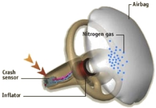
On the verification of timed systems...
... and beyond

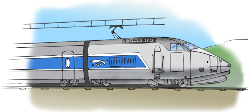
Patricia Bouyer-Decitre

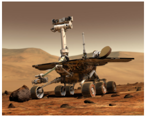
LSV, CNRS & ENS Paris-Saclay, France

Time-dependent systems

- We are interested in timed systems

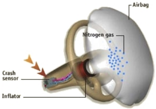


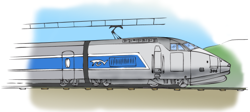


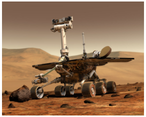


Time-dependent systems

- We are interested in timed systems



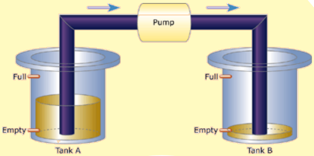




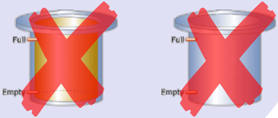
- ... and in their analysis and control

Model-checking and control

system:

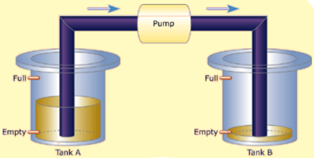


property:

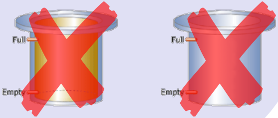


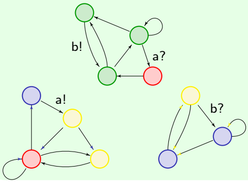
Model-checking and control

system:



property:

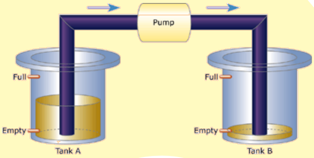




$$\mathbf{AG}(\neg \texttt{B.overfull} \wedge \neg \texttt{B.dried\_up})$$

Model-checking and control

system:



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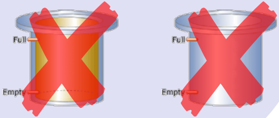
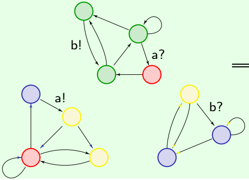


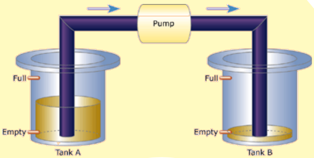


$\Longrightarrow$ algorithm $\Longleftarrow$

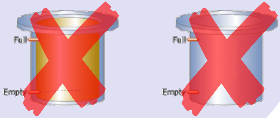
AG($\neg$`B.overfull` $\wedge$ $\neg$`B.dried_up`)

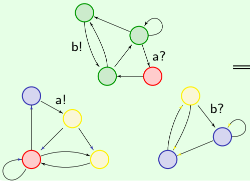
Model-checking and control

system:



property:



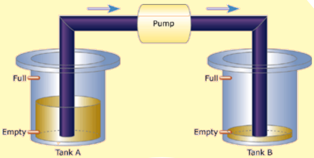


$\Longrightarrow$ model-checking algorithm $\Longleftarrow$ **AG**($\neg$`B.overfull` $\wedge$ $\neg$`B.dried_up`)

yes/no

Model-checking and control

system:



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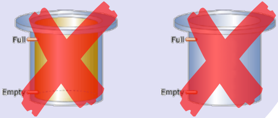
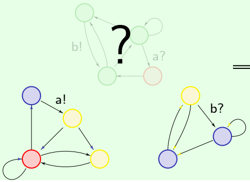
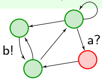




control/synthesis algorithm

AG($\neg$`B.overfull` $\wedge$ $\neg$`B.dried_up`)



An example: The task graph scheduling problem

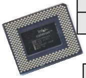
Compute $D\times(C\times(A+B))+(A+B)+(C\times D)$ using two processors:

P_1 (fast):

time	
+	2 picoseconds
×	3 picoseconds

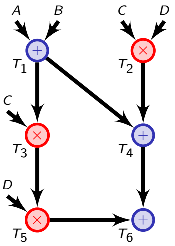
energy	
idle	10 Watt
in use	90 Watts

P_2 (slow):



time	
+	5 picoseconds
×	7 picoseconds

energy	
idle	20 Watts
in use	30 Watts



[BFLM10] Bouyer, Fahrenberg, Larsen, Markey. Quantitative Analysis of Real-Time Systems using Priced Timed Automata *(Communication of the ACM).*

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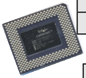

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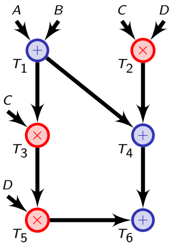

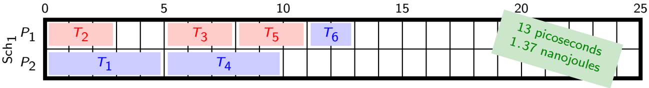

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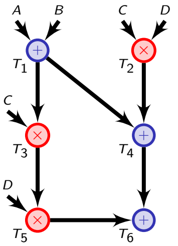

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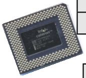
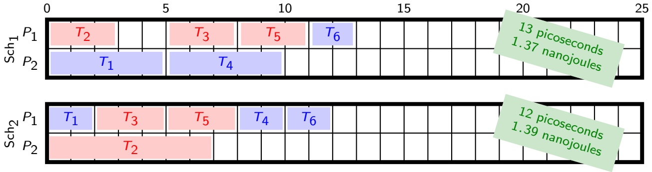
P_2 (slow):



	time
$+$	5 picoseconds
$\times$	7 picoseconds

energy	
idle	20 Watts
in use	30 Watts



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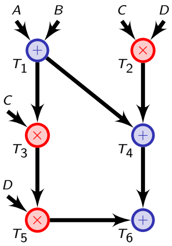

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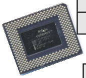

	time
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idle	20 Watts
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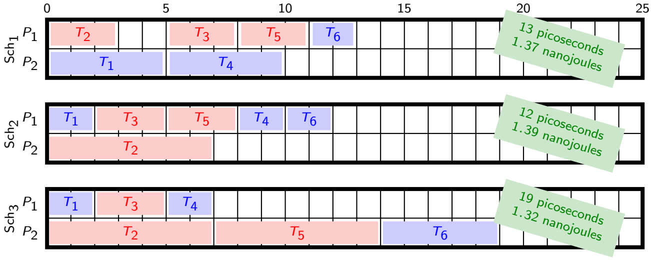

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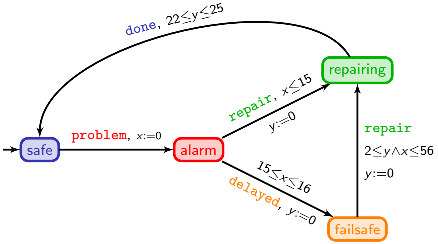
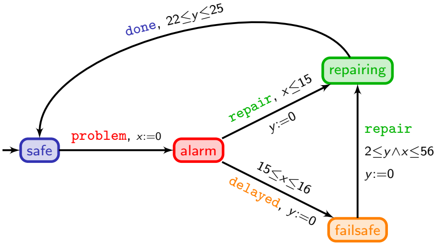
Outline

The model of timed automata



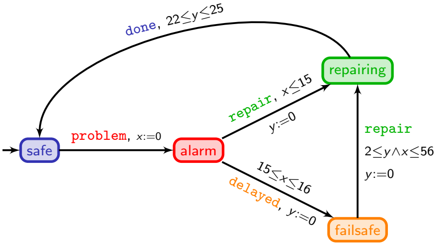
[AD94] Alur, Dill. A Theory of Timed Automata *(Theoretical Computer Science)*.

The model of timed automata



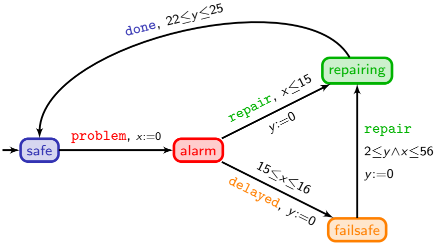
	safe
x	0
y	0

The model of timed automata



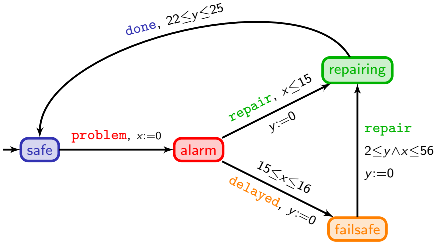
	safe	$\xrightarrow{23}$	safe
x	0		23
y	0		23

The model of timed automata



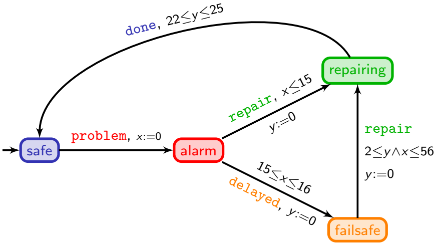
	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm
x	0		23		0
y	0		23		23

The model of timed automata



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm
x	0		23		0		15.6
y	0		23		23		38.6

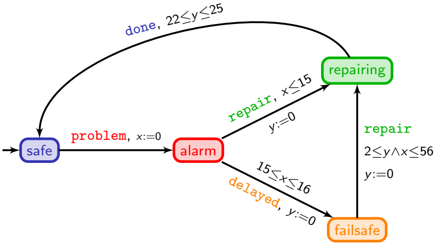
The model of timed automata



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\text{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\text{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe
⋯	15.6
	0

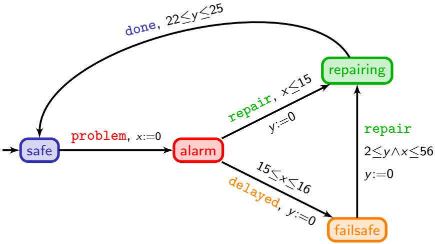
The model of timed automata



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe		
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe
⋯	15.6		17.9
	0		2.3

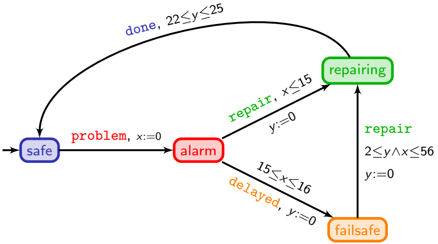
The model of timed automata



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	···
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing
···	15.6		17.9		17.9
	0		2.3		0

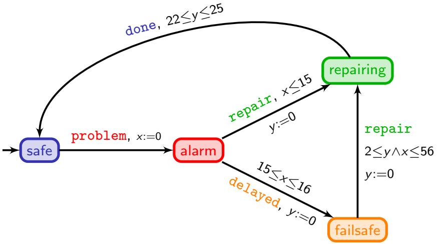
The model of timed automata



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing
⋯	15.6		17.9		17.9		40
	0		2.3		0		22.1

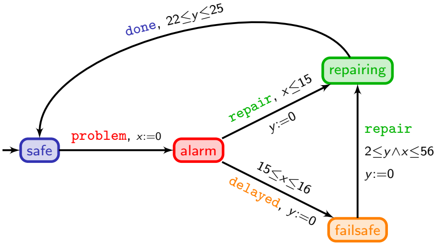
The model of timed automata



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe		
x	0		23		0		15.6		15.6		···
y	0		23		23		38.6		0		

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\texttt{done}}$	safe
···	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

The model of timed automata

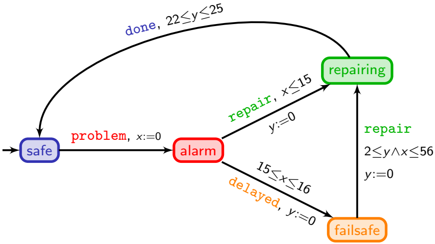


(clock) valuation

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\texttt{done}}$	safe
⋯	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

The model of timed automata



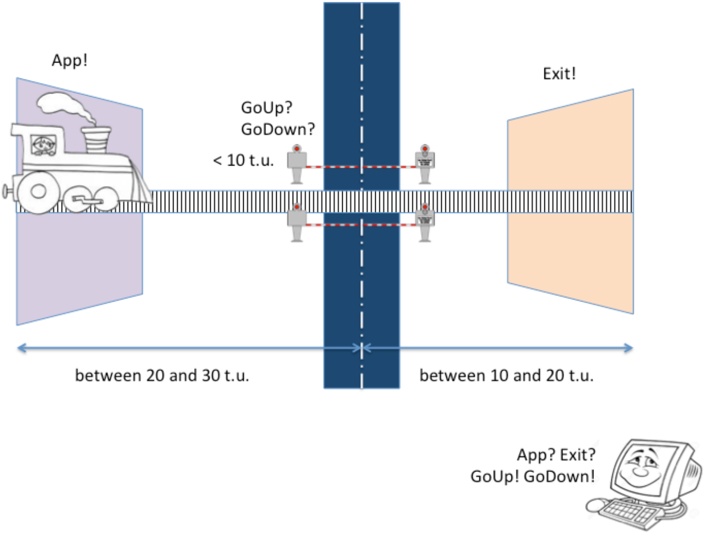
(clock) valuation

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	···
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\texttt{done}}$	safe
···	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

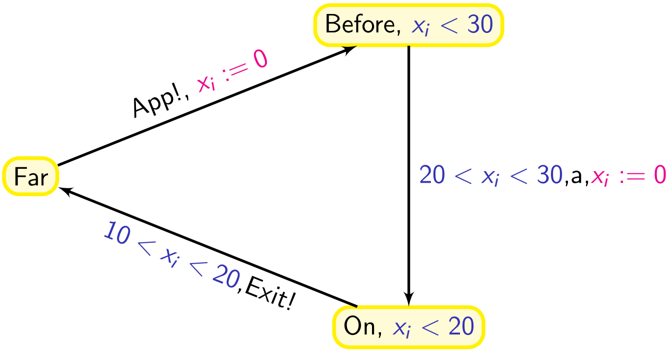
This run reads the timed word
$(\texttt{problem}, 23)(\texttt{delayed}, 38.6)(\texttt{repair}, 40.9)(\texttt{done}, 63)$

The train crossing example



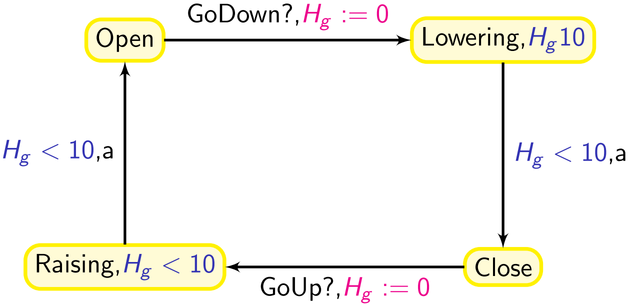
Modelling the train crossing example

Train$_i$ with $i = 1, 2, \ldots$



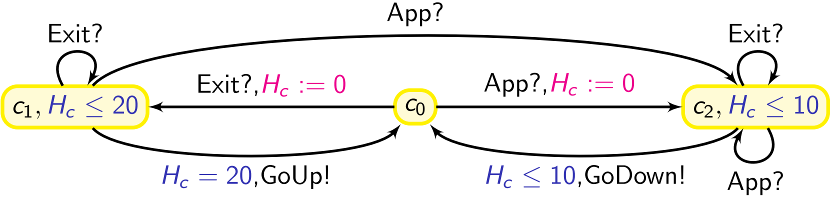
The train crossing example – cont'd

The gate:



The train crossing example – cont'd

The controller:



The train crossing example – cont'd

We use the synchronization function f:

Train_1	Train_2	Gate	Controller	
App!	.	.	App?	App
.	App!	.	App?	App
Exit!	.	.	Exit?	Exit
.	Exit!	.	Exit?	Exit
a	.	.	.	a
.	a	.	.	a
.	.	a	.	a
.	.	GoUp?	GoUp!	GoUp
.	.	GoDown?	GoDown!	GoDown

to define the parallel composition $(\text{Train}_1 \parallel \text{Train}_2 \parallel \text{Gate} \parallel \text{Controller})$

NB: the parallel composition does not add expressive power!

The train crossing example – cont'd

Some properties one could check:

- Is the gate closed when a train crosses the road?

The train crossing example – cont'd

Some properties one could check:

- Is the gate closed when a train crosses the road?
- Is the gate always closed for less than 5 minutes?

Back to the task graph scheduling problem

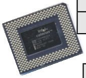
Compute $D\times(C\times(A+B))+(A+B)+(C\times D)$ using two processors:

P_1 (fast):

time	
$+$	2 picoseconds
$\times$	3 picoseconds

energy	
idle	10 Watt
in use	90 Watts

P_2 (slow):



time	
$+$	5 picoseconds
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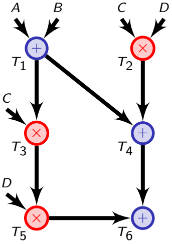
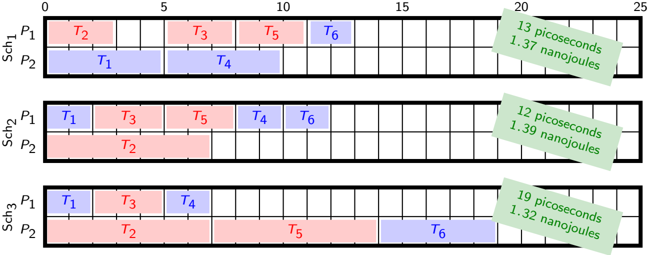
idle	20 Watts
in use	30 Watts





Modelling the task graph scheduling problem

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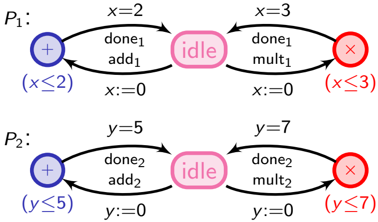
- Processors



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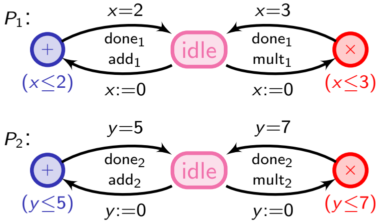
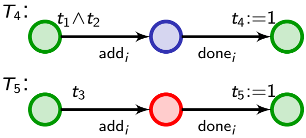


- Tasks



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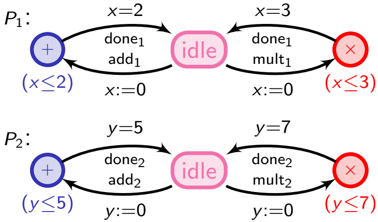

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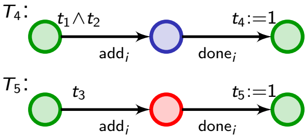

$\rightsquigarrow$ build the synchronized product of all these automata

$$(P_1 \parallel P_2) \parallel_s (T_1 \parallel T_2 \parallel \cdots \parallel T_6)$$

Modelling the task graph scheduling problem

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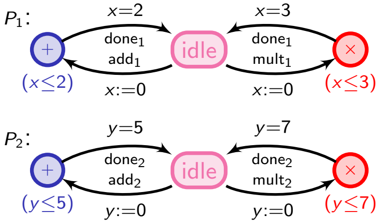

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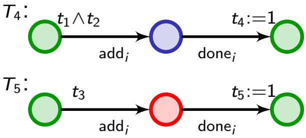

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A schedule: a path in the global system which reaches $t_1 \wedge \cdots \wedge t_6$

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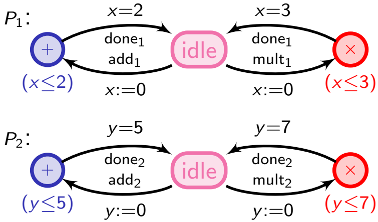

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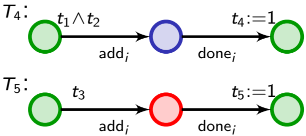

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Questions one can ask

- Can the computation be made in no more than 10 time units?
- Is there a scheduling along which no processor is ever idle?
- ...

What we have so far

- A model which can adequately represent systems with real-time constraint...
- ... on which we can ask relevant questions

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Interesting problems

- Which semantics?
  (and be aware of the limits of the choice)
- Algorithms for automatic verification

Discrete-time semantics

...because computers are digital!

[Alur91] Alur. Techniques for automatic verification of real-time systems. *PhD thesis*, 1991.

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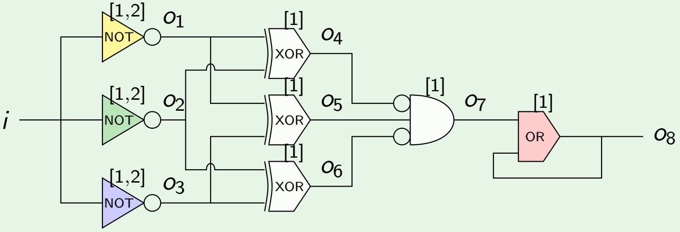
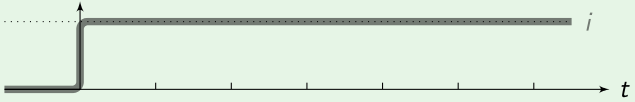
Example [Alur91]



- under discrete-time, the output is always 0:



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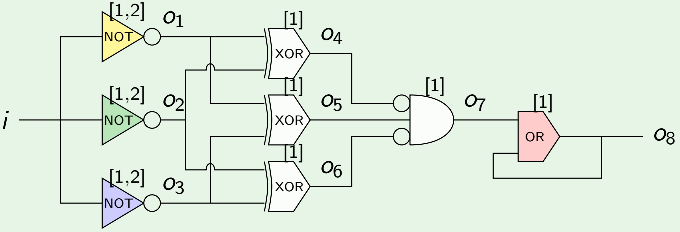

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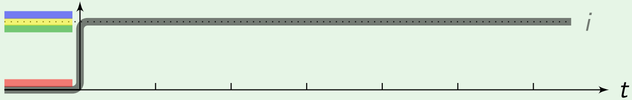

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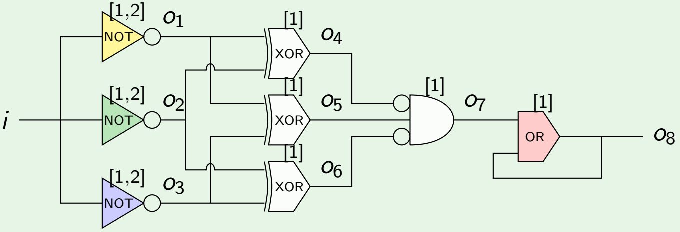

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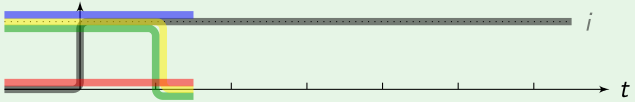

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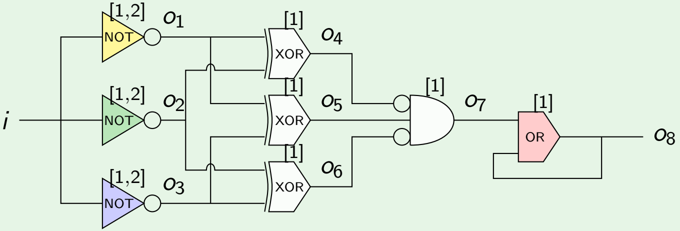

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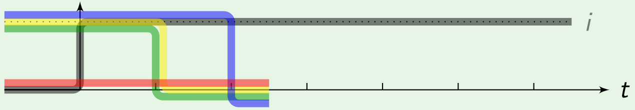

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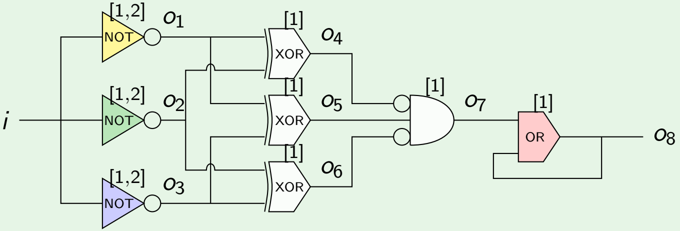

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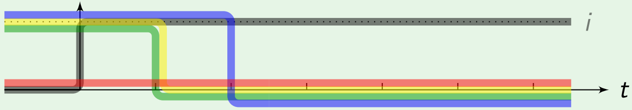

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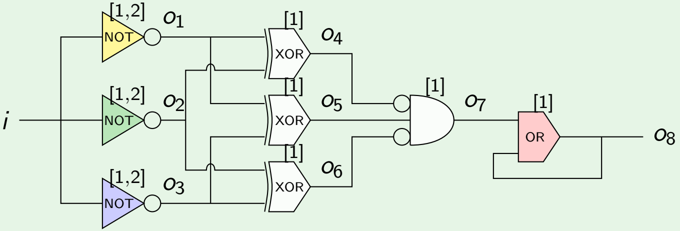
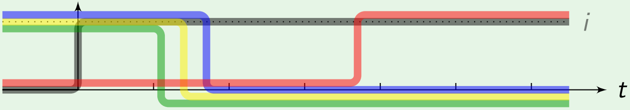


- under continuous-time, the output can be 1:



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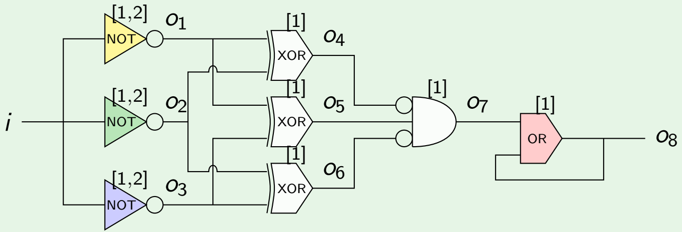

Finding the correct granularity (if one exists) is hard!

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Continuous-time semantics

...real-time models for real-time systems!

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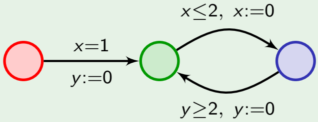

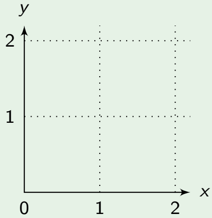

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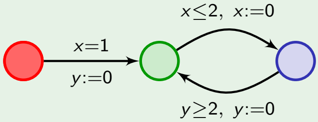

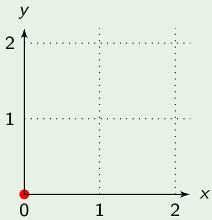

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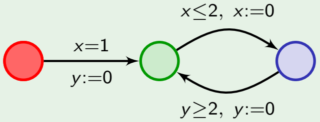

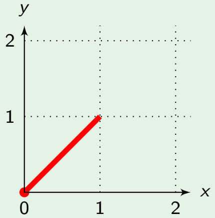

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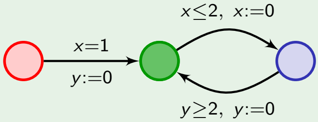

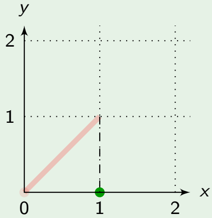

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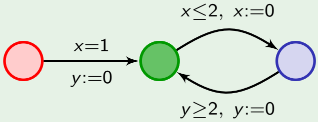

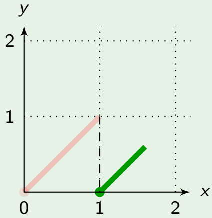

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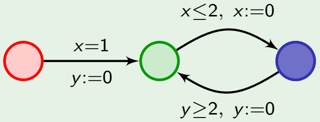

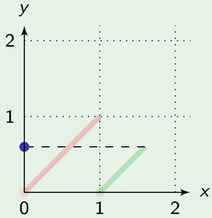

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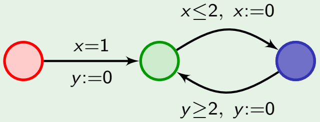

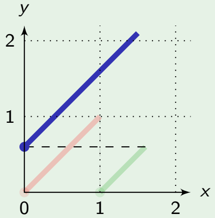

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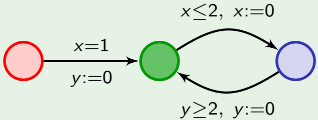

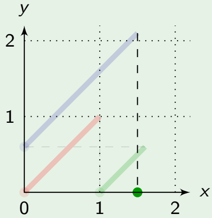

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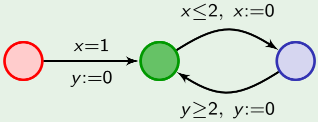

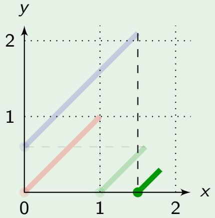

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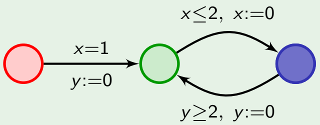

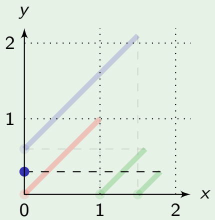

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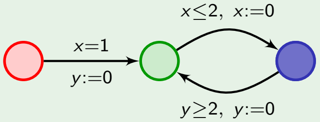

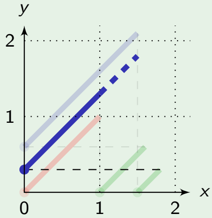

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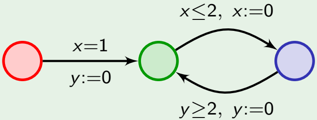

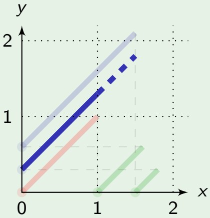

We will focus on the continuous-time semantics, since this is an adequate abstraction of real-time systems

Continuous-time semantics

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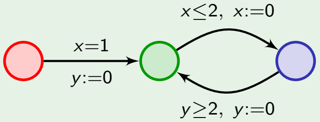
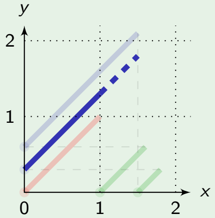
Example





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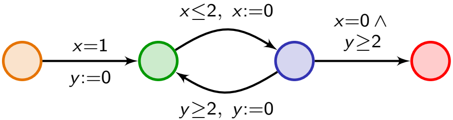
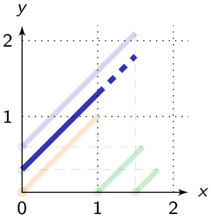
Known limits: robustness issues (we will comment on that later)

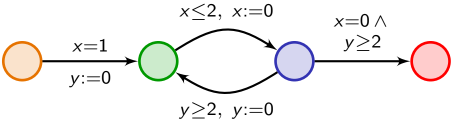
Analyzing timed automata

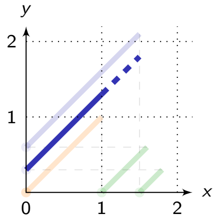




Can we reach state ?

Analyzing timed automata

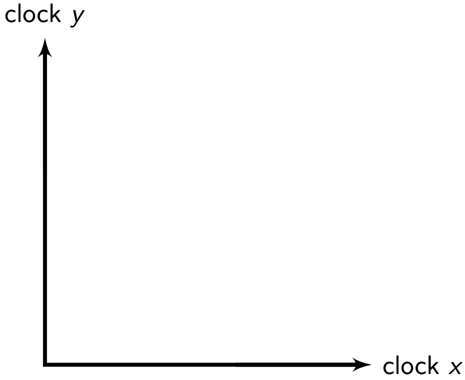




Can we reach state ?

- Problem: the set of configurations is infinite
  $\rightsquigarrow$ classical methods for finite-state systems cannot be applied
- Positive key point: variables (clocks) increase at the same speed

Crux idea: Region abstraction



[AD94] Alur, Dill. A theory of timed automata *(Theoretical Computer Science)*.

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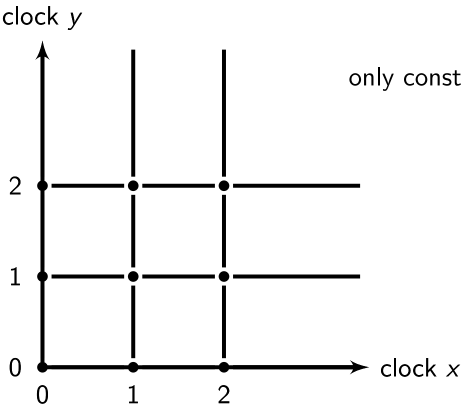

only constraints: $x \sim c$ with $c \in \{0,1,2\}$
$y \sim c$ with $c \in \{0,1,2\}$

- "compatibility" between regions and constraints

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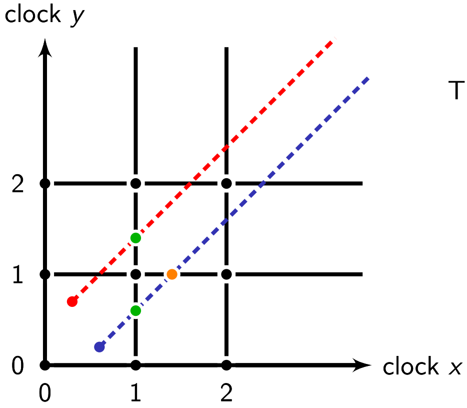

The path

- can be fired from
- cannot be fired from

- "compatibility" between regions and constraints
- "compatibility" between regions and time elapsing

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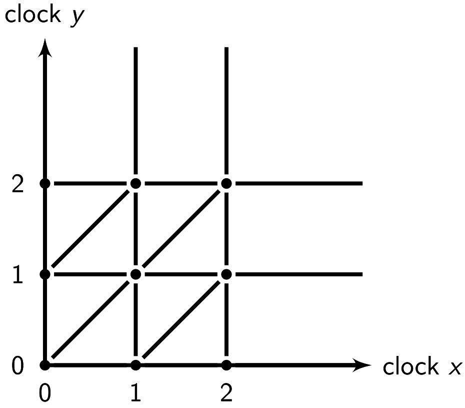

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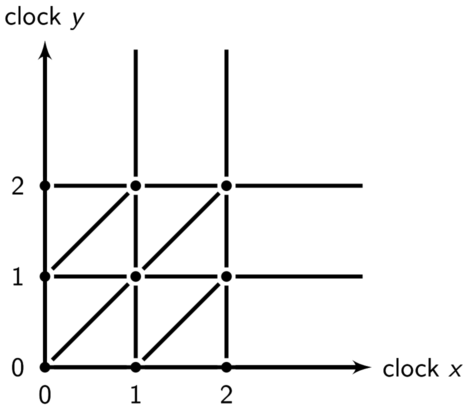

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$\rightsquigarrow$ an equivalence of finite index

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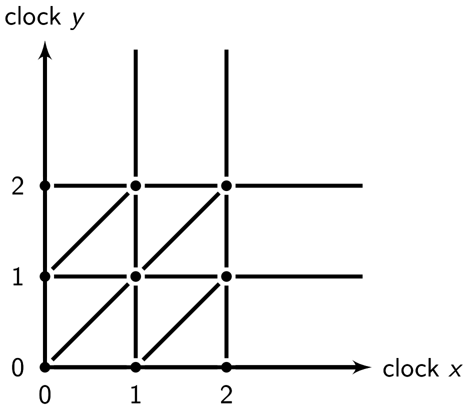

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$\rightsquigarrow$ an equivalence of finite index
a time-abstract bisimulation

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Time-abstract bisimulation

This is a relation between • and • such that:

Time-abstract bisimulation

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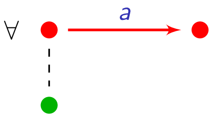

Time-abstract bisimulation

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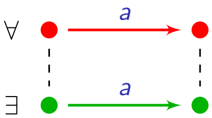

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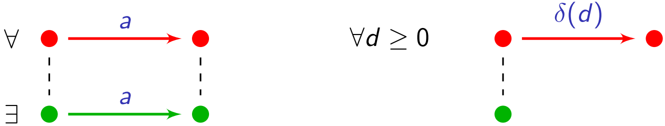

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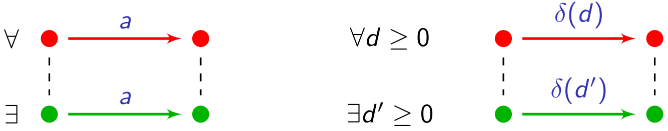

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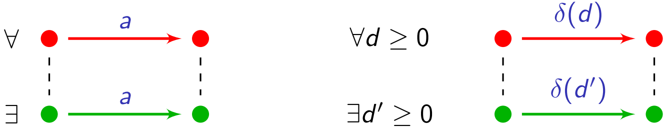

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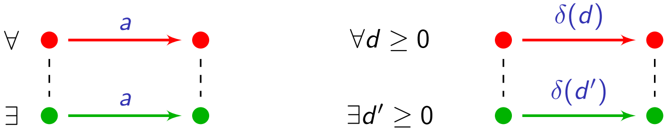

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Consequence

$$\forall \quad (\ell_1, v_1) \xrightarrow{d_1, a_1} (\ell_2, v_2) \xrightarrow{d_2, a_2} (\ell_3, v_3) \xrightarrow{d_3, a_3} \quad \cdots$$

Time-abstract bisimulation

This is a relation between • and • such that:

$$\begin{tikzcd} \forall & \bullet \arrow[r, "a"] \arrow[d, dash, dashed] & \bullet \arrow[d, dash, dashed] \\ \exists & \bullet \arrow[r, "a"] & \bullet \end{tikzcd} \qquad \begin{tikzcd} \forall d \geq 0 & \bullet \arrow[r, "\delta(d)"] \arrow[d, dash, dashed] & \bullet \arrow[d, dash, dashed] \\ \exists d' \geq 0 & \bullet \arrow[r, "\delta(d')"] & \bullet \end{tikzcd}$$

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$$\begin{tikzcd} \forall & (\ell_1, v_1) \arrow[r, "d_1, a_1"] \arrow[d] & (\ell_2, v_2) \arrow[r, "d_2, a_2"] \arrow[d] & (\ell_3, v_3) \arrow[r, "d_3, a_3"] \arrow[d] & \cdots \\ & (\ell_1, R_1) \arrow[r, "a_1"] & (\ell_2, R_2) \arrow[r, "a_2"] & (\ell_3, R_3) \arrow[r, "a_3"] & \cdots \end{tikzcd} \quad \text{with } v_i \in R_i$$

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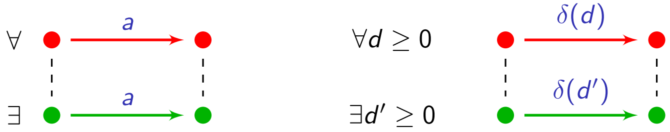

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\end{tikzcd}$$

$\forall v_1' \in R_1$

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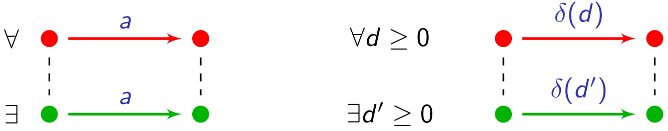

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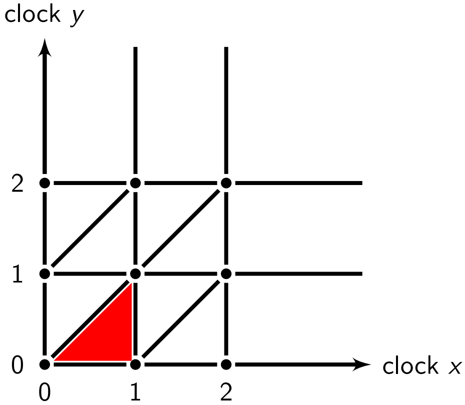
\forall v'_1 \in R_1 \ \exists & (\ell_1, v'_1) \arrow[r, "d'_1{,}a_1"] & (\ell_2, v'_2) \arrow[r, "d'_2{,}a_2"] & (\ell_3, v'_3) \arrow[r, "d'_3{,}a_3"] & \cdots & \text{with } v'_i \in R_i
\end{tikzcd}
$$

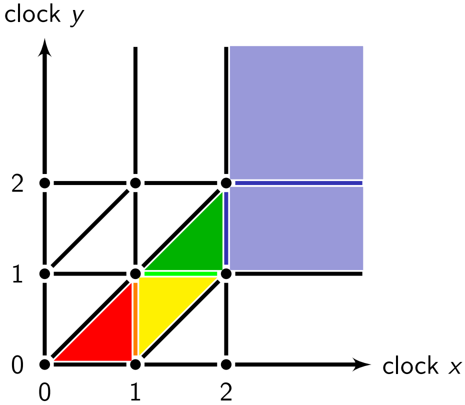
The region abstraction



- region R defined by:

$$\begin{cases} 0 < x < 1 \\ 0 < y < 1 \\ y < x \end{cases}$$

The region abstraction



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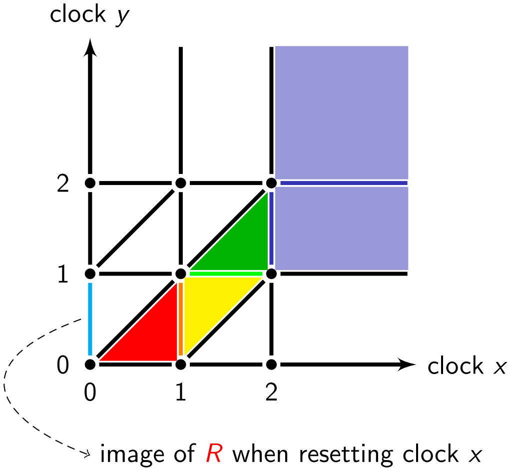

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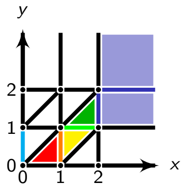
The construction of the region graph

It “mimicks” the behaviours of the clocks.



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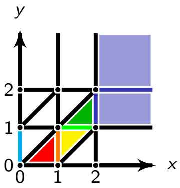
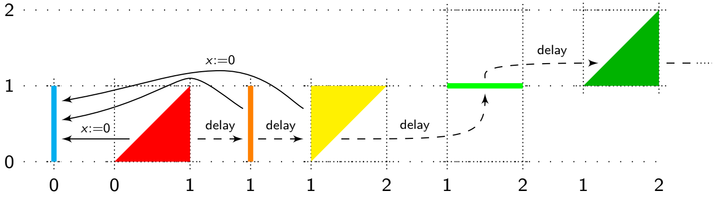
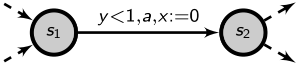


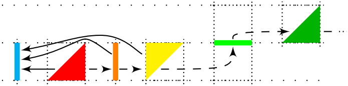


Region automaton $\equiv$ finite bisimulation quotient



timed automaton

$\otimes$



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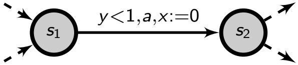

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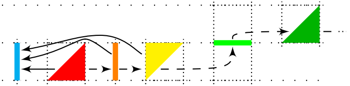
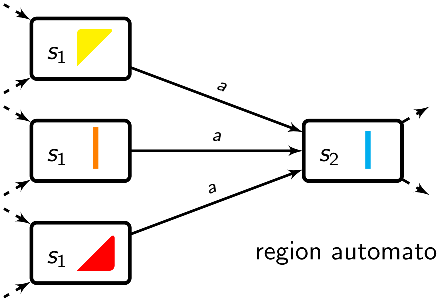


region graph



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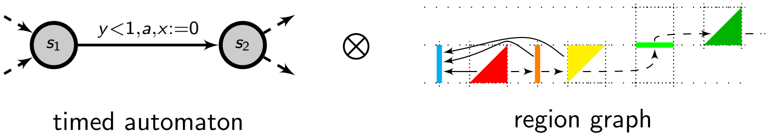
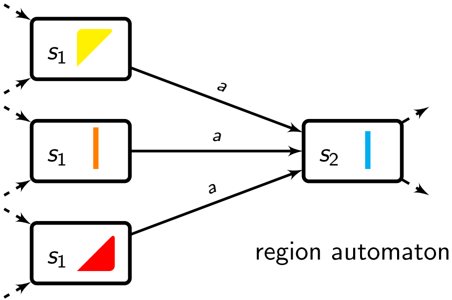
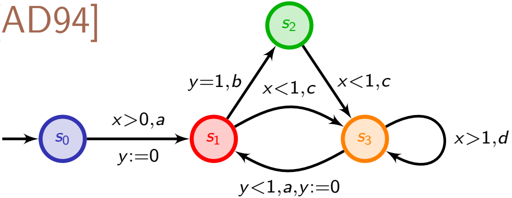




language(reg. aut.) = UNTIME(language(timed aut.))

An example [AD94]



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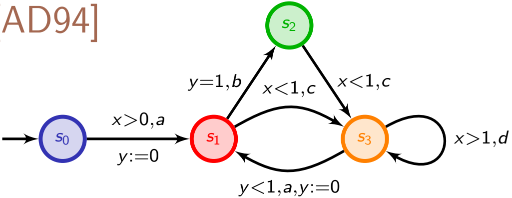

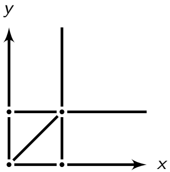

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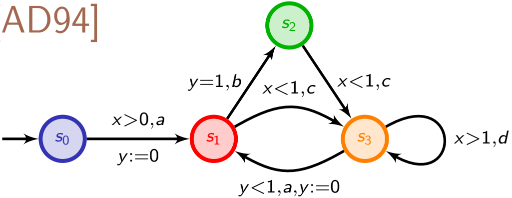
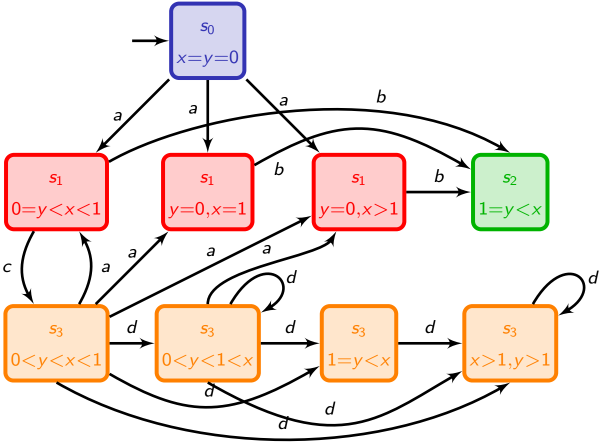
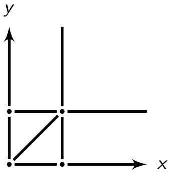
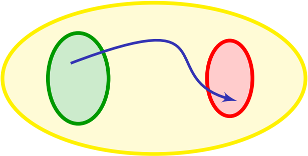
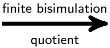


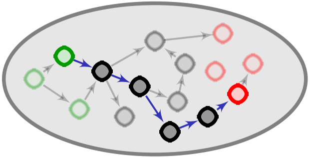




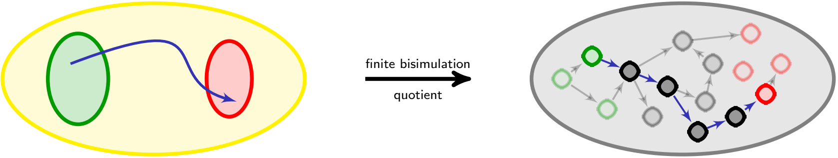


timed automaton





large (but finite) automaton
(region automaton)



timed automaton

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(region automaton)

- large: exponential in the number of clocks and in the constants (if encoded in binary). The number of regions is:

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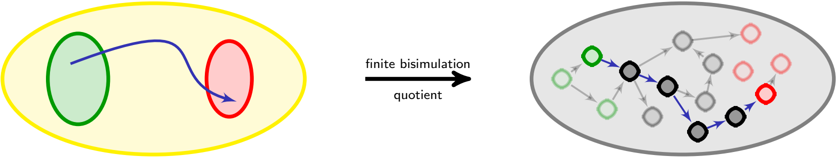

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    - liveness properties (Büchi/ω-regular properties)
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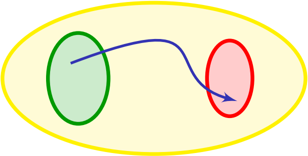
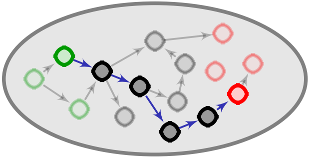


finite bisimulation

quotient



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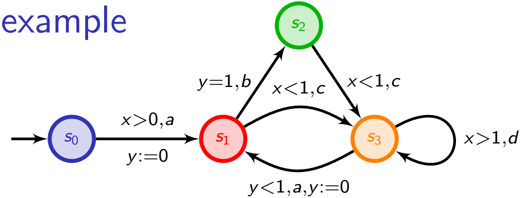
- Problems with Zeno behaviours?
  (infinitely many actions in bounded time)

Back to the example



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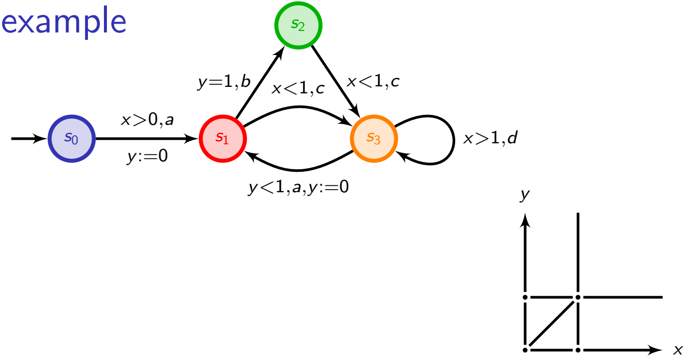

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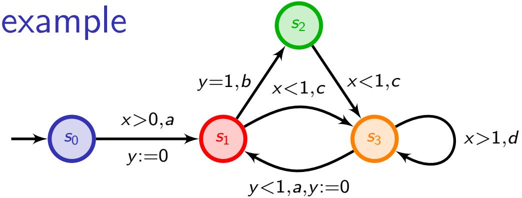

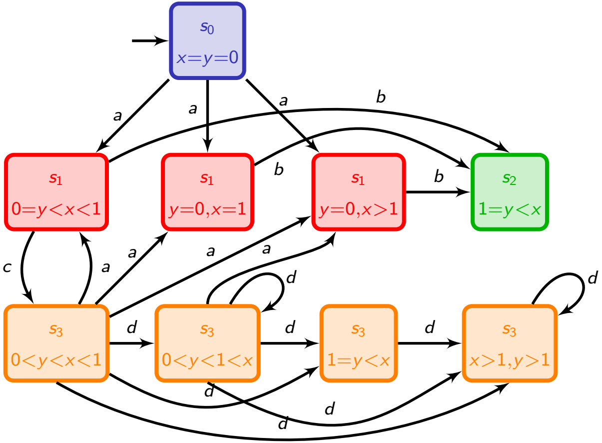

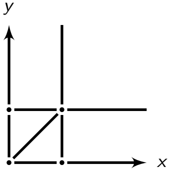

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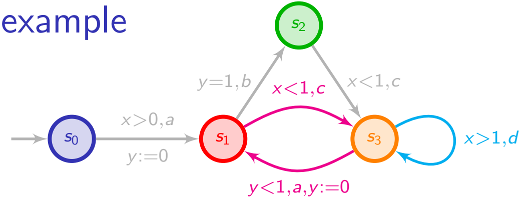
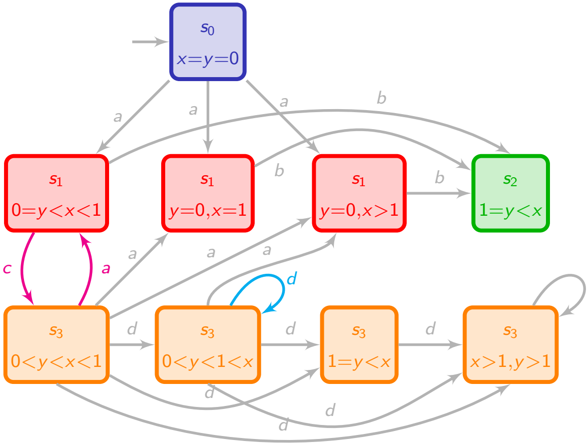
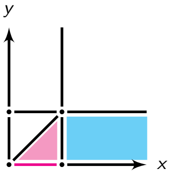
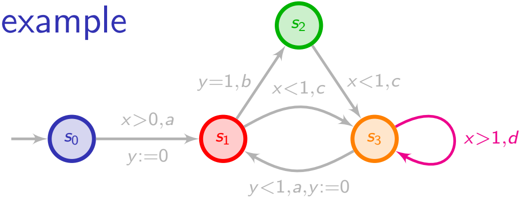


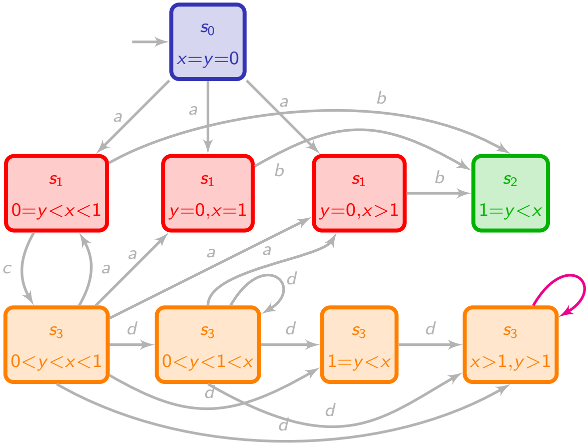


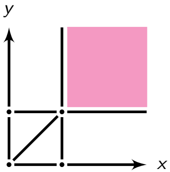


Zeno cycles

Back to the example







Cycles with non-Zeno behaviours

Complexity issues

Theorem [AD90,AD94]

The emptiness problem for timed automata is decidable and PSPACE-complete. It even holds for two-clock timed automata [FJ13]. It is NLOGSPACE-complete for one-clock timed automata [LMS04].

[AD90] Alur, Dill. Automata for modeling real-time systems *(ICALP'90)*.
[AD94] Alur, Dill. A theory of timed automata *(Theoretical Computer Science)*.
[LMS04] Laroussinie, Markey, Schnoebelen. Model checking timed automata with one or two clocks *(CONCUR'04)*.
[FJ13] Fearnley, Jurdziński. Reachability in two-clock timed automata is PSPACE-complete *(ICALP'13)*.

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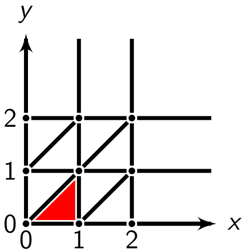

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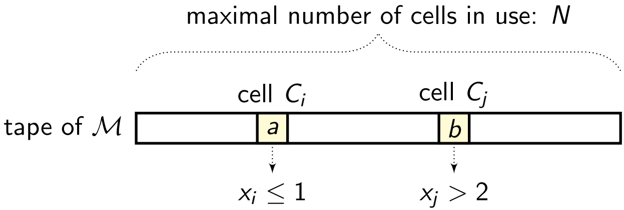
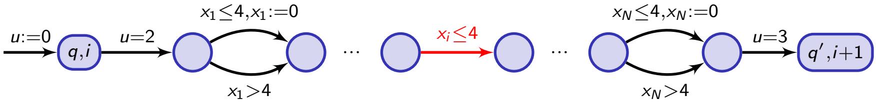


Example of the simulation of a rule $(q, a, b, q', \rightarrow)$:



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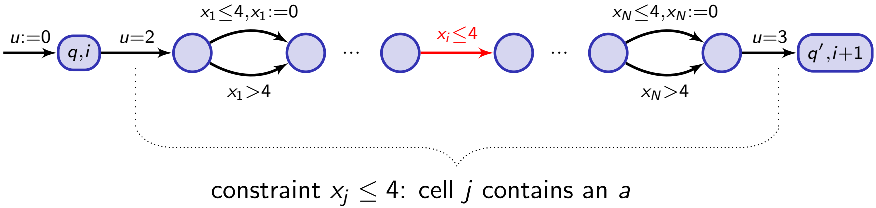

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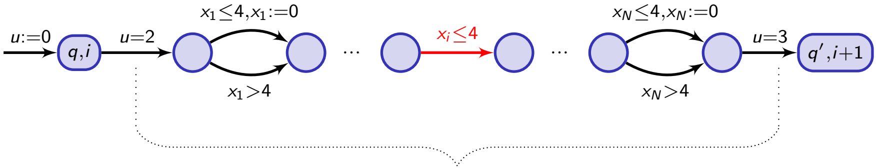

constraint $x_j \leq 4$: cell j contains an a
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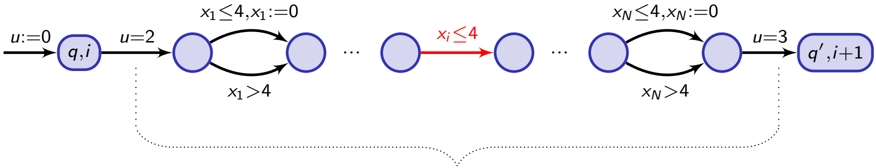

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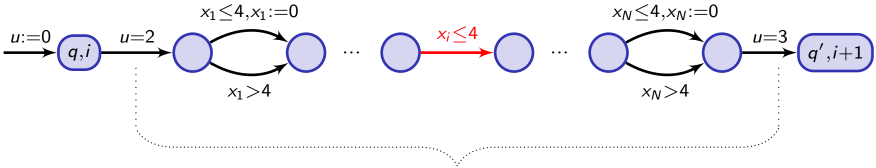

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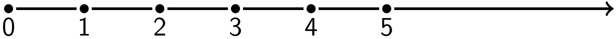
no reset of clock x_j: the new content is a b

The case of single-clock timed automata



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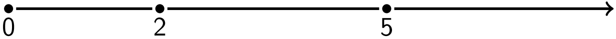

if only constants 0, 2 and 5 are used

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- Note however that it might be hard to prove there is a finite bisimulation quotient!

What about the practice?

- the region automaton is never computed
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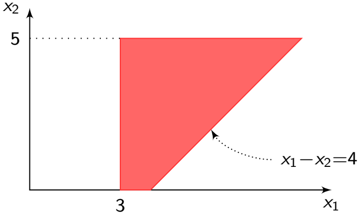
- Symbolic representation: zones

$$Z \; = \; (x_1 \geq 3) \; \wedge \; (x_2 \leq 5) \; \wedge \; (x_1 - x_2 \leq 4)$$



$$\begin{array}{cc} & \begin{array}{ccc} x_0 & x_1 & x_2 \end{array} \\ \begin{array}{c} x_0 \\ x_1 \\ x_2 \end{array} & \begin{pmatrix} \infty & -3 & \infty \\ \infty & \infty & 4 \\ 5 & \infty & \infty \end{pmatrix} \end{array}$$

DBM: Difference Bound Matrice [BM83,Dill89]

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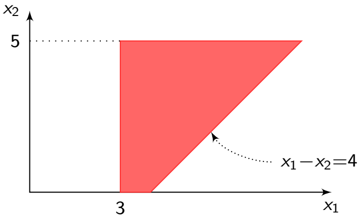

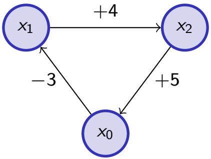

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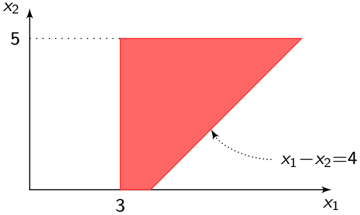

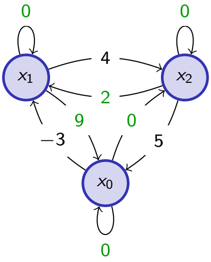

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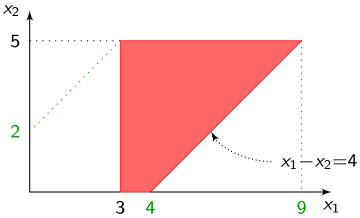

$$\begin{array}{c} \\ x_0 \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{array}{ccc} x_0 & x_1 & x_2 \end{array} \\ \begin{pmatrix} 0 & -3 & 0 \\ 9 & 0 & 4 \\ 5 & 2 & 0 \end{pmatrix} \end{array}$$

"normal form"

What about the practice?

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- Symbolic representation: zones
- Needs of (correct) extrapolation operators... [Bou04,BBLP06]

[Bou04] Bouyer. Forward analysis of updatable timed automata *(Formal Methods in System Design)*.
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- ... or clever inclusion tests [HSW12,HSW13]

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[HSW12] Herbreteau, Srivathsan, Walukiewicz. Better abstractions for timed automata *(LICS'12)*.
[HSW13] Herbreteau, Srivathsan, Walukiewicz. Lazy abstractions for timed automata *(CAV'13)*

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- resets of clocks to 0 only; we can reset to integral values as well
  - more involved updates can be used as well, but they don't interact very well with diagonal constraints. So one needs to be careful

Limits of the model

- Any slight extension of the model is undecidable:
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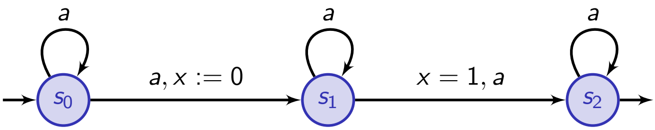
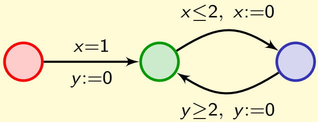
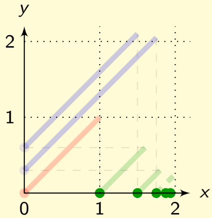
- One cannot complement, determinize timed automata



An important issue: Robustness and implementability





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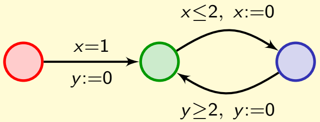

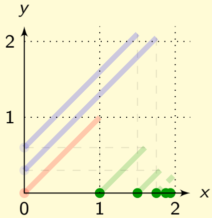

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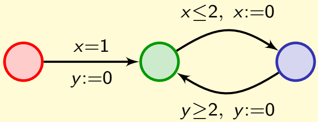

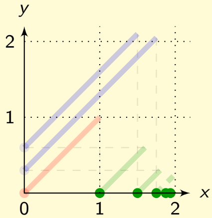

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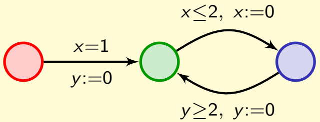

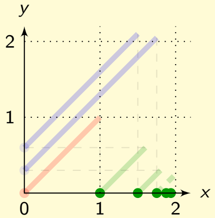

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Can we implement such a strategy??

No. But we can detect such behaviours, and give conditions for implementations!

[BMS13] Bouyer, Markey, Sankur. Robustness in timed automata *(RP'13)*.

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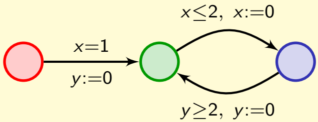

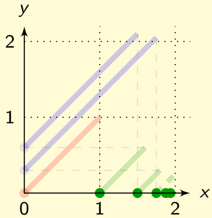

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A survey: [BMS13]

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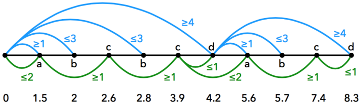
Theoretical recent developments

- Tree automata technics for timed automata analysis [AGK16,AGKS17]



  - Write behaviours as graphs with timing constraints
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[AGK16] Akshay, Gastin, Krishna. Analyzing Timed Systems Using Tree Automata *(CONCUR'16)*.
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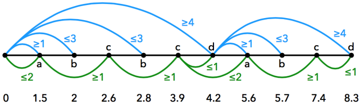

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- Compute and use the reachability relation [CJ99,QSW17]

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[CJ99] Comon, Jurski. Timed Automata and the Theory of Real Numbers *(CONCUR'99)*.
[QSW17] Quaas, Shirmohammadi, Worrell. Revisiting Reachability in Timed Automata *(LICS'17)*.

Outline

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Back to the task-graph scheduling problem

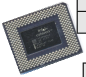
Compute $D\times(C\times(A+B))+(A+B)+(C\times D)$ using two processors:

P_1 (fast):

time	
$+$	2 picoseconds
$\times$	3 picoseconds

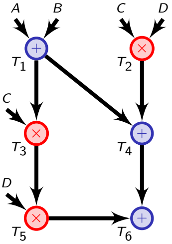
energy	
idle	10 Watt
in use	90 Watts

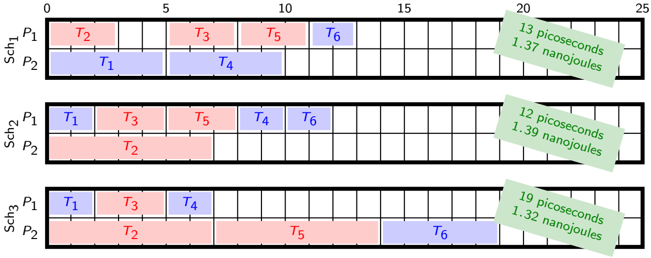
P_2 (slow):



time	
$+$	5 picoseconds
$\times$	7 picoseconds

energy	
idle	20 Watts
in use	30 Watts





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P_2 (slow):

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energy	
idle	20 Watts

How to model energy consumption?

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$$\begin{array}{ll} & \text{a discrete control (the mode of the system)} \\ + & \text{continuous evolution of the variables within a mode} \end{array}$$

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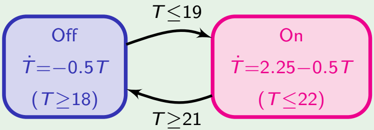

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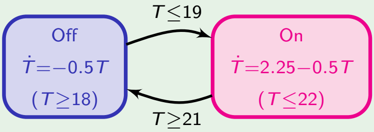

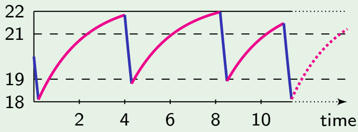

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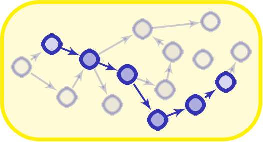

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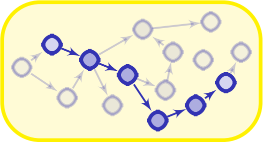

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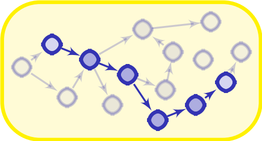

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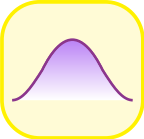

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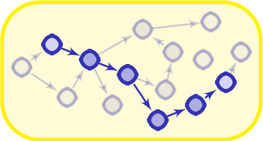

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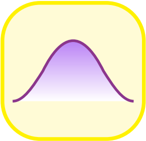

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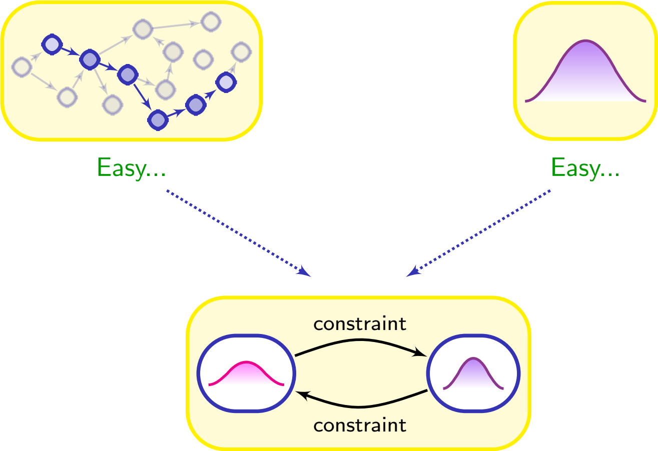

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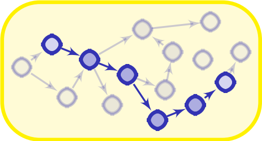

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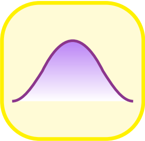

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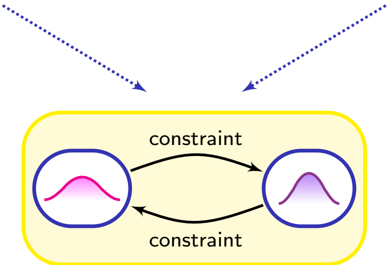

Hard!

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The reachability problem is undecidable in hybrid automata. Even for the simplest, the so-called stopwatch automata (clocks can be stopped).

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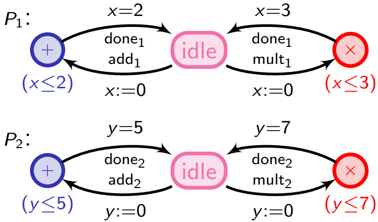
- An alternative: weighted/priced timed automata [ALP01,BFH+01]
  $\rightsquigarrow$ hybrid variables do not constrain the system
  hybrid variables are observer variables

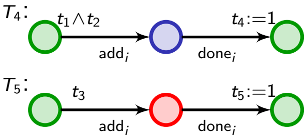
[HKPV95] Henzinger, Kopke, Puri, Varaiya. What's decidable wbout hybrid automata? *(SToC'95)*.
[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
[BFH+01] Behrmann, Fehnker, Hune, Larsen, Pettersson, Romijn, Vaandrager. Minimum-cost reachability in priced timed automata *(HSCC'01)*.

Modelling the task graph scheduling problem

- Processors

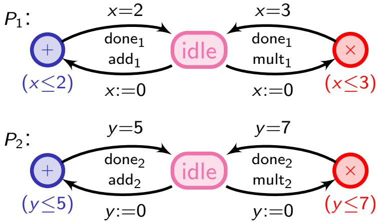


- Tasks

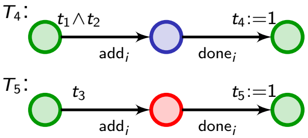


Modelling the task graph scheduling problem

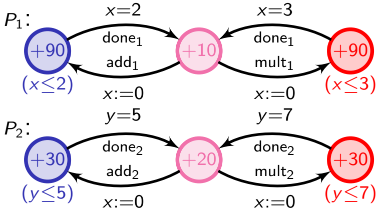
- Processors



- Tasks

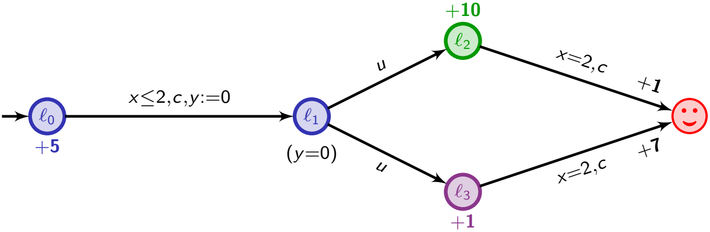


- Modelling energy



A good schedule is a path in the product automaton with a low cost

Weighted/priced timed automata [ALP01,BFH+01]



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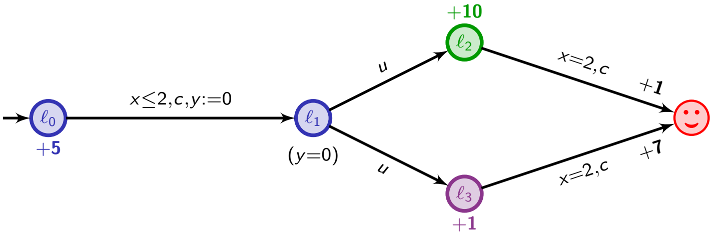
Weighted/priced timed automata [ALP01,BFH+01]



	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

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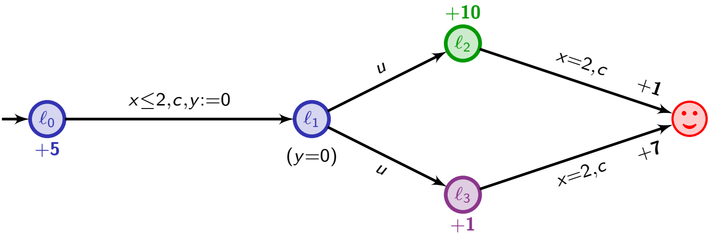
Weighted/priced timed automata [ALP01,BFH+01]



	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost :

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
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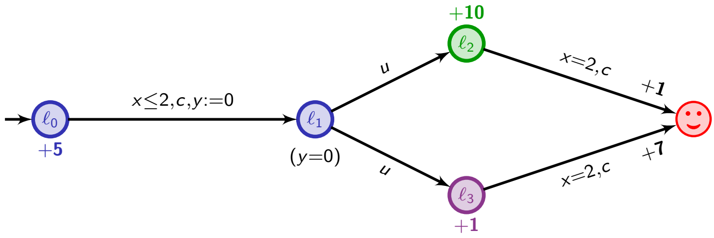
Weighted/priced timed automata [ALP01,BFH+01]



$$
\begin{array}{ccccccccccccc}
 & \ell_0 & \xrightarrow{1.3} & \ell_0 & \xrightarrow{c} & \ell_1 & \xrightarrow{u} & \ell_3 & \xrightarrow{0.7} & \ell_3 & \xrightarrow{c} & ☺ \\
x & 0 & & 1.3 & & 1.3 & & 1.3 & & 2 & & \\
y & 0 & & 1.3 & & 0 & & 0 & & 0.7 & &
\end{array}
$$

cost : 6.5

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
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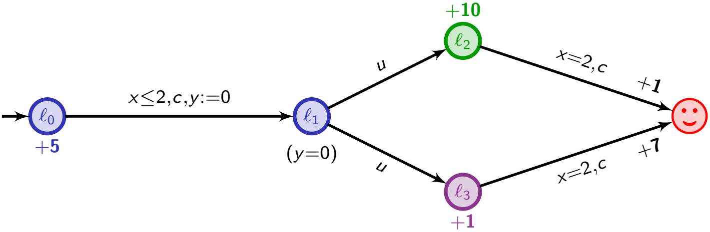
Weighted/priced timed automata [ALP01,BFH+01]



	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : 6.5 + 0

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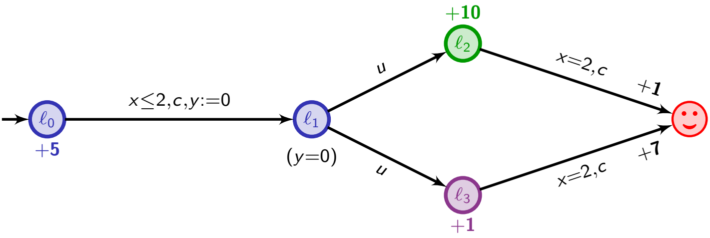
Weighted/priced timed automata [ALP01,BFH+01]



$$
\begin{array}{lcccccccccccc}
 & \ell_0 & \xrightarrow{1.3} & \ell_0 & \xrightarrow{c} & \ell_1 & \xrightarrow{u} & \ell_3 & \xrightarrow{0.7} & \ell_3 & \xrightarrow{c} & ☺ \\
x & 0 & & 1.3 & & 1.3 & & 1.3 & & 2 & & \\
y & 0 & & 1.3 & & 0 & & 0 & & 0.7 & &
\end{array}
$$

cost : 6.5 + 0 + 0

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
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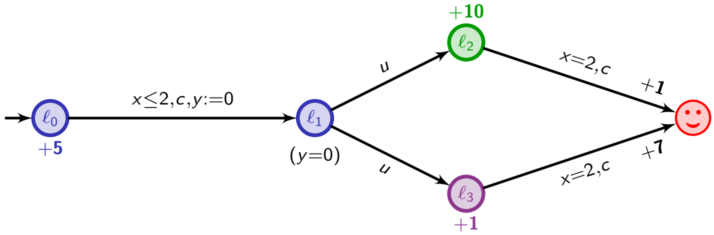
Weighted/priced timed automata [ALP01,BFH+01]



	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		
cost :		6.5	+	0	+	0	+	0.7			

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
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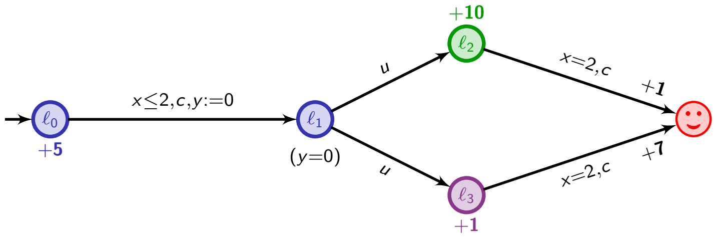
Weighted/priced timed automata [ALP01,BFH+01]



	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : 6.5 + 0 + 0 + 0.7 + 7

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
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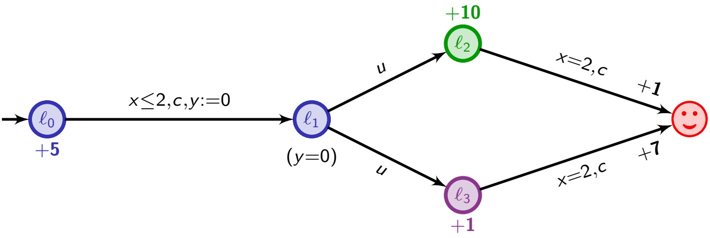
Weighted/priced timed automata [ALP01,BFH+01]



	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : 6.5 + 0 + 0 + 0.7 + 7 = 14.2

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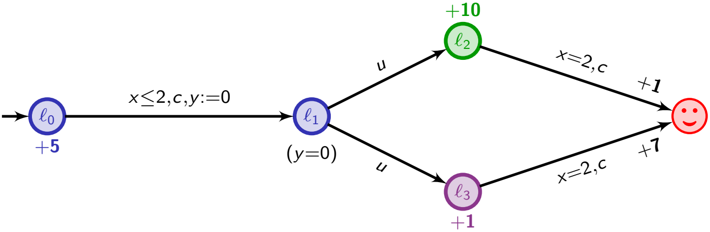
Weighted/priced timed automata [ALP01,BFH+01]



Question: what is the optimal cost for reaching 🙂 ?

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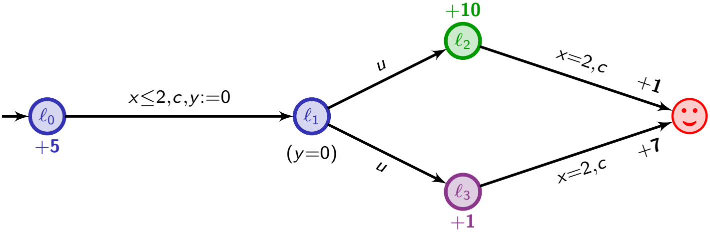
Weighted/priced timed automata [ALP01,BFH+01]



Question: what is the optimal cost for reaching 🙂?

$$5t + 10(2 - t) + 1$$

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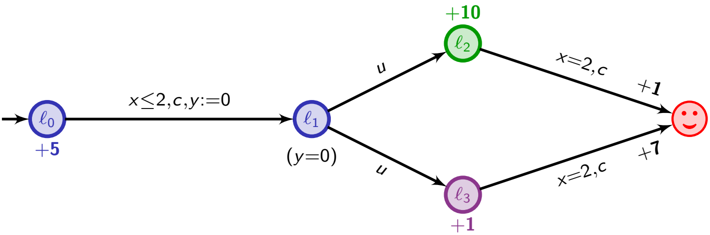
Weighted/priced timed automata [ALP01,BFH+01]



Question: what is the optimal cost for reaching 🙂?

$$5t + 10(2-t) + 1 \;,\; 5t + (2-t) + 7$$

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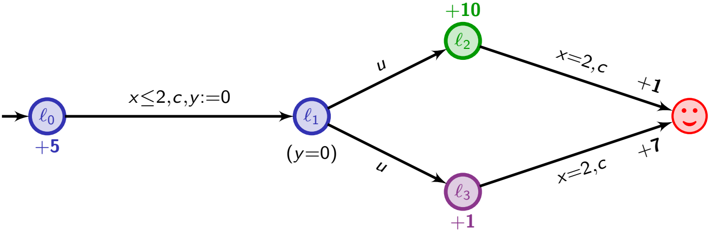
Weighted/priced timed automata [ALP01,BFH+01]



Question: what is the optimal cost for reaching ☺ ?

$$\min\ (\ 5t + 10(2-t) + 1\ ,\ 5t + (2-t) + 7\ )$$

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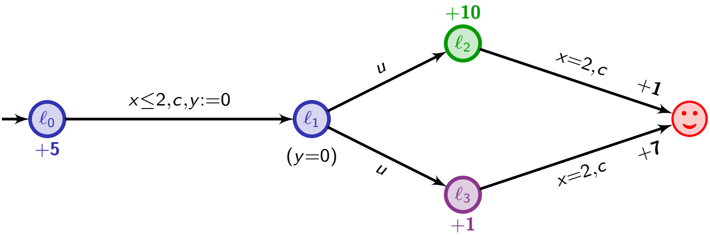
Weighted/priced timed automata [ALP01,BFH+01]



Question: what is the optimal cost for reaching 🙂?

$$\inf_{0\leq t\leq 2} \min\left(5t+10(2-t)+1 \, , \, 5t+(2-t)+7 \right) = 9$$

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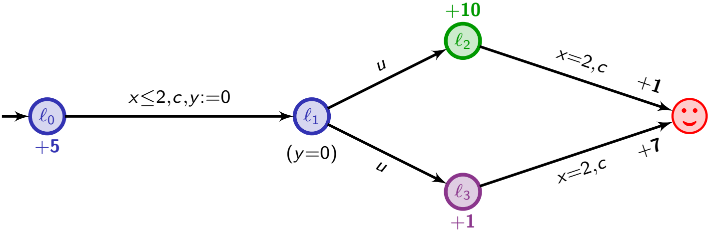
Weighted/priced timed automata [ALP01,BFH+01]



Question: what is the optimal cost for reaching ☺?

$$\inf_{0\leq t\leq 2} \min\left(\, 5t + 10(2-t) + 1 \,,\, 5t + (2-t) + 7 \,\right) = 9$$

$\rightsquigarrow$ *strategy:* leave immediately ℓ_0, go to ℓ_3, and wait there 2 t.u.

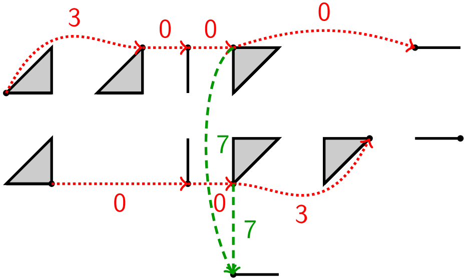
[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
[BFH+01] Behrmann, Fehnker, Hune, Larsen, Pettersson, Romijn, Vaandrager. Minimum-cost reachability in priced timed automata *(HSCC'01)*.

Optimal-cost reachability

Theorem [ALP01,BFH+01,BBBR07]

In weighted timed automata, the optimal cost is an integer and can be computed in PSPACE.

- Technical tool: a refinement of the regions, the corner-point abstraction



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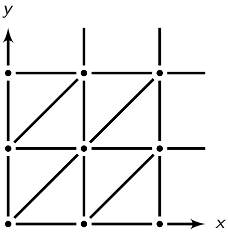
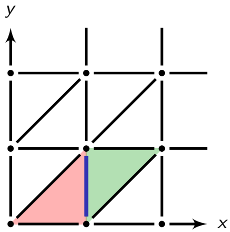
[BBBR07] Bouyer, Brihaye, Bruyère, Raskin. On the optimal reachability problem *(Formal Methods in System Design)*.

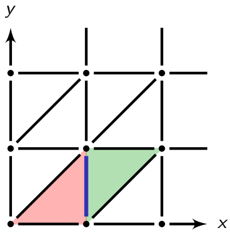
Technical tool: the corner-point abstraction



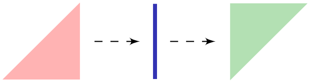
Technical tool: the corner-point abstraction



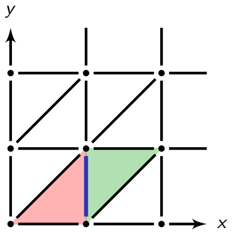
Technical tool: the corner-point abstraction



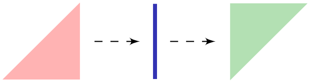
Abstract time successors:



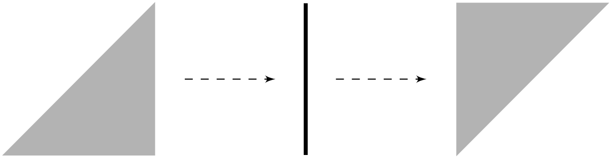
Technical tool: the corner-point abstraction



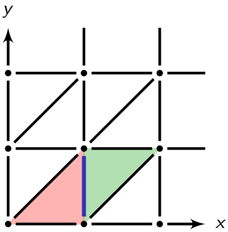
Abstract time successors:



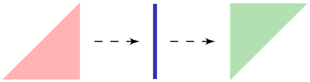
Concrete time successors:



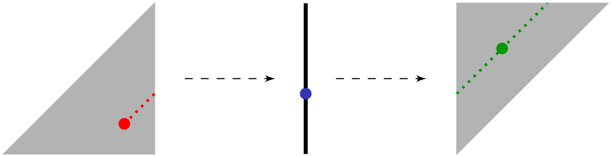
Technical tool: the corner-point abstraction



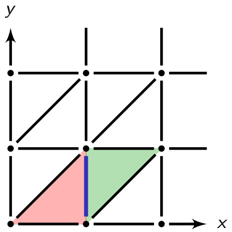
Abstract time successors:



Concrete time successors:



Technical tool: the corner-point abstraction



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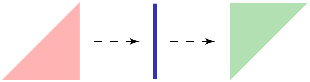

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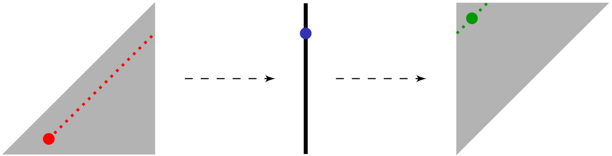
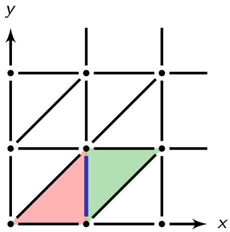


Technical tool: the corner-point abstraction



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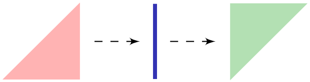

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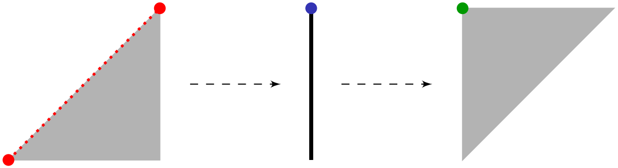

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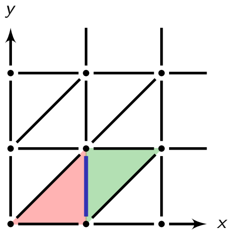

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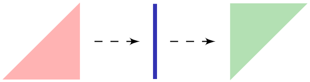
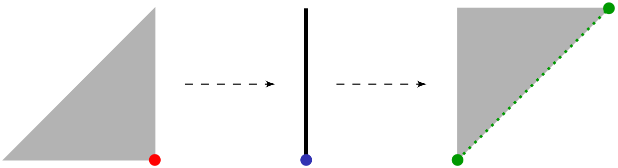
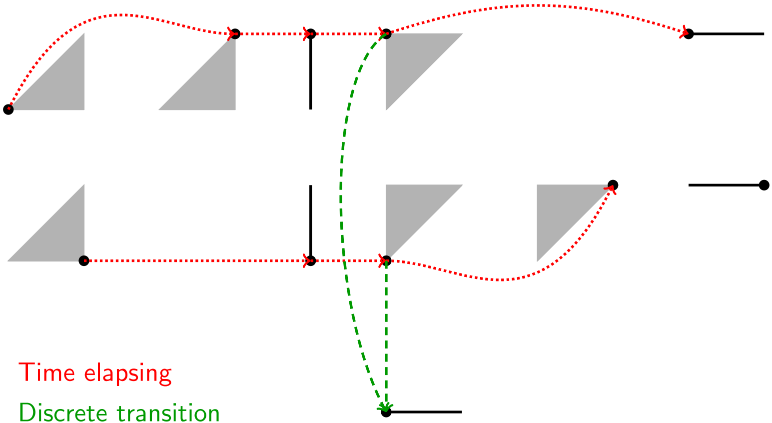


Concrete time successors:



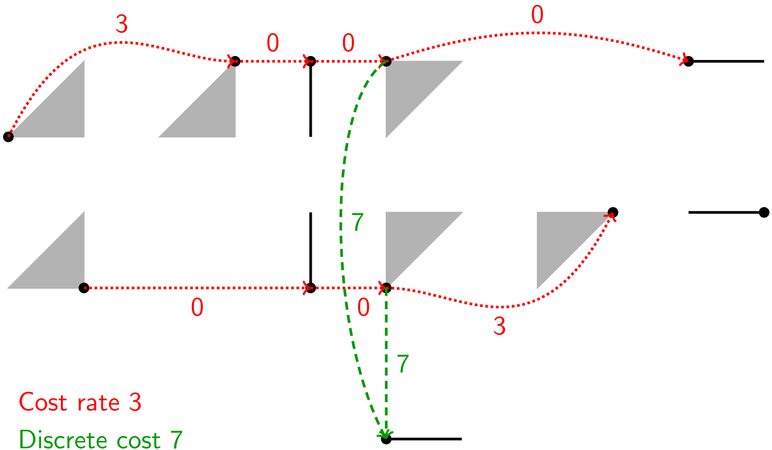
Technical tool: the corner-point abstraction



Time elapsing

Discrete transition

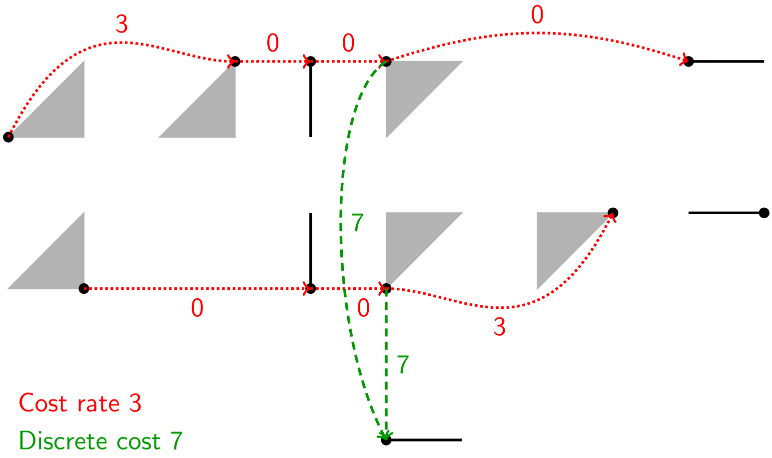
Technical tool: the corner-point abstraction



Cost rate 3

Discrete cost 7

Technical tool: the corner-point abstraction



Optimal cost in the weighted graph
= optimal cost in the weighted timed automaton!

From timed to discrete behaviours

Optimal reachability as a linear programming problem

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$\circ \xrightarrow{t_1} \circ \xrightarrow{t_2} \circ \xrightarrow{t_3} \circ \xrightarrow{t_4} \circ \xrightarrow{t_5} \circ \quad \cdots$$

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$\circ \xrightarrow{t_1} \circ \xrightarrow[x \leq 2]{t_2} \circ \xrightarrow{t_3} \circ \xrightarrow{t_4} \circ \xrightarrow{t_5} \circ \quad \cdots \quad \left\{ \begin{array}{l} t_1 + t_2 \leq 2 \\ \\ \end{array} \right.$$

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$\circ \xrightarrow[y:=0]{t_1} \circ \xrightarrow[x\leq 2]{t_2} \circ \xrightarrow{t_3} \circ \xrightarrow[y\geq 5]{t_4} \circ \xrightarrow{t_5} \circ \quad \cdots \quad \begin{cases} t_1+t_2\leq 2 \\ t_2+t_3+t_4\geq 5 \end{cases}$$

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$\begin{array}{ccccccccccc} & T_1 & & T_2 & & T_3 & & T_4 & & T_5 & \\ \circ & \xrightarrow[y:=0]{t_1} & \circ & \xrightarrow[x\leq 2]{t_2} & \circ & \xrightarrow{t_3} & \circ & \xrightarrow[y\geq 5]{t_4} & \circ & \xrightarrow{t_5} & \circ \quad \cdots \end{array} \quad \begin{cases} t_1+t_2\leq 2 & T_2\leq 2 \\ t_2+t_3+t_4\geq 5 & T_4-T_1\geq 5 \end{cases}$$

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$T_1 \qquad T_2 \qquad T_3 \qquad T_4 \qquad T_5$$

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Lemma

Let Z be a bounded zone and f be a function

$$f : (T_1, ..., T_n) \mapsto \sum_{i=1}^{n} c_i T_i + c$$

well-defined on $\overline{Z}$. Then $inf_Z f$ is obtained on the border of $\overline{Z}$ with integer coordinates.

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$\begin{array}{ccccc} T_1 & T_2 & T_3 & T_4 & T_5 \end{array}$$

$$\circ \xrightarrow[y:=0]{t_1} \circ \xrightarrow[x\leq 2]{t_2} \circ \xrightarrow{t_3} \circ \xrightarrow[y\geq 5]{t_4} \circ \xrightarrow{t_5} \circ \quad \cdots \quad \begin{cases} t_1+t_2\leq 2 & T_2\leq 2 \\ t_2+t_3+t_4\geq 5 & T_4-T_1\geq 5 \end{cases}$$

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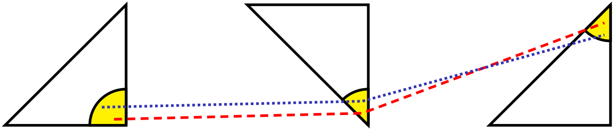
$\rightsquigarrow$ for every finite path π in $\mathcal{A}$, there exists a path Π in $\mathcal{A}_{\mathrm{cp}}$ such that

$$\mathrm{cost}(\Pi) \leq \mathrm{cost}(\pi)$$

[Π is a "corner-point projection" of π]

From discrete to timed behaviours

Approximation of abstract paths:



For any path Π of $\mathcal{A}_{\mathsf{cp}}$,

From discrete to timed behaviours

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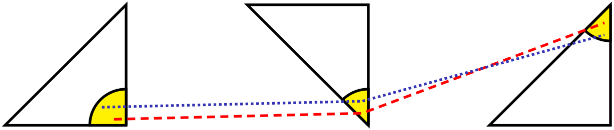

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From discrete to timed behaviours

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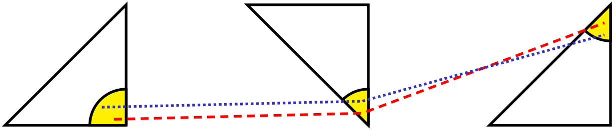

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$$\|\Pi - \pi_\varepsilon\|_\infty < \varepsilon$$

From discrete to timed behaviours

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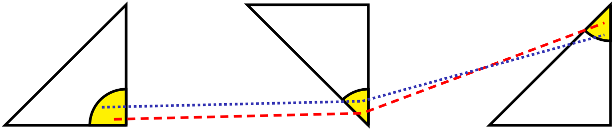

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$$\|\Pi - \pi_\varepsilon\|_\infty < \varepsilon \Rightarrow |\mathsf{cost}(\Pi) - \mathsf{cost}(\pi_\varepsilon)| < \eta$$

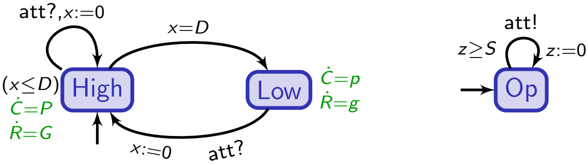
Use of the corner-point abstraction

It is a very interesting abstraction, that can be used in several other contexts:

- for mean-cost optimization [BBL04,BBL08]
- for discounted-cost optimization [FL08]
- for all concavely-priced timed automata [JT08]
- for deciding frequency objectives [BBBS11,Sta12]
- ...

[BBL04] Bouyer, Brinksma, Larsen. Staying Alive As Cheaply As Possible *(HSCC'04)*.
[BBL08] Bouyer, Brinksma, Larsen. Optimal infinite scheduling for multi-priced timed automata *(Formal Methods in System Designs)*.
[FL08] Fahrenberg, Larsen. Discount-optimal infinite runs in priced timed automata *(INFINITY'08)*.
[JT08] Judziński, Trivedi. Concavely-priced timed automata *(FORMATS'08)*.
[BBBS11] Bertrand, Bouyer, Brihaye, Stainer. Emptiness and universality problems in timed automata with positive frequency *(ICALP'11)*.
[Sta12] Stainer. Frequencies in forgetful timed automata *(FORMATS'12)*.

Going further 1: mean-cost optimization



[BBL08] Bouyer, Brinksma, Larsen. Optimal infinite scheduling for multi-priced timed automata *(Formal Methods in System Designs)*.

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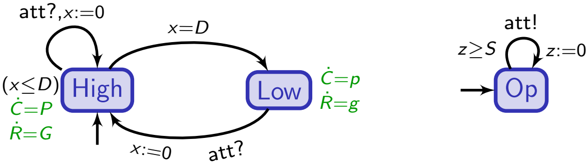

$\rightsquigarrow$ compute optimal infinite schedules that minimize

$$\text{mean-cost}(\pi) = \limsup_{n \to +\infty} \frac{\text{cost}(\pi_n)}{\text{reward}(\pi_n)}$$

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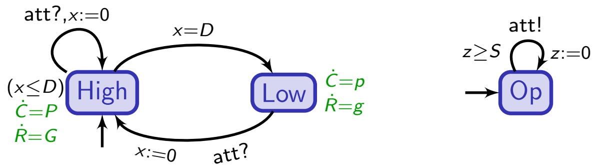

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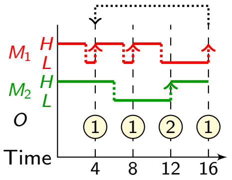
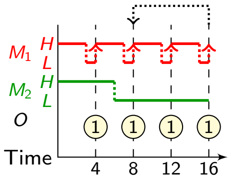


Schedule with ratio ≈1.455



Schedule with ratio ≈1.478

[BBL08] Bouyer, Brinksma, Larsen. Optimal infinite scheduling for multi-priced timed automata *(Formal Methods in System Designs)*.

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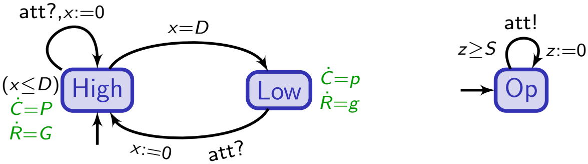

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Theorem [BBL08]

In weighted timed automata, the optimal mean-cost can be compute in PSPACE.

$\rightsquigarrow$ the corner-point abstraction can be used

[BBL08] Bouyer, Brinksma, Larsen. Optimal infinite scheduling for multi-priced timed automata *(Formal Methods in System Designs)*.

From timed to discrete behaviours

- **Finite behaviours:** based on the following property

Lemma

Let Z be a bounded zone and f be a function

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$\leadsto$ for every finite path π in $\mathcal{A}$, there exists a path Π in $\mathcal{A}_{\text{cp}}$ s.t.
mean-cost$(\Pi) \leq$ mean-cost(π)

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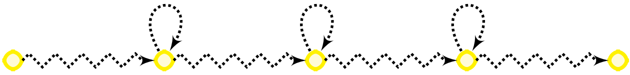
- **Infinite behaviours:** decompose each sufficiently long projection into cycles:



The (acyclic) linear part will be negligible!

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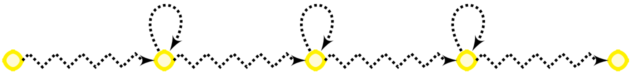
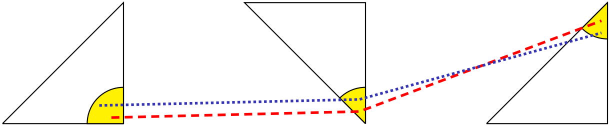


The (acyclic) linear part will be negligible!

$\leadsto$ the optimal cycle of $\mathcal{A}_{\mathrm{cp}}$ is better than any infinite path of $\mathcal{A}$!

From discrete to timed behaviours

Approximation of abstract paths:



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From discrete to timed behaviours

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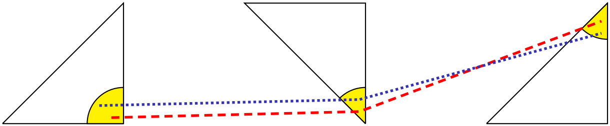

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From discrete to timed behaviours

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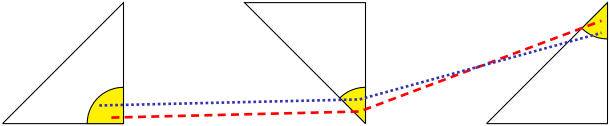

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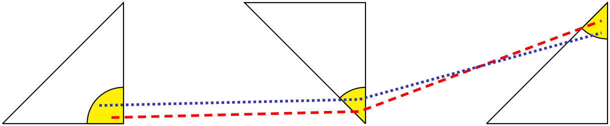

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Going further 2: concavely-priced cost functions

$\rightsquigarrow$ A general abstract framework for quantitative timed systems

Theorem [JT08]

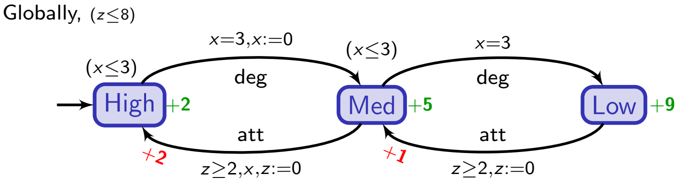
In concavely-priced timed automata, optimal cost is computable, if we restrict to quasi-concave cost functions. For the following cost functions, the (decision) problem is even PSPACE-complete:

- optimal-time and optimal-cost reachability;
- optimal discrete discounted cost;
- optimal mean-cost.

$\rightsquigarrow$ the corner-point abstraction can be used

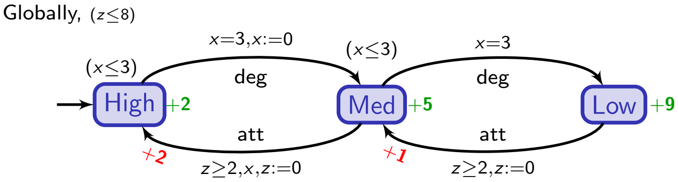
[JT08] Judziński, Trivedi. Concavely-priced timed automata *(FORMATS'08)*.

Going further 3: discounted-time cost optimization



[FL08] Fahrenberg, Larsen. Discount-optimal infinite runs in priced timed automata *(INFINITY'08)*.

Going further 3: discounted-time cost optimization



$\rightsquigarrow$ compute optimal infinite schedules that minimize
discounted cost over time

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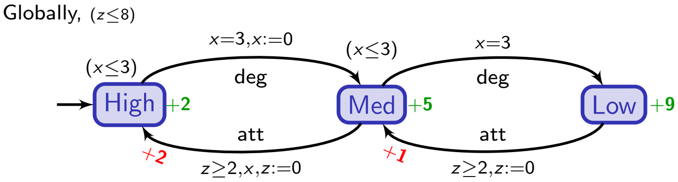

$\rightsquigarrow$ compute optimal infinite schedules that minimize

$$\text{discounted-cost}_\lambda(\pi) = \sum_{n \geq 0} \lambda^{T_n} \int_{t=0}^{\tau_{n+1}} \lambda^t \text{cost}(\ell_n)\, \mathrm{d}t + \lambda^{T_{n+1}} \text{cost}(\ell_n \xrightarrow{a_{n+1}} \ell_{n+1})$$

if $\pi = (\ell_0, v_0) \xrightarrow{\tau_1, a_1} (\ell_1, v_1) \xrightarrow{\tau_2, a_2} \cdots$ and $T_n = \sum_{i \leq n} \tau_i$

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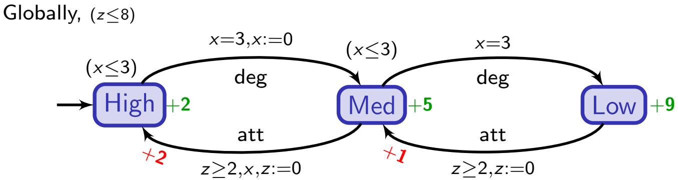
Going further 3: discounted-time cost optimization



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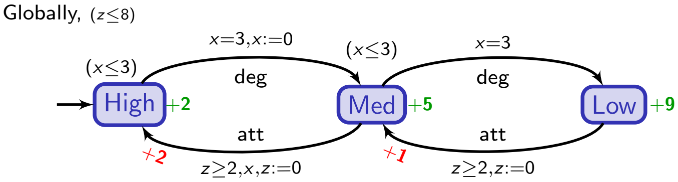

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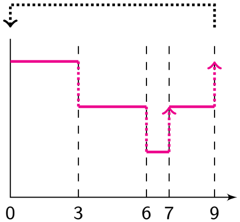
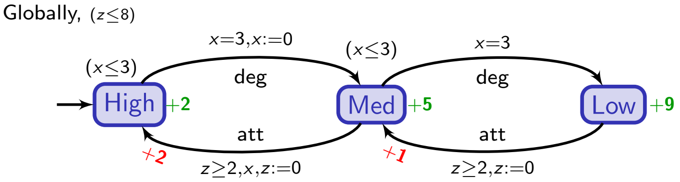


if $\lambda = e^{-1}$, the discounted cost of that infinite schedule is ≈ 2.16

[FL08] Fahrenberg, Larsen. Discount-optimal infinite runs in priced timed automata *(INFINITY'08)*.

Going further 3: discounted-time cost optimization



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Theorem [FL08]

In weighted timed automata, the optimal discounted cost is computable in EXPTIME.

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And symbolically?

- Non-obvious in general...

[LBB+01] Larsen, Behrmann, Brinksma, Fehnker, Hune, Pettersson, Romijn. As cheap as possible: Efficient cost- optimal reachability for priced timed automata *(CAV'01)*.

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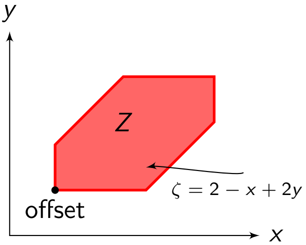
Priced zones

priced zone = zone + affine cost function

☺ efficient representation: DBM + offset cost + affine coefficient for each clock



Represented by: zone Z
offset cost: $+4$
rate for x: -1
rate for y: $+2$

[LBB+01] Larsen, Behrmann, Brinksma, Fehnker, Hune, Pettersson, Romijn. As cheap as possible: Efficient cost- optimal reachability for priced timed automata *(CAV'01)*.

Results

Theorem [LBB+01,RLS06]

The forward algorithm with standard inclusion is correct and terminates for **bounded** timed automata with non-negative costs.

Termination: well-quasi-order on priced zones

[LBB+01] Larsen, Behrmann, Brinksma, Fehnker, Hune, Pettersson, Romijn. As cheap as possible: Efficient cost- optimal reachability for priced timed automata *(CAV'01)*.

[RLS06] Rasmussen, Larsen, Subramani. On using priced timed automata to achieve optimal scheduling *(Formal Methods in System Design)*.

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- $\mathcal{Z} \sqsubseteq_M \mathcal{Z}'$ reduces to several bilevel linear optimization problems

Theorem [BCM16]

The forward algorithm with inclusion test $\sqsubseteq_M$ is correct and terminates for timed automata with some conditions on the cost.
It is always better than standard inclusion for bounded timed automata.

[LBB+01] Larsen, Behrmann, Brinksma, Fehnker, Hune, Pettersson, Romijn. As cheap as possible: Efficient cost- optimal reachability for priced timed automata *(CAV'01)*.
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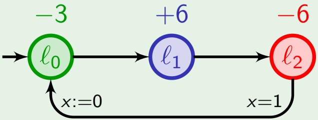
Further problems: Energy management

Example



Further problems: Energy management

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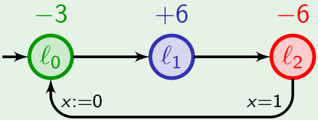

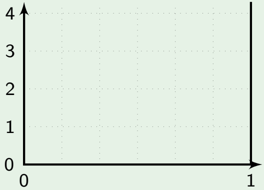

"energy is ≥ 0"

- Lower-bound problem (**L**)

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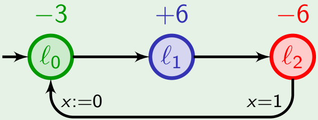

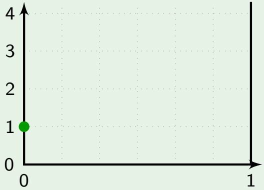

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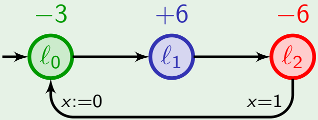

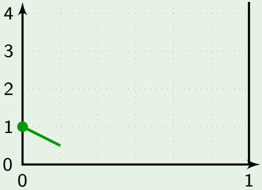

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Further problems: Energy management

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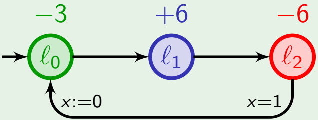
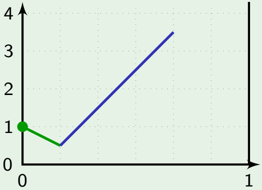


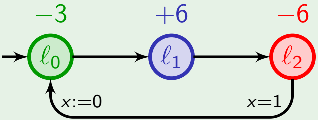


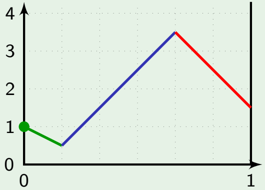
"energy is ≥ 0"

- Lower-bound problem (**L**)

Further problems: Energy management

Example





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Further problems: Energy management

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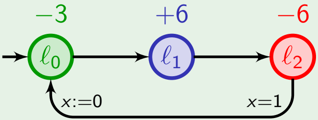
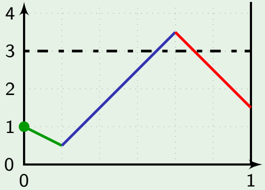


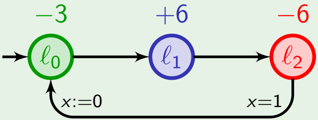


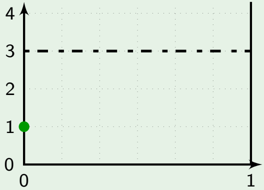
"energy is in [0,3]"

- Lower-bound problem (**L**)
- Lower-and-upper-bound problem (**L+U**)

Further problems: Energy management

Example



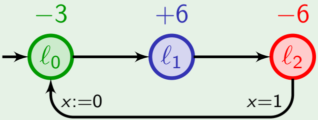


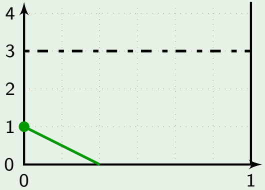
"energy is in [0,3]"

- Lower-bound problem (**L**)
- Lower-and-upper-bound problem (**L+U**)

Further problems: Energy management

Example





"energy is in [0,3]"

- Lower-bound problem (**L**)
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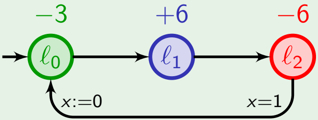
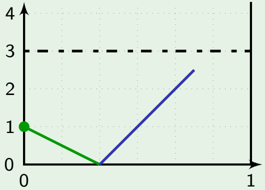
Further problems: Energy management

Example





"energy is in [0,3]"

- Lower-bound problem (**L**)
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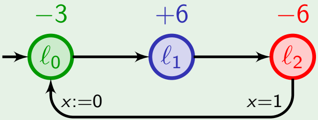
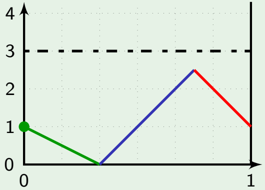
Further problems: Energy management

Example





"energy is in [0,3]"

- Lower-bound problem (**L**)
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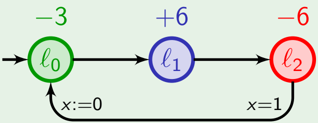
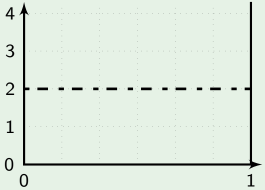
Further problems: Energy management

Example



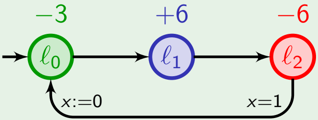


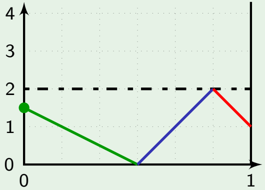
"energy is in [0,2]"

- Lower-bound problem (**L**)
- Lower-and-upper-bound problem (**L+U**)

Further problems: Energy management

Example



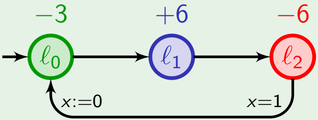


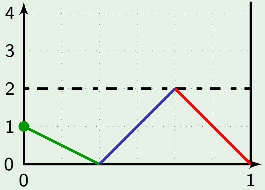
"energy is in [0,2]"

- Lower-bound problem (**L**)
- Lower-and-upper-bound problem (**L+U**)

Further problems: Energy management

Example



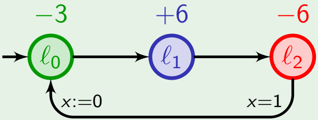


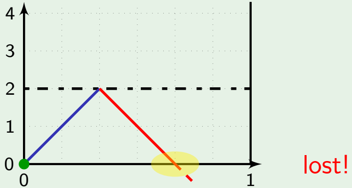
"energy is in [0,2]"

- Lower-bound problem (**L**)
- Lower-and-upper-bound problem (**L+U**)

Further problems: Energy management

Example



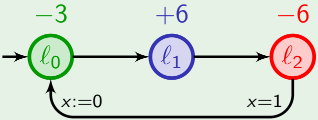


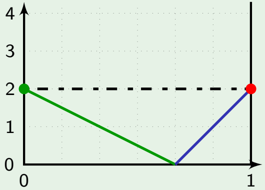
"energy is in [0,2]"

- Lower-bound problem (**L**)
- Lower-and-upper-bound problem (**L+U**)

Further problems: Energy management

Example



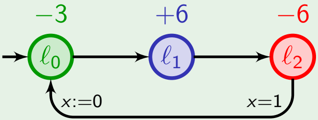


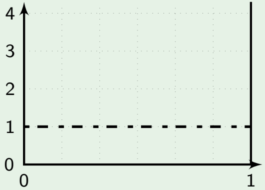
"energy is in [0,2]"

- Lower-bound problem (**L**)
- Lower-and-upper-bound problem (**L+U**)

Further problems: Energy management

Example



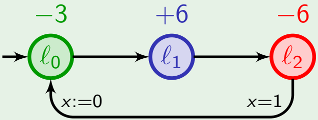


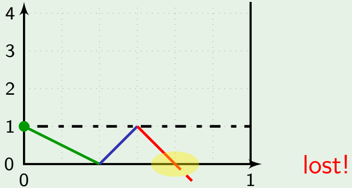
"energy is in [0,1]"

- Lower-bound problem (**L**)
- Lower-and-upper-bound problem (**L+U**)

Further problems: Energy management

Example



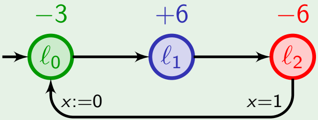


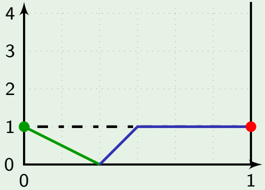
"energy is in [0,1]"

- Lower-bound problem (**L**)
- Lower-and-upper-bound problem (**L+U**)

Further problems: Energy management

Example





"energy is in [0,1] with a weak upper bound"

- Lower-bound problem (**L**)
- Lower-and-upper-bound problem (**L+U**)
- Lower-and-weak-upper-bound problem (**L+W**)

The **L**-problem: use the corner-point abstraction?

Idea: delay in the most profitable location

$\rightsquigarrow$ the corner-point abstraction

The **L**-problem: use the corner-point abstraction?

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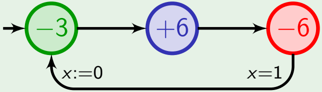

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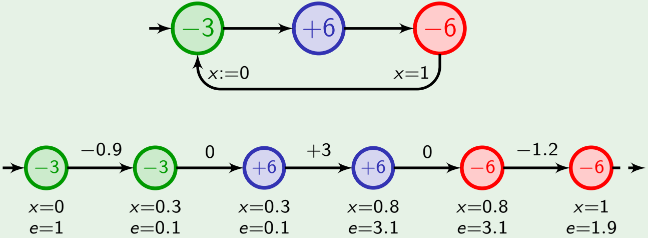

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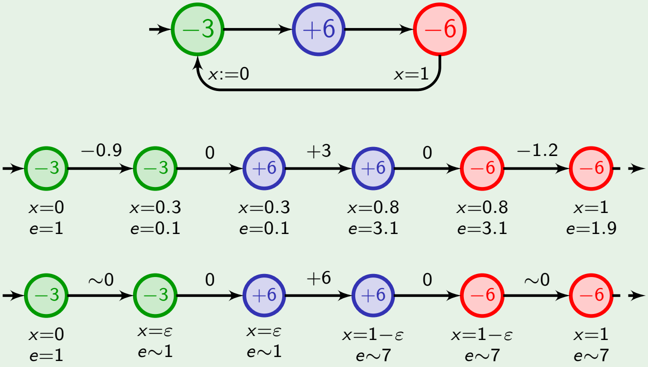

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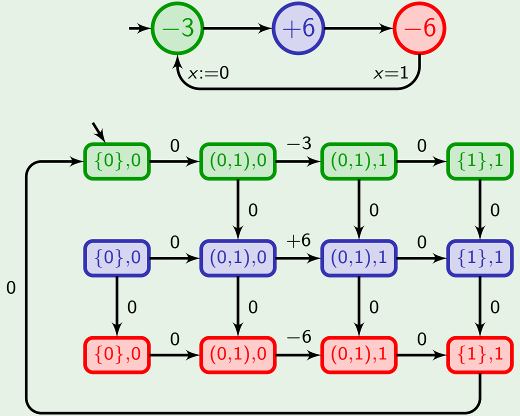

The **L**-problem: use the corner-point abstraction?

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Theorem [BFLMS08]

The corner-point abstraction is sound and complete for single-clock WTA with no discrete costs. Hence the existential **L**-problem is in PTIME in that case.

[BFLMS08] Bouyer, Fahrenberg, Larsen, Markey, Srba. Infinite runs in weighted timed automata with energy constraints *(FORMATS'08)*.

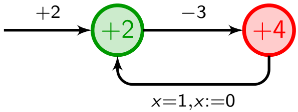
The **L**-problem: use the corner-point abstraction?

Idea: delay in the most profitable location

$\leadsto$ the corner-point abstraction

Remark

The corner-point abstraction is not correct with discrete costs.



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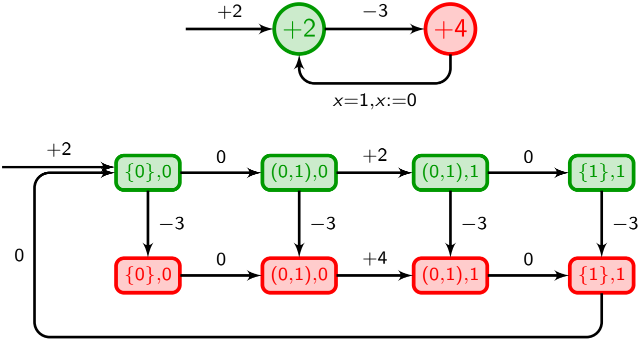

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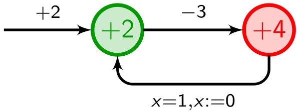

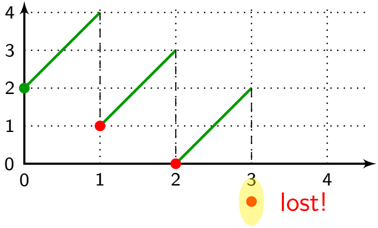

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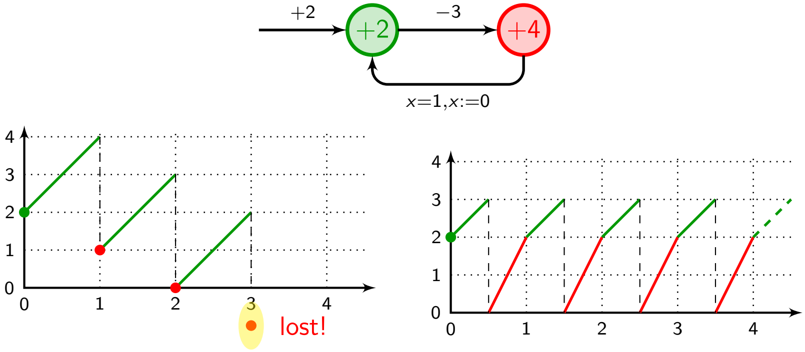

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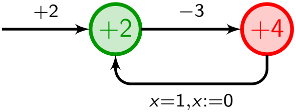
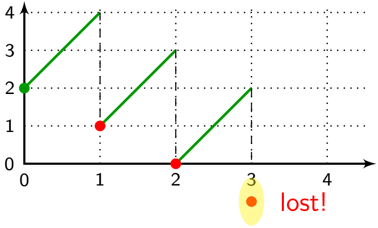
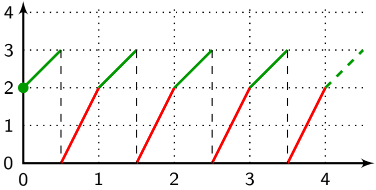


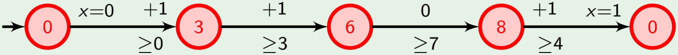




$\rightsquigarrow$ requires new developments!

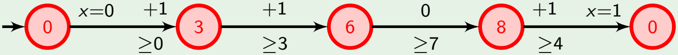
The **L**-problem: computing optimal delays

Example



The **L**-problem: computing optimal delays

Example

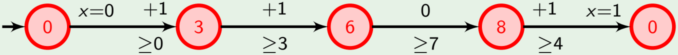


t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};

The **L**-problem: computing optimal delays

Example

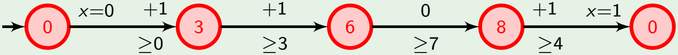


t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—				—

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
- compute optimal possible delays t^* in ℓ_1 to ℓ_{n-1};

The **L**-problem: computing optimal delays

Example

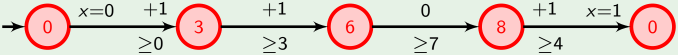


t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—			0	—

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
- compute optimal possible delays t^* in ℓ_1 to ℓ_{n-1};

The **L**-problem: computing optimal delays

Example



t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—		$\frac{1}{2}$	0	—

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
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The **L**-problem: computing optimal delays

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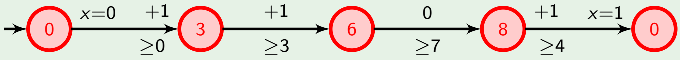

t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
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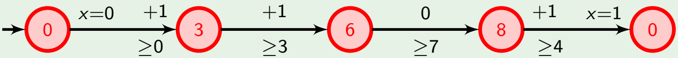
Example



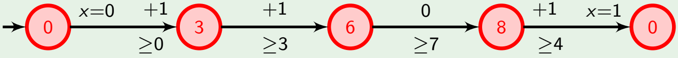
t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—

minimal initial credit required: $\frac{1}{2}$, yields final credit 8.

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
- compute optimal possible delays t^* in ℓ_1 to ℓ_{n-1};

The **L**-problem: computing optimal delays

Example

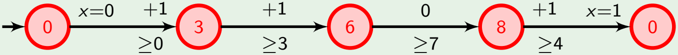


t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
- compute optimal possible delays t^* in ℓ_1 to ℓ_{n-1};
- compute other points on the energy function curve.

The **L**-problem: computing optimal delays

Example



t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—

initial credit
$\frac{1}{2} + \delta$

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
- compute optimal possible delays t^* in ℓ_1 to ℓ_{n-1};
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The **L**-problem: computing optimal delays

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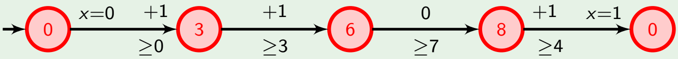

t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—

initial credit $\frac{1}{2} + \delta$ $\quad \frac{1}{2} - \frac{\delta}{3}$

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
- compute optimal possible delays t^* in ℓ_1 to ℓ_{n-1};
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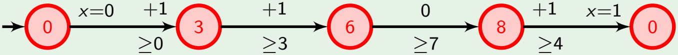
The **L**-problem: computing optimal delays

Example

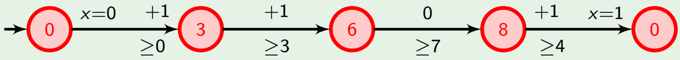


t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—
initial credit	$\frac{1}{2} + \delta$	$\frac{1}{2} - \frac{\delta}{3}$	$\frac{1}{2}$		

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
- compute optimal possible delays t^* in ℓ_1 to ℓ_{n-1};
- compute other points on the energy function curve.

The L-problem: computing optimal delays

Example



t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—
initial credit $\frac{1}{2}+\delta$		$\frac{1}{2}-\frac{\delta}{3}$	$\frac{1}{2}$	$\frac{\delta}{3}$	

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
- compute optimal possible delays t^* in ℓ_1 to ℓ_{n-1};
- compute other points on the energy function curve.

The **L**-problem: computing optimal delays

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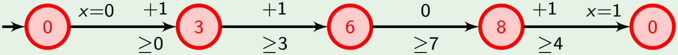

t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—
	initial credit $\frac{1}{2}+\delta$	$\frac{1}{2}-\frac{\delta}{3}$	$\frac{1}{2}$	$\frac{\delta}{3}$	final credit $8+\frac{8}{3}\delta$

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
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The **L**-problem: computing optimal delays

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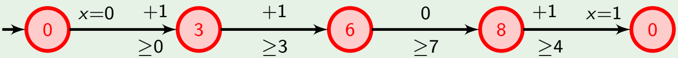

t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—
	initial credit 2	0	$\frac{1}{2}$	$\frac{1}{2}$	final credit 12

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
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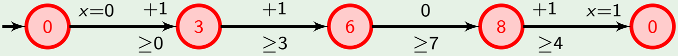

t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—

initial credit $2+\delta$

0

$\frac{1}{2}-\frac{\delta}{6}$

$\frac{1}{2}+\frac{\delta}{6}$

final credit $12+\frac{8}{6}\delta$

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
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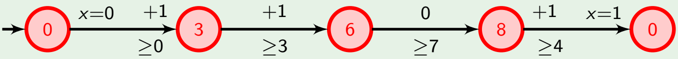

t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—
	initial credit 5	0	0	1	final credit 16

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
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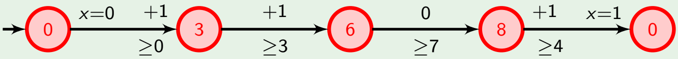

t_{opt}:	—	$\frac{2}{3}$	$\frac{1}{2}$	—	—
t^*:	—	$\frac{1}{2}$	$\frac{1}{2}$	0	—
initial credit $5+\delta$		0	0	1	final credit $16+\delta$

- compute optimal delays t_{opt} in ℓ_1 to ℓ_{n-1};
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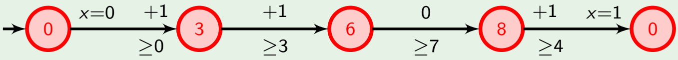

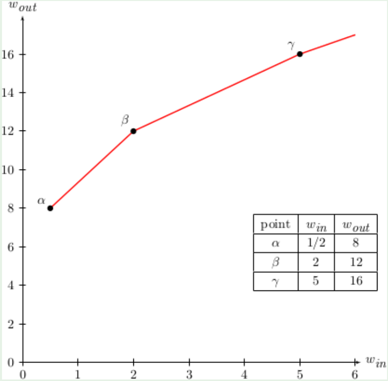

The **L**-problem: concluding

Theorem

Optimization, reachability and existence of infinite runs satisfying the constraint ≥ 0 can be decided in EXPTIME in single-clock WTA

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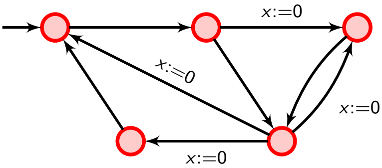
- transform the automaton into an automaton with energy functions;



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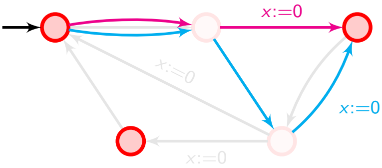

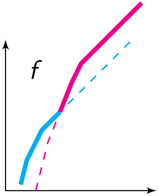

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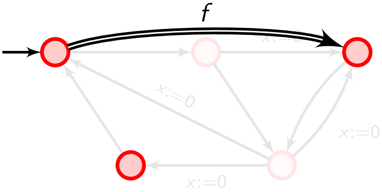

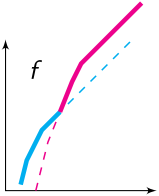

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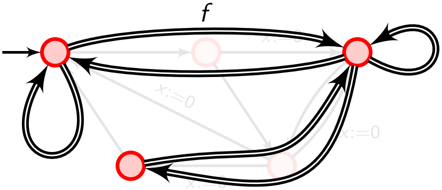

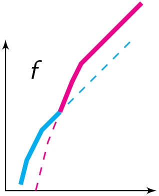

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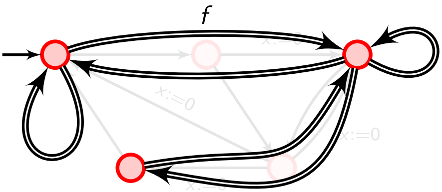
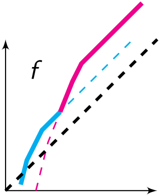




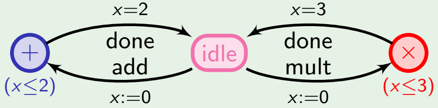
- check if simple cycles can be iterated (or if a Zeno cycle can be reached...)

Outline

Why (timed) games?

- to model uncertainty

Example of a processor in the taskgraph example



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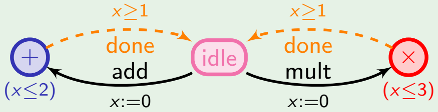

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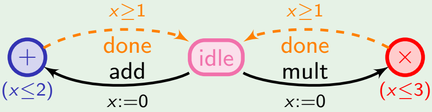
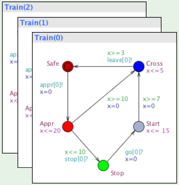
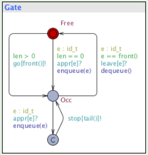


- to model an interaction with the environment

Example of the gate in the train/gate example





Why (timed) games?

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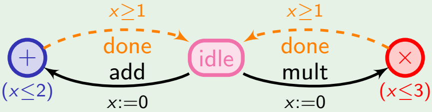
Example of a processor in the taskgraph example



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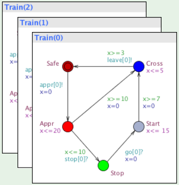

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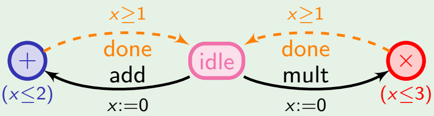

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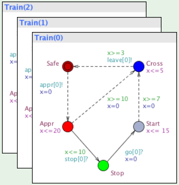
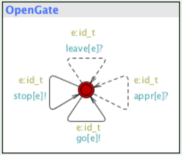
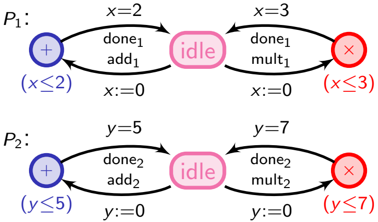


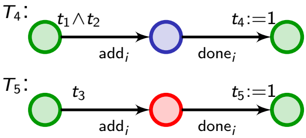


Modelling the task graph scheduling problem

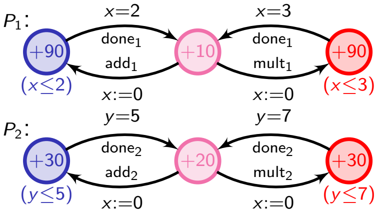
- Processors



- Tasks



- Modelling energy



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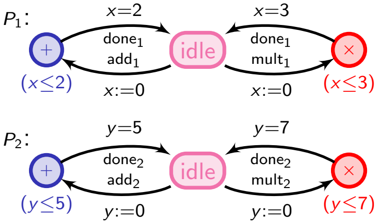

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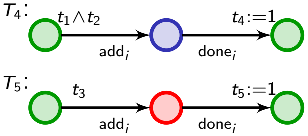

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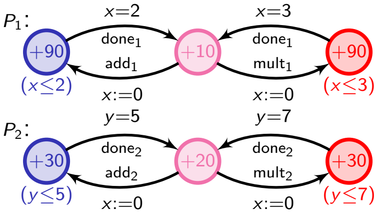
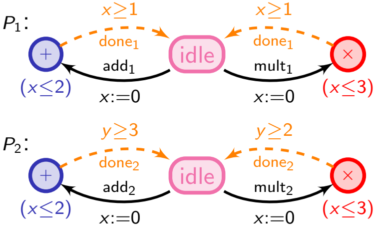


- Modelling uncertainty



Modelling the task graph scheduling problem

- Processors

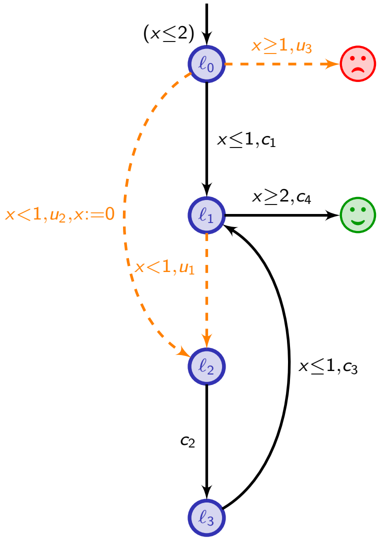
- Tasks

A (good) schedule is a strategy in the product game (with a low cost)

- Modelling energy

- Modelling uncertainty

An example of a timed game



Rule of the game

- Aim: avoid ☹ and reach ☺

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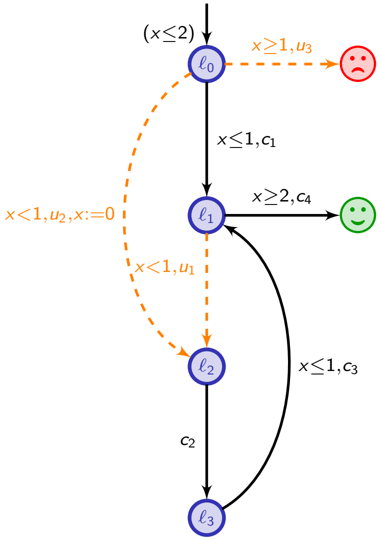

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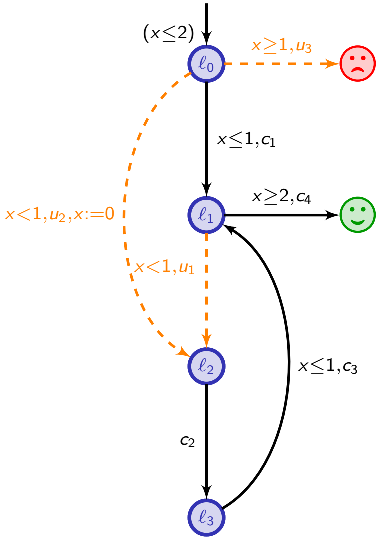

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$$f : \text{history} \mapsto (\text{delay}, \text{cont. transition})$$

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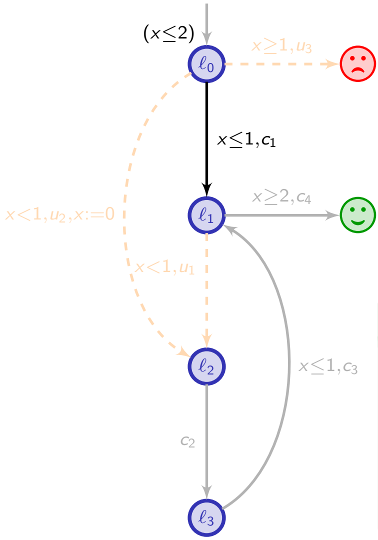

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A (memoryless) winning strategy

- from $(\ell_0, 0)$, play $(0.5, c_1)$

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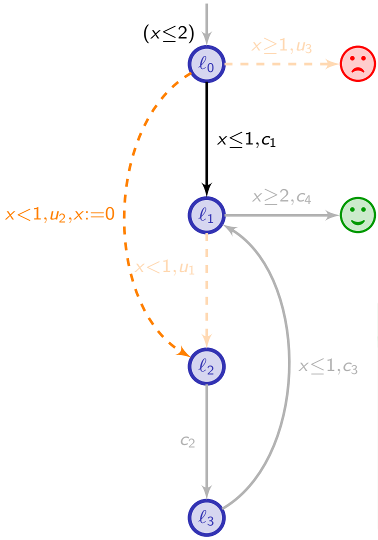

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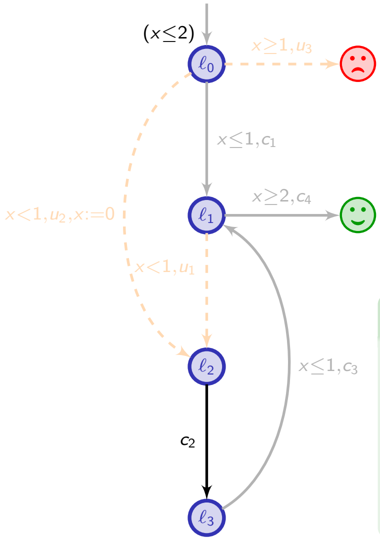

Rule of the game

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- from $(\ell_0, 0)$, play $(0.5, c_1)$
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- from $(\ell_2, \star)$, play $(1 - \star, c_2)$

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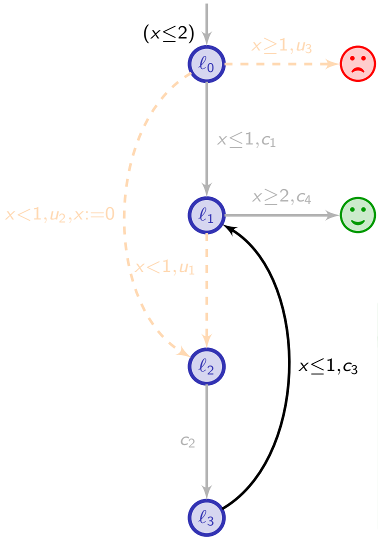

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- from $(\ell_3, 1)$, play $(0, c_3)$

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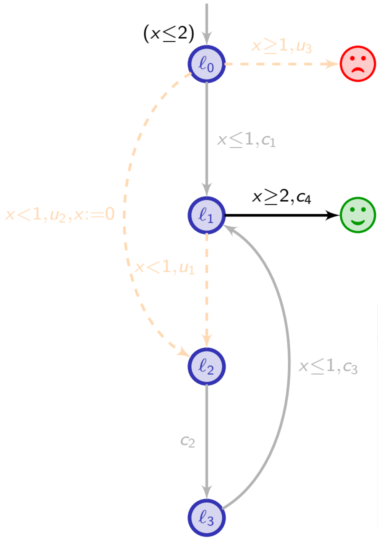

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- from $(\ell_3, 1)$, play $(0, c_3)$
- from $(\ell_1, 1)$, play $(1, c_4)$

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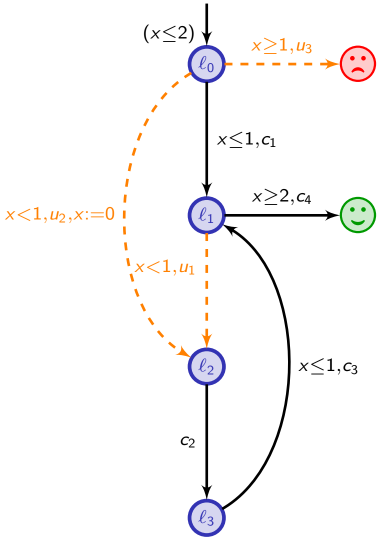

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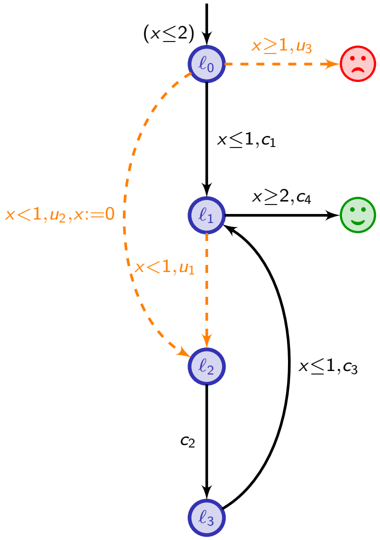

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- Does there exist a winning strategy?

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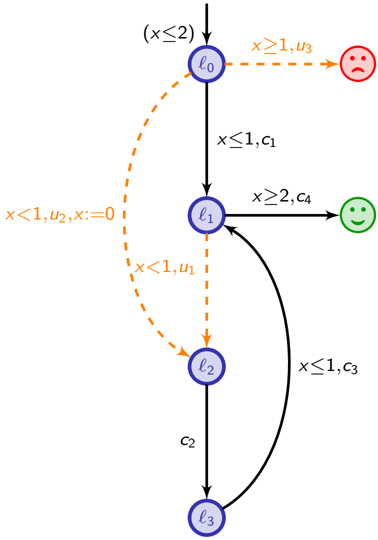

Rule of the game

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- How do we play? According to a strategy:

$$f : \text{history} \mapsto (\text{delay}, \text{cont. transition})$$

Problems to be considered

- Does there exist a winning strategy?
- If yes, compute one (as simple as possible).

Decidability of timed games

Theorem [AMPS98,HK99]

Reachability and safety timed games are decidable and EXPTIME-complete. Furthermore memoryless and “region-based” strategies are sufficient.

[AMPS98] Asarin, Maler, Pnueli, Sifakis. Controller synthesis for timed automata *(SSC'98)*.
[HK99] Henzinger, Kopke. Discrete-time control for rectangular hybrid automata *(Theoretical Computer Science)*.

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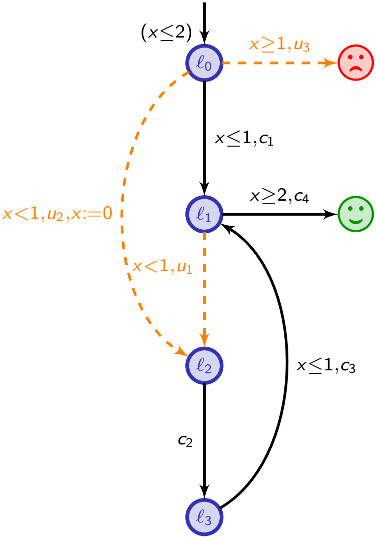
Theorem [AM99,BHPR07,JT07]

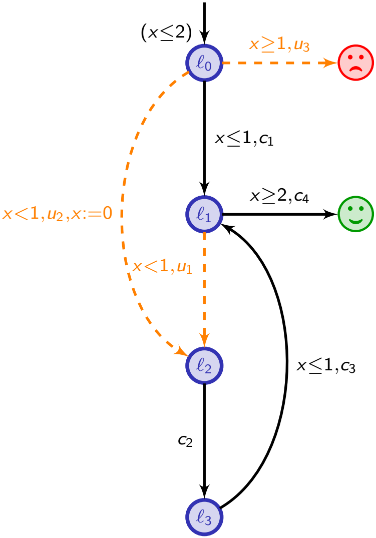
Optimal-time reachability timed games are decidable and EXPTIME-complete.

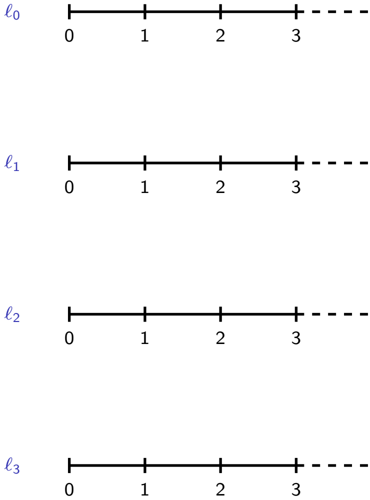
[AM99] Asarin, Maler. As soon as possible: time optimal control for timed automata *(HSCC'99)*.
[BHPR07] Brihaye, Henzinger, Prabhu, Raskin. Minimum-time reachability in timed games *(ICALP'07)*.
[JT07] Jurdziński, Trivedi. Reachability-time games on timed automata *(ICALP'07)*.

Back to the example: computing winning states



Back to the example: computing winning states





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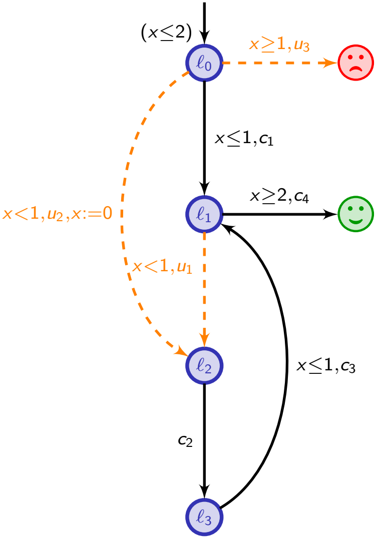

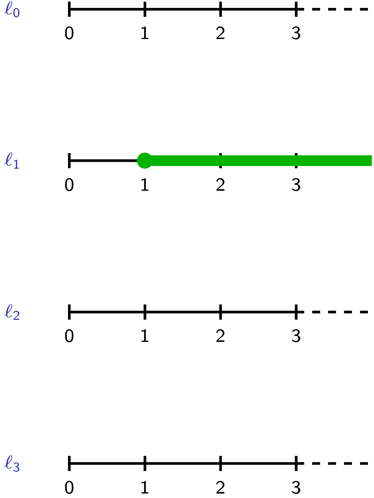

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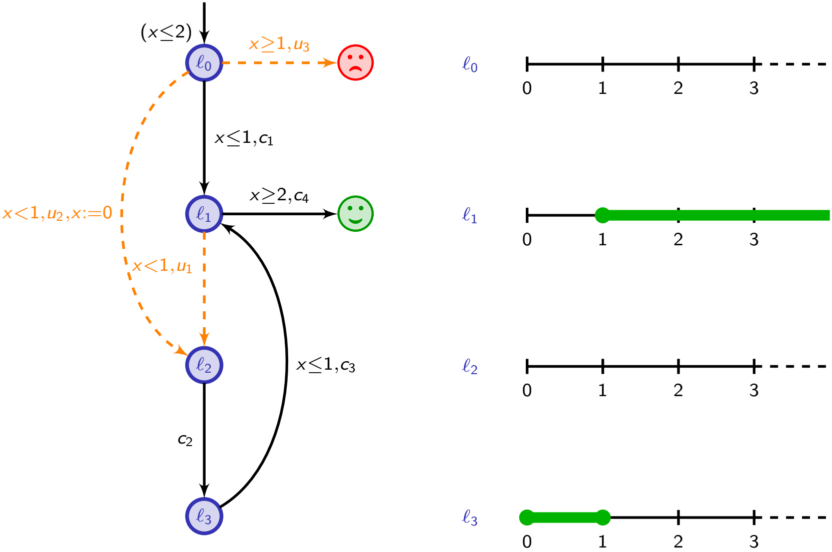

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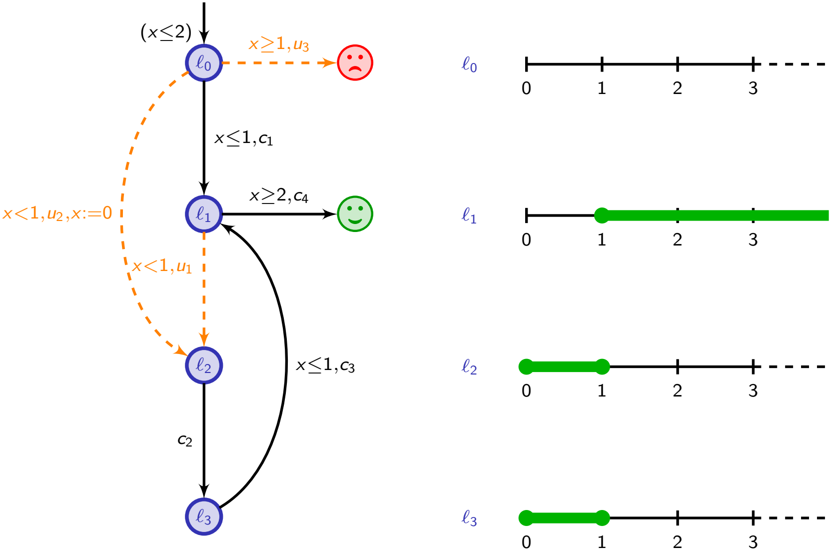

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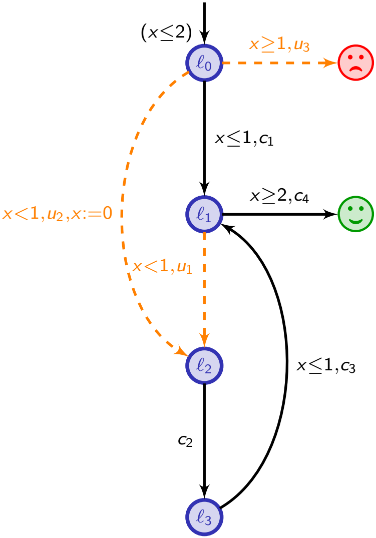

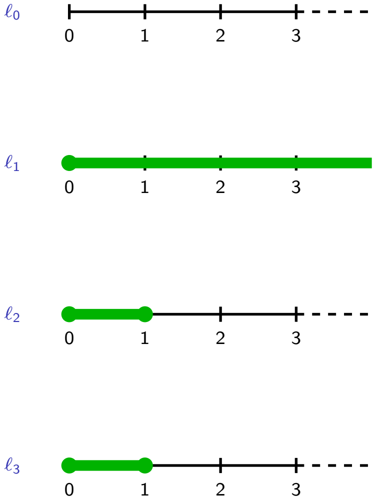

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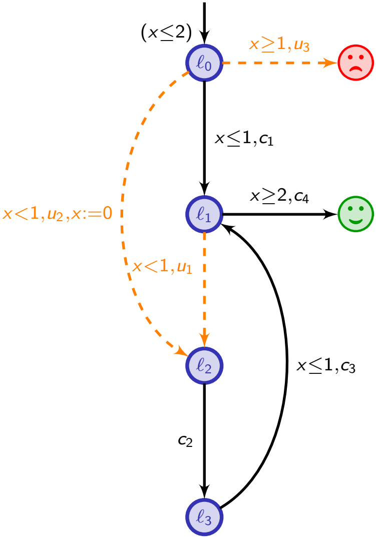

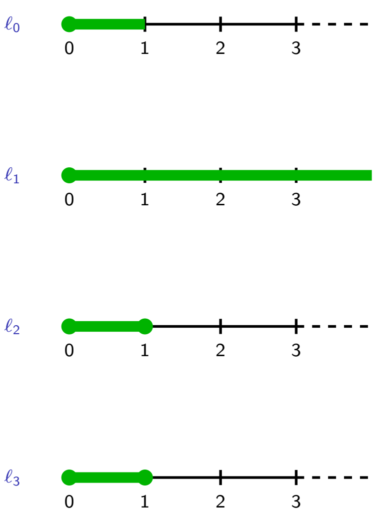

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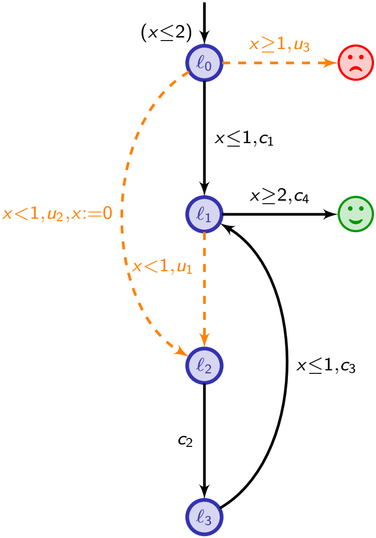

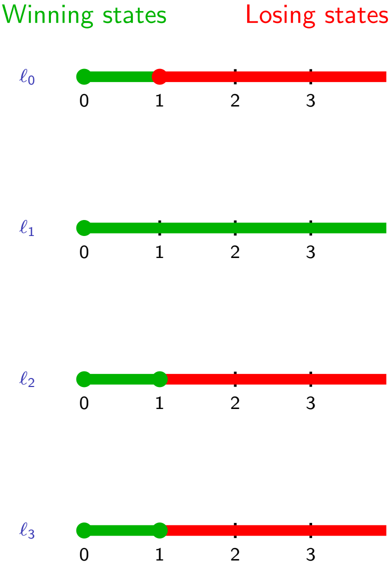

Decidability *via* attractors

Skip attractors

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- $\mathsf{Pred}^a(X) = \{\bullet \mid \bullet \xrightarrow{a} \bullet \in X\}$
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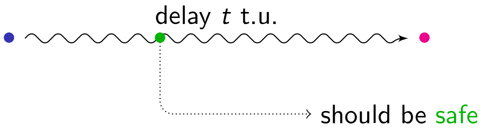

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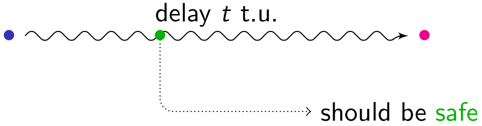

$$\mathrm{Pred}_\delta(X, \mathsf{Safe}) = \{\bullet \mid \exists t \geq 0,\ \bullet \xrightarrow{\delta(t)} \bullet$$
$$\text{and } \forall 0 \leq t' \leq t,\ \bullet \xrightarrow{\delta(t')} \bullet \in \mathsf{Safe}\}$$

Timed games with a reachability objective

We write:

$$\pi(X) = X \cup \mathsf{Pred}_\delta(\mathsf{cPred}(X), \neg\mathsf{uPred}(\neg X))$$

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$$\begin{array}{rcl} \mathsf{Attr}_n(\smiley) & = & \pi(\mathsf{Attr}_{n-1}(\smiley)) \\ & = & \pi^n(\smiley) \end{array}$$

Stability w.r.t. regions

- if X is a union of regions, then:
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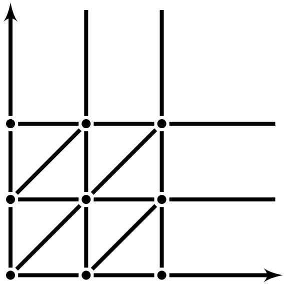

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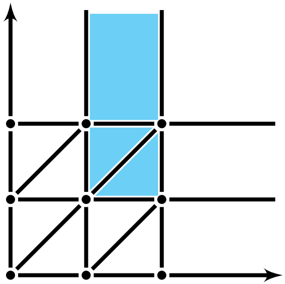

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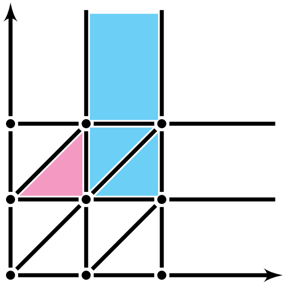

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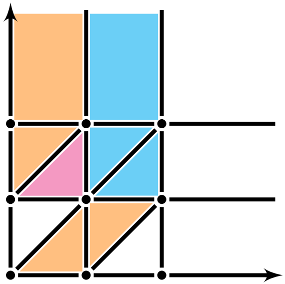

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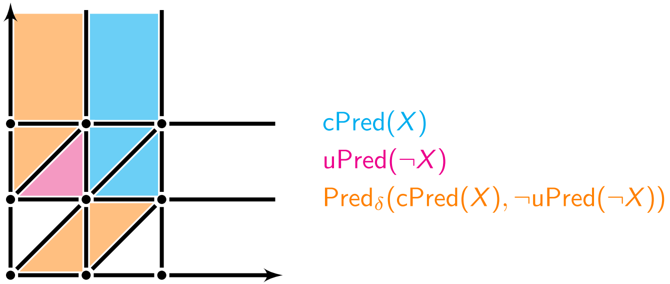

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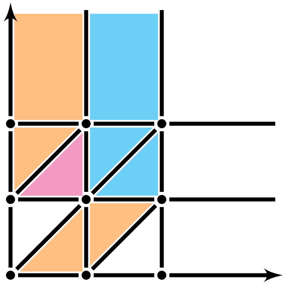

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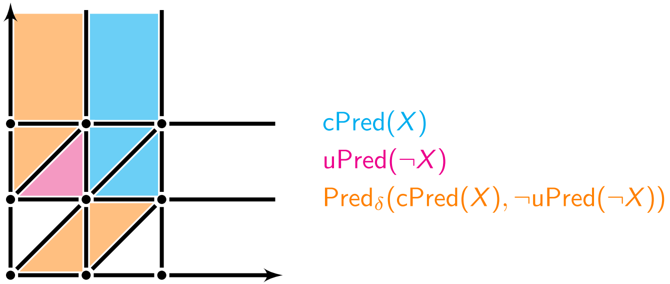

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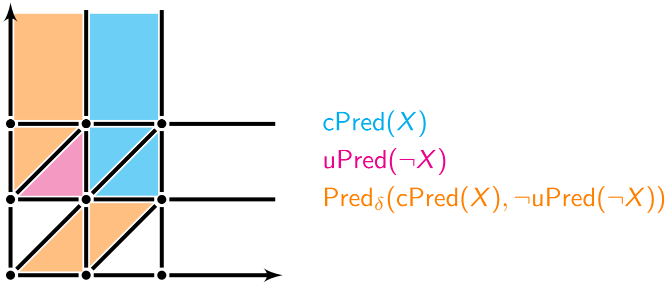

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... and is correct

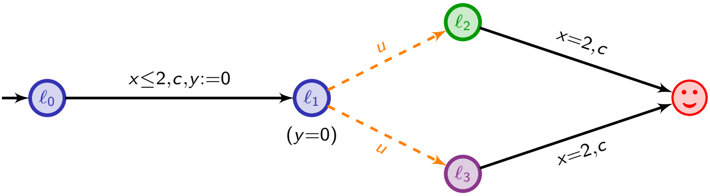
And in practice?

- A zone-based forward algorithm with backtracking [CDF+05,BCD+07]

[CDF+05] Cassez, David, Fleury, Larsen, Lime. Efficient On-the-Fly Algorithms for the Analysis of Timed Games *(CONCUR'05)*.
[BCD+07] Behrmann, Cougnard, David, Fleury, Larsen, Lime. UPPAAL-Tiga: Time for Playing Games! *(CAV'07)*.

Outline

A simple timed game



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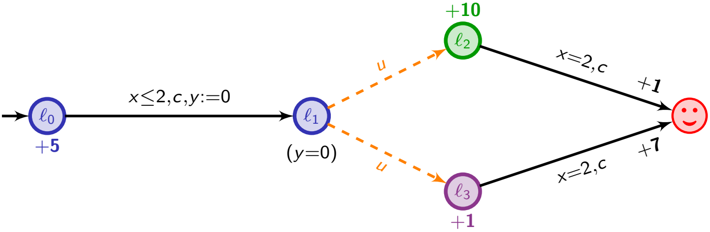

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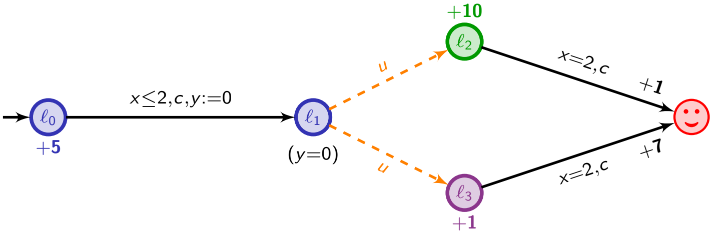

Question: what is the optimal cost we can ensure while reaching 🙂 ?

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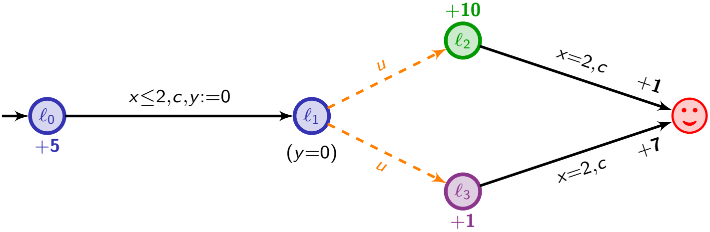

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$$5t + 10(2-t) + 1$$

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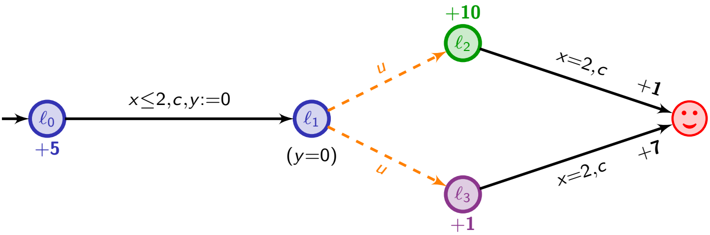

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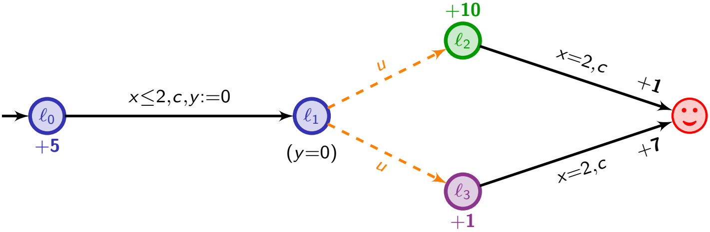
$$5t + 10(2-t) + 1 \ , \ 5t + (2-t) + 7$$

A simple weighted timed game



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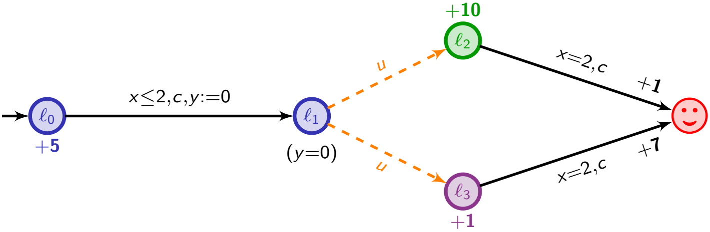

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$$\inf_{0 \leq t \leq 2} \max\left(5t + 10(2-t) + 1 \,,\, 5t + (2-t) + 7 \right) = 14 + \frac{1}{3}$$

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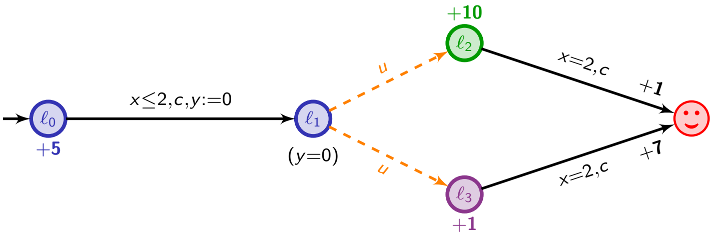

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$\rightsquigarrow$ *strategy:* wait in ℓ_0, and when $t = \frac{4}{3}$, go to ℓ_1

Optimal reachability in weighted timed games (1)

This topic has been fairly hot these last fifteen years...

[LMM02,ABM04,BCFL04,BBR05,BBM06,BLMR06,Rut11,HIM13,BGK+14]

[LMM02] La Torre, Mukhopadhyay, Murano. Optimal-reachability and control for acyclic weighted timed automata *(TCS@02)*.
[ABM04] Alur, Bernardsky, Madhusudan. Optimal reachability in weighted timed games *(ICALP'04)*.
[BCFL04] Bouyer, Cassez, Fleury, Larsen. Optimal strategies in priced timed game automata *(FSTTCS'04)*.
[BBR05] Brihaye, Bruyère, Raskin. On optimal timed strategies *(FORMATS'05)*.
[BBM06] Bouyer, Brihaye, Markey. Improved undecidability results on weighted timed automata *(Information Processing Letters)*.
[BLMR06] Bouyer, Larsen, Markey, Rasmussen. Almost-optimal strategies in one-clock priced timed automata *(FSTTCS'06)*.
[Rut11] Rutkowski. Two-player reachability-price games on single-clock timed automata *(QAPL'11)*.
[HIM13] Hansen, Ibsen-Jensen, Miltersen. A faster algorithm for solving one-clock priced timed games *(CONCUR'13)*.
[BGK+14] Brihaye, Geeraerts, Krishna, Manasa, Monmege, Trivedi. Adding Negative Prices to Priced Timed Games *(CONCUR'14)*.

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Tree-like weighted timed games can be solved in 2EXPTIME.

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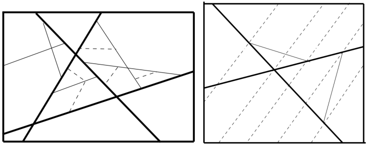
[LMM02]

Tree-like weighted timed games can be solved in 2EXPTIME.

[ABM04,BCFL04]

Depth-k weighted timed games can be solved in EXPTIME. There is a symbolic algorithm to solve weighted timed games **with a strongly non-Zeno cost**.



Optimal reachability in weighted timed games (2)

[BBR05,BBM06,BJM15]

In weighted timed games, the optimal cost (and the value) cannot be computed, as soon as games have three clocks or more.

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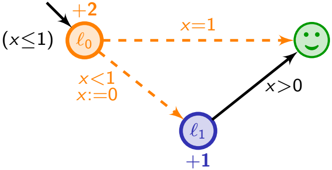
[BLMR06,Rut11,HIM13,BGK+14]

Turn-based optimal timed games are decidable in EXPTIME (resp. PTIME) when automata have a single clock (resp. with two rates). They are PTIME-hard.

What is easier with a single clock?

- Memoryless strategies can be non-optimal...



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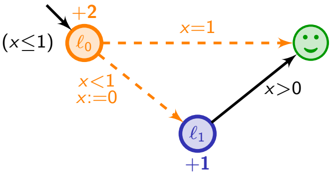

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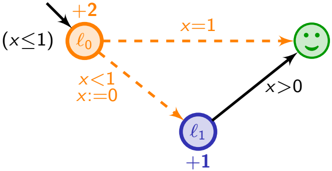

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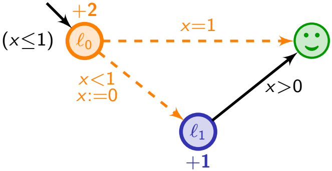

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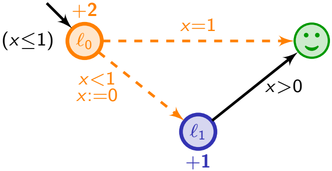

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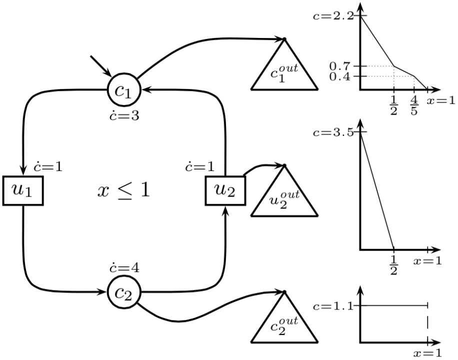
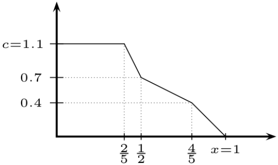
- Rather involved proofs of correctness





$$\sigma(c_2, x) = \begin{cases} c_2^{out} & \text{if } 0 \leq x < 2/5 \\ c_2 & \text{if } 2/5 \leq x < 1/2 \\ u_2 & \text{if } 1/2 \leq x \leq 1 \end{cases}$$

Computing the optimal cost: why is that hard?

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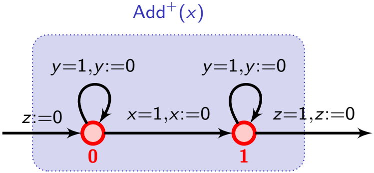
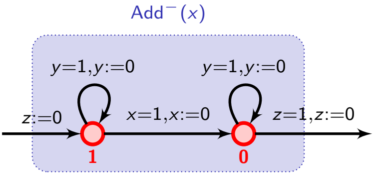


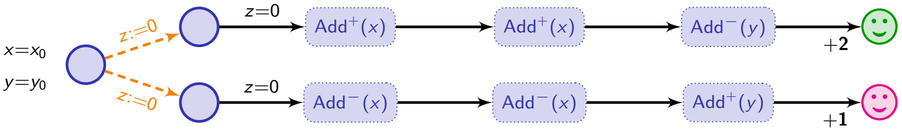
The cost is increased by x_0



The cost is increased by $1-x_0$

Computing the optimal cost: why is that hard?

Given two clocks x and y, we can check whether $y = 2x$.



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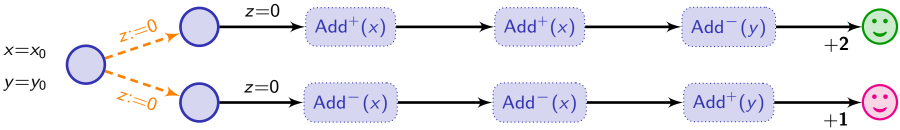

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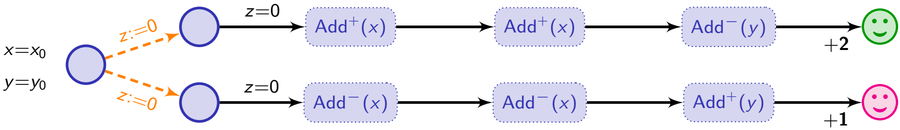

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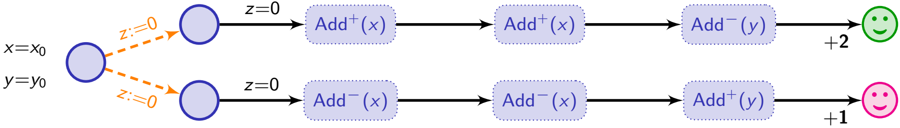

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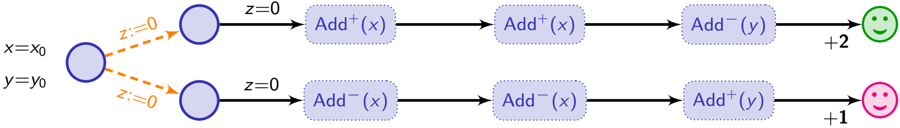

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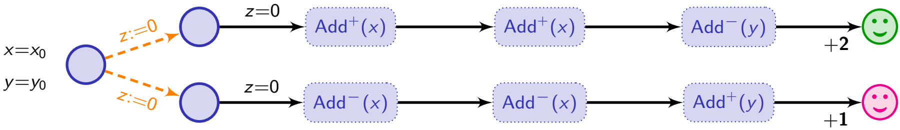

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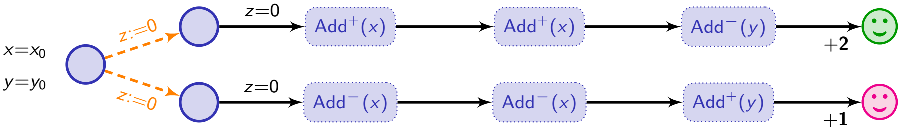

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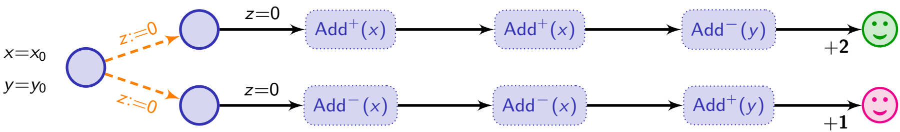

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- Player 1 has a winning strategy with cost ≤ 3 iff $y_0 = 2x_0$

Computing the optimal cost: why is that hard?

Player 1 will simulate a two-counter machine:

- each instruction is encoded as a module;
- the counter values c_1 and c_2 are encoded by two clocks:

$$x = \frac{1}{2^{c_1}} \qquad \text{and} \qquad y = \frac{1}{2^{c_2}}$$

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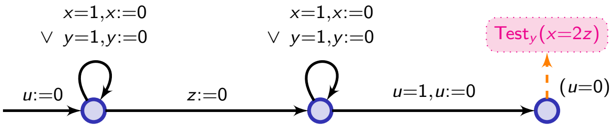
Globally, $(x \leq 1, y \leq 1, u \leq 1)$



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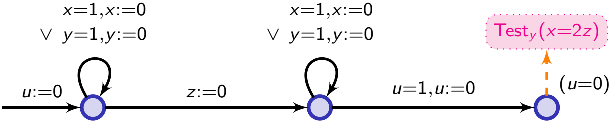

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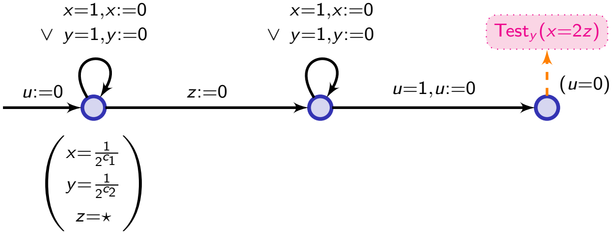

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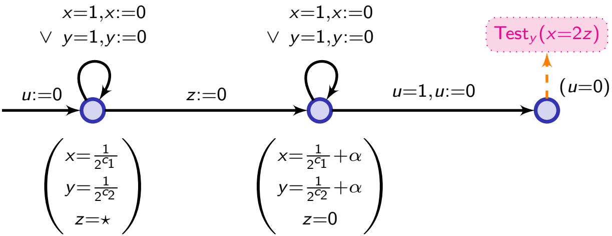

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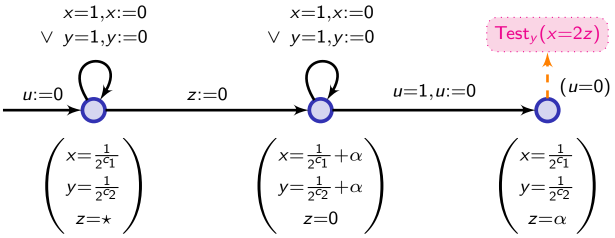

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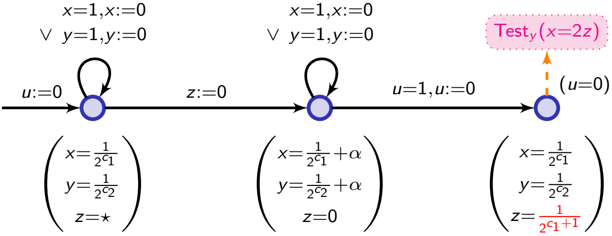
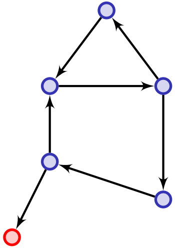


Shape of the reduction



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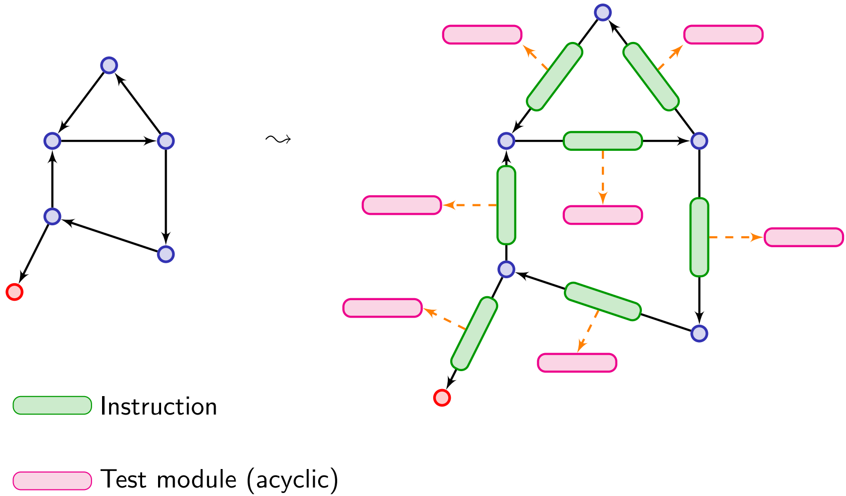

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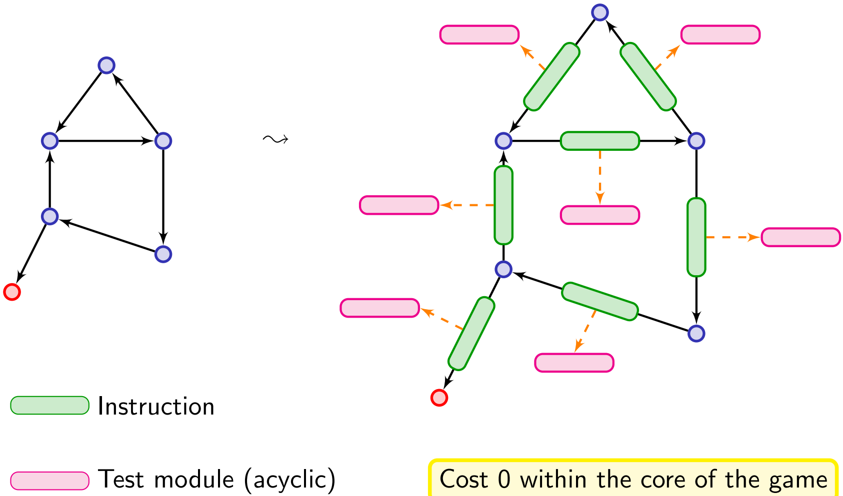

Cost 0 within the core of the game

Some further subtlety

Value of the game = infimum of all costs of strategies

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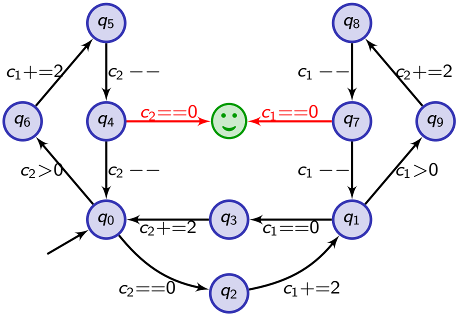

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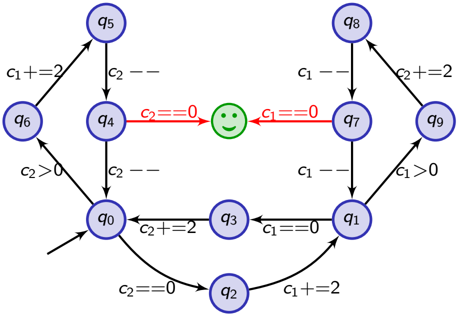
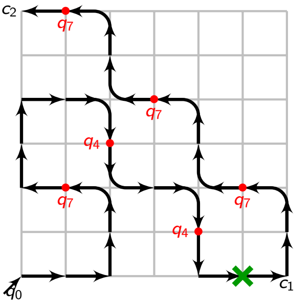
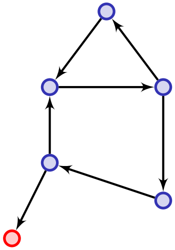
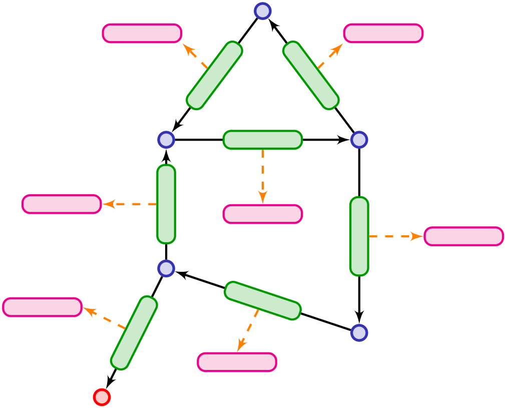




A snapshot on the undecidability proof



$\rightsquigarrow$



Instruction

Test module

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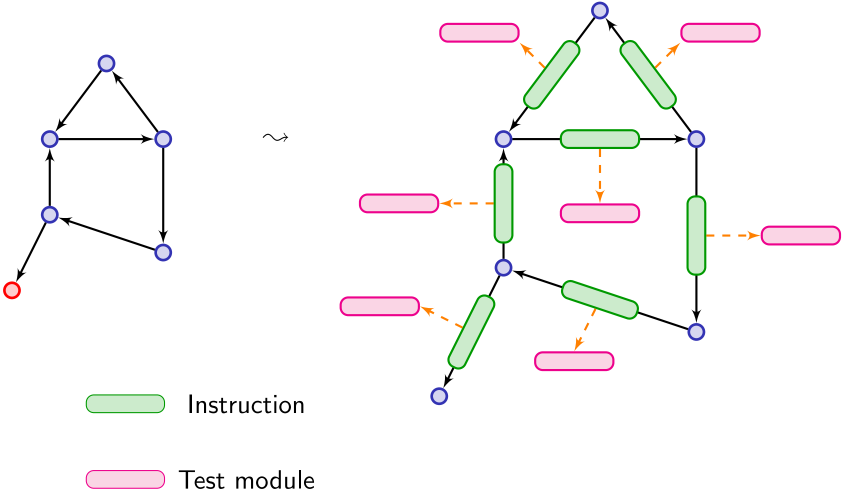

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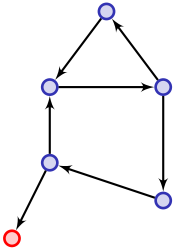

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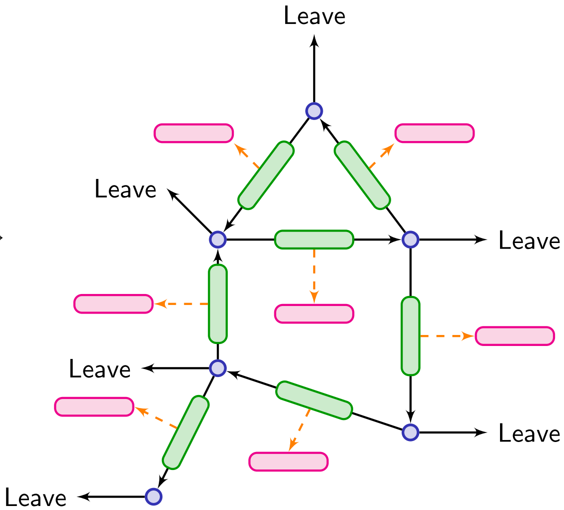
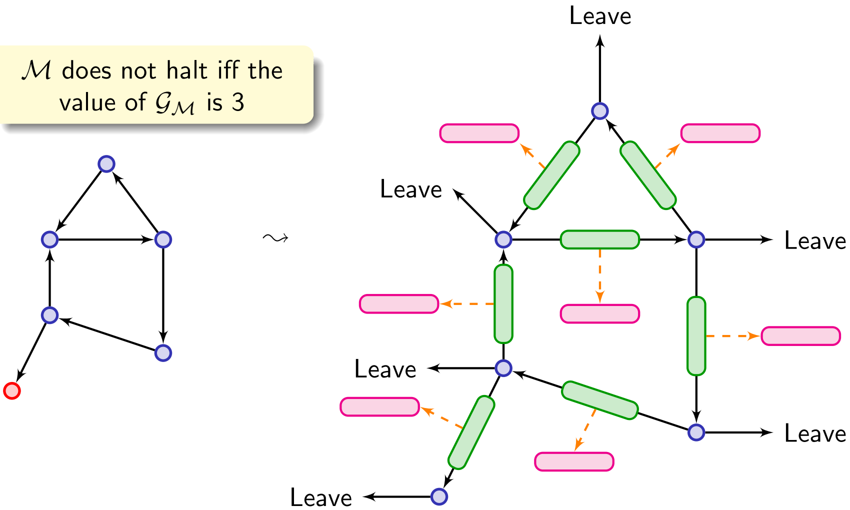


Leave with cost $3 + 1/2^n$ (n: length of the path)

A snapshot on the undecidability proof

$\mathcal{M}$ does not halt iff the value of $\mathcal{G}_{\mathcal{M}}$ is 3



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... when cost is strongly non-zeno. [AM04,BCFL04]

There is $\kappa > 0$ s.t. for every region cycle C, for every real run ϱ read on C,

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[BJM15] Bouyer, Jaziri, Markey. On the value problem in weighted timed games *(CONCUR'15)*.

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- Almost-optimality in practice should be sufficient
- Even when we know how to compute the value, we are only able to synthesize almost-optimal strategies...

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Approximation of the optimal cost

Theorem

Let $\mathcal{G}$ be a weighted timed game, in which the cost is almost-strongly non-zeno. For every $\epsilon > 0$, one can compute:

- two values v_ϵ^- and v_ϵ^+ such that

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Skip approximation scheme

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$\rightsquigarrow$ This is not possible here
There might be runs with prefixes of arbitrary length and cost 0 (e.g. the game of the undecidability proof)

Idea for approximation

Idea

Only partially unfold the game:

- Keep components with cost 0 untouched – we call it the kernel
- Unfold the rest of the game

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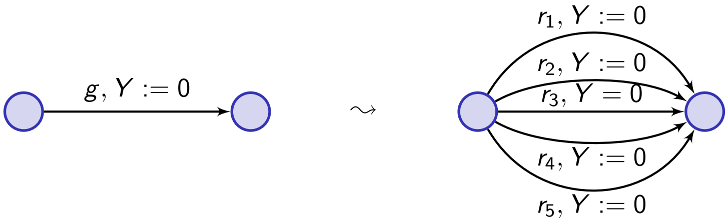
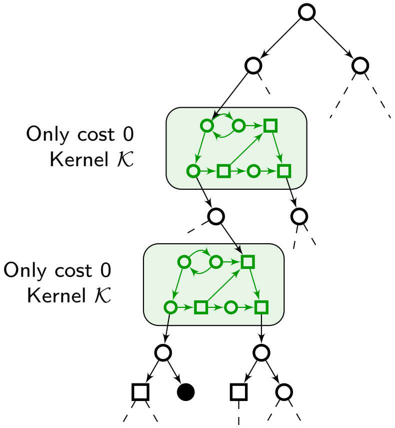
First: split the game along regions!



Idea of the proof: Semi-unfolding



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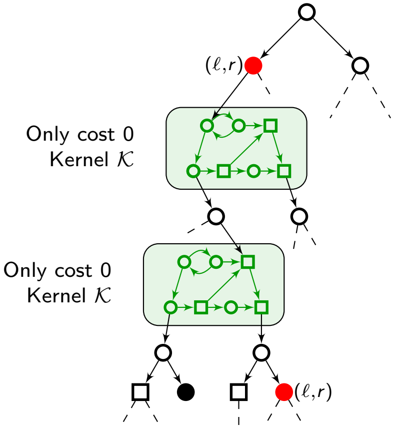

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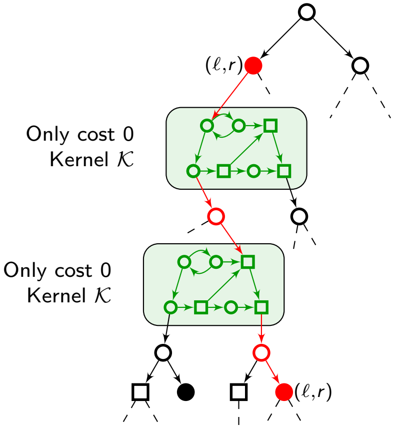

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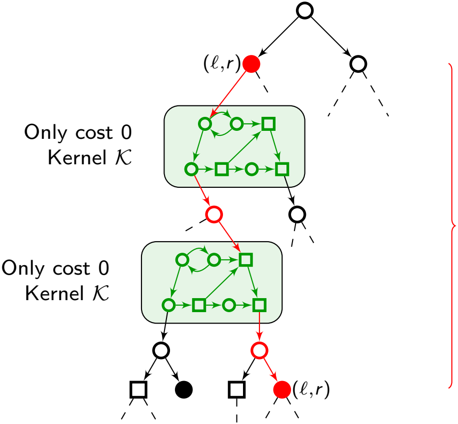
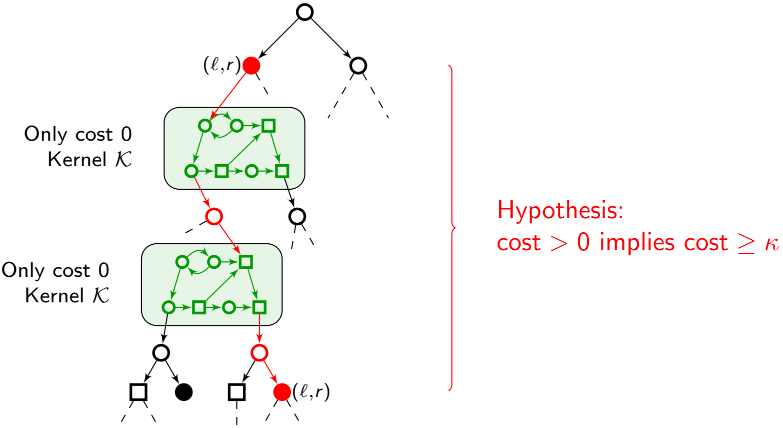


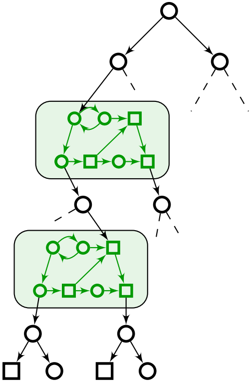
Hypothesis:
cost > 0 implies cost $\geq \kappa$

Idea of the proof: Semi-unfolding



Conclusion: we can stop unfolding the game after finitely many steps

Approximation scheme



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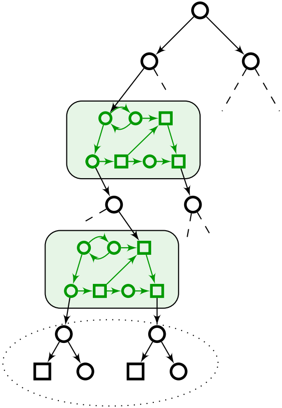

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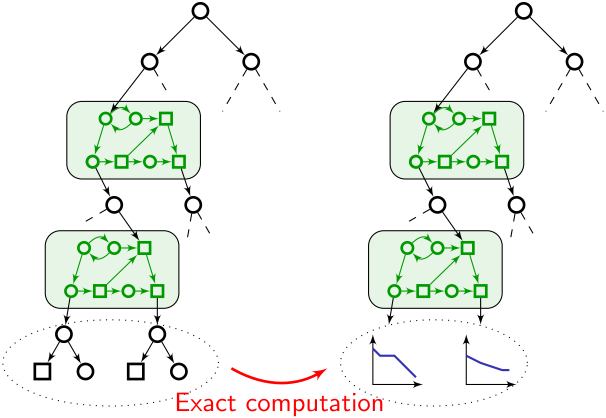

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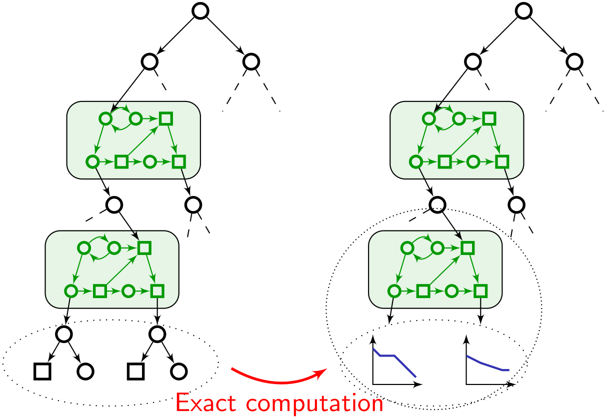

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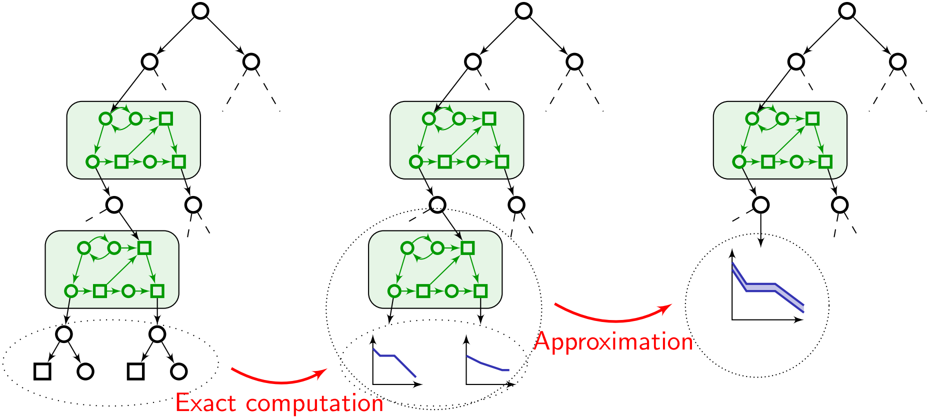
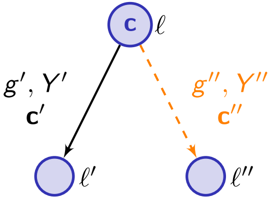


First step: Tree-like parts

$\rightsquigarrow$ Goes back to [LMM02]

[LMM02] La Torre, Mukhopadhyay, Murano. Optimal-reachability and control for acyclic weighted timed automata *(TCS@02)*.

First step: Tree-like parts



$\rightsquigarrow$ Goes back to [LMM02]

[LMM02] La Torre, Mukhopadhyay, Murano. Optimal-reachability and control for acyclic weighted timed automata *(TCS@02)*.

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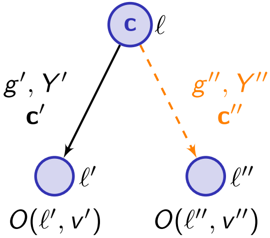

$O(\ell, v) =$

[LMM02] La Torre, Mukhopadhyay, Murano. Optimal-reachability and control for acyclic weighted timed automata *(TCS@02)*.

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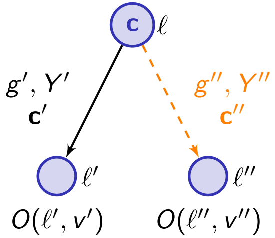

$$O(\ell, v) = \inf_{t' \mid v+t' \models g'}$$

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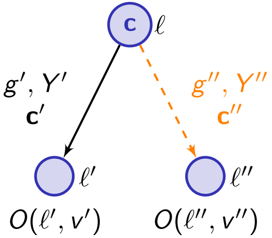

$$O(\ell, v) = \inf_{t' \mid v+t' \models g'} \max(\quad , \quad)$$

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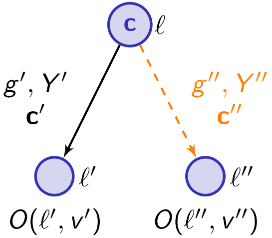

$$O(\ell, v) = \inf_{t' \mid v+t' \models g'} \max\big((\alpha), \quad\big)$$

$$(\alpha) = t'\mathbf{c} + \mathbf{c}' + O(\ell', v')$$

$v' = [Y' \leftarrow 0](v + t')$

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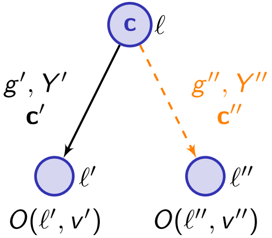

$$O(\ell, v) = \inf_{t' \mid v+t' \models g'} \max\big((\alpha), (\beta)\big)$$

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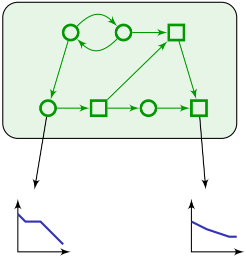
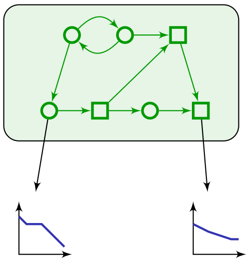
Second step: Kernels



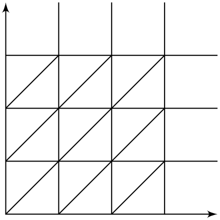
Output cost functions f

Second step: Kernels



Output cost functions f

1. Refine the regions such that f differs of at most ϵ within a small region



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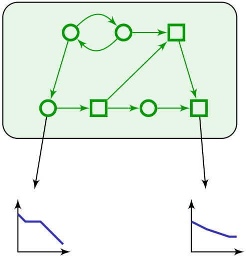

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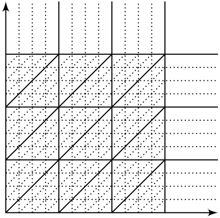

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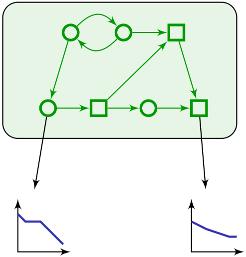

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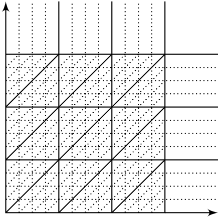

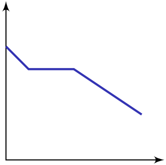

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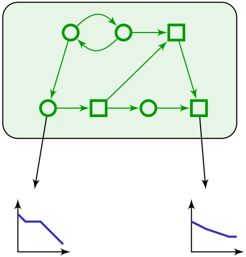

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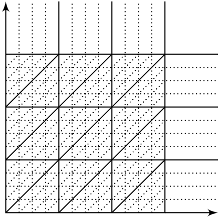
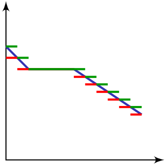
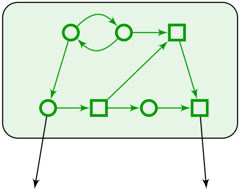


2. Under- and over-approximate by piecewise constant functions f_ϵ^- and f_ϵ^+



Second step: Kernels

3. Refine/split the kernel along the new small regions and fix f_ϵ^- or f_ϵ^+, write f_ϵ



f_ϵ: constant $\qquad f_\epsilon$: constant

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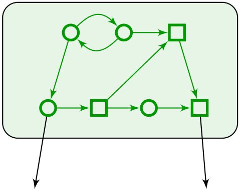

f_ϵ: constant f_ϵ: constant

3. Refine/split the kernel along the new small regions and fix f_ϵ^- or f_ϵ^+, write f_ϵ
4. Since cost is 0 everywhere, the resulting game is nothing more than a reachability timed game with an order on target (output) edges (given by f_ϵ)

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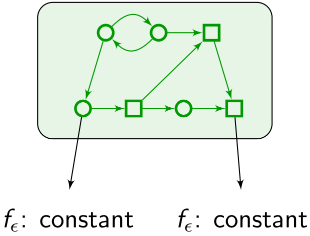

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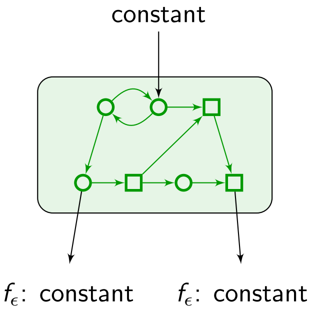

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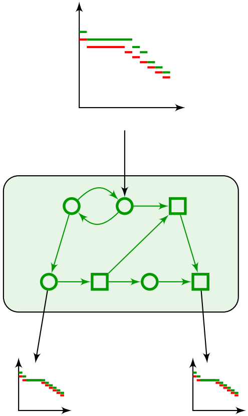

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$\rightsquigarrow$ We have computed ϵ-approximations of the optimal cost, which are constant within small regions. Corresponding strategies can be inferred

Outline

Tools for (weighted) timed automata and games

- Many tools and prototypes everywhere on earth...

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- Tool-suite Uppaal, developed in Aalborg (Denmark) and originally Uppsala (Sweden) since 1995
  - Uppaal for timed automata
  - Uppaal-TiGa for timed games
  - Uppaal-Cora for weighted timed automata

Uppaal url: http://www.uppaal.org

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TiAMo = Timed Automata Model-checker

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- Aims at being a platform for experiments (open source!)
- Aims at asserting and comparing algorithms

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- In the future: TiAMo will merge with TChecker (developed by Frédéric Herbreteau (LaBRI, France))

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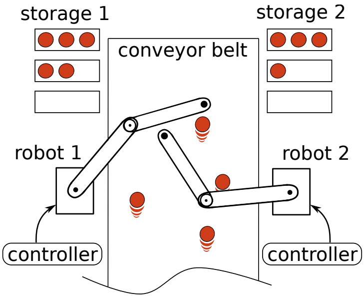
Outline

1. Timed automata
2. Timed temporal logics
3. Weighted timed automata
4. Timed games
5. Weighted timed games
6. Tools
7. Towards applying all this theory to robotic systems
8. Conclusion

Example problem, objective and approach



[BMPS15] Bouyer, Markey, Perrin, Schlehuber-Caissier. Timed-Automata Abstraction of Switched Dynamical Systems Using Control Funnels *(FORMATS'15)*.

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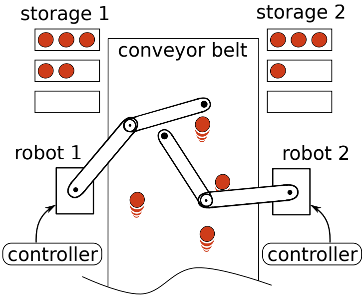

- Infinitely many configurations
- Complex behaviour
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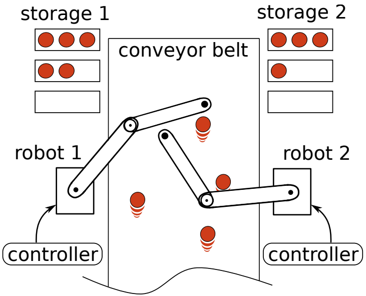

Goal: Synthesize a controller:

- Which robot handles an object
- How to avoid collision
- Don't miss any object

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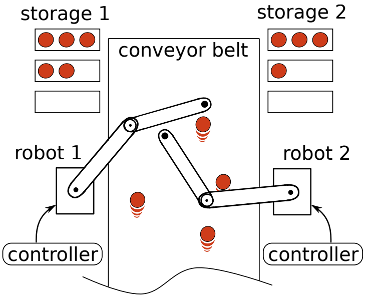

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Approach:

- Discretization of the behaviour via a fixed set of continuous controllers
- Create an abstraction and use previous results

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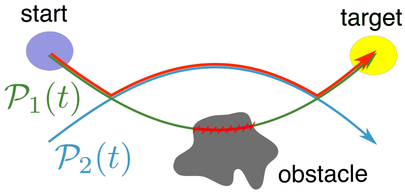
Our approach

Simplistic idea: fixed set of reference trajectories + property



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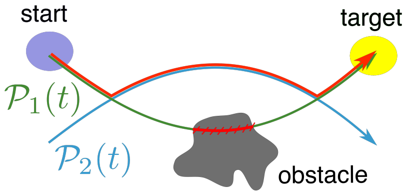
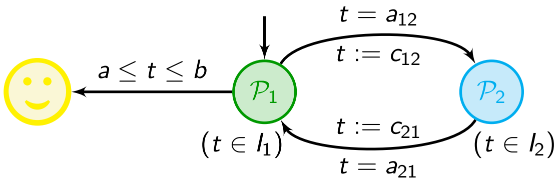


Corresponding timed automaton:



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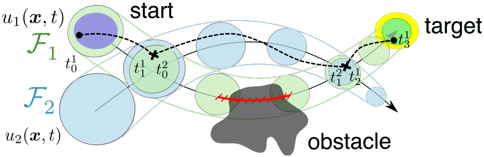
More realistic idea: fixed set of funnels for control law + property



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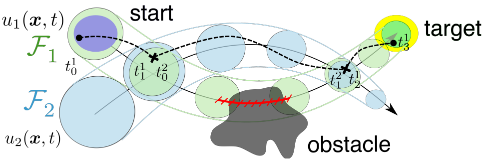

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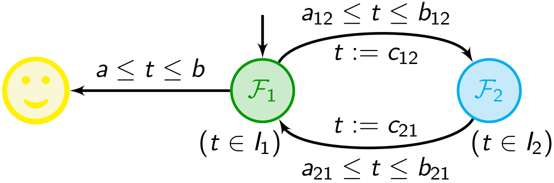
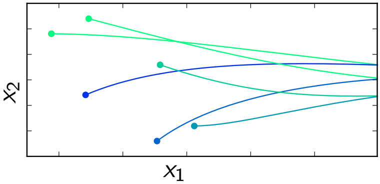


Control funnels

System with continuous dynamics $\dot{\boldsymbol{x}} = f(\boldsymbol{x}, t)$



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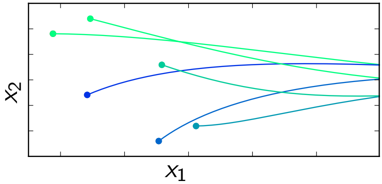

A (control) funnel is a trajectory $\mathcal{F}(t)$ of a set in the state space such that, for any trajectory $\boldsymbol{x}(t)$ of the dynamical system:

$$\forall t_0 \in \mathbb{R},\ \boldsymbol{x}(t_0) \in \mathcal{F}(t_0) \Rightarrow \forall t \geq t_0,\ x(t) \in \mathcal{F}(t)$$

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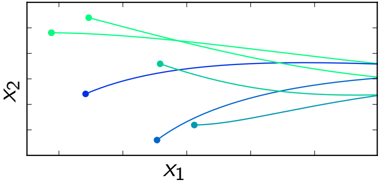

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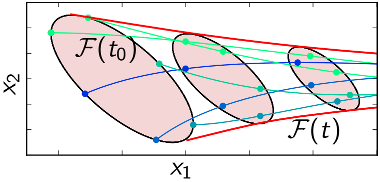
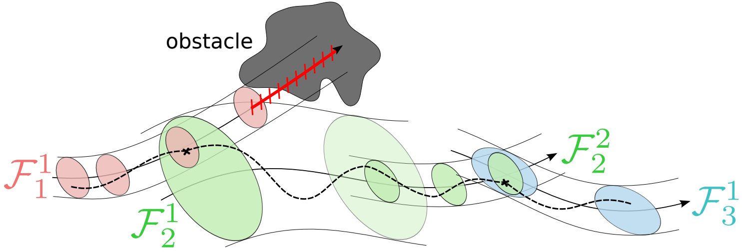


Example



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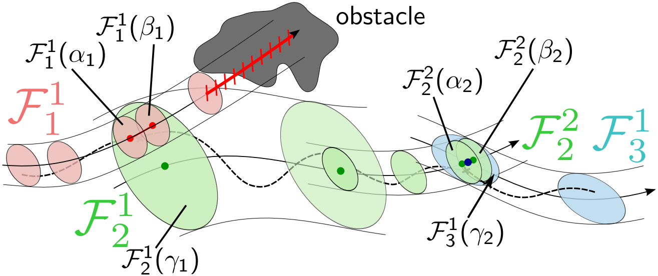
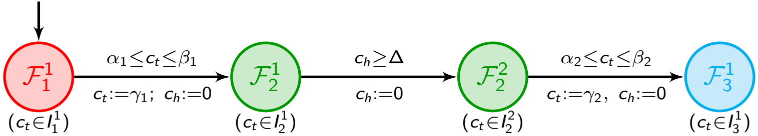
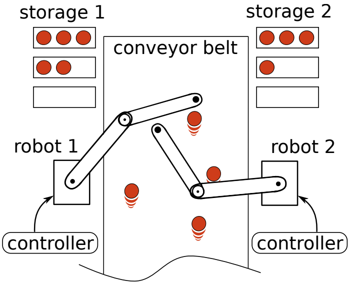


c_t: positional clock; c_h: local clock



Summary



storage 1
storage 2
conveyor belt
robot 1
robot 2
controller
controller

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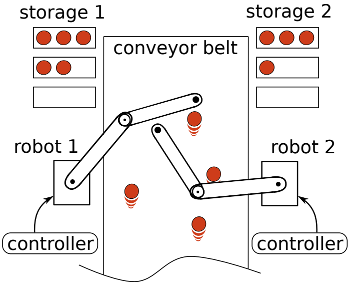

$\rightsquigarrow$ (huge) timed automata/games (with weights), with few clocks

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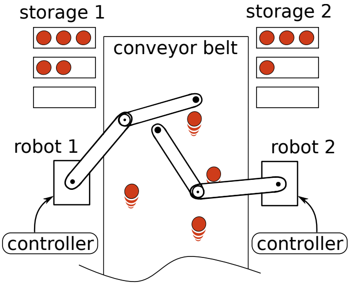

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$\leftarrow$ winning (optimal) strategy

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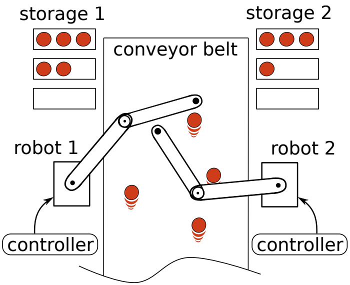
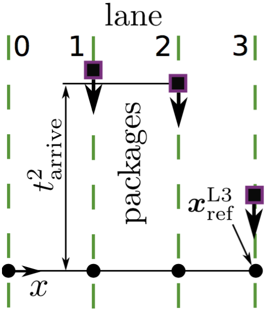


$\rightsquigarrow$ (huge) timed automata/games (with weights), with few clocks

safe (good) controller $\leftarrow$ winning (optimal) strategy

A pick-and-place example

1d point mass



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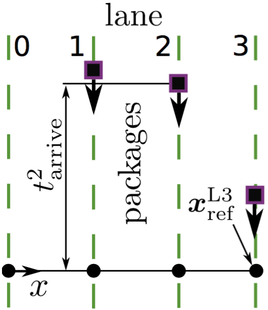
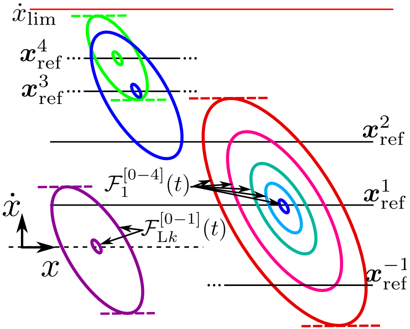
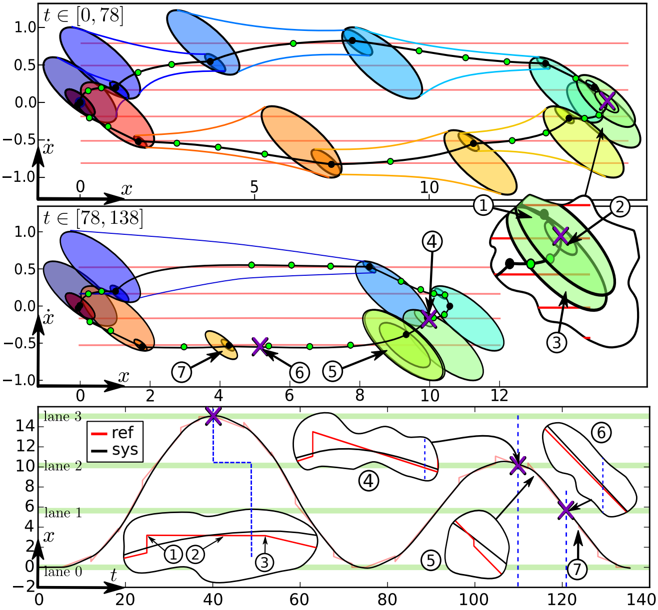
1d point mass



Funnel system




$t \in [0, 78]$
$t \in [78, 138]$
$\dot{x}$
x
t
lane 3
lane 2
lane 1
lane 0
ref
sys
1
2
3
4
5
6
7

Current challenges

For control people

- Handle more non-linear systems (automatically build control funnels)

Current challenges

For control people

- Handle more non-linear systems (automatically build control funnels)

For us

- Does not scale up very well so far (huge timed automata models)
    - Build the model on-demand?
      But, can we give guarantees (optimality) when only part of the model has been built?
    - Develop specific algorithms for the special timed automata we construct?
- Implement efficient approx. algorithm for weighted timed games

Outline

Conclusion

Summary of the talk

- Basics of timed automata verification
- Relevant extensions for applications: weights, games, mix of both
  - We looked at decidability and limits
  - We mentioned algorithmics and tools
- Timed automata can be used as abstractions for more complex systems

Conclusion

Current challenges

- Various theoretical issues
  - Decidability and approximability of weighted timed automata and games
  - New approaches (tree automata, reachability relations) might give a new light on the verification of timed systems
  - Robustness and implementability
- Continue working on algorithms and tools

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  - Implementation of (weighted) timed games (good data structures, abstractions, etc.)
  - More applications with specific challenges (e.g. robotic problems)