Timed games

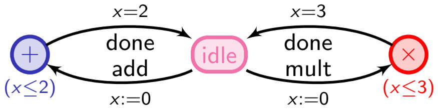
Patricia Bouyer-Decitre

LSV, CNRS & ENS Cachan, France

Why (timed) games?

- to model uncertainty

Example of a processor in the taskgraph example



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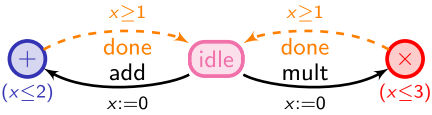

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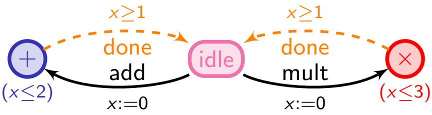
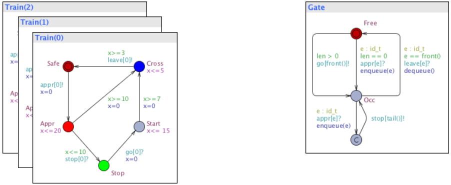


- to model an interaction with an environment

Example of the gate in the train/gate example



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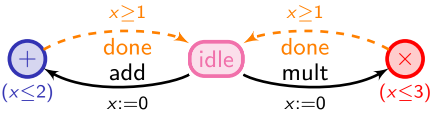

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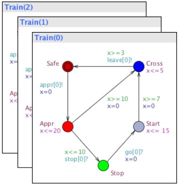

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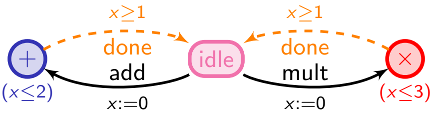

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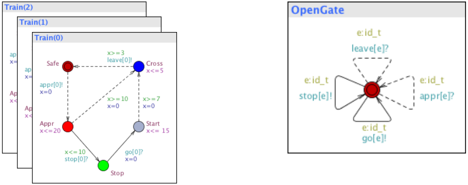

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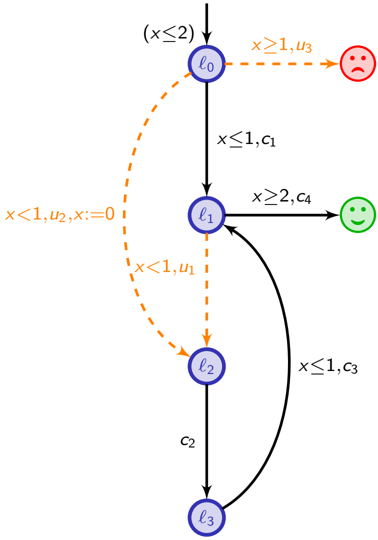

Rule of the game

- Aim: avoid ☹ and reach ☺

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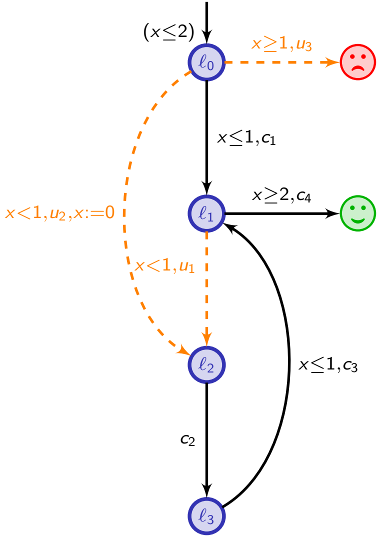

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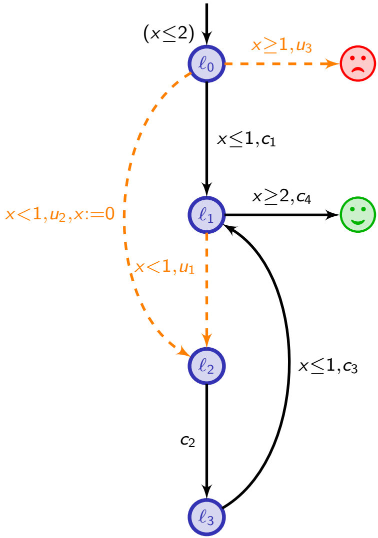

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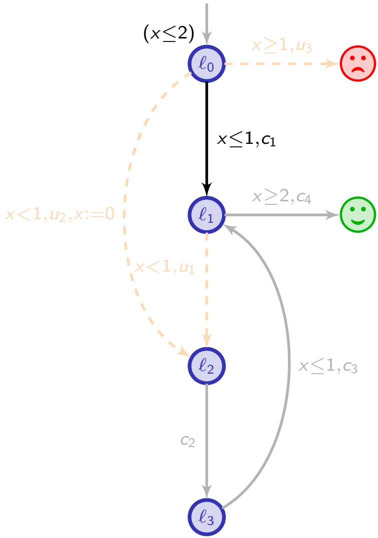

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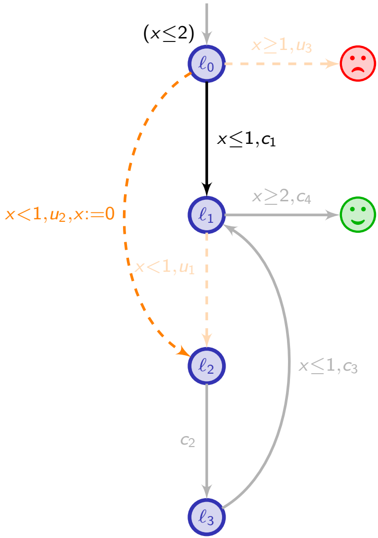

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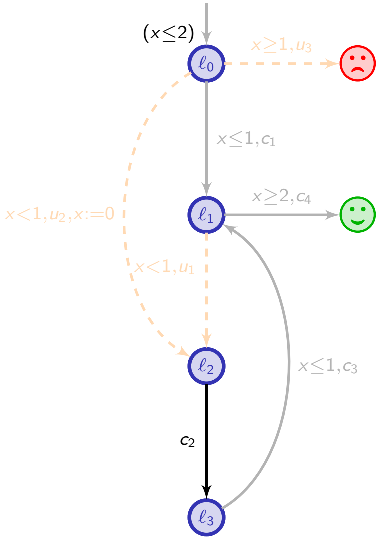

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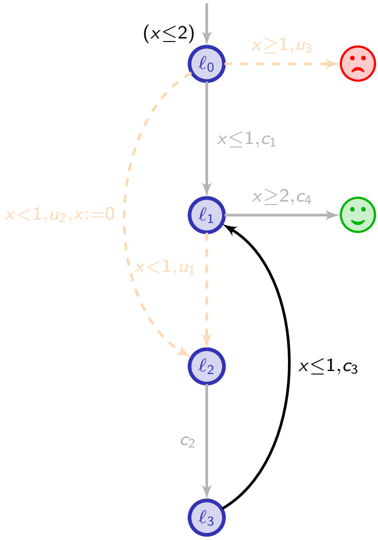

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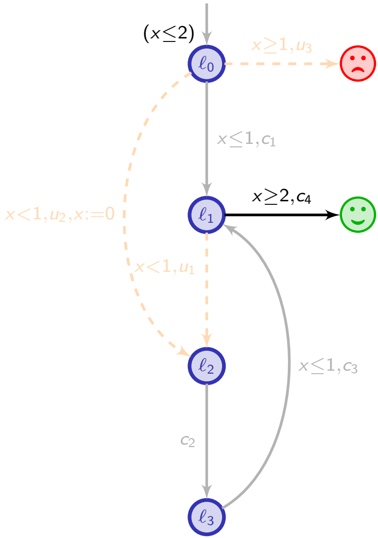

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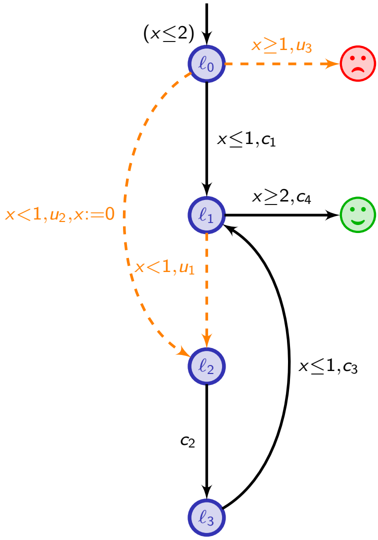

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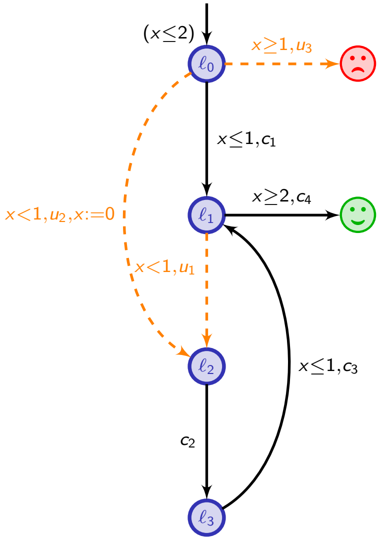

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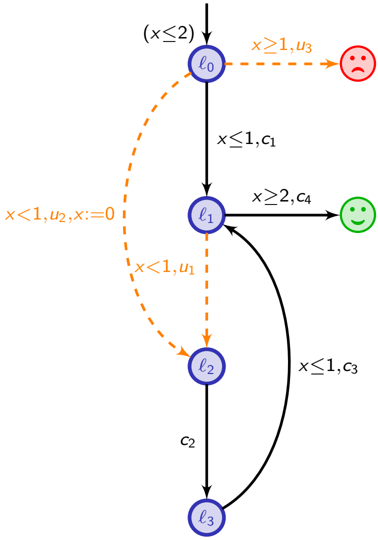

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- Does there exist a winning strategy?
- If yes, compute one (as simple as possible).

Decidability of timed games

Theorem [AMPS98,HK99]

Reachability and safety timed games are decidable and EXPTIME-complete. Furthermore memoryless and "region-based" strategies are sufficient.

[AMPS98] Asarin, Maler, Pnueli, Sifakis. Controller synthesis for timed automata *(SSC'98)*.
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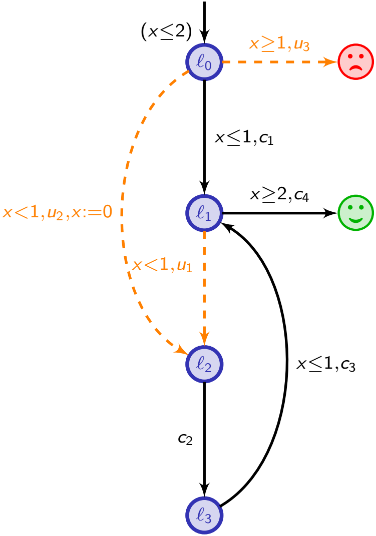
Theorem [AM99,BHPR07,JT07]

Optimal-time reachability timed games are decidable and EXPTIME-complete.

[AM99] Asarin, Maler. As soon as possible: time optimal control for timed automata *(HSCC'99)*.
[BHPR07] Brihaye, Henzinger, Prabhu, Raskin. Minimum-time reachability in timed games *(ICALP'07)*.
[JT07] Jurdziński, Trivedi. Reachability-time games on timed automata *(ICALP'07)*.

Back to the example: computing winning states



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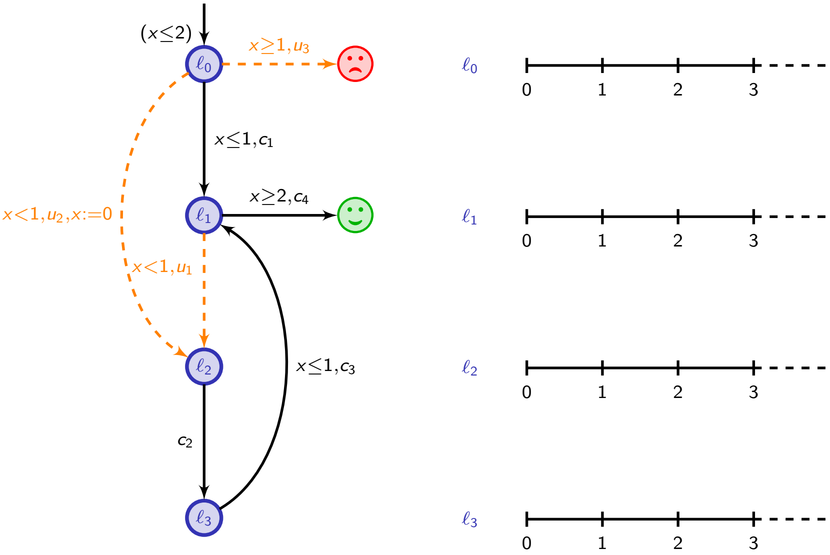

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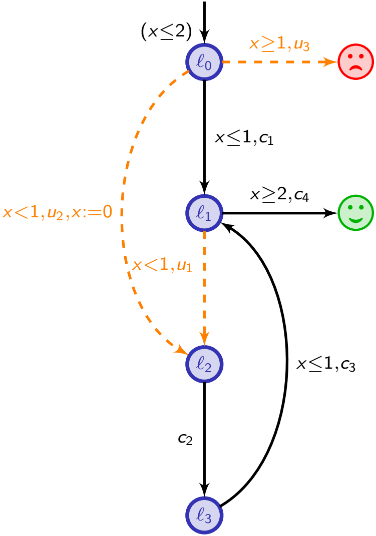

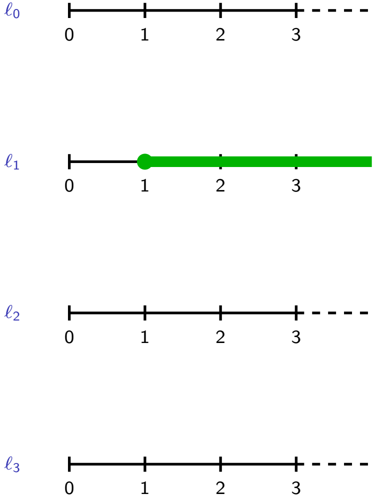

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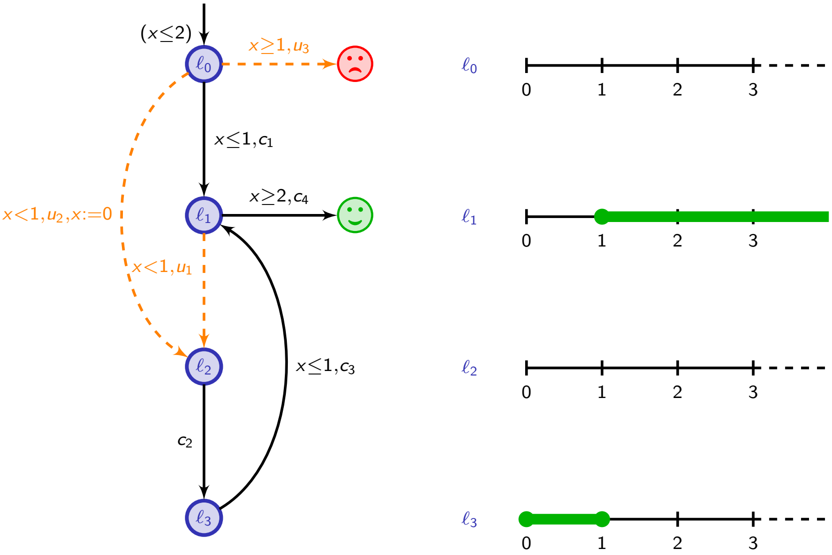

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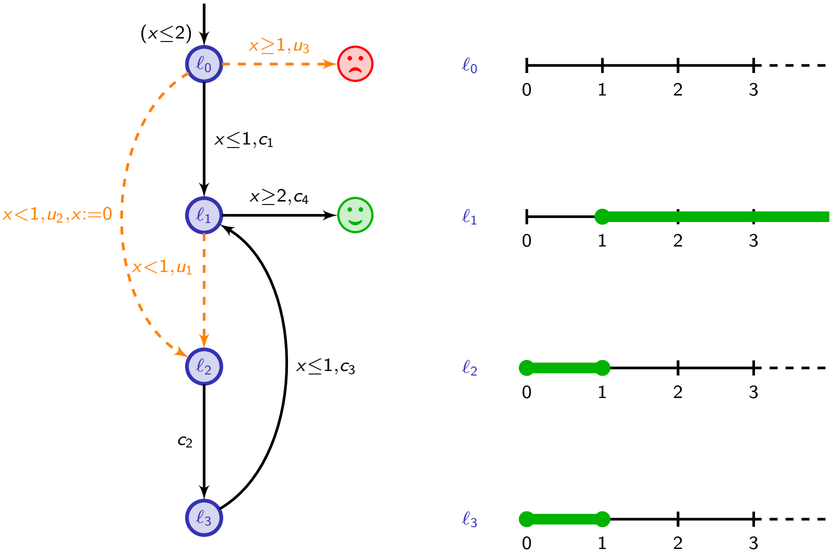

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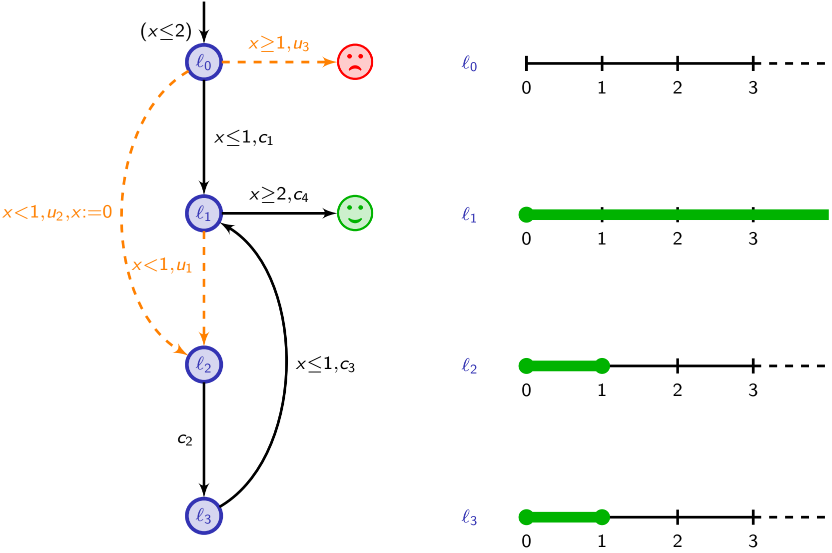

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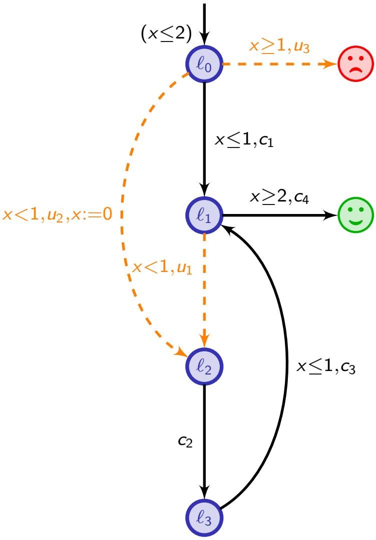

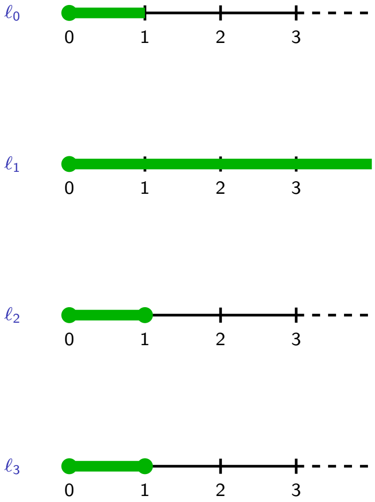

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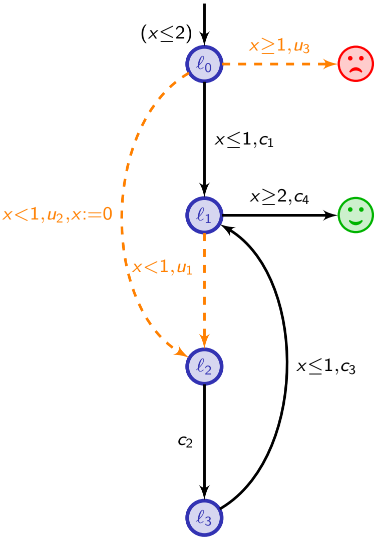

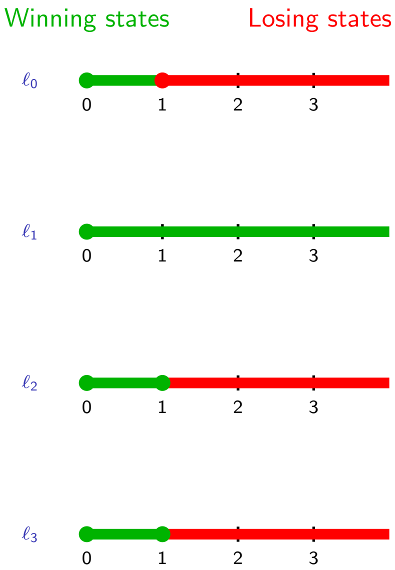

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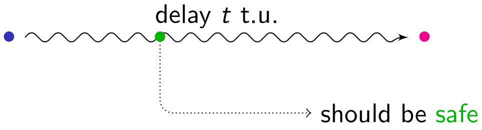

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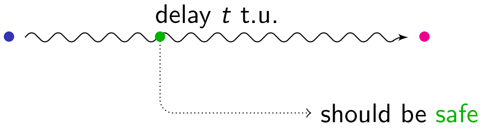

$$\mathsf{Pred}_\delta(X, \mathsf{Safe}) = \{\bullet \mid \exists t \geq 0,\ \bullet \xrightarrow{\delta(t)} \bullet$$

$$\text{and } \forall 0 \leq t' \leq t,\ \bullet \xrightarrow{\delta(t')} \bullet \in \mathsf{Safe}\}$$

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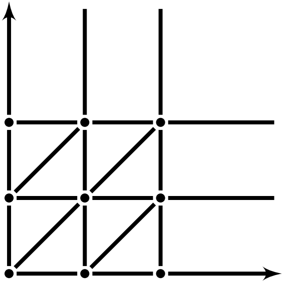

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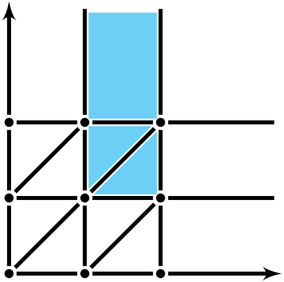

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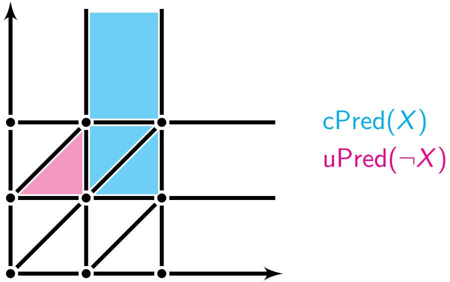

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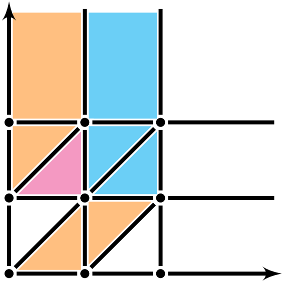

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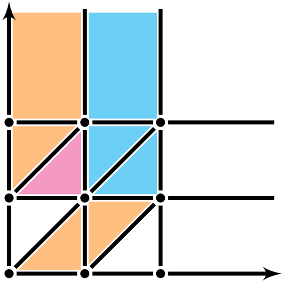

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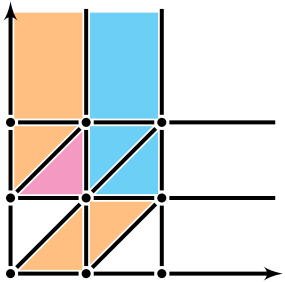

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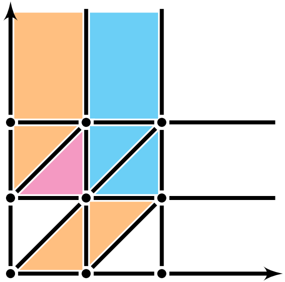

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$\rightsquigarrow$ the computation of π^*(☺) terminates!

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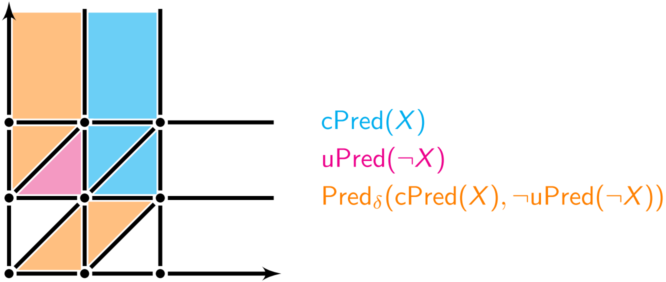

(but it does not preserve zones...)

$\rightsquigarrow$ the computation of $\pi^*(☺)$ terminates!
... and is correct

Timed games with a safety objective

- We can use operator $\widetilde{\pi}$ defined by

$$\widetilde{\pi}(X) = \mathsf{Pred}_\delta(X \cap \mathsf{cPred}(X), \neg \mathsf{uPred}(\neg X))$$

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- It is also stable w.r.t. regions.

Some remarks

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- are asymmetric
  - the environment can preempt any decision of the controller
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Alternative models [AFH+03,BLMO07]

- concurrent and symmetric games
- some incorporate non-Zenoness in the winning condition

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... and they may not represent a proper interaction with an environment ☹

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Application of timed games to strong timed bisimulation

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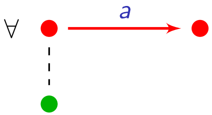

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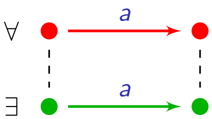

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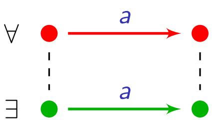

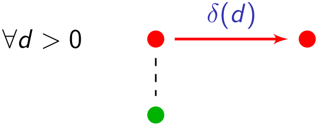

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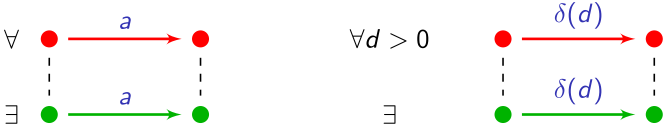

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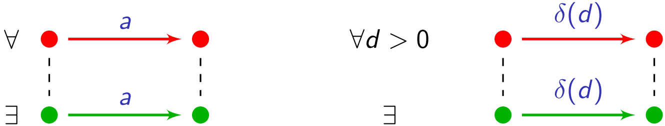

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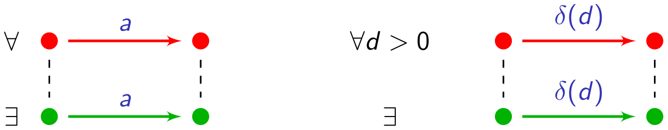

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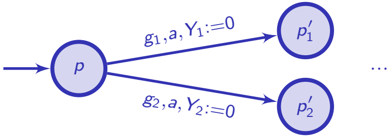
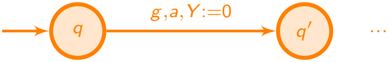
Theorem

Strong timed (bi)simulation between timed automata is decidable and EXPTIME-complete.

timed automaton $\mathcal{A}$



timed automaton $\mathcal{B}$



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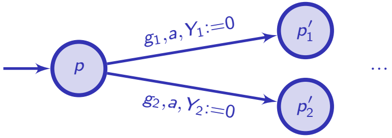

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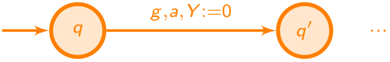

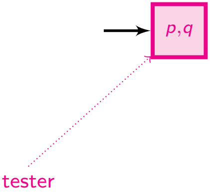

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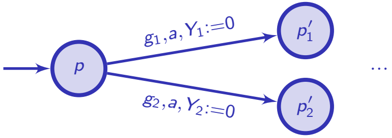

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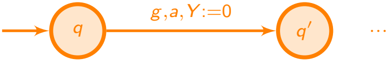

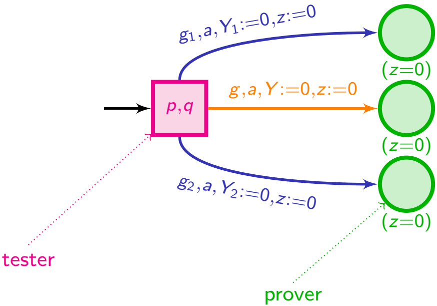

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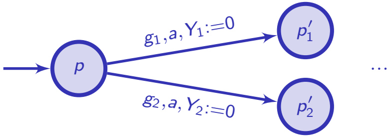

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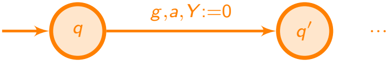
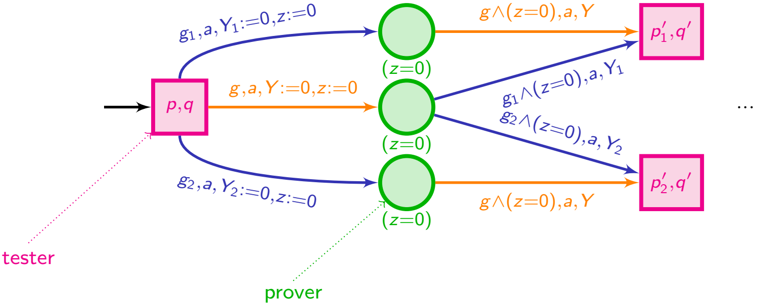
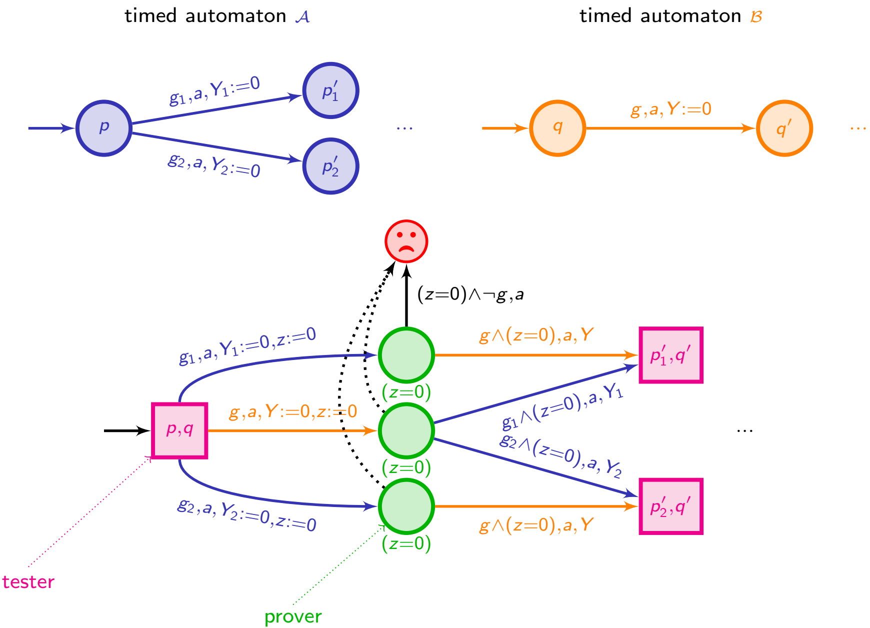
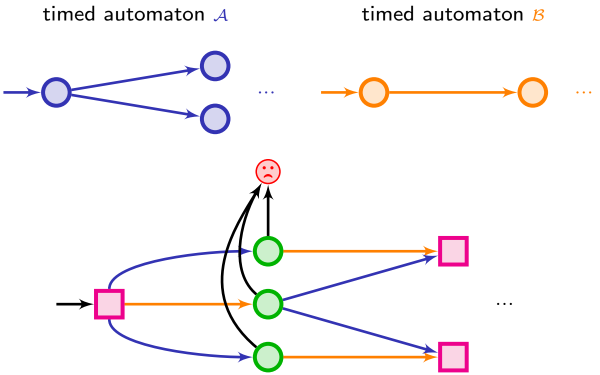





timed automaton $\mathcal{A}$
$g_1, a, Y_1{:=}0$
$g_2, a, Y_2{:=}0$
p
p_1'
p_2'
...
timed automaton $\mathcal{B}$
$g, a, Y{:=}0$
q
q'
...
$(z{=}0) \wedge \neg g, a$
$g_1, a, Y_1{:=}0, z{:=}0$
$g, a, Y{:=}0, z{:=}0$
$g_2, a, Y_2{:=}0, z{:=}0$
p, q
$(z{=}0)$
$(z{=}0)$
$(z{=}0)$
$g \wedge (z{=}0), a, Y$
$g_1 \wedge (z{=}0), a, Y_1$
$g_2 \wedge (z{=}0), a, Y_2$
$g \wedge (z{=}0), a, Y$
p_1', q'
p_2', q'
...
tester
prover



$\mathcal{A}$ and $\mathcal{B}$ are strongly timed bisimilar

iff

the prover has a winning strategy to avoid

What else?

- Implementation: Uppaal-Tiga implements a forward algorithm to compute winning states and winning strategies [CDF+05,BCD+07]

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- Quantitative constraints, see the next lecture!

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