Timed automata – Decidability issues

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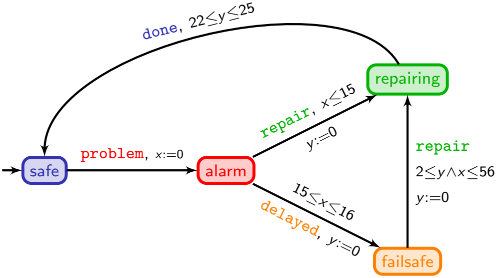
An example of a timed automaton



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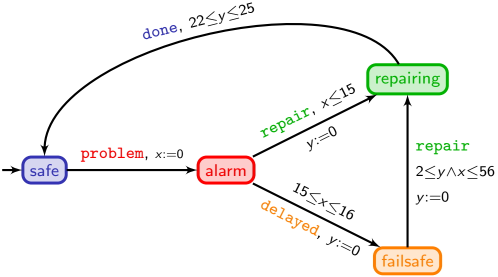

	safe
x	0
y	0

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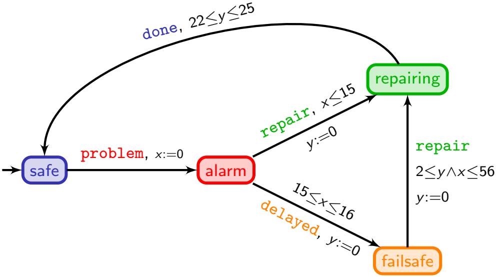

	safe	$\xrightarrow{23}$	safe
x	0		23
y	0		23

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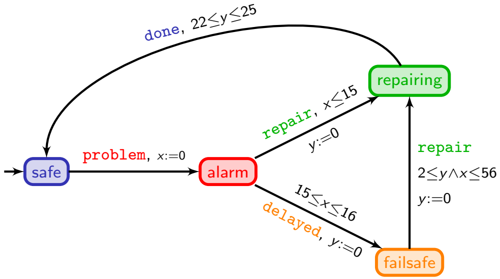

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm
x	0		23		0
y	0		23		23

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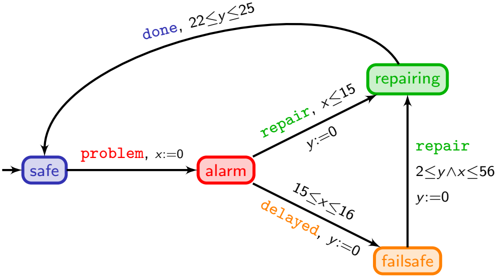

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm
x	0		23		0		15.6
y	0		23		23		38.6

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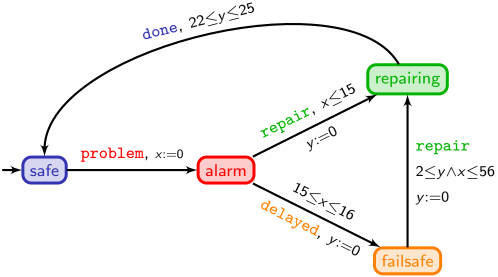

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe
⋯	15.6
	0

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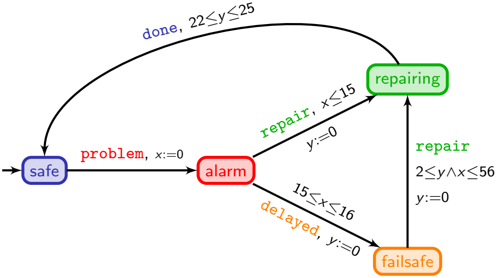

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe
⋯	15.6		17.9
	0		2.3

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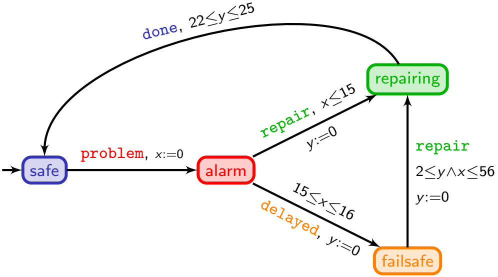

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing
⋯	15.6		17.9		17.9
	0		2.3		0

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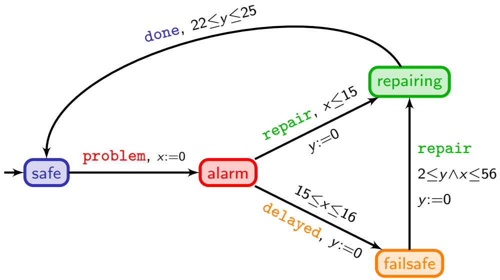
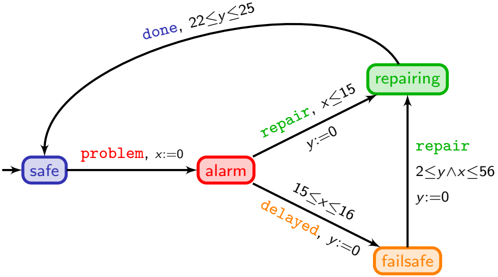


$$\begin{array}{cccccccccccc}
 & \text{safe} & \xrightarrow{23} & \text{safe} & \xrightarrow{\texttt{problem}} & \text{alarm} & \xrightarrow{15.6} & \text{alarm} & \xrightarrow{\texttt{delayed}} & \text{failsafe} & \\
\text{x} & 0 & & 23 & & 0 & & 15.6 & & 15.6 & \cdots \\
\text{y} & 0 & & 23 & & 23 & & 38.6 & & 0 &
\end{array}$$

$$\begin{array}{cccccccc}
 & \text{failsafe} & \xrightarrow{2.3} & \text{failsafe} & \xrightarrow{\texttt{repair}} & \text{repairing} & \xrightarrow{22.1} & \text{repairing} \\
\cdots & 15.6 & & 17.9 & & 17.9 & & 40 \\
 & 0 & & 2.3 & & 0 & & 22.1
\end{array}$$

An example of a timed automaton



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x & 0 & & 23 & & 0 & & 15.6 & & 15.6 & \cdots \\
y & 0 & & 23 & & 23 & & 38.6 & & 0 &
\end{array}$$

$$\begin{array}{cccccccccccc}
 & \text{failsafe} & \xrightarrow{2.3} & \text{failsafe} & \xrightarrow{\texttt{repair}} & \text{repairing} & \xrightarrow{22.1} & \text{repairing} & \xrightarrow{\texttt{done}} & \text{safe} \\
\cdots & 15.6 & & 17.9 & & 17.9 & & 40 & & 40 \\
 & 0 & & 2.3 & & 0 & & 22.1 & & 22.1
\end{array}$$

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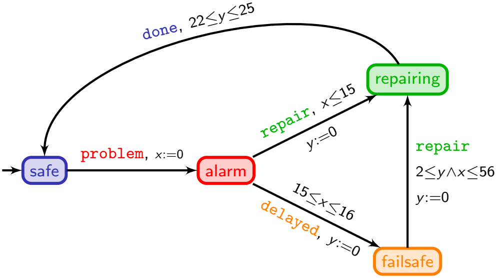

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	…
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\texttt{done}}$	safe
…	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

This run reads the timed word
$(\texttt{problem}, 23)(\texttt{delayed}, 38.6)(\texttt{repair}, 40.9), (\texttt{done}, 63)$.

Outline

1. Decidability of basic properties
2. Equivalence (or preorder) checking
3. Some extensions of timed automata

Verification

Emptiness problem

Is the language accepted by a timed automaton empty?

- basic reachability/safety properties (final states)
- basic liveness properties (ω-regular conditions)

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  $\rightsquigarrow$ classical methods for finite-state systems cannot be applied

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Theorem [AD90,AD94]

The emptiness problem for timed automata is decidable and PSPACE-complete.

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Method: construct a finite abstraction

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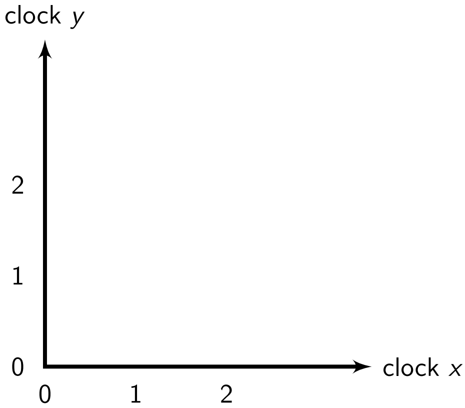
The region abstraction



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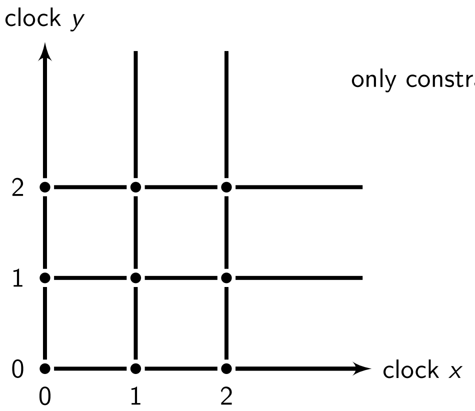
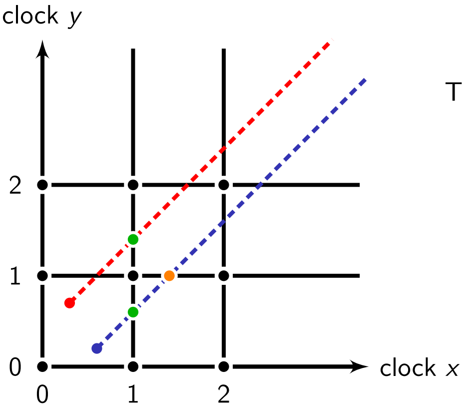


only constraints: $x \sim c$ with $c \in \{0, 1, 2\}$
$y \sim c$ with $c \in \{0, 1, 2\}$

- "compatibility" between regions and constraints

The region abstraction

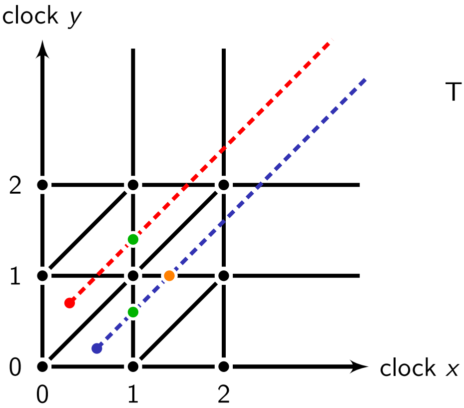


The path 

- can be fired from
- cannot be fired from

- "compatibility" between regions and constraints
- "compatibility" between regions and time elapsing

The region abstraction



The path 

- can be fired from ●
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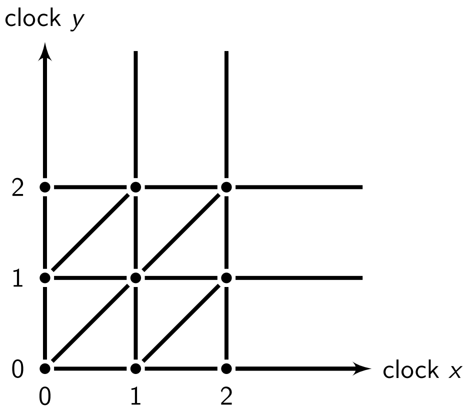
The region abstraction



- "compatibility" between regions and constraints
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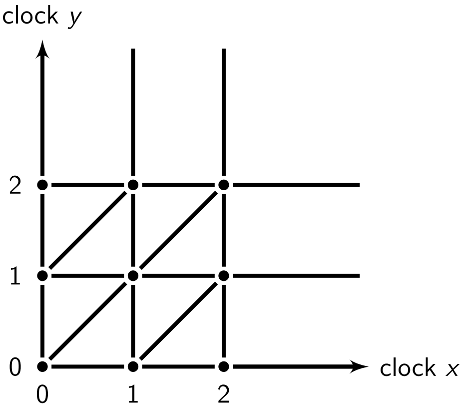
$\rightsquigarrow$ an equivalence of finite index

The region abstraction



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$\rightsquigarrow$ an equivalence of finite index
a time-abstract bisimulation

Time-abstract bisimulation

This is a relation between • and • such that:

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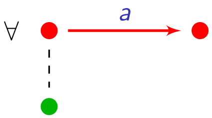

Time-abstract bisimulation

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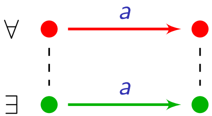

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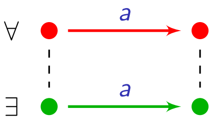
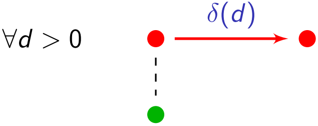




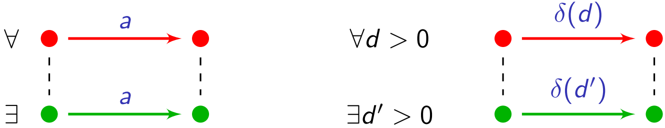
Time-abstract bisimulation

This is a relation between • and • such that:

$$\begin{tikzcd} \forall & \bullet \arrow[r, "a"] \arrow[d, dashed, no head] & \bullet \arrow[d, dashed, no head] \\ \exists & \bullet \arrow[r, "a"] & \bullet \end{tikzcd} \qquad \begin{tikzcd} \forall d > 0 & \bullet \arrow[r, "\delta(d)"] \arrow[d, dashed, no head] & \bullet \arrow[d, dashed, no head] \\ \exists d' > 0 & \bullet \arrow[r, "\delta(d')"] & \bullet \end{tikzcd}$$

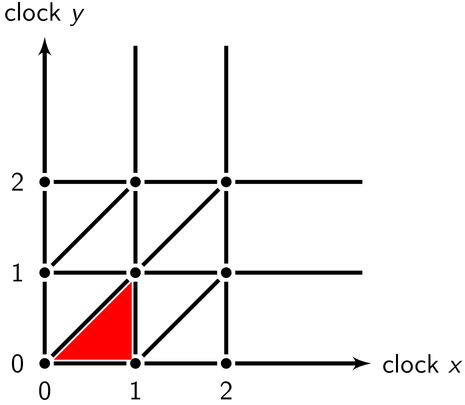
Time-abstract bisimulation

This is a relation between • and • such that:



... and vice-versa (swap • and •).

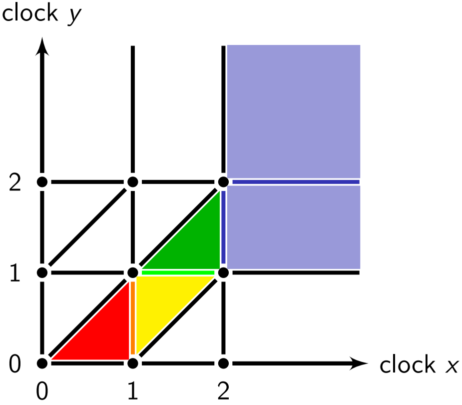
The region abstraction (2)



- region R defined by:

$$\begin{cases} 0 < x < 1 \\ 0 < y < 1 \\ y < x \end{cases}$$

The region abstraction (2)



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- time successors of R

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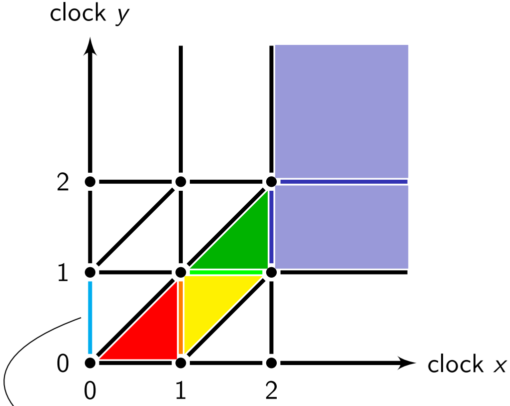


- region R defined by:

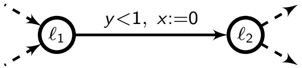
$$\begin{cases} 0 < x < 1 \\ 0 < y < 1 \\ y < x \end{cases}$$

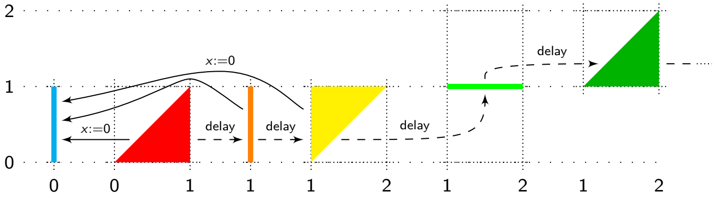
- time successors of R

image of R when resetting clock x

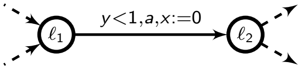
The construction of the region graph

It "mimicks" the behaviours of the clocks.



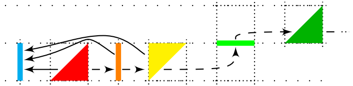


Region automaton $\equiv$ finite bisimulation quotient



timed automaton

$\otimes$



region graph

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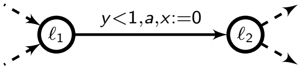

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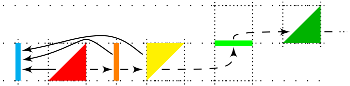

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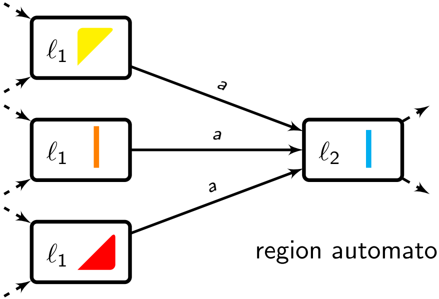

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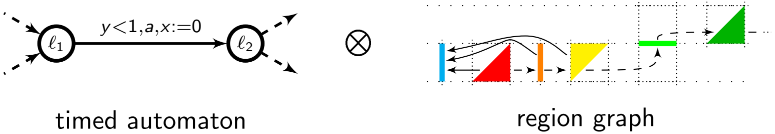
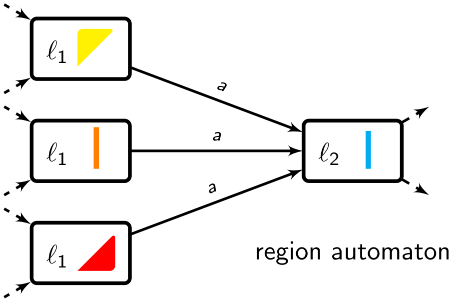
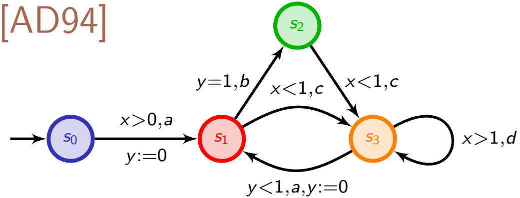




$\mathcal{L}(\text{reg. aut.}) = \text{UNTIME}(\mathcal{L}(\text{timed aut.}))$

An example [AD94]



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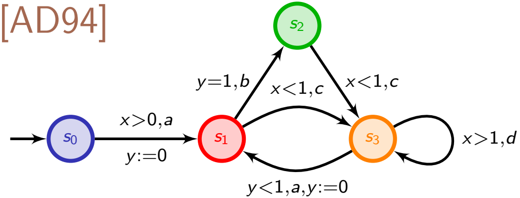

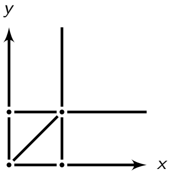

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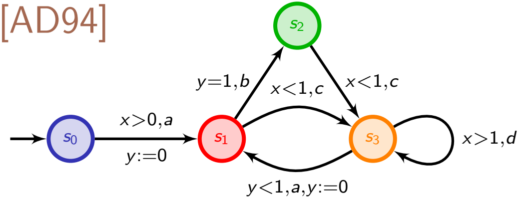
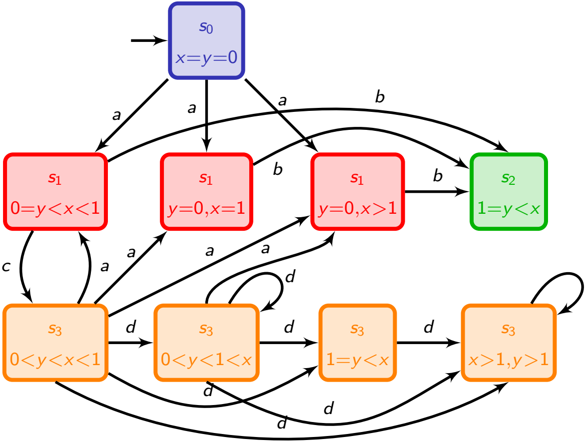
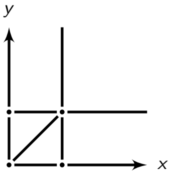
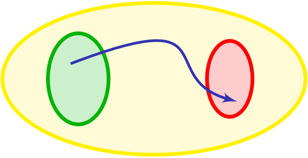
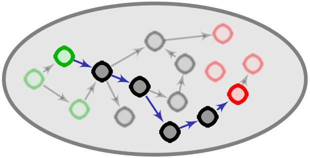




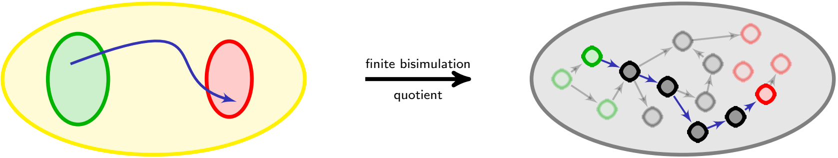






timed automaton

large (but finite) automaton
(region automaton)

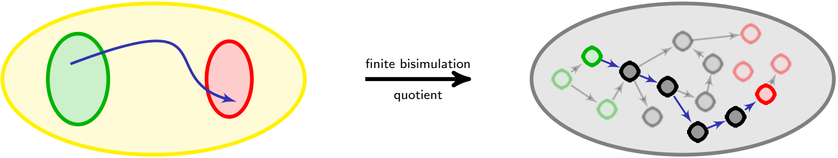


timed automaton

large (but finite) automaton
(region automaton)

- large: exponential in the number of clocks and in the constants (if encoded in binary). The number of regions is:

$$\prod_{x \in X} (2M_x + 2) \cdot |X|! \cdot 2^{|X|}$$



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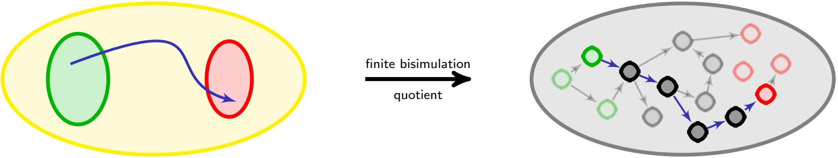

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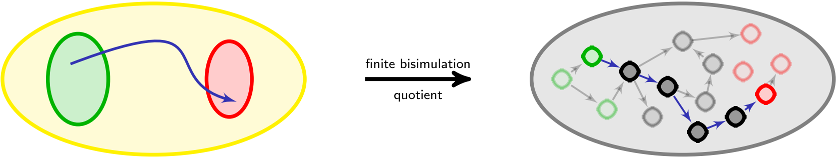

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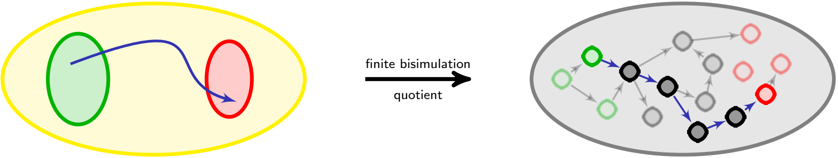
  - liveness properties (like Büchi properties)



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- It can be used to check for:
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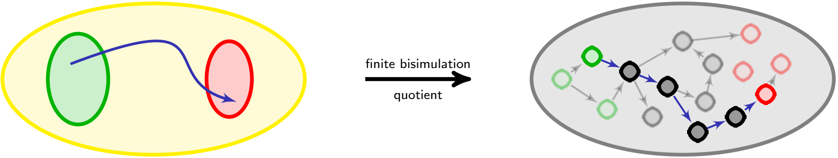
  - liveness properties (like Büchi properties)

$\rightsquigarrow$ problems with Zeno behaviours?

(infinitely many actions in bounded time)



timed automaton

large (but finite) automaton
(region automaton)

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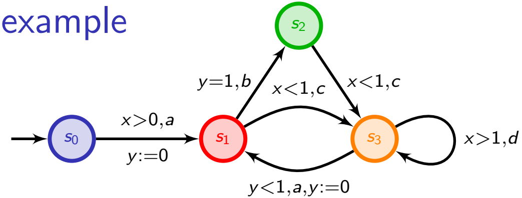
NB: standard problem in timed automata...

Back to the example



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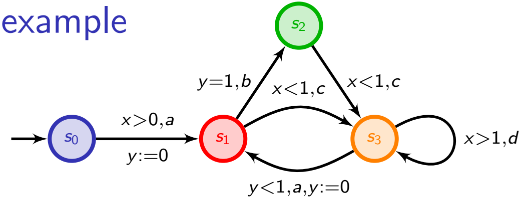

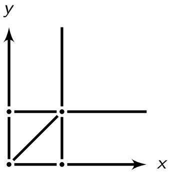

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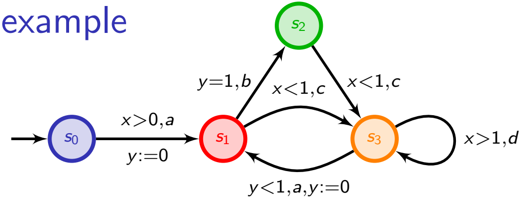

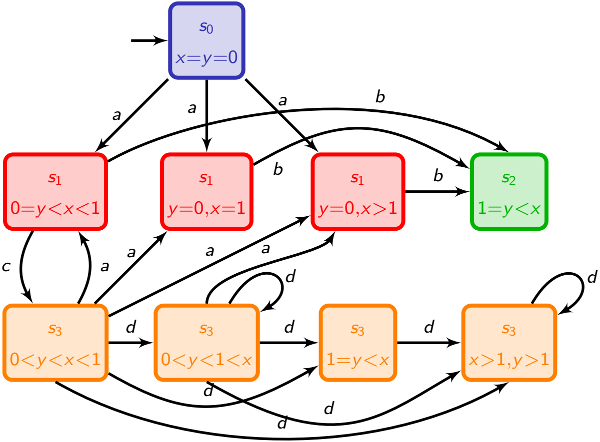

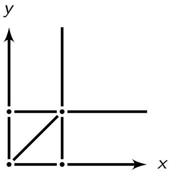

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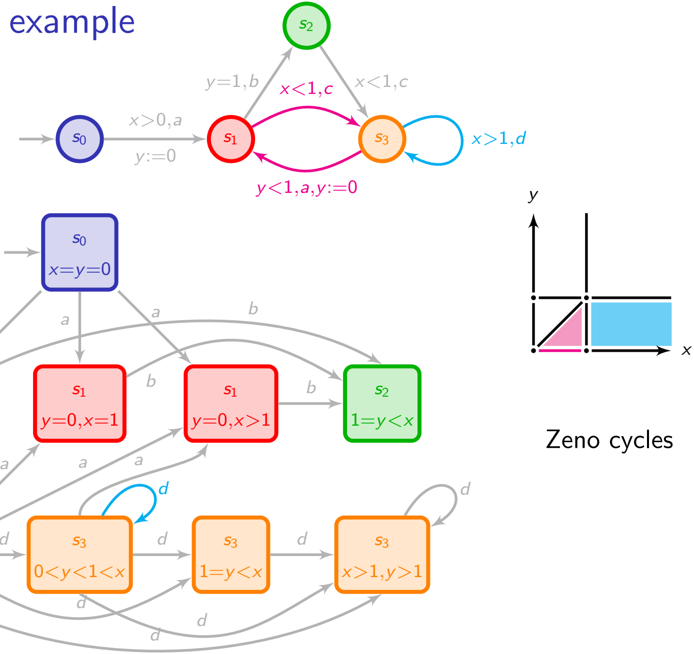
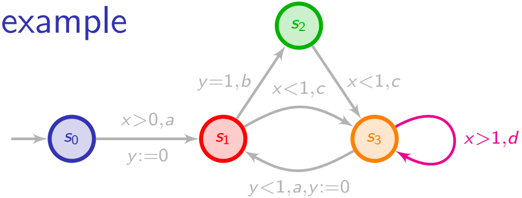
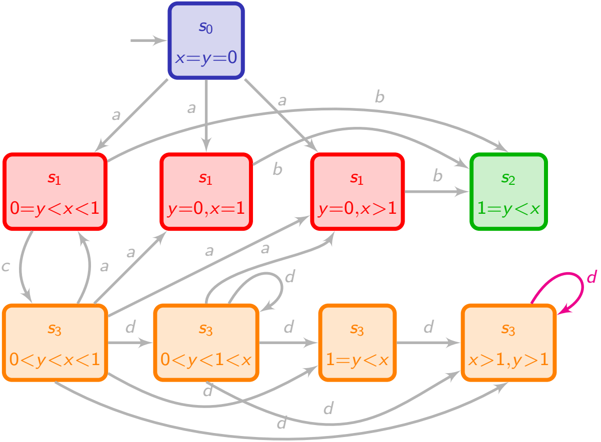
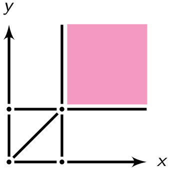


Zeno cycles

Back to the example







Cycles with non-Zeno behaviours

Outline

1. Decidability of basic properties
2. Equivalence (or preorder) checking
3. Some extensions of timed automata

Strong timed (bi)simulation

This is a relation between • and • such that:

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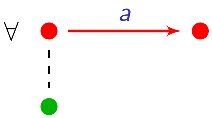

Strong timed (bi)simulation

This is a relation between • and • such that:

$$
\begin{tikzcd}
\forall & \bullet \arrow[r, "a"] \arrow[d, dash, dashed] & \bullet \arrow[d, dash, dashed] \\
\exists & \bullet \arrow[r, "a"] & \bullet
\end{tikzcd}
$$

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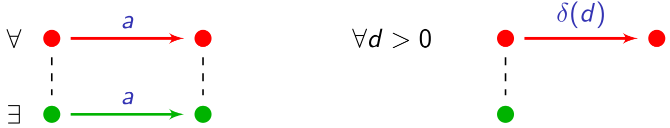

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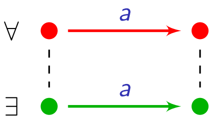

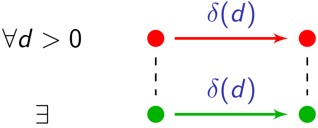

Strong timed (bi)simulation

This is a relation between $\bullet$ and $\bullet$ such that:

$$\begin{tikzcd}
\forall & \bullet \arrow[r, "a"] \arrow[d, dash, dashed] & \bullet \arrow[d, dash, dashed] & & \forall d > 0 & \bullet \arrow[r, "\delta(d)"] \arrow[d, dash, dashed] & \bullet \arrow[d, dash, dashed] \\
\exists & \bullet \arrow[r, "a"] & \bullet & & \exists & \bullet \arrow[r, "\delta(d)"] & \bullet
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... and vice-versa (swap $\bullet$ and $\bullet$) for the bisimulation relation.

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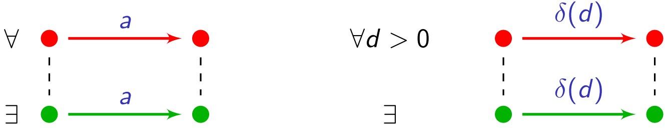

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Theorem

Strong timed (bi)simulation between timed automata is decidable and EXPTIME-complete.

(see later for a simple proof of the upper bound)

Language (or trace) equivalence and inclusion

Question

Given two timed automata $\mathcal{A}$ and $\mathcal{B}$, is $L(\mathcal{A}) = L(\mathcal{B})$ (resp. $L(\mathcal{A}) \subseteq L(\mathcal{B})$)?

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Theorem [AD90,AD94]

Language equivalence and language inclusion are undecidable in timed automata.

... as a special case of the universality problem (are all timed words accepted by the automaton?).

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$\rightsquigarrow$ Proof by reduction from the recurring problem of a two-counter machine

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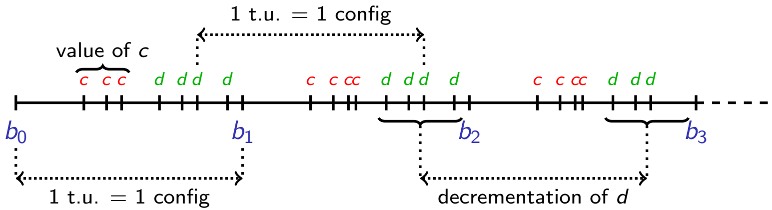

- one configuration is encoded in one time unit
- number of c's: value of counter c
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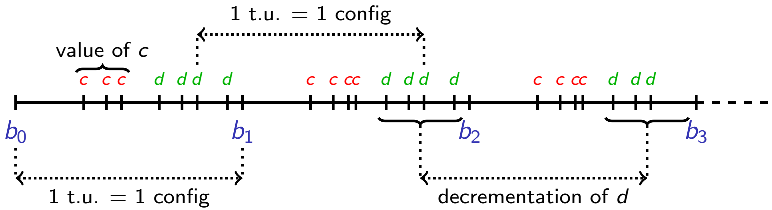

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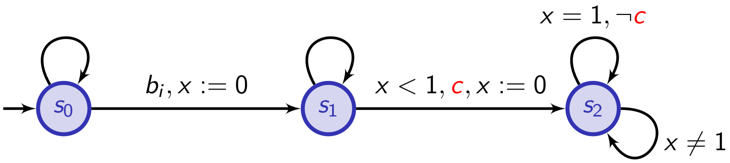
$\rightsquigarrow$ We encode "non-behaviours" of a two-counter machine

Example

Module to check that if instruction i does not decrease counter c, then all actions c appearing less than 1 t.u. after b_i has to be followed by an other c 1 t.u. later.



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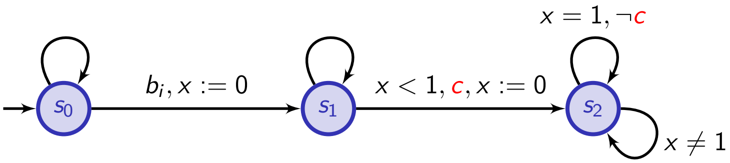

The union of all small modules is not universal
iff
The two-counter machine has a recurring computation

Bad news

- Language inclusion is undecidable [AD90,AD94]
  (Bad news for the application to verification)
- Complementability is undecidable [Tri03,Fin06]
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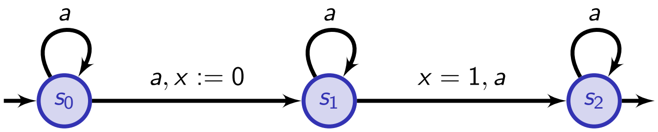
An example of non-determinizable/non-complementable timed aut.:



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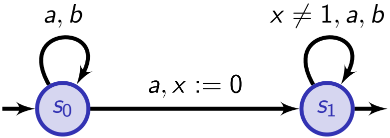

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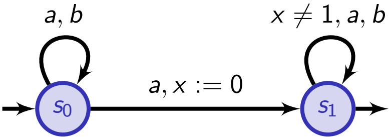

UNTIME $\left(\overline{L} \cap \{(a^* b^*, \tau) \mid \text{all } a's \text{ happen before 1 and no two } a's \text{ simultaneously}\}\right)$ is not regular **(exercise!)**

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What if we extend the clock constraints?

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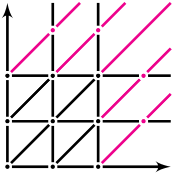

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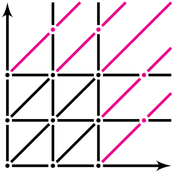

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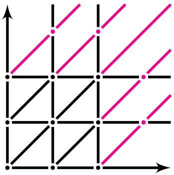

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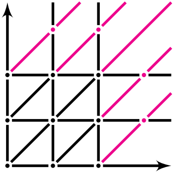

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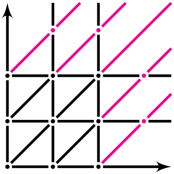

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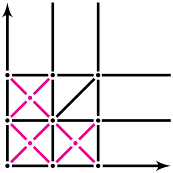

is a time-abstract bisimulation (when two clocks x and y and constraints $x + y \sim c$)!

What if we allow more operations on clocks?

- that can be transfer operations (*i.e.* $x := y$), or reinitialization operations (*i.e.* $x := 4$), or ... [BDFP04]

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	simple constraints	+ diagonal constraints
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$x := y + c$		
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$x :< c$		
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$x :\sim y + c$		
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$\rightsquigarrow$ need of being very careful when using more operations on clocks!

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A note on hybrid automata (see more on Friday)

$$\begin{array}{ll} & \text{a discrete control (the mode of the system)} \\ + & \text{continuous evolution of the variables within a mode} \end{array}$$

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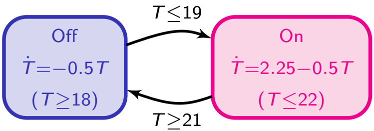

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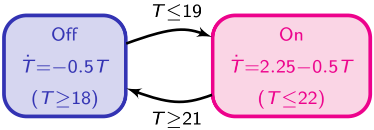

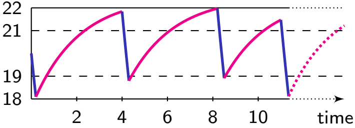

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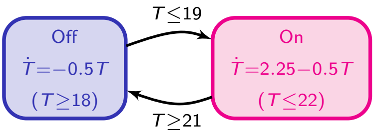

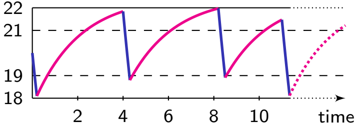

Theorem [HKPV95]

The reachability problem is undecidable in hybrid automata, even for stopwatch automata.

(stopwatch automata: timed automata in which clocks can be stopped)

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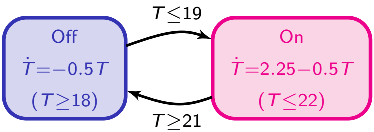

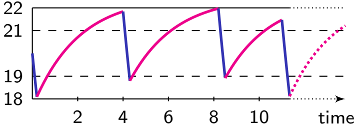

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A relevant question

Is there something between timed automata and hybrid automata which is decidable?

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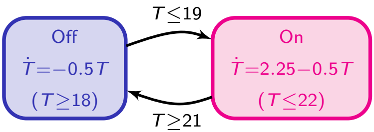

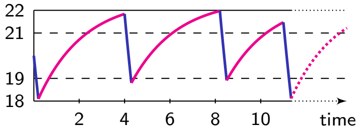

Theorem [HKPV95]

The reachability problem is undecidable in hybrid automata, even for stopwatch automata.

(stopwatch automata: timed automata in which clocks can be stopped)

A relevant question

Is there something between timed automata and hybrid automata which is decidable? $\rightsquigarrow$ See Nicolas' afternoon lecture