An introduction to timed automata

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Outline

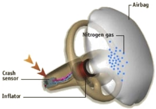
1. Introduction
   - Timed automata
   - Examples
2. Decidability of basic properties
   - The region abstraction
   - Extensions of timed automata
   - Weighted timed automata
3. Implementation and tools
4. Other verification problems
   - Equivalence (or preorder) checking
   - Verification of timed temporal logics (short)
5. Timed control
   - Timed games
   - Weighted timed games
6. Conclusion

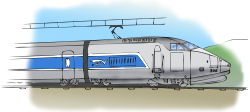
Time-dependent systems

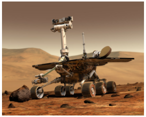
- We are interested in timed systems

Time-dependent systems

- We are interested in timed systems

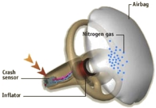


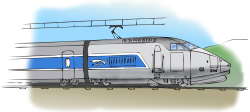


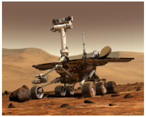


Time-dependent systems

- We are interested in timed systems



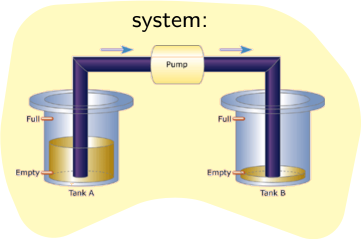


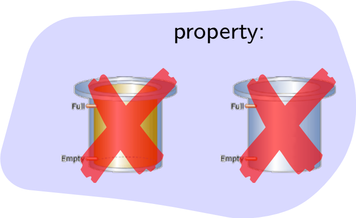


- and in their correctness

"Will the airbag open within 5ms after the car crashes?"
"Will the robot explore a given area without getting out of energy?"

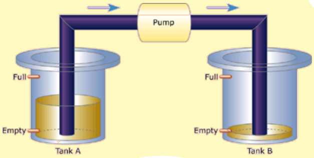
Model-checking and control



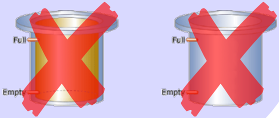


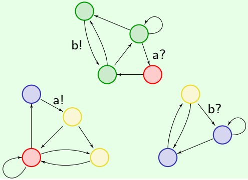
Model-checking and control

system:



property:

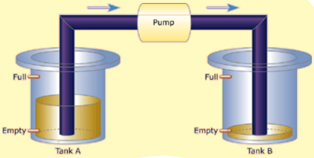




$\mathbf{AG}(\neg \texttt{B.overfull} \wedge \neg \texttt{B.dried\_up})$

Model-checking and control

system:



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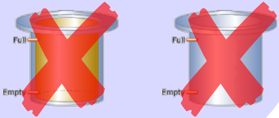
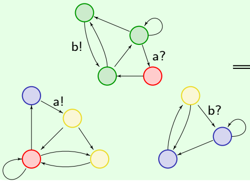


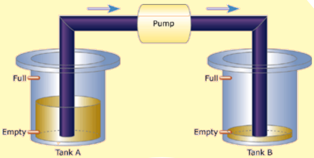


algorithm

AG($\neg$`B.overfull`
$\wedge$ $\neg$`B.dried_up`)

Model-checking and control

system:



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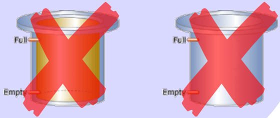
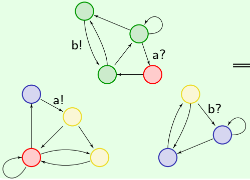




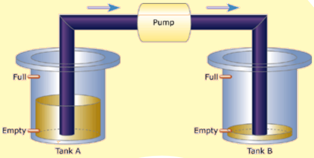
model-checking algorithm

$\mathbf{AG}(\neg\texttt{B.overfull} \wedge \neg\texttt{B.dried\_up})$

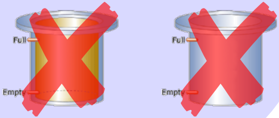
yes/no

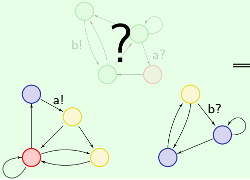
Model-checking and control

system:



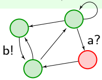
property:





control/synthesis algorithm

$\mathbf{AG}(\neg \texttt{B.overfull} \wedge \neg \texttt{B.dried\_up})$



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Reasoning about real-time systems

A plethora of models

- timed circuits,
- time(d) Petri nets,
- timed automata,
- timed process algebra,
- ...

Reasoning about real-time systems

A plethora of models

- timed circuits,
- time(d) Petri nets,
- **timed automata,**
- timed process algebra,
- . . .

[AD94] Alur, Dill. A Theory of Timed Automata. *Theor. Comp. Science*, 1994.

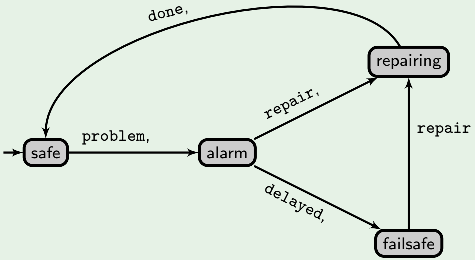
Reasoning about real-time systems

The model of timed automata [AD94]

A timed automaton is made of

- a finite automaton-based structure

Example



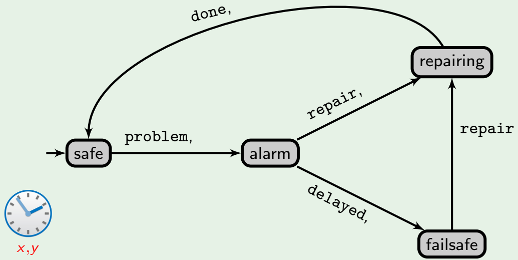
Reasoning about real-time systems

The model of timed automata [AD94]

A timed automaton is made of

- a finite automaton-based structure
- a set of clocks

Example



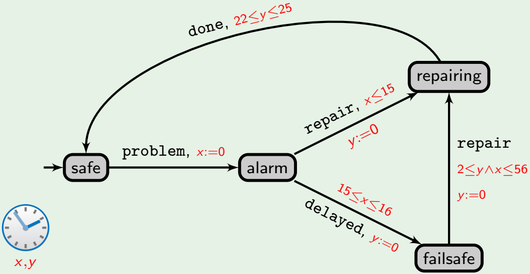
Reasoning about real-time systems

The model of timed automata [AD94]

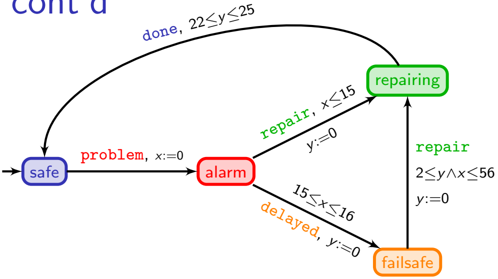
A timed automaton is made of

- a finite automaton-based structure
- a set of clocks
- timing constraints and clock resets on transitions

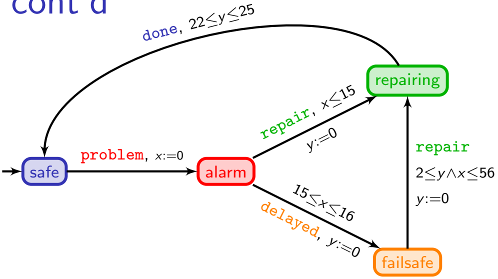
Example



Example – cont'd

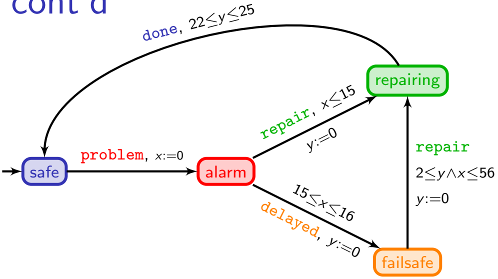


Example – cont'd



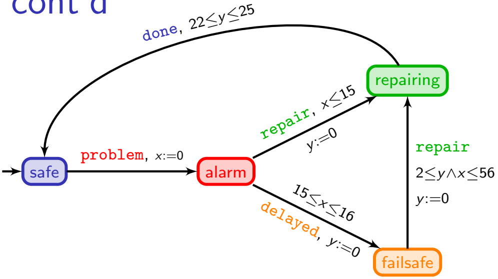
	safe
x	0
y	0

Example – cont'd



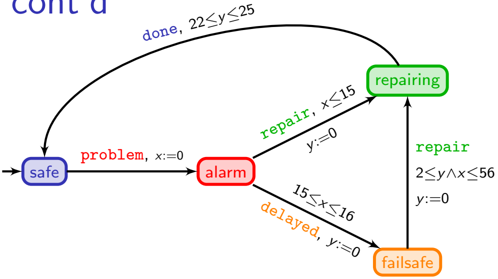
	safe	$\xrightarrow{23}$	safe
x	0		23
y	0		23

Example – cont'd



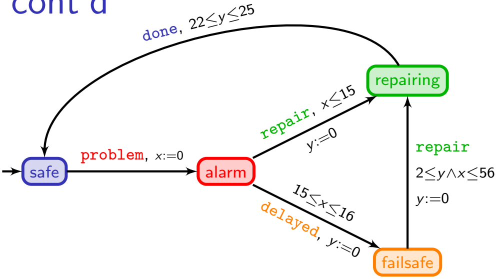
	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm
x	0		23		0
y	0		23		23

Example – cont'd



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm
x	0		23		0		15.6
y	0		23		23		38.6

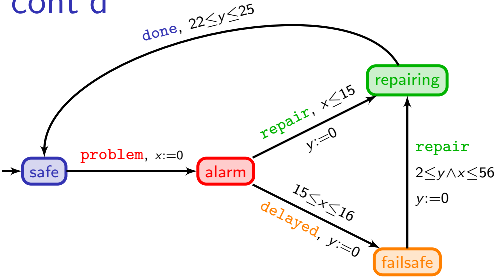
Example – cont'd



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe
⋯	15.6
	0

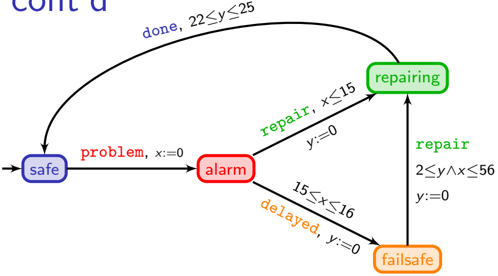
Example – cont'd



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe
⋯	15.6		17.9
	0		2.3

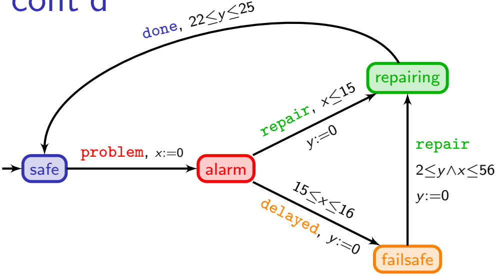
Example – cont'd



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\text{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\text{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	···
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\text{repair}}$	repairing
···	15.6		17.9		17.9
	0		2.3		0

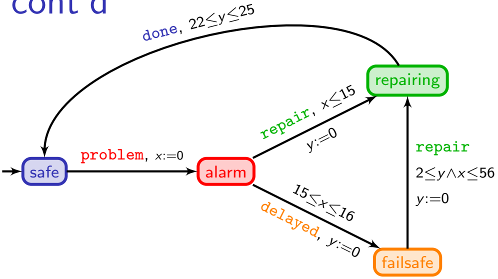
Example – cont'd



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	$\cdots$
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing
$\cdots$	15.6		17.9		17.9		40
	0		2.3		0		22.1

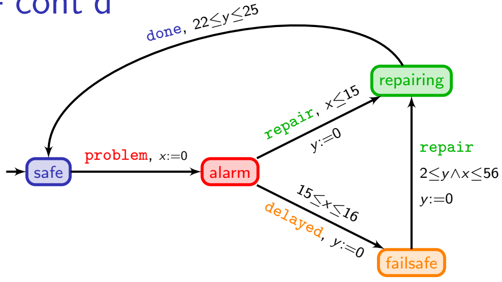
Example – cont'd



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe		
x	0		23		0		15.6		15.6		···
y	0		23		23		38.6		0		

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\texttt{done}}$	safe
···	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

Example – cont'd

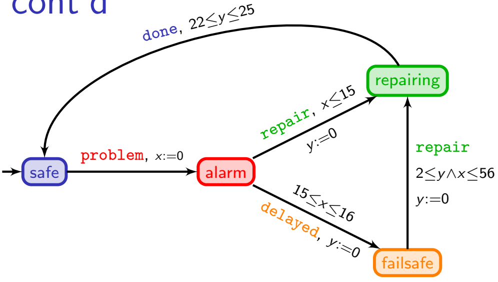


valuation

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\texttt{done}}$	safe
⋯	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

Example – cont'd



valuation

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe		
x	0		23		0		15.6		15.6		···
y	0		23		23		38.6		0		

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\texttt{done}}$	safe
···	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

This run reads the timed word
(`problem`, 23)(`delayed`, 38.6)(`repair`, 40.9)(`done`, 63).

Discrete-time semantics

...because computers are digital!

[Alur91] Techniques for automatic verification of real-time systems. *PhD thesis*, 1991.

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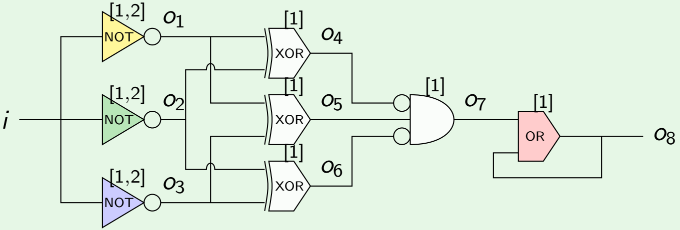
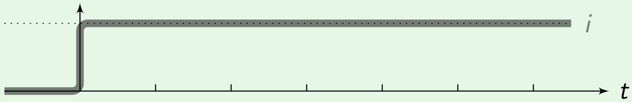


- under discrete-time, the output is always 0:



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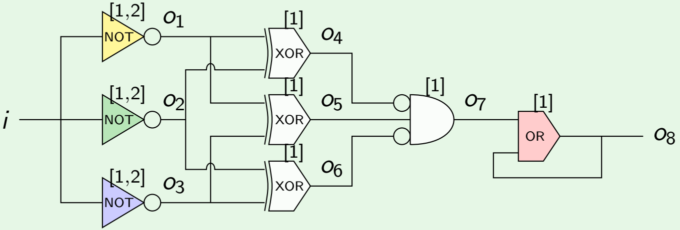

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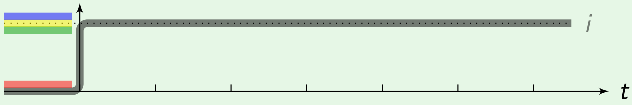

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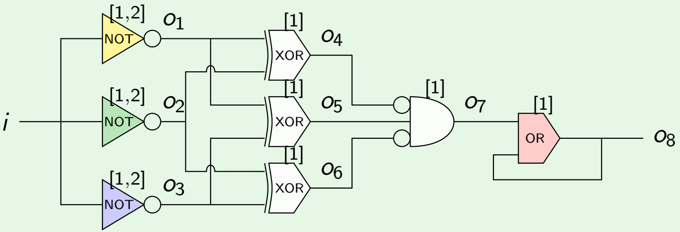

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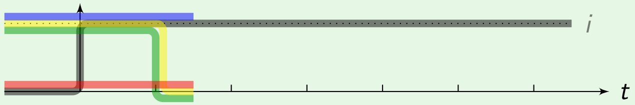

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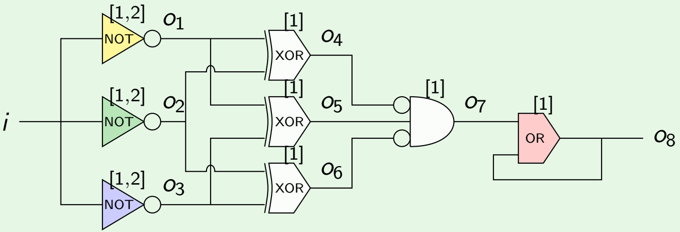

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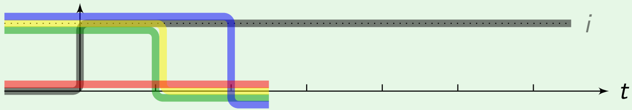

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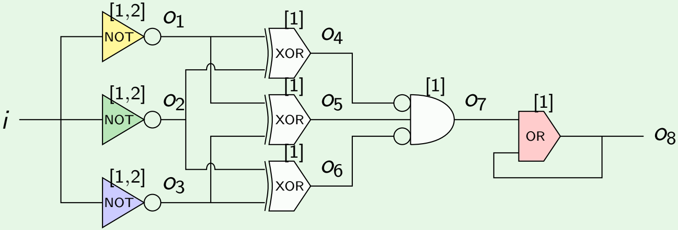

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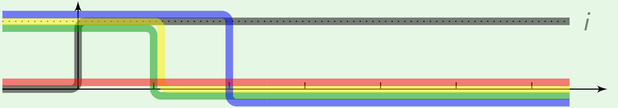

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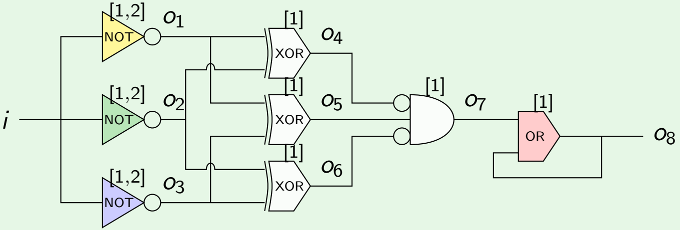
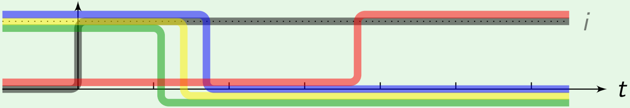


- under continuous-time, the output can be 1:



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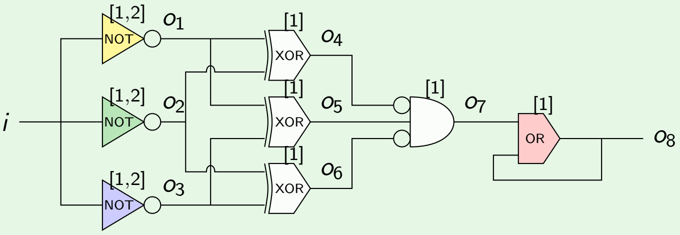

Finding the correct granularity (if one exists) is hard!

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Continuous-time semantics

...real-time models for real-time systems!

Continuous-time semantics

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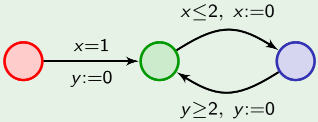
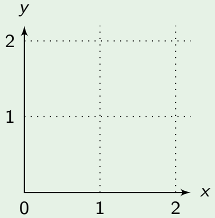
Example

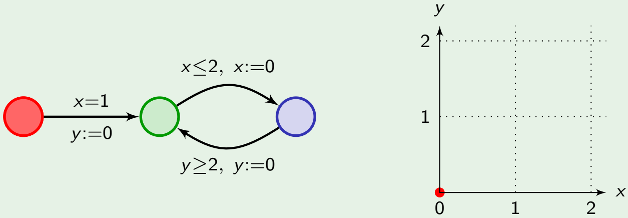




Continuous-time semantics

...real-time models for real-time systems!

Example



Continuous-time semantics

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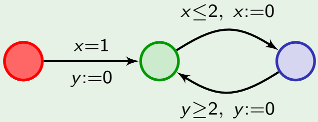

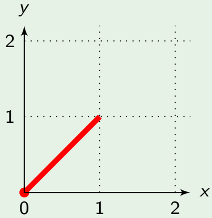

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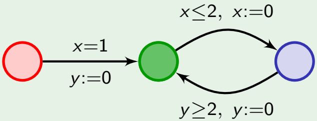

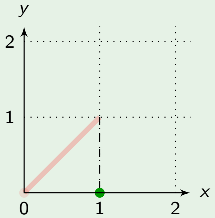

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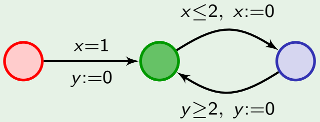

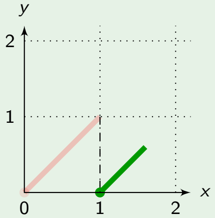

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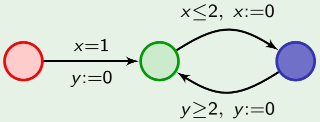

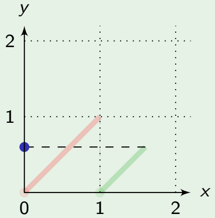

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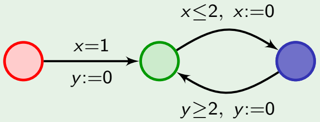

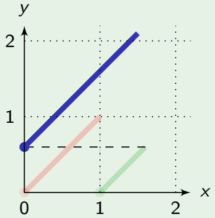

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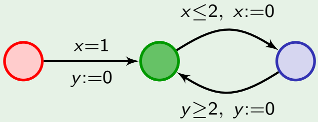

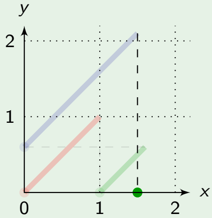

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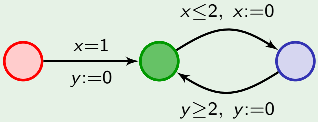

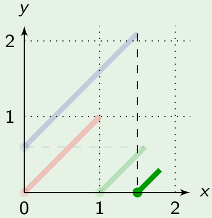

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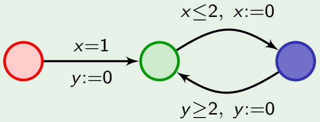

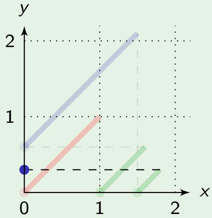

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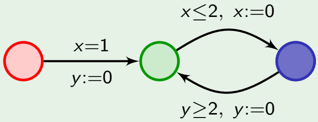

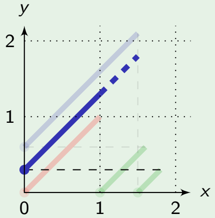

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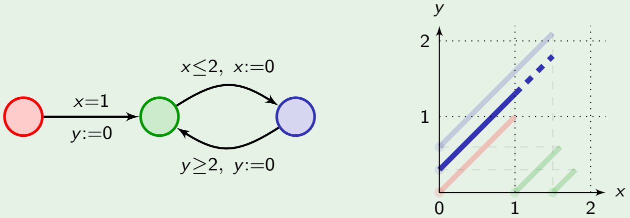

We will focus on the continuous-time semantics, and discuss further its relevance at the end of the tutorial

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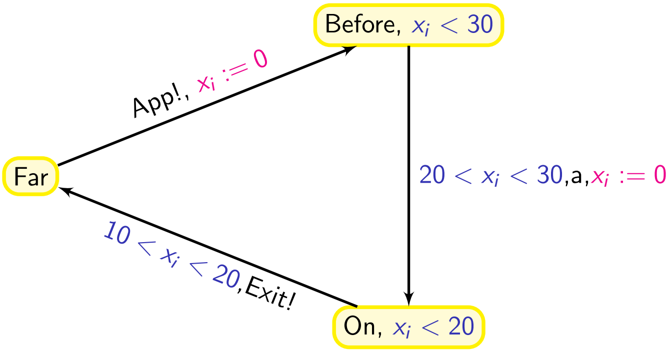
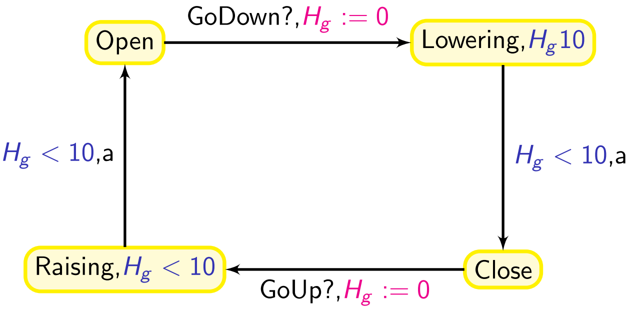
The train crossing example

Train$_i$ with $i = 1, 2, \ldots$



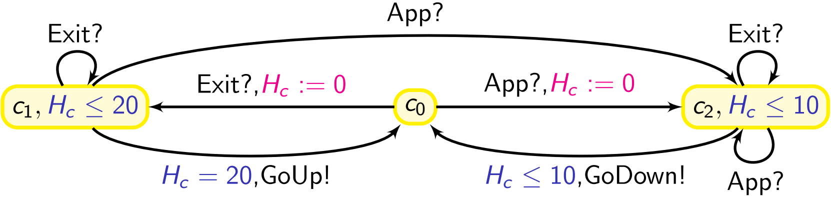
The train crossing example – cont'd

The gate:



The train crossing example – cont'd

The controller:



The train crossing example – cont'd

We use the synchronization function f:

Train_1	Train_2	Gate	Controller	
App!	.	.	App?	App
.	App!	.	App?	App
Exit!	.	.	Exit?	Exit
.	Exit!	.	Exit?	Exit
a	.	.	.	a
.	a	.	.	a
.	.	a	.	a
.	.	GoUp?	GoUp!	GoUp
.	.	GoDown?	GoDown!	GoDown

to define the parallel composition $(\text{Train}_1 \parallel \text{Train}_2 \parallel \text{Gate} \parallel \text{Controller})$

NB: the parallel composition does not add expressive power!

The train crossing example – cont'd

Some properties one could check:

- Is the gate closed when a train crosses the road?

The train crossing example – cont'd

Some properties one could check:

- Is the gate closed when a train crosses the road?
- Is the gate always closed for less than 5 minutes?

Another example: A mutual exclusion protocol

A mutual exclusion protocol with a shared variable *id* [AL94].

[AL94] Abadí, Lamport. An old-fashioned recipe for real time *(ACM Transactions on Programming Languages and Systems)*.

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A mutual exclusion protocol with a shared variable *id* [AL94].

Process i:

a : await ($id = 0$);
b : set id to i;
c : await ($id = i$);
d : enter critical section.

$\rightsquigarrow$ a max. delay k_1 between a and b
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$\rightsquigarrow$ See the demo with the tool Uppaal
(can be downloaded on `http://www.uppaal.com/`)

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Another example: The task graph scheduling problem

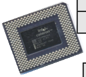
Compute $D \times (C \times (A+B)) + (A+B) + (C \times D)$ using two processors:

P_1 (fast):

time	
$+$	2 picoseconds
$\times$	3 picoseconds

energy	
idle	10 Watt
in use	90 Watts

P_2 (slow):



time	
$+$	5 picoseconds
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idle	20 Watts
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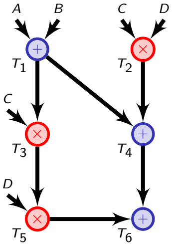

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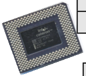

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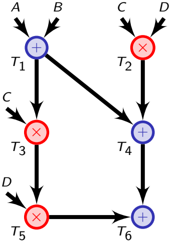

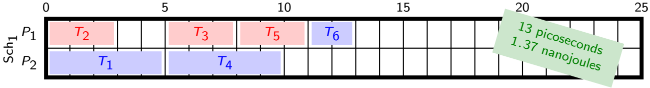

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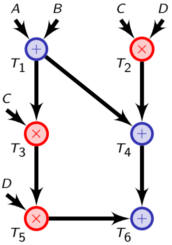

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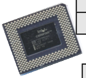

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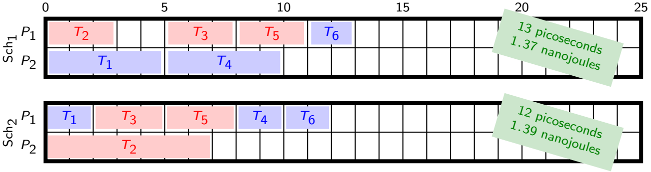

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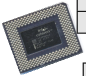

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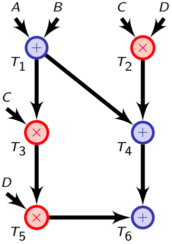

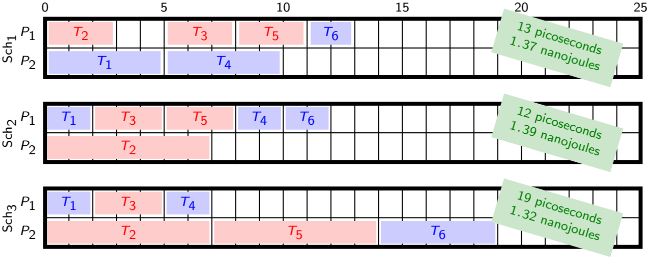

Modelling the task graph scheduling problem

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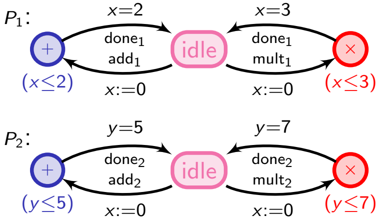

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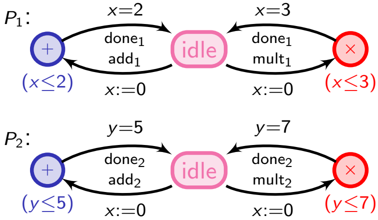

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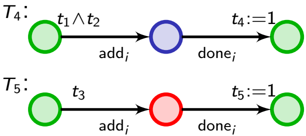

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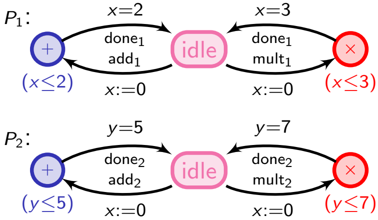

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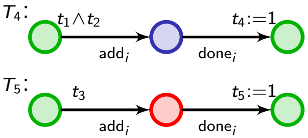

$\rightsquigarrow$ build the synchronized product of all these automata

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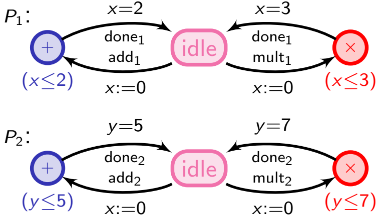

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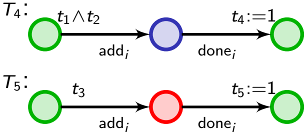

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A schedule: a path in the global system which reaches $t_1 \wedge \cdots \wedge t_6$

Modelling the task graph scheduling problem

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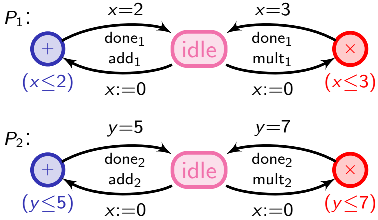

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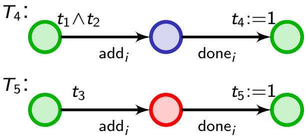

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Questions one can ask

- Can the computation be made in no more than 10 time units?
- Is there a scheduling along which no processor is ever idle?
- ...

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Verification

Basic verification problems

- basic reachability/safety properties
- basic liveness properties

Verification

Basic verification problems

- basic reachability/safety properties (final states)
- basic liveness properties (ω-regular conditions)

Is the language accepted by a timed automaton empty?

Verification

Basic verification problems

- Problem: the set of configurations is infinite
  $\leadsto$ classical methods for finite-state systems cannot be applied

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- Problem: the set of configurations is infinite
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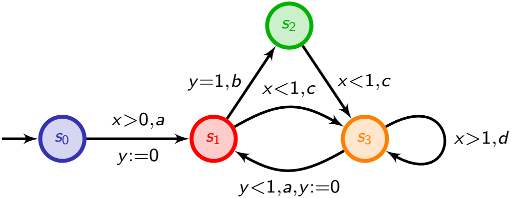
Theorem [AD90,AD94]

The emptiness problem for timed automata is decidable and PSPACE-complete.

[AD90] Alur, Dill. Automata for modeling real-time systems *(ICALP'90)*.
[AD94] Alur, Dill. A theory of timed automata *(Theoretical Computer Science)*.

An example [AD94]



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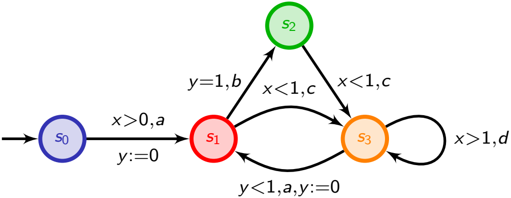

Starting at s_0, can we visit s_2 and then s_3?

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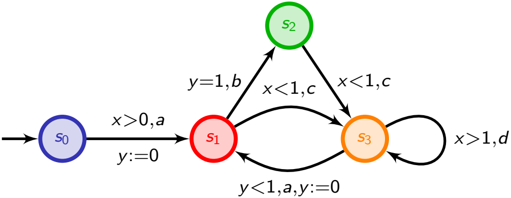

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Method: construct a finite abstraction

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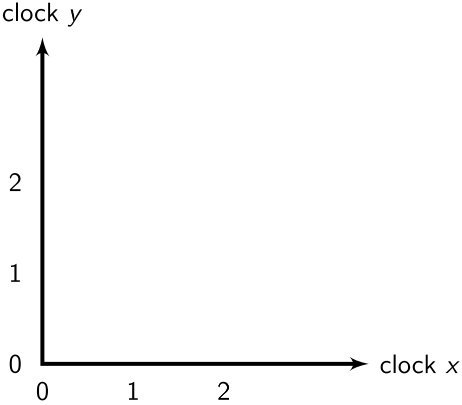
The region abstraction



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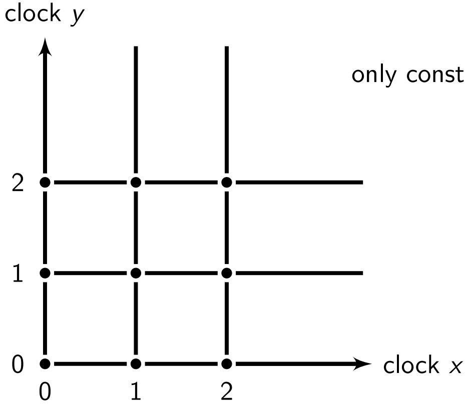
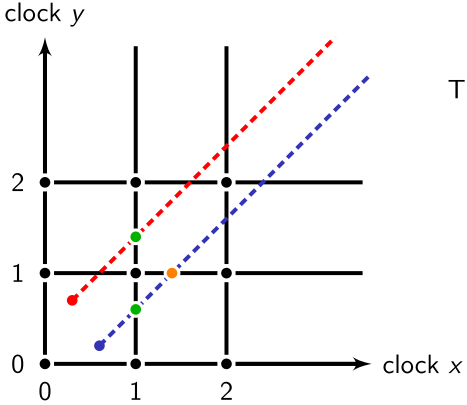


only constraints: $x \sim c$ with $c \in \{0, 1, 2\}$
$y \sim c$ with $c \in \{0, 1, 2\}$

- "compatibility" between regions and constraints

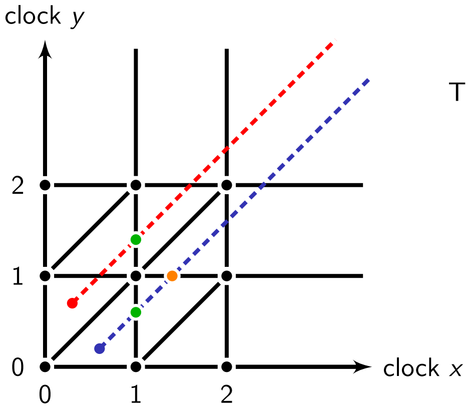
The region abstraction



The path
- can be fired from
- cannot be fired from

- "compatibility" between regions and constraints
- "compatibility" between regions and time elapsing

The region abstraction



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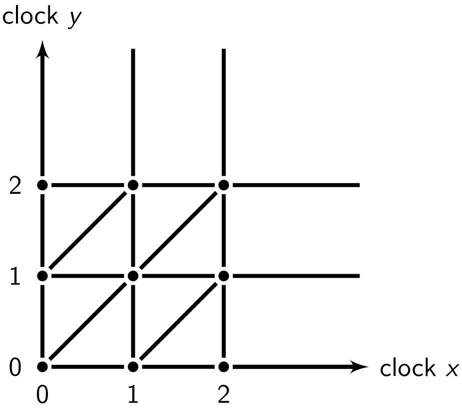

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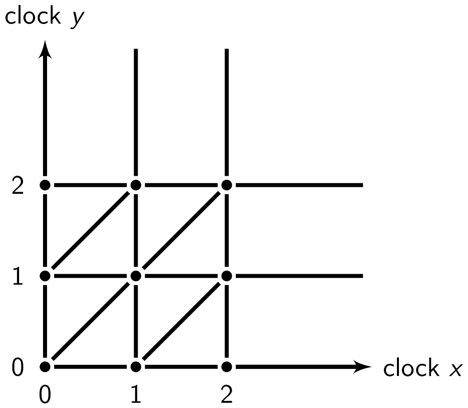
$\rightsquigarrow$ an equivalence of finite index

The region abstraction



- "compatibility" between regions and constraints
- "compatibility" between regions and time elapsing

$\rightsquigarrow$ an equivalence of finite index
a time-abstract bisimulation

Time-abstract bisimulation

This is a relation between • and • such that:

Time-abstract bisimulation

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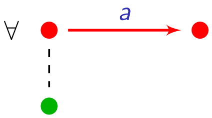

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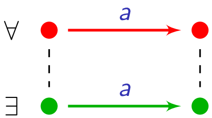

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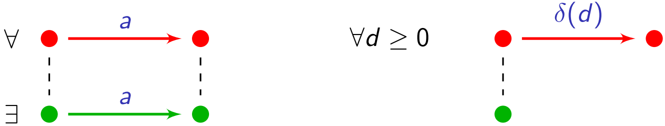

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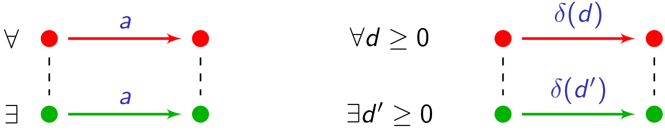

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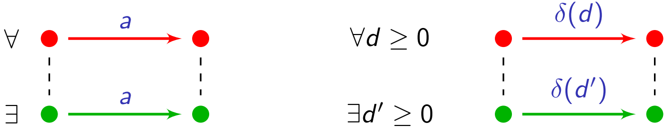

... and vice-versa (swap • and •).

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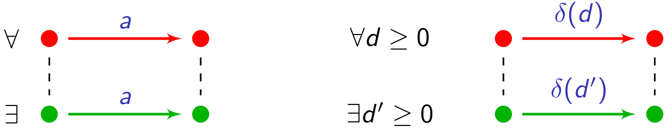

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Consequence

$$\forall \quad (\ell_1, v_1) \xrightarrow{d_1,a_1} (\ell_2, v_2) \xrightarrow{d_2,a_2} (\ell_3, v_3) \xrightarrow{d_3,a_3} \quad \cdots$$

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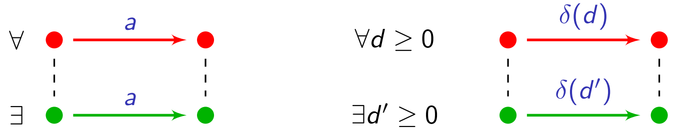

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Consequence

$$
\forall \quad
\begin{tikzcd}
(\ell_1, v_1) \arrow[r, "{d_1, a_1}"] \arrow[d] & (\ell_2, v_2) \arrow[r, "{d_2, a_2}"] \arrow[d] & (\ell_3, v_3) \arrow[r, "{d_3, a_3}"] \arrow[d] & \cdots \\
(\ell_1, R_1) \arrow[r, "a_1"] & (\ell_2, R_2) \arrow[r, "a_2"] & (\ell_3, R_3) \arrow[r, "a_3"] & \cdots
\end{tikzcd}
\quad \text{with } v_i \in R_i
$$

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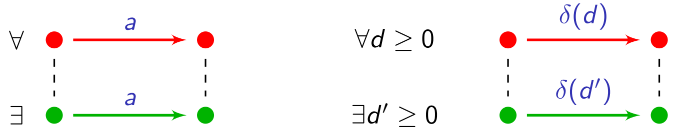

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\forall & (\ell_1, v_1) & \xrightarrow{d_1, a_1} & (\ell_2, v_2) & \xrightarrow{d_2, a_2} & (\ell_3, v_3) & \xrightarrow{d_3, a_3} & \cdots & & \\
 & \downarrow & & \downarrow & & \downarrow & & & & \\
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$\forall v'_1 \in R_1$

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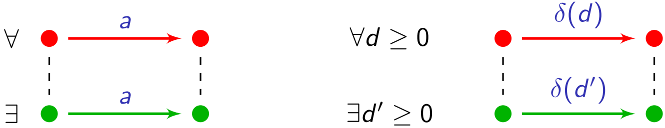

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 & \downarrow & & \downarrow & & \downarrow & & & & \\
\forall v'_1 \in R_1 \; \exists & (\ell_1, v'_1) & \xrightarrow{d'_1, a_1} & (\ell_2, v'_2) & \xrightarrow{d'_2, a_2} & (\ell_3, v'_3) & \xrightarrow{d'_3, a_3} & \cdots & & \text{with } v'_i \in R_i
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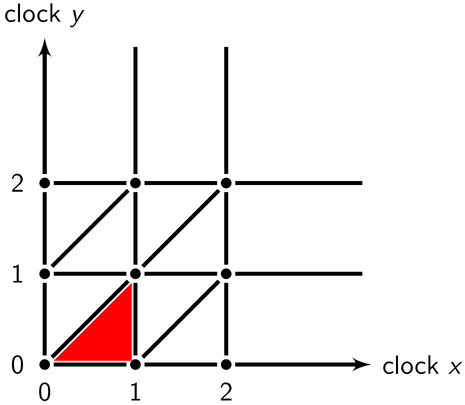
The region abstraction



- region R defined by:

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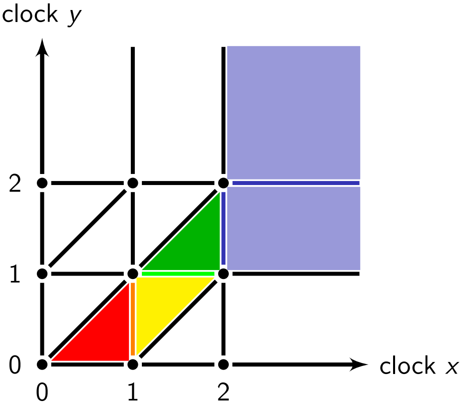
The region abstraction



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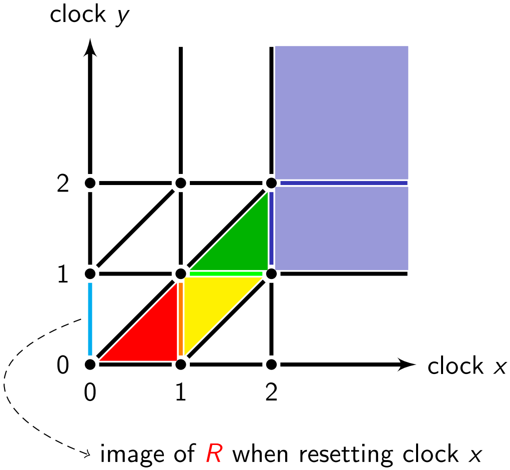

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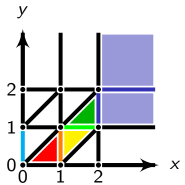
The construction of the region graph

It "mimicks" the behaviours of the clocks.



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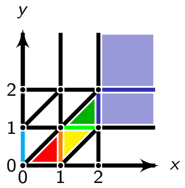
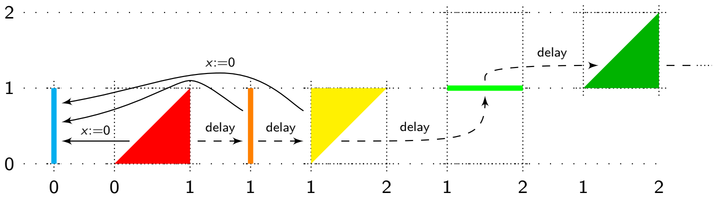
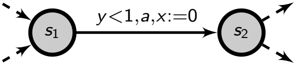


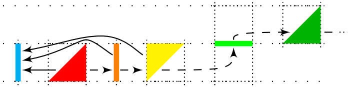


Region automaton $\equiv$ finite bisimulation quotient



timed automaton

$\otimes$



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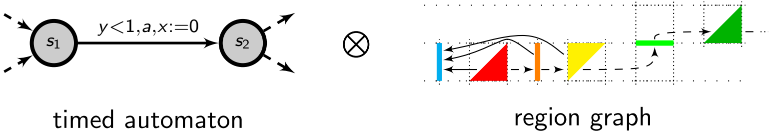

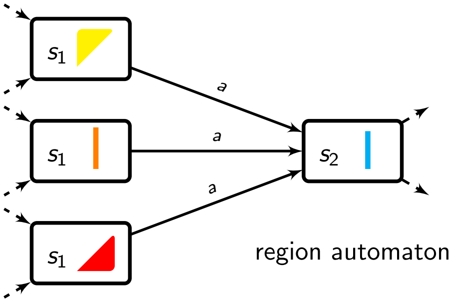

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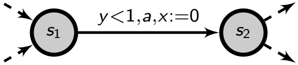

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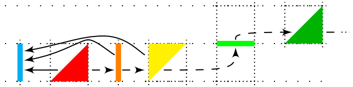

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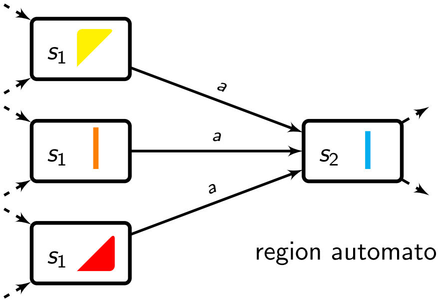
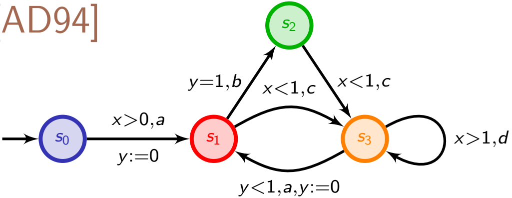


region automaton

language(reg. aut.) = UNTIME(language(timed aut.))

An example [AD94]



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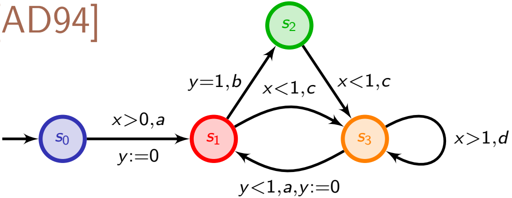

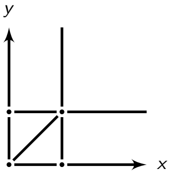

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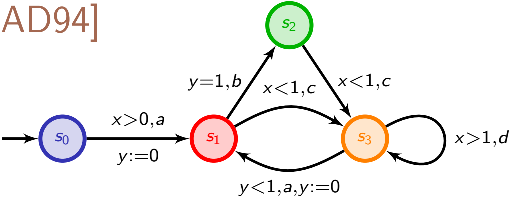
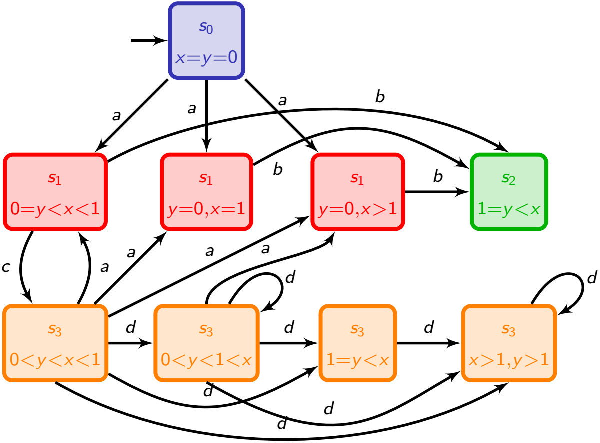
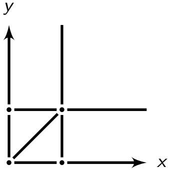
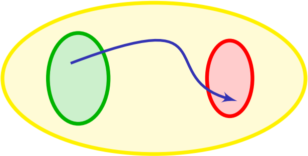
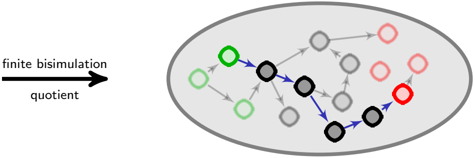




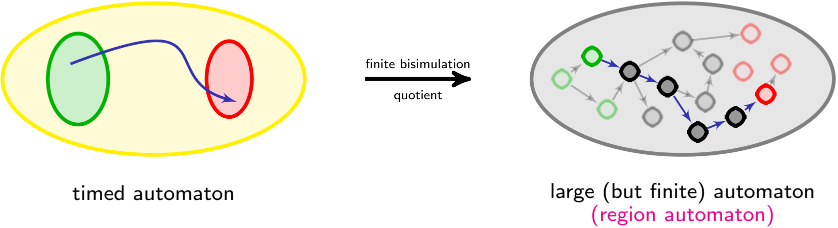




timed automaton

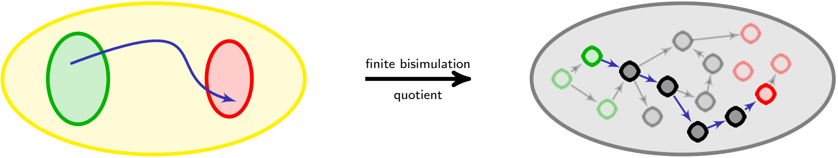


large (but finite) automaton
(region automaton)



- large: exponential in the number of clocks and in the constants (if encoded in binary). The number of regions is:

$$\prod_{x \in X} (2M_x + 2) \cdot |X|! \cdot 2^{|X|}$$



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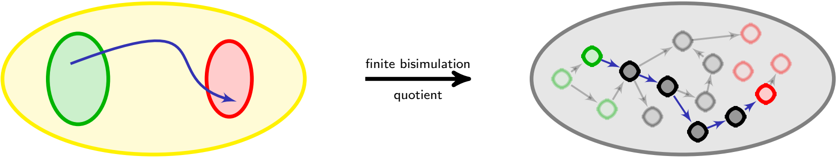
- It can be used to check for:
  - reachability/safety properties
  - liveness properties (Büchi/ω-regular properties)
  - LTL properties



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- It can be used to check for:
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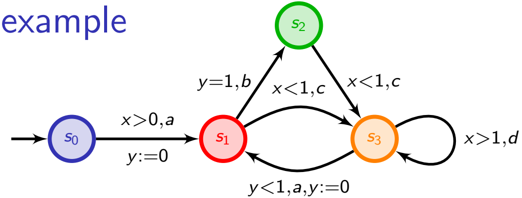
- Problems with Zeno behaviours?
  (infinitely many actions in bounded time)

Back to the example



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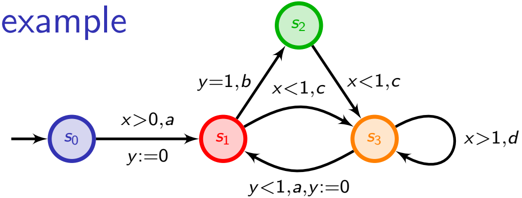

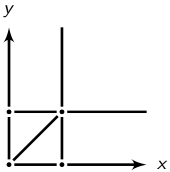

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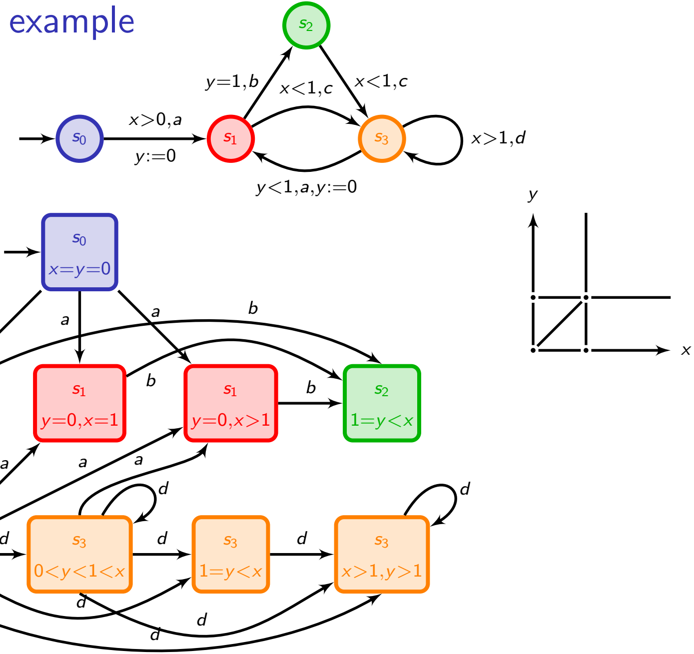

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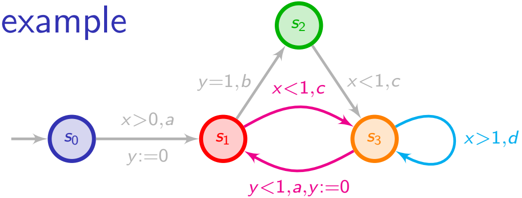
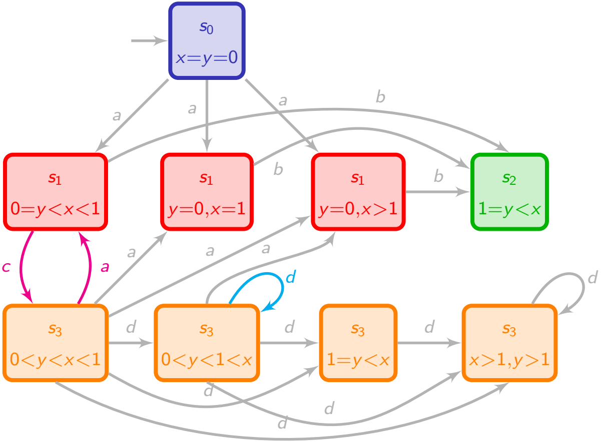
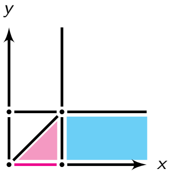
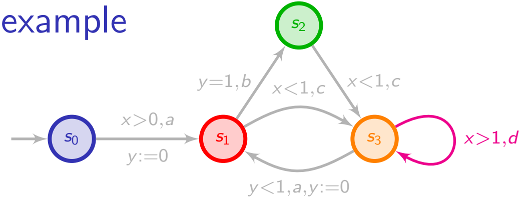


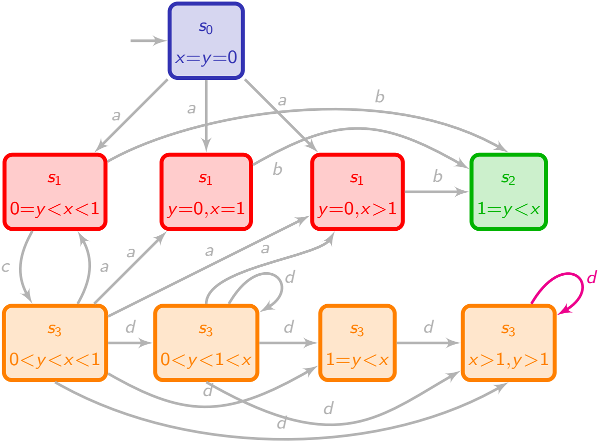


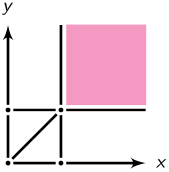


Zeno cycles

Back to the example







Cycles with
non-Zeno behaviours

Complexity issues

Theorem [AD90,AD94]

The emptiness problem for timed automata is decidable and PSPACE-complete. It even holds for two-clock timed automata [FJ13]. It is NLOGSPACE-complete for one-clock timed automata [LMS04].

[AD90] Alur, Dill. Automata for modeling real-time systems *(ICALP'90)*.
[AD94] Alur, Dill. A theory of timed automata *(Theoretical Computer Science)*.
[LMS04] Laroussinie, Markey, Schnoebelen. Model checking timed automata with one or two clocks *(CONCUR'04)*.
[FJ13] Fearnley, Jurdziński. Reachability in two-clock timed automata is PSPACE-complete *(ICALP'13)*.

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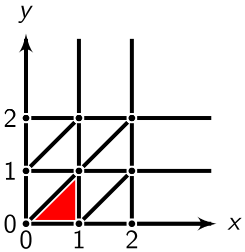

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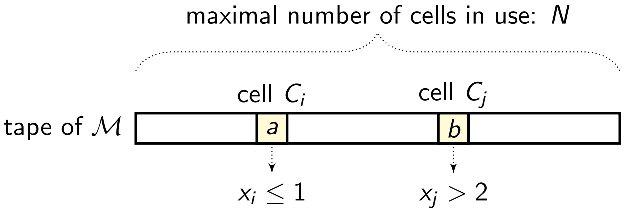
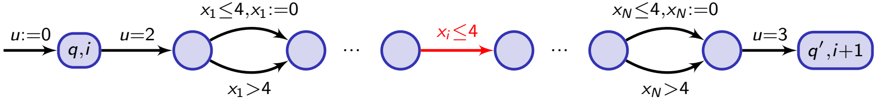


Example of the simulation of a rule $(q, a, b, q', \rightarrow)$:



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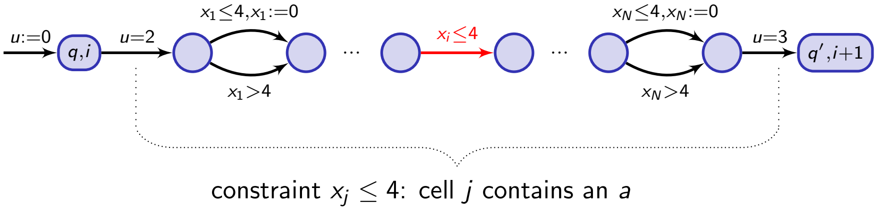

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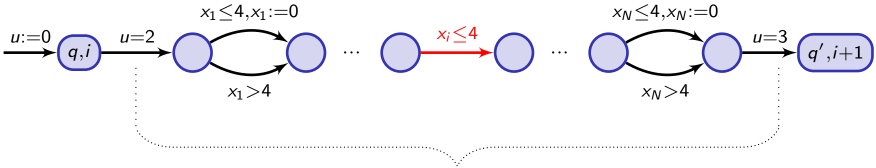

constraint $x_j \leq 4$: cell j contains an a
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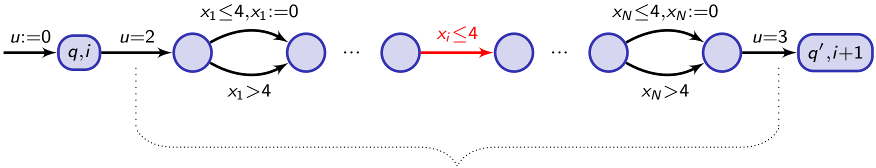

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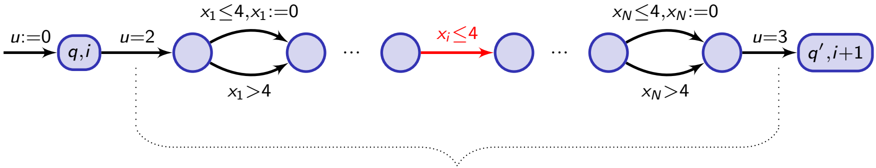

constraint $x_j \leq 4$: cell j contains an a
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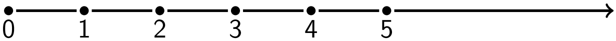
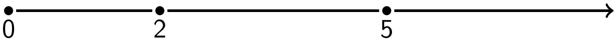
reset of clock x_j: the new content is an a
no reset of clock x_j: the new content is a b

The case of single-clock timed automata



The case of single-clock timed automata



if only constants 0, 2 and 5 are used

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- Note however that it might be hard to prove there is a finite bisimulation quotient!

Outline

1 Introduction
- Timed automata
- Examples

2 Decidability of basic properties
- The region abstraction
- Extensions of timed automata
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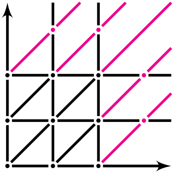

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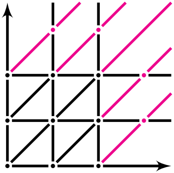

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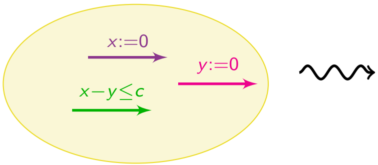
Removing diagonal constraints

copy where "$x-y \leq c$"

Assume $c \geq 0$



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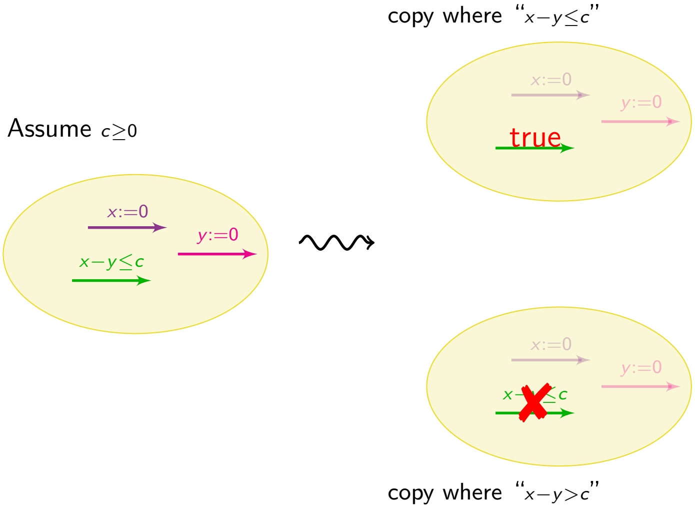

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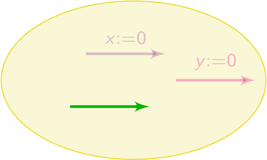

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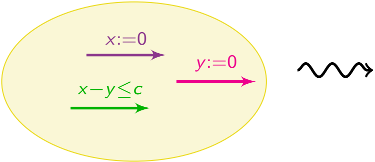

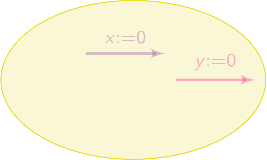

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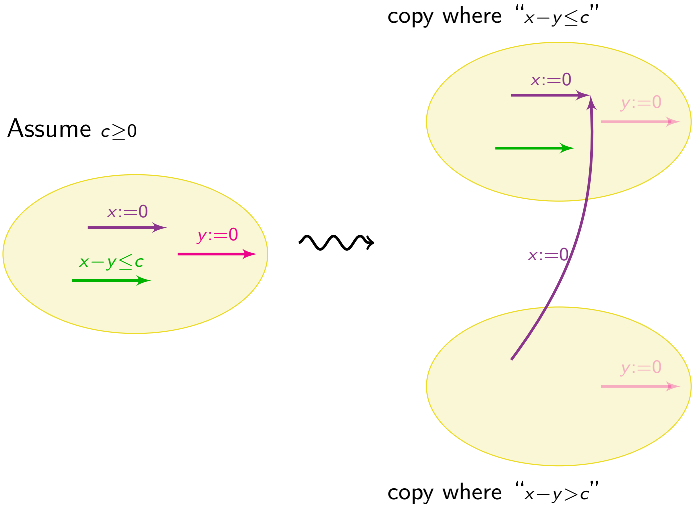

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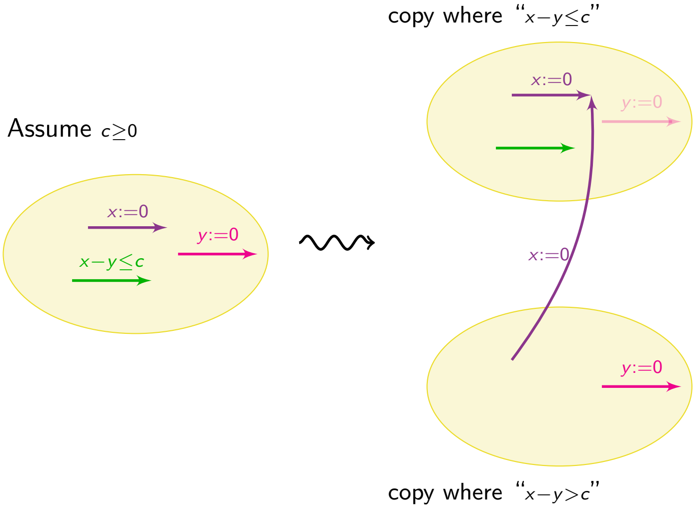

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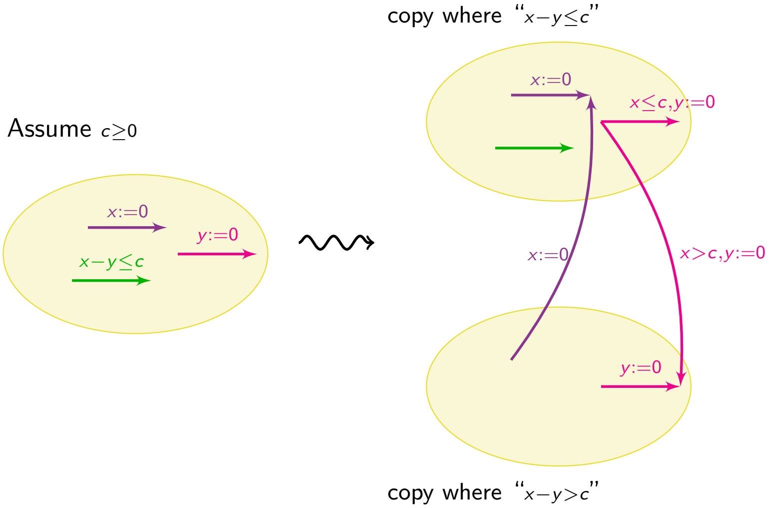

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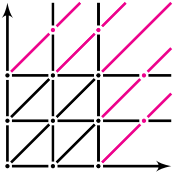

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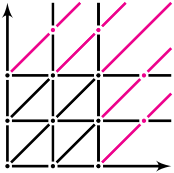

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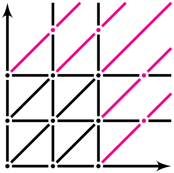

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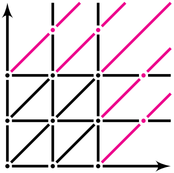

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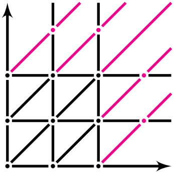

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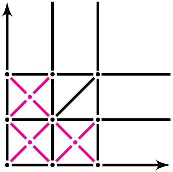

is a time-abstract bisimulation (when two clocks x and y and constraints $x + y \sim c$)!

What if we allow more operations on clocks?

- that can be transfer operations (*i.e.* $x := y$), or reinitialization operations (*i.e.* $x := 4$), or ... [BDFP04]

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	simple constraints	+ diagonal constraints
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$x :< c$		
$x :> c$		
$x :\sim y + c$		
$y + c <: x :< y + d$		
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$\rightsquigarrow$ need of being very careful when using more operations on clocks!

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Visually...

The example of decrement updates $x := x - 1$

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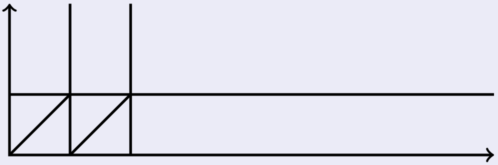

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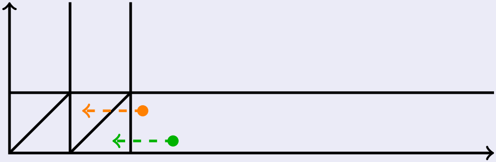

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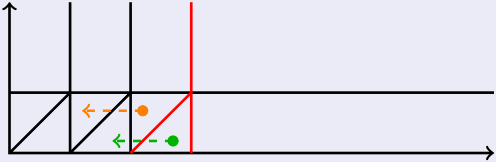

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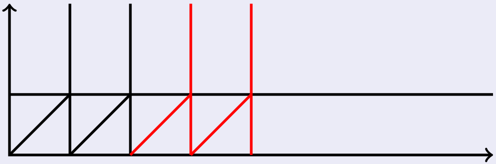

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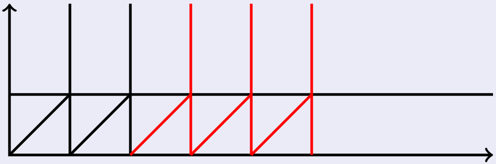

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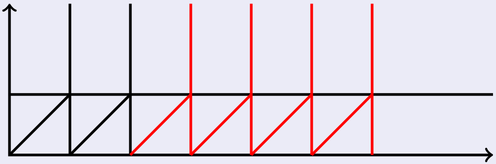

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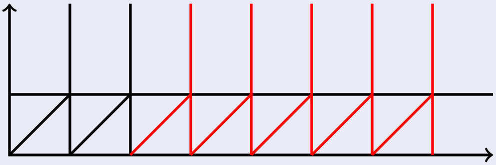

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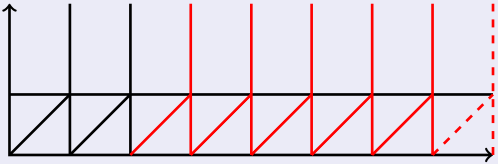

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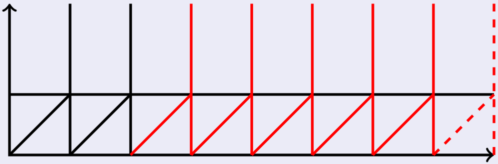

$\rightsquigarrow$ an infinite number of regions

And formally...

- We can simulate a two-counter machine!

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Definition

A two-counter machine is a finite set of instructions over two counters (c and d):

- Incrementation:
  `(p):` $c := c + 1$; `goto (q)`
- Decrementation:
  `(p): if` $c > 0$ `then` $c := c - 1$; `goto (q) else goto (r)`

Theorem [Minsky 67]

The halting and recurring problems for two counter machines are undecidable.

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- We can simulate a two-counter machine!
- Clocks x and y store the two counters...

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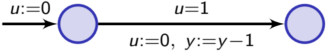
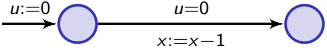


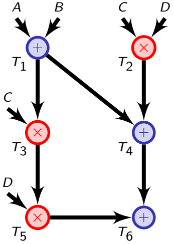
Increment x



Decrement x

Back to the task-graph scheduling problem

Compute $D\times(C\times(A+B))+(A+B)+(C\times D)$ using two processors:

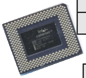


P_1 (fast):

time	
$+$	2 picoseconds
$\times$	3 picoseconds

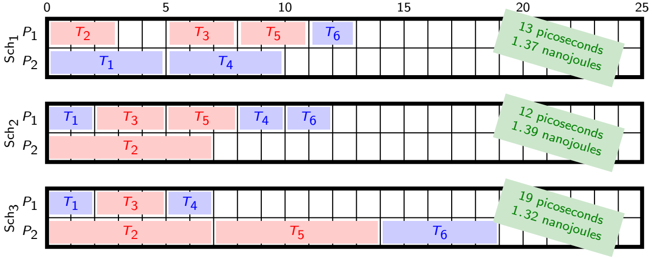
energy	
idle	10 Watt
in use	90 Watts

P_2 (slow):



time	
$+$	5 picoseconds
$\times$	7 picoseconds

energy	
idle	20 Watts
in use	30 Watts



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energy	
idle	10 Watt

P_2 (slow):

time	
+	5 picoseconds
×	7 picoseconds

energy	
idle	20 Watts

How to model energy consumption?

A note on hybrid automata (see more on Thursday)

a discrete control (the mode of the system)

$+$ continuous evolution of the variables within a mode

[HKPV95] Henzinger, Kopke, Puri, Varaiya. What's decidable wbout hybrid automata? *(SToC'95)*.

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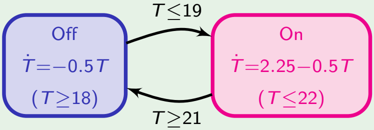
The thermostat example



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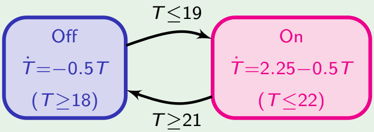

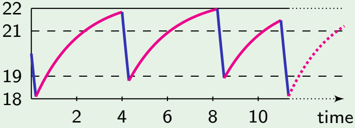

Theorem [HKPV95]

The reachability problem is undecidable in hybrid automata, even for stopwatch automata.

(stopwatch automata: timed automata in which clocks can be stopped)

[HKPV95] Henzinger, Kopke, Puri, Varaiya. What's decidable wbout hybrid automata? *(SToC'95)*.

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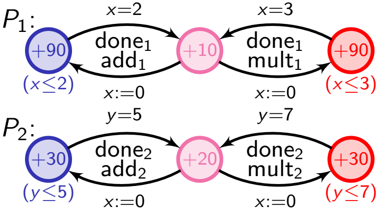
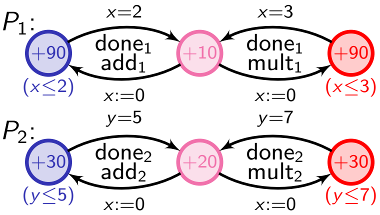
Weighted timed automata



Weighted timed automata



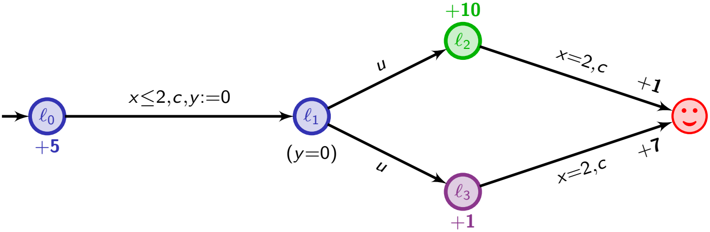
The model of weighted automata

hybrid variables are observer variables (they do not constrain a priori the system)

$\rightsquigarrow$ models energy consumption, bandwidth, price to pay, etc.

Skip

Weighted/priced timed automata [ALP01,BFH+01]



[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
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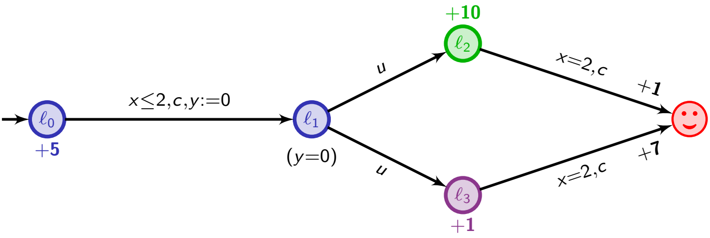
Weighted/priced timed automata [ALP01,BFH+01]



	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

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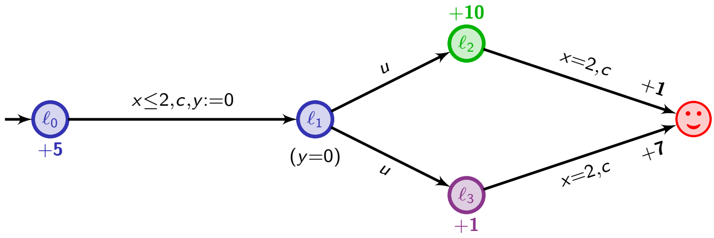
Weighted/priced timed automata [ALP01,BFH+01]



$$
\begin{array}{ccccccccccc}
 & \ell_0 & \xrightarrow{1.3} & \ell_0 & \xrightarrow{c} & \ell_1 & \xrightarrow{u} & \ell_3 & \xrightarrow{0.7} & \ell_3 & \xrightarrow{c} & ☺ \\
x & 0 & & 1.3 & & 1.3 & & 1.3 & & 2 & & \\
y & 0 & & 1.3 & & 0 & & 0 & & 0.7 & &
\end{array}
$$

cost :

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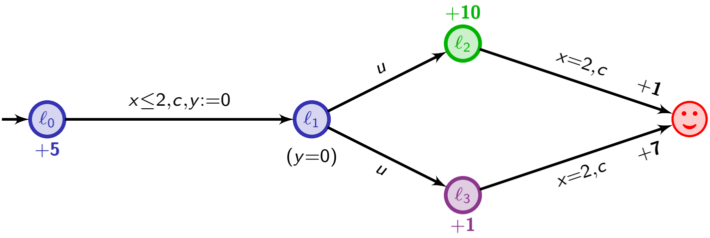

	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : 6.5

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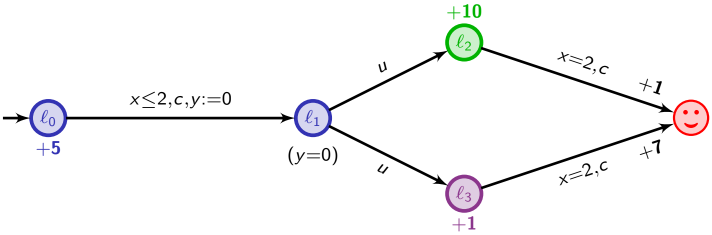

	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : 6.5 + 0

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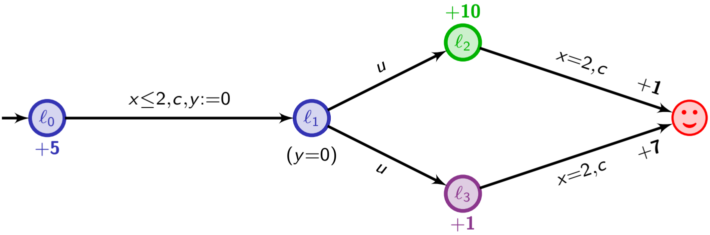

	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : $6.5 + 0 + 0$

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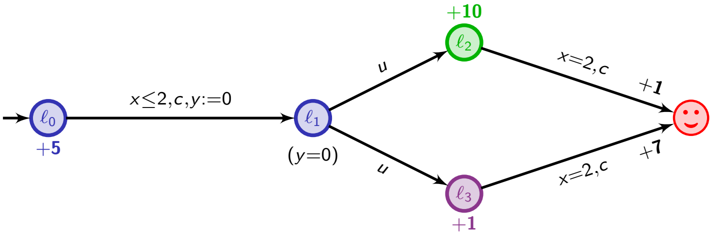

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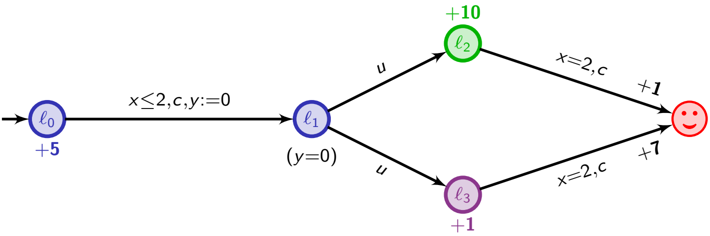

	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : 6.5 + 0 + 0 + 0.7 + 7

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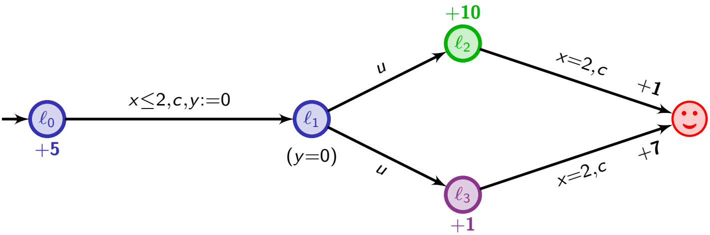

	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : $6.5 + 0 + 0 + 0.7 + 7 = 14.2$

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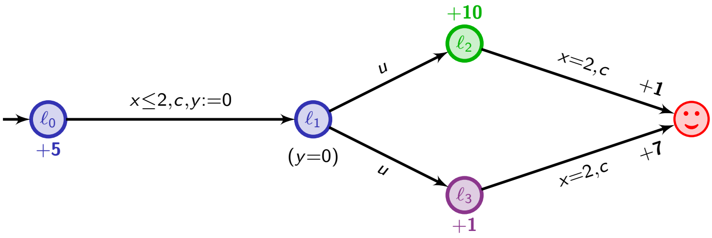

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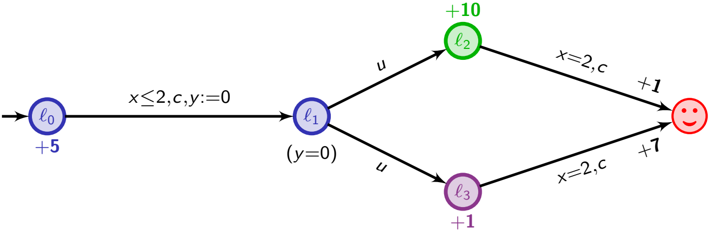

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$$5t + 10(2 - t) + 1$$

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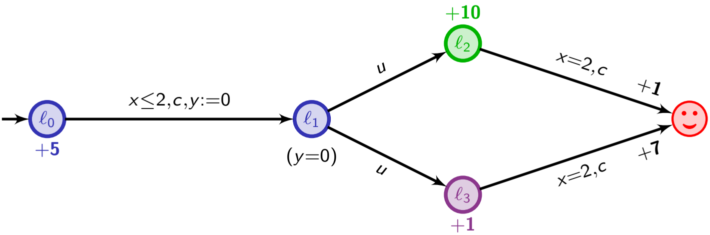

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$$5t + 10(2-t) + 1 \;,\; 5t + (2-t) + 7$$

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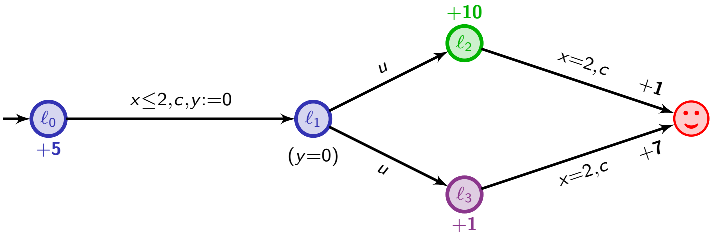

Question: what is the optimal cost for reaching ☺ ?

$$\min\left(\,5t + 10(2-t) + 1\,,\; 5t + (2-t) + 7\,\right)$$

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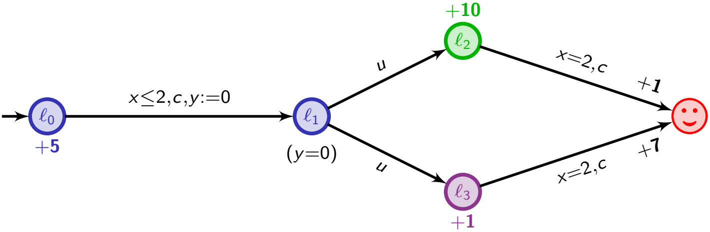

Question: what is the optimal cost for reaching ☺ ?

$$\inf_{0\leq t\leq 2} \min\left(\,5t+10(2-t)+1\,,\;5t+(2-t)+7\,\right)=9$$

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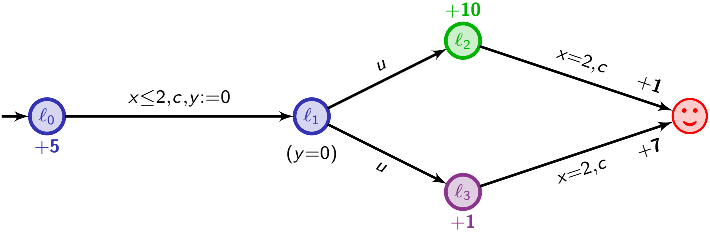

Question: what is the optimal cost for reaching 🙂?

$$\inf_{0 \leq t \leq 2} \min\left(5t + 10(2-t) + 1 \; , \; 5t + (2-t) + 7 \right) = 9$$

$\leadsto$ *strategy:* leave immediately ℓ_0, go to ℓ_3, and wait there 2 t.u.

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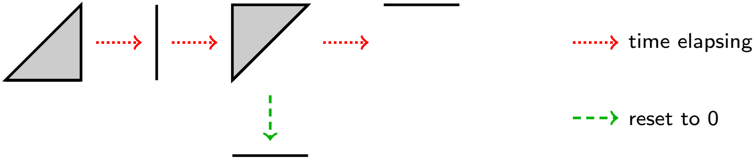
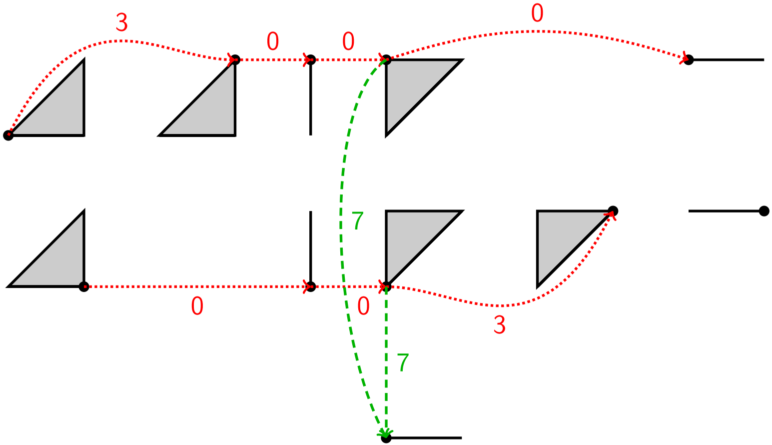
The region abstraction is not fine enough



The corner-point abstraction



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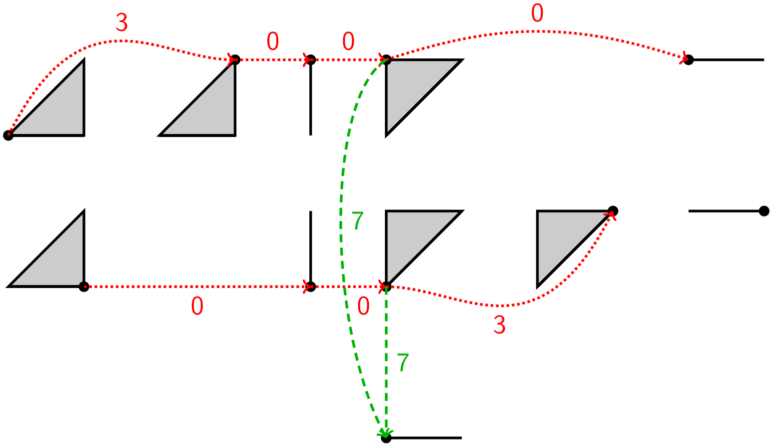

We can somehow discretize the behaviours...

From timed to discrete behaviours

Optimal reachability as a linear programming problem

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$\circ \xrightarrow{t_1} \circ \xrightarrow{t_2} \circ \xrightarrow{t_3} \circ \xrightarrow{t_4} \circ \xrightarrow{t_5} \circ \quad \cdots$$

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$\circ \xrightarrow{t_1} \circ \xrightarrow[x \leq 2]{t_2} \circ \xrightarrow{t_3} \circ \xrightarrow{t_4} \circ \xrightarrow{t_5} \circ \quad \cdots \qquad \begin{cases} t_1 + t_2 \leq 2 \\ \end{cases}$$

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$\circ \xrightarrow[y:=0]{t_1} \circ \xrightarrow[x\leq 2]{t_2} \circ \xrightarrow{t_3} \circ \xrightarrow[y\geq 5]{t_4} \circ \xrightarrow{t_5} \circ \quad \cdots \qquad \begin{cases} t_1+t_2\leq 2 \\ t_2+t_3+t_4\geq 5 \end{cases}$$

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Lemma

Let Z be a bounded constraint as above and f be a function

$$f : (t_1, ..., t_n) \mapsto \sum_{i=1}^{n} c_i t_i + c$$

well-defined on $\overline{Z}$. Then $inf_Z f$ is obtained on the border of $\overline{Z}$ with integer coordinates.

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well-defined on $\overline{Z}$. Then $inf_Z f$ is obtained on the border of $\overline{Z}$ with integer coordinates.

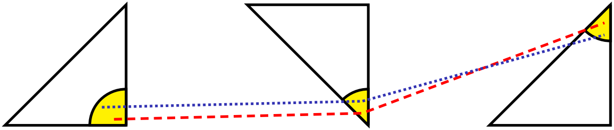
$\rightsquigarrow$ for every finite path π in $\mathcal{A}$, there exists a path Π in $\mathcal{A}_{\mathrm{cp}}$ such that

$$\mathrm{cost}(\Pi) \leq \mathrm{cost}(\pi)$$

[Π is a "corner-point projection" of π]

From discrete to timed behaviours

Approximation of abstract paths:



For any path Π of $\mathcal{A}_{\mathsf{cp}}$,

From discrete to timed behaviours

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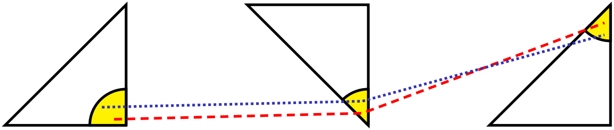
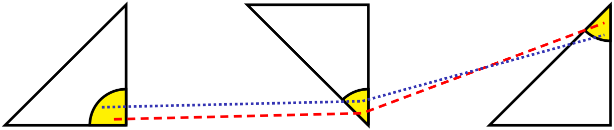


For any path Π of $\mathcal{A}_{\mathsf{cp}}$, for any $\varepsilon > 0$,

From discrete to timed behaviours

Approximation of abstract paths:

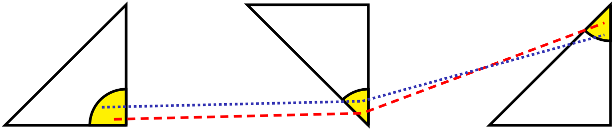


For any path Π of $\mathcal{A}_{\mathsf{cp}}$, for any $\varepsilon > 0$, there exists a path π_ε of $\mathcal{A}$ s.t.

$$\|\Pi - \pi_\varepsilon\|_\infty < \varepsilon$$

From discrete to timed behaviours

Approximation of abstract paths:



For any path Π of $\mathcal{A}_{\mathsf{cp}}$, for any $\varepsilon > 0$, there exists a path π_ε of $\mathcal{A}$ s.t.

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For every $\eta > 0$, there exists $\varepsilon > 0$ s.t.

$$\|\Pi - \pi_\varepsilon\|_\infty < \varepsilon \Rightarrow |\mathsf{cost}(\Pi) - \mathsf{cost}(\pi_\varepsilon)| < \eta$$

Optimal-cost reachability

Theorem [ALP01,BFH+01,BBBR07]

The optimal-cost reachability problem is decidable (and PSPACE-complete) in weighted timed automata.

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
[BFH+01] Behrmann, Fehnker, Hune, Larsen, Pettersson, Romijn, Vaandrager. Minimum-cost reachability in priced timed automata *(HSCC'01)*.
[BBBR07] Bouyer, Brihaye, Bruyère, Raskin. On the optimal reachability problem *(Formal Methods in System Design)*.

Further problems of interest

Relevant questions

- Optimization questions:
  - optimal reachability
  - optimal average consumption
  - ...
- Management of resources:
  - a lower bound global constraint (your bank account)
  - a lower and an upper bound global constraint (the tank of your car, the pressure in a pump)
  - ...

$\leadsto$ lots of developments, many open problems

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[BFLMS08] Bouyer, Fahrenberg, Larsen, Markey, Srba. Infinite runs in weighted timed automata with energy constraints *(FORMATS'08)*.
[BFLM10] Bouyer, Fahrenberg, Larsen, Markey. Timed automata with observers under energy constraints *(HSCC'10)*.
[BFM12] Bouyer, Larsen, Markey. Lower-bound constrained runs in weighted timed automata *(QEST'12)*.

Outline

1. Introduction
   - Timed automata
   - Examples
2. Decidability of basic properties
   - The region abstraction
   - Extensions of timed automata
   - Weighted timed automata
3. Implementation and tools
4. Other verification problems
   - Equivalence (or preorder) checking
   - Verification of timed temporal logics (short)
5. Timed control
   - Timed games
   - Weighted timed games
6. Conclusion

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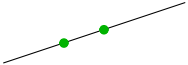

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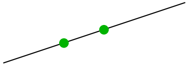

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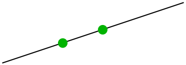

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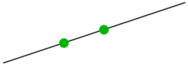

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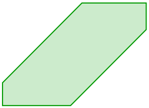

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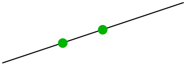

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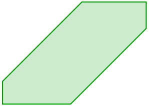

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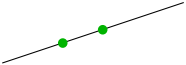

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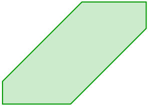
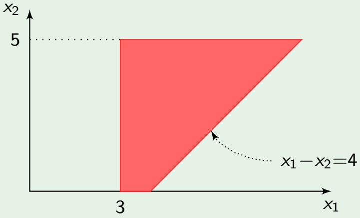


- Need of abstractions, heuristics, etc...

Zones: A symbolic representation for timed systems

Example of a zone and its DBM representation

$$Z \;=\; (x_1 \geq 3) \;\wedge\; (x_2 \leq 5) \;\wedge\; (x_1 - x_2 \leq 4)$$



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[BM83] Berthomieu, Menasche. An enumerative approach for analyzing time Petri nets *World Comupter Congress*.
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Zones: A symbolic representation for timed systems

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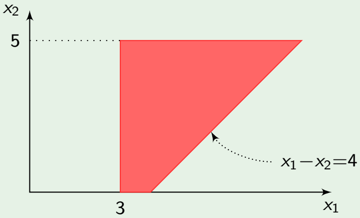

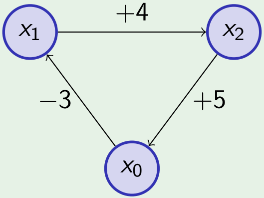

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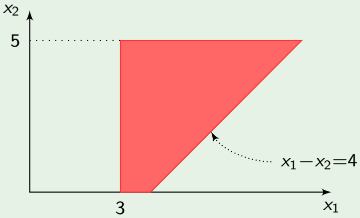

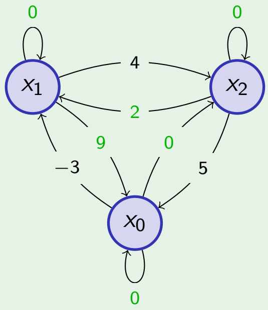

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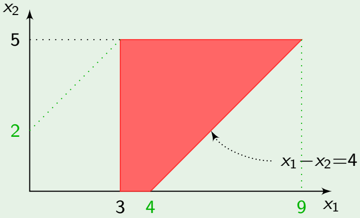

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Backward computation

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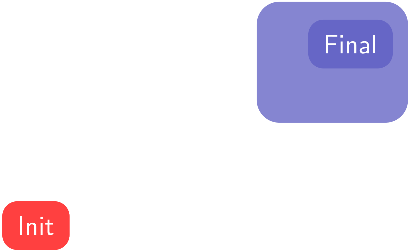

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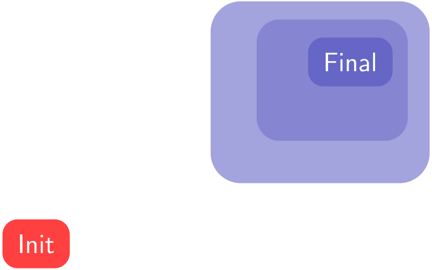

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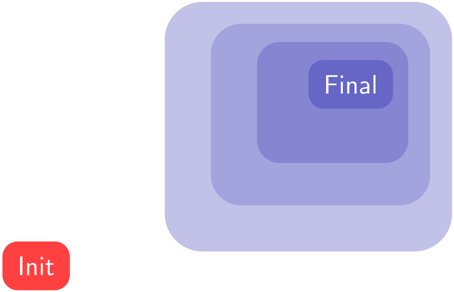
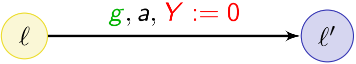


Backward computation

Note on the backward analysis of timed automata



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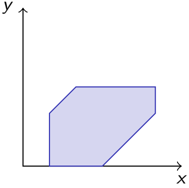

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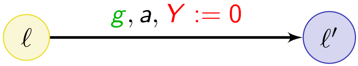

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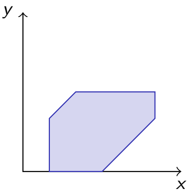

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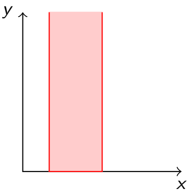

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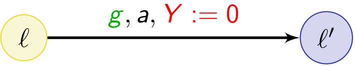

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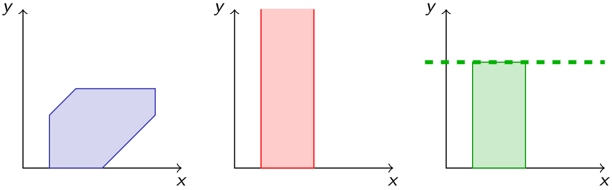

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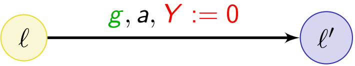

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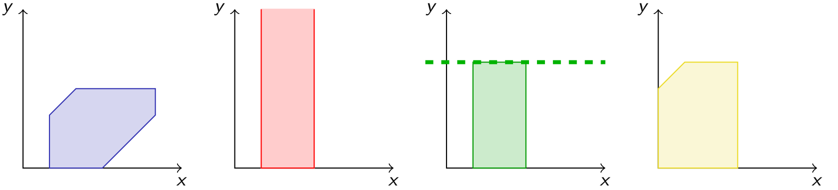

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Note on the backward analysis (cont.)

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The backward computation terminates

Because of the bisimulation property of the region abstraction:

"Every set of valuations which is computed along the backward computation is a finite union of regions"

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Let R be a region. Assume:

- $v \in \overleftarrow{R}$ (for ex. $v + t \in R$)
- $v' \equiv_{\text{reg.}} v$

There exists t' s.t. $v' + t' \equiv_{\text{reg.}} v + t$, which implies that $v' + t' \in R$ and thus $v' \in \overleftarrow{R}$.

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However the backward computation is not appropriate to manipulate other variables (think for instance of assignment $i := j.k + l$)

Forward computation

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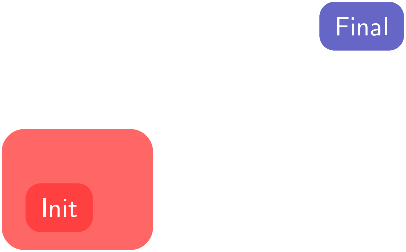

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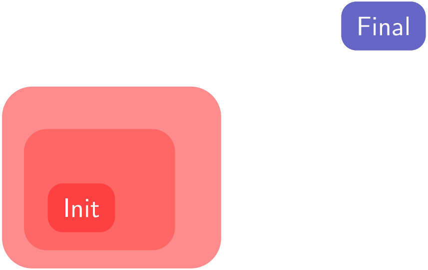

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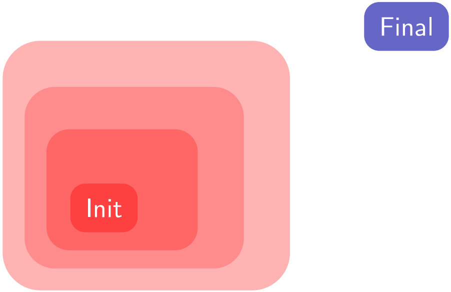

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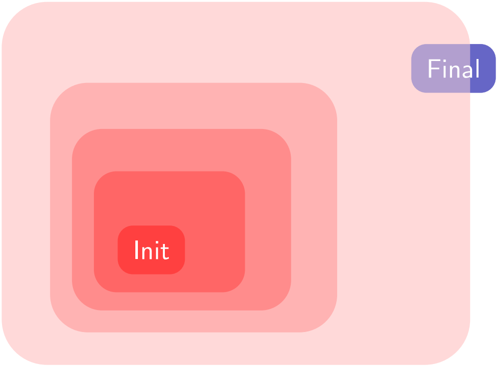

Forward analysis of timed automata

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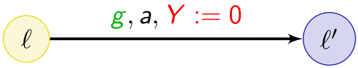

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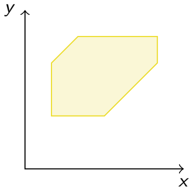

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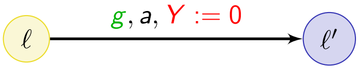

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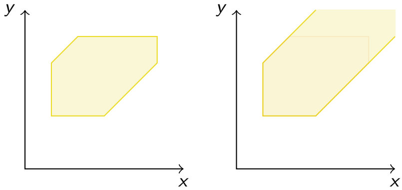

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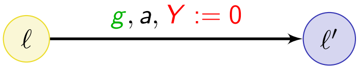

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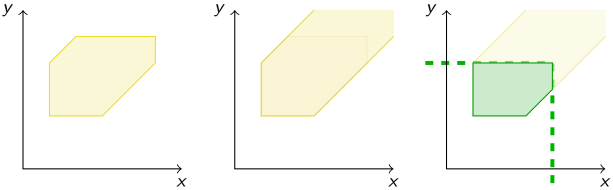

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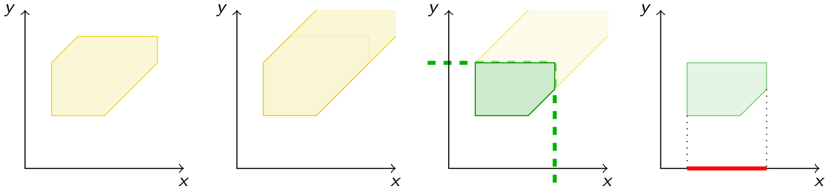

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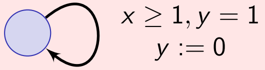

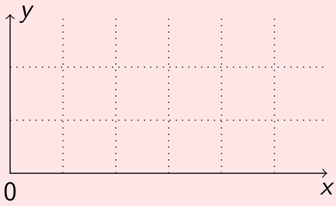

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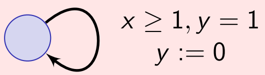

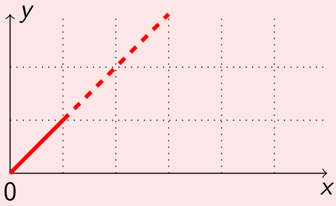

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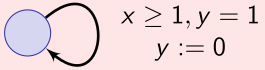

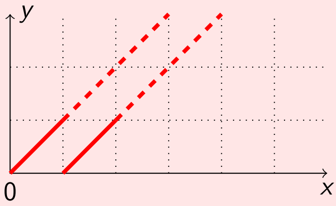

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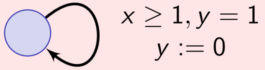

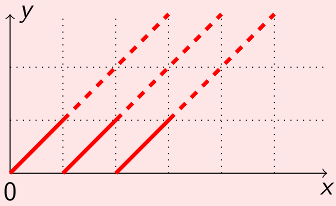

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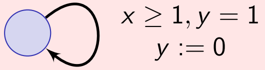

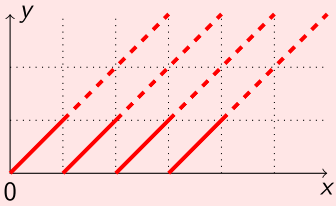

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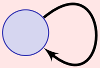
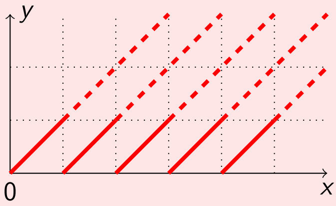


$$x \geq 1, y = 1$$
$$y := 0$$



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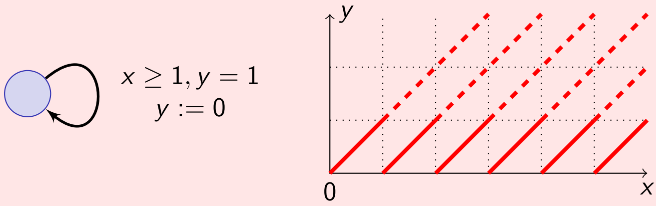
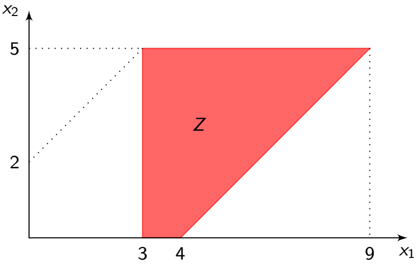


$\rightsquigarrow$ an infinite number of steps...

An abstraction: the extrapolation operator

$\mathsf{Approx}_2(Z)$: "the smallest zone containing Z that is defined only with constants no more than 2"



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$\rightsquigarrow$ The extrapolation operator ensures termination of the computation!

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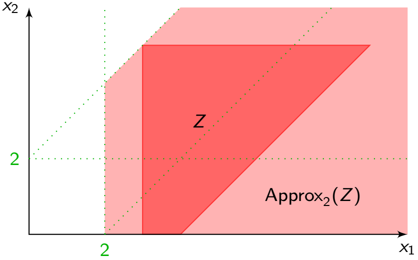

$$\begin{pmatrix} 0 & -3 & 0 \\ 9 & 0 & 4 \\ 5 & 2 & 0 \end{pmatrix} \xRightarrow{\mathsf{Approx}_2} \begin{pmatrix} 0 & -2 & 0 \\ \infty & 0 & \infty \\ \infty & 2 & 0 \end{pmatrix}$$

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The extrapolation: correctness

Theorem

The algorithm using the extrapolation w.r.t. the maximal constant is correct for timed automata with only rectangular constraints.
Note: the hypothesis on the constraints is crucial.

[Bou04] Bouyer. Forward analysis of updatable timed automata *(Formal Methods in System Design)*.

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The algorithm using the extrapolation w.r.t. the maximal constant is correct for timed automata with only rectangular constraints.
Note: the hypothesis on the constraints is crucial.

- Implemented in tools like Uppaal, Kronos, RT-Spin...
- Successfully used on many real-life examples

[Bou04] Bouyer. Forward analysis of updatable timed automata *(Formal Methods in System Design)*.

Improving the classical algorithm

- the extrapolation operator can be made coarser:
  - local extrapolation constants [BBFL03];
  - distinguish between lower- and upper-bounded contraints [BBLP03,BBLP06,HSW12,HSW13]

[BBFL03] Behrmann, Bouyer, Fleury, Larsen. Static Guard Analysis in Timed Automata Verification *(TACAS'03)*.
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This is a relation between • and • such that:

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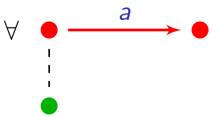

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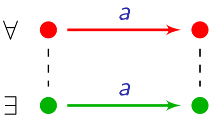

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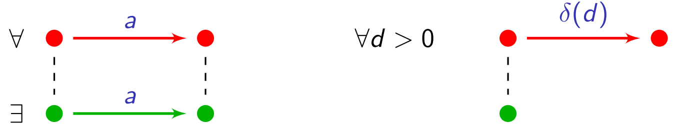

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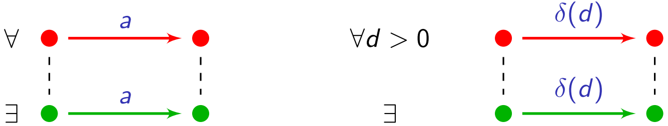

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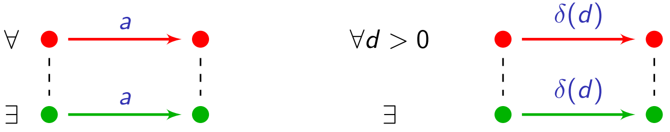

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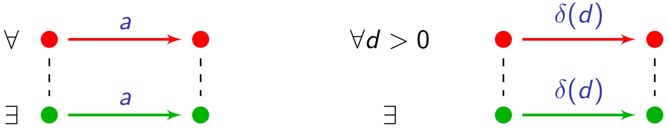

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Theorem

Strong timed (bi)simulation between timed automata is decidable and EXPTIME-complete.

(see later for a simple proof of the upper bound)

Language (or trace) equivalence and inclusion

Question

Given two timed automata $\mathcal{A}$ and $\mathcal{B}$, is $L(\mathcal{A}) = L(\mathcal{B})$ (resp. $L(\mathcal{A}) \subseteq L(\mathcal{B})$)?

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Theorem [AD90,AD94]

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$\rightsquigarrow$ Proof by reduction from the recurring problem of a two-counter machine

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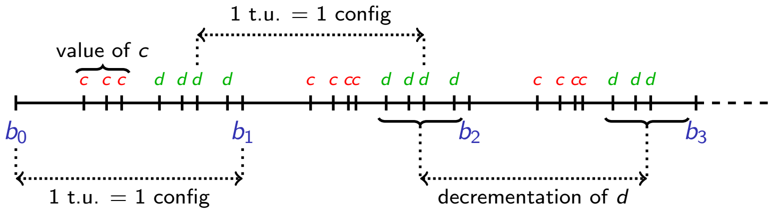

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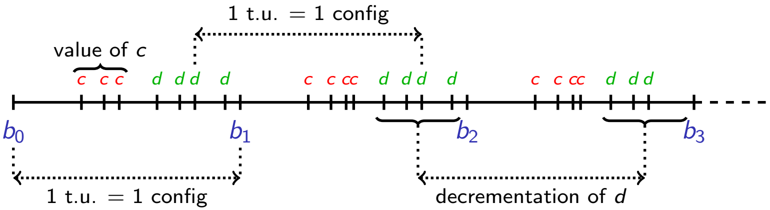

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$\rightsquigarrow$ We encode "non-behaviours" of a two-counter machine

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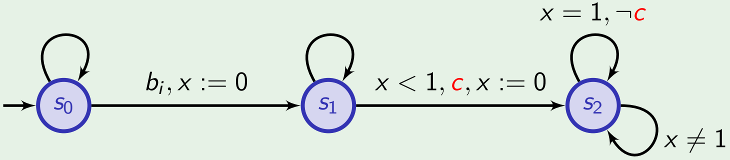
Module to check that if instruction i does not decrease counter c, then all actions c appearing less than 1 t.u. after b_i has to be followed by an other c 1 t.u. later.



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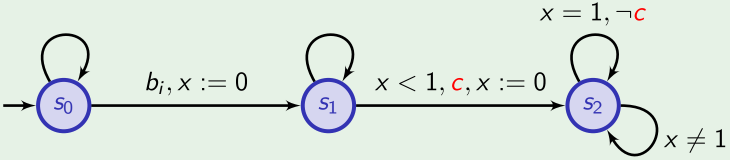

The union of all small modules is not universal
iff
The two-counter machine has a recurring computation

Bad consequences

- Language inclusion is undecidable [AD90,AD94]
  (Bad news for the application to verification)
- Complementability is undecidable [Tri03,Fin06]
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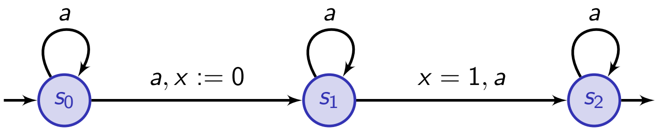

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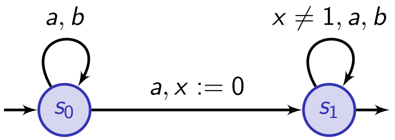

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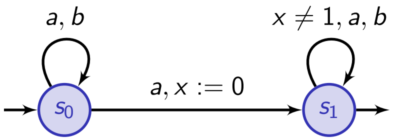

UNTIME $\left(\overline{L} \cap \{(a^* b^*, \tau) \mid \text{all } a's \text{ happen before 1 and no two } a's \text{ simultaneously}\}\right)$ is not regular **(exercise!)**

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Timed temporal logics

- Branching-time: TCTL

$$\text{TCTL} \ni \varphi \ ::= \ a \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \mathbf{E}\varphi\, \mathbf{U}_I\, \varphi$$

where I is an interval with integral bounds.

- Linear-time: MTL [Koy90]

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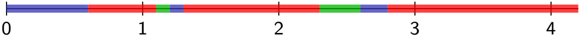

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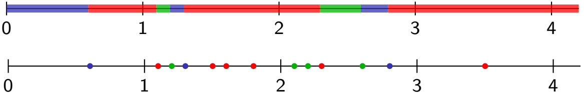

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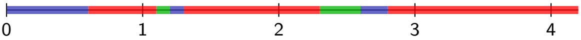

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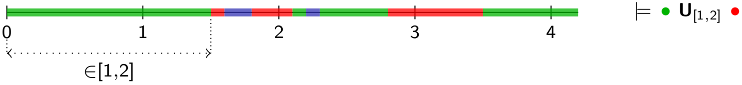

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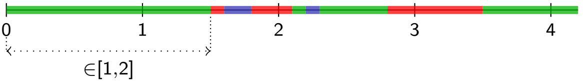
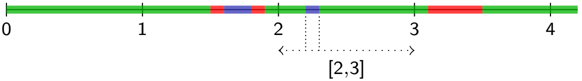


$\models \bullet\ \mathbf{U}_{[1,2]}\ \bullet$



$\not\models\ \mathbf{G}_{[2,3]}\ \bullet$

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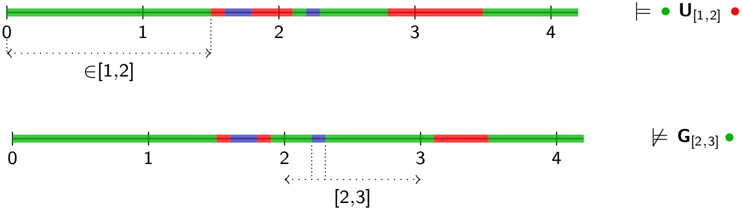

- "Every problem is followed within 56 time units by an alarm"

$$\mathbf{G}(\texttt{problem} \rightarrow \mathbf{F}_{\leq 56}\,\texttt{alarm})$$

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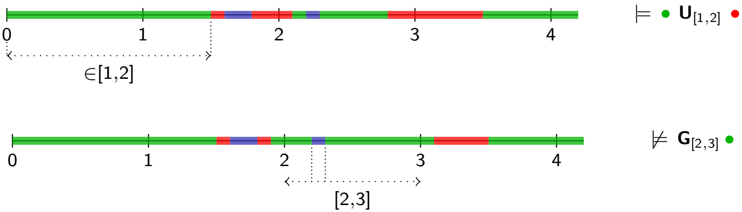

- "Every problem is followed within 56 time units by an alarm"

$$\mathbf{G}(\mathtt{problem} \rightarrow \mathbf{F}_{\leq 56}\,\mathtt{alarm})$$

- "Each time there is a problem, it is either repaired within the next 15 time units, or an alarm rings during 3 time units 12 time units later"

$$\mathbf{G}(\mathtt{problem} \rightarrow (\mathbf{F}_{\leq 15}\,\mathtt{repair} \vee \mathbf{G}_{[12,15)}\,\mathtt{alarm}))$$

Model-checking timed temporal logics

Branching-time logic TCTL [ACD93]

The model-checking of TCTL is PSPACE-complete!
(The region abstraction can be used, with an extra clock for the formula)

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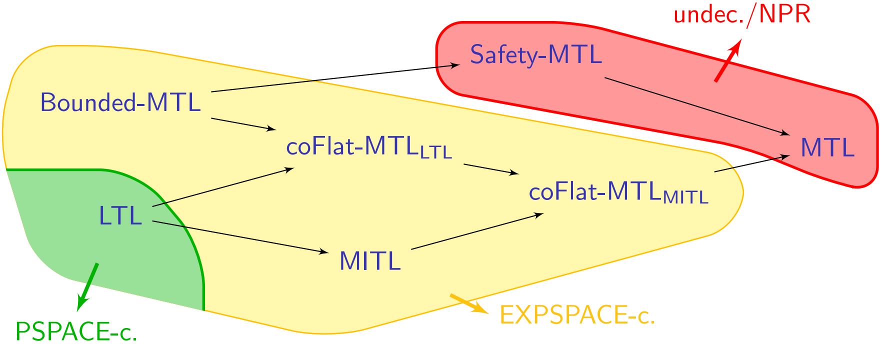

Technics: alternating timed automata, channel machines, small-model properties

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[BMOW08] Bouyer, Markey, Ouaknine, Worrell. On Expressiveness and Complexity in Real-time Model Checking *(ICALP'08)*.
[BMOW07] Bouyer, Markey, Ouaknine, Worrell. The cost of punctuality *(LICS'07)*.
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Example

$$\mathbf{G}(\bullet \to \mathbf{F}_{=1} \bullet)$$

- each time an • occurs, start a new clock, and check that a • occurs 1 time unit later
- this requires an unbounded number of clocks

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The logic MITL

- Bans "punctual" constraints
- Consequences:
  - ☞ we can bound the variability of signals
  - ☞ an MITL formula defines a timed regular language

Formula $\mathbf{G}_{(0,1)}(a \rightarrow \mathbf{F}_{[1,2]}\, b)$

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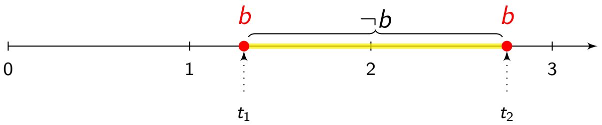

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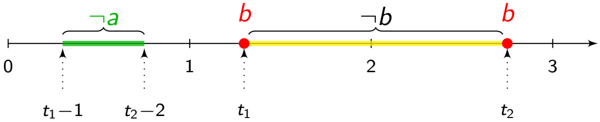

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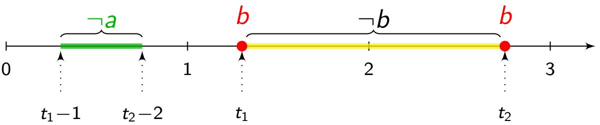

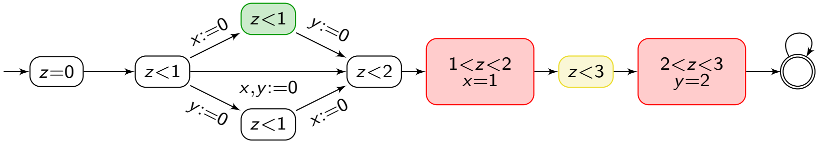

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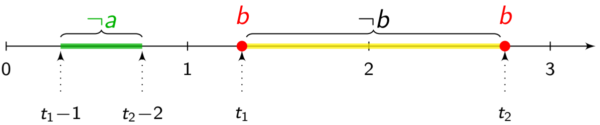

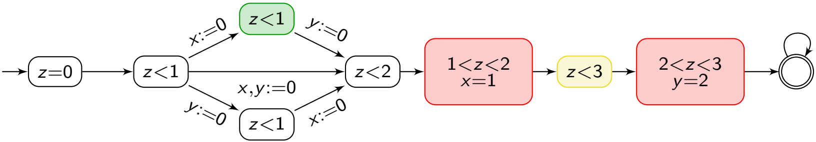

☞ This idea can be extended to any formula in MITL

Outline

1. Introduction
   - Timed automata
   - Examples
2. Decidability of basic properties
   - The region abstraction
   - Extensions of timed automata
   - Weighted timed automata
3. Implementation and tools
4. Other verification problems
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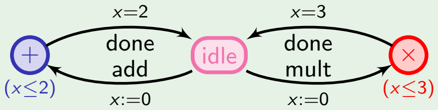
6. Conclusion

Why (timed) games?

- to model uncertainty

Example of a processor in the taskgraph example



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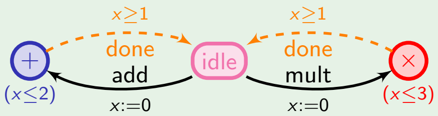

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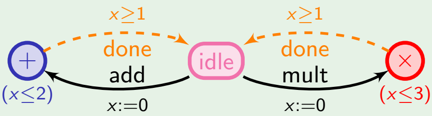
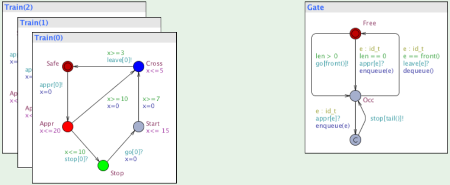


- to model an interaction with the environment

Example of the gate in the train/gate example



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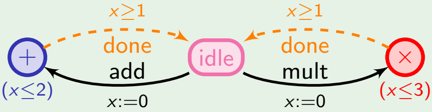

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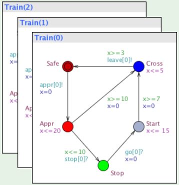

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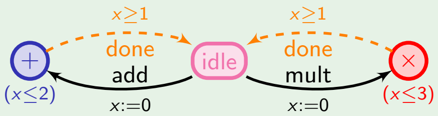

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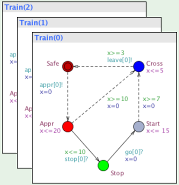

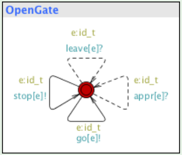

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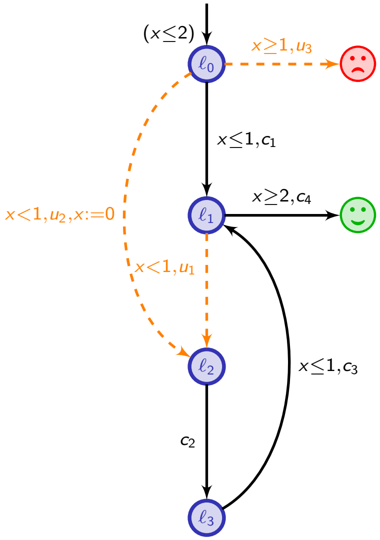

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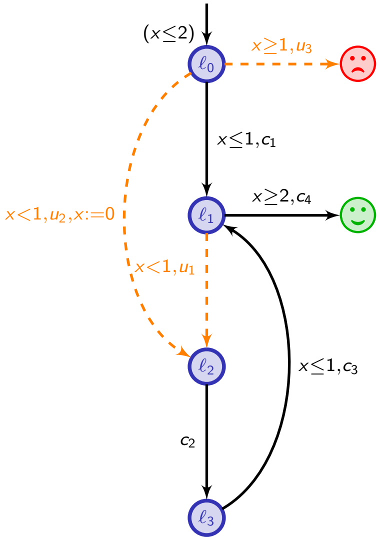

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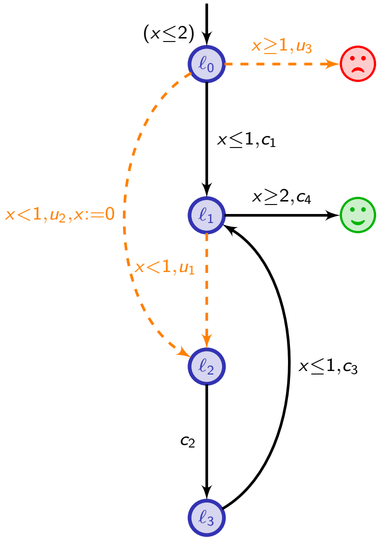

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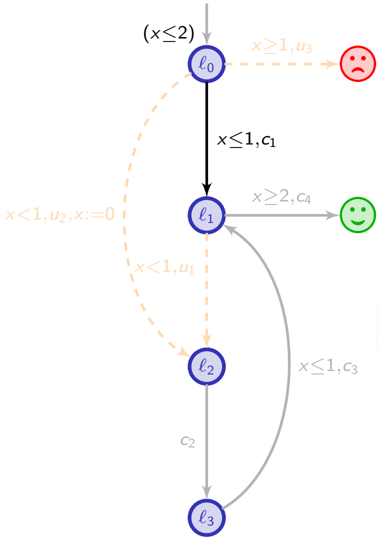

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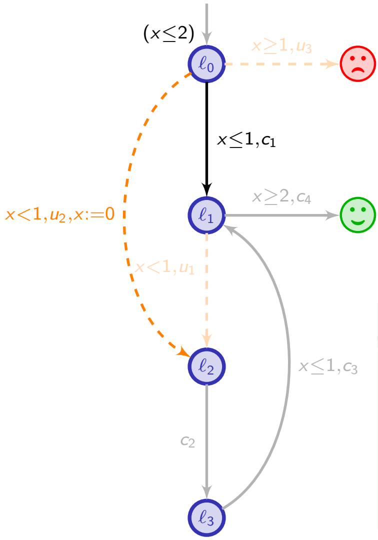

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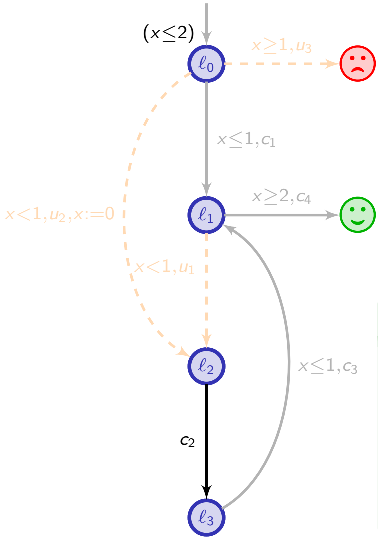

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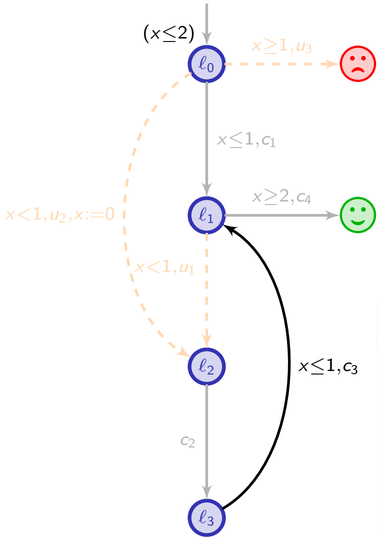

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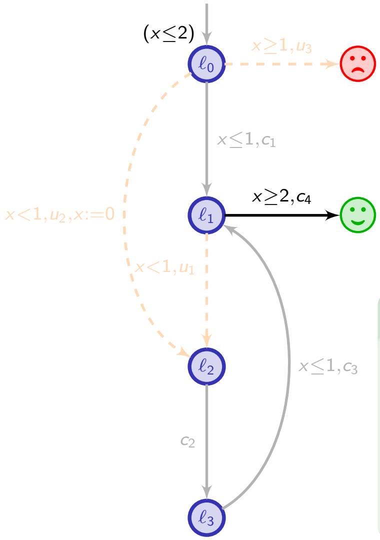

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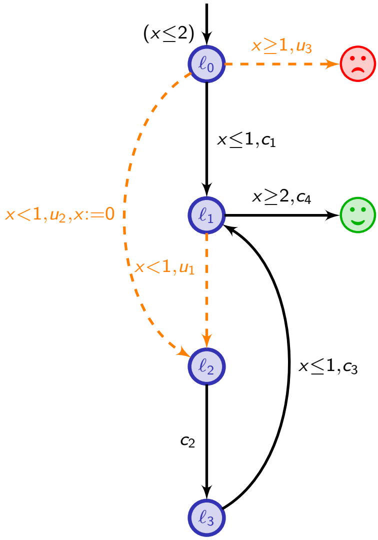

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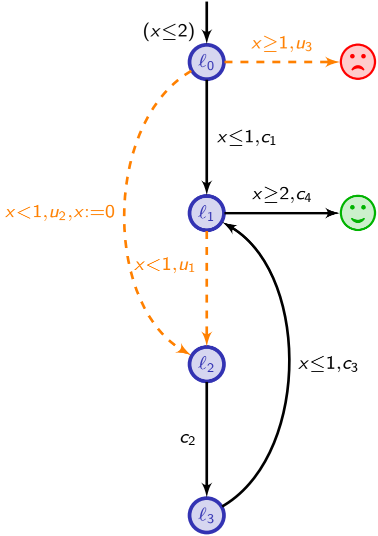

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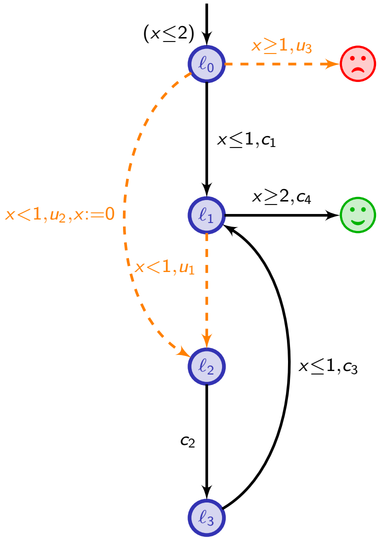

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- If yes, compute one (as simple as possible).

Decidability of timed games

Theorem [AMPS98,HK99]

Reachability and safety timed games are decidable and EXPTIME-complete. Furthermore memoryless and “region-based” strategies are sufficient.

[AMPS98] Asarin, Maler, Pnueli, Sifakis. Controller synthesis for timed automata *(SSC'98)*.
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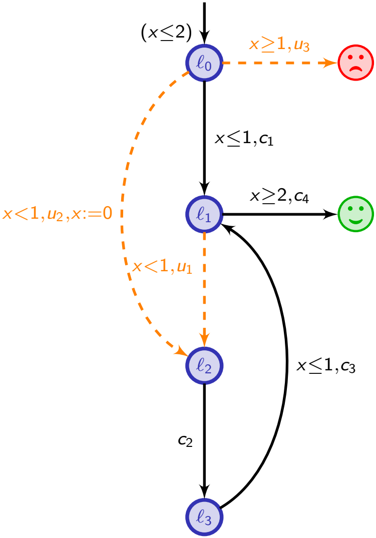
Theorem [AM99,BHPR07,JT07]

Optimal-time reachability timed games are decidable and EXPTIME-complete.

[AM99] Asarin, Maler. As soon as possible: time optimal control for timed automata *(HSCC'99)*.
[BHPR07] Brihaye, Henzinger, Prabhu, Raskin. Minimum-time reachability in timed games *(ICALP'07)*.
[JT07] Jurdziński, Trivedi. Reachability-time games on timed automata *(ICALP'07)*.

Back to the example: computing winning states



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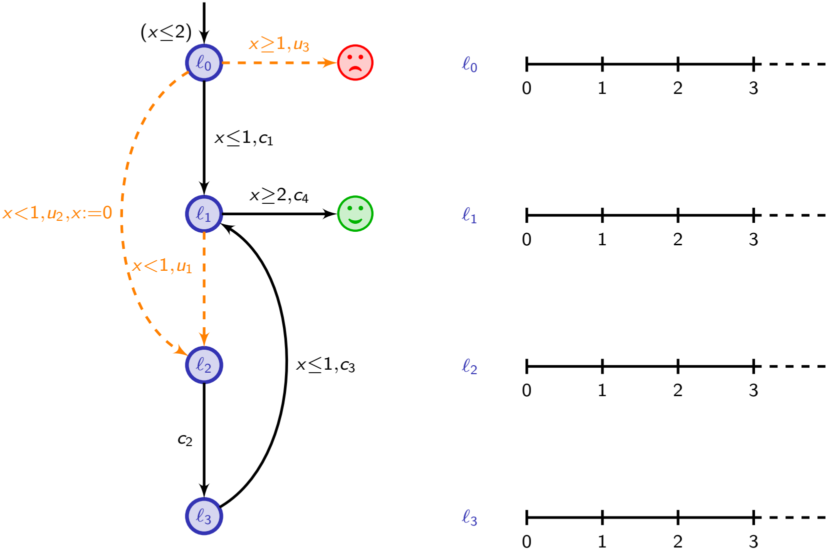

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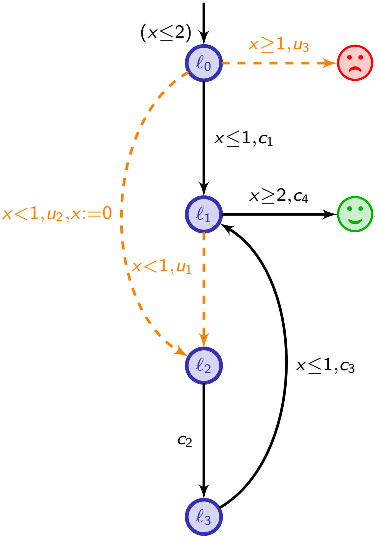

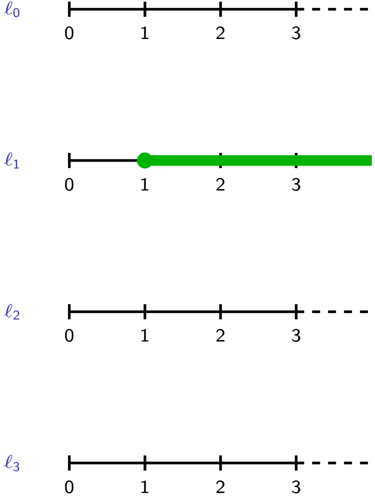

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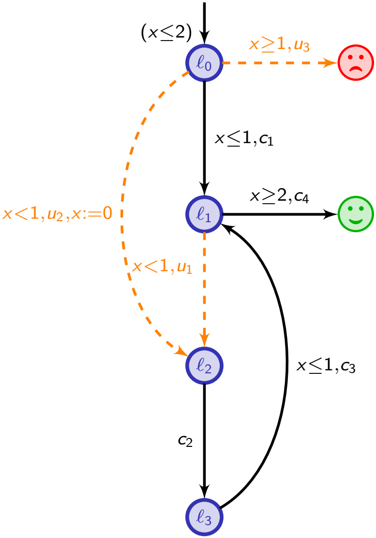

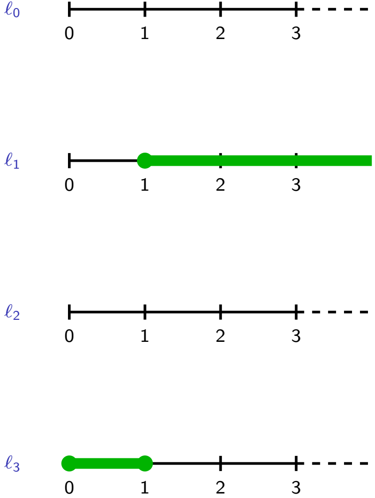

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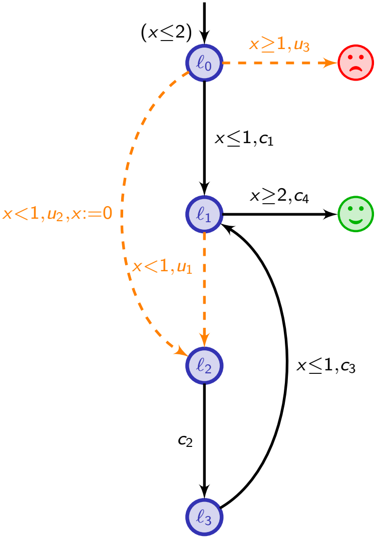

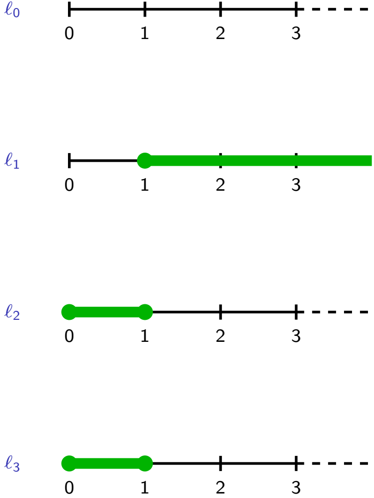

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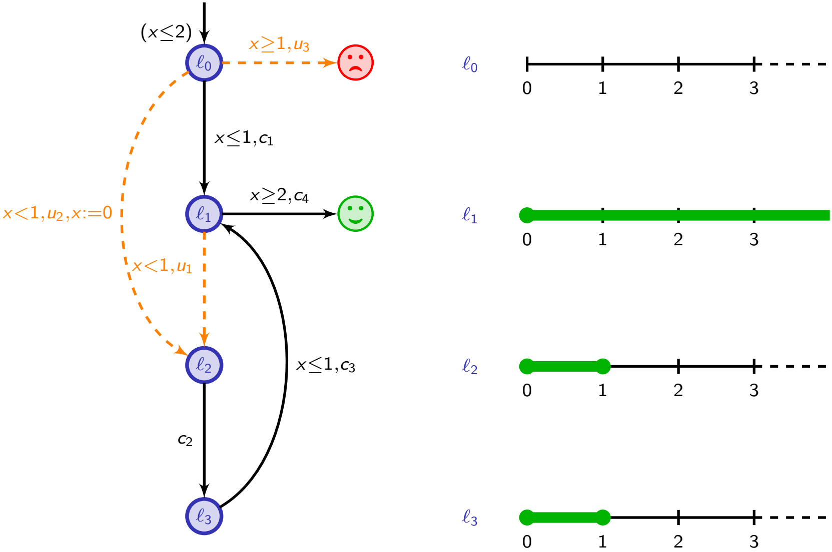

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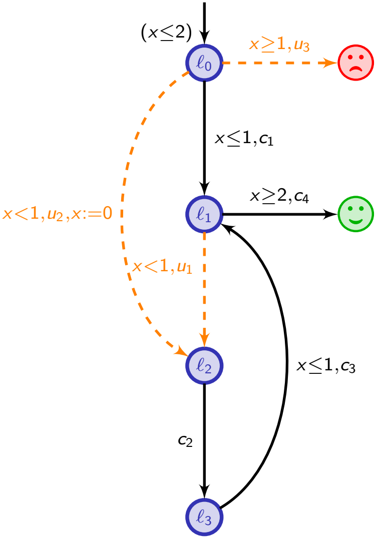

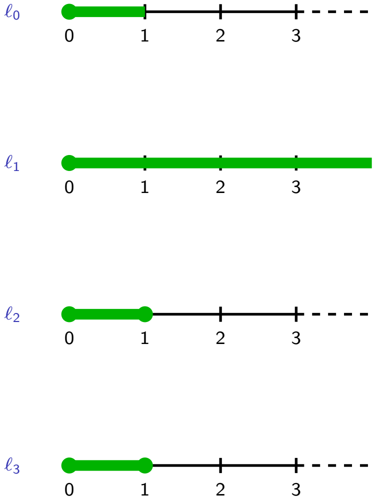

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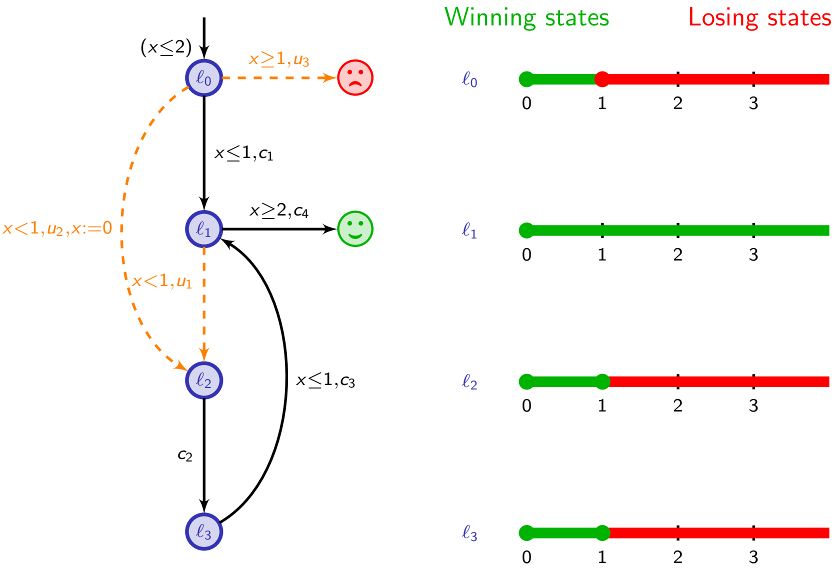

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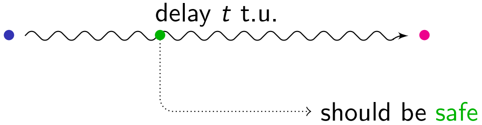

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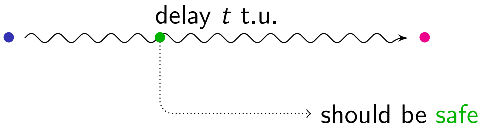

$$\mathrm{Pred}_\delta(X, \mathrm{Safe}) = \{\bullet \mid \exists t \geq 0,\ \bullet \xrightarrow{\delta(t)} \bullet$$
$$\text{and } \forall 0 \leq t' \leq t,\ \bullet \xrightarrow{\delta(t')} \bullet \in \mathrm{Safe}\}$$

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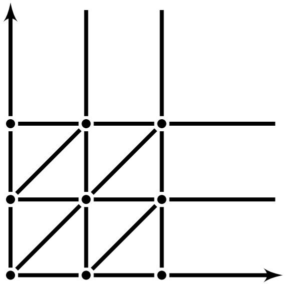

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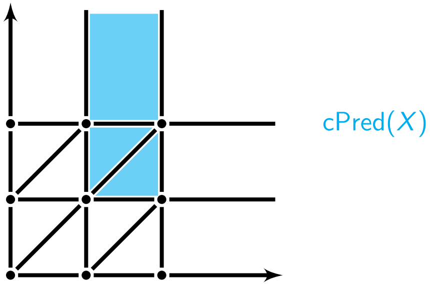

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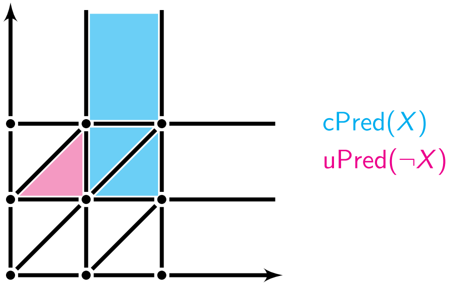

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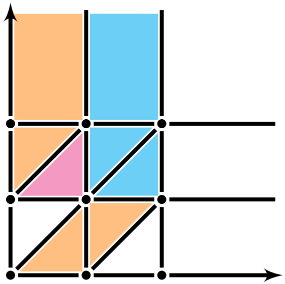

$\mathrm{cPred}(X)$

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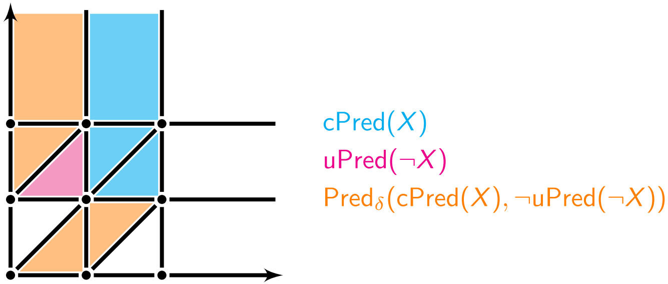

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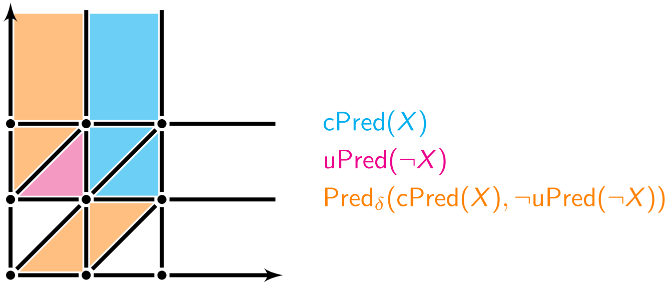

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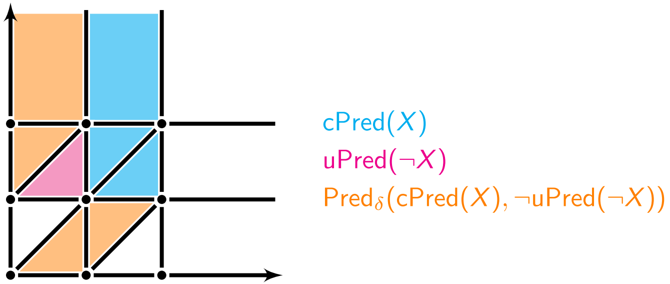

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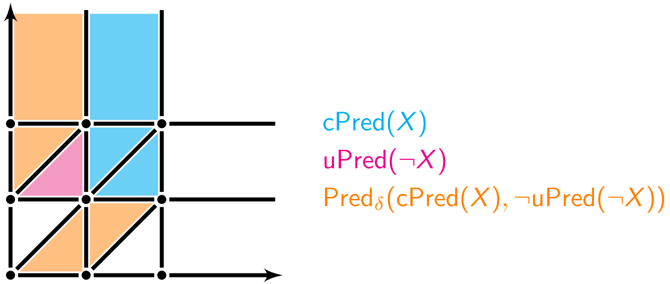

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... and is correct

Timed games with a safety objective

- We can use operator $\widetilde{\pi}$ defined by

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- are asymmetric
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Implementation

Uppaal-Tiga implements a forward algorithm to compute winning states and winning strategies [CDF+05,BCD+07]

[CDF+05] Cassez, David, Fleury, Larsen, Lime. Efficient on-the-fly algorithms for the analysis of timed games *(CONCUR'05)*.
[BCD+07] Berhmann, Cougnard, David, Fleury, Larsen, Lime. Uppaal-Tiga: Time for playing games! *(CAV'07)*.

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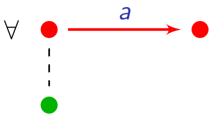

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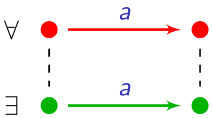

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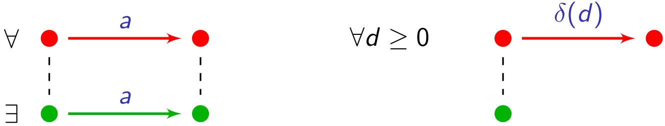

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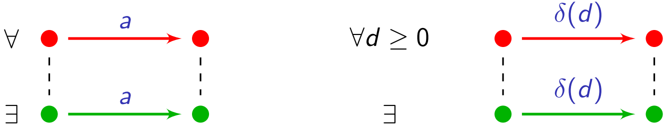

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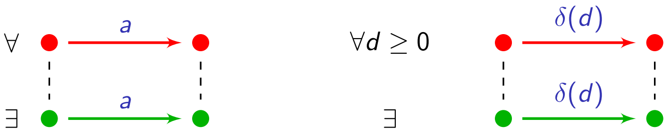

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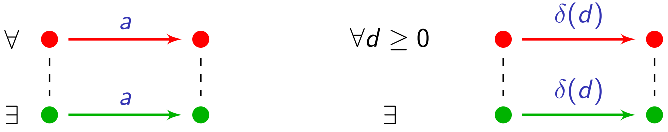

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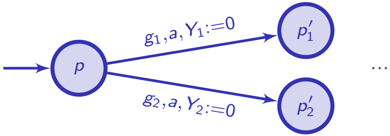
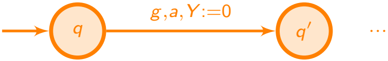
Theorem

Strong timed (bi)simulation between timed automata is decidable and EXPTIME-complete.

timed automaton $\mathcal{A}$



timed automaton $\mathcal{B}$



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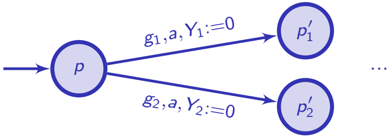

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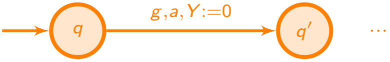

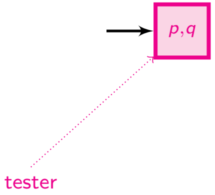

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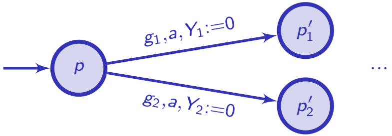

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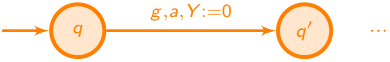

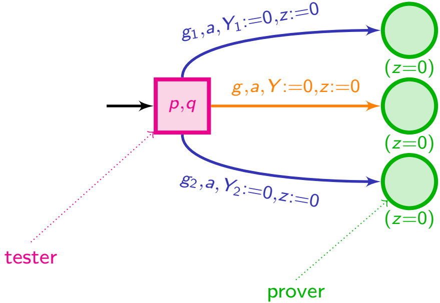

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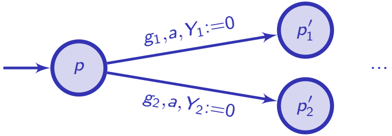

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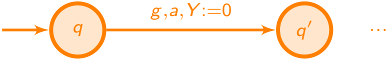
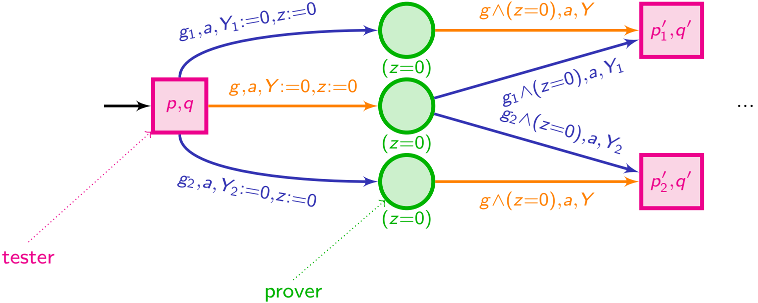
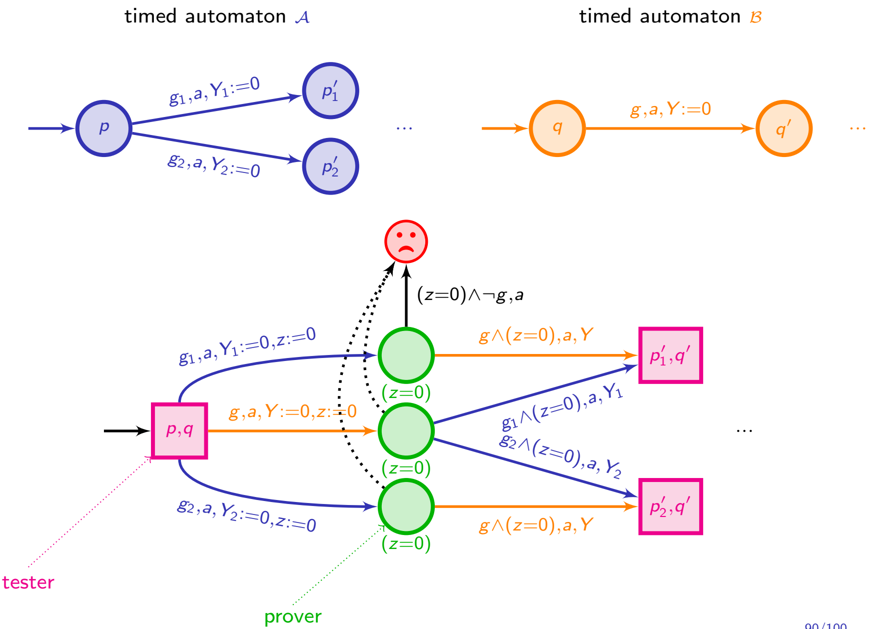
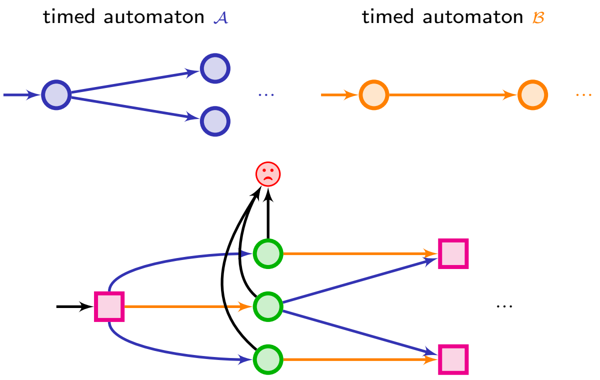





timed automaton $\mathcal{A}$
timed automaton $\mathcal{B}$
p
$g_1, a, Y_1{:=}0$
p'_1
$g_2, a, Y_2{:=}0$
p'_2
q
$g, a, Y{:=}0$
q'
$(z{=}0) \wedge \neg g, a$
$g_1, a, Y_1{:=}0, z{:=}0$
$g \wedge (z{=}0), a, Y$
p'_1, q'
$(z{=}0)$
p, q
$g, a, Y{:=}0, z{:=}0$
$g_1 \wedge (z{=}0), a, Y_1$
$g_2 \wedge (z{=}0), a, Y_2$
$g_2, a, Y_2{:=}0, z{:=}0$
p'_2, q'
tester
prover



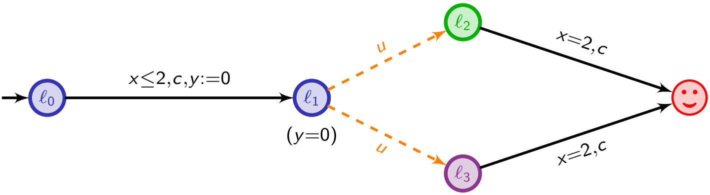
$\mathcal{A}$ and $\mathcal{B}$ are strongly timed bisimilar
iff
the prover has a winning strategy to avoid ☹

Outline

1. Introduction
   - Timed automata
   - Examples
2. Decidability of basic properties
   - The region abstraction
   - Extensions of timed automata
   - Weighted timed automata
3. Implementation and tools
4. Other verification problems
   - Equivalence (or preorder) checking
   - Verification of timed temporal logics (short)
5. Timed control
   - Timed games
   - Weighted timed games
6. Conclusion

A simple weighted timed game

Skip



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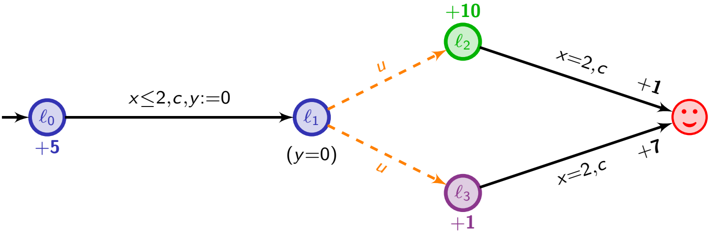

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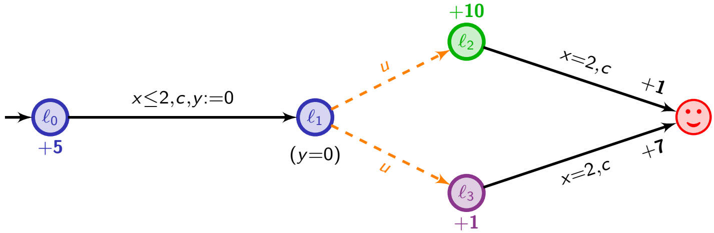

Question: what is the optimal cost we can ensure while reaching ?

A simple weighted timed game

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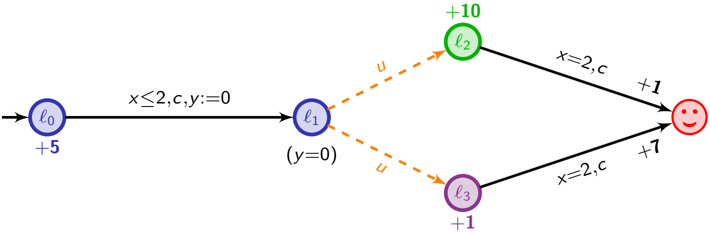

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$$5t + 10(2 - t) + 1$$

A simple weighted timed game

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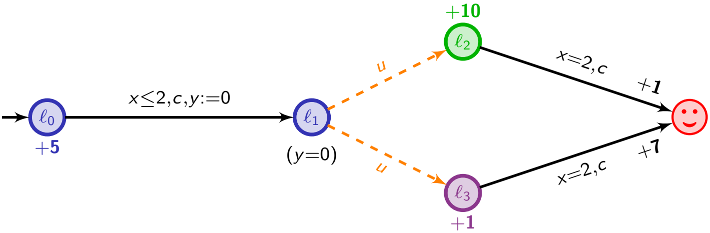

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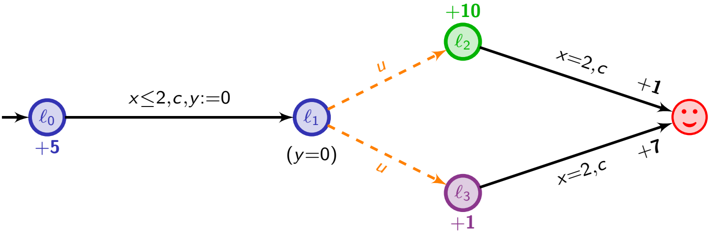
$$5t + 10(2-t) + 1\ ,\ 5t + (2-t) + 7$$

A simple weighted timed game



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A simple weighted timed game

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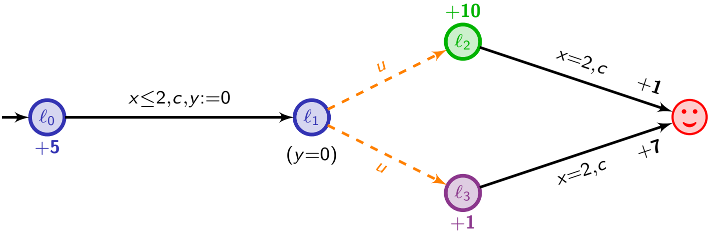

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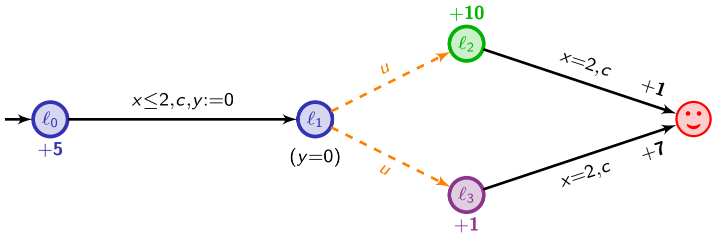
$$\inf_{0 \leq t \leq 2} \max\left(5t + 10(2-t) + 1 \,,\, 5t + (2-t) + 7 \right) = 14 + \frac{1}{3}$$

A simple weighted timed game

Skip



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$\rightsquigarrow$ *strategy:* wait in ℓ_0, and when $t = \frac{4}{3}$, go to ℓ_1

Optimal reachability in weighted timed games

This topic has been fairly hot these last ten years...
[LMM02,ABM04,BCFL04,BBR05,BBM06,BLMR06,Rut11,HIM13,BGK+14]

[LMM02] La Torre, Mukhopadhyay, Murano. Optimal-reachability and control for acyclic weighted timed automata *(TCS@02)*.
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Theorem [BLMR06,Rut11,HIM13,BGK+14]

Turn-based optimal timed games are decidable in EXPTIME (resp. PTIME) when automata have a single clock (with two rates). They are PTIME-hard.

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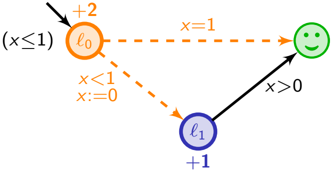

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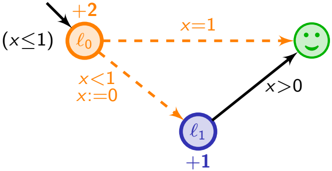

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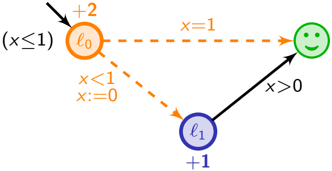

- However, by unfolding and removing one by one the locations,we can synthesize memoryless almost-optimal winning strategies.
- Rather involved proof of correctness for a simple algorithm.

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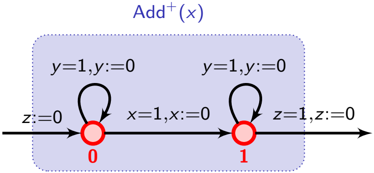
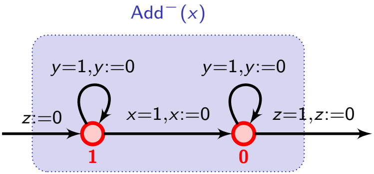


The cost is increased by x_0



The cost is increased by $1-x_0$

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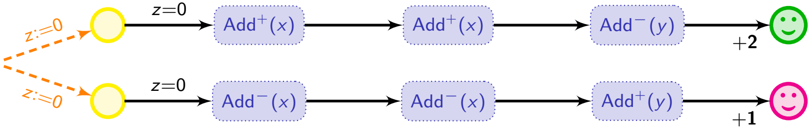

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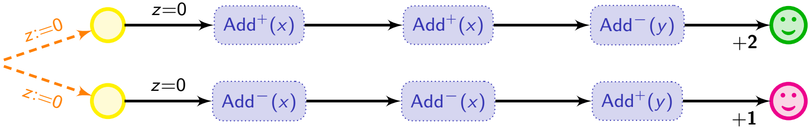

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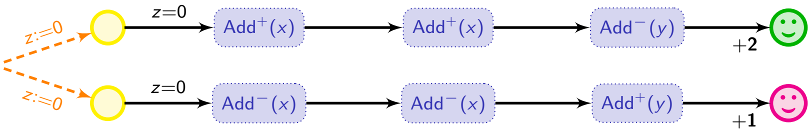
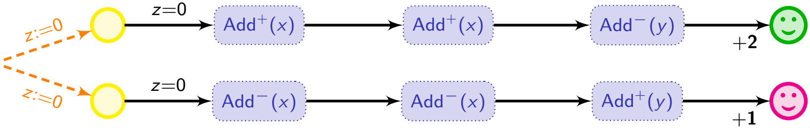


- In , cost $= 2x_0 + (1 - y_0) + 2$
  In , cost $= 2(1 - x_0) + y_0 + 1$

The negative side: why is that hard?

Given two clocks x and y, we can check whether $y = 2x$.



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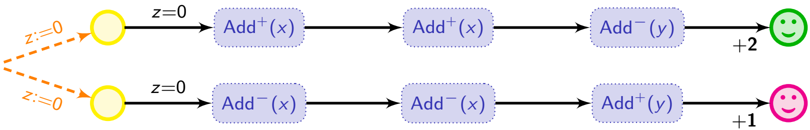

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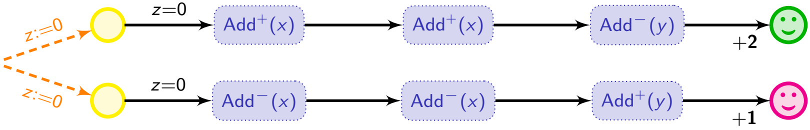

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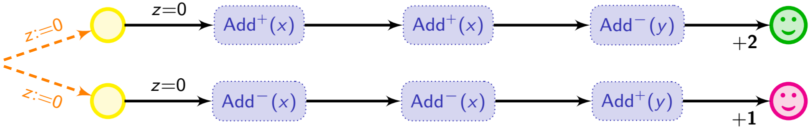

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- Player 1 has a winning strategy with cost ≤ 3 iff $y_0 = 2x_0$

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Player 1 will simulate a two-counter machine:

- each instruction is encoded as a module;
- the counter values c_1 and c_2 are encoded by two clocks:

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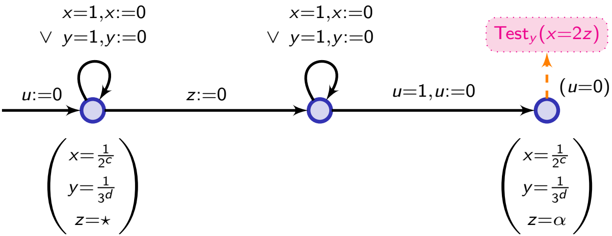
Globally, $(x\leq 1, y\leq 1, u\leq 1)$



Outline

1. Introduction
   - Timed automata
   - Examples
2. Decidability of basic properties
   - The region abstraction
   - Extensions of timed automata
   - Weighted timed automata
3. Implementation and tools
4. Other verification problems
   - Equivalence (or preorder) checking
   - Verification of timed temporal logics (short)
5. Timed control
   - Timed games
   - Weighted timed games
6. Conclusion

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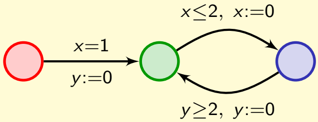
- Going further in the use of timed automata in verification...
  ... requires to think about the accurateness of the (mathematical) model we analyze w.r.t. the real-world system

Just a taste...



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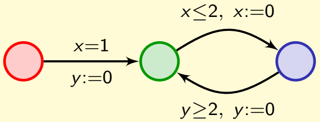

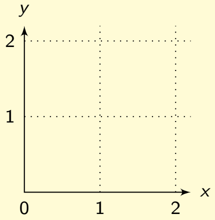

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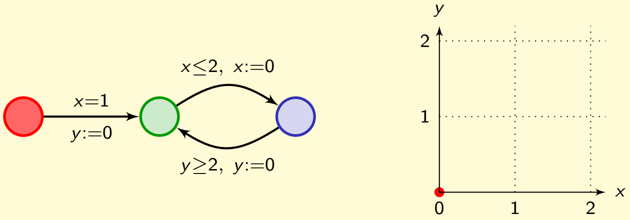

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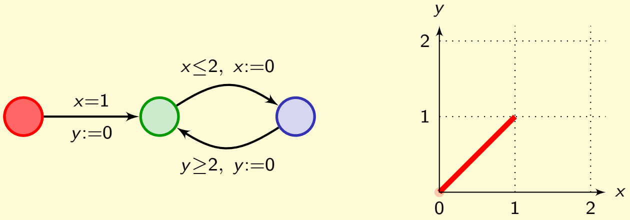

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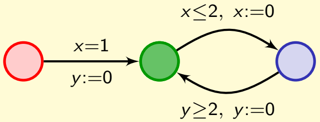

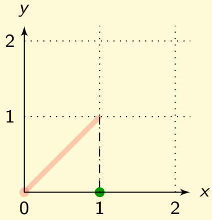

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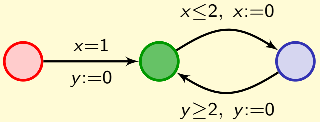

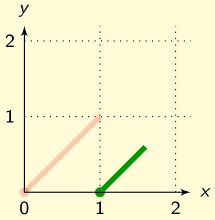

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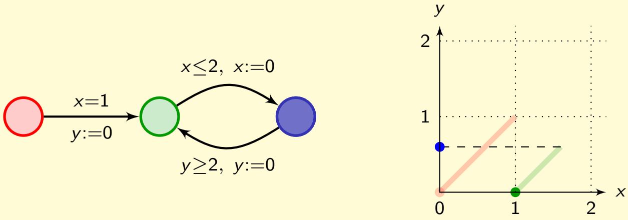

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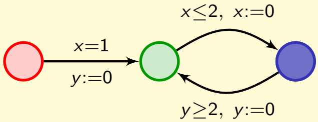

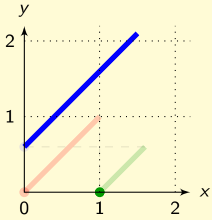

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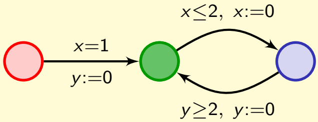

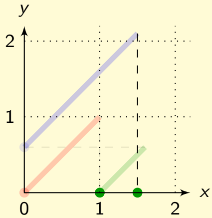

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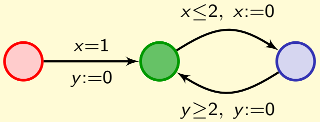

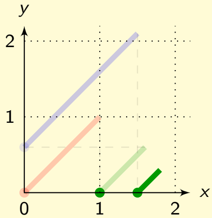

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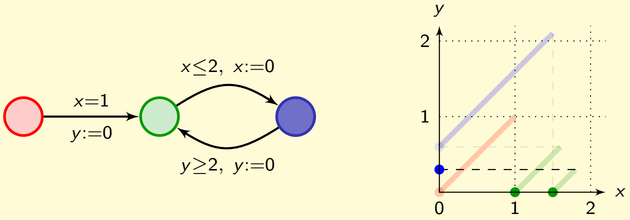

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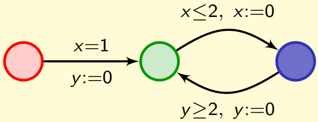

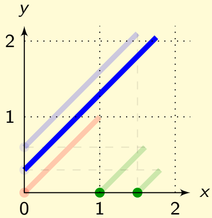

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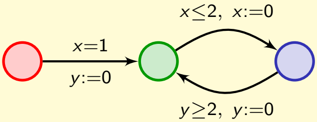

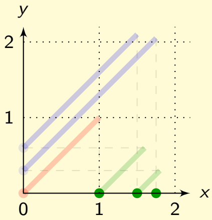

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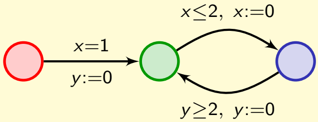

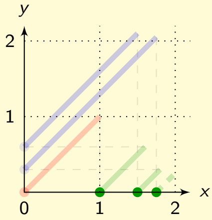

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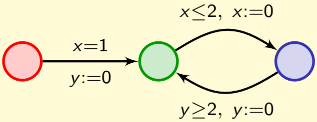

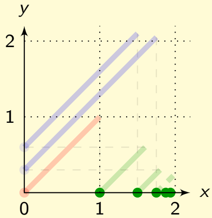

$\rightsquigarrow$ Value of clock x when hitting ◯ is converging, even though global time diverges

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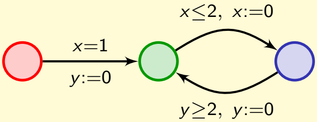

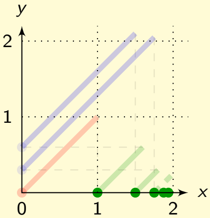

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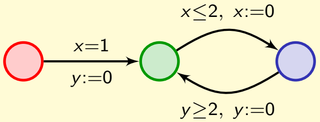

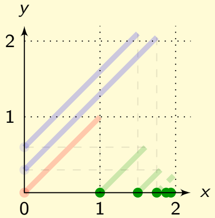

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☞ lecture of Pierre-Alain tomorrow afternoon!