On Expressiveness and Complexity in Real-time Model Checking

Patricia Bouyer Nicolas Markey
LSV, CNRS, ENS Cachan, France

Joël Ouaknine James Worrell
Oxford University, UK

Outline

1. Introduction

2. The logic MTL

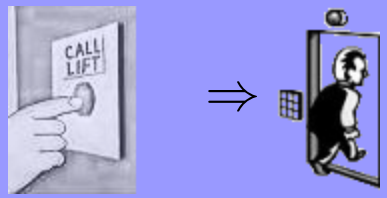
3. Towards tractable fragments of MTL

4. Conclusion

Model-checking

system:

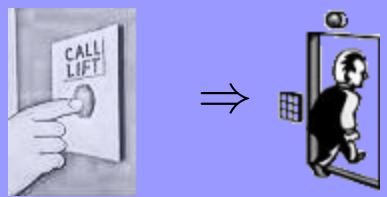
property:

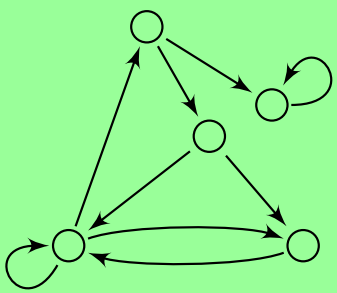


Model-checking

system:

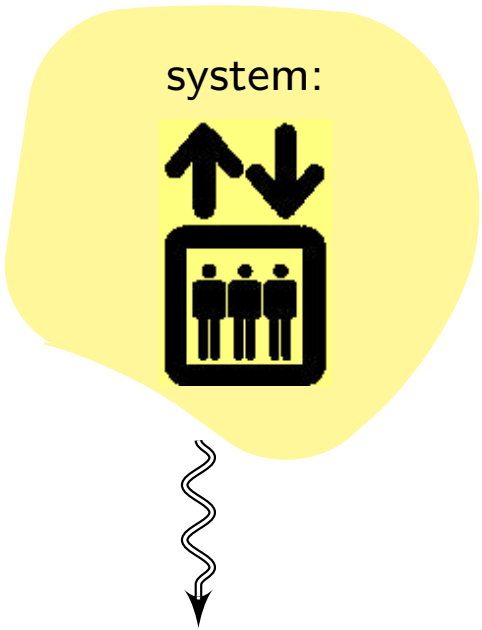
property:

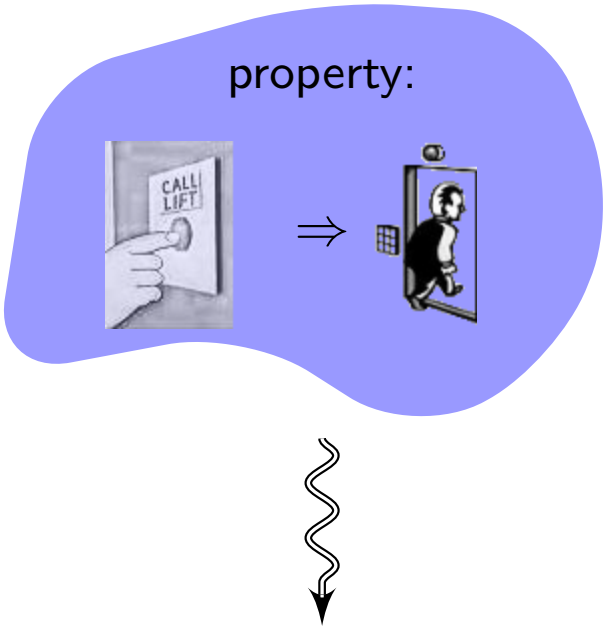


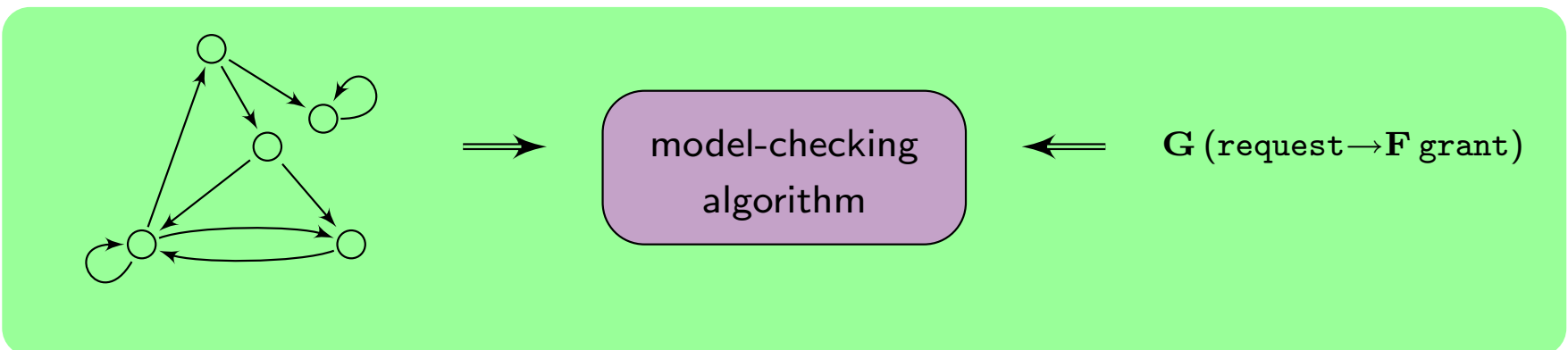


$$\mathbf{G}\,(\texttt{request} \rightarrow \mathbf{F}\,\texttt{grant})$$

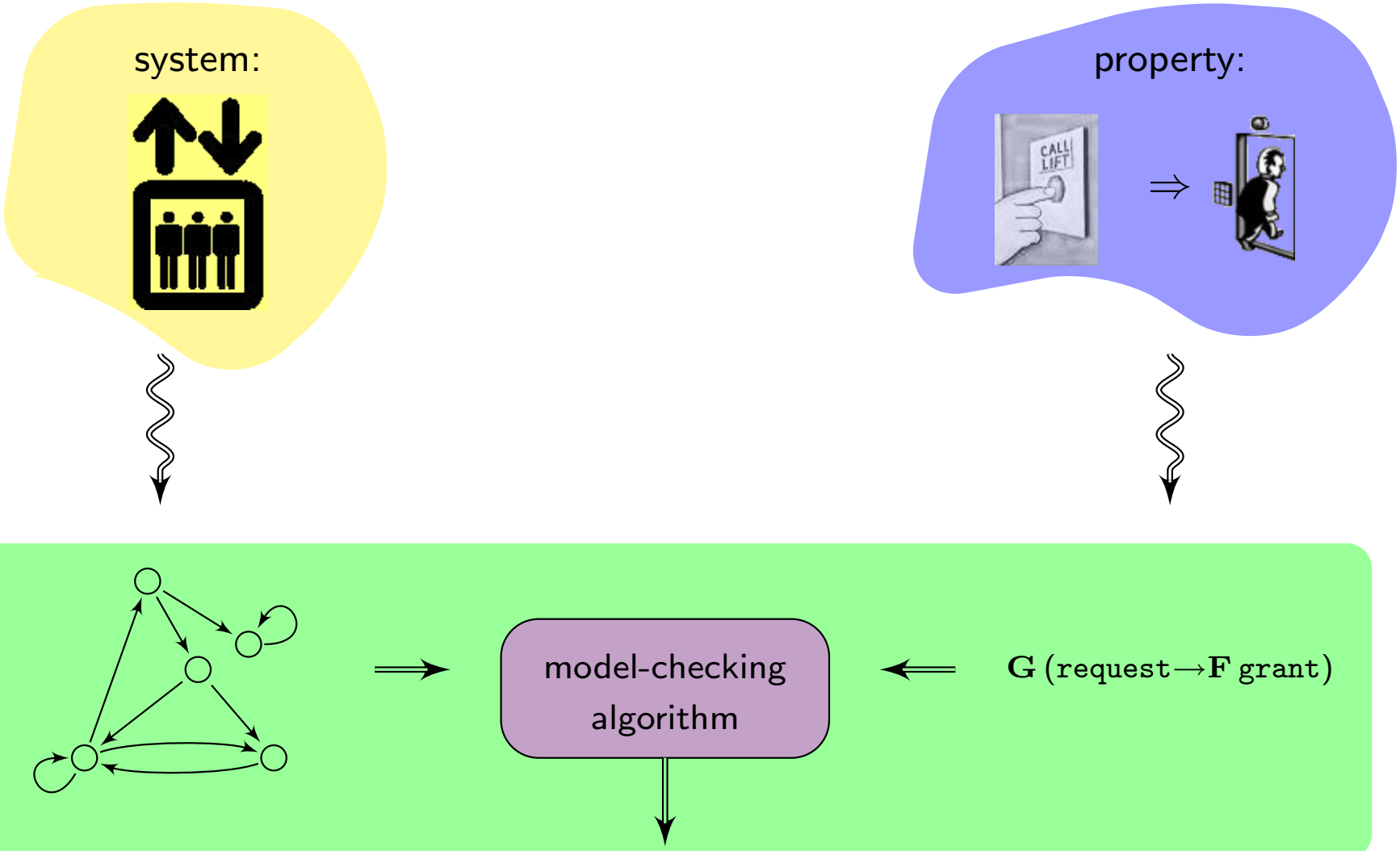
Model-checking







Model-checking



Towards real-time model-checking

- Classical theory:
  - Finite automata, temporal logics, etc...
  - Discrete model of time (ordered sequences of states/actions)
  - Theoretical foundations rather well understood

Towards real-time model-checking

- Classical theory:
  - Finite automata, temporal logics, etc...
  - Discrete model of time (ordered sequences of states/actions)
  - Theoretical foundations rather well understood

[Tra95]: "Lift the 'classical' [theory] to real-time systems."

[Tra95] Trakhtenbrot. Origins and metamorphoses of the Trinity: Logic, nets, automata (LICS, 1995).

Towards real-time model-checking

- Classical theory:
  - Finite automata, temporal logics, etc...
  - Discrete model of time (ordered sequences of states/actions)
  - Theoretical foundations rather well understood

[Tra95]: "Lift the 'classical' [theory] to real-time systems."

- Real-time theory:
  - Timed automata, timed temporal logics, etc...
  - Quantitative and dense model of time
  - Theoretical foundations under active development

[Tra95] Trakhtenbrot. Origins and metamorphoses of the Trinity: Logic, nets, automata (LICS, 1995).

The untimed (linear-time) framework

LTL: linear-time temporal logic [Pnu77]

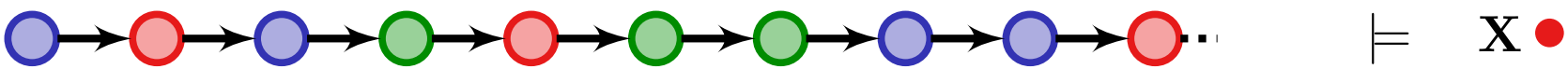
$$\text{LTL} \ni \varphi \quad ::= \quad \bullet \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \neg\varphi \mid \mathbf{X}\,\varphi \mid \varphi\,\mathbf{U}\,\varphi$$

[Pnu77] Pnueli. The temporal logic of programs (FOCS'77).

The untimed (linear-time) framework

LTL: linear-time temporal logic [Pnu77]

$$\text{LTL} \ni \varphi \quad ::= \quad \bullet \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \neg\varphi \mid \mathbf{X}\,\varphi \mid \varphi\,\mathbf{U}\,\varphi$$



[Pnu77] Pnueli. The temporal logic of programs (FOCS'77).

The untimed (linear-time) framework

LTL: linear-time temporal logic [Pnu77]

$$\text{LTL} \ni \varphi \quad ::= \quad \bullet \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \neg\varphi \mid \mathbf{X}\,\varphi \mid \varphi\,\mathbf{U}\,\varphi$$

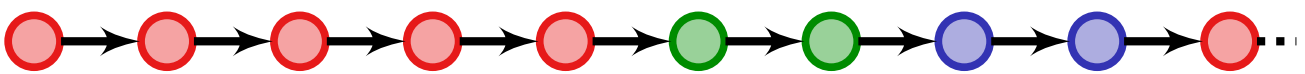
[Pnu77] Pnueli. The temporal logic of programs (FOCS'77).

The untimed (linear-time) framework

LTL: linear-time temporal logic [Pnu77]

$$\text{LTL} \ni \varphi \quad ::= \quad \bullet \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \neg\varphi \mid \mathbf{X}\,\varphi \mid \varphi\,\mathbf{U}\,\varphi$$

$\models \quad \mathbf{X} \bullet$



$\models \quad \bullet\,\mathbf{U}\,\bullet$

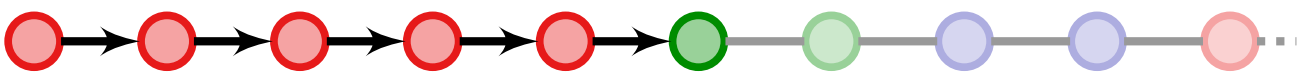
[Pnu77] Pnueli. The temporal logic of programs (FOCS'77).

The untimed (linear-time) framework

LTL: linear-time temporal logic [Pnu77]

$$\text{LTL} \ni \varphi \quad ::= \quad \bullet \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \neg\varphi \mid \mathbf{X}\,\varphi \mid \varphi\,\mathbf{U}\,\varphi$$

$\models \quad \mathbf{X} \bullet$



$\models \quad \bullet\,\mathbf{U}\,\bullet$

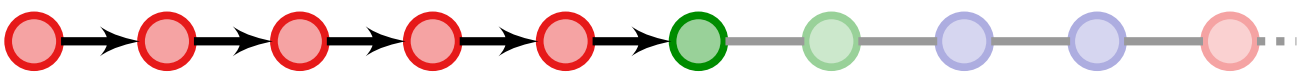
[Pnu77] Pnueli. The temporal logic of programs (FOCS'77).

The untimed (linear-time) framework

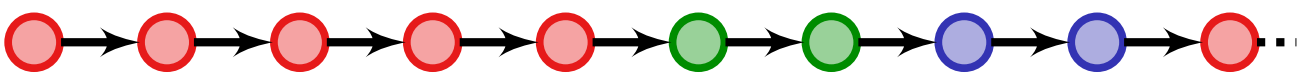
LTL: linear-time temporal logic [Pnu77]

$$\text{LTL} \ni \varphi \quad ::= \quad \bullet \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \neg\varphi \mid \mathbf{X}\,\varphi \mid \varphi\,\mathbf{U}\,\varphi$$

$\models \quad \mathbf{X}\bullet$



$\models \quad \bullet\,\mathbf{U}\,\bullet$



$\models \quad \mathbf{F}\bullet \equiv \mathtt{tt}\,\mathbf{U}\,\bullet$

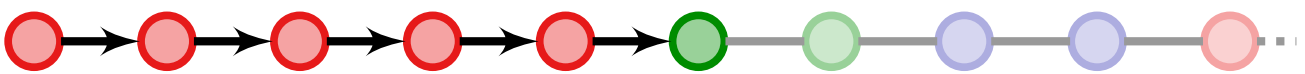
[Pnu77] Pnueli. The temporal logic of programs (FOCS'77).

The untimed (linear-time) framework

LTL: linear-time temporal logic [Pnu77]

$$\text{LTL} \ni \varphi \quad ::= \quad \bullet \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \neg\varphi \mid \mathbf{X}\,\varphi \mid \varphi\,\mathbf{U}\,\varphi$$

$\models \quad \mathbf{X}\bullet$



$\models \quad \bullet\,\mathbf{U}\,\bullet$

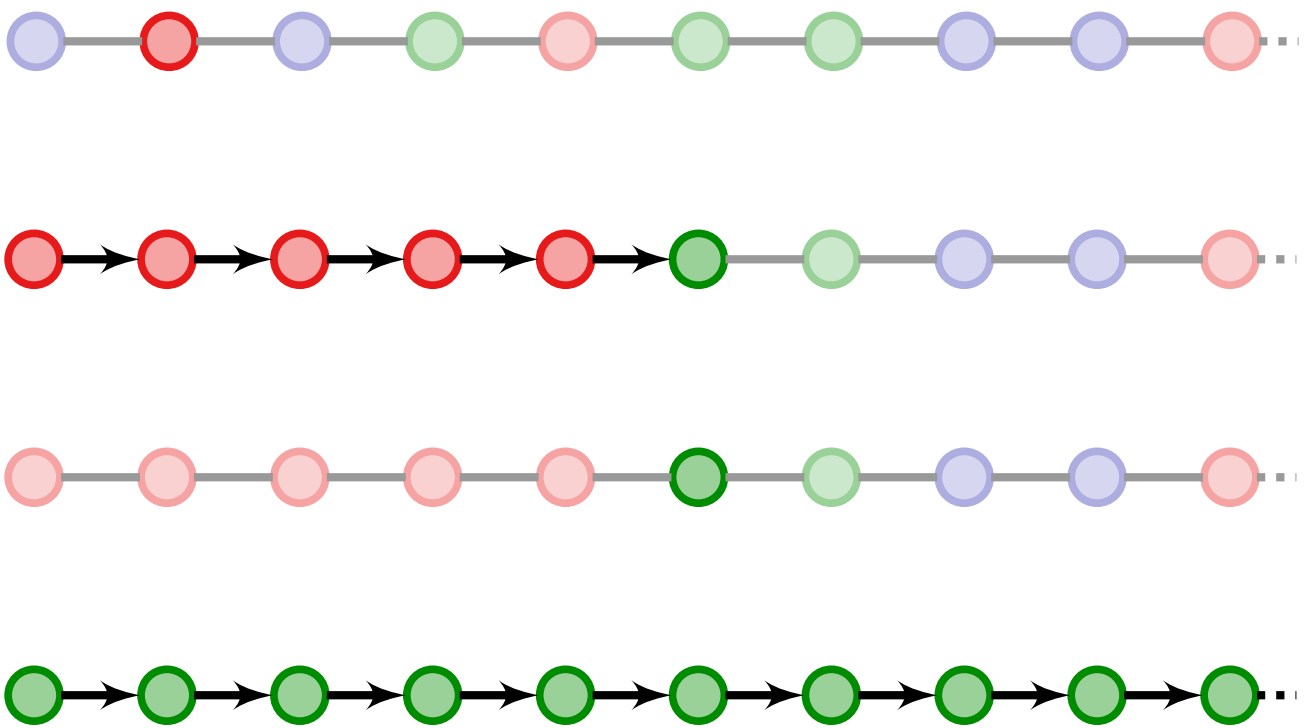
$\models \quad \mathbf{F}\bullet \equiv \mathtt{tt}\,\mathbf{U}\,\bullet$

[Pnu77] Pnueli. The temporal logic of programs (FOCS'77).

The untimed (linear-time) framework

LTL: linear-time temporal logic [Pnu77]

$$\text{LTL} \ni \varphi \quad ::= \quad \bullet \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \neg\varphi \mid \mathbf{X}\,\varphi \mid \varphi\,\mathbf{U}\,\varphi$$



[Pnu77] Pnueli. The temporal logic of programs (FOCS'77).

The untimed (linear-time) framework

LTL: linear-time temporal logic [Pnu77]

$$\text{LTL} \ni \varphi \quad ::= \quad \bullet \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \neg\varphi \mid \mathbf{X}\,\varphi \mid \varphi\,\mathbf{U}\,\varphi$$

- response property:

$$\mathbf{G}\,(\bullet \rightarrow \mathbf{F}\,\bullet)$$

[Pnu77] Pnueli. The temporal logic of programs (FOCS'77).

The untimed (linear-time) framework

LTL: linear-time temporal logic [Pnu77]

$$\text{LTL} \ni \varphi \quad ::= \quad \bullet \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \neg\varphi \mid \mathbf{X}\,\varphi \mid \varphi\,\mathbf{U}\,\varphi$$

- response property:

$$\mathbf{G}\,(\bullet \rightarrow \mathbf{F}\,\bullet)$$

- liveness property:

$$\mathbf{G}\,\mathbf{F}\,\bullet$$

[Pnu77] Pnueli. The temporal logic of programs (FOCS'77).

The untimed (linear-time) framework

LTL: linear-time temporal logic [Pnu77]

$$\text{LTL} \ni \varphi \quad ::= \quad \bullet \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \neg\varphi \mid \mathbf{X}\,\varphi \mid \varphi\,\mathbf{U}\,\varphi$$

- response property:

$$\mathbf{G}\,(\bullet \rightarrow \mathbf{F}\,\bullet)$$

- liveness property:

$$\mathbf{G}\,\mathbf{F}\,\bullet$$

- safety property:

$$\mathbf{G}\,\neg\bullet$$

[Pnu77] Pnueli. The temporal logic of programs (FOCS'77).

The untimed (linear-time) framework

LTL: linear-time temporal logic [Pnu77]

$$\text{LTL} \ni \varphi \quad ::= \quad \bullet \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \neg\varphi \mid \mathbf{X}\,\varphi \mid \varphi\,\mathbf{U}\,\varphi$$

- response property:

$$\mathbf{G}\,(\bullet \rightarrow \mathbf{F}\,\bullet)$$

- liveness property:

$$\mathbf{G}\,\mathbf{F}\,\bullet$$

- safety property:

$$\mathbf{G}\,\neg\bullet$$

- a more complex property:

$$(\bullet \wedge (\mathbf{F}\,\bullet \vee \mathbf{G}\,\bullet))\,\mathbf{U}\,\bullet$$

[Pnu77] Pnueli. The temporal logic of programs (FOCS'77).

The complexity of LTL

- The satisfiability problem:
  - *Input:* an LTL formula φ
  - *Output:* is there a model of φ? If yes, describe one.

[SC85] Sistla, Clarke. The complexity of propositional linear temporal logics (JACM'85).

The complexity of LTL

- The satisfiability problem:
  - *Input:* an LTL formula φ
  - *Output:* is there a model of φ? If yes, describe one.
- The model-checking problem:
  - *Input:* a finite automaton $\mathcal{A}$, an LTL formula φ
  - *Output:* does $\mathcal{A}$ satisfy φ?
    (do all behaviours of $\mathcal{A}$ satisfy φ?)

[SC85] Sistla, Clarke. The complexity of propositional linear temporal logics (JACM'85).

The complexity of LTL

- The satisfiability problem:
  - *Input:* an LTL formula φ
  - *Output:* is there a model of φ? If yes, describe one.
- The model-checking problem:
  - *Input:* a finite automaton $\mathcal{A}$, an LTL formula φ
  - *Output:* does $\mathcal{A}$ satisfy φ?
    (do all behaviours of $\mathcal{A}$ satisfy φ?)

Theorem [SC85]

These two problems are PSPACE-complete.

[SC85] Sistla, Clarke. The complexity of propositional linear temporal logics (JACM'85).

The complexity of LTL

- The satisfiability problem:
  - *Input:* an LTL formula φ
  - *Output:* is there a model of φ? If yes, describe one.
- The model-checking problem:
  - *Input:* a finite automaton $\mathcal{A}$, an LTL formula φ
  - *Output:* does $\mathcal{A}$ satisfy φ?
    (do all behaviours of $\mathcal{A}$ satisfy φ?)

Theorem [SC85]

These two problems are PSPACE-complete.

☞ it's time to extend to quantitative specifications

[SC85] Sistla, Clarke. The complexity of propositional linear temporal logics (JACM'85).

Outline

1. Introduction

2. The logic MTL

3. Towards tractable fragments of MTL

4. Conclusion

Metric temporal logic (MTL)

[Koy90]

$$\text{MTL} \ni \varphi \quad ::= \quad \bullet \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \varphi \, \mathbf{U}_I \, \varphi$$

where I is an interval with integral bounds.

[Koy90] Koymans. Specifying real-time properties with metric temporal logic (Real-time systems, 1990).

Metric temporal logic (MTL)

[Koy90]

$$\text{MTL} \ni \varphi \quad ::= \quad \bullet \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \varphi \, \mathbf{U}_I \, \varphi$$

where I is an interval with integral bounds.

- This is a timed extension of LTL

[Koy90] Koymans. Specifying real-time properties with metric temporal logic (Real-time systems, 1990).

Metric temporal logic (MTL)

[Koy90]

$$\text{MTL} \ni \varphi \quad ::= \quad \bullet \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \varphi \, \mathbf{U}_I \, \varphi$$

where I is an interval with integral bounds.

- This is a timed extension of LTL
- There are several semantics for MTL

[Koy90] Koymans. Specifying real-time properties with metric temporal logic (Real-time systems, 1990).

Metric temporal logic (MTL)

[Koy90]

$$\text{MTL} \ni \varphi \quad ::= \quad \bullet \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \varphi \, \mathbf{U}_I \, \varphi$$

where I is an interval with integral bounds.

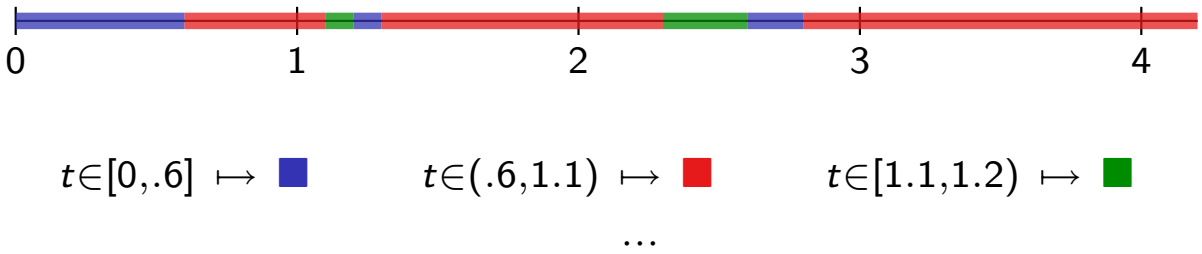
- This is a timed extension of LTL
- There are several semantics for MTL

☞ we focus on the continuous semantics...

[Koy90] Koymans. Specifying real-time properties with metric temporal logic (Real-time systems, 1990).

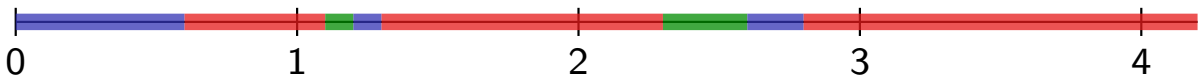
The semantics of MTL

MTL formulas are interpreted over signals:



The semantics of MTL

MTL formulas are interpreted over signals:



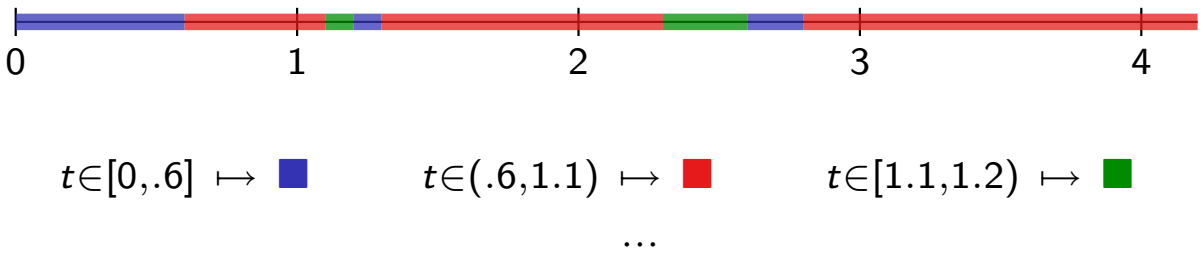
$t \in [0,.6] \mapsto$ ■ $t \in (.6,1.1) \mapsto$ ■ $t \in [1.1,1.2) \mapsto$ ■

...

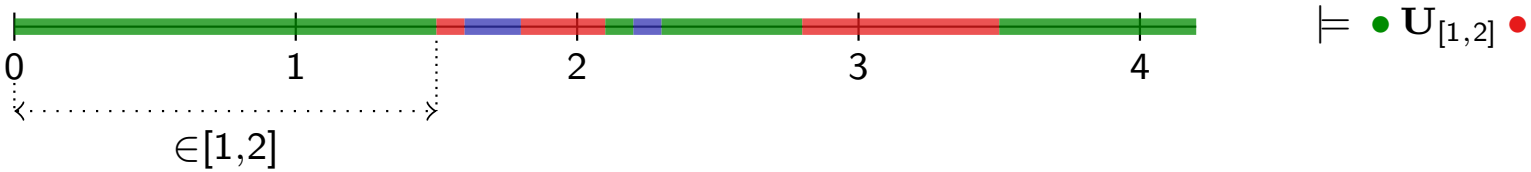
☞ the system is observed continuously

The semantics of MTL

MTL formulas are interpreted over signals:

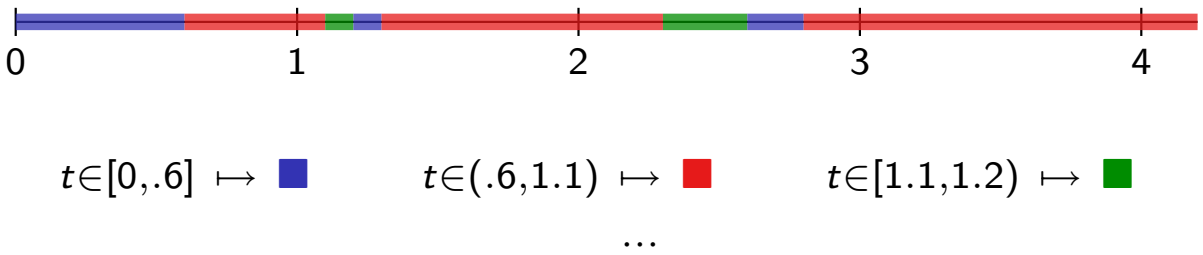


☞ the system is observed continuously

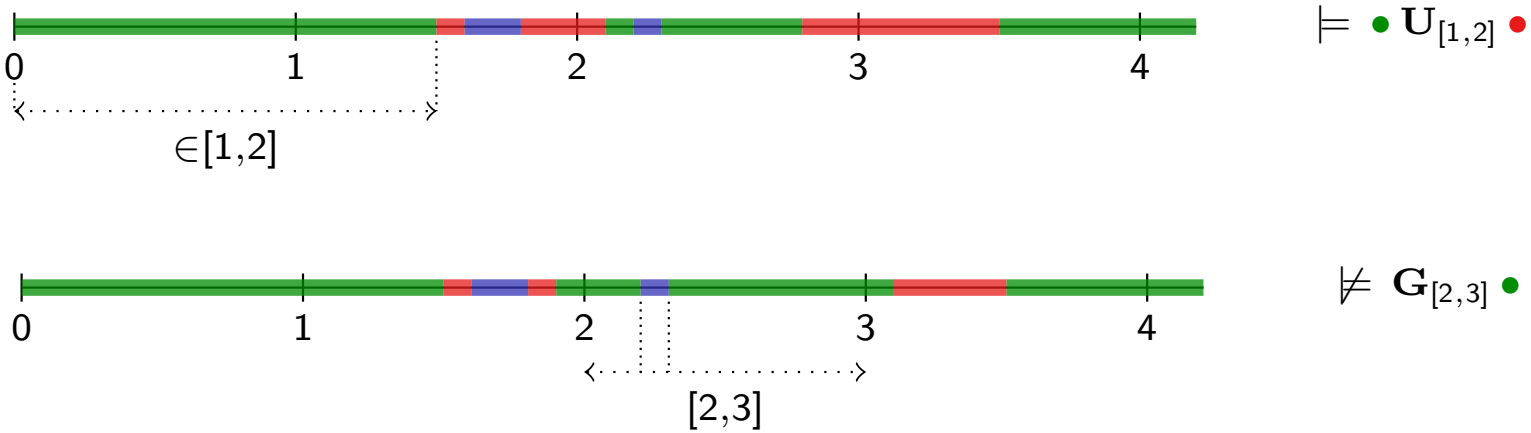


The semantics of MTL

MTL formulas are interpreted over signals:



☞ the system is observed continuously



The decidability of MTL

- The satisfiability problem:
  - *Input:* an MTL formula φ
  - *Output:* is there a model of φ? If yes, describe one.

The decidability of MTL

- The satisfiability problem:
  - *Input:* an MTL formula φ
  - *Output:* is there a model of φ? If yes, describe one.
- The model-checking problem:
  - *Input:* a timed automaton $\mathcal{A}$, an MTL formula φ
  - *Output:* does $\mathcal{A}$ satisfy φ?

The decidability of MTL

- The satisfiability problem:
  - *Input:* an MTL formula φ
  - *Output:* is there a model of φ? If yes, describe one.
- The model-checking problem:
  - *Input:* a timed automaton $\mathcal{A}$, an MTL formula φ
  - *Output:* does $\mathcal{A}$ satisfy φ?

Theorem [AH92,OW05,OW06]

The model-checking and the satisfiability problems for MTL is undecidable for almost all semantics, except for the most restrictive one (in which case, it is non-primitive recursive...).

[AH92] Alur, Henzinger. Logics and models of real-time: A survey (REX'91).
[OW05] Ouaknine, Worrell. On the decidability of metric temporal logic (LICS'05).
[OW06] Ouaknine, Worrell. On metric temporal logic and faulty Turing machines (FoSSaCS'06).

The decidability of MTL

- The satisfiability problem:
  - *Input:* an MTL formula φ
  - *Output:* is there a model of φ? If yes, describe one.
- The model-checking problem:
  - *Input:* a timed automaton $\mathcal{A}$, an MTL formula φ
  - *Output:* does $\mathcal{A}$ satisfy φ?

Theorem [AH92,OW05,OW06]

The model-checking and the satisfiability problems for MTL is undecidable for almost all semantics, except for the most restrictive one (in which case, it is non-primitive recursive...).

☞ a quest to more tractable fragments of MTL

[AH92] Alur, Henzinger. Logics and models of real-time: A survey (REX'91).
[OW05] Ouaknine, Worrell. On the decidability of metric temporal logic (LICS'05).
[OW06] Ouaknine, Worrell. On metric temporal logic and faulty Turing machines (FoSSaCS'06).

The decidability of MTL

- The satisfiability problem:
  - *Input:* an MTL formula φ
  - *Output:* is there a model of φ? If yes, describe one.
- The model-checking problem:
  - *Input:* a timed automaton $\mathcal{A}$, an MTL formula φ
  - *Output:* does $\mathcal{A}$ satisfy φ?

Theorem [AH92,OW05,OW06]

The model-checking and the satisfiability problems for MTL is undecidable for almost all semantics, except for the most restrictive one (in which case, it is non-primitive recursive...).

☞ a quest to more tractable fragments of MTL

ban punctuality?

[AH92] Alur, Henzinger. Logics and models of real-time: A survey (REX'91).
[OW05] Ouaknine, Worrell. On the decidability of metric temporal logic (LICS'05).
[OW06] Ouaknine, Worrell. On metric temporal logic and faulty Turing machines (FoSSaCS'06).

The decidability of MTL

- The satisfiability problem:
  - *Input:* an MTL formula φ
  - *Output:* is there a model of φ? If yes, describe one.
- The model-checking problem:
  - *Input:* a timed automaton $\mathcal{A}$, an MTL formula φ
  - *Output:* does $\mathcal{A}$ satisfy φ?

Theorem [AH92,OW05,OW06]

The model-checking and the satisfiability problems for MTL is undecidable for almost all semantics, except for the most restrictive one (in which case, it is non-primitive recursive...).

☞ a quest to more tractable fragments of MTL

{ ban punctuality?
{ or not ban punctuality?

[AH92] Alur, Henzinger. Logics and models of real-time: A survey (REX'91).
[OW05] Ouaknine, Worrell. On the decidability of metric temporal logic (LICS'05).
[OW06] Ouaknine, Worrell. On metric temporal logic and faulty Turing machines (FoSSaCS'06).

Outline

1. Introduction

2. The logic MTL

3. Towards tractable fragments of MTL

4. Conclusion

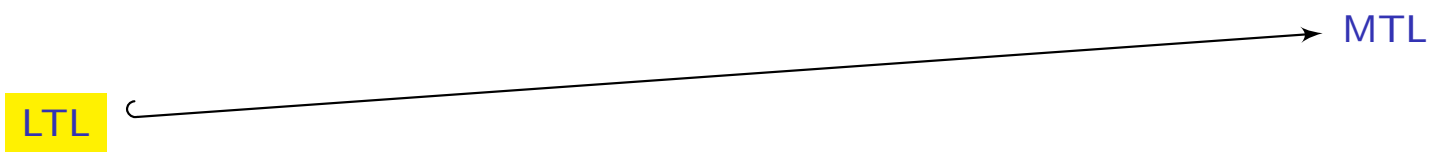
Interesting fragments of MTL

$$\text{MTL} \ni \varphi \ ::= \ \bullet \ \mid \ \neg\bullet \ \mid \ \varphi \vee \varphi \ \mid \ \varphi \wedge \varphi \ \mid \ \varphi \, \mathbf{U}_I \, \varphi \ \mid \ \varphi \, \widetilde{\mathbf{U}}_I \, \varphi$$

MTL

Interesting fragments of MTL

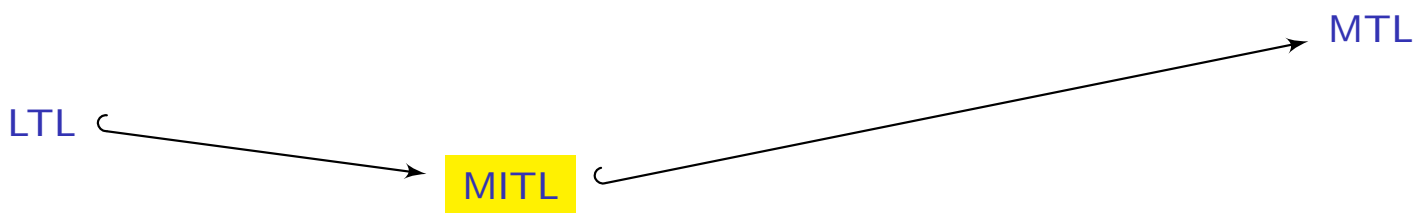
$$\text{LTL} \ni \varphi \ ::= \ \bullet \ \mid \ \neg\bullet \ \mid \ \varphi \vee \varphi \ \mid \ \varphi \wedge \varphi \ \mid \ \varphi \,\mathbf{U}\, \varphi \ \mid \ \varphi \,\widetilde{\mathbf{U}}\, \varphi$$



Interesting fragments of MTL

$$\text{MITL} \ni \varphi \ ::= \ \bullet \ \mid \ \neg\bullet \ \mid \ \varphi \vee \varphi \ \mid \ \varphi \wedge \varphi \ \mid \ \varphi \, \mathbf{U}_I \, \varphi \ \mid \ \varphi \, \widetilde{\mathbf{U}}_I \, \varphi$$

with I non-singular, *i.e.*, with no "punctuality"

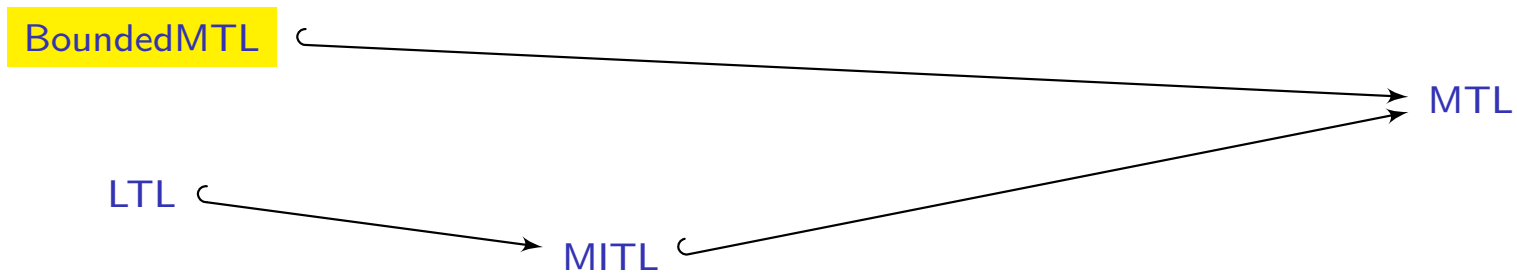


[AFH96] Alur, Feder Henzinger. The benefits of relaxing punctuality (JACM'96).

Interesting fragments of MTL

$$\text{BoundedMTL} \ni \varphi ::= \bullet \mid \neg\bullet \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \varphi \,\mathbf{U}_I\, \varphi \mid \varphi \,\widetilde{\mathbf{U}}_I\, \varphi$$

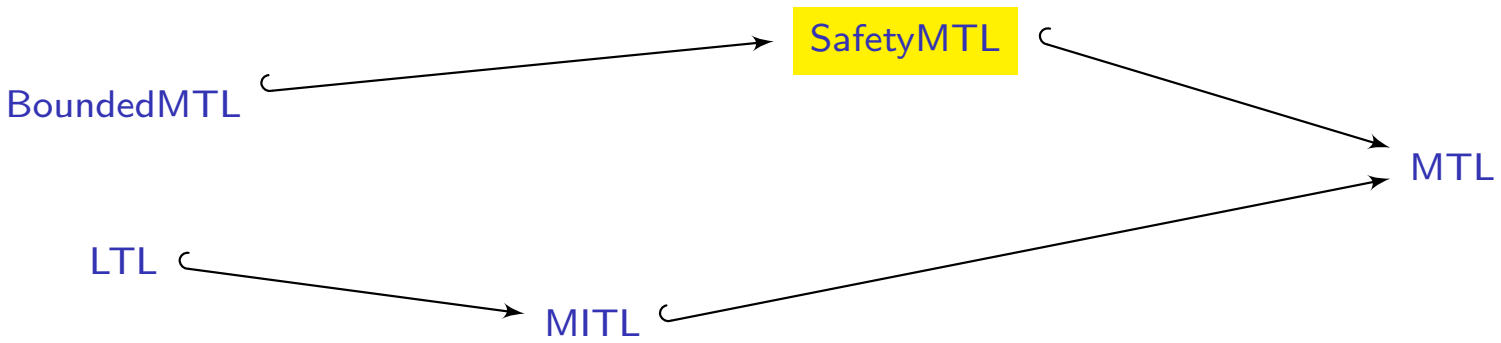
with I bounded



Interesting fragments of MTL

$$\text{SafetyMTL} \ni \varphi \ ::= \ \bullet \ \mid \ \neg\bullet \ \mid \ \varphi \vee \varphi \ \mid \ \varphi \wedge \varphi \ \mid \ \varphi \, \mathbf{U}_J \, \varphi \ \mid \ \varphi \, \widetilde{\mathbf{U}}_I \, \varphi$$

with J bounded



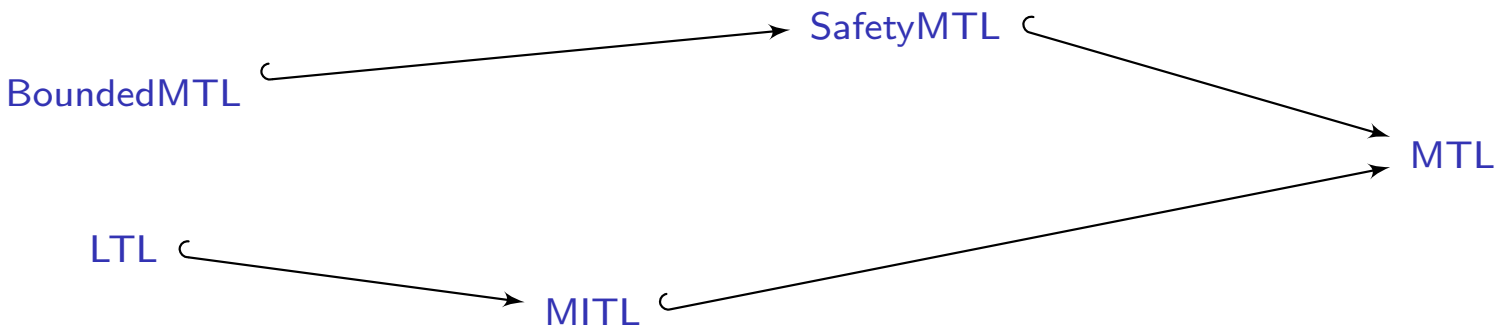
$$\text{BoundedMTL} + \text{Invariance} \subseteq \text{SafetyMTL}$$

[OW05] Ouaknine, Worrell. On the decidability of metric temporal logic (LICS'05).

Interesting fragments of MTL

$$\text{FlatMTL} \ni \varphi \ ::= \ \bullet \ \mid \ \neg\bullet \ \mid \ \varphi \vee \varphi \ \mid \ \varphi \wedge \varphi \ \mid \ \psi \, \mathbf{U}_I \, \varphi \ \mid \ \varphi \, \widetilde{\mathbf{U}}_I \, \psi$$

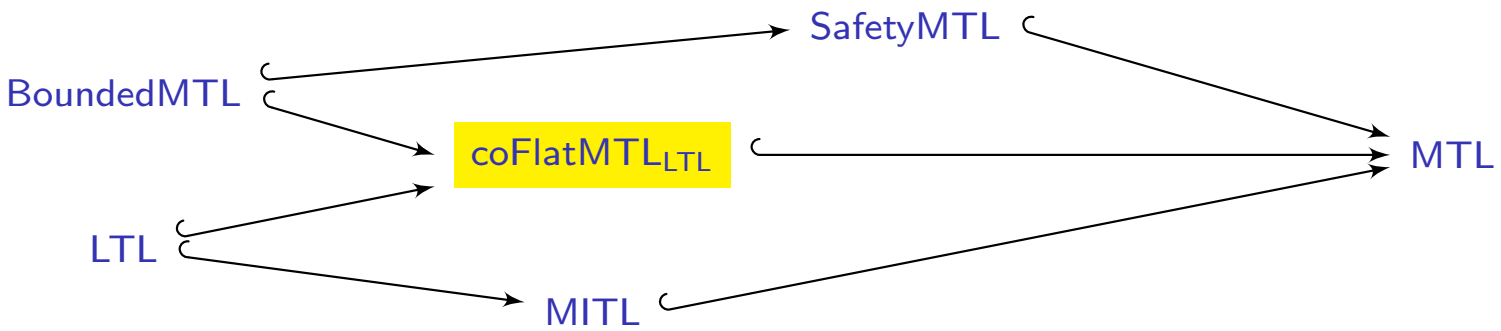
with I unbounded $\Rightarrow \psi \in \text{LTL}$



Interesting fragments of MTL

$$\text{coFlatMTL}_{\text{LTL}} \ni \varphi ::= \bullet \mid \neg\bullet \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \varphi \,\mathbf{U}_I\, \psi \mid \psi \,\widetilde{\mathbf{U}}_I\, \varphi$$

with I unbounded $\Rightarrow \psi \in \text{LTL}$



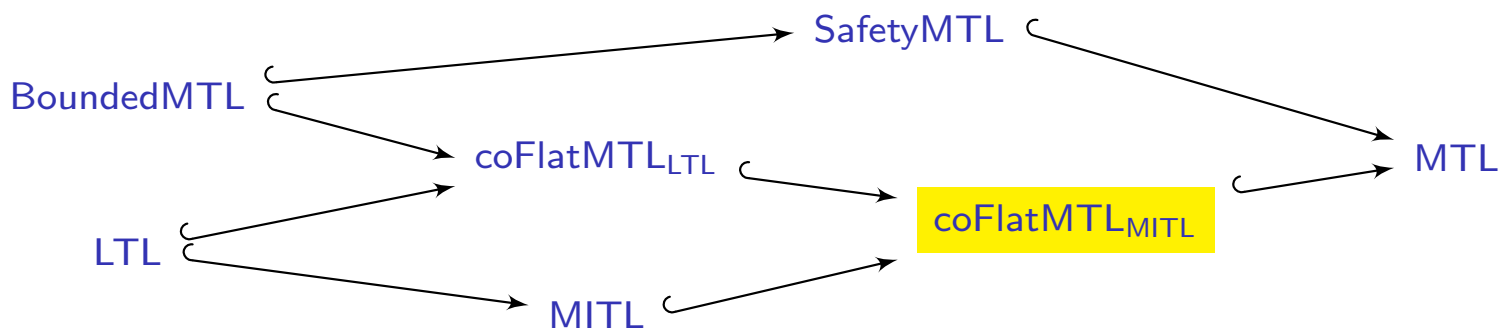
$$\text{BoundedMTL} + \text{Invariance} \subseteq \text{coFlatMTL}_{\text{LTL}}$$

[BMOW07] Bouyer, Markey, Ouaknine, Worrell. The cost of punctuality (LICS'07).

Interesting fragments of MTL

$$\mathsf{coFlatMTL}_{\mathsf{MITL}} \ni \varphi ::= \bullet \mid \neg\bullet \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \varphi \,\mathbf{U}_I\, \psi \mid \psi \,\widetilde{\mathbf{U}}_I\, \varphi$$

with I unbounded $\Rightarrow \psi \in \mathsf{MITL}$



This talk!

What can we express?

- MITL *vs* BoundedMTL

What can we express?

- MITL *vs* BoundedMTL
  - A dual point-of-view:

What can we express?

- MITL *vs* BoundedMTL
  - A dual point-of-view:

    MITL: ban small intervals (*no punctuality!*)

What can we express?

- MITL *vs* BoundedMTL
  - A dual point-of-view:

    MITL: ban small intervals (*no punctuality!*)

    BoundedMTL: ban large intervals

What can we express?

- MITL *vs* BoundedMTL
  - A dual point-of-view:

    MITL: ban small intervals (*no punctuality!*)

    BoundedMTL: ban large intervals
  - MITL defines regular languages [AFH96]

What can we express?

- MITL *vs* BoundedMTL
  - A dual point-of-view:

    MITL: ban small intervals (*no punctuality!*)

    BoundedMTL: ban large intervals
  - MITL defines regular languages [AFH96]
    The formula $\mathbf{G}_{(0,1)}\,(\bullet \rightarrow \mathbf{F}_{=1}\,\bullet)$ is in BoundedMTL and defines the non-regular language $\{\bullet^m\bullet^n \mid m \leq n\}$.

What can we express?

- MITL *vs* BoundedMTL
  - A dual point-of-view:

    MITL: ban small intervals (*no punctuality!*)

    BoundedMTL: ban large intervals
  - MITL defines regular languages [AFH96]
    The formula $\mathbf{G}_{(0,1)}\,(\bullet \rightarrow \mathbf{F}_{=1}\,\bullet)$ is in BoundedMTL and defines the non-regular language $\{\bullet^m\bullet^n \mid m \leq n\}$.
  - What is missing?

What can we express?

- MITL *vs* BoundedMTL
  - A dual point-of-view:

    MITL: ban small intervals (*no punctuality!*)

    BoundedMTL: ban large intervals
  - MITL defines regular languages [AFH96]
    The formula $\mathbf{G}_{(0,1)}\,(\bullet \rightarrow \mathbf{F}_{=1}\,\bullet)$ is in BoundedMTL and defines the non-regular language $\{\bullet^m\bullet^n \mid m \leq n\}$.
  - What is missing?

    MITL: punctuality ☺

    (is punctuality really to be banned?)

What can we express?

- MITL *vs* BoundedMTL
  - A dual point-of-view:

    MITL: ban small intervals (*no punctuality!*)

    BoundedMTL: ban large intervals
  - MITL defines regular languages [AFH96]
    The formula $\mathbf{G}_{(0,1)}\,(\bullet \rightarrow \mathbf{F}_{=1}\,\bullet)$ is in BoundedMTL and defines the non-regular language $\{\bullet^m\bullet^n \mid m \leq n\}$.
  - What is missing?

    MITL: punctuality ☺

    (is punctuality really to be banned?)

    BoundedMTL: global invariance

What can we express?

- MITL *vs* BoundedMTL
  - A dual point-of-view:
    MITL: ban small intervals (*no punctuality!*)
    BoundedMTL: ban large intervals
  - MITL defines regular languages [AFH96]
    The formula $\mathbf{G}_{(0,1)}\,(\bullet \rightarrow \mathbf{F}_{=1}\,\bullet)$ is in BoundedMTL and defines the non-regular language $\{\bullet^m\bullet^n \mid m \leq n\}$.
  - What is missing?
    MITL: punctuality ☺
    (is punctuality really to be banned?)
    BoundedMTL: global invariance

- What's the point with $\text{coFlatMTL}_{\text{MITL}}$?

What can we express?

- MITL *vs* BoundedMTL
  - A dual point-of-view:
    MITL: ban small intervals (*no punctuality!*)
    BoundedMTL: ban large intervals
  - MITL defines regular languages [AFH96]
    The formula $\mathbf{G}_{(0,1)}\,(\bullet \rightarrow \mathbf{F}_{=1}\,\bullet)$ is in BoundedMTL and defines the non-regular language $\{\bullet^m\bullet^n \mid m \leq n\}$.
  - What is missing?
    MITL: punctuality ☺
    (is punctuality really to be banned?)
    BoundedMTL: global invariance

- What's the point with $\text{coFlatMTL}_{\text{MITL}}$?
  - Includes both MITL and BoundedMTL

What can we express?

- MITL *vs* BoundedMTL
  - A dual point-of-view:
    MITL: ban small intervals (*no punctuality!*)
    BoundedMTL: ban large intervals
  - MITL defines regular languages [AFH96]
    The formula $\mathbf{G}_{(0,1)}\,(\bullet \rightarrow \mathbf{F}_{=1}\,\bullet)$ is in BoundedMTL and defines the non-regular language $\{\bullet^m\bullet^n \mid m \leq n\}$.
  - What is missing?
    MITL: punctuality ☺
    (is punctuality really to be banned?)
    BoundedMTL: global invariance

- What's the point with $\text{coFlatMTL}_{\text{MITL}}$?
  - Includes both MITL and BoundedMTL
  - Allows to express global invariance, bounded response, some punctuality

What can we express?

- MITL *vs* BoundedMTL
  - A dual point-of-view:
    MITL: ban small intervals (*no punctuality!*)
    BoundedMTL: ban large intervals
  - MITL defines regular languages [AFH96]
    The formula $\mathbf{G}_{(0,1)}\,(\bullet \rightarrow \mathbf{F}_{=1}\,\bullet)$ is in BoundedMTL and defines the non-regular language $\{\bullet^m\bullet^n \mid m \leq n\}$.
  - What is missing?
    MITL: punctuality ☺
    (is punctuality really to be banned?)
    BoundedMTL: global invariance

- What's the point with $\text{coFlatMTL}_{\text{MITL}}$?
  - Includes both MITL and BoundedMTL
  - Allows to express global invariance, bounded response, some punctuality
  - For instance, the formula

$$\mathbf{G}\,(\bullet \rightarrow \mathbf{F}_{=1}\,\bullet)$$

is in $\text{coFlatMTL}_{\text{MITL}}$, but neither in MITL, nor in BoundedMTL

Complexity results for the model-checking problem

constants:	binary encoding	unary encoding
LTL	PSPACE-complete [SC85]	
MITL	EXPSPACE-complete [AFH96]	PSPACE-complete [HR05]
BoundedMTL	EXPSPACE-complete [BMOW07]	
$\text{coFlatMTL}_{\text{LTL}}$	EXPSPACE-complete [BMOW07]	
SafetyMTL	decidable, NPR [OW05]	
$\text{coFlatMTL}_{\text{MITL}}$		

[SC85] Sistla, Clarke. The complexity of propositional linear temporal logics (JACM'85).
[AFH96] Alur, Feder, Henzinger. The benefits of relaxing punctuality (JACM'96).
[HR05] Hirshfeld, Rabinovich. Timed formulas and decidable metric temporal logic (I&C'05).
[BMOW07] Bouyer, Markey, Ouaknine, Worrell. The cost of punctuality (LICS'07).
[OW05] Ouaknine, Worrell. On the decidability of metric temporal logic (LICS'05).

Complexity results for the model-checking problem

constants:	binary encoding	unary encoding
LTL	PSPACE-complete [SC85]	
MITL	EXPSPACE-complete [AFH96]	PSPACE-complete [HR05]
BoundedMTL	EXPSPACE-complete [BMOW07]	PSPACE-complete
$\text{coFlatMTL}_{\text{LTL}}$	EXPSPACE-complete [BMOW07]	EXPSPACE-complete
SafetyMTL	decidable, NPR [OW05]	
$\text{coFlatMTL}_{\text{MITL}}$	EXPSPACE-complete	EXPSPACE-complete

[SC85] Sistla, Clarke. The complexity of propositional linear temporal logics (JACM'85).
[AFH96] Alur, Feder, Henzinger. The benefits of relaxing punctuality (JACM'96).
[HR05] Hirshfeld, Rabinovich. Timed formulas and decidable metric temporal logic (I&C'05).
[BMOW07] Bouyer, Markey, Ouaknine, Worrell. The cost of punctuality (LICS'07).
[OW05] Ouaknine, Worrell. On the decidability of metric temporal logic (LICS'05).

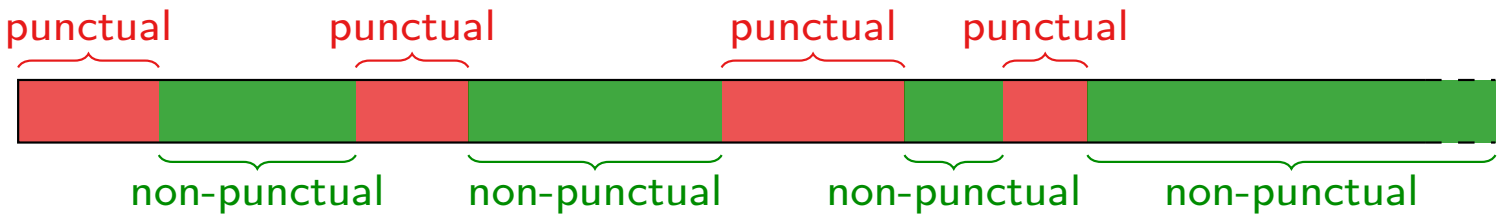
Model-checking coFlatMTL$_{\text{MITL}}$

$\equiv$ satisfiability of FlatMTL$_{\text{MITL}}$

Model-checking coFlatMTL$_{\text{MITL}}$

$\equiv$ satisfiability of FlatMTL$_{\text{MITL}}$

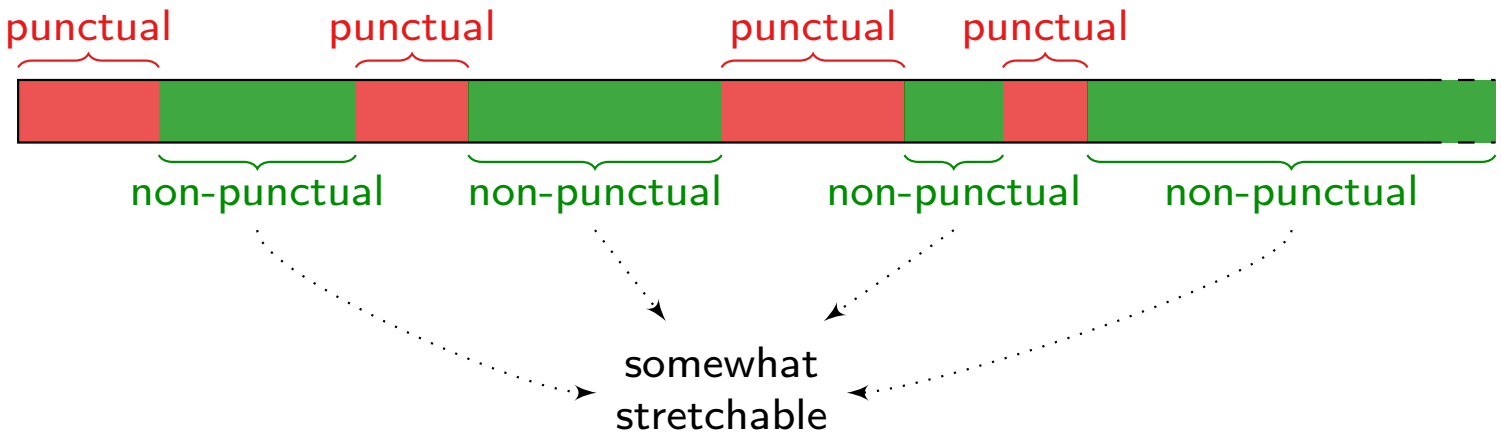
- Any satisfiable FlatMTL$_{\text{MITL}}$-formula φ has a model such that:



Model-checking coFlatMTL$_{\text{MITL}}$

$\equiv$ satisfiability of FlatMTL$_{\text{MITL}}$

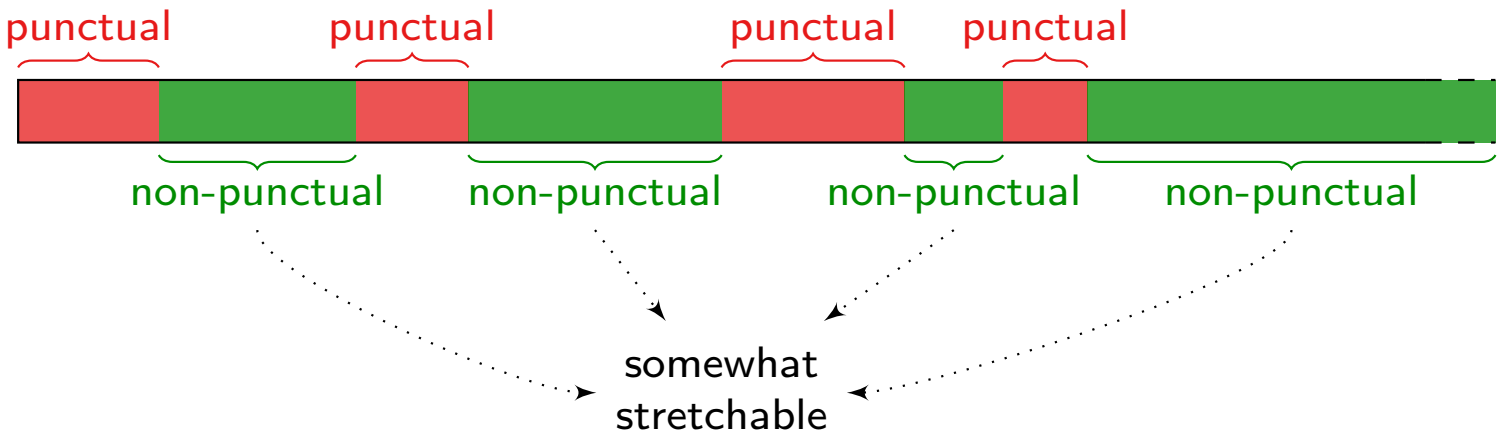
- Any satisfiable FlatMTL$_{\text{MITL}}$-formula φ has a model such that:



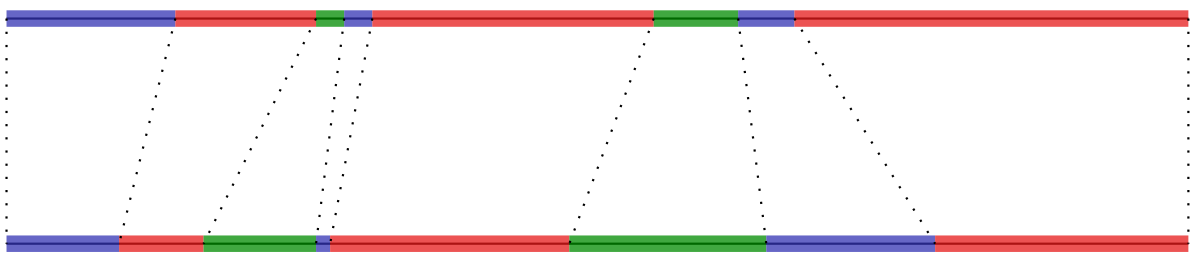
Model-checking coFlatMTL$_{\text{MITL}}$

$\equiv$ satisfiability of FlatMTL$_{\text{MITL}}$

- Any satisfiable FlatMTL$_{\text{MITL}}$-formula φ has a model such that:



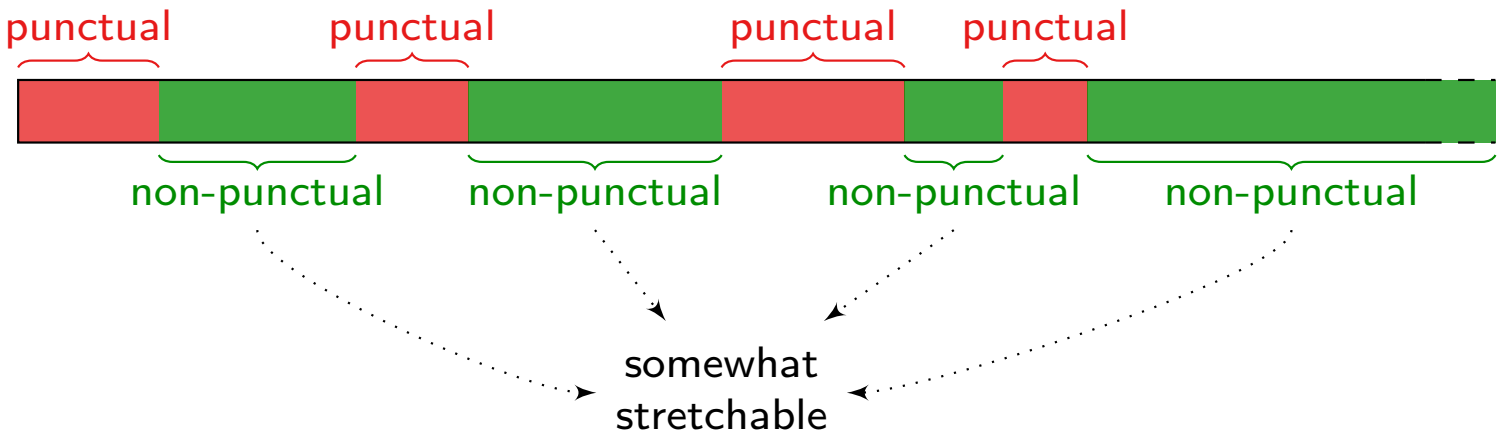
- Stretchable signal:



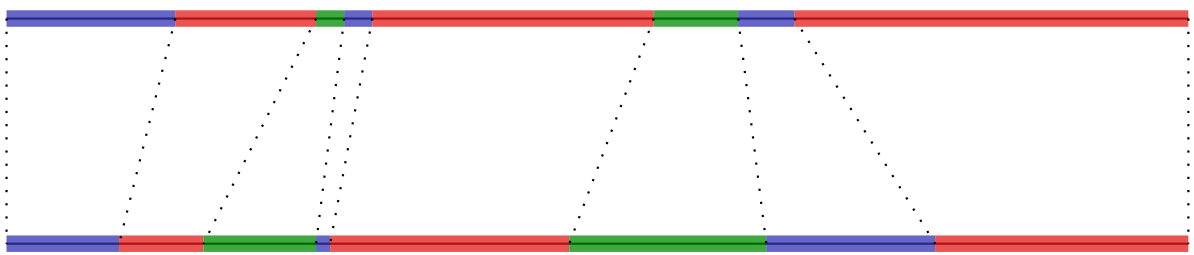
Model-checking coFlatMTL$_{\text{MITL}}$

$\equiv$ satisfiability of FlatMTL$_{\text{MITL}}$

- Any satisfiable FlatMTL$_{\text{MITL}}$-formula φ has a model such that:



- Stretchable signal:

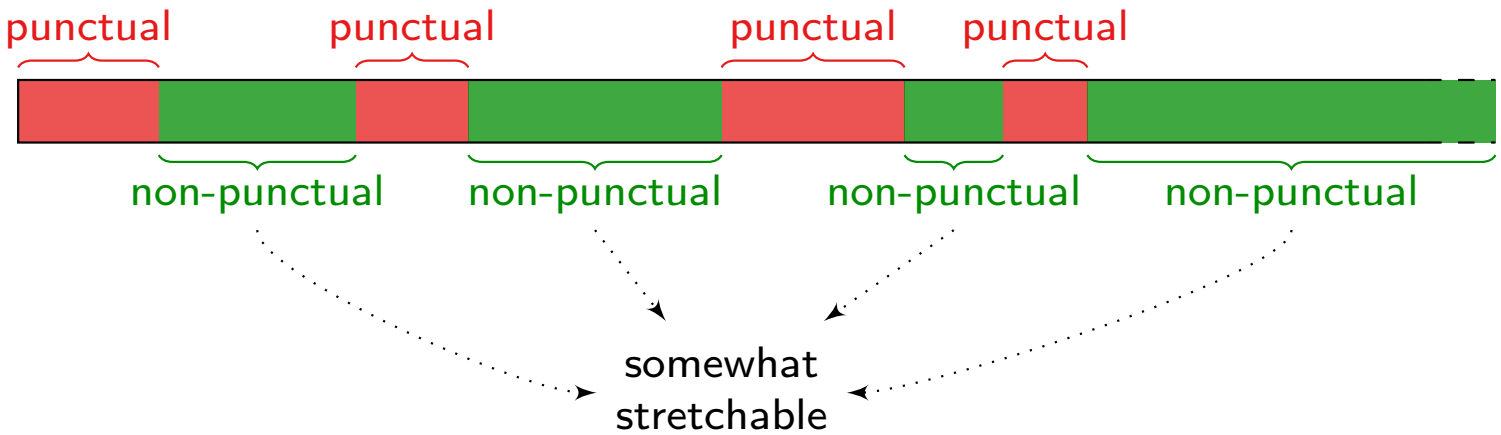


  - Any model of an LTL formula is stretchable.

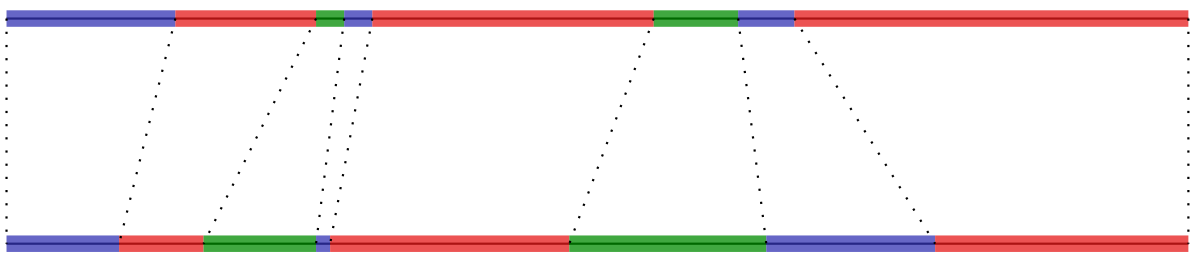
Model-checking coFlatMTL$_{\text{MITL}}$

$\equiv$ satisfiability of FlatMTL$_{\text{MITL}}$

- Any satisfiable FlatMTL$_{\text{MITL}}$-formula φ has a model such that:



- Stretchable signal:



  - Any model of an LTL formula is stretchable.
  - Any model of an MITL formula is *somewhat* stretchable.

- non-punctual part:

  somewhat stretchable

  $\longrightarrow$ transformed into a stretchable part using extra atomic propositions

  $\longrightarrow$ transform into LTL constraints

- non-punctual part:
  somewhat stretchable
  $\longrightarrow$ transformed into a stretchable part using extra atomic propositions
  $\longrightarrow$ transform into LTL constraints

- punctual part:

- non-punctual part:
  somewhat stretchable
  $\longrightarrow$ transformed into a stretchable part using extra atomic propositions
  $\longrightarrow$ transform into LTL constraints

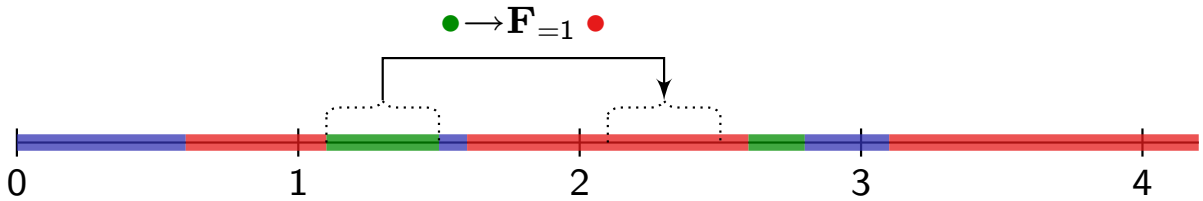
- punctual part: non-stretchable... ☹

- non-punctual part:
  somewhat stretchable
  $\longrightarrow$ transformed into a stretchable part using extra atomic propositions
  $\longrightarrow$ transform into LTL constraints

- punctual part: non-stretchable... ☹
  but not too long... ☺

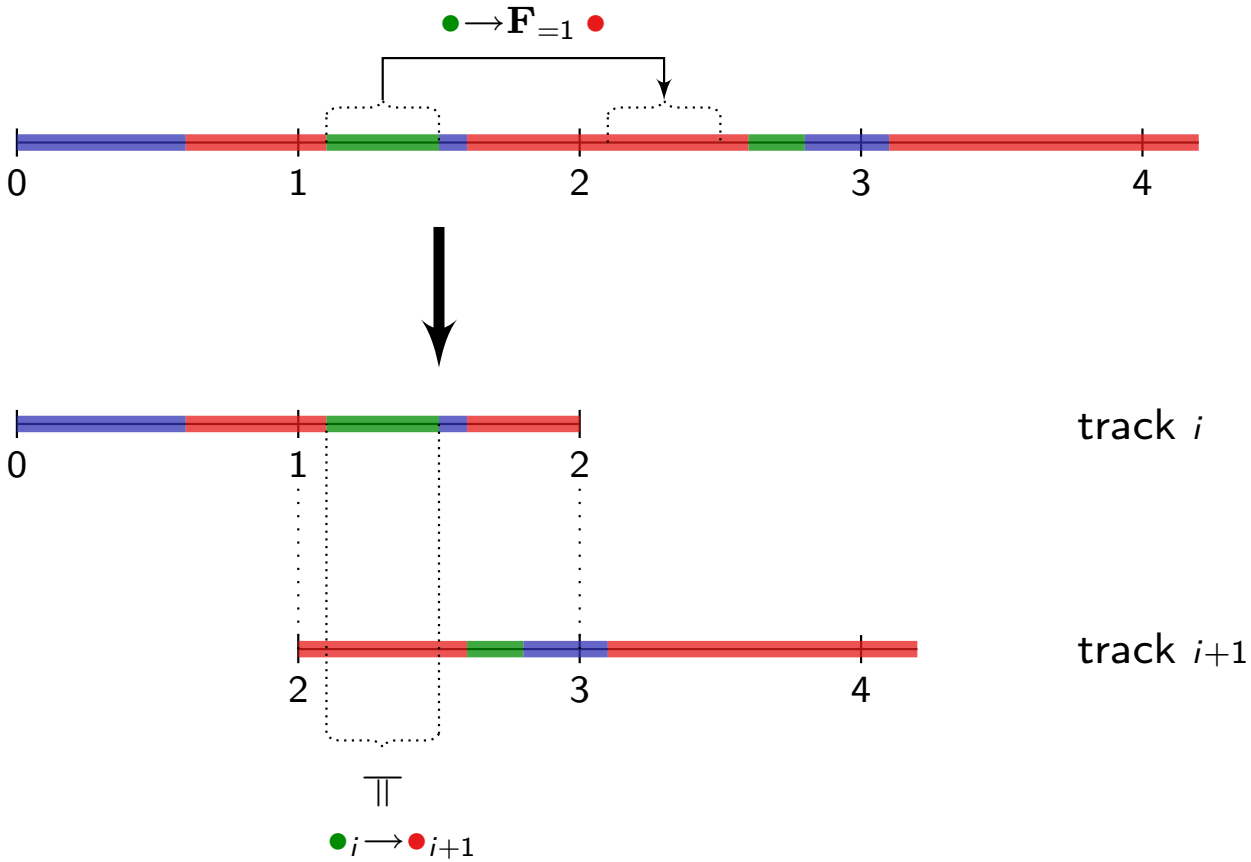
- non-punctual part:
  somewhat stretchable
  $\longrightarrow$ transformed into a stretchable part using extra atomic propositions
  $\longrightarrow$ transform into LTL constraints

- punctual part: non-stretchable... ☹
  but not too long... ☺

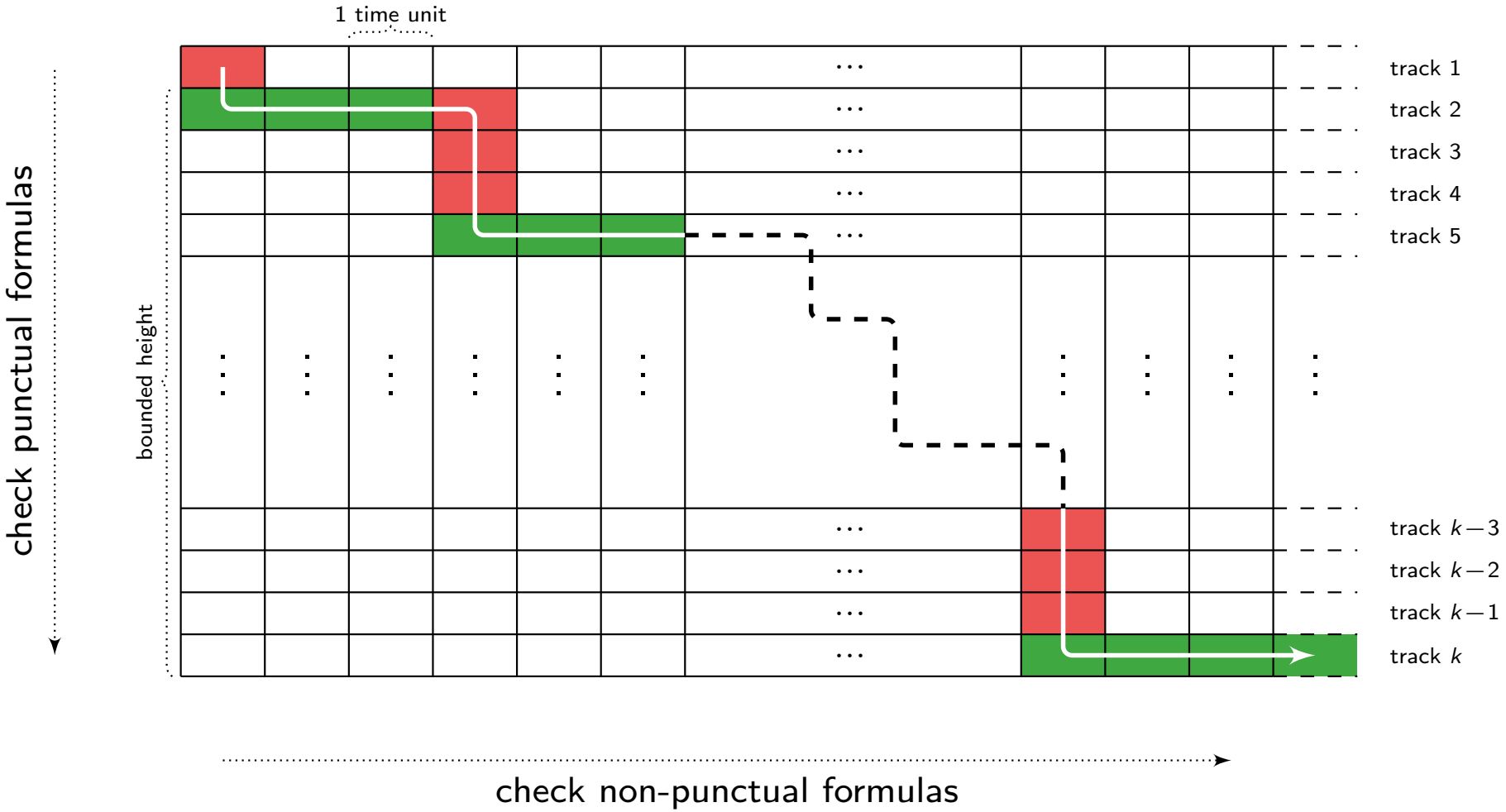


- non-punctual part:
  somewhat stretchable
  $\longrightarrow$ transformed into a stretchable part using extra atomic propositions
  $\longrightarrow$ transform into LTL constraints

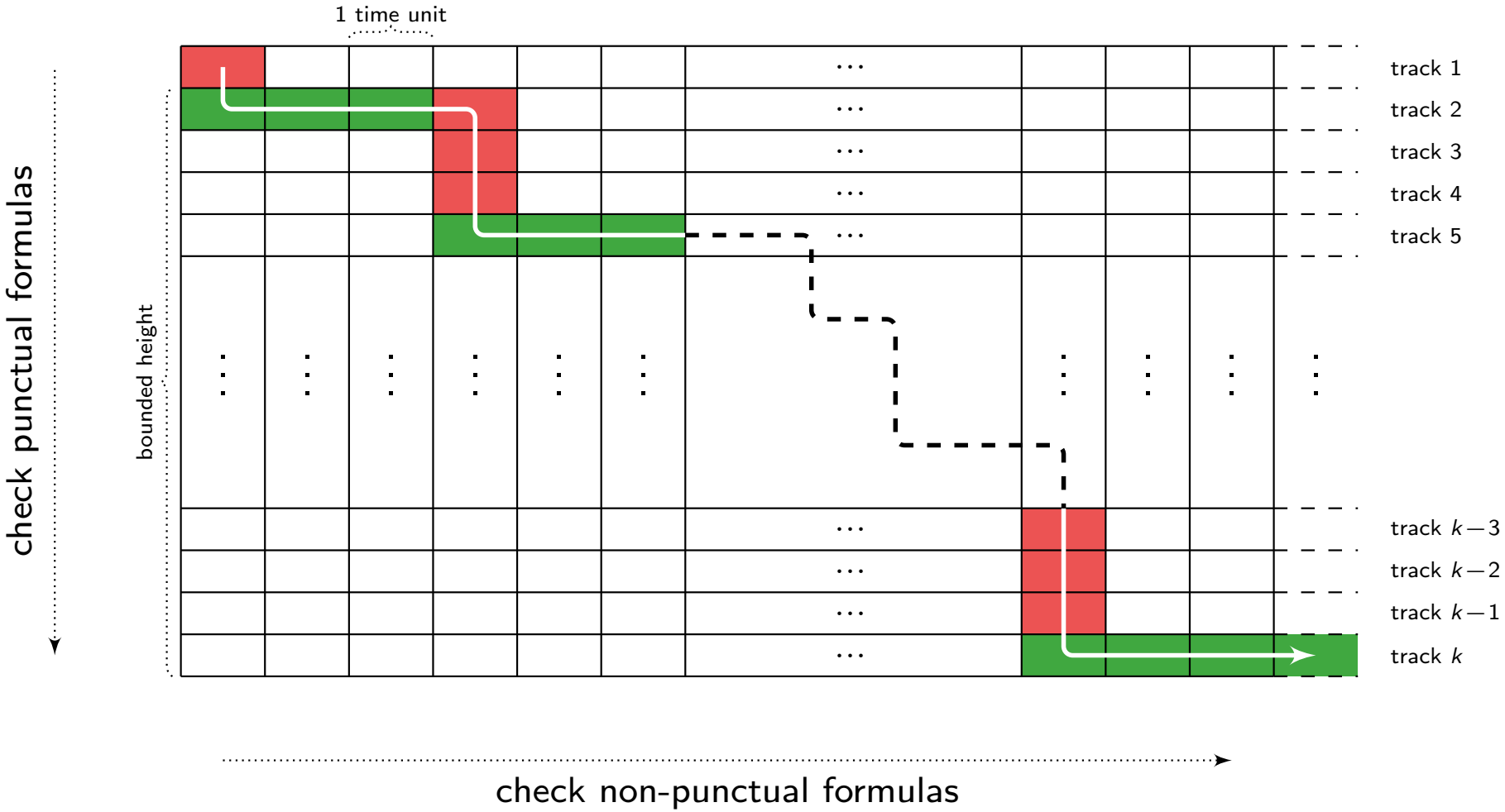
- punctual part: non-stretchable... ☹
  but not too long... ☺



A tableau satisfiability problem



A tableau satisfiability problem



☞ transform into sat. prob. for LTL+Past over $\mathbb{R}_+$ (PSPACE: [Rey04])

[Rey04] Reynolds. The complexity of the temporal logic over the reals (Subm.'04).

Outline

1. Introduction

2. The logic MTL

3. Towards tractable fragments of MTL

4. Conclusion

Conclusion

- We have proposed a subclass of MTL called $\text{coFlatMTL}_{\text{MITL}}$ s.t.
  - its model-checking problem is EXPSPACE-complete;
  - it includes most known timed temporal languages that can be efficiently model-checked.
- Our tableau construction $\equiv$ small-model property
- Note:
  - $\text{coFlatMTL}_{\text{MITL}}$ is not closed under negation;
  - The satisfiability problem for $\text{coFlatMTL}_{\text{MITL}}$ is undecidable.

Further investigations

- Can we apply such ideas to branching-time logics?
- Can we find some more practical algorithms?