Real-time Model Checking

— Priced timed automata —

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March 3, 2010

Time is not always sufficient

Timed automata are (rather) well understood – Can we go further?

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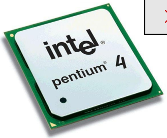
Compute $D\times(C\times(A+B))+(A+B)+(C\times D)$ using two processors:

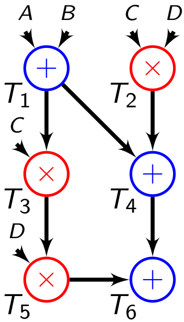
P_1 (fast):

	time
$+$	2 picosec.
$\times$	3 picosec.



P_2 (slow):

	time
$+$	5 picosec.
$\times$	7 picosec.



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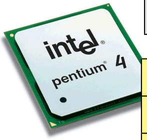

time	
$+$	2 picosec.
$\times$	3 picosec.

energy	
idle	10 W
in use	90 W

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energy	
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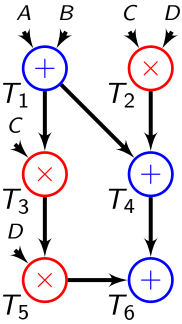

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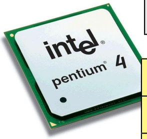

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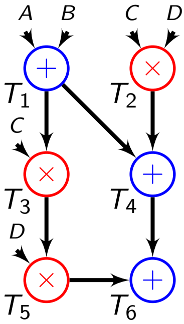

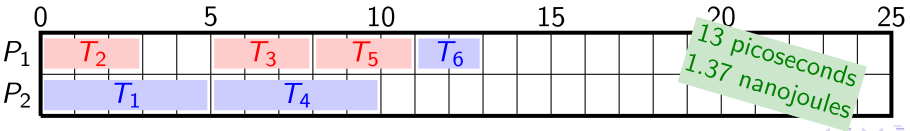

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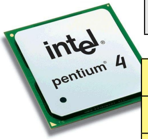

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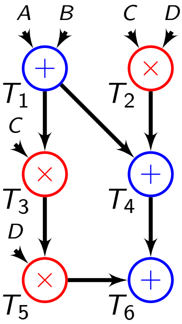

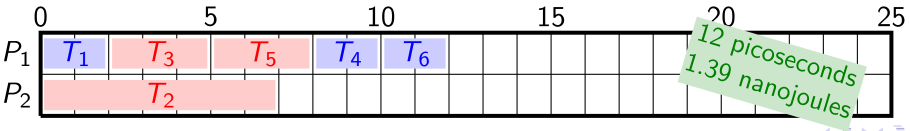

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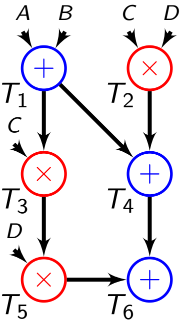
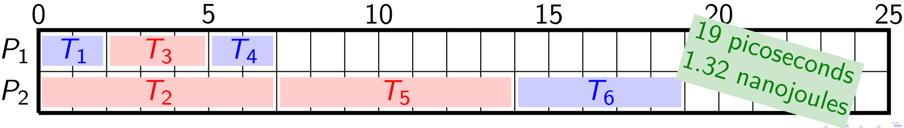
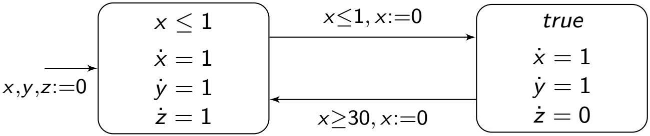




Time is not always sufficient

- hybrid automata: timed automata augmented with variables whose derivative is not constant.

$\leadsto$ examples: leaking gas burner, water-level monitor, ...



Theorem

Reachability is undecidable (even for timed automata with one stopwatch).

Refs: [1] Henzinger, Kopke, Puri, Varaiya. *What's Decidable about Hybrid Automata?* (1995).

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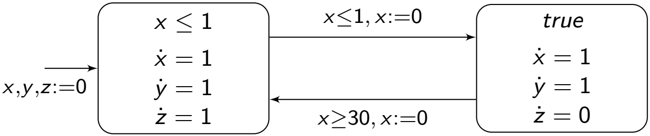

- timed automata with observers: similar to hybrid automata, but the behavior only depends on clock variables.

Refs: [1] Alur, La Torre, Pappas. *Optimal Paths in Weighted Timed Automata* (2001).
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Outline of the talk

1. Introduction
2. Timed automata with observers
3. Resource-optimization problems
   - Optimal reachabililty
   - Weighted temporal logics
   - Optimal strategies
4. Resource-management problems
5. Conclusions and perspectives

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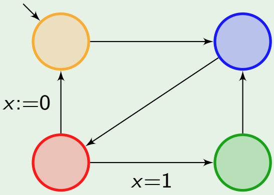
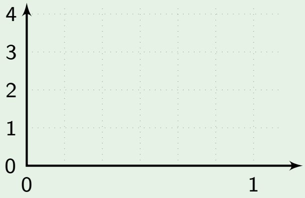
Timed automata with (linear) observers

Example





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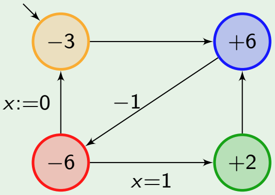

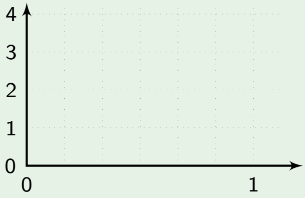

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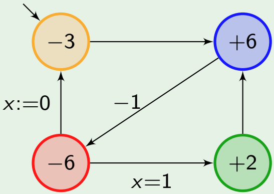

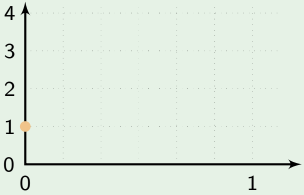

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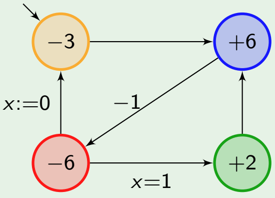

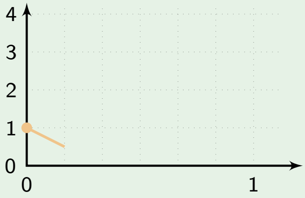

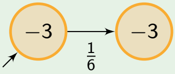

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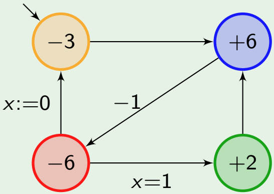

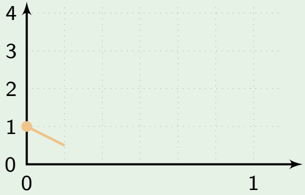

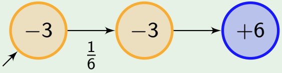

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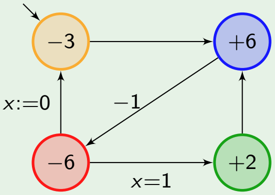

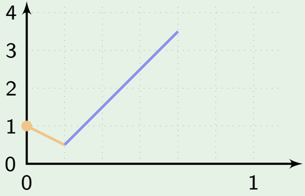

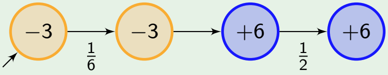

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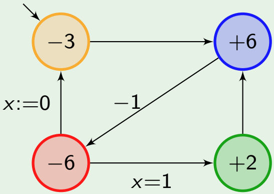

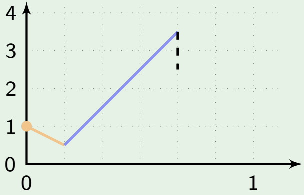

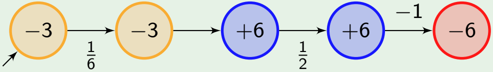

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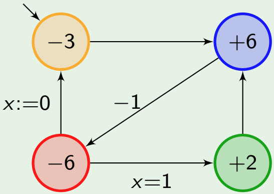

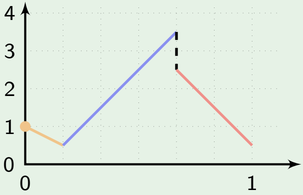

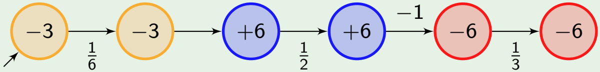

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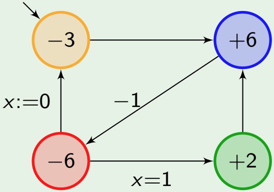

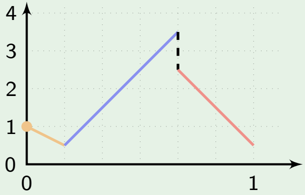

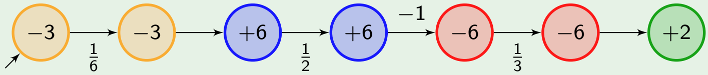

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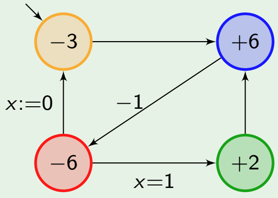

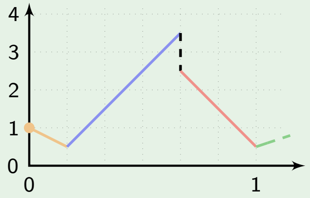

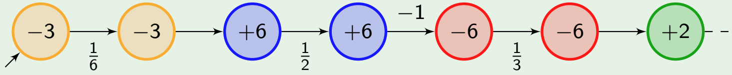

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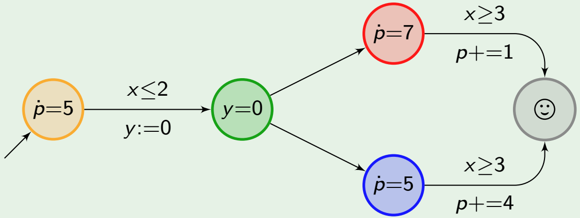

Optimal reachability

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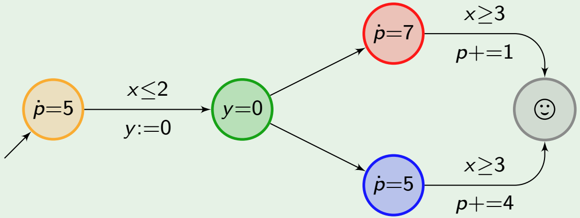
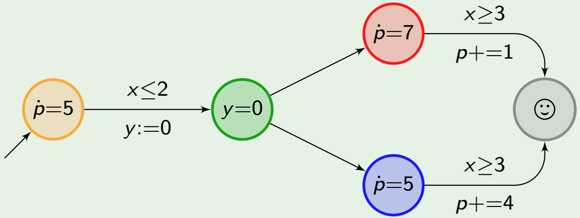


Minimal cost for reaching ☺:

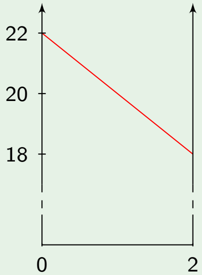
Optimal reachability

Example



Minimal cost for reaching ☺:

$$5t + 7(3-t) + 1$$



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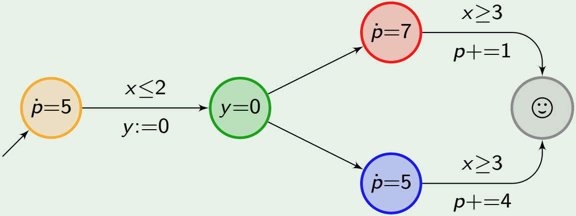
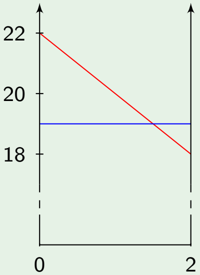


Minimal cost for reaching ☺:

$$5t + 7(3 - t) + 1$$
$$5t + 5(3 - t) + 4$$



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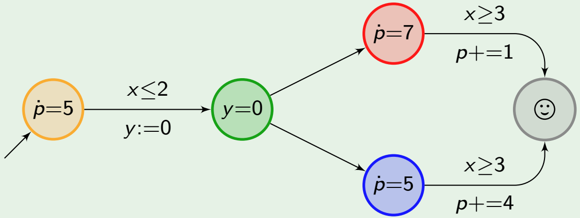
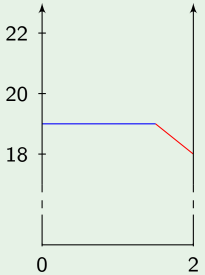


Minimal cost for reaching ☺:

$$\min \begin{pmatrix} 5t + 7(3-t) + 1 \\ 5t + 5(3-t) + 4 \end{pmatrix}$$



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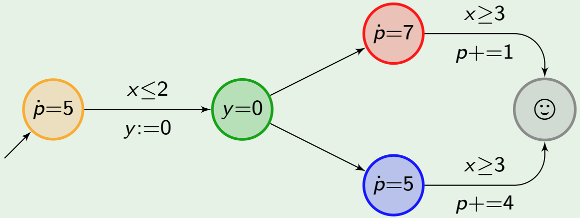
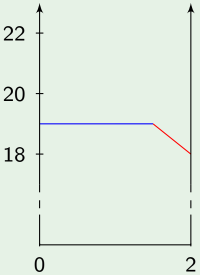


Minimal cost for reaching ☺:

$$\inf_{0 \le t \le 2} \min \begin{pmatrix} 5t + 7(3-t) + 1 \\ 5t + 5(3-t) + 4 \end{pmatrix}$$



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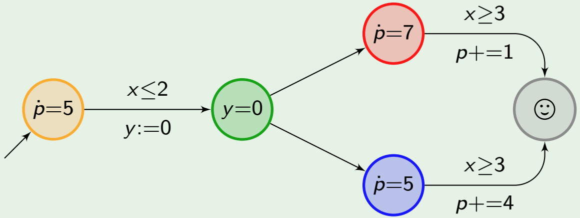
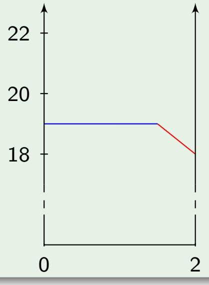


Minimal cost for reaching ☺:

$$\inf_{0\leq t\leq 2} \min \begin{pmatrix} 5t+7(3-t)+1 \\ 5t+5(3-t)+4 \end{pmatrix} = 18$$



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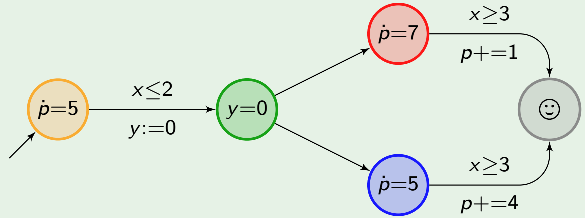

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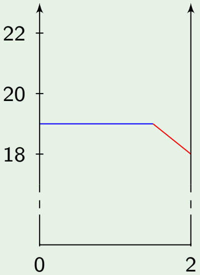
$$\inf_{0\leq t\leq 2} \min \begin{pmatrix} 5t + 7(3-t) + 1 \\ 5t + 5(3-t) + 4 \end{pmatrix} = 18$$

The optimal *schedule* consists in

- waiting 2 time units in ◯;
- going through ◯.



Optimal reachability

Theorem

Optimal reachability in priced timed automata is PSPACE-complete.

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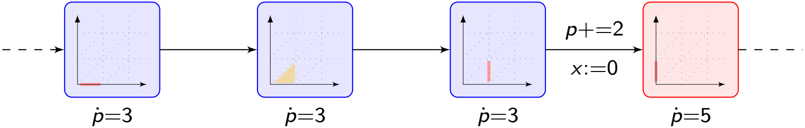
- The region abstraction is not fine enough:



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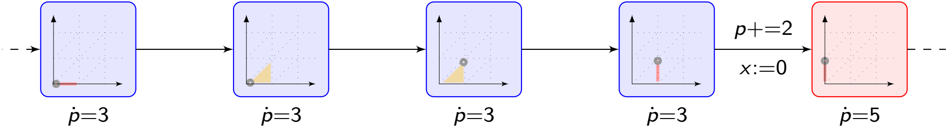

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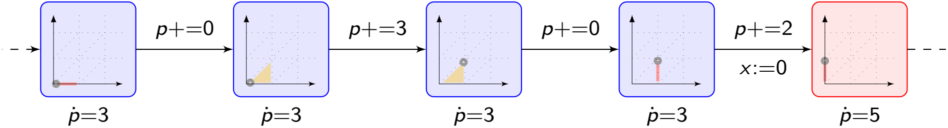

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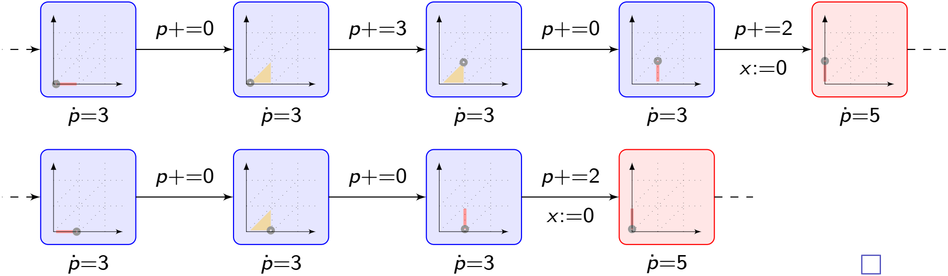

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2. Timed automata with observers
3. Resource-optimization problems
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Weighted temporal logic

Example

Decorate temporal modalities with constraints on cost:

Weighted temporal logic

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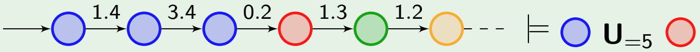

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Weighted temporal logic

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Weighted temporal logic

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Undecidability results

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WMTL model-checking is undecidable.

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Undecidability results

Theorem

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Proof.

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Theorem

WCTL model-checking is undecidable.

Proof.

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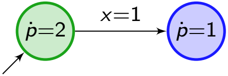

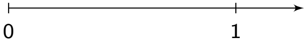

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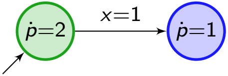

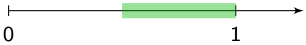

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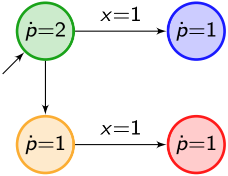

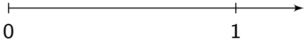

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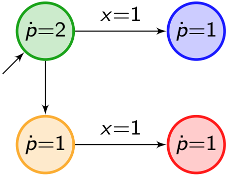

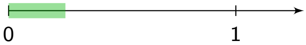

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Decidable subcases

Theorem

WCTL model-checking is PSPACE-complete on 1-clock weighted timed automata.

Proof.

- region-based algorithm;
- but region are not fine enough:
- Refine regions: granularity $1/M^{|\varphi|}$ is sufficient.

□

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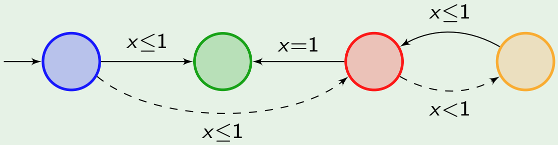
Weighted timed games

Example

Timed games can also be extended with weights:



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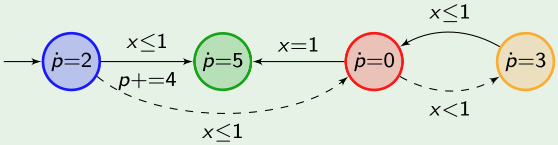

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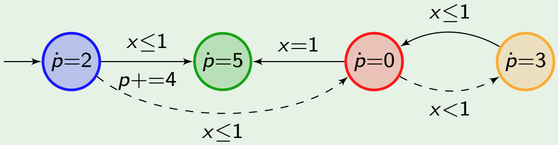
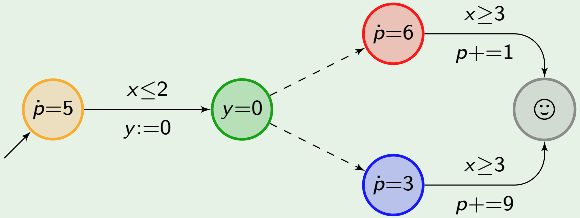


- A strategy for a player indicates which (action or delay) transition to play;
- A strategy is winning if all its outcomes are.

Optimal winning strategy

Example



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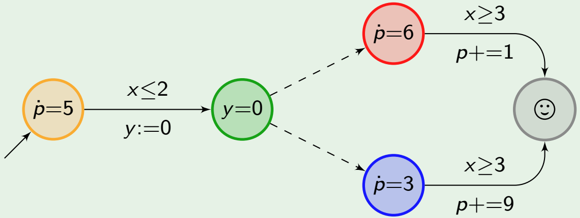

Minimal cost for reaching ☺:

Optimal winning strategy

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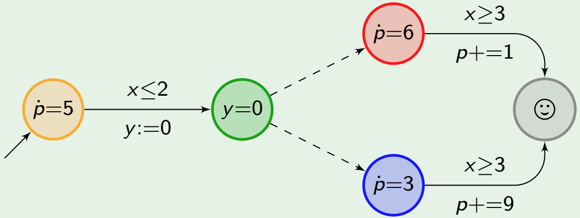
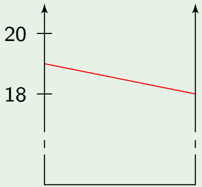


Minimal cost for reaching ☺:

$$5t + 6(3 - t) + 1$$



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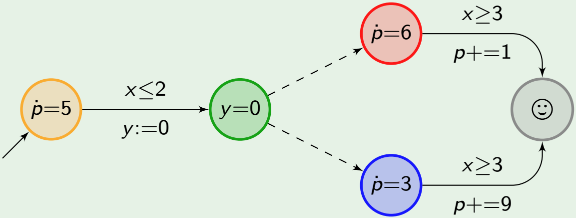
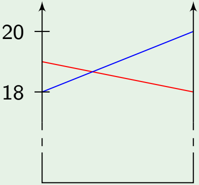


Minimal cost for reaching ☺:

$$5t + 6(3-t) + 1$$
$$5t + 3(3-t) + 9$$



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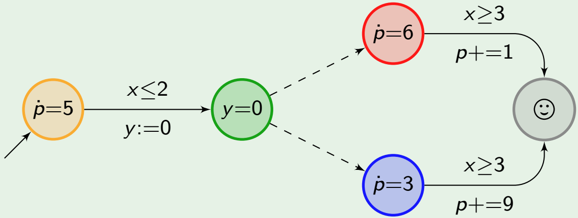
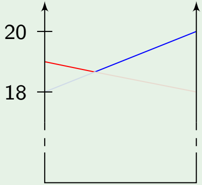


Minimal cost for reaching ☺:

$$\max \begin{pmatrix} 5t + 6(3-t) + 1 \\ 5t + 3(3-t) + 9 \end{pmatrix}$$



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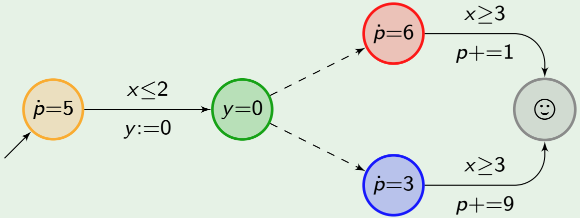
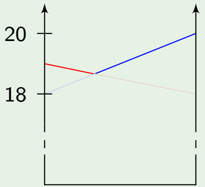


Minimal cost for reaching ☺:

$$\inf_{0 \le t \le 2} \max \begin{pmatrix} 5t + 6(3-t) + 1 \\ 5t + 3(3-t) + 9 \end{pmatrix}$$



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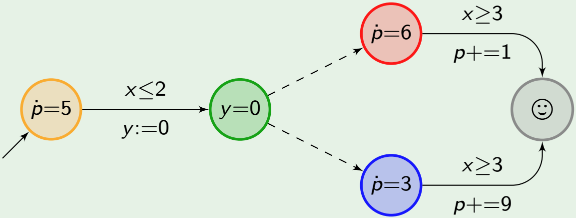

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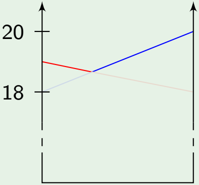

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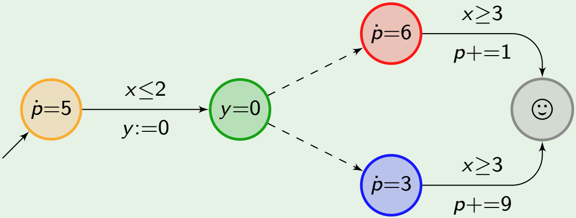

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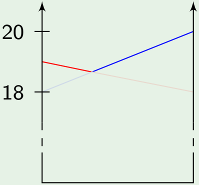

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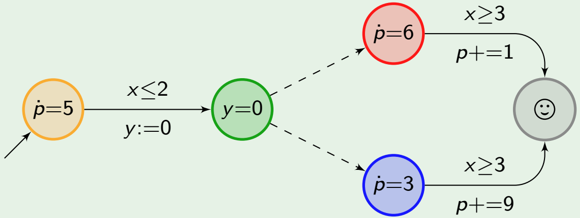

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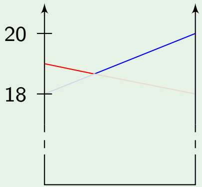

Corollary

Regions are not sufficient for solving priced timed games.

Computing optimal winning strategies is undecidable

Theorem

Computing optimal strategies in priced timed games is undecidable.

Refs: [1] Bouyer, Brihaye, M. *Improved Undecidability Results on Weighted Timed Automata* (2006).

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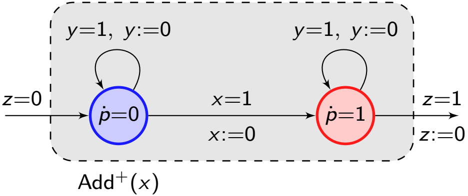

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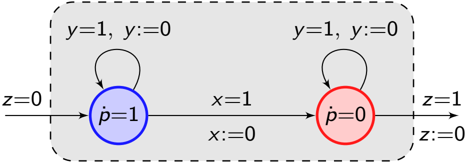
- Adding $1 - x$ to the cost:



$\mathrm{Add}^{-}(x)$

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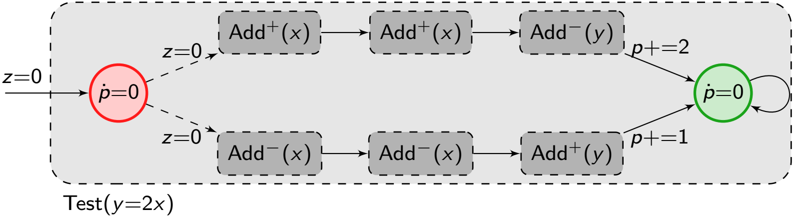

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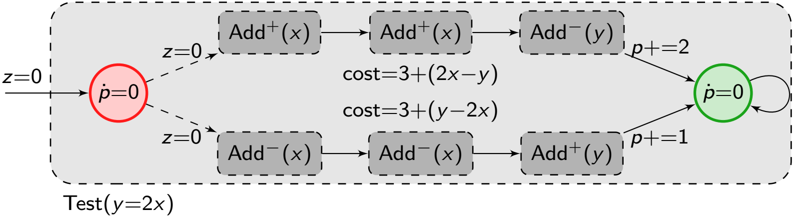

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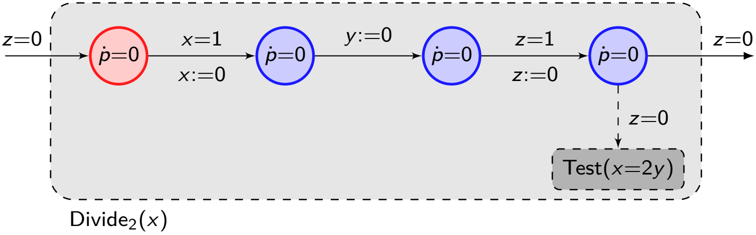

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- encode counter c_1 as $x_1 = 2^{-c_1}$ and counter c_2 as $x_2 = 3^{-c_1}$;
- by cleverly juggling with clocks, we can achieve this encoding with three clocks.

□

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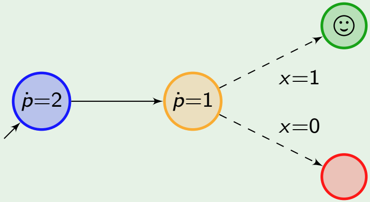
Turn-based 1-clock priced timed games are decidable

Example

- Optimal strategies do not always exist:



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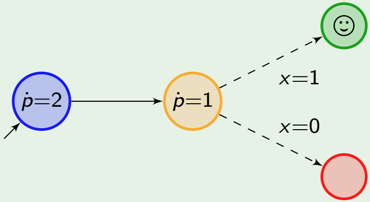
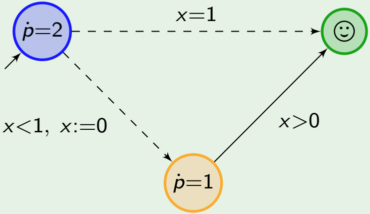


- Optimal strategies may require memory:



Turn-based 1-clock priced timed games are decidable

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Turn-based 1-clock priced timed games always admit ε-optimal winning strategies, and such strategies can be computed.

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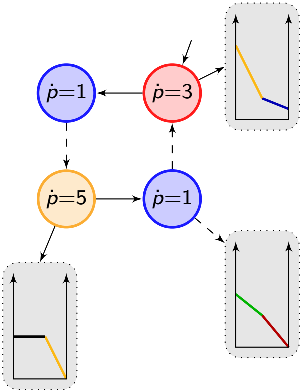

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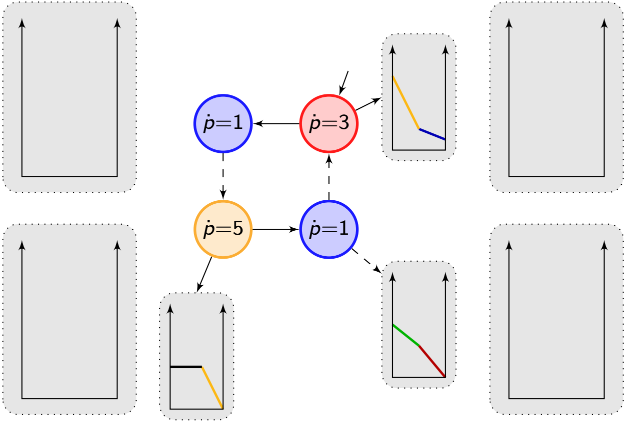

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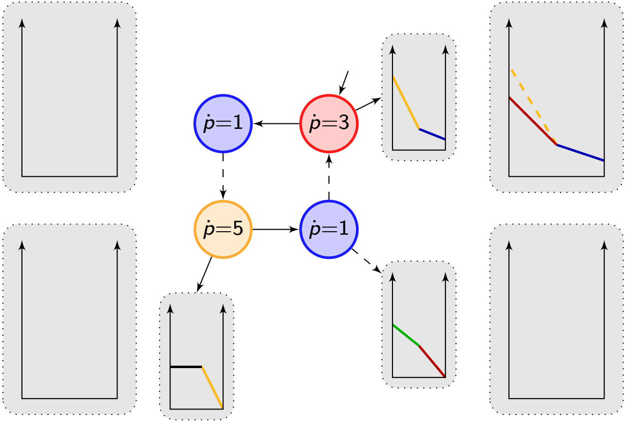

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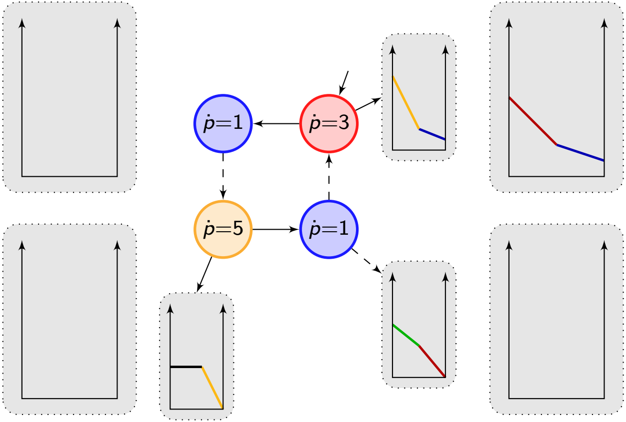

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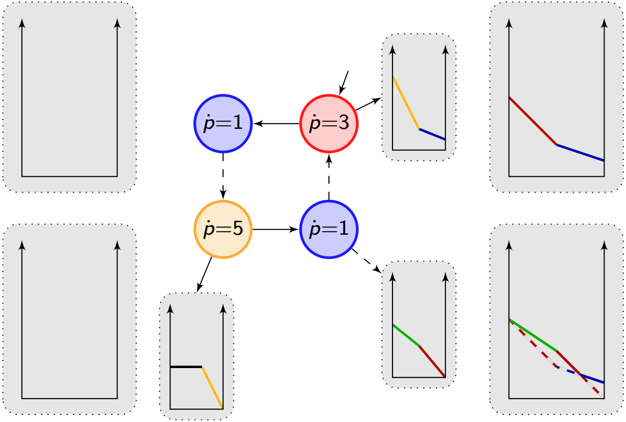

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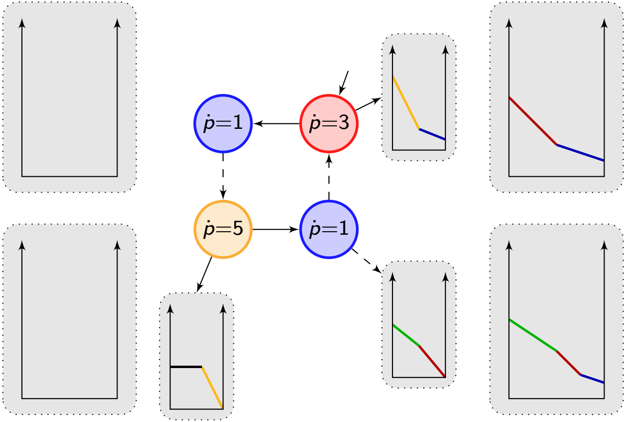

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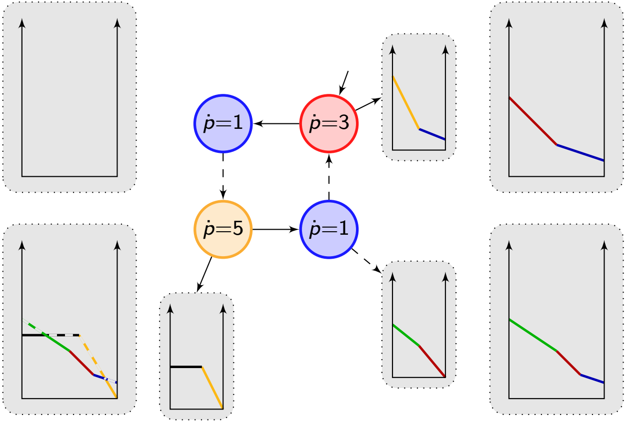

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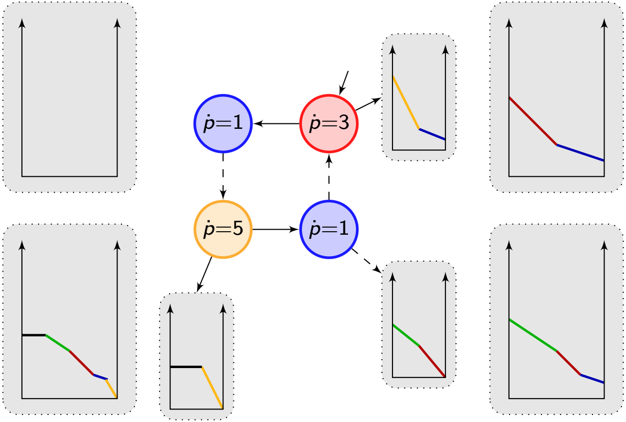

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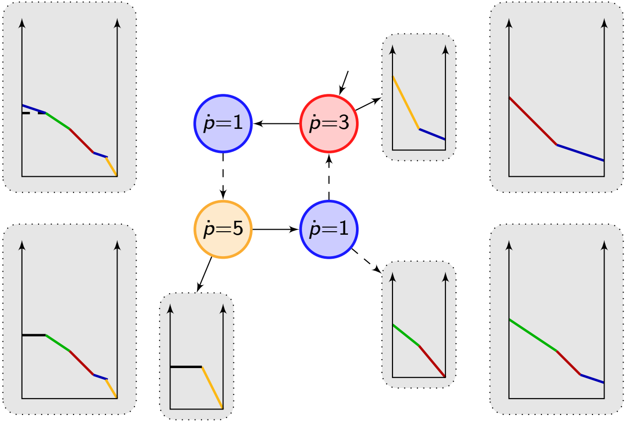

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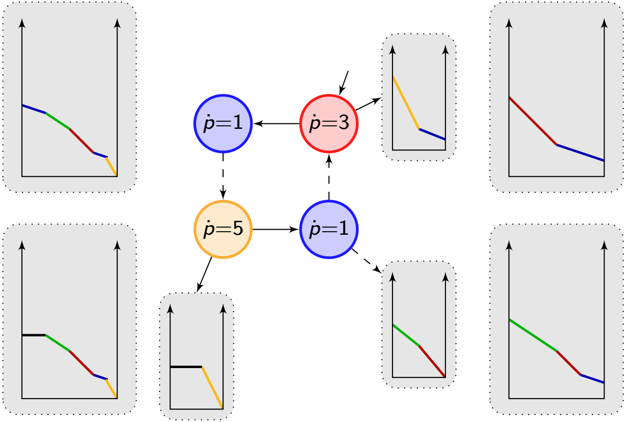

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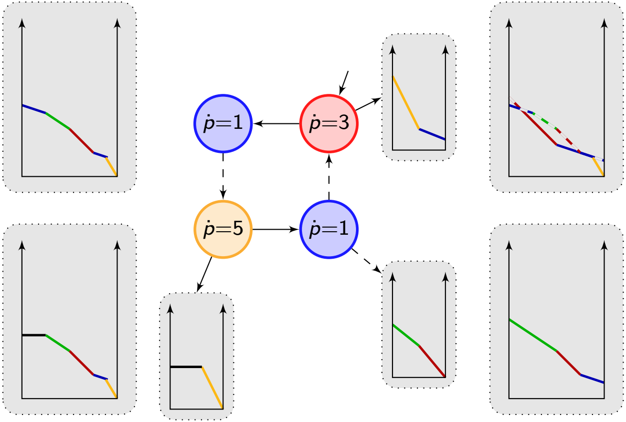

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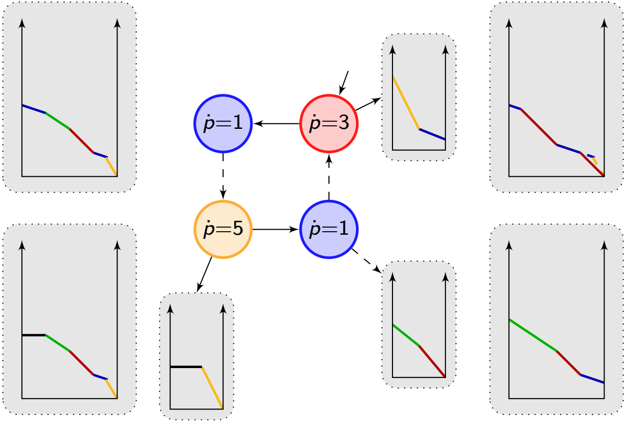

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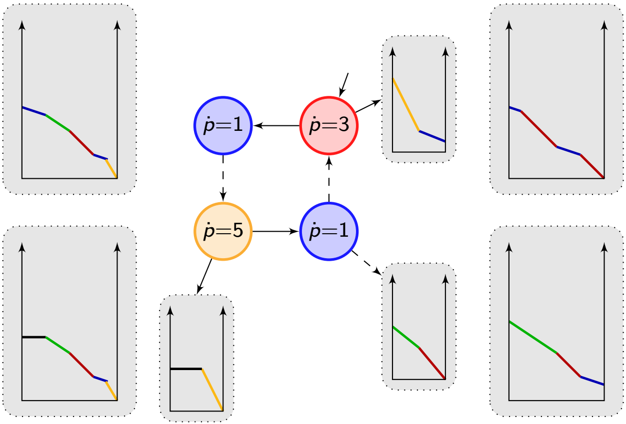

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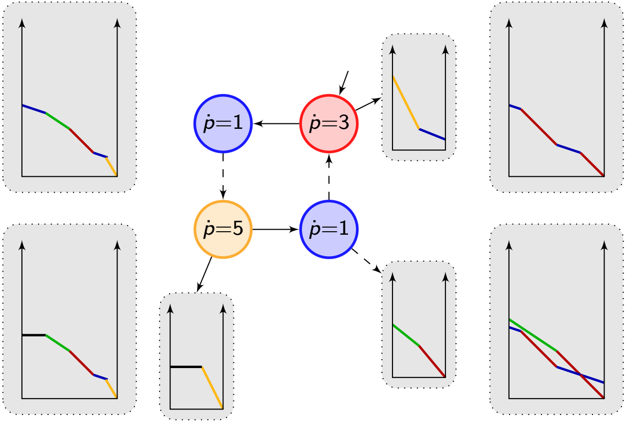

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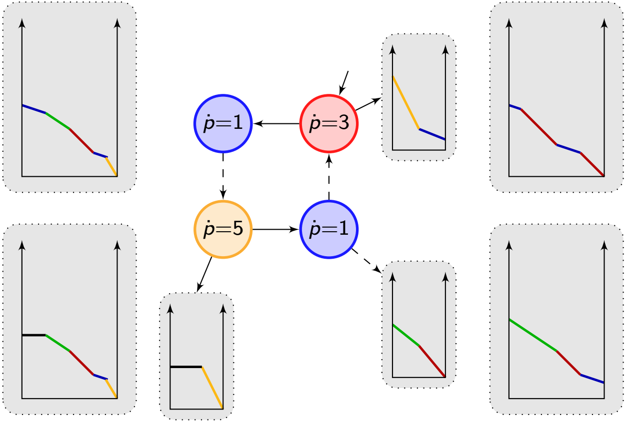

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Proof.

- The procedure terminates;
- There is a positive granularity for with the region abstraction is correct;
- The optimal cost functions are piecewise affine, continuous, decreasing functions. Their slopes are rates of the automaton.

□

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Outline of the talk

1. Introduction

2. Timed automata with observers

3. Resource-optimization problems
   - Optimal reachabililty
   - Weighted temporal logics
   - Optimal strategies

4. Resource-management problems

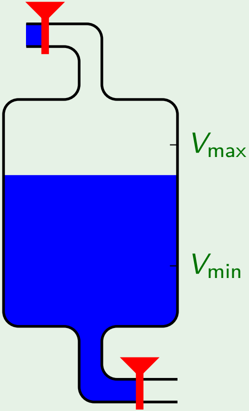
5. Conclusions and perspectives

Managing resources

Example

In some cases, resources can both be consumed and regained.

The aim is then to keep the level of resources within given bounds.



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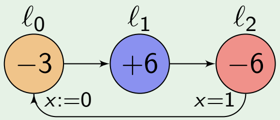

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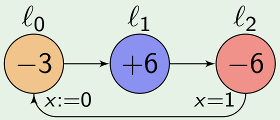

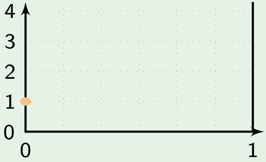

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Managing resources

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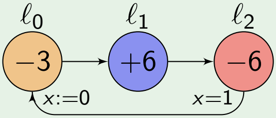

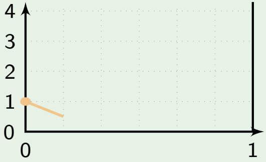

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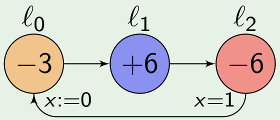

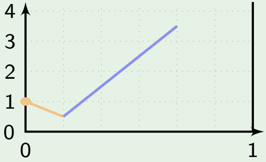

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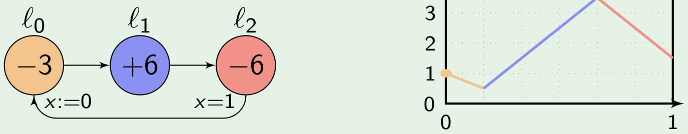

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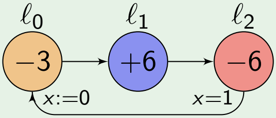

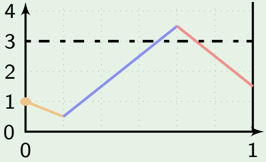

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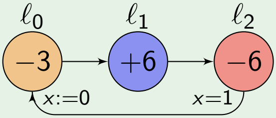

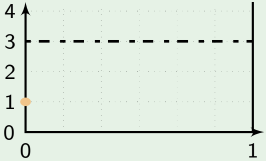

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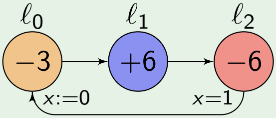

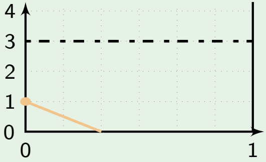

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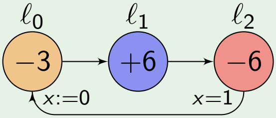

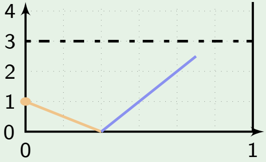

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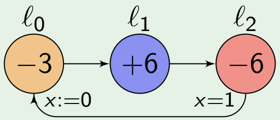

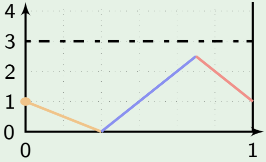

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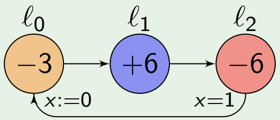

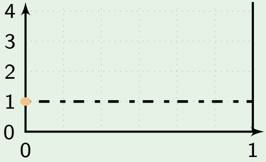

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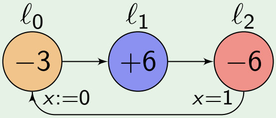

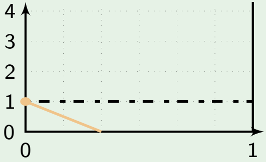

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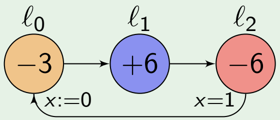

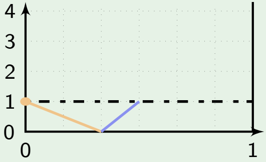

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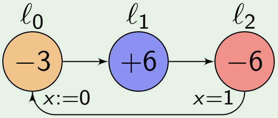

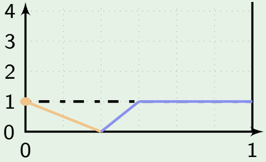

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Results in the untimed case

Theorem

In the untimed case, the following results hold:

	existential problem	***universal problem***	***games***
Lower bound	$\in$ PTIME	$\in$ PTIME	$\in$ UP $\cap$ coUP PTIME-*hard*
Lower bound, finite capacity	$\in$ PTIME	$\in$ PTIME	$\in$ NP PTIME-*hard*
Interval	$\in$ PSPACE NP-*hard*	$\in$ PTIME	EXPTIME-*c.*

Refs: [1] Bouyer, Fahrenberg, Larsen, M., Srba. *Infinite Runs in Weighted Timed Automata with Energy Constraints* (2008).

Results in the 1-clock case

Theorem

In the 1-clock case, the existence of an infinite run with resource level above a given lower bound is decidable in EXPTIME.

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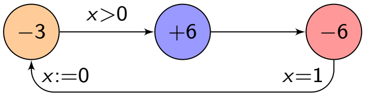
- Corner-point abstraction:



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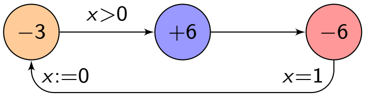

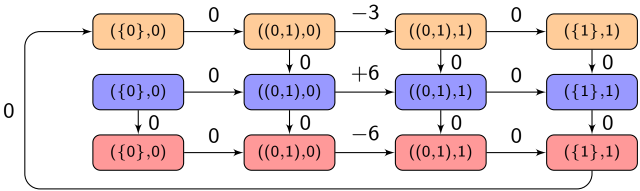

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Results in the 1-clock case

Theorem

In the 1-clock case, the existence of an infinite run with resource level above a given lower bound is decidable in EXPTIME.

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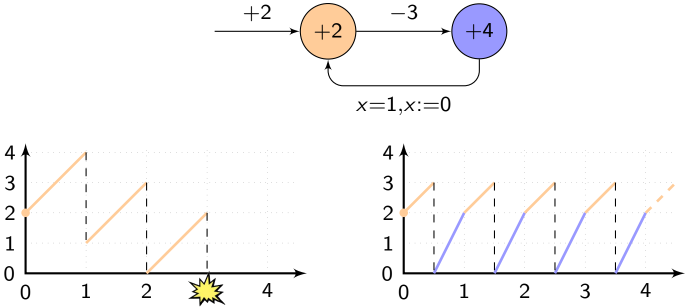

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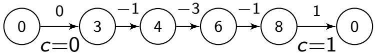

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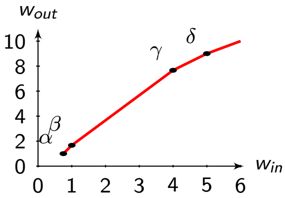

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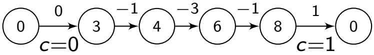

  - compute optimal final resource-level along a non-resetting path;
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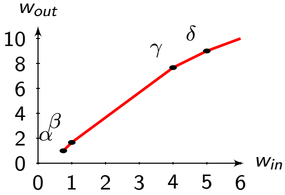

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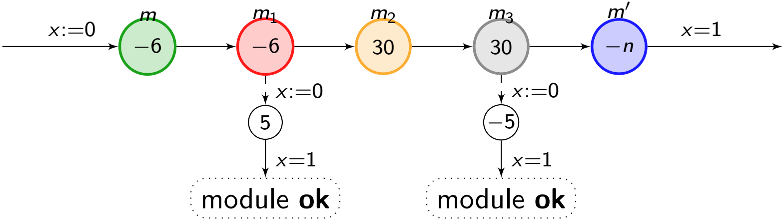

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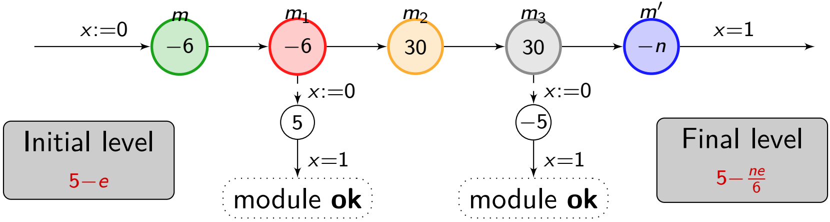

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Outline of the talk

1. Introduction
2. Timed automata with observers
3. Resource-optimization problems
   - Optimal reachabililty
   - Weighted temporal logics
   - Optimal strategies
4. Resource-management problems
5. Conclusions and perspectives

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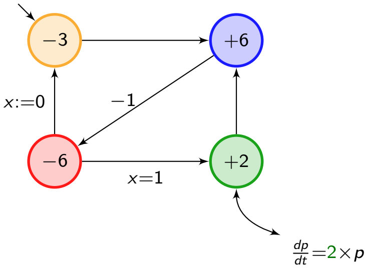

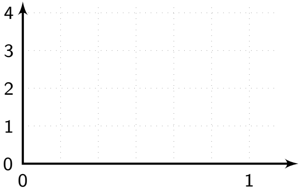

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