Real-Time and Hybrid Systems

Patricia Bouyer

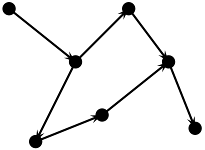
LSV – CNRS & ENS de Cachan

Winter School MOVEP'04

Model-checking

Does the system satisfy the property?

Modelling



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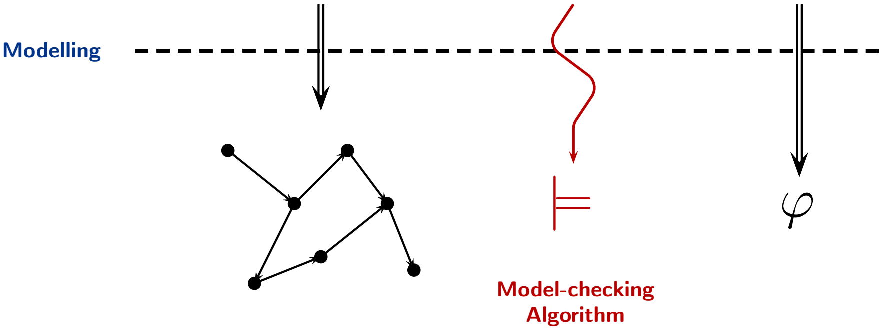

Time!

Context: verification of embedded critical systems

Time

- naturally appears in real systems
- appears in properties (for ex. bounded response time)

➜ Need of models and specification languages integrating timing aspects

Outline

1. **About time semantics**
2. Timed automata, decidability issues
3. Some extensions of the model
4. Implementation of timed automata
5. Conclusion & bibliography

Adding timing informations

- **Untimed case:** sequence of observable events
  a: send message b: receive message

$$a\ b\ a\ b\ a\ b\ a\ b\ a\ b\ \cdots = (a\ b)^\omega$$

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$$(a, d_1)\ (b, d_2)\ (a, d_3)\ (b, d_4)\ (a, d_5)\ (b, d_6)\ \cdots$$

d_1: date at which the first a occurs
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  - **Discrete-time semantics:** dates are *e.g.* taken in N
    Ex: $(a, 1)(b, 3)(c, 4)(a, 6)$
  - **Dense-time semantics:** dates are *e.g.* taken in Q^+, or in R^+
    Ex: $(a, 1.28).(b, 3.1).(c, 3.98)(a, 6.13)$

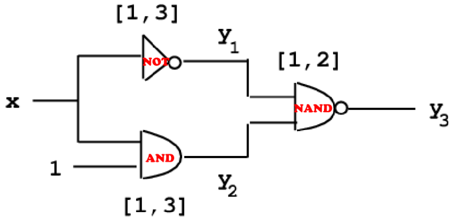
A case for dense-time

Time domain: discrete (*e.g.* N) or dense (*e.g.* Q^+)

- A compositionality problem with discrete time
- Dense-time is a more general model than discrete time
- But, can we not always discretize?

A digital circuit [Alur 91]

Discussion in the context of reachability problems for asynchronous digital circuits [Brzozowski, Seger 1991]



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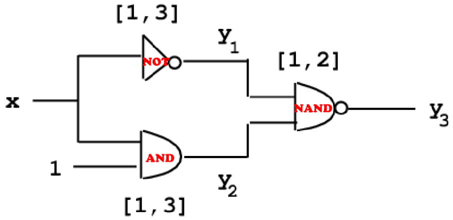

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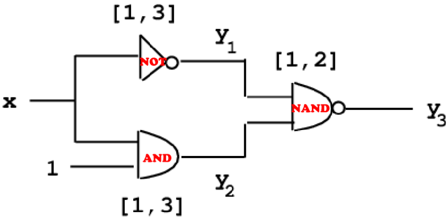

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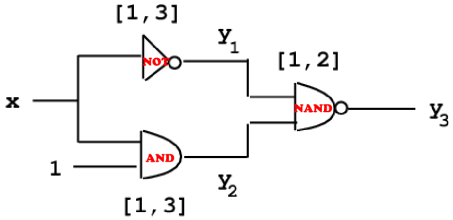

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However, many possible behaviours, e.g.

$$[101] \xrightarrow[1.2]{y_2} [111] \xrightarrow[2.5]{y_3} [110] \xrightarrow[2.8]{y_1} [010] \xrightarrow[4.5]{y_3} [011]$$

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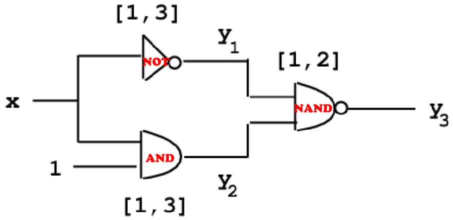

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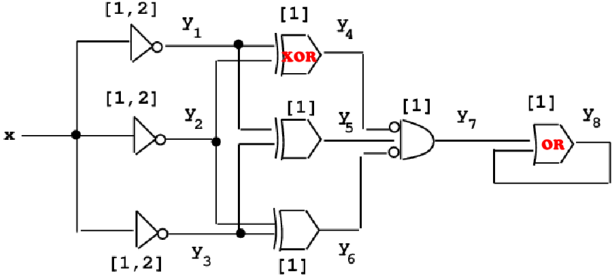
Reachable configurations: {[101], [111], [110], [010], [011], [001]}

Is discretizing sufficient? An example [Alur 91]



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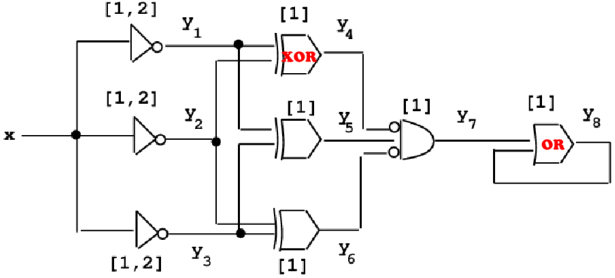

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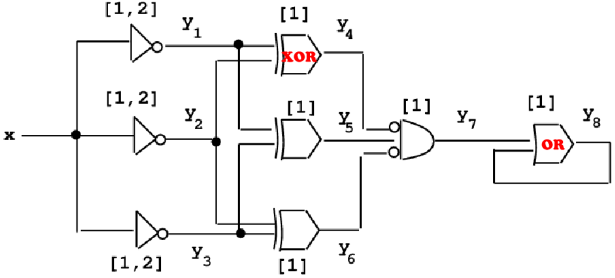

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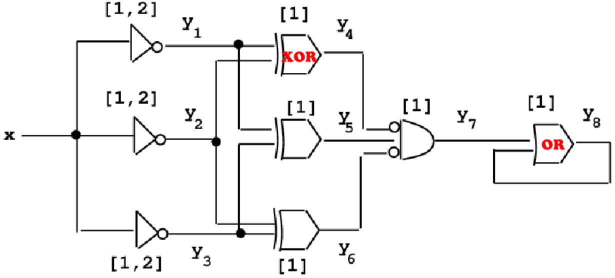

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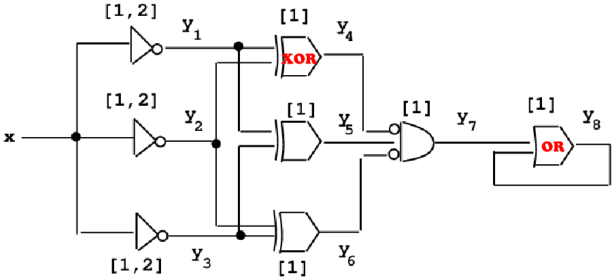

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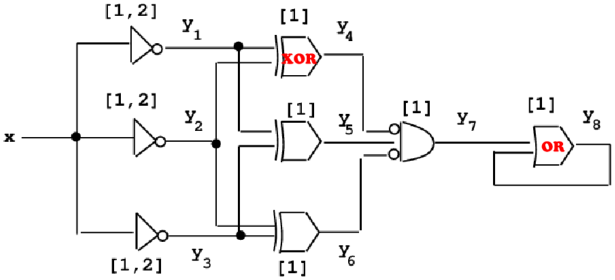

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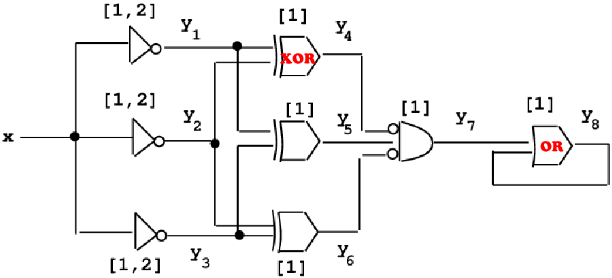

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[Brzozowski Seger 1991]

Theorem: for every $k \geq 1$, there exists a digital circuit such that the reachability set of states in dense-time is strictly larger than the one in discrete time (with granularity $\frac{1}{k}$).

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Claim: finding a correct granularity is as difficult as computing the set of reachable states in dense-time

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Claim: finding a correct granularity is as difficult as computing the set of reachable states in dense-time

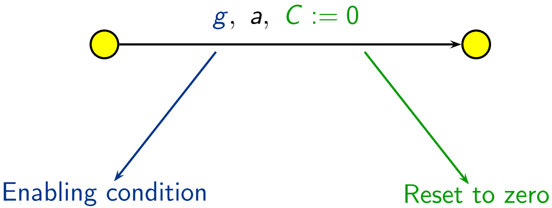
Further counter-example: there exist systems for which no granularity exists

(see later)

Outline

Timed automata [Alur & Dill 90's]

- A finite control structure + variables (clocks)
- A transition is of the form:



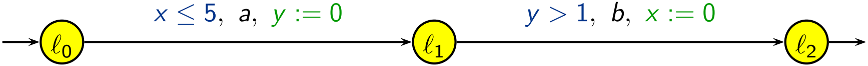
- An enabling condition (or guard) is:

$$g \ ::= \ x \sim c \ \mid \ g \wedge g$$

where $\sim \in \{<, \leq, =, \geq, >\}$

Timed automata (example)

x, y : clocks



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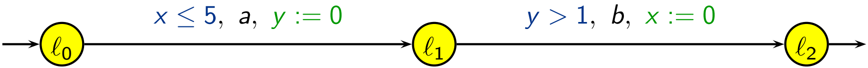

$$
\begin{array}{cccccccccc}
 & \ell_0 & \xrightarrow{\delta(4.1)} & \ell_0 & \xrightarrow{a} & \ell_1 & \xrightarrow{\delta(1.4)} & \ell_1 & \xrightarrow{b} & \ell_2 \\
x & 0 & & 4.1 & & 4.1 & & 5.5 & & 0 \\
y & 0 & & 4.1 & & 0 & & 1.4 & & 1.4
\end{array}
$$

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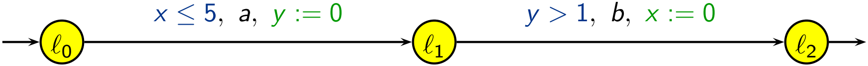

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(clock) valuation

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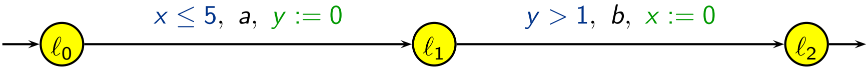

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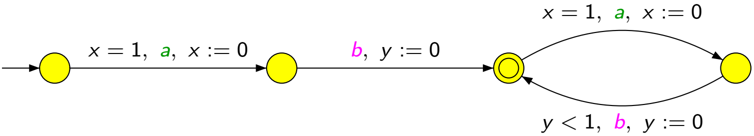
(clock) valuation

➜ timed word $(a, 4.1)(b, 5.5)$

Timed automata semantics

- $\mathcal{A} = (\Sigma, L, X, \longrightarrow)$ is a TA
- **Configurations:** $(\ell, v) \in L \times T^X$ where T is the time domain
- **Timed Transition System:**
  - **action transition**: $(\ell, v) \xrightarrow{a} (\ell', v')$ if $\exists \ell \xrightarrow{g,a,r} \ell' \in \mathcal{A}$ s.t. $\begin{cases} v \models g \\ v' = v[r \leftarrow 0] \end{cases}$
  - **delay transition**: $(\ell, v) \xrightarrow{\delta(d)} (\ell, v + d)$ if $d \in T$

Discrete *vs* dense-time semantics



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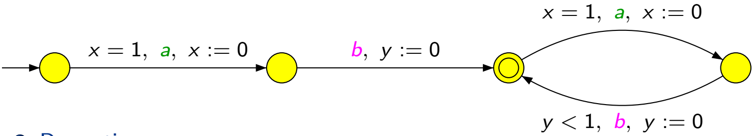

- Dense-time:
  $L_{dense} = \{((ab)^\omega, \tau) \mid \forall i,\ \tau_{2i-1} = i \text{ and } \tau_{2i} - \tau_{2i-1} > \tau_{2i+2} - \tau_{2i+1}\}$

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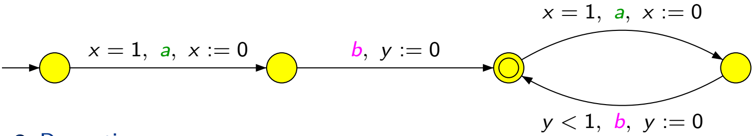

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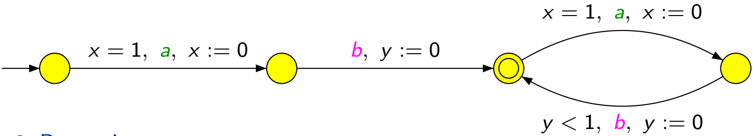

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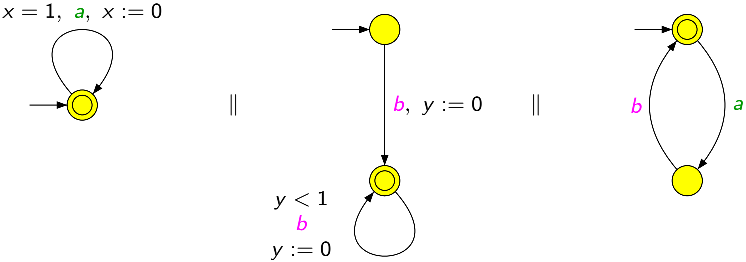
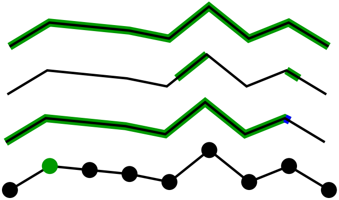


Classical verification problems

- reachability of a control state
- $\mathcal{S} \sim \mathcal{S}'$: bisimulation, etc...
- $L(\mathcal{S}) \subseteq L(\mathcal{S}')$: language inclusion
- $\mathcal{S} \models \varphi$ for some formula φ: model-checking
- $\mathcal{S} \parallel A_T$ + reachability: testing automata
- ...

Classical temporal logics

Path formulas:



$G\phi$ « Always »

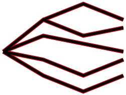
$F\phi$ « Eventually »

$\phi U \phi'$ « Until »

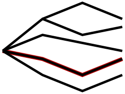
$X\phi$ « Next »

State formulas:

$A\psi$



$E\psi$



➜ LTL: Linear Temporal Logic **[Pnueli 1977]**,
CTL: Computation Tree Logic **[Emerson, Clarke 1982]**

Adding time to temporal logics

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"any problem is followed by an alarm"

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How can we express:

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- Temporal logics with **subscripts**. ex: $CTL + \left| \begin{array}{l} E\varphi U_{\sim k}\psi \\ A\varphi U_{\sim k}\psi \end{array} \right.$

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- Temporal logics with **subscripts**.

$$AG(problem \Rightarrow AF_{\leq 20}\ alarm)$$

- Temporal logics with **clocks**.

$$AG(problem \Rightarrow (x \texttt{ in } AF(x \leq 20 \wedge alarm)))$$

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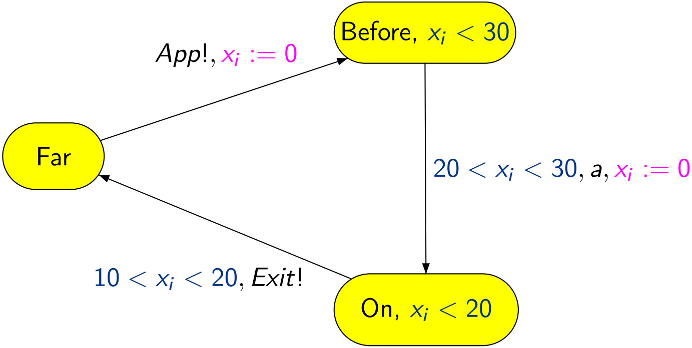
➜ **TCTL**: Timed CTL **[ACD90,ACD93,HNSY94]**

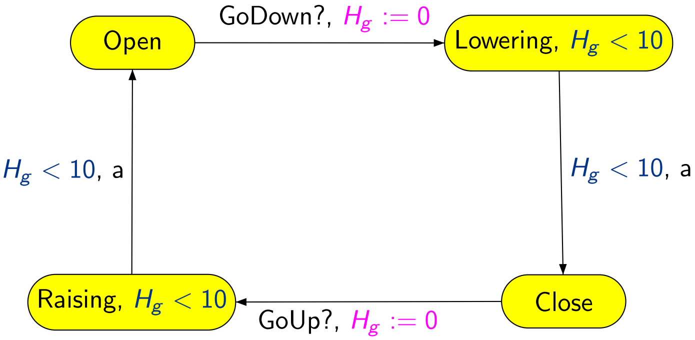
The train crossing example (1)

Train$_i$ with $i = 1, 2, \ldots$



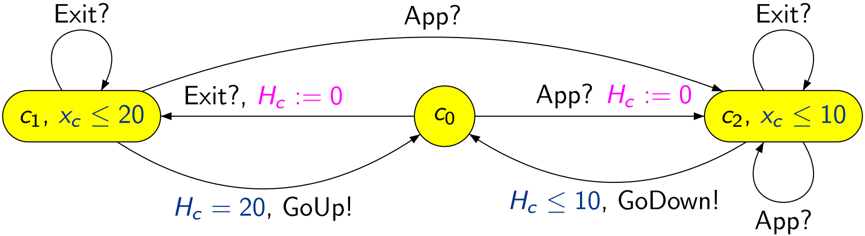
The train crossing example (2)

The gate:



The train crossing example (3)

The controller:



The train crossing example (4)

We use the synchronization function f:

Train_1	Train_2	Gate	Controller	
$App!$	.	.	$App?$	App
.	$App!$	.	$App?$	App
$Exit!$	.	.	$Exit?$	$Exit$
.	$Exit!$	.	$Exit?$	$Exit$
a	.	.	.	a
.	a	.	.	a
.	.	a	.	a
.	.	$GoUp?$	$GoUp!$	$GoUp$
.	.	$GoDown?$	$GoDown!$	$GoDown$

to define the parallel composition $(\text{Train}_1 \parallel \text{Train}_2 \parallel \text{Gate} \parallel \text{Controller})$

***NB*:** the parallel composition does not add expressive power!

The train crossing example (5)

Some properties one could check:

- Is the gate closed when a train crosses the road?

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Verification

Emptiness problem: is the language accepted by a timed automaton empty?

- reachability properties (final states)
- basic liveness properties (Büchi (or other) conditions)

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Note: This is also the case for the discrete semantics.

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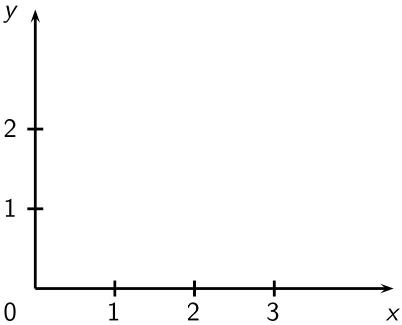
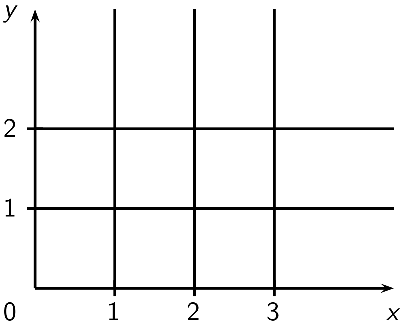
Method: construct a finite abstraction

The region abstraction



Equivalence of finite index

The region abstraction



Equivalence of finite index

- "compatibility" between regions and constraints

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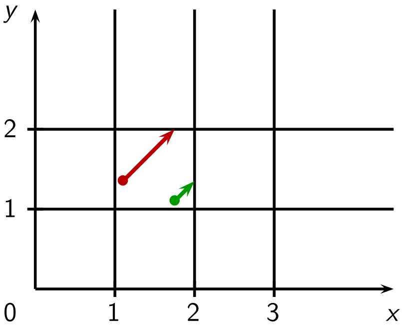

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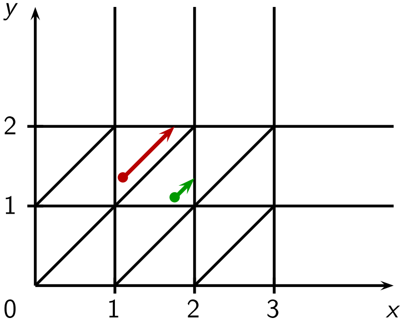

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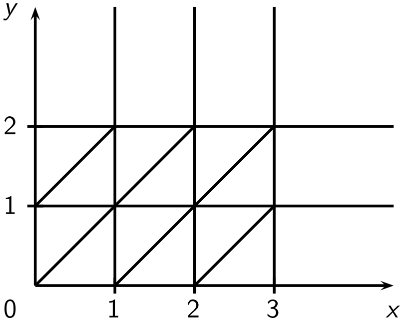

Equivalence of finite index

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➜ a bisimulation property

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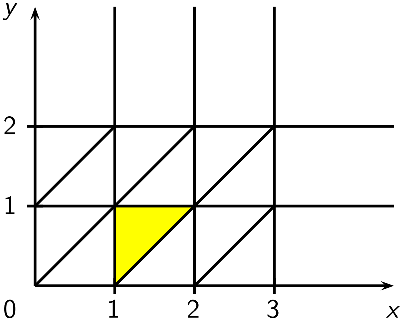

Equivalence of finite index

region defined by

$I_x =]1;2[$, $I_y =]0;1[$

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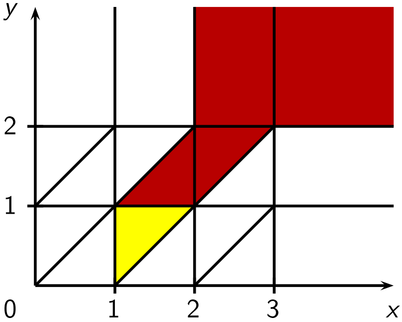

Equivalence of finite index

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successor regions

- "compatibility" between regions and constraints
- "compatibility" between regions and time elapsing

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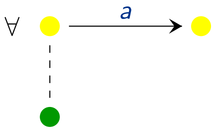
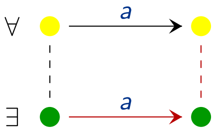
Time-abstract bisimulation



Time-abstract bisimulation



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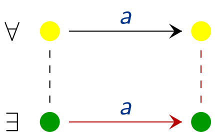

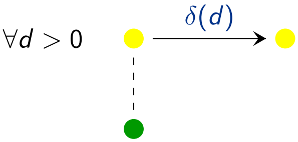

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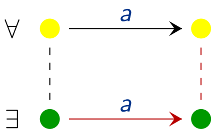

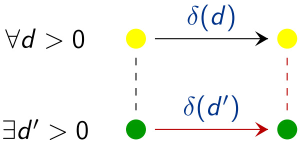

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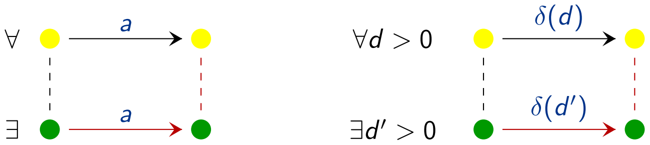
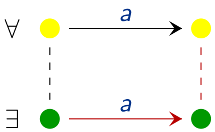
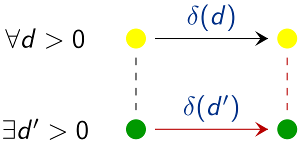


$$(\ell_0, v_0) \xrightarrow{a_1, t_1} (\ell_1, v_1) \xrightarrow{a_2, t_2} (\ell_2, v_2) \xrightarrow{a_3, t_3} \dots$$

Time-abstract bisimulation





$$\begin{tikzcd}
(\ell_0, v_0) \arrow[r, "a_1, t_1"] \arrow[d] & (\ell_1, v_1) \arrow[r, "a_2, t_2"] \arrow[d] & (\ell_2, v_2) \arrow[r, "a_3, t_3"] \arrow[d] & \dots \\
(\ell_0, R_0) \arrow[r, "a_1"] & (\ell_1, R_1) \arrow[r, "a_2"] & (\ell_2, R_2) \arrow[r, "a_3"] & \dots
\end{tikzcd}$$

with $v_i \in R_i$ for all i.

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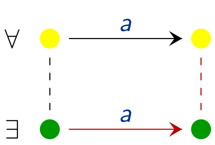

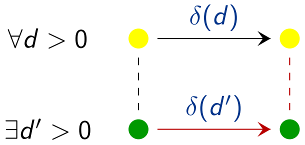

$$\begin{array}{ccccccc}
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\updownarrow & & \updownarrow & & \updownarrow & & \\
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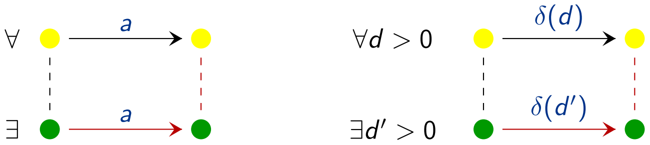

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\end{array}
$$

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Remark: Real-time properties can not be checked with a time-abstract bisimulation. For TCTL, a clock associated with the formula needs to be added.

The region automaton

timed automaton $\otimes$ region abstraction

$\ell \xrightarrow{g,a,C:=0} \ell'$ is transformed into:

$(\ell, R) \xrightarrow{a} (\ell', R')$ if there exists $R'' \in \mathrm{Succ}_t^*(R)$ s.t.

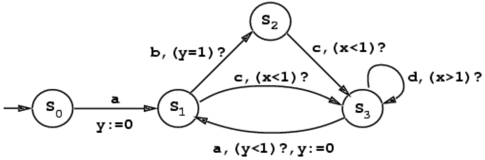
- $R'' \subseteq g$
- $[C \leftarrow 0]R'' \subseteq R'$

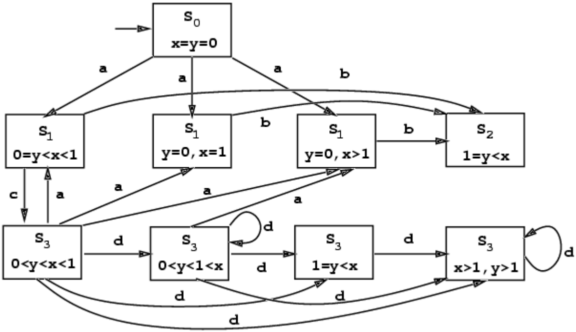
➜ **time-abstract bisimulation**

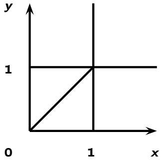
$\mathcal{L}$**(reg. aut.) = UNTIME(**$\mathcal{L}$**(timed aut.))**

where $\mathrm{UNTIME}((a_1, t_1)(a_2, t_2)\ldots) = a_1 a_2 \ldots$

An example [AD 90's]







PSPACE-easyness

¡ The size of the region graph is in $\mathcal{O}(|X|!.2^{|X|})$!

- **One configuration:** a discrete location + a region

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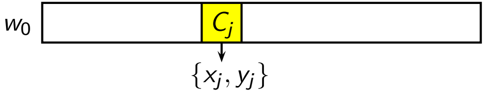
- By guessing a path: needs only to store two configurations

➜ in NPSPACE, thus in PSPACE

PSPACE-hardness

$$\left.\begin{array}{c}\mathcal{M}\text{ LBTM}\\ w_0 \in \{a,b\}^*\end{array}\right\} \rightsquigarrow$$ $A_{\mathcal{M},w_0}$ s.t. $\mathcal{M}$ accepts w_0 iff the final state of $A_{\mathcal{M},w_0}$ is reachable



C_j contains an "a" if $x_j = y_j$
C_j contains a "b" if $x_j < y_j$

(these conditions are invariant by time elapsing)

➜ proof taken in **[Aceto & Laroussinie 2002]**

PSPACE-hardness (cont.)

If $q \xrightarrow{\alpha,\alpha',\delta} q'$ is a transition of $\mathcal{M}$, then for each position i of the tape, we have a transition

$$(q,i) \xrightarrow{g,r:=0} (q',i')$$

where:

- g is $x_i = y_i$ (resp. $x_i < y_i$) if $\alpha = a$ (resp. $\alpha = b$)
- $r = \{x_i, y_i\}$ (resp. $r = \{x_i\}$) if $\alpha' = a$ (resp. $\alpha' = b$)
- $i' = i + 1$ (resp. $i' = i - 1$) if δ is right and $i < n$ (resp. left)

Enforcing time elapsing: on each transition, add the condition $t = 1$ and clock t is reset.

Initialization: init $\xrightarrow{t=1, r_0:=0} (q_0, 1)$ where $r_0 = \{x_i \mid w_0[i] = b\} \cup \{t\}$

Termination: $(q_f, i) \longrightarrow$ end

Consequence of region automata construction

Region automata: correct finite abstraction for checking reachability/Büchi-like properties

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However, everything can not be reduced to finite automata...

A model not far from undecidability

- Universality is undecidable **[Alur & Dill 90's]**
- Inclusion is undecidable **[Alur & Dill 90's]**
- Determinizability is undecidable **[Tripakis 2003]**
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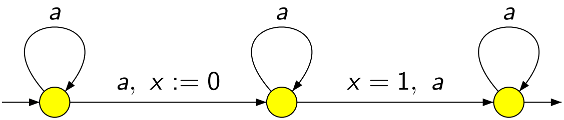
An example of non-determinizable/non-complementable timed aut.:



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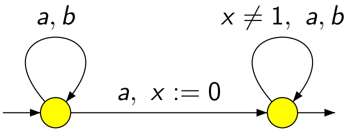
[Alur,Madhusudan 2004]



UNTIME $\left(\overline{L} \cap \{(a^* b^*, \tau) \mid \text{all } a's \text{ happen before 1 and no two } a's \text{ simultaneously}\}\right)$ is not regular **(exercise!)**

Partial conclusion

➜ a timed model interesting for verification purposes

Numerous works have been (and are) devoted to:

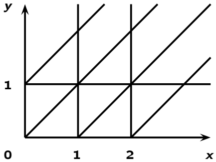
- the "theoretical" comprehension of timed automata (*cf* **[Asarin 2004]**)
- extensions of the model (to ease modelling)
  - expressiveness
  - analyzability
- algorithmic problems and implementation

Outline

Role of diagonal constraints

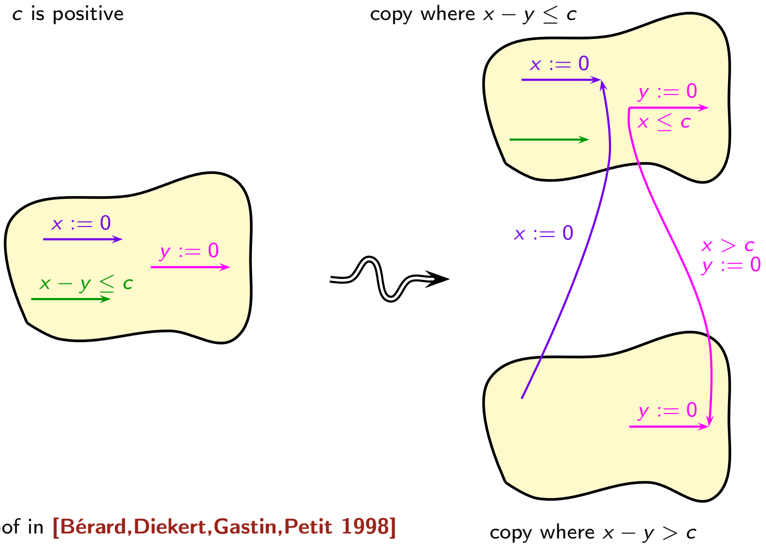
$$\boxed{x - y \sim c \quad \text{and} \quad x \sim c}$$

- **Decidability:** yes, using the region abstraction



- **Expressiveness:** no additional expressive power

Role of diagonal constraints (cont.)



➜ proof in **[Bérard,Diekert,Gastin,Petit 1998]**

Role of diagonal constraints (cont.)

Open question: is this construction "optimal"?

In the sense that timed automata with diagonal constraints are exponentially more concise than diagonal-free timed automata.

Adding silent actions

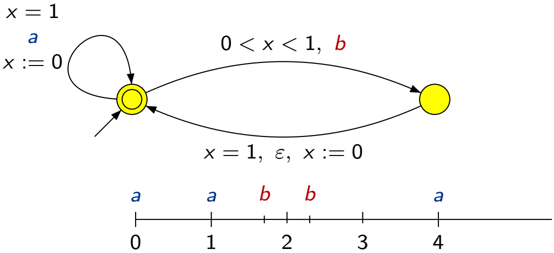
$$\xrightarrow{g, \varepsilon, C := 0}$$

[Bérard,Diekert,Gastin,Petit 1998]

- **Decidability:** yes

  (actions have no influence on region automaton construction)

- **Expressiveness:** strictly more expressive!



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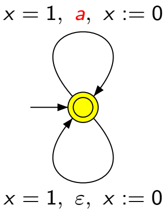
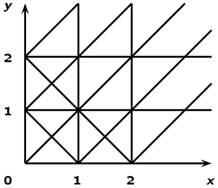


Adding constraints of the form $x + y \sim c$

$\boxed{x + y \sim c \quad \text{and} \quad x \sim c}$ **[Bérard,Dufourd 2000]**

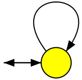
- **Decidability:** - for two clocks, decidable using the abstraction



  - for four clocks (or more), undecidable!

- **Expressiveness:** more expressive! (even using two clocks)

$$\{(a^n, t_1 \dots t_n) \mid n \geq 1 \text{ and } t_i = 1 - \tfrac{1}{2^i}\}$$



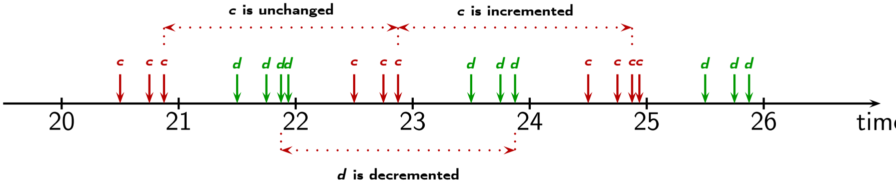
The two-counter machine

Definition. A two-counter machine is a finite set of instructions over two counters (x and y):

- Incrementation:
  `(p):` $x := x + 1$; `goto (q)`
- Decrementation:
  `(p):` `if` $x > 0$ `then` $x := x - 1$; `goto (q) else goto (r)`

Theorem. **[Minsky 67]** The halting problem for two counter machines is undecidable.

Undecidability proof



➜ simulation of
- decrementation of a counter
- incrementation of a counter

We will use 4 clocks:

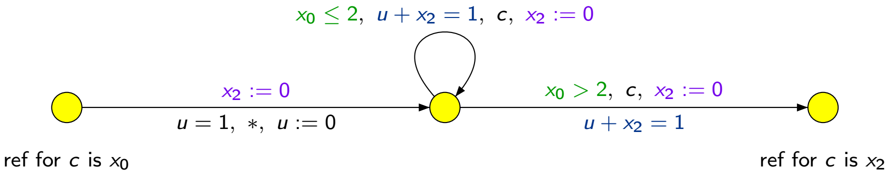
- u, "tic" clock (each time unit)
- x_0, x_1, x_2: reference clocks for the two counters

"x_i reference for c" $\equiv$ "the last time x_i has been reset is the last time action c has been performed"

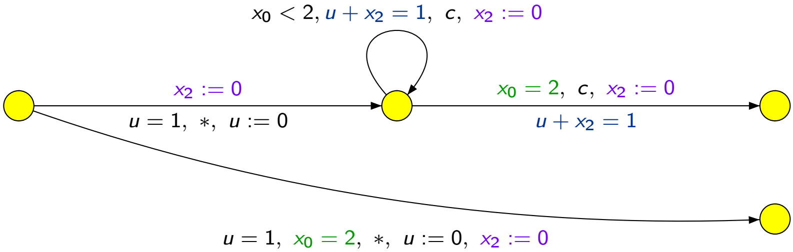
[Bérard,Dufourd 2000]

Undecidability proof (cont.)

- **Incrementation of counter c:**

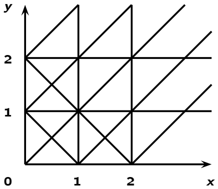


- **Decrementation of counter c:**



Adding constraints of the form $x + y \sim c$

- Two clocks: decidable using the abstraction



- Four clocks (or more): undecidable!

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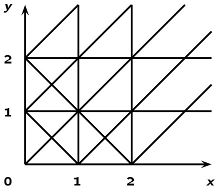


- Three clocks: **open question**

- Four clocks (or more): undecidable!

Adding new operations on clocks

Several types of updates: $x := y + c$, $x :< c$, $x :> c$, etc...

Adding new operations on clocks

Several types of updates: $x := y + c$, $x :< c$, $x :> c$, etc...

- The general model is undecidable.
  (simulation of a two-counter machine)

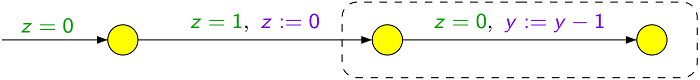
Adding new operations on clocks

Several types of updates: $x := y + c$, $x :< c$, $x :> c$, etc...

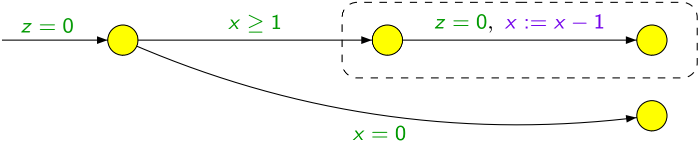
- The general model is undecidable.
  (simulation of a two-counter machine)

- Only decrementation also leads to undecidability

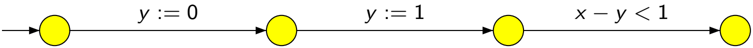
  - **Incrementation of counter x**



  - **Decrementation of counter x**



Decidability



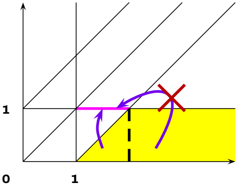


image by $y := 1$

➜ the bisimulation property is not met

The classical region automaton construction is not correct.

Decidability (cont.)

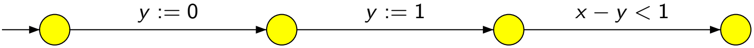
$\mathcal{A}$ $\rightsquigarrow$ Diophantine linear inequations system
$\rightsquigarrow$ is there a solution?
$\rightsquigarrow$ if yes, belongs to a decidable class

Examples:

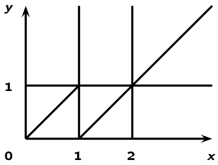
- constraint $x \sim c$ — $c \leq \max_x$
- constraint $x - y \sim c$ — $c \leq \max_{x,y}$
- update $x :\sim y + c$ — $\max_x \leq \max_y + c$
  and for each clock z, $\max_{x,z} \geq \max_{y,z} + c$, $\max_{z,x} \geq \max_{z,y} - c$
- update $x :< c$ — $c \leq \max_x$
  and for each clock z, $\max_z \geq c + \max_{z,x}$

The constants $(\max_x)$ and $(\max_{x,y})$ define a set of regions.

Decidability (cont.)



$$\left\{\begin{array}{l} \max_y \geq 0 \\ \max_x \geq 0 + \max_{x,y} \\ \max_y \geq 1 \\ \max_x \geq 1 + \max_{x,y} \\ \max_{x,y} \geq 1 \end{array}\right. \quad \text{implies} \quad \left\{\begin{array}{l} \max_x = 2 \\ \max_y = 1 \\ \max_{x,y} = 1 \\ \max_{y,x} = -1 \end{array}\right.$$

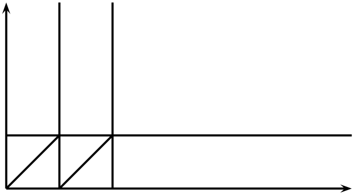


The bisimulation property is met.

What's wrong when undecidable?

Decrementation $x := x - 1$

$$\max_x \leq \max_x - 1$$



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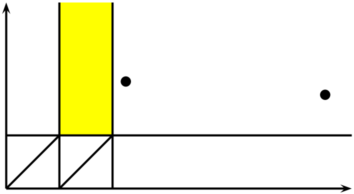

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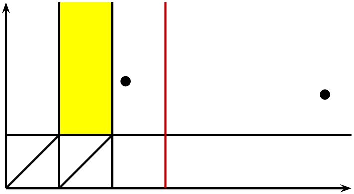

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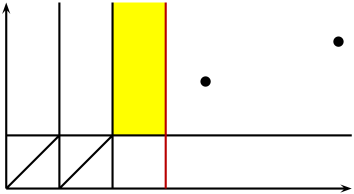

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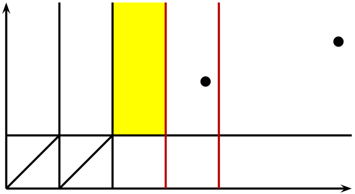

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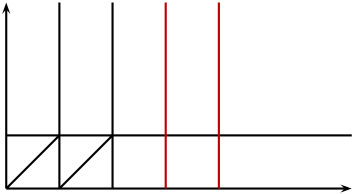

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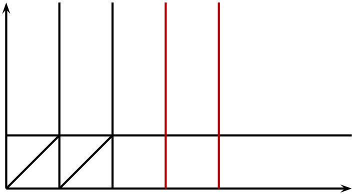

etc...

Decidability (cont.)

<table>
<tr><th></th><th>Diagonal-free constraints</th><th>General constraints</th></tr>
<tr><td>$x := c,\ x := y$</td><td rowspan="3">PSPACE-complete</td><td>PSPACE-complete</td></tr>
<tr><td>$x := x + 1$</td><td rowspan="3">Undecidable</td></tr>
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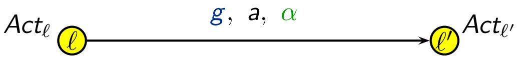
<tr><td>$y + c <: x :< y + d$</td></tr>
<tr><td>$y + c <: x :< z + d$</td><td>Undecidable</td></tr>
</table>

[Bouyer,Dufourd,Fleury,Petit 2000]

Linear hybrid automata [Henzinger 1996]

- A finite control structure + a set X of *dynamic variables*
- A transition is of the form:



- g is a linear constraint on variables
- α is a jump condition, *i.e.* an affine update of the form $X' = A.X + B$
- in each state, an activity function assigning a slope to each variable (for each $x \in X$, $Act(x) \in [\ell, u]$)

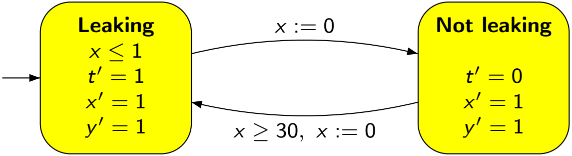
LHA semantics

- $\mathcal{H} = (\Sigma, L, X, Act \longrightarrow)$ is a LHA
- **Configurations:** $(\ell, v) \in L \times T^X$ where T is the domain
- **Timed Transition System:**
  - **action transition**: $(\ell, v) \xrightarrow{a} (\ell', v')$ if $\exists \ell \xrightarrow{g,a,J} \ell' \in \mathcal{A}$ s.t. $\begin{cases} v \models g \\ v' = \alpha(v) \end{cases}$
  - **delay transition**: $(\ell, v) \xrightarrow{\delta(d)} (\ell, v + d.Act(\ell))$ if $d \in T$

Linear hybrid automata (example)

The **gas burner** may leak. **[ACHH93]**

- each time a leakage is detected, it is repaired or stopped in less than 1s
- two leakages are separated by at least 30s



Is it possible that the gas burner leaks during a time greater than $\frac{1}{20}$ of the global time after the 60 first minutes?

$$\mathrm{AG}\,(y \geq 60 \implies 20t \leq y)$$

What about decidability?

➜ almost everything is undecidable
[Henzinger,Kopke,Puri,Varaiya 98]

Theorem. The class of LHA with clocks and only one variable having possibly two slopes $k_1 \neq k_2$ is undecidable.

Theorem. The class of *stopwatch* automata is undecidable.

One of the "largest" classes of LHA which are decidable is the class of initialized rectangular automata.

Outline

Notice

The region automaton is not used for implementation:

- suffers from a combinatorics explosion
  (the number of regions is exponential in the number of clocks)
- no really adapted data structure

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...but **on-the-fly technics** are prefered.

Reachability analysis

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  compute the successors of initial configurations

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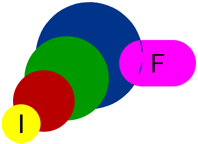

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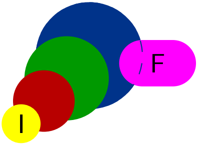

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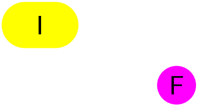

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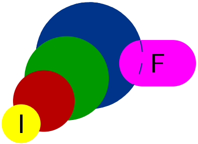

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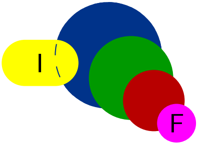

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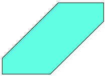

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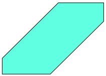

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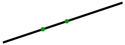

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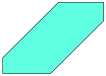

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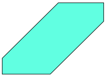


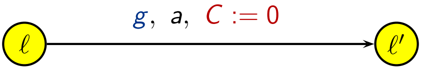
- Need of abstractions, heuristics, etc...

Examples of systems: counter automata, pushdown systems, linear hybrid automata, timed automata, etc...

An example of computation with HyTech

```
command:  /usr/local/bin/hytech gas_burner
=================================================================
HyTech:  symbolic model checker for embedded systems
Version 1.04f (last modified 1/24/02) from v1.04a of 12/6/96
For more info:
    email:  hytech@eecs.berkeley.edu
    http://www.eecs.berkeley.edu/~tah/HyTech
Warning:  Input has changed from version 1.00(a).  Use -i for more info
=================================================================

Backward computation
Number of iterations required for reachability:  6
System satisfies non-leaking duration property

Location:  not_leaking
x >= 0 & t >= 3 & y <= 20t & y >= 0
| x + 20t >= y + 11 & y <= 20t + 19 & t >= 2 & x >= 0 & y >= 0
| y >= 0 & t >= 1 & x + 20t >= y + 22 & y <= 20t + 8 & x >= 0
| y >= 0 & x + 20t >= y + 33 & 20t >= y + 3 & x >= 0
Location:  leaking
19x + y <= 20t + 19 & y >= x + 59 & x <= 1 & x >= 0
| t >= x + 2 & x <= 1 & y >= 0 & 19x + y <= 20t + 19 & x >= 0
| t >= x + 1 & x <= 1 & y >= 0 & 19x + y <= 20t + 8 & x >= 0
| 20t >= 19x + y + 3 & y >= 0 & x <= 1 & x >= 0

=================================================================
Max memory used = 0 pages = 0 bytes = 0.00 MB
Time spent = 0.02u + 0.00s = 0.02 sec total

=================================================================
```

Note on the backward analysis of TA



$\overleftarrow{[C \leftarrow 0]^{-1}(Z \cap (C = 0)) \cap g}$ $\qquad\qquad$ Z

Note on the backward analysis of TA

$$\ell \xrightarrow{g,\ a,\ C := 0} \ell'$$

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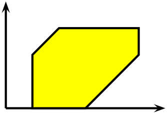

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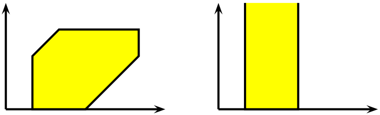

Z $\qquad$ $[C \leftarrow 0]^{-1}(Z \cap (C = 0))$

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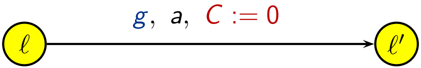

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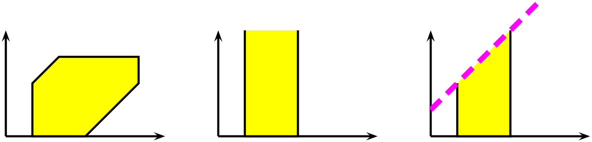

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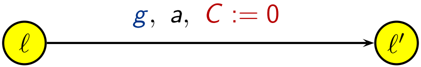

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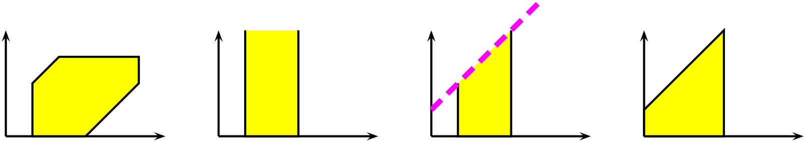

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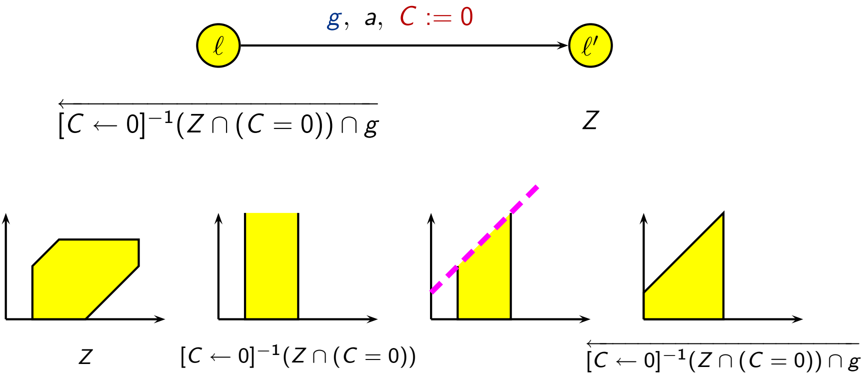

The exact backward computation terminates and is correct!

Note on the backward analysis (cont.)

If $\mathcal{A}$ is a timed automaton, we construct its corresponding set of regions.

Because of the bisimulation property, we get that:

> "Every set of valuations which is computed along the backward computation is a finite union of regions"

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Let R be a region. Assume:

- $v \in \overleftarrow{R}$ (for ex. $v + t \in R$)
- $v' \equiv_{\mathbf{reg.}} v$

There exists t' s.t. $v' + t' \equiv_{\mathbf{reg.}} v + t$, which implies that $v' + t' \in R$ and thus $v' \in \overleftarrow{R}$.

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If $\mathcal{A}$ is a timed automaton, we construct its corresponding set of regions.

Because of the bisimulation property, we get that:

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But, the backward computation is not so nice, when also dealing with integer variables...

$$i := j.k + \ell.m$$

Forward analysis of timed automata

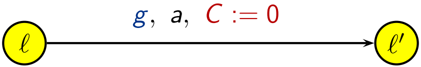
$$\ell \xrightarrow{\quad g,\ a,\ C := 0 \quad} \ell'$$

zones $\qquad Z \qquad\qquad [C \leftarrow 0](\overrightarrow{Z} \cap g)$

A zone is a set of valuations defined by a clock constraint

$$\varphi \ ::= \ x \sim c \ \mid \ x - y \sim c \ \mid \ \varphi \wedge \varphi$$

Forward analysis of timed automata



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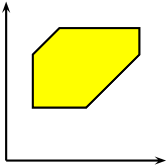

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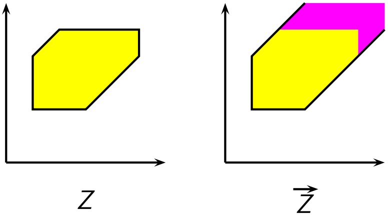

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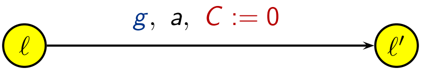

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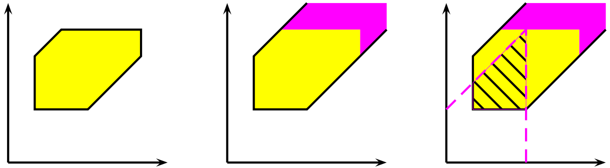
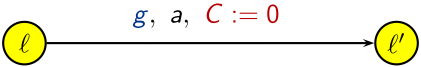


$Z \quad\quad \overrightarrow{Z} \quad\quad \overrightarrow{Z} \cap g$

Forward analysis of timed automata



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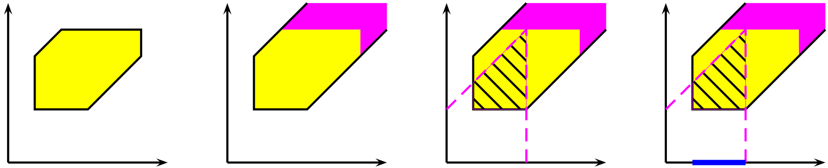

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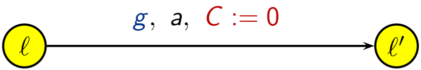

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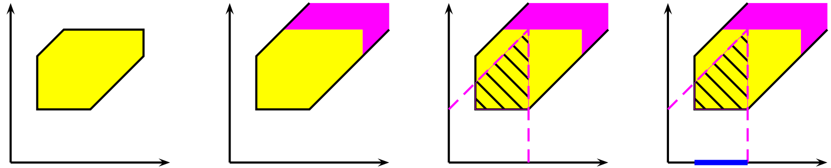

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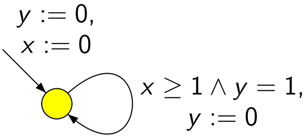
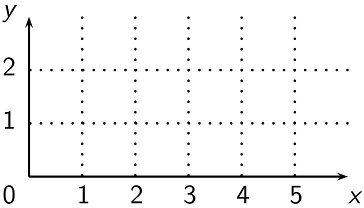
➜ a termination problem

Non termination of the forward analysis





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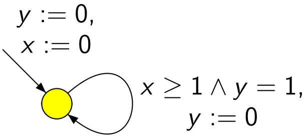

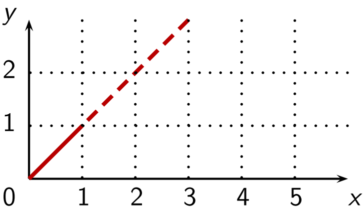

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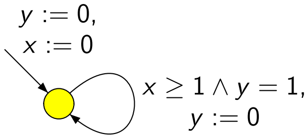

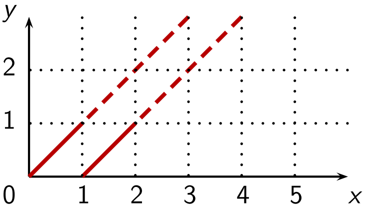

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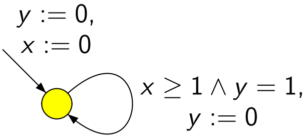

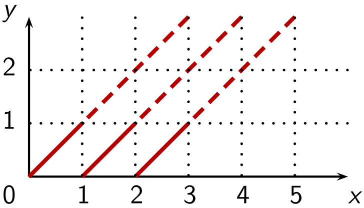

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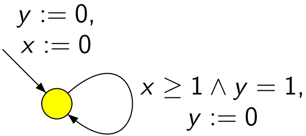

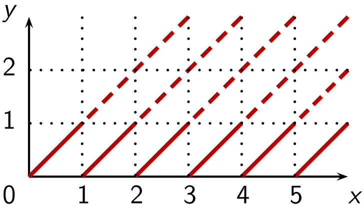

➜ an infinite number of steps...

"Solutions" to this problem

(f.ex. in **[Larsen,Pettersson,Yi 1997]** or in **[Daws,Tripakis 1998]**)

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- **activity**: eliminate redundant clocks **[Daws,Yovine 1996]**

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$$q \xrightarrow{g,a,C:=0} q' \quad \text{implies} \quad \mathrm{Act}(q) = \mathrm{clocks}(g) \cup (\mathrm{Act}(q') \setminus C)$$

...

"Solutions" to this problem (cont.)

- **convex-hull approximation**: if Z and Z' are computed then we overapproximate using "$Z \sqcup Z'$".

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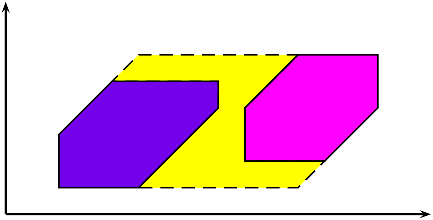

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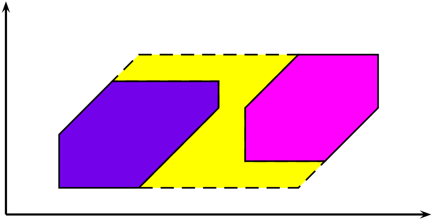

- **extrapolation**, a widening operator on zones

The DBM data structure

DBM (Difference Bounded Matrice) data structure

[Berthomieu, Menasche 1983] [Dill 1989]

$$(x_1 \geq 3) \ \wedge \ (x_2 \leq 5) \ \wedge \ (x_1 - x_2 \leq 4)$$

$$\begin{array}{c c} & \begin{array}{ccc} x_0 & x_1 & x_2 \end{array} \\ \begin{array}{c} x_0 \\ x_1 \\ x_2 \end{array} & \begin{pmatrix} +\infty & -3 & +\infty \\ +\infty & +\infty & 4 \\ 5 & +\infty & +\infty \end{pmatrix} \end{array}$$

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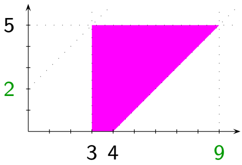
- Existence of a normal form



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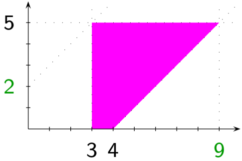

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- All previous operations on zones can be computed using DBMs

The extrapolation operator

Fix an integer k ("$*$" represents an integer between $-k$ and $+k$)

$$\begin{pmatrix} * & \boxed{> k} & * \\ * & * & * \\ \boxed{< -k} & * & * \end{pmatrix} \quad \leadsto \quad \begin{pmatrix} * & \boxed{+\infty} & * \\ * & * & * \\ \boxed{-k} & * & * \end{pmatrix}$$

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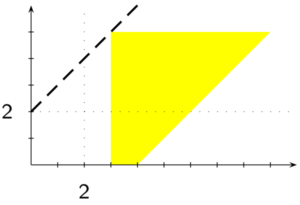

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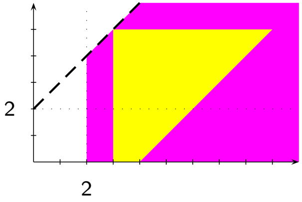

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Classical algorithm, focus on correctness

Challenge: choose a **good** constant for the extrapolation so that the forward computation is correct.

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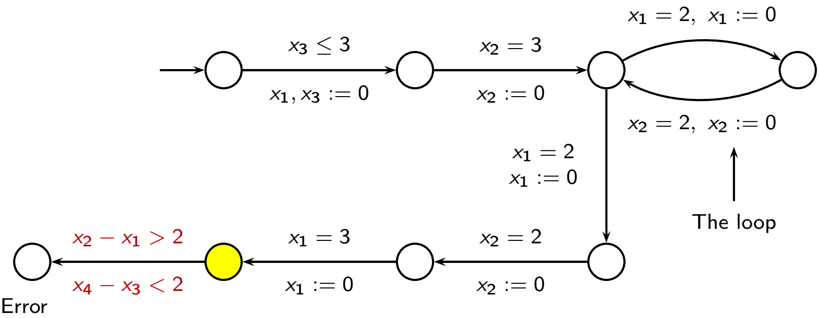
Theorem: this algorithm is correct for diagonal-free timed automata.

However, this theorem does not extend to timed automata using diagonal clock constraints...

A problematic automaton



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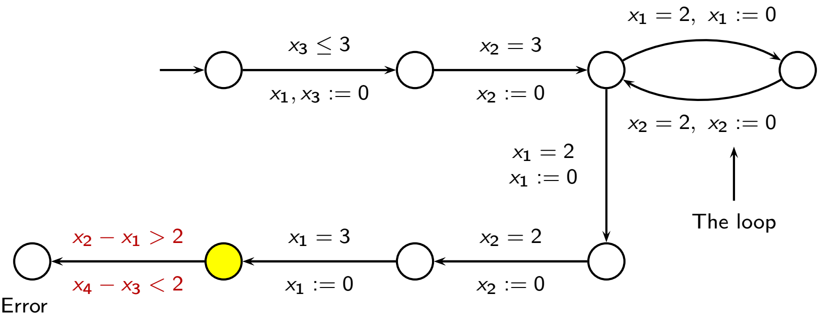

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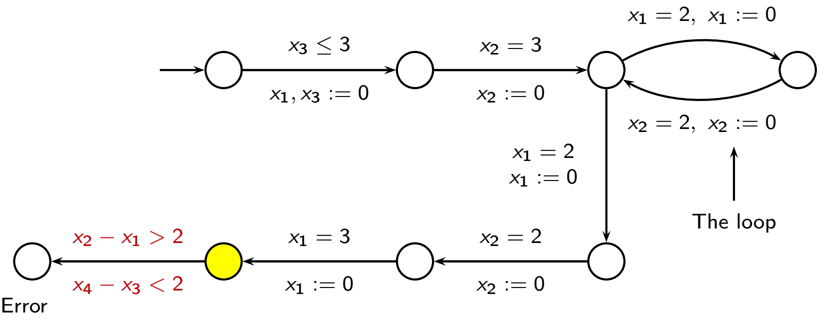

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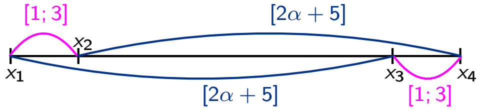
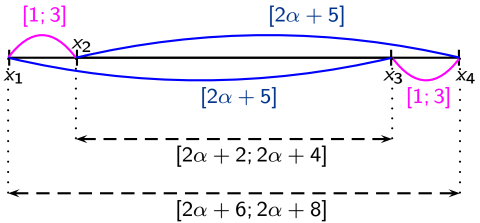


The problematic zone



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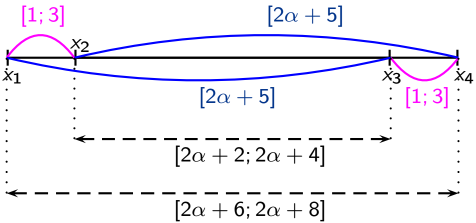

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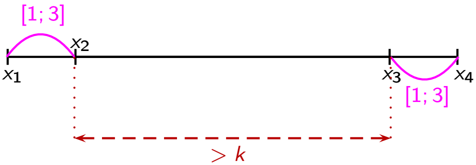
If α is sufficiently large, after extrapolation:



does not imply $x_1 - x_2 = x_3 - x_4$.

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For the previous automaton,

no abstraction operator can satisfy all these criteria!

Why that?

Assume there is a "nice" operator Abs.

The set $\{M \text{ DBM representing a zone } \mathrm{Abs}(Z)\}$ is finite.

➜ k the max. constant defining one of the previous DBMs

We get that, for every zone Z,

$$Z \subseteq \mathrm{Extra}_k(Z) \subseteq \mathrm{Abs}(Z)$$

Problem!

Open questions:
- which conditions can be made weaker?
- find a clever termination criterium?
- use an other data structure than zones/DBMs?

Outline

Discussion on complexity

[Alur 1991, Alur Henzinger 1994, Alur Courcoubetis Dill 1993, Aceto Laroussinie 2002]

	Kripke structures S	Timed automaton A
Reachability	NLOGSPACE-complete	
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Tools for timed systems: Uppaal, HyTech, Kronos, etc...

Conclusion & Further Work

- Decidability is quite well understood.
- Needs to understand better the **geometry** of the reachable state space.
    - clever (and correct) implementation of timed automata
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To be continued...

- Some other current challenges:
    - controller synthesis
    - implementability issues (program synthesis)
    - optimal computations (see Kim's talk)
    - ...

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Kronos: `http://www-verimag.imag.fr/TEMPORISE/kronos/`

Uppaal: `http://www.uppaal.com/`