# Stochastic timed automata and beyond

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Joint work with Nathalie Bertrand, Thomas Brihaye, Pierre Carlier

# Purpose of this work

- Study stochastic real-time systems, and more generally stochastic processes
- ... with a model-checking perspective

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We want to design algorithms for verifying properties of (complex) stochastic real-time systems!

# Models with time and probabilities

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- Probabilistic timed automata [KNSS99]
  $\rightsquigarrow$ only discrete probabilities over edges
- Continuous probabilistic timed automata [KNSS00]
  $\rightsquigarrow$ resets of clocks are randomized, but only few results
- Stochastic timed automata [BBB+14]

[KNSS99] Kwiatkowska, Norman, Segala, Sproston. Automatic verification of real-time systems with discrete probability distributions *(ARTS'99)*.
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## Other related models

- Continuous-time Markov chains (CTMCs)
- Generalized semi-Markov processes (GSMPs) [BKKR11]
- Process algebras (like Modest) [DK05,BDHK06]


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[BDHK06] Bohnenkamp, D'Argenio, Hermanns, Katoen. MODEST: A compositional modeling formalism for hard and softly timed systems *(IEEE Trans. Software Engineering)*.

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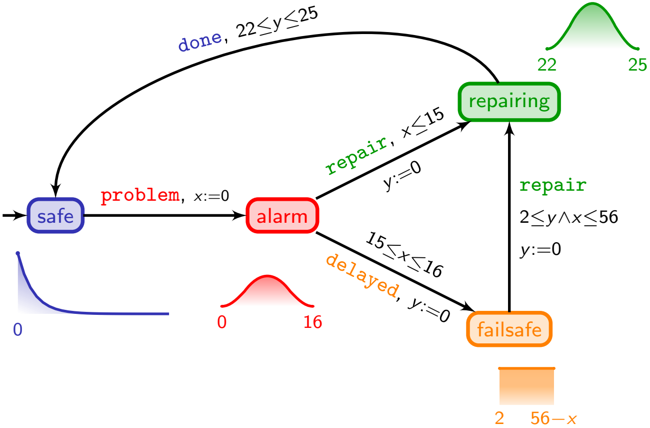

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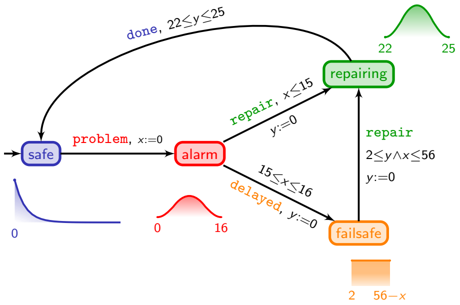


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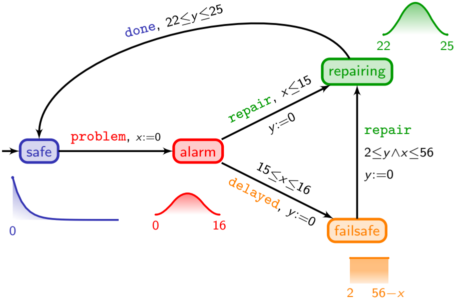


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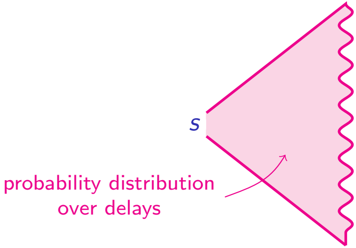

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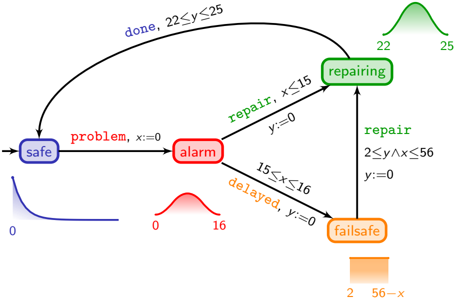


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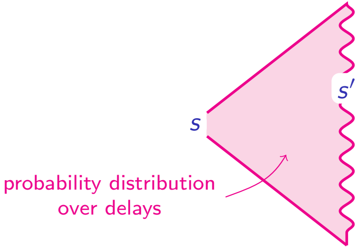

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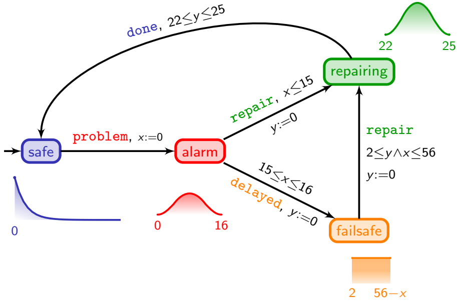
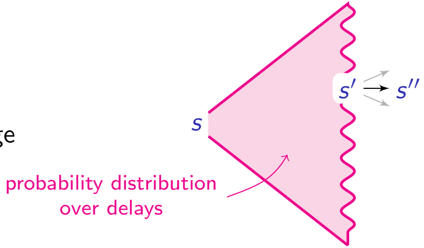



From state $s = (\ell, v)$:

- randomly choose a delay
- then randomly select an edge



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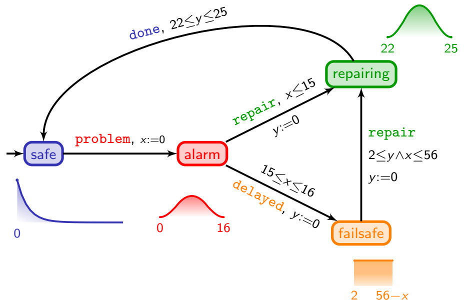
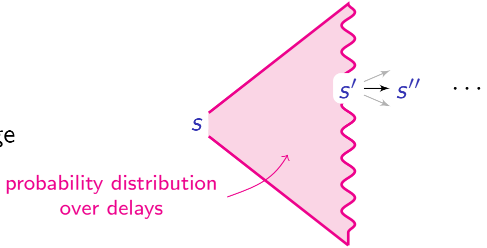



From state $s = (\ell, v)$:

- randomly choose a delay
- then randomly select an edge
- then continue



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- Quantitative analysis: What is the probability for an STA to satisfy some property?

# Stochastic timed automata (3)

## Historical overview

- Almost-sure analysis of safety properties [BBBBG07]

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# Decisive Markov chains [ABM07]

## Denumerable Markov chain $\mathcal{M}$

- $B$ set of states
- $\widetilde{B} = \{s \text{ state} \mid s \models \mathbf{A}\,\mathbf{G}\,\neg B\}$
- $\mathcal{M}$ decisive w.r.t. $B$ if for every $s$, $\mathbb{P}_{\mathcal{M}}(s \models \mathbf{F}\,B \vee \mathbf{F}\,\widetilde{B}) = 1$

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- Markov chains generated by stochastic lossy channel systems are decisive w.r.t. every set of configurations

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## What are they good for?

Provided they satisfy nice effectiveness properties:

- one can decide almost-sure satisfaction of (repeated) reachability properties

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- one can design approximation schemes for (repeated) reachability properties

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Decisiveness ensured by the existence of a finite attractor

- Finite attractor $=$ finite set $A$ such that for every state $s$, $\mathbb{P}(s \models \mathbf{F}\,A) = 1$

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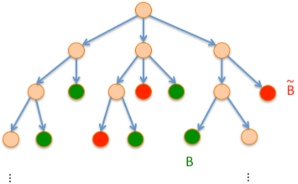
# Example




### Approximation scheme for reachability properties

For every $n \in \mathbb{N}$:

$$\begin{cases} p_n^{\text{Yes}} & = & \mathbb{P}_{\mathcal{M}}(\mathbf{F}_{\leq n}\, B) \\ p_n^{\text{No}} & = & \mathbb{P}_{\mathcal{M}}(\neg B \;\mathbf{U}_{\leq n}\; \widetilde{B}) \end{cases}$$

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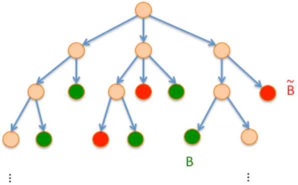


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## Result

Assuming $\mathcal{M}$ is decisive w.r.t. $B$, the two sequences $(p_n^{\text{Yes}})_n$ and $(1 - p_n^{\text{No}})_n$ are adjacent and converge to $\mathbb{P}_{\mathcal{M}}(\mathbf{F}\ B)$.

## How to extend this idea to a continuous state-space?

# More abstract model

### Stochastic transition system $\mathcal{T} = (S, \Sigma, \kappa)$

- $(S, \Sigma)$ a measurable space (more or less)
- $\kappa : S \times \Sigma \to [0, 1]$ is the Markov kernel of $\mathcal{T}$
  - for every $s \in S$, $\kappa(s, \cdot)$ is a probability measure
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## Examples

- discrete Markov chains
- continuous-time Markov chains
- generalized semi-Markov processes
- stochastic timed automata
- ...

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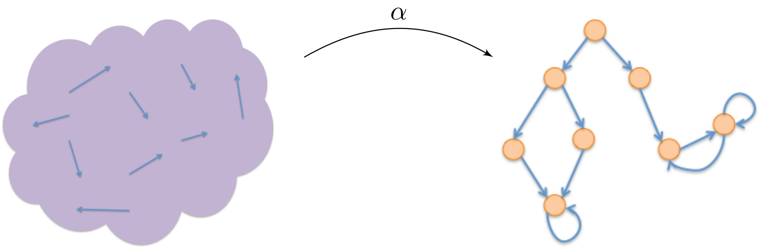
Via an abstraction!



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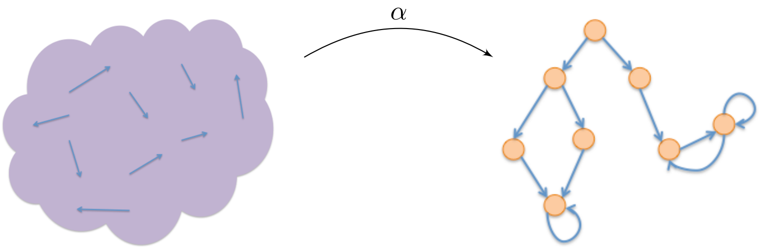


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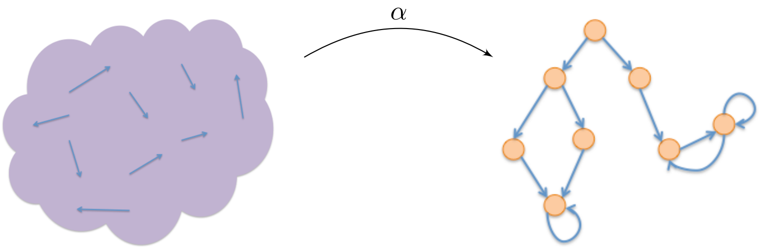
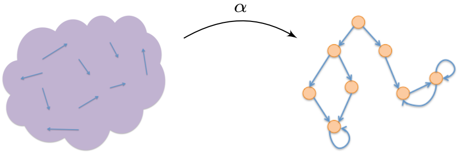



- Notions of soundness and completeness can be formally defined
- Sound and complete abstractions allow to transfer properties and algorithms between the abstract and the concrete model.

# Main property




## Strong fairness result (†)

Assume that $\mathcal{T}_1 \xrightarrow{\alpha} \mathcal{T}_2$, and that $\mathcal{T}_2$ is discrete.

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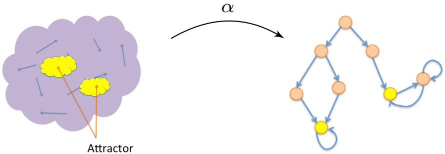


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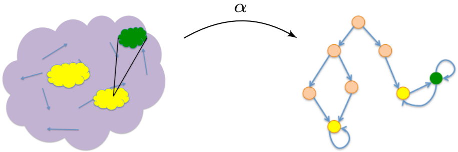


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Assume that $A_2$ is a finite attractor of $\mathcal{T}_2$ such that $\alpha^{-1}(A_2)$ is an attractor for $\mathcal{T}_1$.
Assume furthermore that there exists $\epsilon > 0$ and $k \in \mathbb{N}$ such that for every $s_2 \in A_2$, for every $B_2 \in \Sigma_2$, writing $B_1 = \alpha^{-1}(B_2)$, for every $s_1 \in \alpha^{-1}(s_2)$:

- either $\mathbb{P}_{\mathcal{T}_1}(s_1 \models \mathbf{F}\, B_1) = 0$
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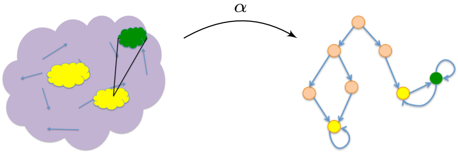


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Then this is really great!

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## Nice properties which are satisfied

If $(\dagger)$ holds for $\mathcal{T}_1$ and $\mathcal{T}_2$:

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## Nice properties which are satisfied

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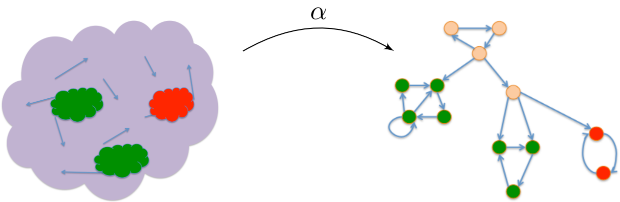
If $(\dagger)$ holds for $\mathcal{T}_1 \ltimes \mathcal{M}$ and $\mathcal{T}_2 \ltimes \mathcal{M}$ ($\mathcal{M}$: det. Muller automaton):

- One can decide qualitative satisfaction of property $\mathcal{M}$ in $\mathcal{T}_1$
- One can approximate the probability of satisfying property $\mathcal{M}$ in $\mathcal{T}_1$

... under effectiveness properties...

# Illustration

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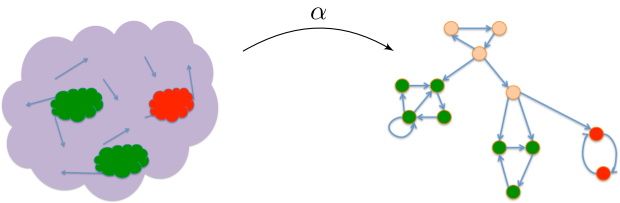


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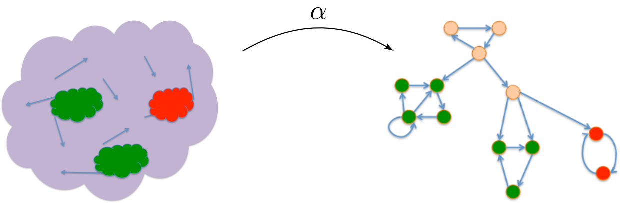


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- Approx. algorithm for reachability properties can be applied to $\mathcal{T}_1 \ltimes \mathcal{M}$ and $\displaystyle\bigcup_{\substack{C\ \mathcal{F}\text{-good BSCC} \\ \text{of } \mathcal{T}_2 \ltimes \mathcal{M}}} \alpha^{-1}(C)$

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- ... and even more!

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$$\{(\ell, v) \mid v \text{ is } \epsilon\text{-separated}\}$$

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- A generic approach to the analysis of continuous stochastic systems
  - Algorithms for qualitative analysis
  - Approximation schemes for quantitative analysis
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## Further work

- Application to more classes of systems
  - Try to fit existing approximation results in our context
  - Further examples: timed lossy channel systems
- Convergence speed of the approximation schemes
- Extend to systems with non-determinism (and more...)
- Compositional approach