Average-energy games

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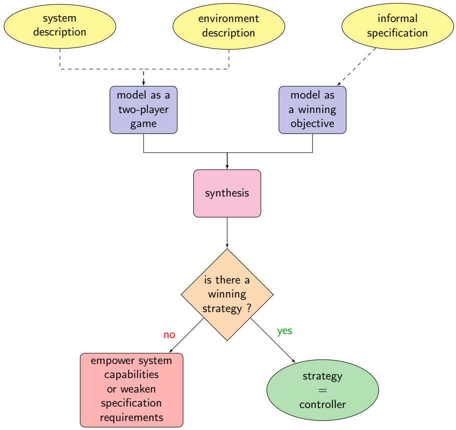
Based on joint works with:

N. Markey, M. Randour
K.G. Larsen, S. Laursen
(GandALF'15 / Acta Informatica)

P. Hofman, N. Markey
M. Randour, M. Zimmermann
(FoSSaCS'17)

Thanks to Mickael for his slides!

General context: strategy synthesis in quantitative games



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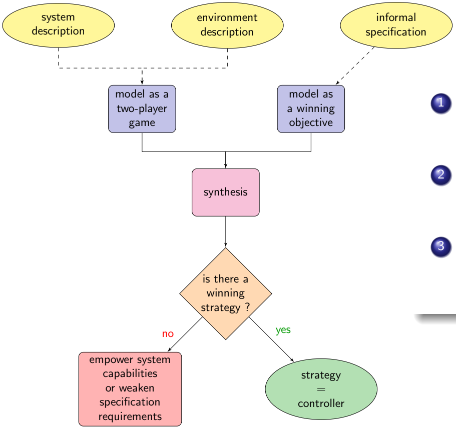
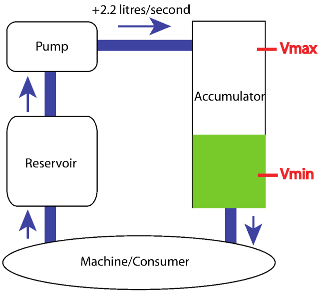


1. Can we decide if a winning strategy exists?
2. How complex such a strategy needs to be? Simpler is better...
3. Can we synthesize one efficiently?

$\leadsto$ Depends on the winning objective

Motivating example

Hydac oil pump industrial case study (Quasimodo research project)



Goals

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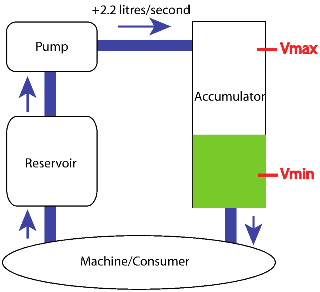

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1. Keep the oil level in the safe zone
   $\hookrightarrow$ Energy objective with lower and upper bounds: EG_{LU}

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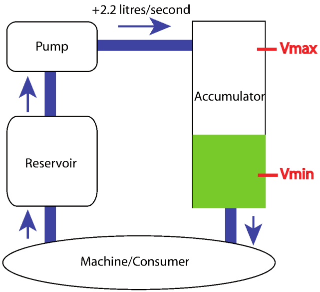

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   ↪ Average-energy objective: AE

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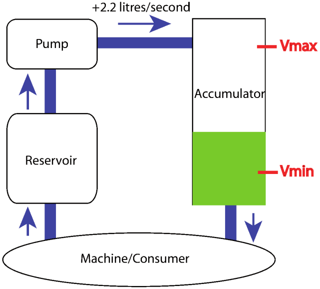

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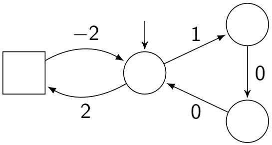
1. Keep the oil level in the safe zone
   $\hookrightarrow$ Energy objective with lower and upper bounds: EG_{LU}
2. Minimize the average oil level
   $\hookrightarrow$ Average-energy objective: AE
3. Conjunction: AE_{LU}

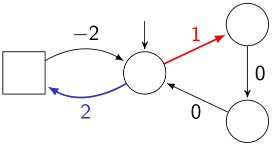
Outline

Average-energy: illustration



- Two-player turn-based games with integer weights

Average-energy: illustration



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- Focus on two memoryless strategies

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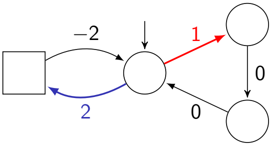

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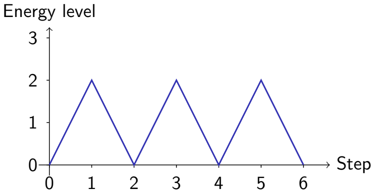

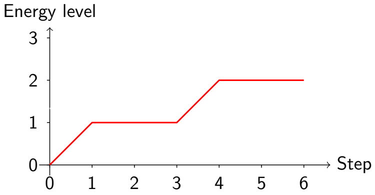

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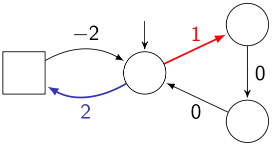

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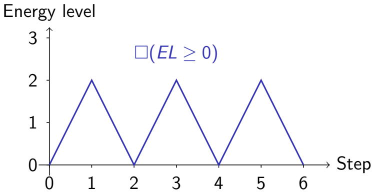
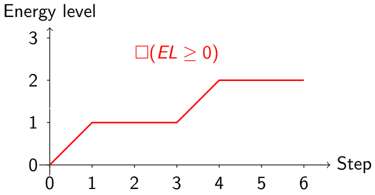
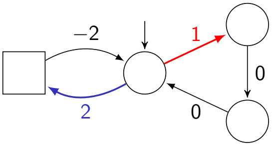




Energy objective (EG_L/EG_{LU}): e.g., always maintain $EL \geq 0$

Average-energy: illustration



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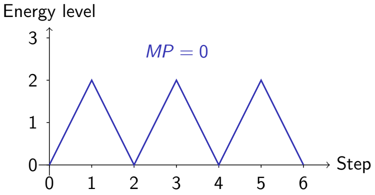
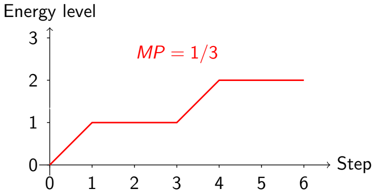
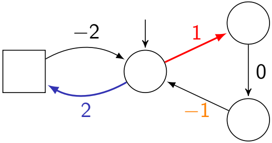




Mean-payoff (MP): long-run average payoff per transition

Average-energy: illustration



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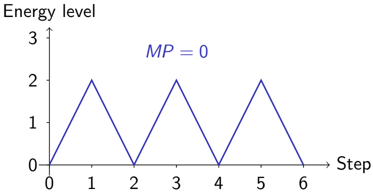
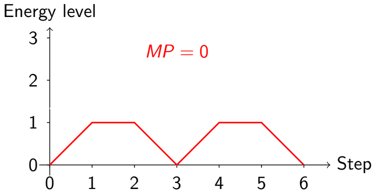
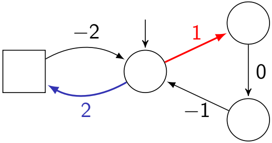




Mean-payoff (MP): long-run average payoff per transition

$\Longrightarrow$ Let's change the weights of our game

Average-energy: illustration



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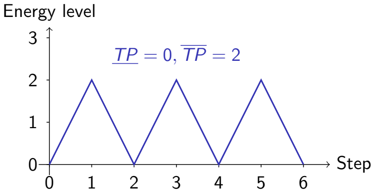

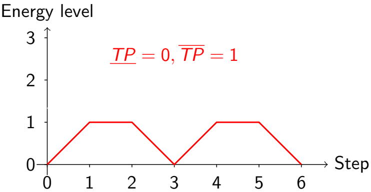

Total-payoff (TP) refines MP in the case $MP = 0$ by looking at high/low points of the sequence

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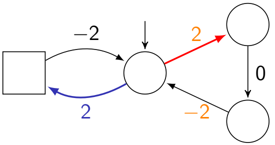

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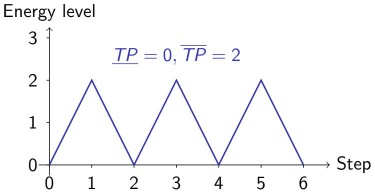

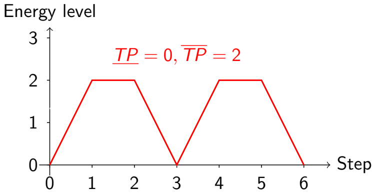

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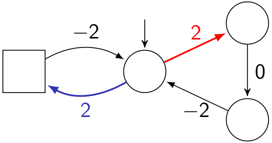
$\Longrightarrow$ Let's change the weights again

Average-energy: illustration



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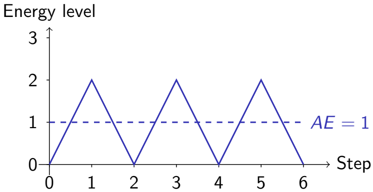

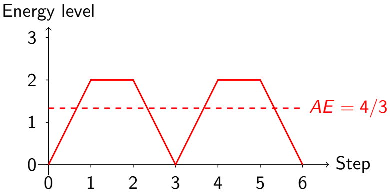

Average-energy (AE) further refines TP: average EL along a play

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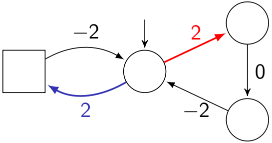

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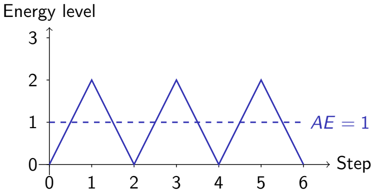

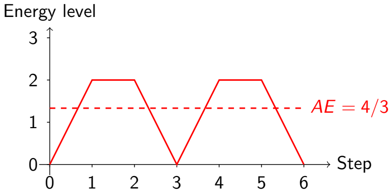

Average-energy (AE) further refines TP: average EL along a play

$\Longrightarrow$ Natural concept (cf. case study)

Average-energy: overview

Objective	**1-player**	**2-player**	**memory**
MP	P [Kar78]	NP $\cap$ coNP [ZP96]	memoryless [EM79]
TP	P [FV97]	NP $\cap$ coNP [GS09]	memoryless [GZ04]
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EG_{LU}	PSPACE-c. [FJ13]	EXPTIME-c. [BFL+08]	pseudo-polynomial

[Kar78] Karp. A characterization of the minimum cycle mean in a digraph *(Discrete Mathematics)*
[ZP96] Zwick, Paterson. The complexity of mean payoff games on graphs *(Theoretical Computer Science)*
[EM79] Ehrenfeucht, Mycielski. Positional strategies for mean payoff games *(Int. Journal of Game Theory)*
[FV97] Filar, Vrieze. Competitive Markov decision provesses *(Springer)*
[GS09] Gawlitza, Seidl. Games through nested fixpoints *(CAV'09)*
[GZ04] Gimbert, Zielonka. When can you play positionally? *(MFCS'04)*
[BFL+08] Bouyer, Fahrenberg, Larsen, Markey, Srba. Infinite runs in weighted timed automata with energy constraints *(FORMATS'08)*
[CDHS03] Chakrabarti, De Alfaro, Henzinger, Stoelinga. Resource interfaces *(EMSOFT'03)*
[FJ13] Fearnley, Jurdziński. Reachability in two-clock timed automata is PSPACE-complete *(ICALP'13)*

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Techniques

- Classical criteria cannot be applied out-of-the-box [EM79,BSV04,AR14,GZ04,Kop06]

  $\hookrightarrow$ we cannot mix nor shuffle cycles, but only 0-cycles!

[GZ04] Gimbert, Zielonka. When can you play positionally? *(MFCS'04)*
[EM79] Ehrenfeucht, Mycielski. Positional strategies for mean payoff games *(Int. Journal of Game Theory)*
[BSV04] Björklund, Sandberg, Vorobyov. Memoryless determinacy of parity and mean payoff games: A simple proof *(Theoretical Computer Science)*
[AR14] Aminof, Rubin. First cycle games *(SR'14)*
[Kop06] Kopczynski. Half-positional determinacy of infinite games *(ICALP'06)*

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[GZ05] Gimbert, Zielonka. Games where you can play optimally without any memory *(CONCUR'05)*

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- MP-hardness

Outline

1. Average-energy games
2. Average-energy with energy constraints

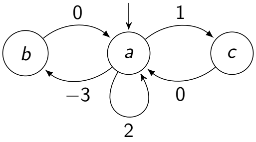
Two settings

1. AE_{LU}: AE with lower (0) and upper ($U \in \mathbb{N}$) bounds
2. AE_{L}: AE with only the lower bound (0)

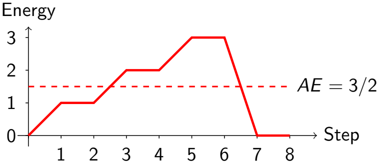
$\hookrightarrow$ Fixed initial credit $c_{\text{init}} = 0$

With LU energy constraints, memory is needed!

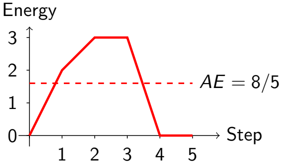
$AE_{LU} \rightsquigarrow$ minimize AE while keeping $EL \in [0, 3]$ (init. $EL = 0$).



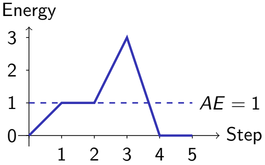
(a) One-player AE_{LU} game.



(b) Play $\pi_1 = (acacacab)^\omega$.



(c) Play $\pi_2 = (aacab)^\omega$.



(d) Play $\pi_3 = (acaab)^\omega$.

Minimal AE with π_3: alternating between the $+1$, $+2$ and -3 cycles

With LU energy constraints, memory is needed!

$AE_{LU} \rightsquigarrow$ minimize AE while keeping $EL \in [0, 3]$ (init. $EL = 0$).

Non-trivial behavior in general!
$\rightsquigarrow$ We need to choose carefully which cycles to play

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Result

The AE_{LU} problem is EXPTIME-complete.

- Reduction from AE_{LU} to AE on pseudo-polynomial game
- Reduction from this AE game to MP game + pseudo-poly. algorithm.

Complexity results for AE_{LU}

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- ... hence the state-space of the game becomes infinite

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One-player case

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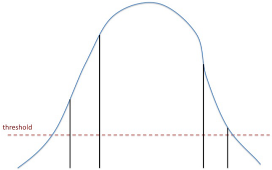

[LLT04] Lafourcade, Lugiez, Treinen. Intruder deduction for AC-like equational theories with homomorphism *(Research report LSV-04-16)*

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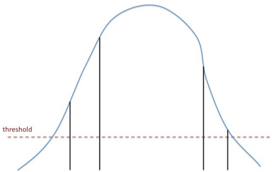

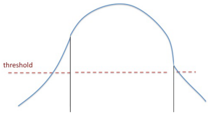

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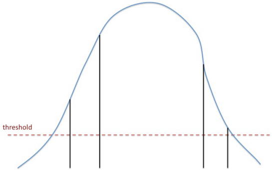
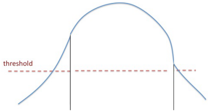




- Results for AE_{LU} apply!

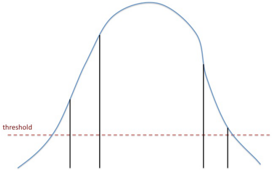
[LLT04] Lafourcade, Lugiez, Treinen. Intruder deduction for AC-like equational theories with homomorphism *(Research report LSV-04-16)*

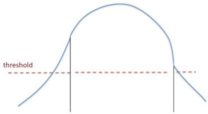
What happens with only L-constraints?

- There is a priori no more an upper bound on the energy level...
- ... hence the state-space of the game becomes infinite

One-player case

- Upper bound on the energy level, thanks to [LLT04]





- Results for AE_{LU} apply!

This cannot easily be extended to two-player games...

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Two-player games with L-constraints

The crux idea

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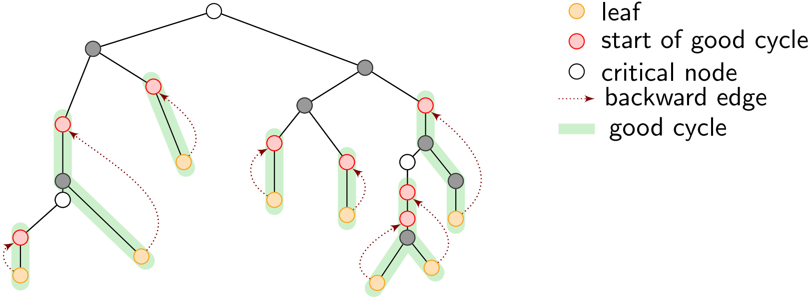
- All (reachable) good cycles with no strict good sub-cycles have length bounded by $8t_1t_2(t+1)^3|S|^2$

Seeing strategies as (finite) trees



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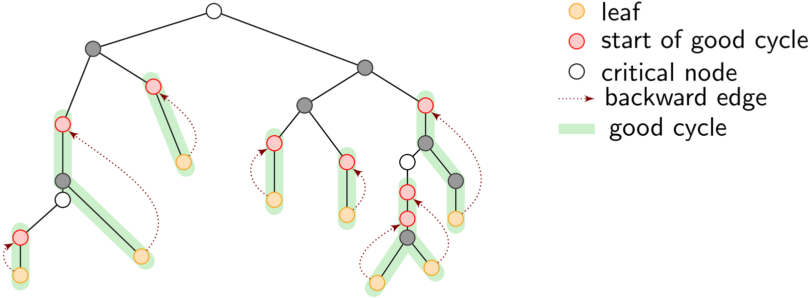

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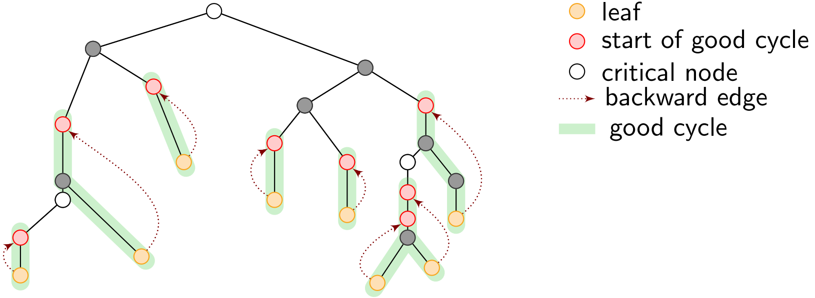
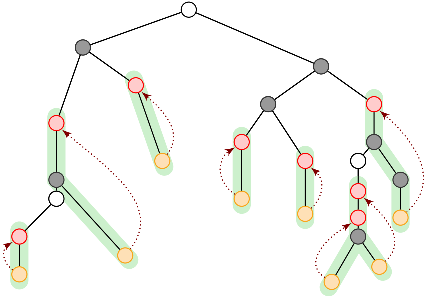


- A finite good tree represents a winning strategy
- Fix a winning strategy, and build a finite tree by “closing” minimal good cycles. We then have a finite good tree, hence a winning strategy!

What is missing?



[Wal01] Walukiewicz. Pushdown processes: Games and model-checking *(Information and Computation)*
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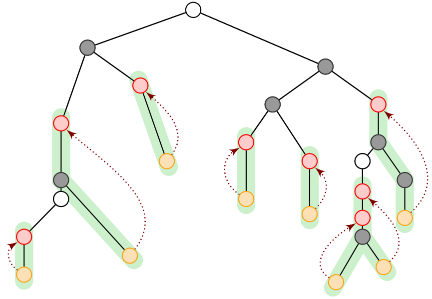
- energy level is bounded in green parts (good cycles)



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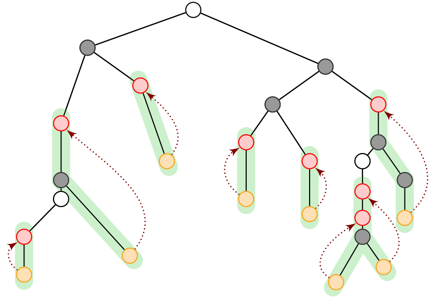

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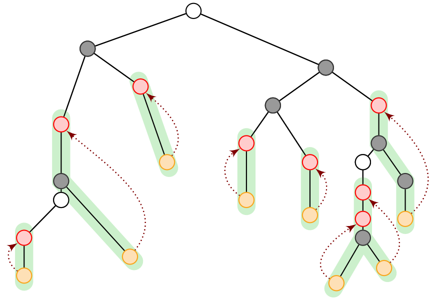

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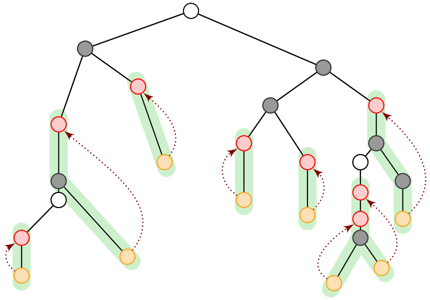

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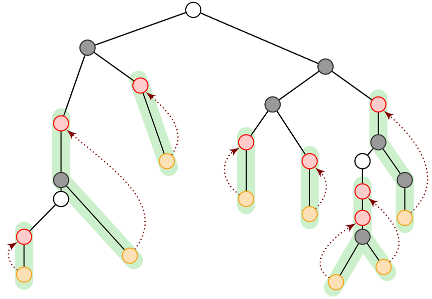
- reduced to AE_{LU} problem!



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With energy constraints: results overview

Objective	**1-player**	**2-player**	**memory**
MP	P [Kar78]	NP ∩ coNP [ZP96]	memoryless [EM79]
TP	P [FV97]	NP ∩ coNP [GS09]	memoryless [GZ04]
EG_L	P [BFL+08]	NP ∩ coNP [CDHS03,BFL+08]	memoryless [CDHS03]
EG_{LU}	PSPACE-c. [FJ13]	EXPTIME-c. [BFL+08]	pseudo-polynomial
AE	P	NP ∩ coNP	memoryless
AE_{LU}	PSPACE-c.	EXPTIME-c.	pseudo-polynomial
AE_L	PSPACE-e./NP-h.	2-EXPTIME-e./EXPSPACE-h.	*super-exp.* (doubly exp.)

$\rightsquigarrow$ Lower bounds for AE_{LU} inferred from [Hun14,Hun15]

[Hun14] Hunter. Reachability in succinct one-counter games *(CoRR abs/1407.1996)*
[Hun15] Hunter. Reachability in succinct one-counter games *(RP'15)*

Conclusion

- "New" quantitative objective
  Appeared in [TV87] as an alternative *total reward* definition but not studied until recently. See also [CP13,BEGM15]
- Yields natural payoff functions
- AE "refines" TP (and MP)
- Same complexity class as EG_L, MP and TP games
- AE_{LU} and AE_L now well-understood

Next...

- Investigate further that payoff function
- Investigate further mean-payoff pushdown games?
  - Undecidable in general
  - By-product of this work: decidable restricted subclass

[TV87] Thuijsman, Vrieze. The bad match; A total reward stochastic game *(IR Spektrum)*
[CP13] Chatterjee, Prabhu. Quantitative timed simulation functions and refinement metrics for real-time systems *(HSCC'13)*
[BEGM15] Boros, Elbassioni, Gurvich, Makino. Markov decision processes and stochastic games with total effective payoff *(STACS'15)*