Quantitative Models for Verification

— A timed-automata-based perspective —

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September 22, 2011

Time-dependent systems

- We are interested in timed systems

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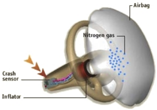
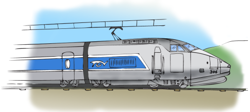
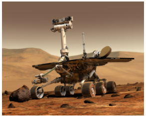
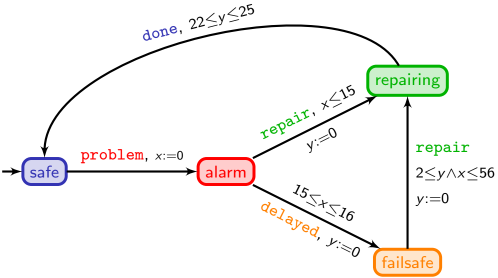






The standard timed automaton model [AD90,AD94]

Example



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

[AD90] Alur, Dill. Automata for modeling real-time systems *(ICALP'90)*.
[AD94] Alur, Dill. A theory of timed automata *(Theoretical Computer Science)*.

Why should we go further?

- Not only time should be modelled, but further (time-dependent, or not) information might be of interest.
- Examples:
  - Interaction
  - Uncertainty
  - Resources
  - ...

Task graph scheduling problems

Compute $D\times(C\times(A+B))+(A+B)+(C\times D)$ using two processors:

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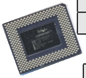
energy	
idle	10 Watt
in use	90 Watts

P_2 (slow):



time	
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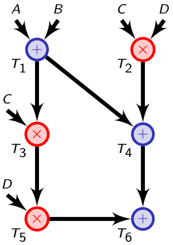

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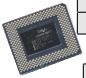

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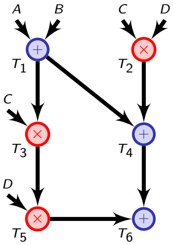

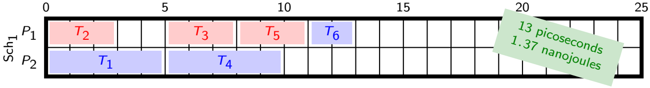

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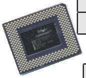

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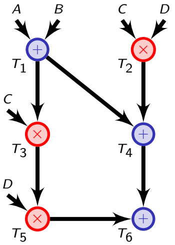

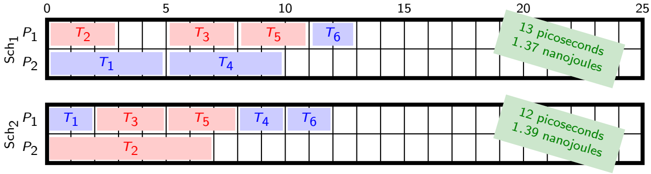

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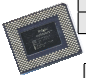

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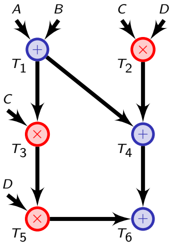
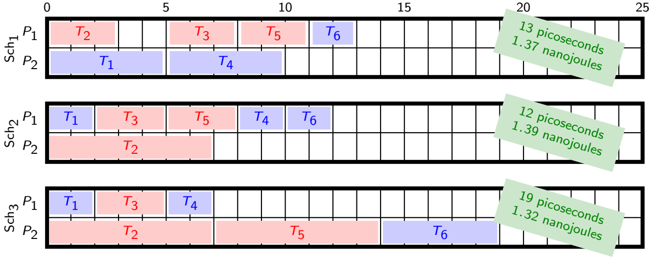
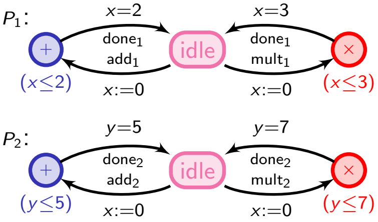




Modelling the task graph scheduling problem

- Processors



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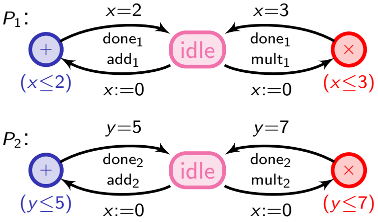

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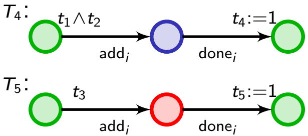

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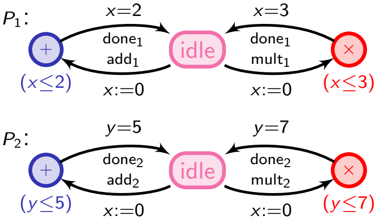

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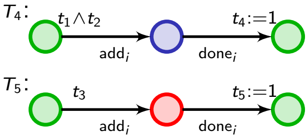


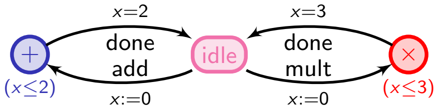
Global system: $(P_1 \parallel P_2) \parallel_s (T_1 \parallel T_2 \parallel \cdots \parallel T_6)$

A schedule: a path in the global system which reaches $t_1 \wedge \cdots \wedge t_6$

Why (timed) games?

- to model uncertainty

Example of a processor in the taskgraph example



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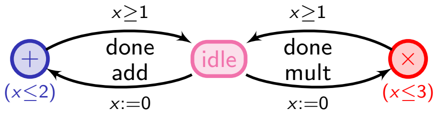

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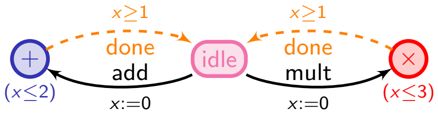

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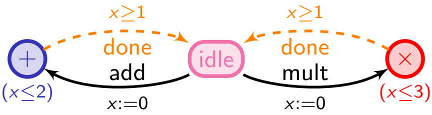
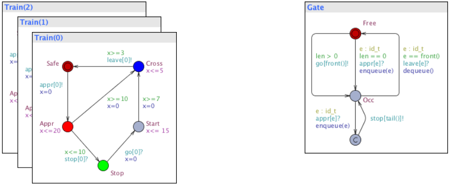


- to model an interaction with an environment

Example of the gate in the train/gate example



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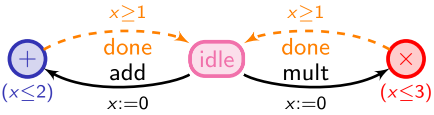

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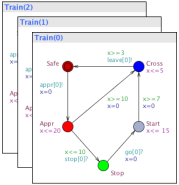

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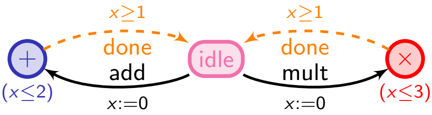

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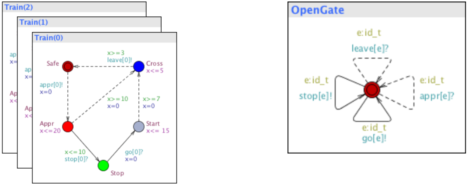

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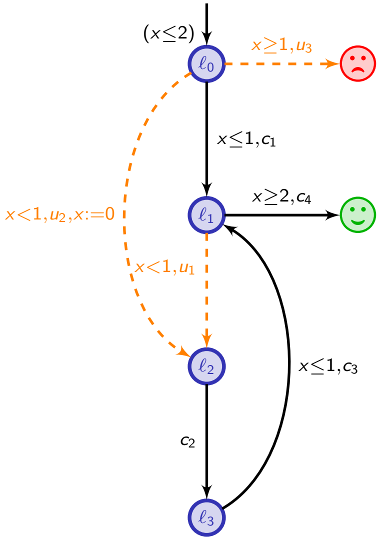

Rule of the game

- Aim: avoid ☹ and reach ☺

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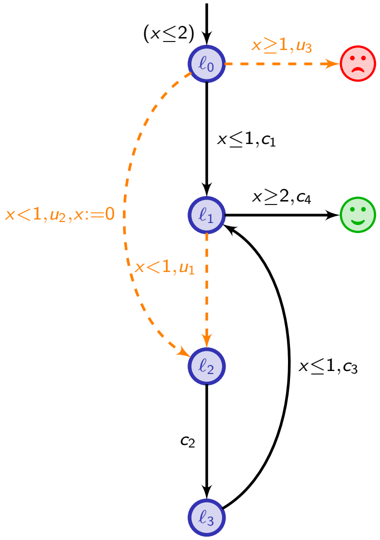

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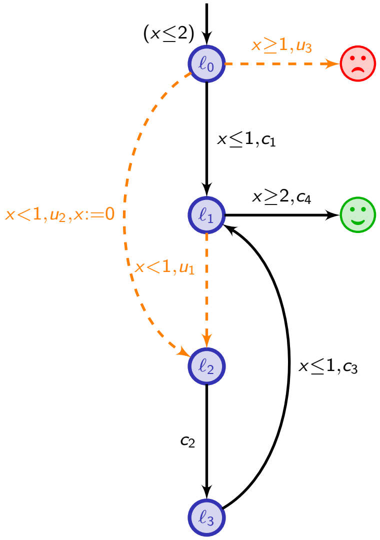

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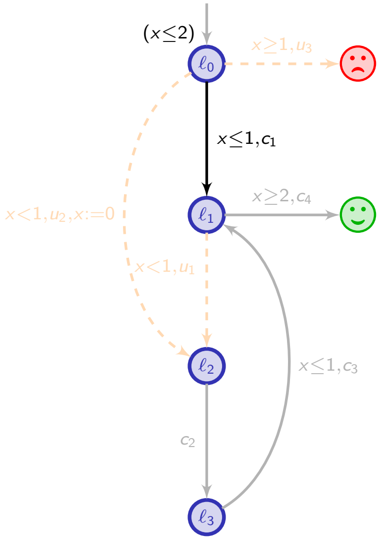

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- from $(\ell_0, 0)$, play $(0.5, c_1)$

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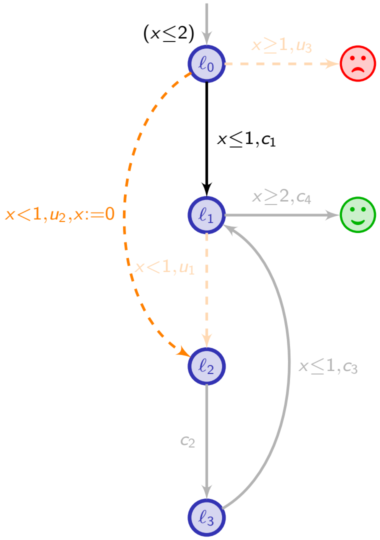

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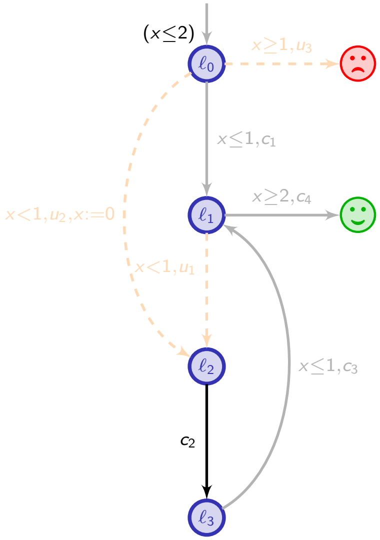

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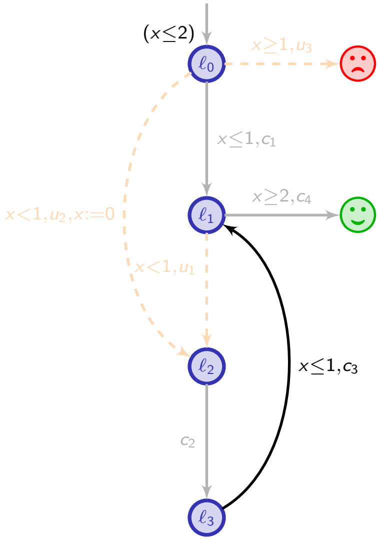

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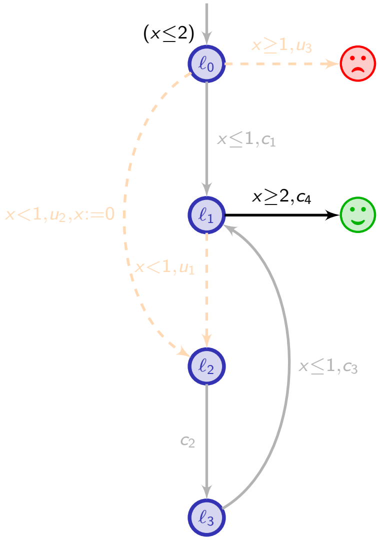

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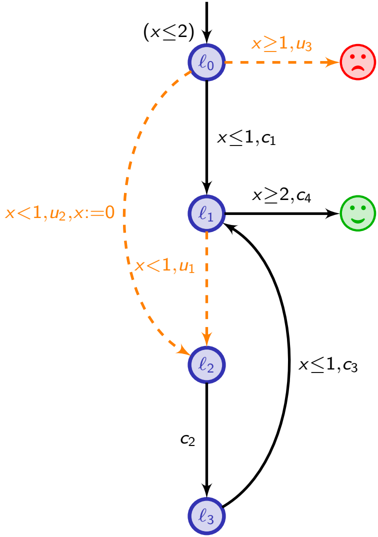

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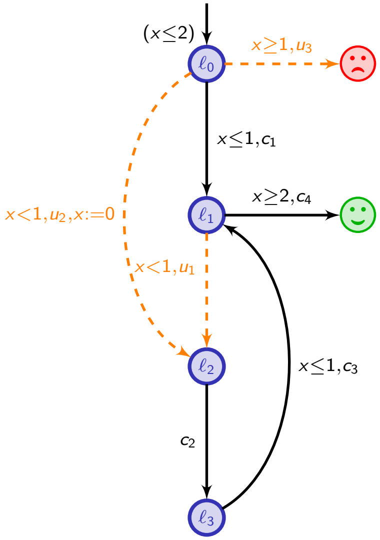

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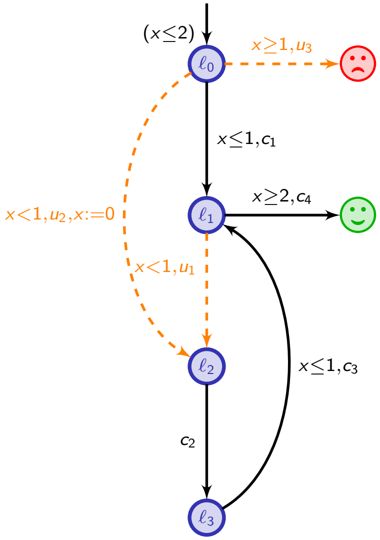

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- If yes, compute one (as simple as possible).

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- to model probabilistic behaviours

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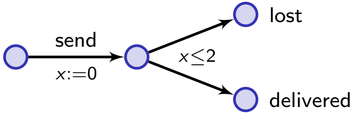
Example of losses when sending messages



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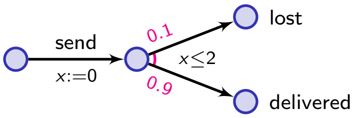

$\rightsquigarrow$ the probabilistic timed automata model used e.g. in PRISM and UPPAAL-PRO [KNSS02]

[KNSS02] Automatic verification of real-time systems with discrete probability distributions *(TCS)*.

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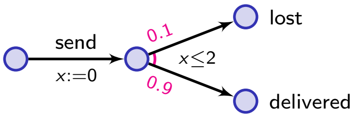

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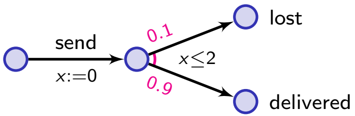

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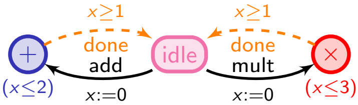

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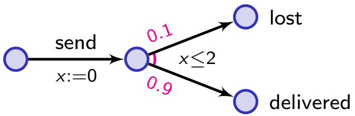

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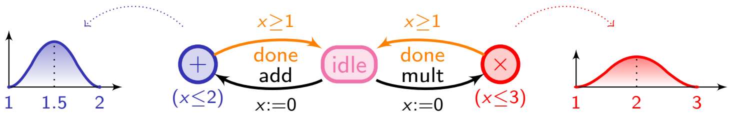
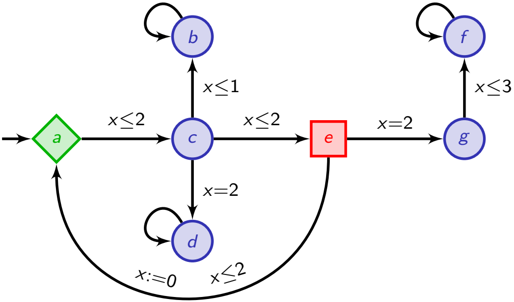


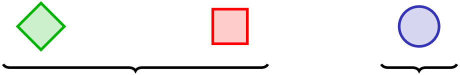
⤳ the stochastic timed automata model [BBB+08,BF09]

[KNSS02] Automatic verification of real-time systems with discrete probability distributions *(TCS)*.
[BBB+08] Baier, Bertrand, Bouyer, Brihaye, Größer. Almost-sure model checking of infinite paths in one-clock timed automata *(LICS'08)*.
[BF09] Bouyer, Forejt. Reachability in stochastic timed games *(ICALP'09)*.

Stochastic timed game: an example



- Timed graph with vertices partitioned among three players:



classical players playing “turn-based” | the Nature stochastic player

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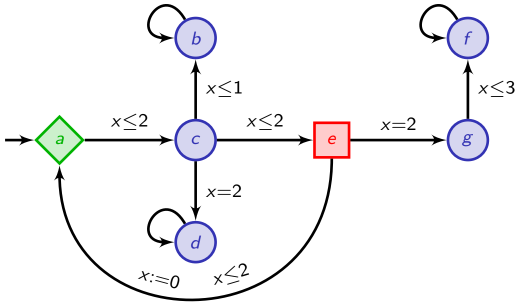

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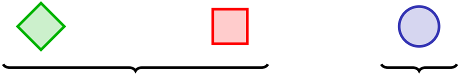

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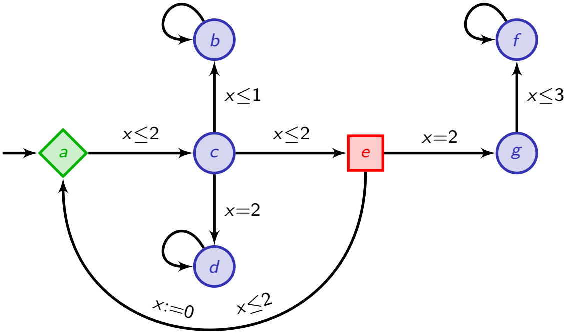
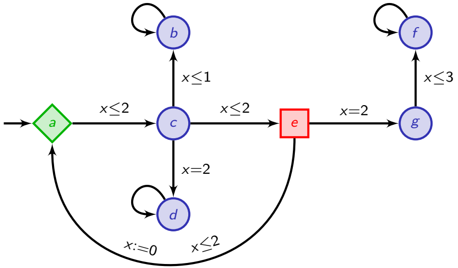
- There are prescribed probability distributions from vertices.

How is this game played?



- Players  and  play according to standard strategies
- Player  plays according to the prescribed probability distributions:
  - choose a delay according to some distribution
  - choose an action according to some discrete distribution

Play, an example



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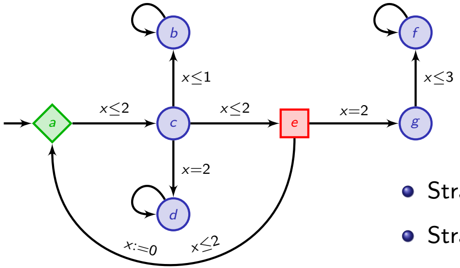

- Strategy for : go to c when $x = 1$
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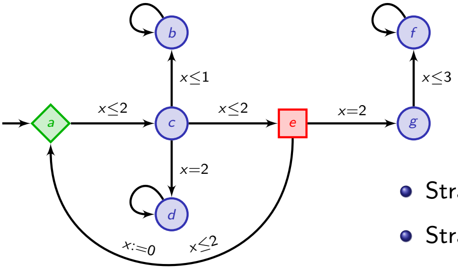

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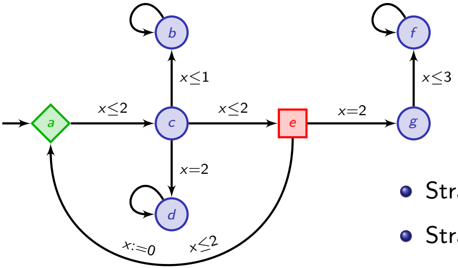

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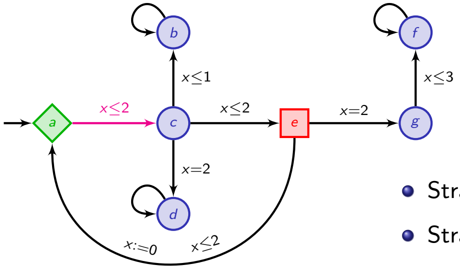

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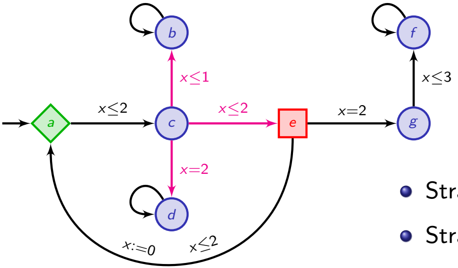

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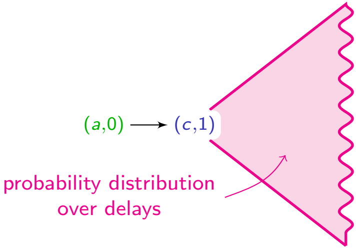

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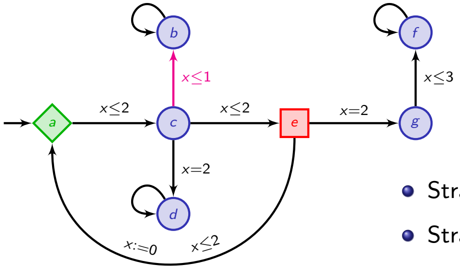

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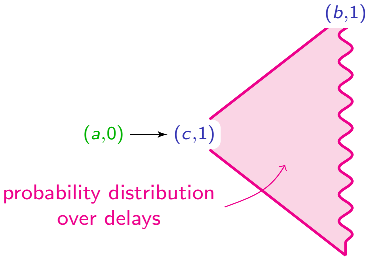

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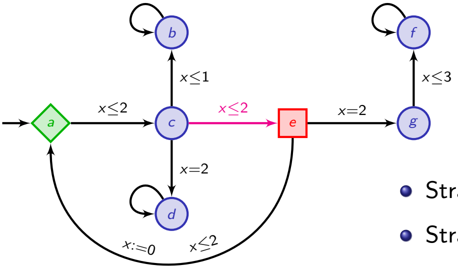

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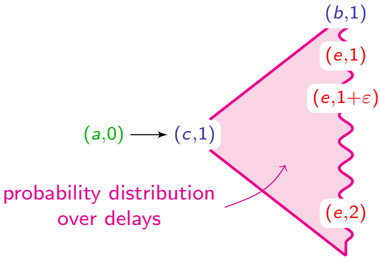

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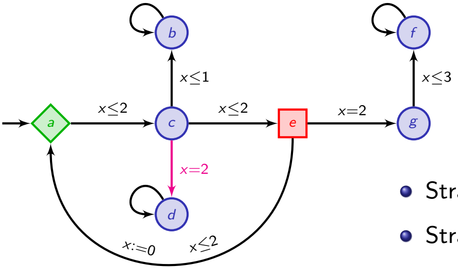

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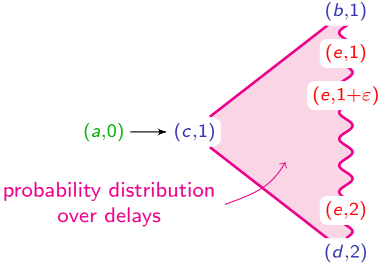

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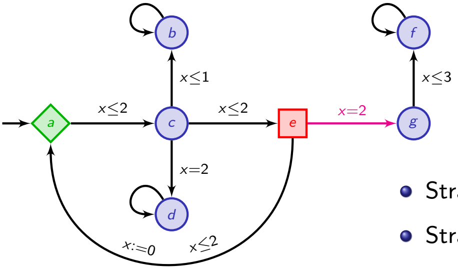

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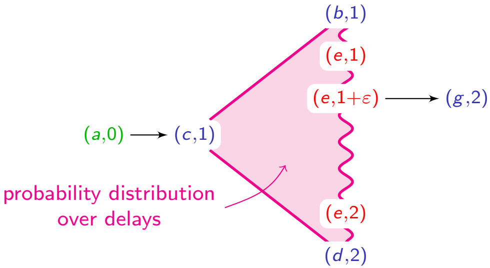

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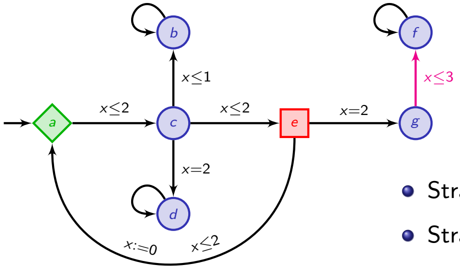

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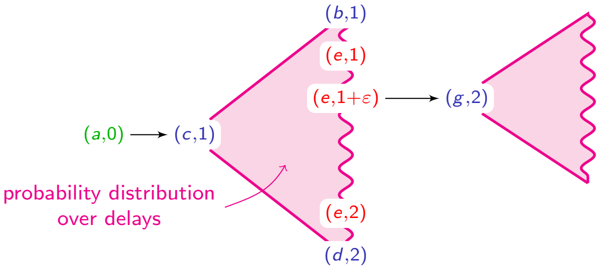

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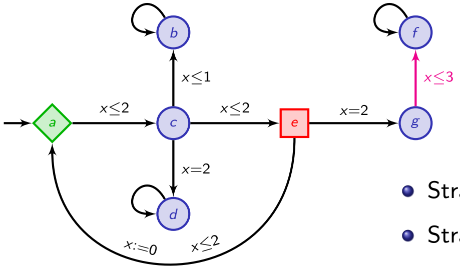

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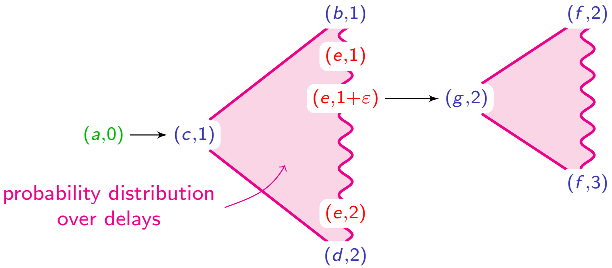
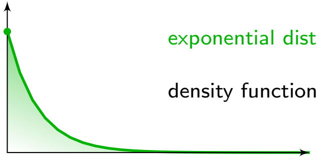


How can we attach probabilities to delays?

- The example of continuous-time Markov chains



exponential distribution

density function $t \mapsto \begin{cases} \lambda \cdot \exp(-\lambda t) & \text{if } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$

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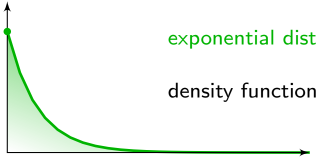

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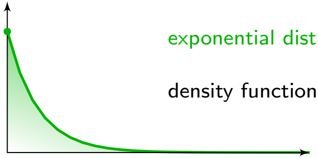

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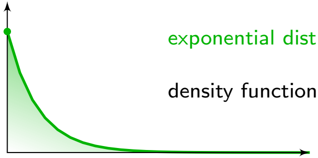

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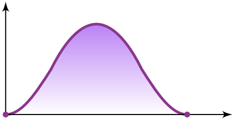

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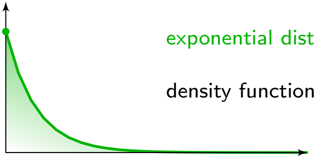

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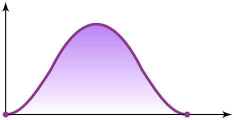

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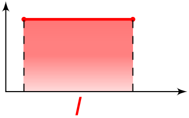

uniform distribution
density function $t \mapsto \begin{cases} \frac{1}{|I|} & \text{if } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$

The semantics for $\frac{1}{2}$-player games in a nutshell

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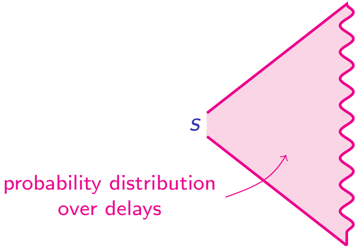

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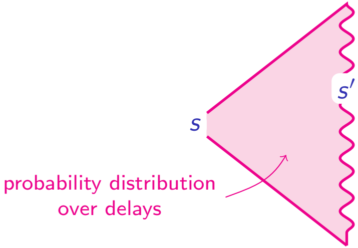

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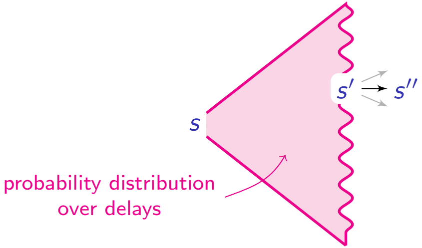

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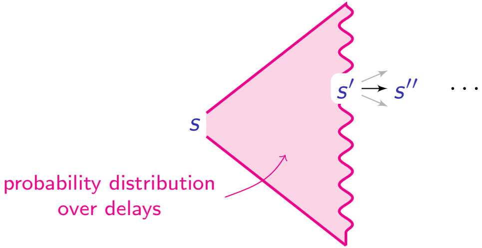

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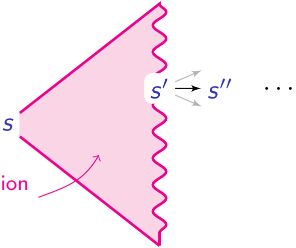

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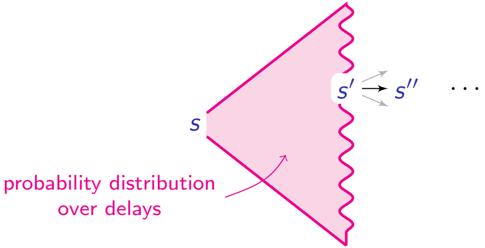

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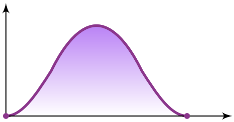

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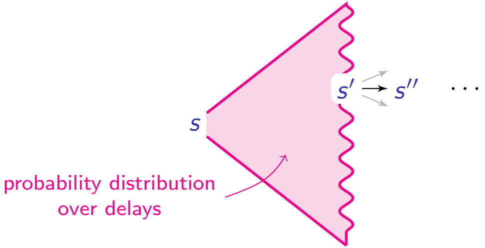

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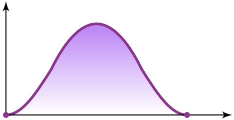

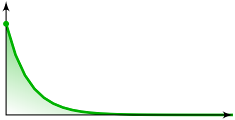

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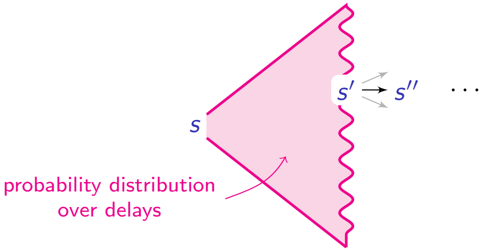

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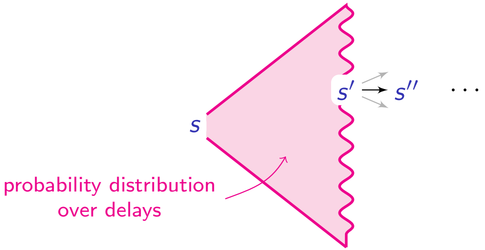

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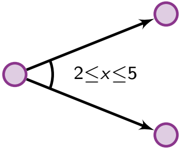
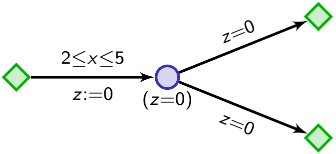
- Probabilistic timed automata = a subclass of the $1\frac{1}{2}$-player games





The synthesis problem

Problem statement

Given a game G, a (linear-time) property φ, a rational threshold $\bowtie r$,

is there a strategy $f_\Diamond$ for player $\Diamond$ s.t.

for all strategies $f_\Box$ of player $\Box$, $\mathbb{P}\left(G_{f_\Diamond, f_\Box} \models \varphi\right) \bowtie r$?

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Number of players

- $2\frac{1}{2}$-player games: $\Diamond$ $\square$ $\bigcirc$
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Number of players

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- Possibility to ask 'performance evaluation' questions [BHHK10]

[BHHK10] Baier, Hermanns, Haverkort, Katoen. Performance Evaluation and Model Checking Join Forces *(CACM)*.

Why model time-dependent resources?

- System resources might be relevant and even crucial information

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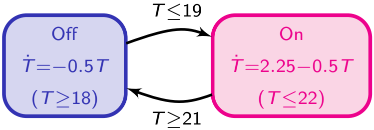

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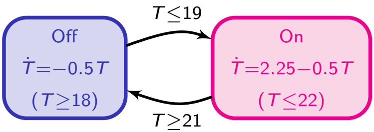

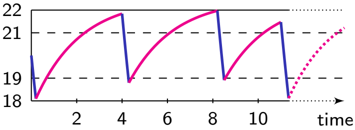

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Theorem [HKPV95]

The reachability problem is undecidable in hybrid automata.

[HKPV95] Henzinger, Kopke, Puri, Varaiya. What's decidable wbout hybrid automata? *(SToC'95)*.

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- An alternative: priced/weighted timed automata [ALP01,BFH+01]

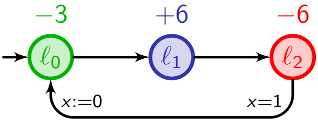
  $\rightsquigarrow$ hybrid variables are observer variables
  (they do not constrain *a priori* the system)

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
[BFH+01] Behrmann, Fehnker, Hune, Larsen, Pettersson, Romijn, Vaandrager. Minimum-cost reachability in priced timed automata *(HSCC'01)*.

A simple example of weighted timed automata (WTA)

[ALP01,BFH+01]

Example (with a linear observer)



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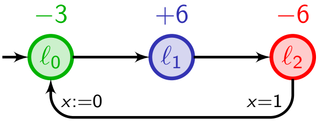

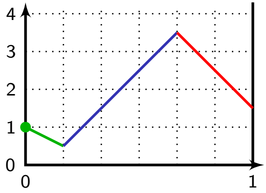

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The taskgraph scheduling example

Compute $D \times (C \times (A+B)) + (A+B) + (C \times D)$ using two processors:

P_1 (fast):

time	
$+$	2 picoseconds
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energy	
idle	10 Watt
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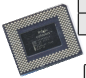
P_2 (slow):



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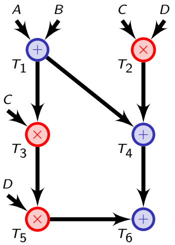

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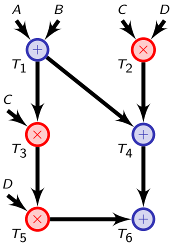

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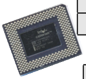

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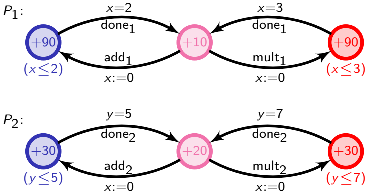
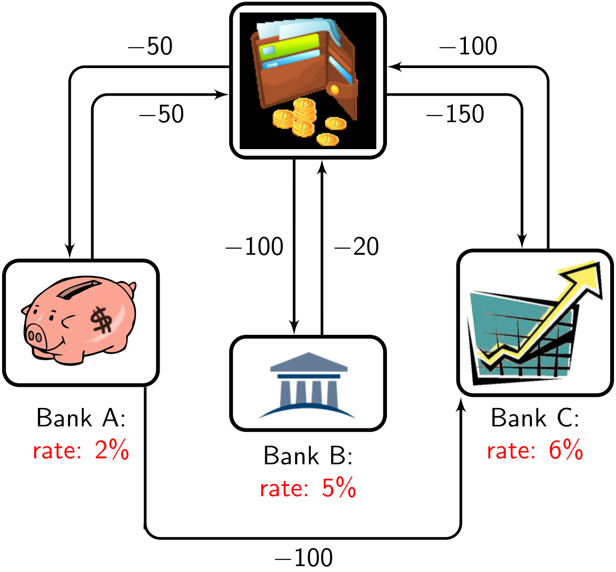


Beyond linear observers...



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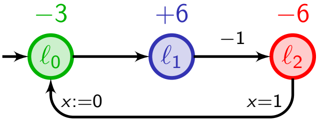

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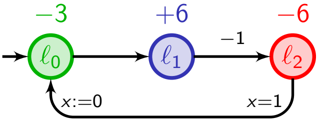

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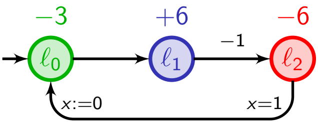

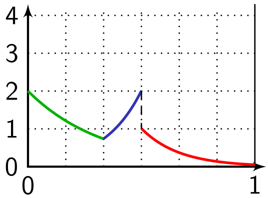

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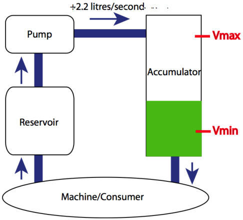
  ⤳ *An example:* an oil pump control system [CJL+09]



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[CJL+09] Cassez, Jessen, Larsen, Raskin, Reynier. Automatic synthesis of robust and optimal controllers - An industrial case study *(HSCC'09)*.

Conclusion

- Timed automata have been proven to be a convenient model for representing real-time systems
- However it is not expressive enough to faithfully represent some important features of systems
  - interaction with the environment (antagonistic, stochastic, cooperative...)
  - modelling of resources or energy
  - probabilities
- A number of extensions have been proposed to adequately represent such features (we can mix them)
  - The algorithmics of such systems is difficult (in general)
  - But a huge effort is put to develop methods for (approximate) verification