \right) = 9$$

⇝ *strategy:* leave immediately ℓ_0, go to ℓ_3, and wait there 2 t.u.

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