Timed Models: From Theory to Implementation

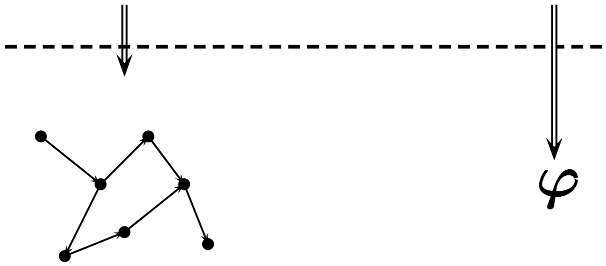
Patricia Bouyer

LSV - CNRS & ENS de Cachan

Model-checking

Does the system satisfy the property?

Modelling



Model-checking

Does the system satisfy the property?

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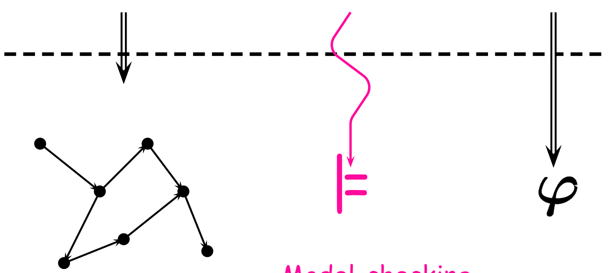


Model-checking Algorithm

Time!

Context: verification of embedded critical systems

Time

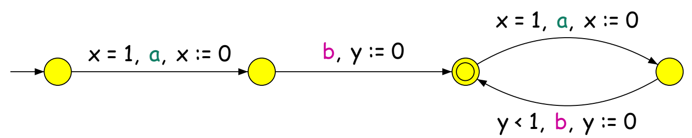
- ✔ naturally appears in real systems
- ✔ appears in properties (for ex. bounded response time)

➜ Need of models and specification languages integrating timing aspects

A Case for Dense-Time

Time domain: discrete (e.g. N) or dense (e.g. Q^+)

- ✔ A compositionality problem with discrete time
- ✔ Dense-time is a more general model than discrete time
- ✔



- Dense-time: $L_{dense} = \{((ab)^\omega, \tau) \mid \forall i,\ \tau_{2i-1} = i \text{ and } \tau_{2i} - \tau_{2i-1} > \tau_{2i+2} - \tau_{2i+1}\}$
- Discrete-time: $L_{discrete} = \emptyset$

Roadmap

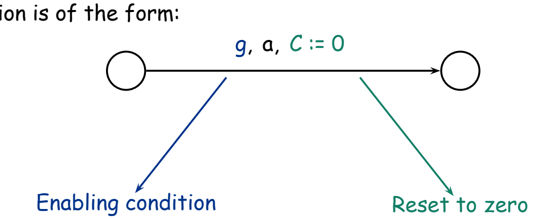
- ✔ **Timed automata, decidability issues**
- ✔ **Some extensions of the model**
- ✔ **Implementation of timed automata**

Timed automata, decidability issues

- ✔ presentation of the model
- ✔ decidability of the model
- ✔ the region automaton construction

Timed automata

- ✔ A finite control structure + variables (clocks)
- ✔ A transition is of the form:



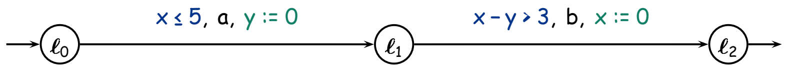
- ✔ An enabling condition (or guard) is:

$$g \ ::= \ x \sim c \ \mid \ x - y \sim c \ \mid \ g \wedge g$$

where $\sim \in \{<, \leq, =, \geq, >\}$

Timed automata (example)

x, y : clocks



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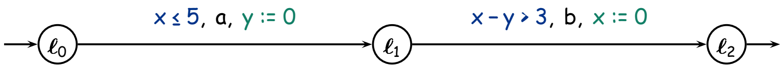

	ℓ_0	$\xrightarrow{\delta(4.1)}$	ℓ_0	$\xrightarrow{a}$	ℓ_1	$\xrightarrow{\delta(1.4)}$	ℓ_1	$\xrightarrow{b}$	ℓ_2
x	0		4.1		4.1		5.5		0
y	0		4.1		0		1.4		1.4

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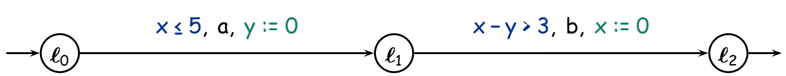

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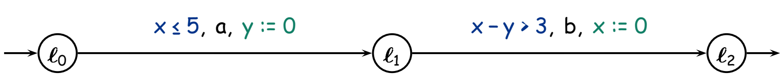

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(clock) valuation

➜ timed word $(a, 4.1)(b, 5.5)$

TA Semantics

- $\mathcal{A} = (\Sigma, L, X, \longrightarrow)$ is a TA
- **Configurations:** $(\ell, v) \in L \times T^X$ where T is the time domain
- **Timed Transition System:**
  - **action transition:** $(\ell, v) \xrightarrow{a} (\ell', v')$ if $\exists \ell \xrightarrow{g,a,r} \ell' \in \mathcal{A}$ s.t. $v \models g$, $v' = v[r \leftarrow 0]$
  - **delay transition:** $(q, v) \xrightarrow{\delta(d)} (q, v + d)$ if $d \in T$

Verification

Emptiness problem: is the language accepted by a timed automaton empty?

- reachability properties (final states)
- basic liveness properties (Büchi (or other) conditions)

Verification

Emptiness problem: is the language accepted by a timed automaton empty?

Problem: the set of configurations is infinite
➜ classical methods can not be applied

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Positive key point: variables (clocks) have the same speed

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Theorem: The emptiness problem for timed automata is decidable.
It is PSPACE-complete. [Alur & Dill 1990's]

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Method: construct a finite abstraction

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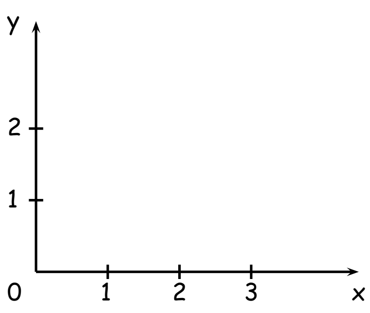
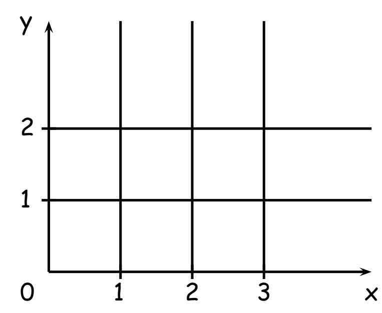
The region abstraction



Equivalence of finite index

The region abstraction



Equivalence of finite index

✔ "compatibility" between regions and constraints

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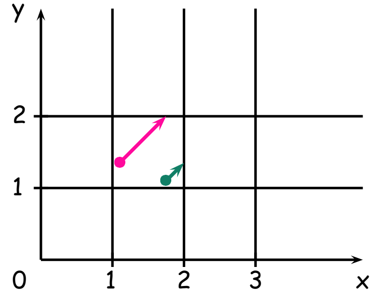

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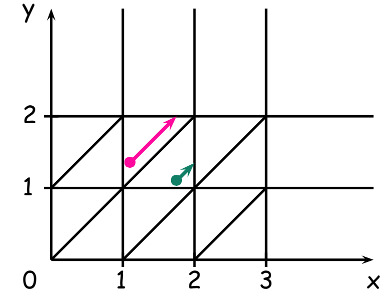

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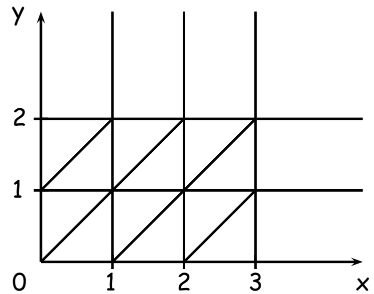

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➜ a bisimulation property

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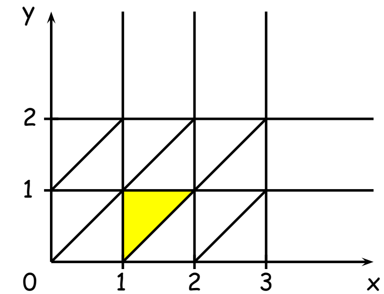

Equivalence of finite index

region defined by
$I_x =]1;2[$, $I_y =]0;1[$
$\{x\} < \{y\}$

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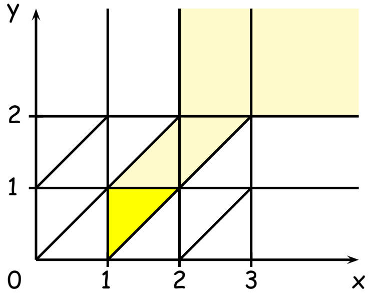

Equivalence of finite index

region defined by
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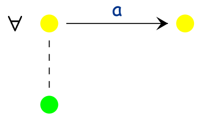
successor regions

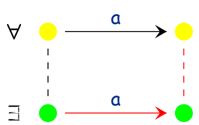
- ✔ "compatibility" between regions and constraints
- ✔ "compatibility" between regions and time elapsing

➜ a bisimulation property

Time-Abstract Bisimulation



Time-Abstract Bisimulation



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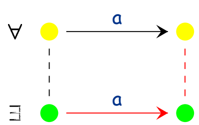

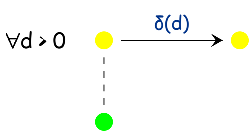

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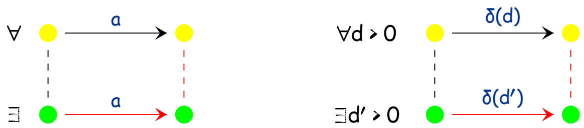

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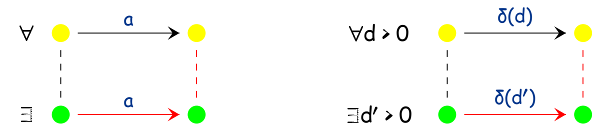

$$(\ell_0, v_0) \xrightarrow{a_1, t_1} (\ell_1, v_1) \xrightarrow{a_2, t_2} (\ell_2, v_2) \xrightarrow{a_3, t_3} \ldots$$

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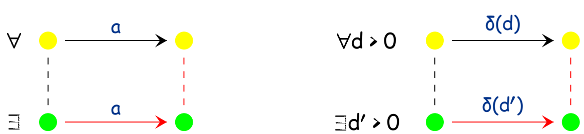

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with $v_i \in R_i$ for all i.

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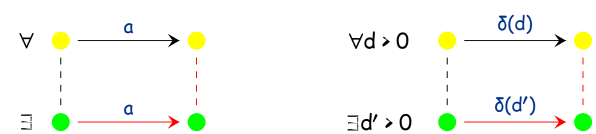

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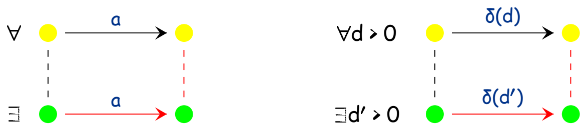

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Remark: We can not check real-time properties with a time-abstract bisimulation. We need to add clocks for the formula we want to check.

The region automaton

timed automaton ⊗ region abstraction

$\ell \xrightarrow{g,a,C:=0} \ell'$ is transformed into:

$(\ell, R) \xrightarrow{a} (\ell', R')$ if there exists $R'' \in \mathrm{Succ}_t^*(R)$ s.t.

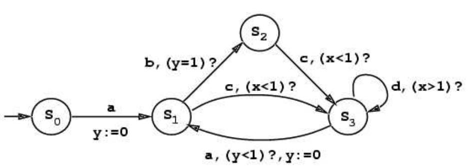
- ✔ $R'' \subseteq g$
- ✔ $[C \leftarrow 0]R'' \subseteq R'$

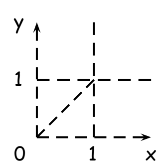
➜ time-abstract bisimulation

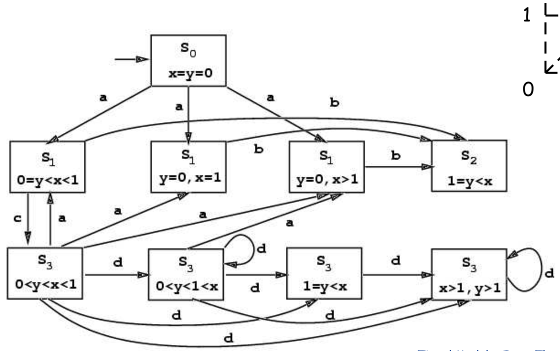
$\mathcal{L}$(reg. aut.) = UNTIME($\mathcal{L}$(timed aut.))

where UNTIME$((a_1, t_1)(a_2, t_2)\dots) = a_1 a_2 \dots$

An example [AD 90's]







PSPACE-Easyness

¡ The size of the region graph is in $\mathcal{O}(|X|!.2^{|X|})$!

- ✔ **One configuration:** a discrete location + a region

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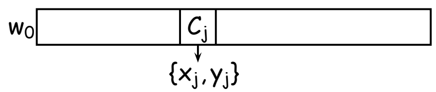
- ✔ By guessing a path: needs only to store two configurations

➜ in NPSPACE, thus in PSPACE

PSPACE-Hardness

$\mathcal{M}$ LBTM
$w_0 \in \{a,b\}^*$ } $\leadsto$ $A_{\mathcal{M},w_0}$ s.t. $\mathcal{M}$ accepts w_0 iff the final state of $A_{\mathcal{M},w_0}$ is reachable



C_j contains a "a" iff $x_j = y_j$

C_j contains a "b" iff $x_j < y_j$

(these conditions are invariant by time elapsing)

➜ proof taken in **[Aceto & Laroussinie 2002]**

PSPACE-Hardness (cont.)

If $q \xrightarrow{a,a',\delta} q'$ is a transition of $\mathcal{M}$, then for each position i of the tape, we have a transition

$$(q,i) \xrightarrow{g,r:=0} (q',i')$$

where:

- ✔ g is $x_i = y_i$ (resp. $x_i < y_i$) if $a = a$ (resp. $a = b$)
- ✔ $r = \{x_i, y_i\}$ (resp. $r = \{x_i\}$) if $a = a$ (resp. $a = b$)
- ✔ $i' = i + 1$ (resp. $i' = i - 1$) if δ is right and $i < n$ (resp. left)

Enforcing time elapsing: on each transition, add the condition $t = 1$ and clock t is reset.

Initialization: init $\xrightarrow{t=1, r_0:=0}$ $(q_0, 1)$ where $r_0 = \{x_i \mid w_0[i] = b\} \cup \{t\}$

Termination: $(q_f, i) \longrightarrow$ end

A Model Not Far From Undecidability

- ✔ Universality is undecidable **[Alur & Dill 90's]**
- ✔ Inclusion is undecidable **[Alur & Dill 90's]**
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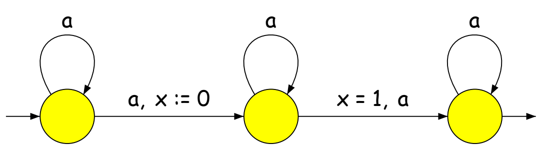
An example of non-determinizable TA:



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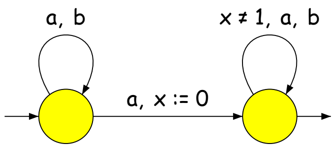
An example of non-complementable TA:



Partial conclusion

➜ a timed model interesting for verification purposes

Numerous works have been (and are) devoted to:

- ✔ the "theoretical" comprehension of timed automata
- ✔ extensions of the model (to ease the modelling)
  - expressiveness
  - analyzability
- ✔ algorithmic problems and implementation

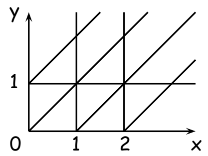
Some extensions of the model

- ✔ adding constraints of the form $x - y \sim c$
- ✔ adding silent actions
- ✔ adding constraints of the form $x + y \sim c$
- ✔ adding new operations on clocks

Adding diagonal constraints

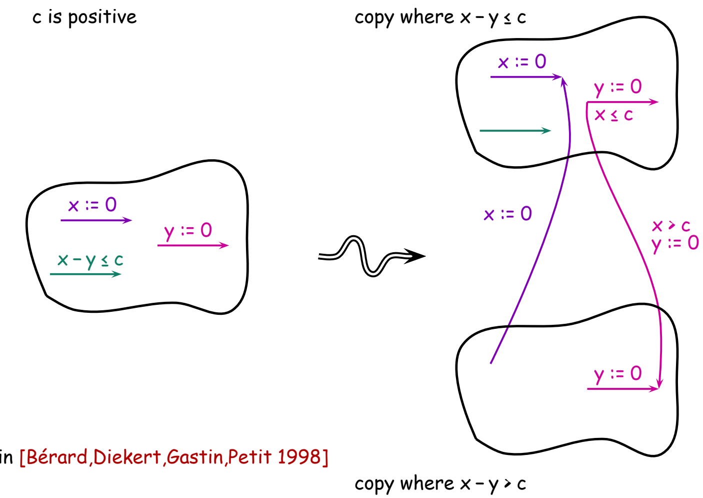
$x - y \sim c$ and $x \sim c$

- ✔ **Decidability:** yes, using the region abstraction



- ✔ **Expressiveness:** no additional expressive power

Adding diagonal constraints (cont.)



➜ proof in [Bérard,Diekert,Gastin,Petit 1998]

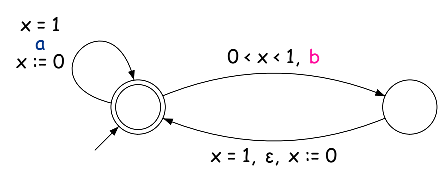
Adding diagonal constraints (cont.)

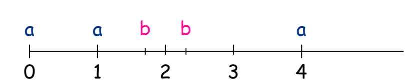
Open question: is this construction "optimal"?
In the sense that timed automata with diagonal constraints are exponentially more concise than diagonal-free timed automata.

Adding silent actions

$g, \varepsilon, C := 0$ [Bérard,Diekert,Gastin,Petit 1998]

- ✔ **Decidability:** yes (actions has no influence on the previous construction)
- ✔ **Expressiveness:** strictly more expressive!

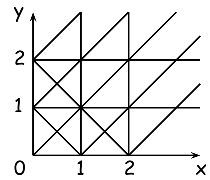




Adding constraints of the form $x + y \sim c$

$x + y \sim c$ and $x \sim c$ [Bérard,Dufourd 2000]

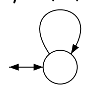
✔ **Decidability:** - for two clocks, decidable using the abstraction



- for four clocks (or more), undecidable!

✔ **Expressiveness:** more expressive! (even using two clocks)

$\{(a^n, t_1 \ldots t_n) \mid n \geq 1 \text{ and } t_i = 1 - \frac{1}{2^i}\}$



The two-counter machine

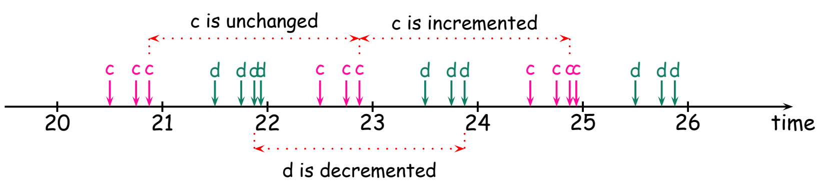
Definition. A two-counter machine is a finite set of instructions over two counters (x and y):

✔ Incrementation:
(p) : $x := x + 1$; goto (q)

✔ Decrementation:
(p) : if $x > 0$ then $x := x - 1$; goto (q) else goto (r)

Theorem. [Minsky 67] The emptiness problem for two counter machines is undecidable.

Undecidability proof



➜ simulation of
- decrement of d
- increment of c

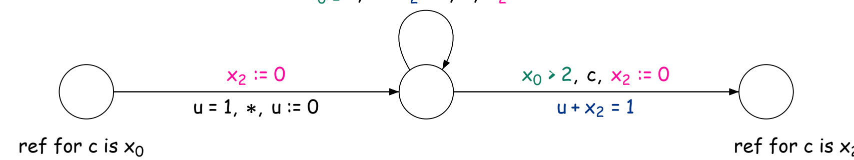
We will use 4 clocks:
- u, "tic" clock (each time unit)
- x_0, x_1, x_2: reference clocks for the two counters

"x_i reference for c" ≡ "the last time x_i has been reset is the last time action c has been performed"

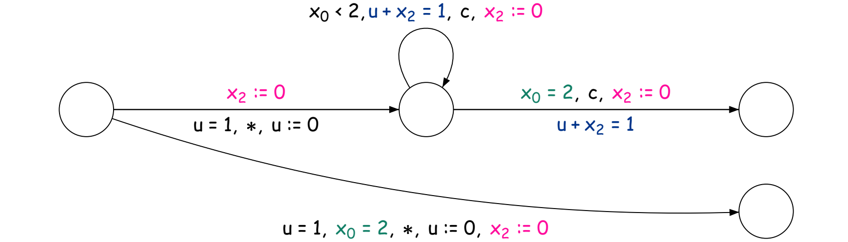
[Bérard,Dufourd 2000]

Undecidability proof (cont.)

✔ **Increment of counter c:**

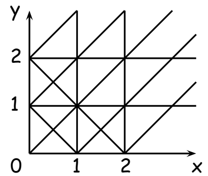


✔ **Decrement of counter c:**



Adding constraints of the form $x + y \sim c$

- ✔ Two clocks: decidable! using the abstraction



- ✔ Four clocks (or more): undecidable!

Adding constraints of the form $x + y \sim c$

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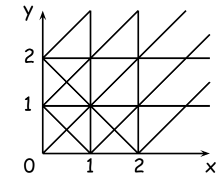

- ✔ Three clocks: **open question**
- ✔ Four clocks (or more): undecidable!

Adding new operations on clocks

Several types of updates: $x := y + c$, $x :< c$, $x :> c$, etc...

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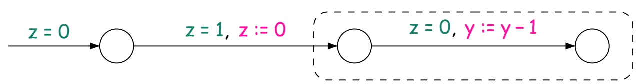
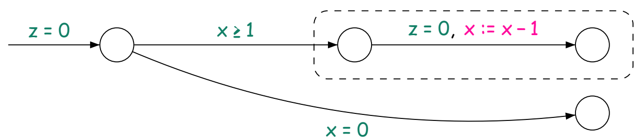
  (simulation of a two-counter machine)

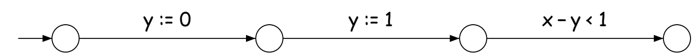
- ✔ Only decrementation also leads to undecidability
  - **Incrementation of counter** x



  - **Decrementation of counter** x



Decidability



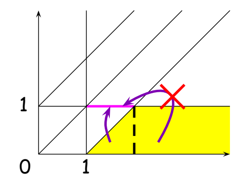


image by $y := 1$

➜ the bisimulation property is not met

The classical region automaton construction is not correct.

Decidability (cont.)

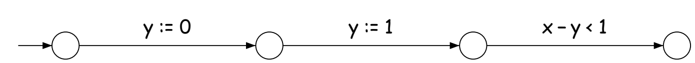
$\mathcal{A}$ ⇝ Diophantine linear inequations system
⇝ is there a solution?
⇝ if yes, belongs to a decidable class

Examples:

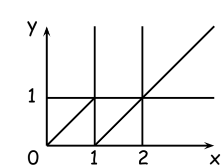
- ✔ constraint $x \sim c$ — $c \le max_x$
- ✔ constraint $x - y \sim c$ — $c \le max_{x,y}$
- ✔ update $x :\sim y + c$ — $max_x \le max_y + c$
  and for each clock z, $max_{x,z} \ge max_{y,z} + c$, $max_{z,x} \ge max_{z,y} - c$
- ✔ update $x :< c$ — $c \le max_x$
  and for each clock z, $max_z \ge c + max_{z,x}$

The constants (max_x) and ($max_{x,y}$) define a set of regions.

Decidability (cont.)



$$\begin{cases} max_y \ge 0 \\ max_x \ge 0 + max_{x,y} \\ max_y \ge 1 \\ max_x \ge 1 + max_{x,y} \\ max_{x,y} \ge 1 \end{cases} \Longrightarrow \begin{cases} max_x = 2 \\ max_y = 1 \\ max_{x,y} = 1 \\ max_{y,x} = -1 \end{cases}$$

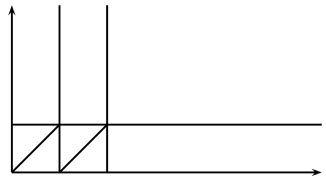


The bisimulation property is met.

What's wrong when undecidable?

Decrementation $x := x - 1$

$max_x \leq max_x - 1$



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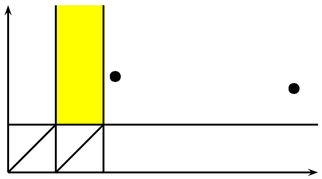

QEST'04 - Tutorial - September 2004 Timed Models: From Theory to Implementation - p. 33

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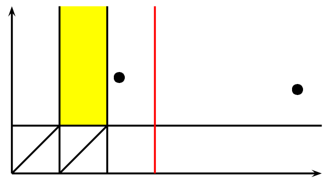

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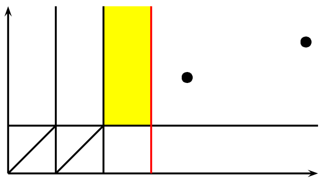

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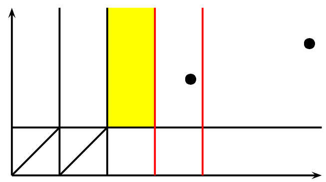

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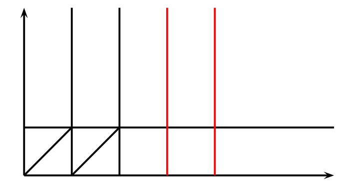

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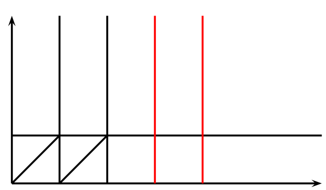

etc...

Decidability (cont.)

	Diagonal-free constraints	General constraints
x := c, x := y	PSPACE-complete	PSPACE-complete
x := x + 1	PSPACE-complete	Undecidable
x := y + c		
x := x − 1	Undecidable	
x :< c	PSPACE-complete	PSPACE-complete
x :> c		Undecidable
x :~ y + c		
y + c <: x :< y + d		
y + c <: x :< z + d	Undecidable	

[Bouyer,Dufourd,Fleury,Petit 2000]

Implementation of Timed Automata

- ✔ analysis algorithms
- ✔ the DBM data structure
- ✔ a bug in the forward analysis

Notice

The region automaton is not used for implementation:

- ✔ suffers from a combinatorics explosion
  (the number of regions is exponential in the number of clocks)
- ✔ no really adapted data structure

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...but **on-the-fly technics** are preferred.

Reachability analysis

- ✔ **forward analysis algorithm:**
  compute the successors of initial configurations

Reachability analysis

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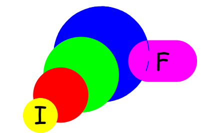

Reachability analysis

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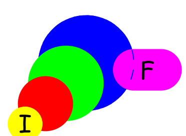

- ✔ **backward analysis algorithm:**
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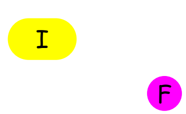

Reachability analysis

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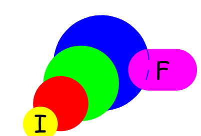

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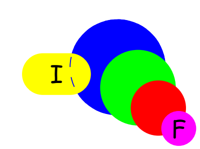
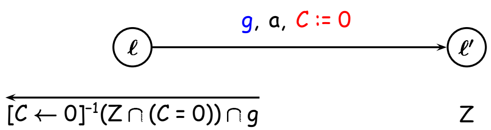


Note on the backward analysis



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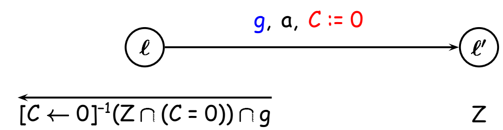

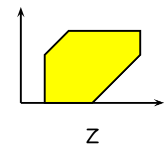

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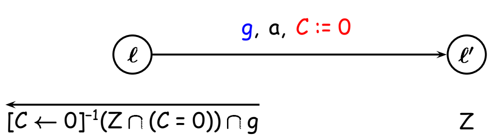

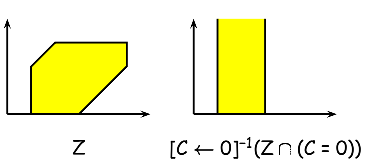

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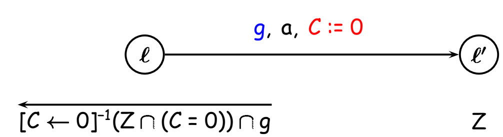

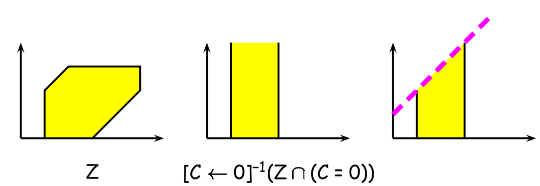

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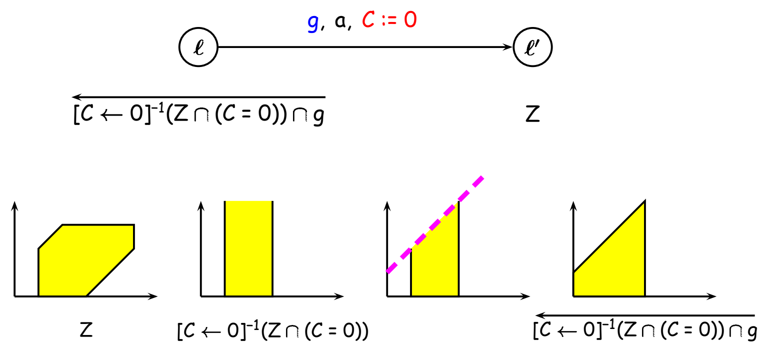

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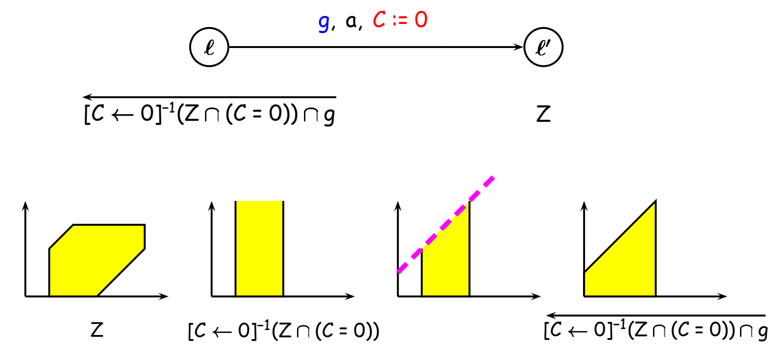

The exact backward computation terminates and is correct!

Note on the backward analysis (cont.)

If $\mathcal{A}$ is a timed automaton, we construct its corresponding set of regions.

Because of the bisimulation property, we get that:

"Every set of valuations which is computed along the backward computation is a finite union of regions"

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If $\mathcal{A}$ is a timed automaton, we construct its corresponding set of regions.

Because of the bisimulation property, we get that:

"Every set of valuations which is computed along the backward computation is a finite union of regions"

Let R be a region. Assume:

- ✔ $v \in \overleftarrow{R}$ (for ex. $v + t \in R$)
- ✔ $v' \equiv_{reg.} v$

There exists t' s.t. $v' + t' \equiv_{reg.} v + t$, which implies that $v' + t' \in R$ and thus $v' \in \overleftarrow{R}$.

Note on the backward analysis (cont.)

If $\mathcal{A}$ is a timed automaton, we construct its corresponding set of regions.

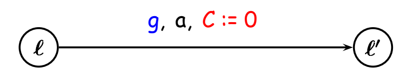
Because of the bisimulation property, we get that:

"Every set of valuations which is computed along the backward computation is a finite union of regions"

But, the backward computation is not so nice, when also dealing with integer variables...

$$i := j.k + \ell.m$$

Forward analysis of TA

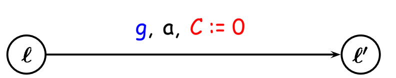


zones $\quad Z \quad [C \leftarrow 0](\overrightarrow{Z} \cap g)$

A zone is a set of valuations defined by a clock constraint

$$\varphi ::= x \sim c \mid x - y \sim c \mid \varphi \wedge \varphi$$

Forward analysis of TA



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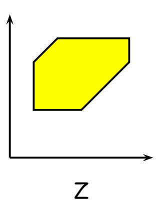

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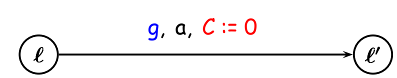

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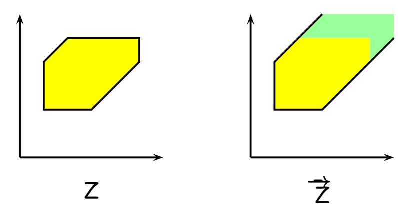

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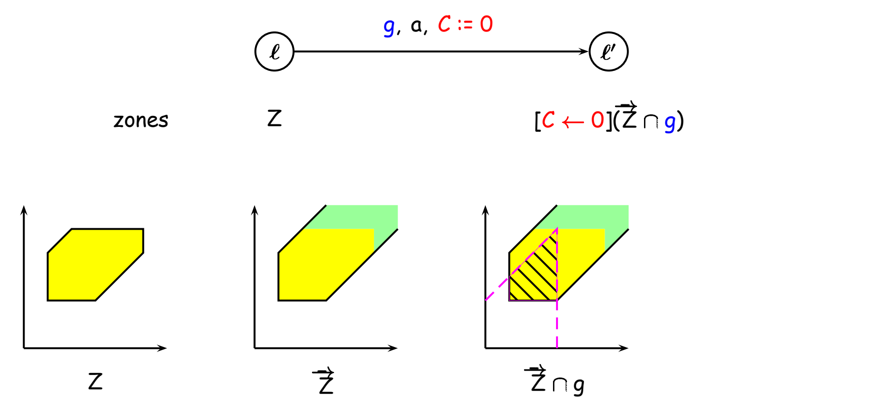

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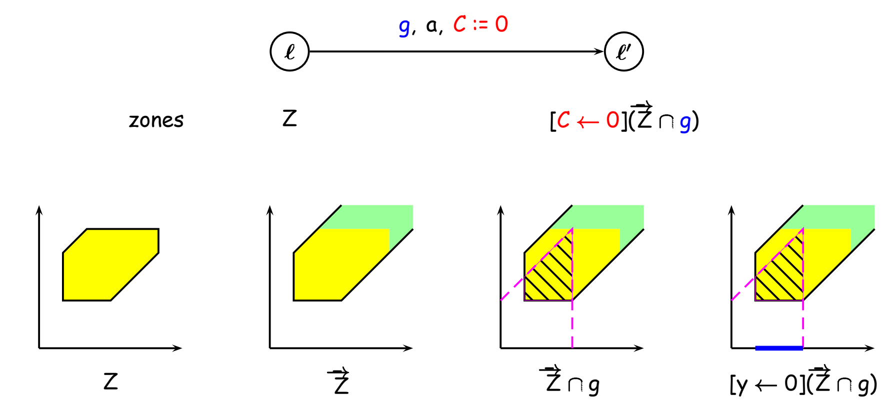

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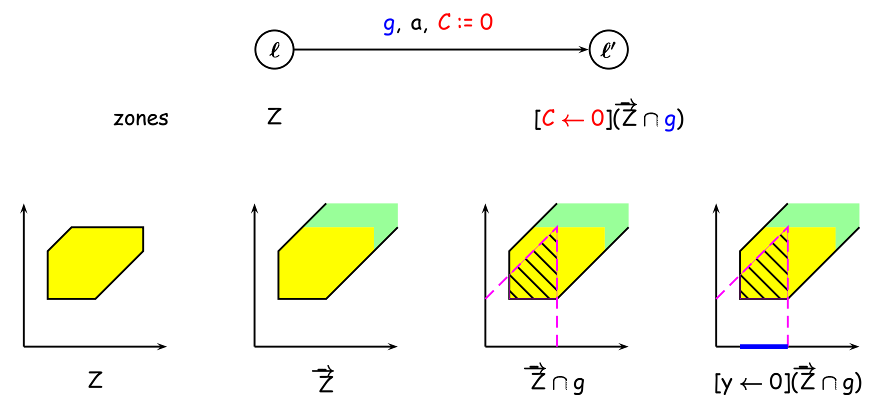
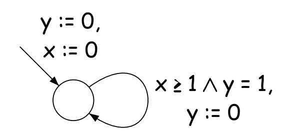
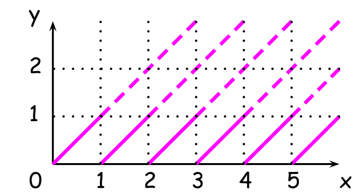


➜ a termination problem

Non termination of the forward analysis





➜ an infinite number of steps...

“Solutions” to this problem

(f.ex. in [Larsen,Pettersson,Yi 1997] or in [Daws,Tripakis 1998])

✔ **inclusion checking**: if $Z \subseteq Z'$ and Z' still handled, then we don't need to handle Z

➜ correct w.r.t. reachability

...

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➜ correct w.r.t. reachability

✔ **activity**: eliminate redundant clocks [Daws,Yovine 1996]

➜ correct w.r.t. reachability

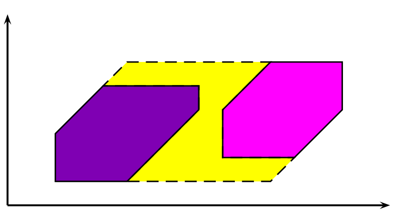
$$q \xrightarrow{g,a,C:=0} q' \quad \Longrightarrow \quad Act(q) = clocks(g) \cup (Act(q') \setminus C)$$

...

“Solutions” to this problem (cont.)

✔ **convex-hull approximation**: if Z and Z' are computed then we overapproximate using “$Z \sqcup Z'$”.

➜ “semi-correct” w.r.t. reachability



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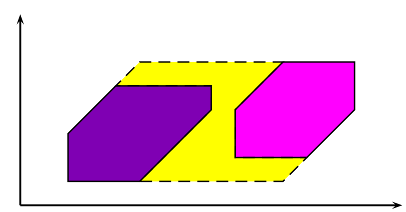

✔ **extrapolation**, a widening operator on zones

The DBM data structure

DBM (Difference Bounded Matrice) data structure [Dill 1989]

$(x_1 \geq 3) \wedge (x_2 \leq 5) \wedge (x_1 - x_2 \leq 4)$

$$\begin{array}{c} \\ x_0 \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{array}{ccc} x_0 & x_1 & x_2 \end{array} \\ \left[\begin{array}{ccc} +\infty & -3 & +\infty \\ +\infty & +\infty & 4 \\ 5 & +\infty & +\infty \end{array}\right] \end{array}$$

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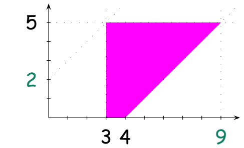
- ✔ Existence of a normal form



$$\left[\begin{array}{ccc} 0 & -3 & 0 \\ 9 & 0 & 4 \\ 5 & 2 & 0 \end{array}\right]$$

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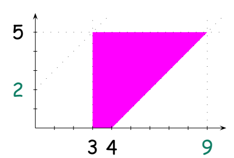
DBM (Difference Bounded Matrice) data structure [Dill 1989]

$(x_1 \geq 3) \wedge (x_2 \leq 5) \wedge (x_1 - x_2 \leq 4)$

$$\begin{array}{c} \\ x_0 \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{array}{ccc} x_0 & x_1 & x_2 \end{array} \\ \left[\begin{array}{ccc} +\infty & -3 & +\infty \\ +\infty & +\infty & 4 \\ 5 & +\infty & +\infty \end{array}\right] \end{array}$$

- ✔ Existence of a normal form



$$\left[\begin{array}{ccc} 0 & -3 & 0 \\ 9 & 0 & 4 \\ 5 & 2 & 0 \end{array}\right]$$

- ✔ All previous operations on zones can be computed using DBMs

The extrapolation operator

Fix an integer k ("$*$" represents an integer between $-k$ and $+k$)

$$\left[\begin{array}{ccc} * & \boxed{>k} & * \\ * & * & * \\ \boxed{<-k} & * & * \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc} * & \boxed{+\infty} & * \\ * & * & * \\ \boxed{-k} & * & * \end{array}\right]$$

- ✔ "intuitively", erase non-relevant constraints

➜ ensures termination

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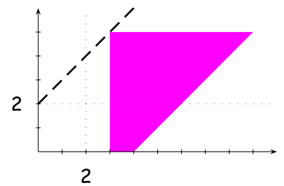

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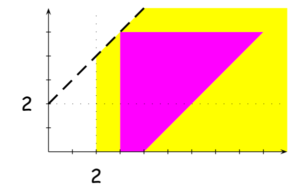

➜ ensures termination

Challenge

Propose a **good** constant for the extrapolation:

- ✔ keep the correctness of the forward computation

Solution by the past: maximal constant appearing in the automaton

- ✔ Several correctness proofs can be found
- ✔ Implemented in tools like UPPAAL, KRONOS, RT-SPIN...
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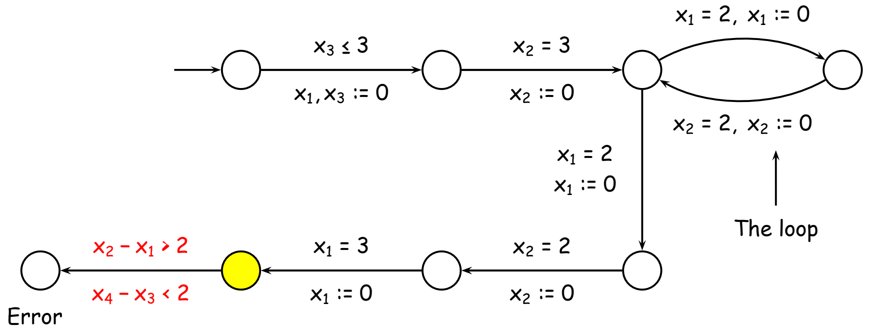
However...

A problematic automaton



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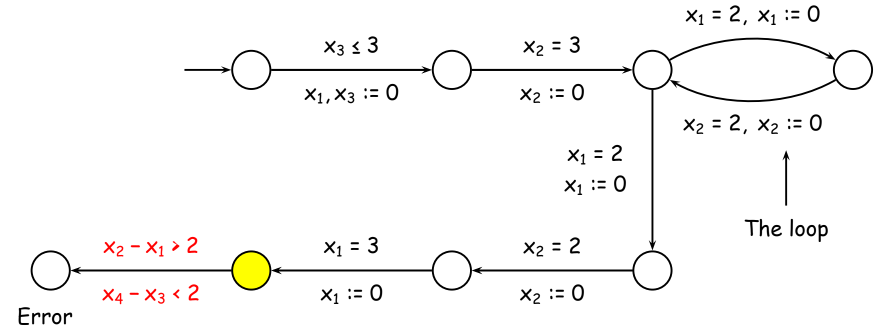

$v(x_1) = 0$
$v(x_2) = d$
$v(x_3) = 2a + 5$
$v(x_4) = 2a + 5 + d$

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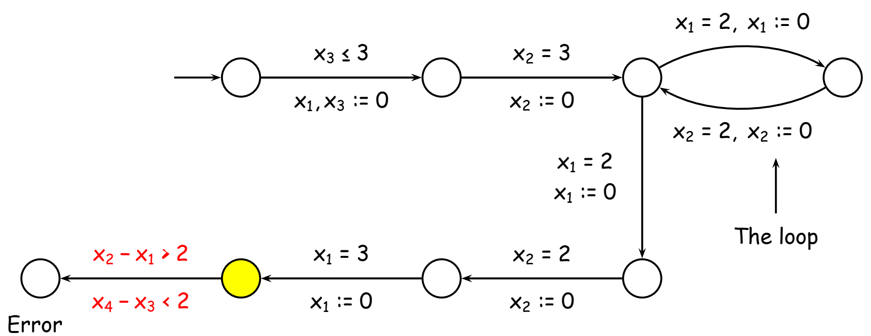

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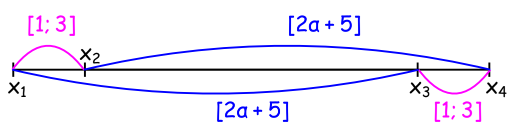
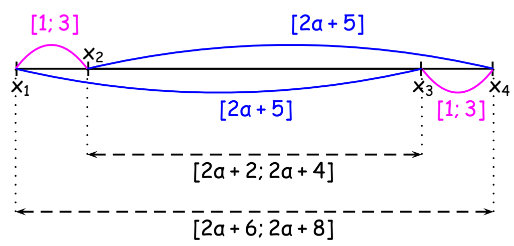


The problematic zone



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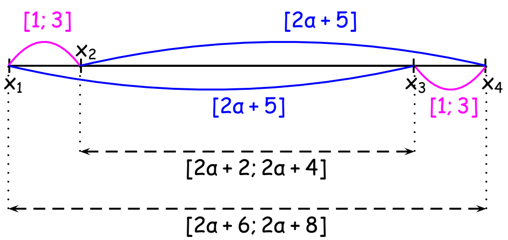
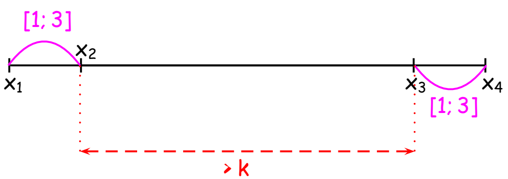


implies $x_1 - x_2 = x_3 - x_4$.

If a is sufficiently large, after extrapolation:



does not imply $x_1 - x_2 = x_3 - x_4$.

General abstractions

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For the previous automaton,

no abstraction operator can satisfy all these criteria!

Why that?

Assume there is a "nice" operator Abs.

The set {M DBM representing a zone Abs(Z)} is finite.

➜ k the max. constant defining one of the previous DBMs

We get that, for every zone Z,

$$Z \subseteq Extra_k(Z) \subseteq Abs(Z)$$

Problem!

Open questions:
- which conditions can be made weaker?
- find a clever termination criterium?
- use an other data structure than zones/DBMs?

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Diagonal-free: only guards $x \sim c$
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General: both guards $x \sim c$ and $x - y \sim c$

Proposition: the classical algorithm is correct for timed automata that use **less than 3 clocks**.

(the constant used is bigger than the maximal constant...)

Conclusion & Further Work

- ✔ Decidability is quite well understood.
- ✔ Needs to understand better the **geometry** of the reachable state space.
- ✔ data structures for both **dense** and **discrete** parts

To be continued...

- ✔ Some other current challenges:
  - controller synthesis
  - implementability issues (program synthesis)

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Uppaal: `http://www.uppaal.com/`