A Probabilistic Semantics for Timed Automata

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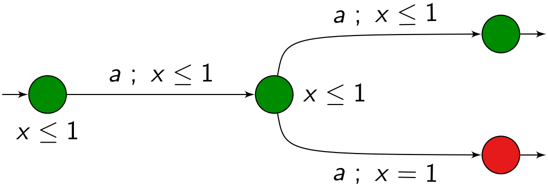
Aim: Use probabilities to “relax” the semantics of timed automata

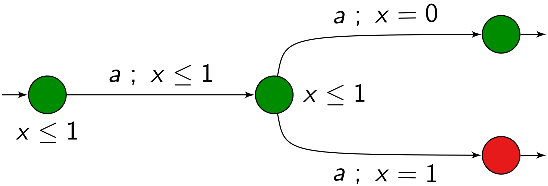
Initial example



Intuition: from the initial state,

this automaton *almost-surely* satisfies “**G** ”

The limits of intuition...



Does it *almost-surely* satisfy "$\mathbf{F}$ ●"?

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$\pi(s \xrightarrow{e_1} \dots \xrightarrow{e_n})$: symbolic path starting in s and firing edges $e_1, \dots, e_n$

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- $\mathbb{P}\left(\pi(s_t \xrightarrow{e_2} \dots \xrightarrow{e_n})\right)$: probability of firing $e_2, \dots, e_n$ after s_t
- $\frac{1}{2}$: normalization factor

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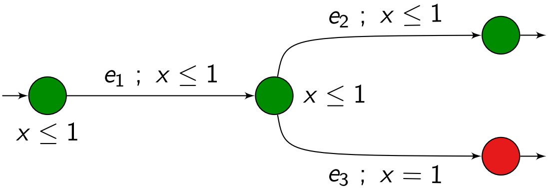

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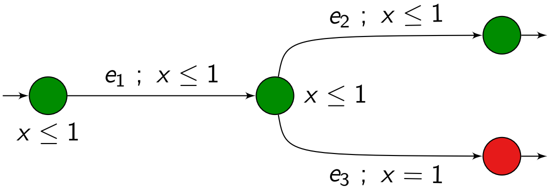

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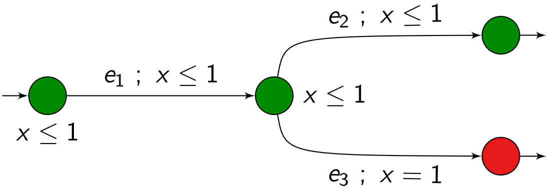

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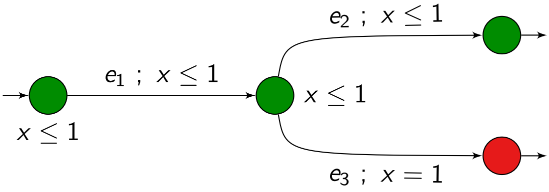

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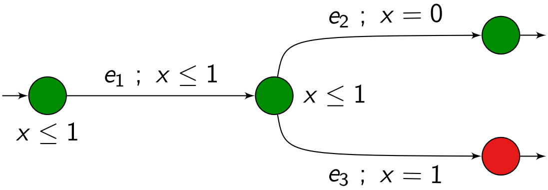

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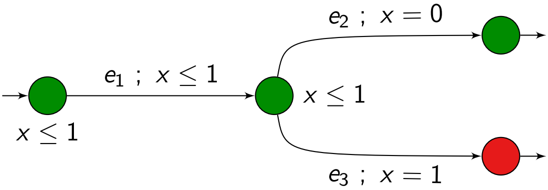

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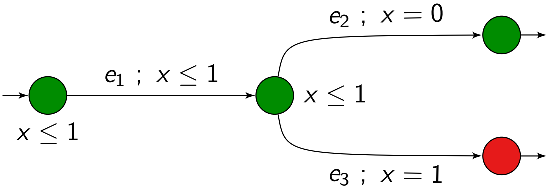

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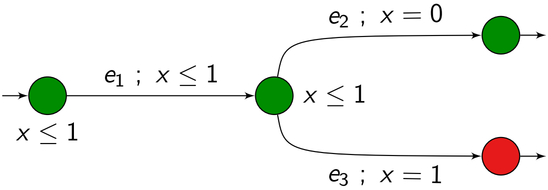

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Properties of $\mathbb{P}$

Lemma

If s is a state, then $\mathbb{P}\left(\bigcup_{n\in\mathbb{N}}\bigcup_{e_1,\dots,e_n}\pi(s \xrightarrow{e_1} \dots \xrightarrow{e_n})\right) = 1$.

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Lemma

If s and s' are region-equivalent, then

$$\mathbb{P}\left(\pi(s \xrightarrow{e_1} \dots \xrightarrow{e_n})\right) > 0 \quad\Leftrightarrow\quad \mathbb{P}\left(\pi(s' \xrightarrow{e_1} \dots \xrightarrow{e_n})\right) > 0.$$

Qualitative probabilistic model-checking

If φ is an LTL formula, then we define:

$$\begin{cases} s_0 \approx\!\!\!\!\mid_{\forall} \varphi & \overset{\text{def}}{\Leftrightarrow} \quad \mathbb{P}\{\varrho \in \mathsf{Runs}(s_0) \mid \varrho \models \varphi\} = 1, \\ s_0 \approx\!\!\!\!\mid_{\exists} \varphi & \overset{\text{def}}{\Leftrightarrow} \quad \mathbb{P}\{\varrho \in \mathsf{Runs}(s_0) \mid \varrho \models \varphi\} > 0. \end{cases}$$

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2. to decide qualitative model-checking.

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These notions are abstract but enjoy a very nice characterization using Banach-Mazur games!

Banach-Mazur games

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Theorems

- Banach-Mazur games are not determined.
- [Oxtoby57] Player 2 has a winning strategy iff C is meager.

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- Topological space $(\mathbb{R}, \mathcal{T})$ with
  $$\text{basis}(\mathcal{T}) = \{\emptyset\} \cup \{(i, i + 2^{-n}) \mid i \in \mathbb{Z},\ n \in \mathbb{N}\} \cup \{\mathbb{R}\},$$
  $\mathcal{B} = \text{basis}(\mathcal{T}) \setminus \{\emptyset\}$.
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  $$\text{basis}(\mathcal{T}) = \{\emptyset\} \cup \{(i, i + 2^{-n}) \mid i \in \mathbb{Z},\ n \in \mathbb{N}\} \cup \{\mathbb{R}\},$$
  $\mathcal{B} = \text{basis}(\mathcal{T}) \setminus \{\emptyset\}$.
  Take $C = (0, 1)$.

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  ex: $\mathbb{R}$ is a Baire space, $\mathbb{Q}$ is not a Baire space.

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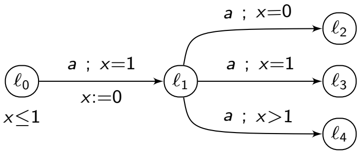

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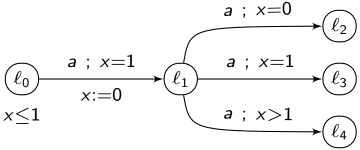

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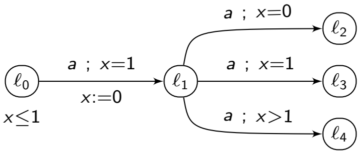

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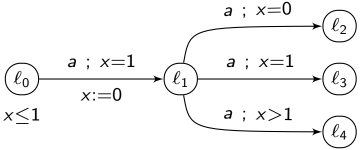

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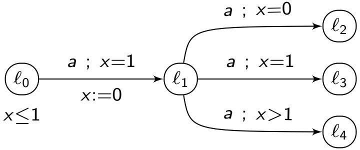

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  - $\pi_2 \cup \pi_3$ is meager, and $\pi_0 \cup \pi_1 \cup \pi_4$ is large.

Probabilistic semantics *vs* topology

- If π is a symbolic path in $R(\mathcal{A})$, then

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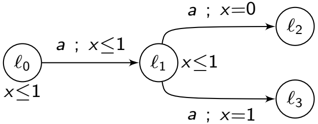

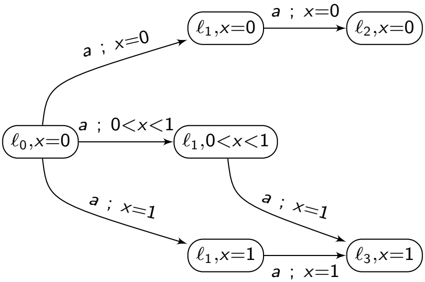

Probabilistic semantics *vs* topology (2)

Theorem

Let $\mathcal{A}$ be a timed automaton, s_0 a state of $\mathcal{A}$, and φ an LTL formula. Then,

$$\begin{array}{rcl} \mathcal{A}, s_0 \mathrel{|}\!\!\approx_\forall \varphi & \stackrel{\text{def}}{\Leftrightarrow} & \text{w.r.t. } \mathbb{P}\text{, almost all paths from } s_0 \text{ in } \mathcal{A} \text{ satisfy } \varphi \\ & \Leftrightarrow & \text{the paths of } R(\mathcal{A}) \text{ from } s_0 \text{ not satisfying } \varphi \text{ have an undefined dimension} \\ & \Leftrightarrow & \text{the set of paths of } R(\mathcal{A}) \text{ from } s_0 \text{ satisfying } \varphi \text{ is (topologically) large} \end{array}$$

(simple application of Banach-Mazur games)

From an algorithmic point-of-view

Theorem

Over finite timed words, the almost-sure ($\models\!\!\!\approx_\forall$) and the positive ($\models\!\!\!\approx_\exists$) LTL model-checking problems over non-blocking timed automata are PSPACE-Complete.

Some remarks

- the probabilistic semantics can be defined for a larger class of systems, for instance hybrid systems with a finite bisimulation quotient

Related works

- Other "probabilistic and timed" (automata-)based models

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- Strong relation with robustness
  - robust timed automata [GHJ97,HR00]
  - robust model-checking [Puri98,DDR04,DDMR04,ALM05,BMR06]

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- a probabilistic semantics for timed automata which removes "unlikely" events,
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- a probabilistic semantics for timed automata which removes "unlikely" events,
- qualitative model-checking has a topological interpretation,
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Further work

- extend to infinite paths,
- quantitative analysis,
- timed objectives,
- ...

Some possible improvements?

- handle accepting states,
- the normalization factor $\frac{1}{2}$ is not completely satisfactory,
- discount time, not the number of transitions,
- ...

Extension to infinite timed words

- definition: straightforward extension to cylinders
- non-trivial to decide...

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For one-clock timed automata,

- we can decide qualitative LTL model-checking
- we have properties like

$$\mathbb{P}(\text{Zeno behaviours}) = 0$$

if the automaton is not "degenerated"