Almost-Sure Model Checking of Infinite Paths in One-Clock Timed Automata

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Outline

1. Introduction

2. A probabilistic semantics

3. Solving the qualitative model-checking problem

4. Conclusion

Motivations

- Timed automata, an idealized mathematical model for real-time systems

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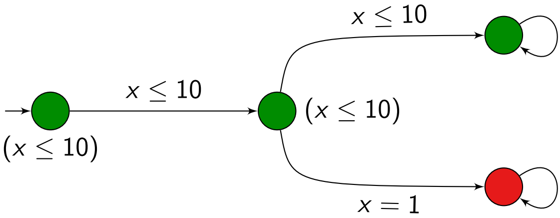
Our aim:

Use probabilities to "relax" the semantics of timed automata

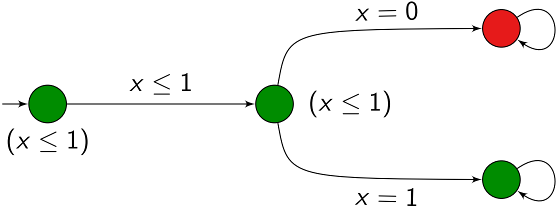
Initial example



Intuition: from the initial state,

this automaton *almost-surely* satisfies "$\mathbf{G}$ green"

A maybe less intuitive example



Does it *almost-surely* satisfy "$\mathbf{G}$ green"?

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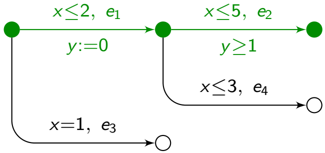

$$\pi(s_0 \xrightarrow{e_1}\xrightarrow{e_2}) = \{s_0 \xrightarrow{\tau_1, e_1} s_1 \xrightarrow{\tau_2, e_2} s_2 \mid \tau_1 \leq 2,\ \tau_1 + \tau_2 \leq 5,\ \tau_2 \geq 1\}$$

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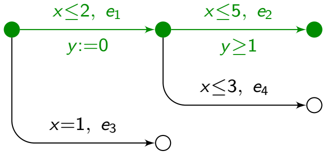

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Idea:

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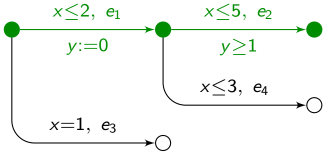

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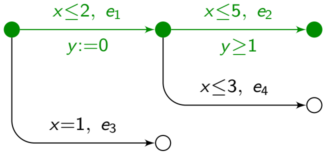

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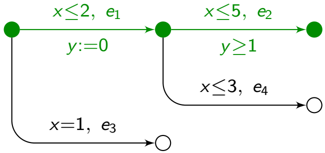

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Idea:

From state s_0:

- randomly choose a delay
- then randomly select an edge
- then continue

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Symbolic path: $\pi\left(s \xrightarrow{e_1} \cdots \xrightarrow{e_n}\right) = \left\{s \xrightarrow{\tau_1, e_1} s_1 \cdots \xrightarrow{\tau_n, e_n} s_n\right\}$

$$\mathbb{P}\left(\pi\left(s \xrightarrow{e_1} \cdots \xrightarrow{e_n}\right)\right) = \int_{t \in I(s, e_1)} p_{s+t}(e_1)\, \mathbb{P}\left(\pi\left(s_t^{e_1} \xrightarrow{e_2} \cdots \xrightarrow{e_n}\right)\right) \mathrm{d}\mu_s(t)$$

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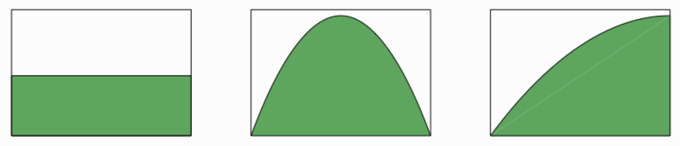

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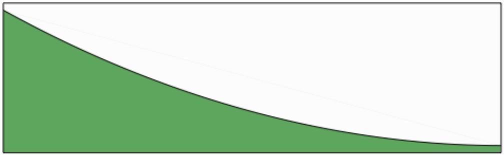

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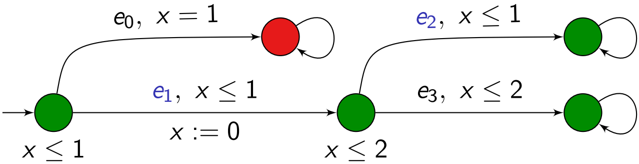
- Example:
  - $\mathrm{Zeno}(s) = \bigcup_{M \in \mathbb{N}} \bigcap_{n \in \mathbb{N}} \bigcup_{(e_1, \cdots, e_n) \in E^n} \mathrm{Cyl}(\pi_{\Sigma_i \tau_i \leq M}(s \xrightarrow{e_1} \cdots \xrightarrow{e_n}))$

An example of computation (with uniform distributions)



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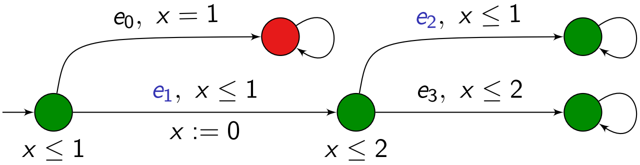

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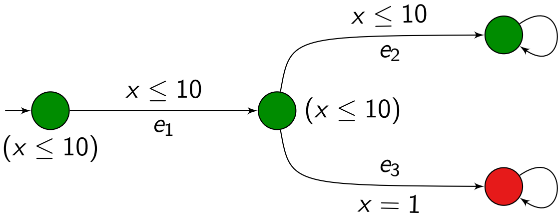
$$\begin{aligned}
\mathbb{P}\Big(\pi(s_0 \xrightarrow{e_1}\xrightarrow{e_2})\Big) &= \int_0^1 \mathbb{P}\Big(\pi(s_1 \xrightarrow{e_2})\Big)\mathrm{d}\mu_{s_0}(t) + \int_1^1 \frac{\mathbb{P}\Big(\pi(s_1 \xrightarrow{e_2})\Big)}{2}\mathrm{d}\mu_{s_0}(t) \\
&= \int_0^1 \int_0^1 \left(\frac{\mathbb{P}\Big(\pi(s_2)\Big)}{2}\mathrm{d}\mu_{s_1}(u)\right)\mathrm{d}\mu_{s_0}(t) \\
&= \int_0^1 \int_0^1 \left(\frac{1}{2}\frac{\mathrm{d}u}{2}\right)\mathrm{d}t \quad = \frac{1}{4}
\end{aligned}$$

Back to the first example



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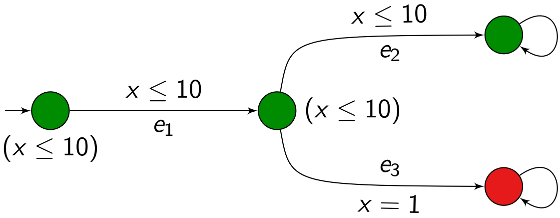

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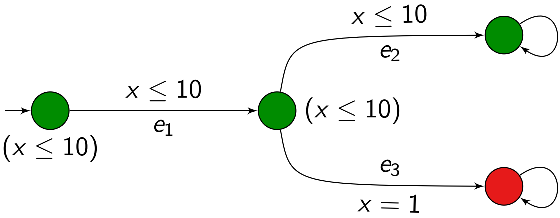

- $\mathbb{P}\Big(\pi\big(s_0 \xrightarrow{e_1}\xrightarrow{e_2}\big)\Big) = 1$
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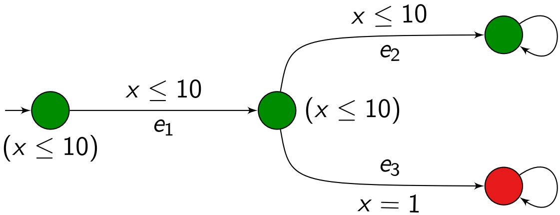

- $\mathbb{P}\Big(\pi\big(s_0 \xrightarrow{e_1}\xrightarrow{e_2}\big)\Big) = 1$
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- $\mathbb{P}\Big(\mathbf{G}\ \text{green}\Big) = 1$

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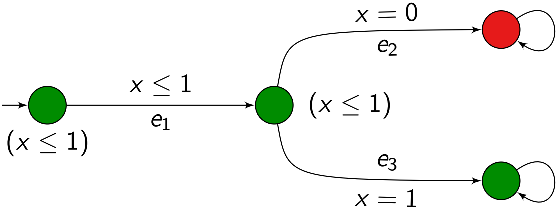

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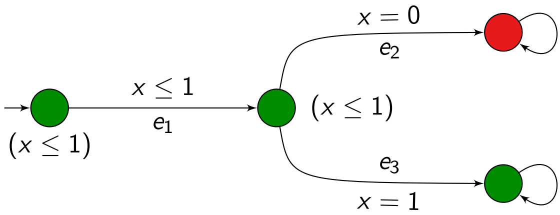

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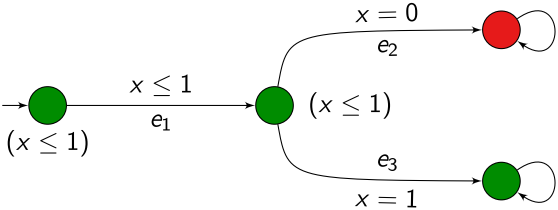

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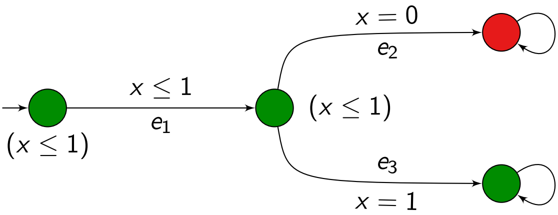

- $\mathbb{P}\Big(\pi\big(s_0 \xrightarrow{e_1}\xrightarrow{e_2}\big)\Big) = 0$
- $\mathbb{P}\Big(\pi\big(s_0 \xrightarrow{e_1}\xrightarrow{e_3}\big)\Big) = 1$
- $\mathbb{P}\Big(\mathbf{G}\ \text{green}\Big) = 1$

Almost-sure model-checking

If φ is an LTL (or ω-regular) property,

$$s \mathrel{|\!\!\approx} \varphi \quad \stackrel{\text{def}}{\Leftrightarrow} \quad \mathbb{P}\Big(\{\varrho \in \mathsf{Runs}(s) \mid \varrho \models \varphi\}\Big) = 1$$

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We want to decide the almost-sure model-checking...
(This is a qualitative model-checking question)

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2. A probabilistic semantics

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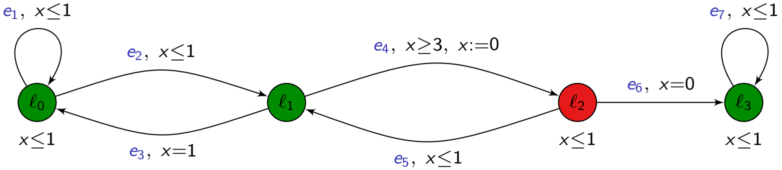
An example



e_1, $x \leq 1$
e_2, $x \leq 1$
e_4, $x \geq 3$, $x := 0$
e_6, $x = 0$
e_7, $x \leq 1$
ℓ_0
ℓ_1
ℓ_2
ℓ_3
$x \leq 1$
e_3, $x = 1$
e_5, $x \leq 1$
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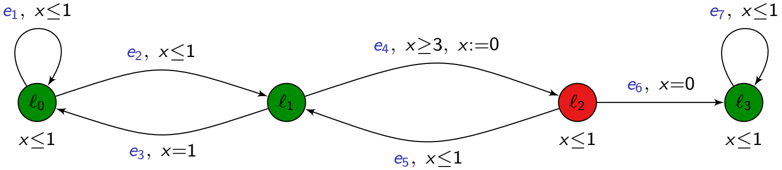

$\mathcal{A} \not\models \mathbf{G}(\text{green} \Rightarrow \mathbf{F}\ \text{red})$

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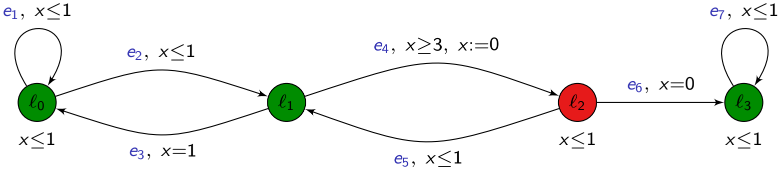

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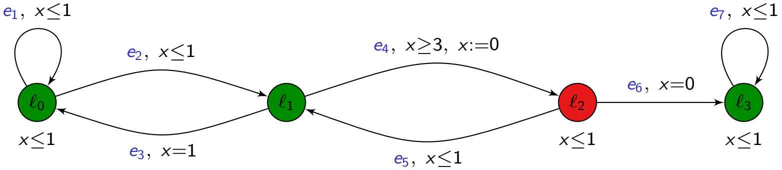

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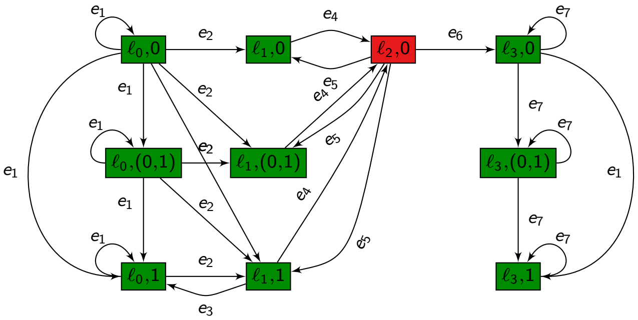
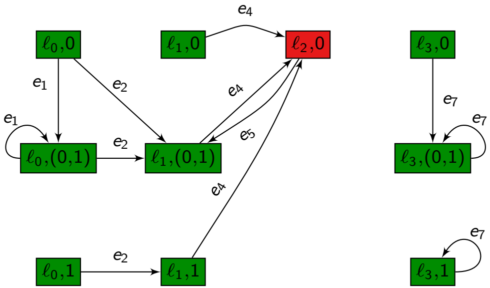
Indeed, almost surely, paths are of the form $e_1^* e_2 \Big(e_4 e_5 \Big)^\omega$

The classical region automaton



The pruned region automaton



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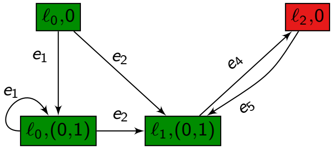

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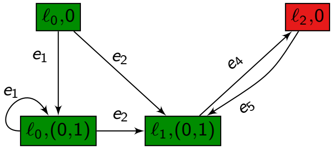

... viewed as a finite Markov chain $MC(\mathcal{A})$

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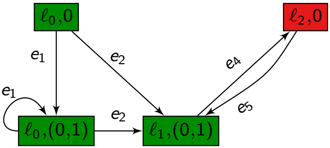

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Theorem

For single-clock timed automata,

$$\mathcal{A} \approx\!\!\!\!| \; \varphi \quad \text{iff} \quad \mathbb{P}(MC(\mathcal{A}) \models \varphi) = 1$$

Result

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For single-clock timed automata, the almost-sure model-checking

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For single-clock timed automata, the almost-sure model-checking

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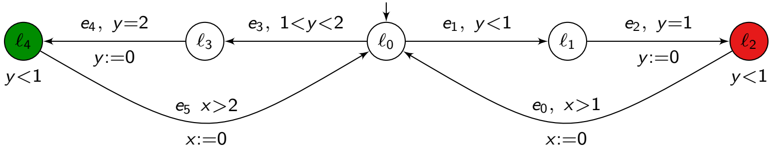
- Complexity:
  - size of single-clock region automata = polynomial [LMS04]
  - apply result of [CSS03] to the finite Markov chain
- Correctness: the proof is rather involved
  - requires the definition of a topology over the set of paths
  - notions of largeness (for proba 1) and meagerness (for proba 0)
  - link between probabilities and topology thanks to the topological games called Banach-Mazur games

An example with two clocks



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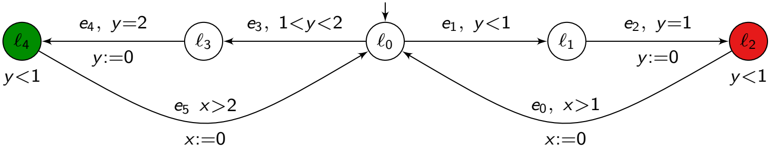

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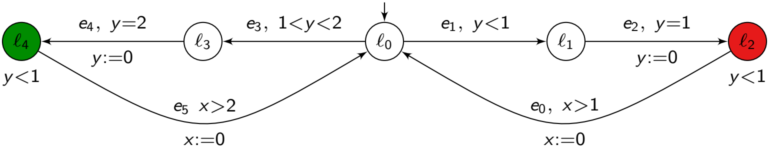

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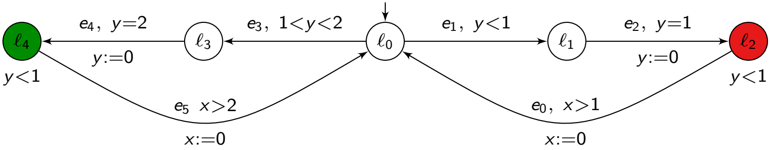

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- However, we can prove that $\mathbb{P}\Big(\mathbf{G}\,\neg\text{red}\Big) > 0$
- There is a *strange* convergence phenomenon: along an execution, if $\delta_i > 0$ is the delay in location ℓ_4, then we have that $\sum_i \delta_i \le 1$

A note on Zeno behaviours

- The set of Zeno behaviours is measurable:

$$\mathsf{Zeno}(s) \;=\; \bigcup_{M \in \mathbb{N}} \bigcap_{n \in \mathbb{N}} \bigcup_{(e_1,\cdots,e_n) \in E^n} \mathsf{Cyl}(\pi(s \xrightarrow{e_1} \cdots \xrightarrow{e_n}))$$

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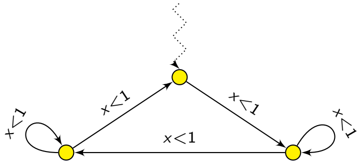

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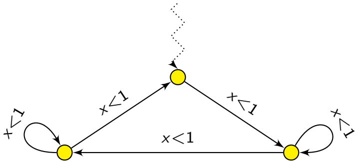
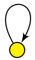


  - an interesting notion of non-Zeno timed automata



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Ongoing work

- our semantics can be viewed as a $\frac{1}{2}$-player game
  $1\frac{1}{2}$- and $2\frac{1}{2}$-player games

  $\leadsto$ further interesting (un)decidability results