Robustness in Timed Automata: A Game-Based Approach

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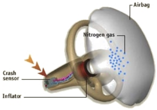
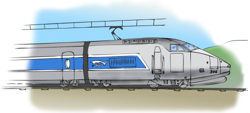
LSV, CNRS & ENS Cachan, France

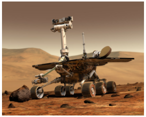
Based on joint works with Nicolas Markey, Pierre-Alain Reynier and Ocan Sankur
Acknowledgment to Nicolas and Ocan for slides

Time-dependent systems

- We are interested in timed systems

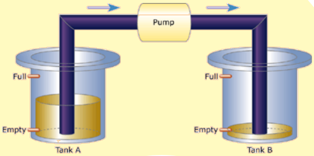






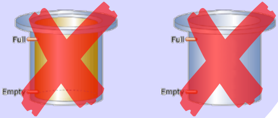
Model-checking and control

system:



[http://www.embedded.com]

property:



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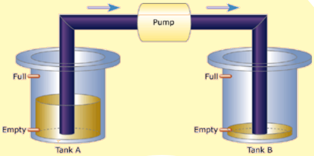

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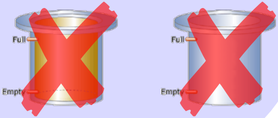
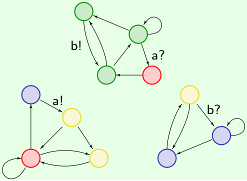


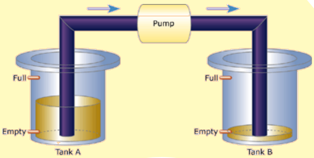


AG(¬`B.overfull`

∧ ¬`B.dried_up`)

Model-checking and control

system:



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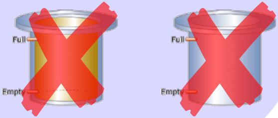
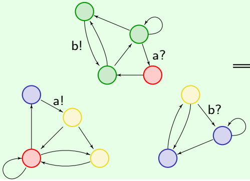


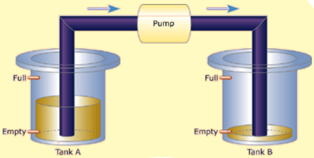


algorithm

$$\mathbf{AG}(\neg\texttt{B.overfull} \wedge \neg\texttt{B.dried\_up})$$

Model-checking and control

system:



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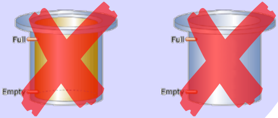
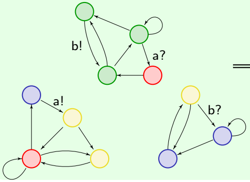




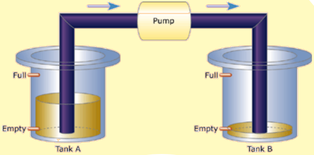
model-checking algorithm

$\mathbf{AG}(\neg\mathtt{B.overfull} \wedge \neg\mathtt{B.dried\_up})$

yes/no

Model-checking and control

system:



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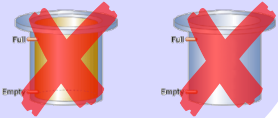
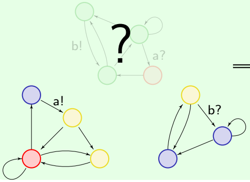
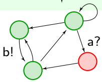
property:





control/synthesis algorithm

$\mathbf{AG}(\neg \texttt{B.overfull} \wedge \neg \texttt{B.dried\_up})$



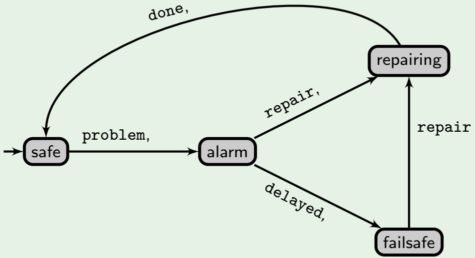
Reasoning about real-time systems

The model of timed automata [AD94]

A timed automaton is made of

- a finite automaton-based structure

Example



[AD94] Alur, Dill: A Theory of Timed Automata. Theor. Comp. Science, 1994.

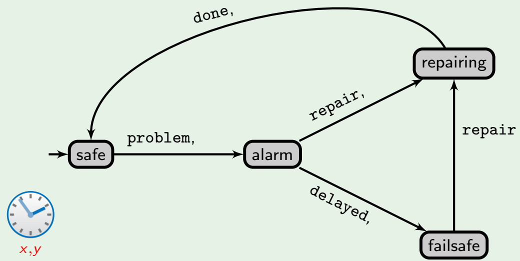
Reasoning about real-time systems

The model of timed automata [AD94]

A timed automaton is made of

- a finite automaton-based structure
- a set of clocks

Example



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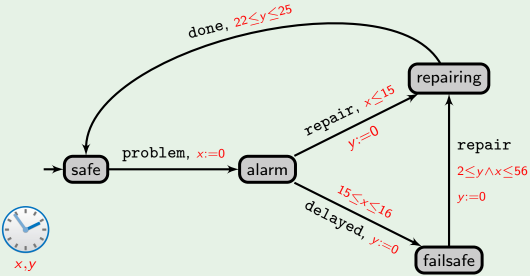
Reasoning about real-time systems

The model of timed automata [AD94]

A timed automaton is made of

- a finite automaton-based structure
- a set of clocks
- timing constraints on transitions

Example



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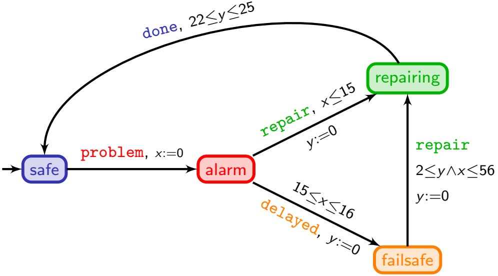
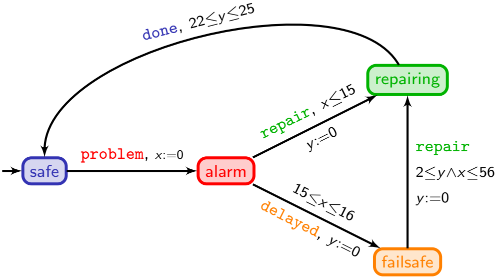
Example – cont'd

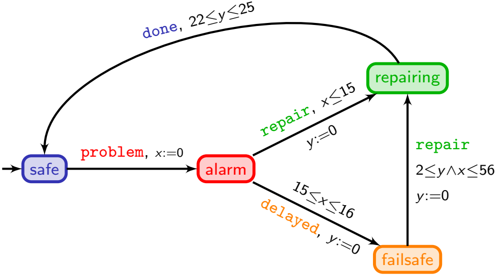


Example – cont'd



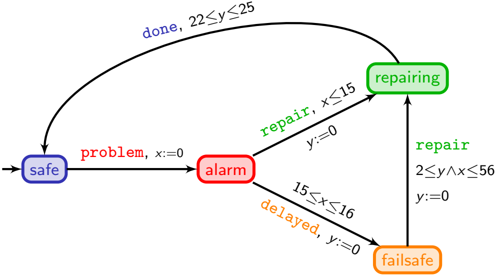
	safe
x	0
y	0

Example – cont'd



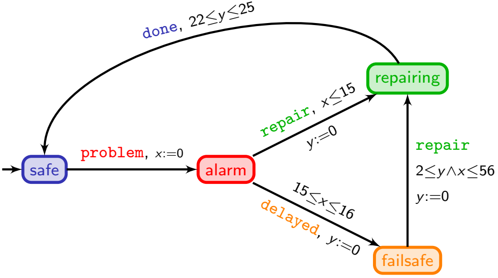
	safe	$\xrightarrow{23}$	safe
x	0		23
y	0		23

Example – cont'd



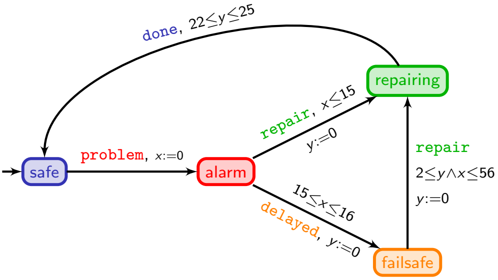
	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm
x	0		23		0
y	0		23		23

Example – cont'd



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm
x	0		23		0		15.6
y	0		23		23		38.6

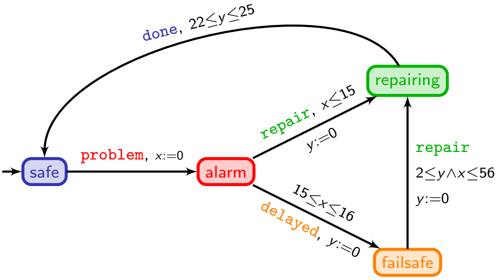
Example – cont'd



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	$\cdots$
y	0		23		23		38.6		0	

	failsafe
$\cdots$	15.6
	0

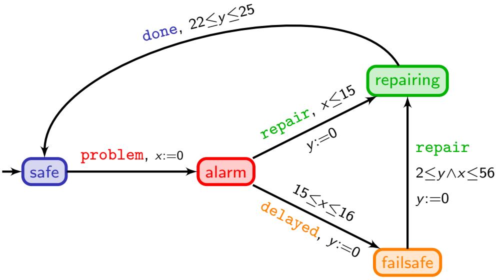
Example – cont'd



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe
⋯	15.6		17.9
	0		2.3

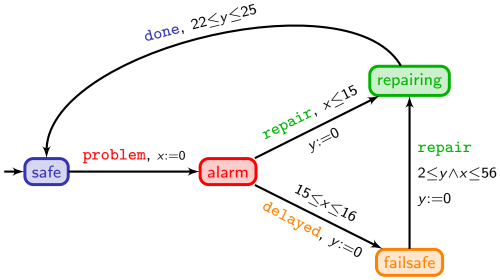
Example – cont'd



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	···
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing
···	15.6		17.9		17.9
	0		2.3		0

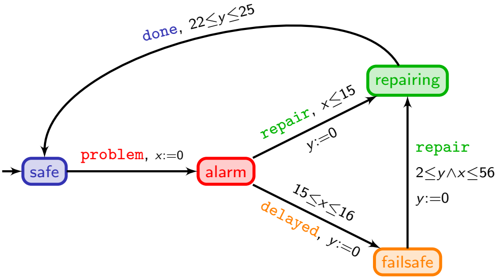
Example – cont'd



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	$\cdots$
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing
$\cdots$	15.6		17.9		17.9		40
	0		2.3		0		22.1

Example – cont'd



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	···
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\texttt{done}}$	safe
···	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

Discrete-time semantics

...because computers are digital!

[Alur91] Techniques for automatic verification of real-time systems. *PhD thesis*, 1991.

Discrete-time semantics

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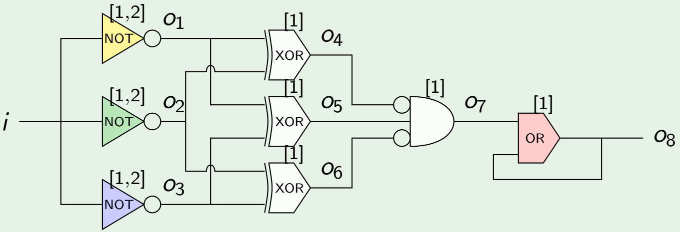
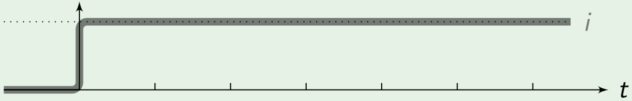
Example [Alur91]



- under discrete-time, the output is always 0:



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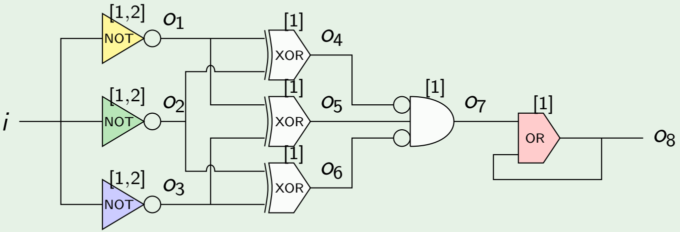

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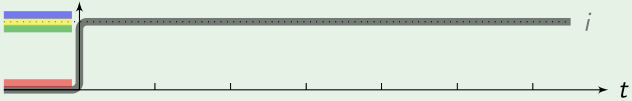

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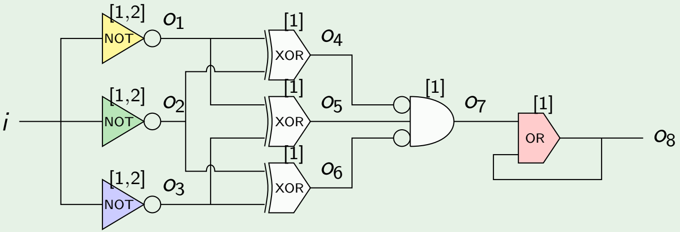

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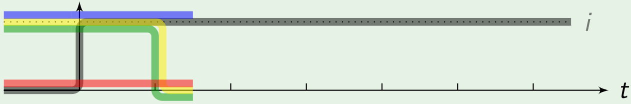

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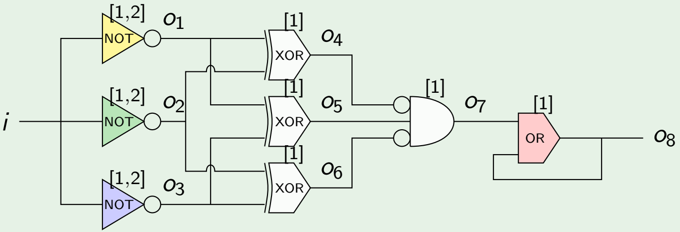

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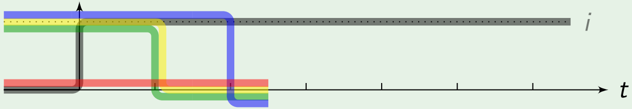

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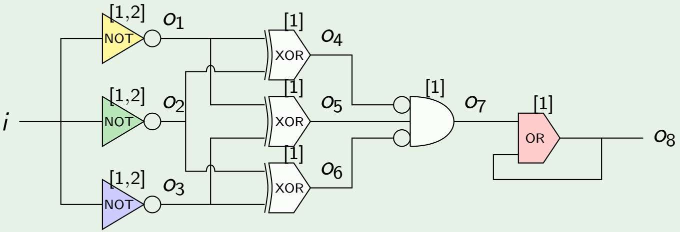

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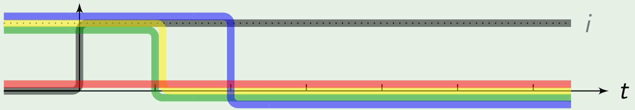

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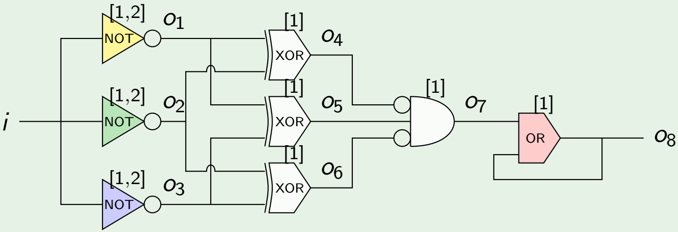
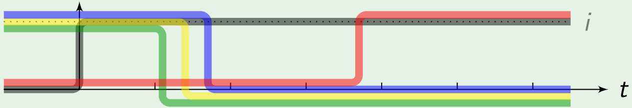


- under continuous-time, the output can be 1:



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Continuous-time semantics

...real-time models for real-time systems!

Continuous-time semantics

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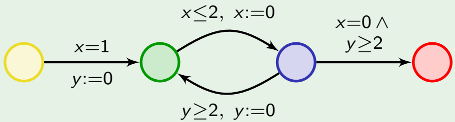
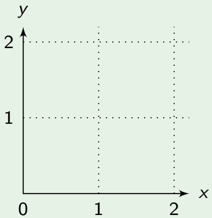
Example





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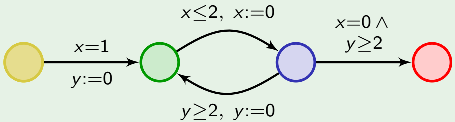
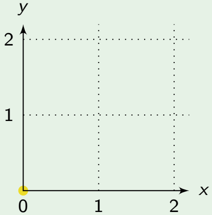
Example





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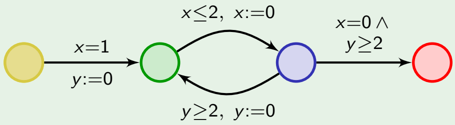

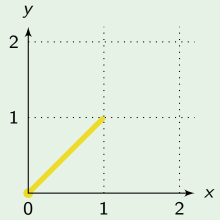

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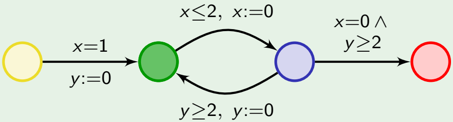

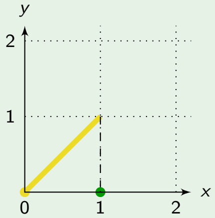

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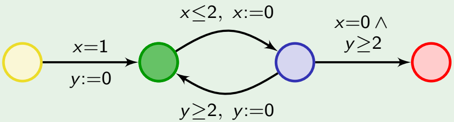

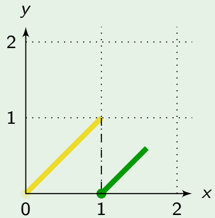

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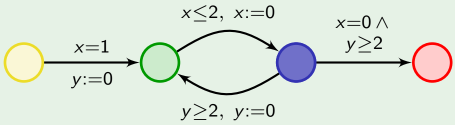

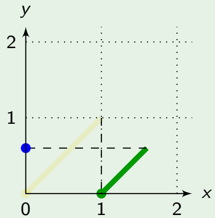

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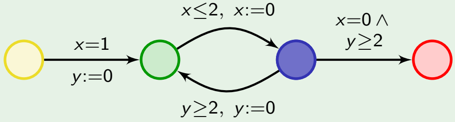

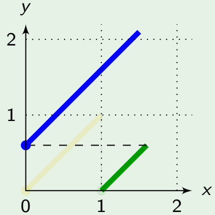

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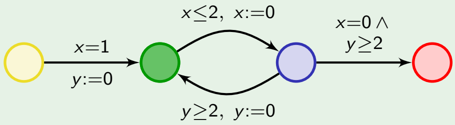
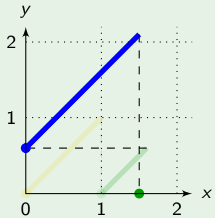
Example





Continuous-time semantics

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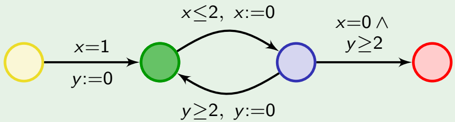

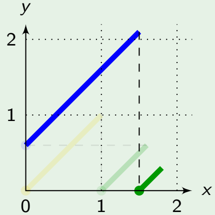

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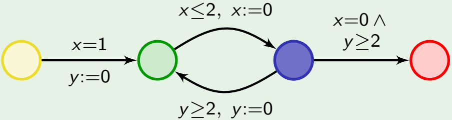

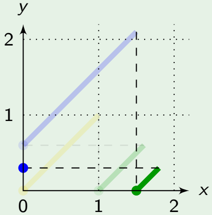

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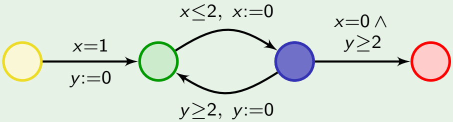

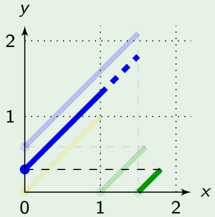

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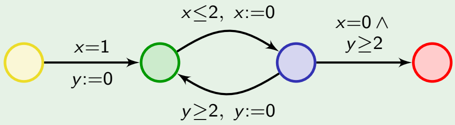

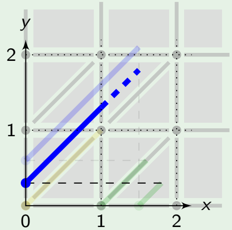

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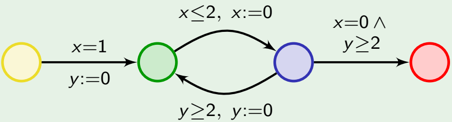

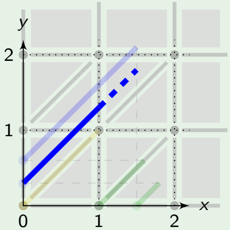

Theorem [AD94]

Reachability in timed automata is decidable (as well as many other important properties). It is PSPACE-complete.

- Technical tool: region abstraction

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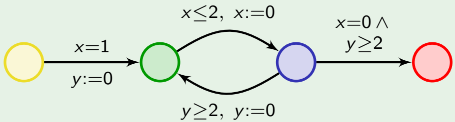

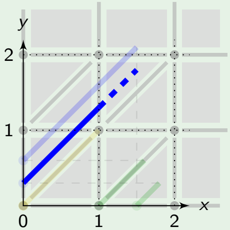

Theorem [AD94]

Reachability in timed automata is decidable (as well as many other important properties). It is PSPACE-complete.

- Technical tool: region abstraction
- Efficient symbolic technics based on zones, implemented in tools

Are we doing the right job?

The continuous-time semantics is
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It is adequate for abstract design and high-level analysis.

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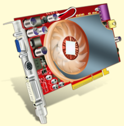
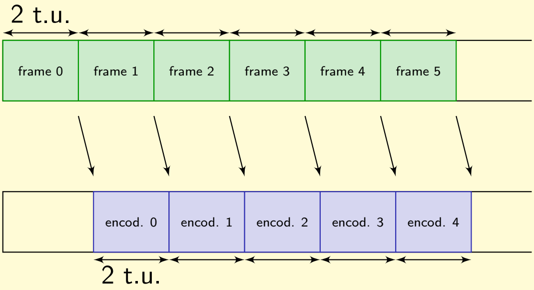
- It may generate timing anomalies
- It does not exclude non-realizable behaviours:
  - not only Zeno behaviours
  - many convergence phenomena are hidden

$\rightsquigarrow$ this requires infinite precision and might not be realizable

Example 1: Imprecision on clock values

Frame capture





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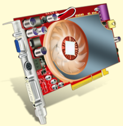

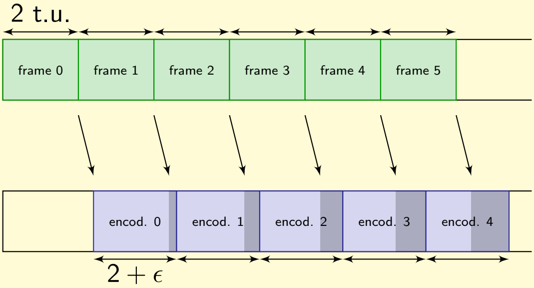

$\rightsquigarrow$ A frame will eventually be skipped

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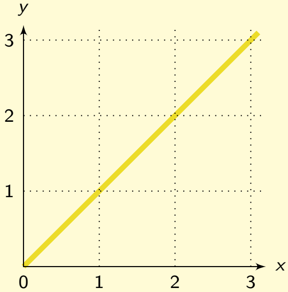
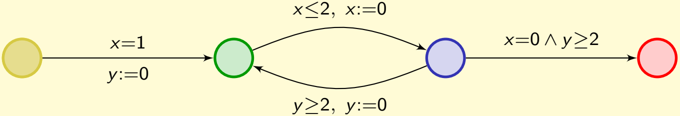
Example 2: Does actual analysis transfer to real world?

Impact of small timing jitters





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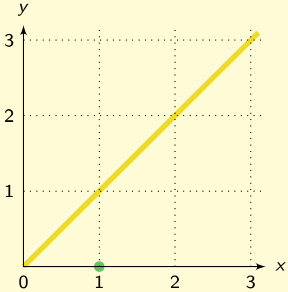

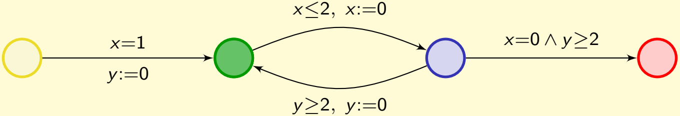

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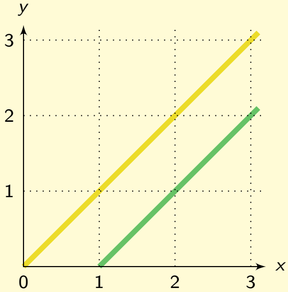

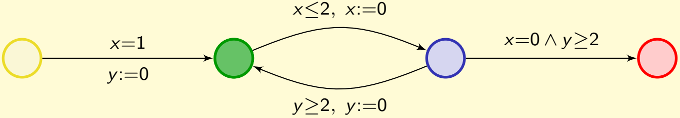

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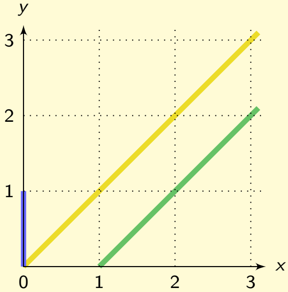

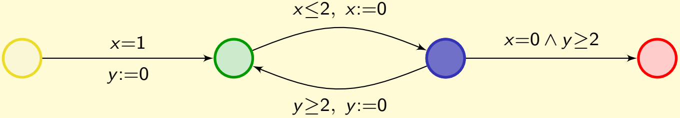

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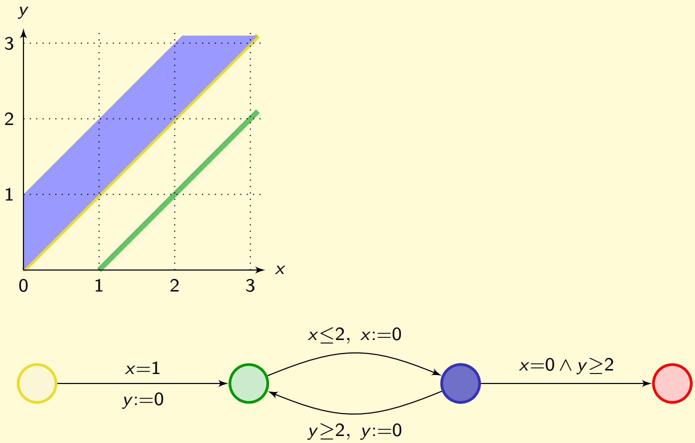

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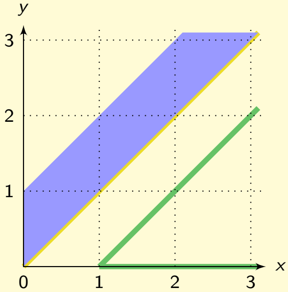

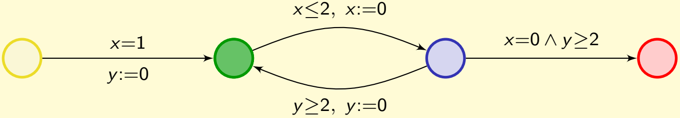

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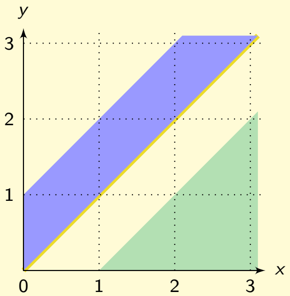

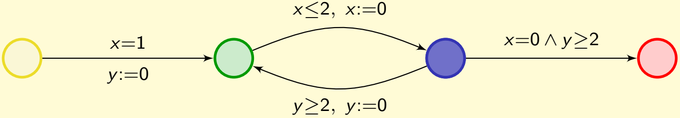

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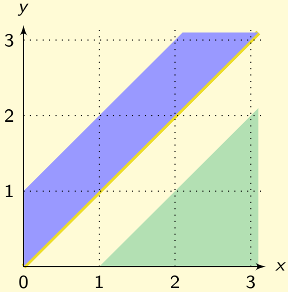

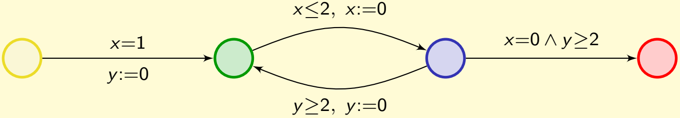

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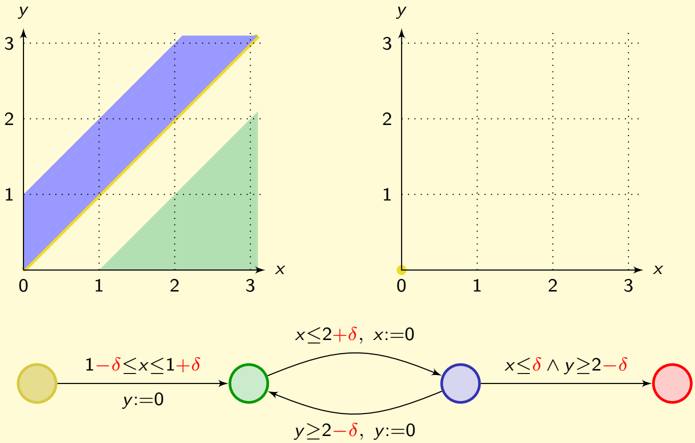

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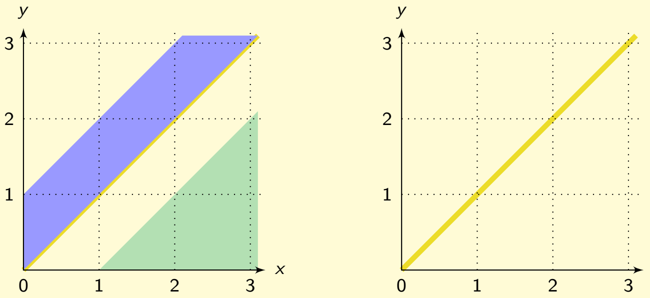

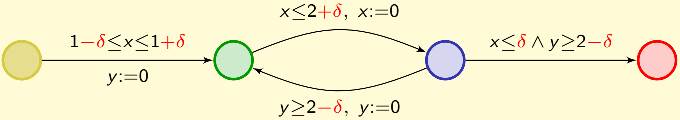

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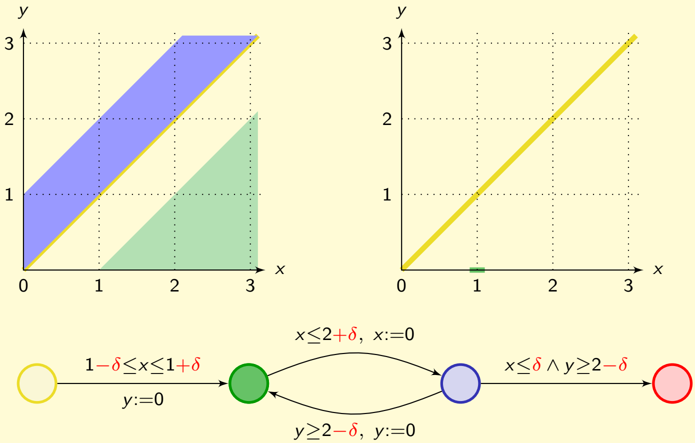

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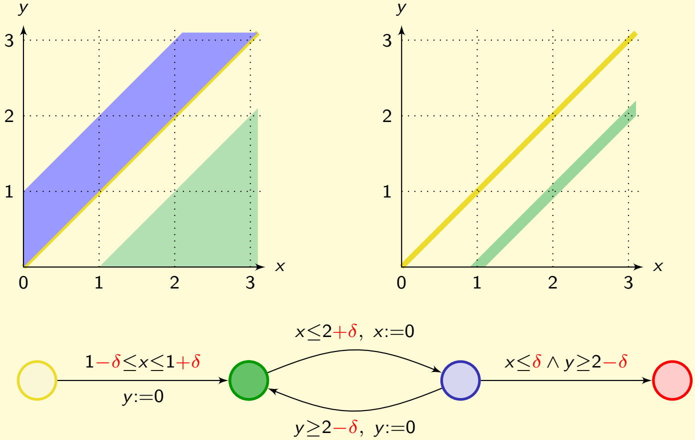

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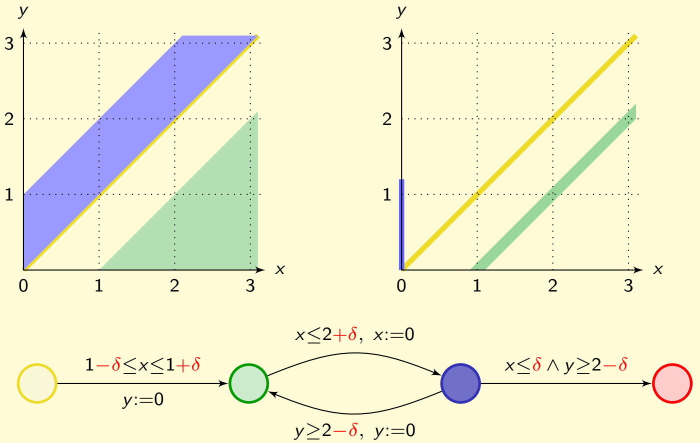

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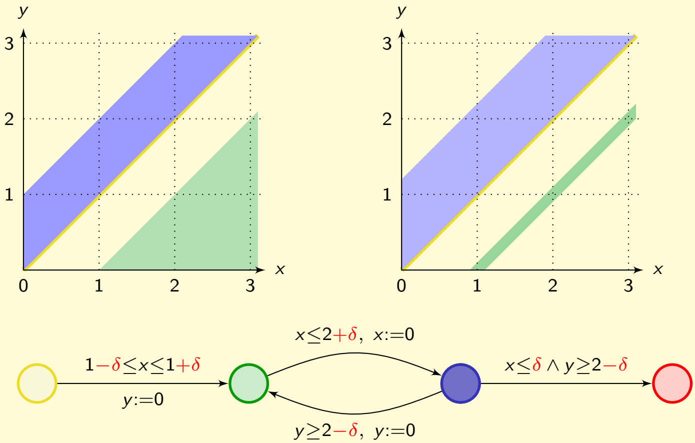

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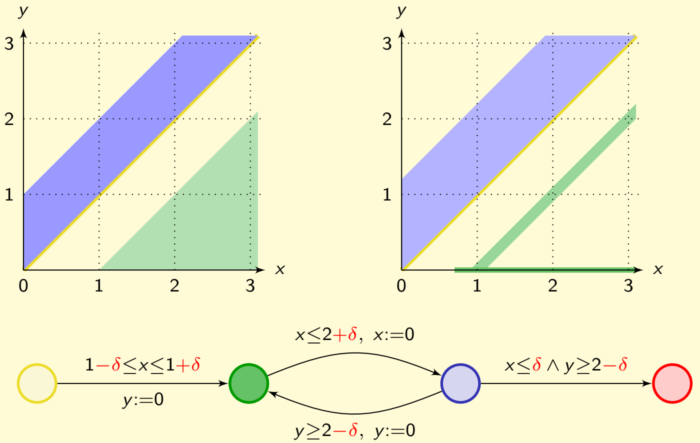

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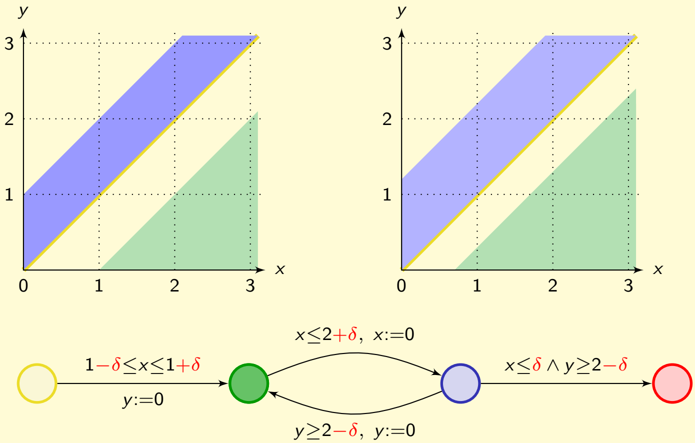

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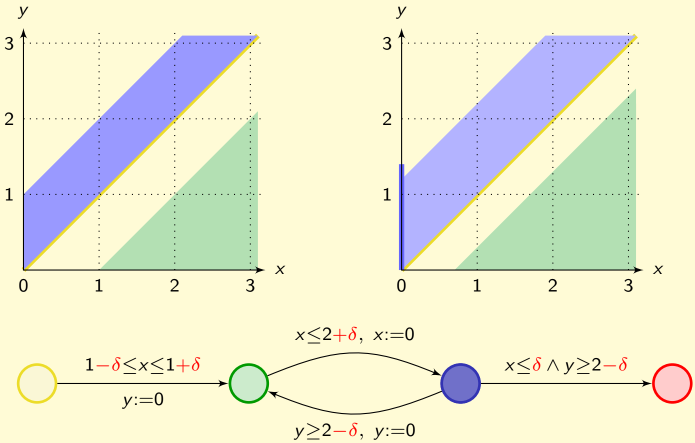

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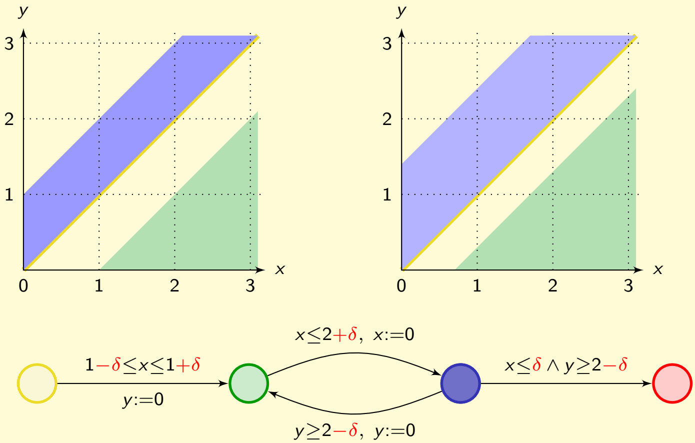

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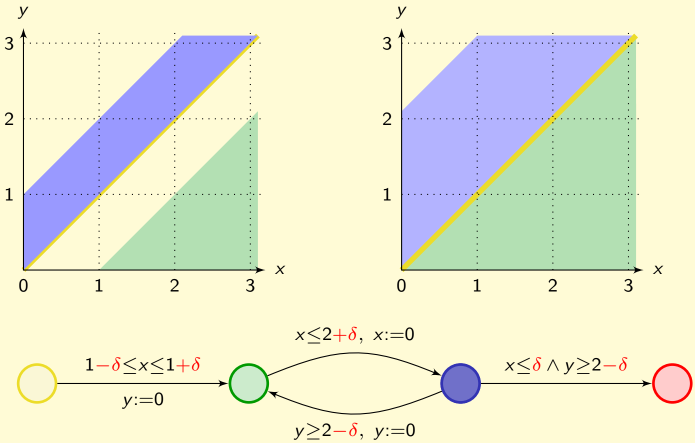

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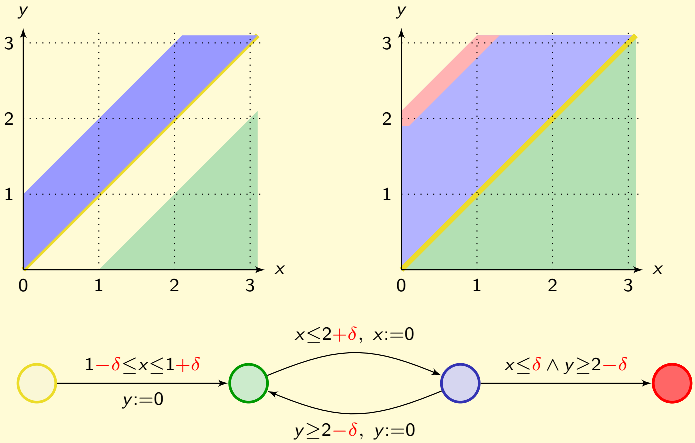

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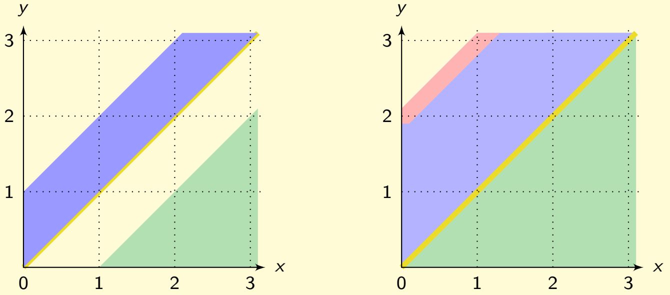

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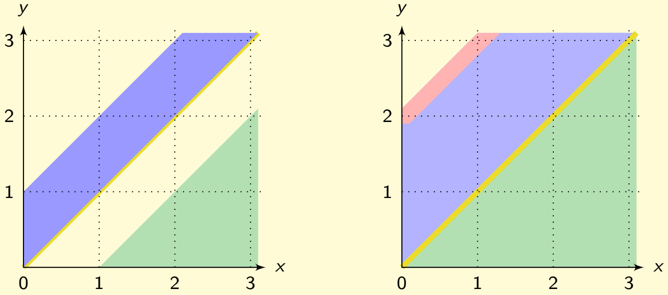

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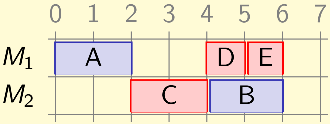
Example 3: Scheduling and timing anomaly

- Scheduling analysis with timed automata [AAM06]
- **Goal:** analyze a *work-conserving* scheduling policy on given scenarios (no machine is idle if a task is waiting for execution)

Example of a scenario



with the dependency constraints: $A \to B$ and $C \to D, E$.

1. A, D, E must be scheduled on machine M_1
2. B, C must be scheduled on machine M_2
3. C starts no sooner than 2 time units

[AAM06] Abdeddaim, Asarin, Maler. Scheduling with timed automata. *Theor. Comp. Science*, 2006.

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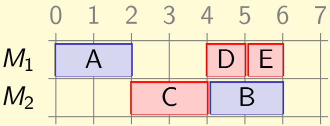
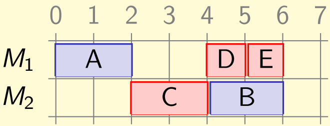


$\rightsquigarrow$ Schedulable in 6 time units

Example 3: Scheduling and timing anomaly

Example of a scenario



⇝ Schedulable in 6 time units

- Unexpectedly, the duration of A drops to 1.999

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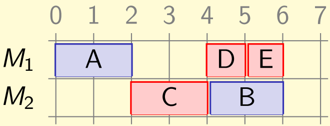

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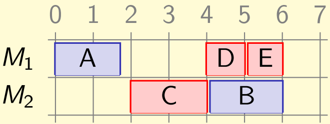

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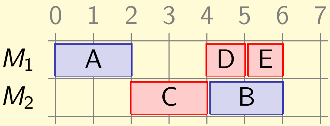

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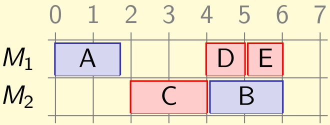

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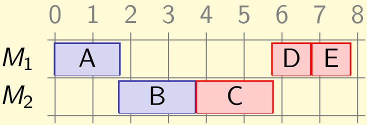

is work-conserving
and completes in 7.999 t.u.

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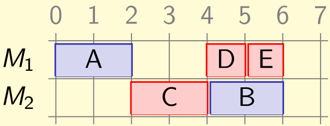

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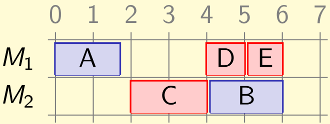

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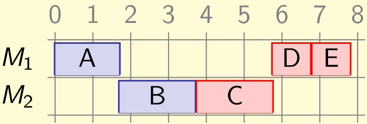

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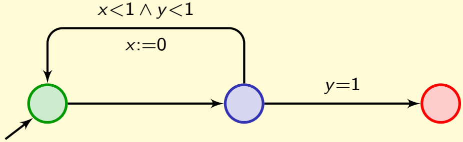
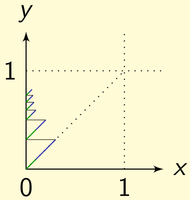
$\rightsquigarrow$ Standard analysis does not capture this **timing anomaly**

Example 4: Zeno behaviours





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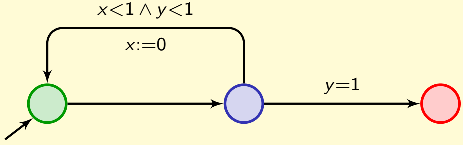

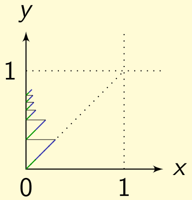

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[HS11] Herbreteau, Srivathsan. Coarse abstractions make Zeno behaviours difficult to detect, *Logic. Meth. Comp. Science*, 2011.

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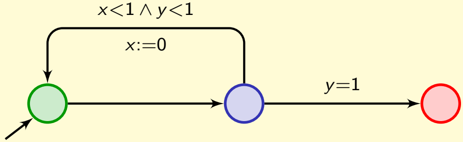

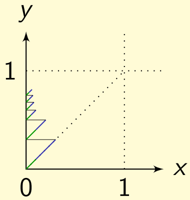

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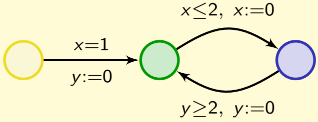
Example 5: More complex convergence phenomena



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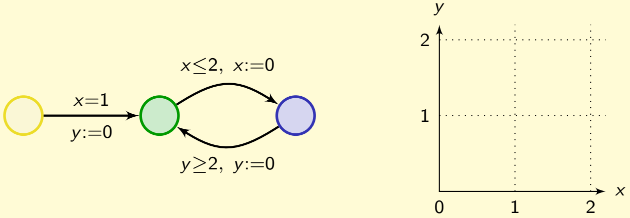

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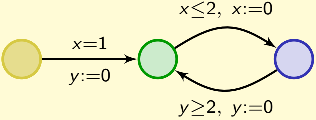

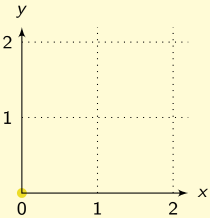

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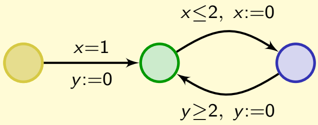

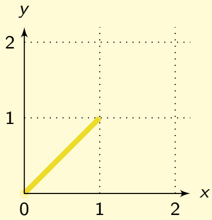

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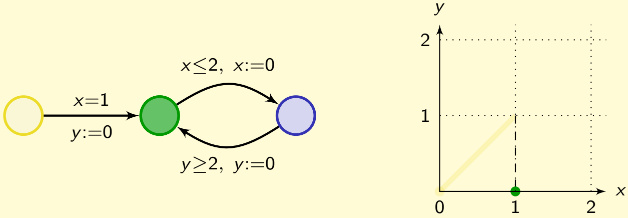

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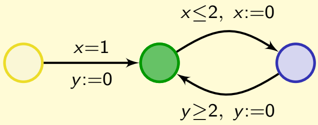

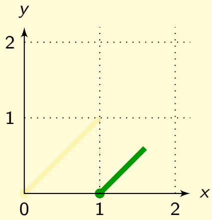

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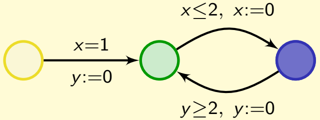

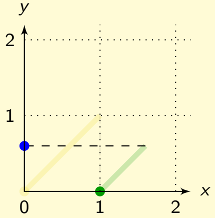

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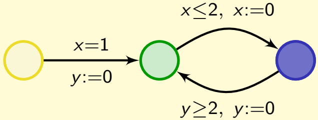

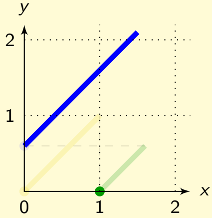

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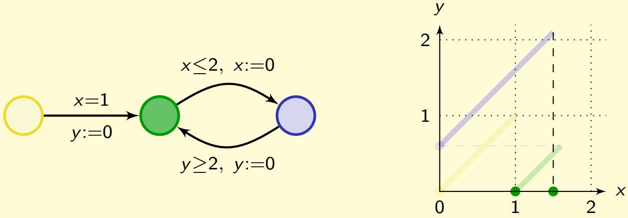

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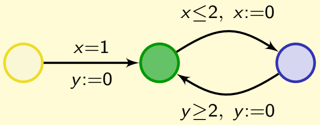

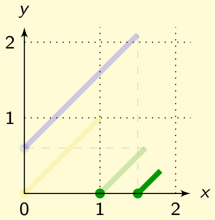

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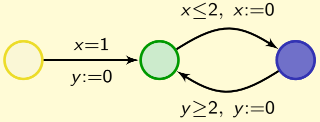

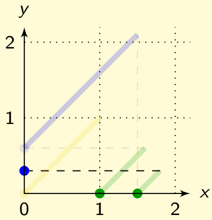

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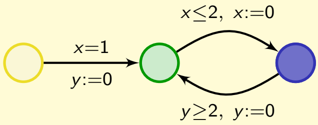

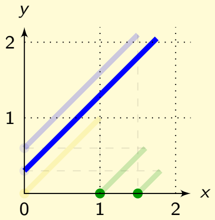

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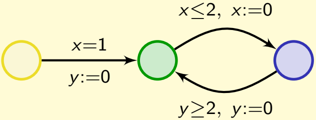

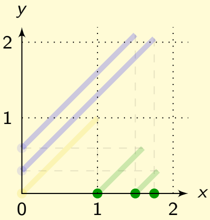

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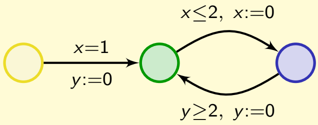

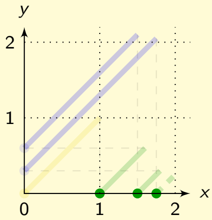

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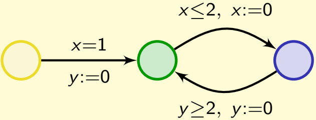

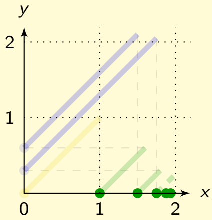

$\rightsquigarrow$ Value of clock x when hitting ◯ is converging,
even though global time diverges

An important issue in timed-automata verification

Add robustness to the theory of timed automata!

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- **Aim:** provide frameworks to build robustly correct systems

We focus on perturbations on time measurements and jitter.

Some hints into the robustness of timed automata

Robust model-checking: the worst-case approach

- Compute an overapproximation of the set of perturbed behaviours (Possibly relate with an implementation)
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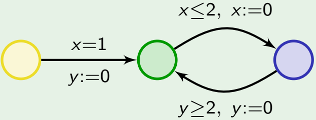
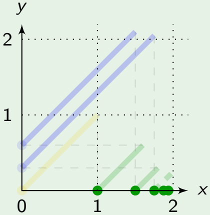
We propose a game-based approach to robust realisability

Realisability

Example (Can we ensure an infinite behaviour?)





Strategy: in location with value x, delay $\frac{2-x}{2}$

Realisability

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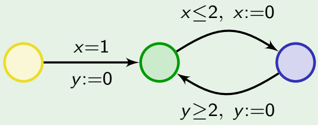

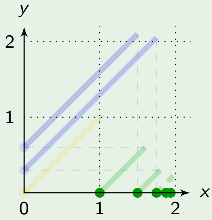

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Robust realisability

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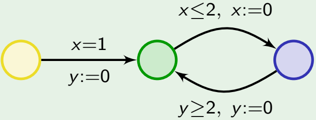

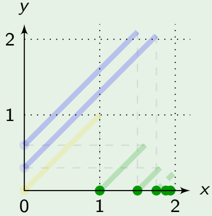

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Robust realisability

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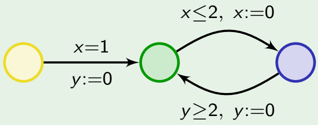
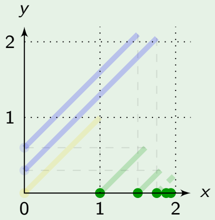




Strategy: in location ◯ with value x, delay $\frac{2-x}{2}$

- This strategy requires infinite precision
- In practice, when x is close to 2, no additional delay is supported: the run is theoretically infinite, but it is actually blocking
- And that is unavoidable

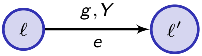
Robust realisability

Idea of robust realisability

- $\rightsquigarrow$ Synthesize strategies that realise some property while tolerating small timing perturbations
- $\rightsquigarrow$ Consequence: remove convergence phenomena

Robust realisability via a game semantics

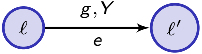
Timed automaton $\mathcal{A}$



Robust realisability via a game semantics

Timed automaton $\mathcal{A}$

Game $\mathcal{G}_\delta(\mathcal{A})$



$\rightsquigarrow$

Game semantics $\mathcal{G}_\delta(\mathcal{A})$ of timed automaton $\mathcal{A}$...

... between Controller and Perturbator:

Robust realisability via a game semantics

Timed automaton $\mathcal{A}$

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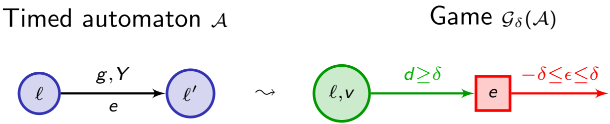

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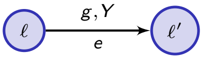

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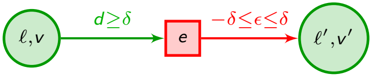

with $v'(v+d+\epsilon)[Y \leftarrow 0]$

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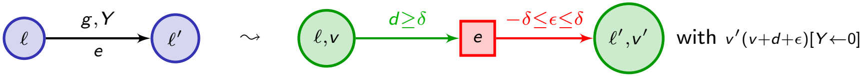

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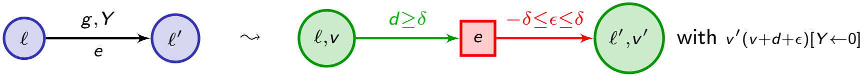
Note: when $\delta = 0$, this is the standard semantics of timed automata.

Robust realisability via a game semantics

Timed automaton $\mathcal{A}$ — Game $\mathcal{G}_\delta(\mathcal{A})$



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- from (ℓ, v), Controller suggests a delay $d \geq \delta$ and a next edge $e = (\ell \xrightarrow{g, Y} \ell')$ that is available after delay d
- Perturbator then chooses a perturbation $\epsilon \in [-\delta; +\delta]$
- Next state is $(\ell', (v + d + \epsilon)[Y \leftarrow 0])$

Note: when $\delta = 0$, this is the standard semantics of timed automata.

A δ-robust strategy for Controller is then a strategy that satisfies the expected property, whatever plays Perturbator.

The excess game semantics

Constraints may not be satisfied after the perturbation:
that is, only $v + d$ should satisfy g

[BMS12] Bouyer, Markey, Sankur. Robust reachability in timed automata: A game-based approach. *ICALP'12*.

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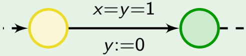

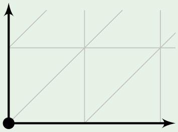

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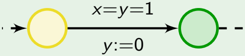

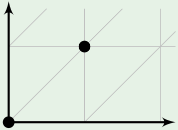

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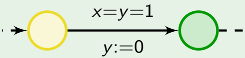

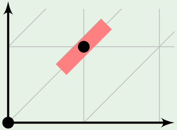

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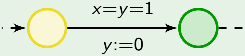

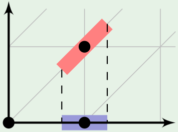

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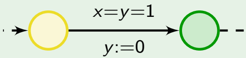

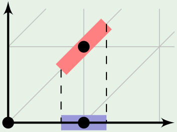

$\rightsquigarrow$ Allows simple design of constraints, ensures divergence of time, avoids convergence phenomena

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The excess game semantics – Algorithmics

The (parameterized) synthesis problem

Synthesize $\delta > 0$ and a δ-robust strategy that achieves a given goal.

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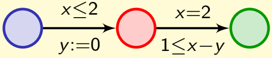

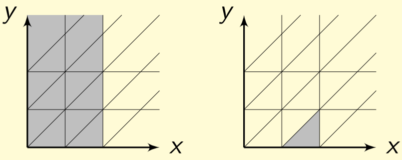

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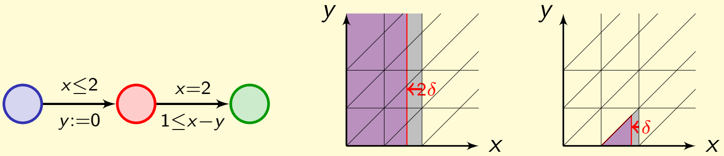

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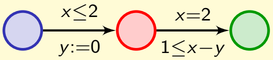
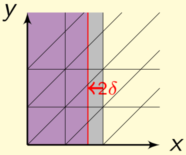
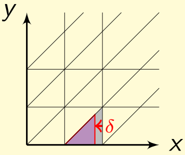
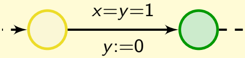
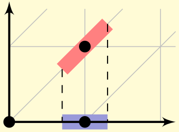






2. New regions become reachable





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Theorem

The parameterized synthesis problem for reachability properties is decidable and EXPTIME-complete. Furthermore, uniform winning strategies (w.r.t. δ) can be computed.

- Technical tools: a region-based refined game abstraction, shrunk DBMs
- ☺ Extends to two-player games (i.e. to real control problems)

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Constraints have to be satisfied after the perturbation: that is,
$v + d + \epsilon$ **should satisfy** g **for every** $\epsilon \in [-\delta; +\delta]$

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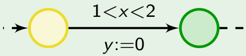

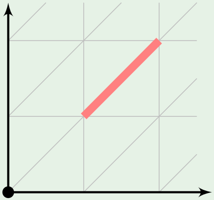

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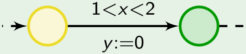

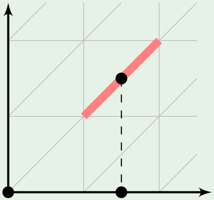

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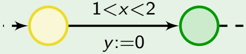

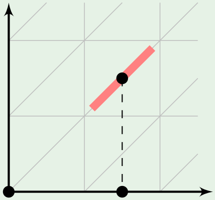

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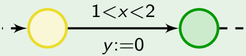

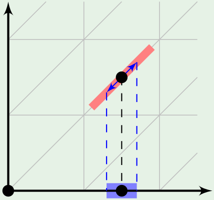

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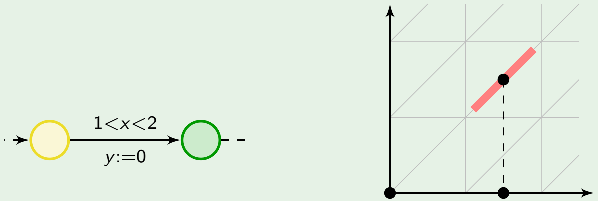

⤳ Strongly ensures timing constraints, ensures divergence of time, prevents converging phenomena

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- ☺ Valid for all ω-regular properties
- ☺ Same complexity as standard verification!
- ☺ Extends partially to timed games [ORS14]

[ORS14] Oualhadj, Reynier, Sankur. Robust strategies in timed games.

The problem consists in finding cycles that do not become blocked.

[AB11] Asarin, Basset. Thin and Thick Timed Regular Languages. *FORMATS'11*.

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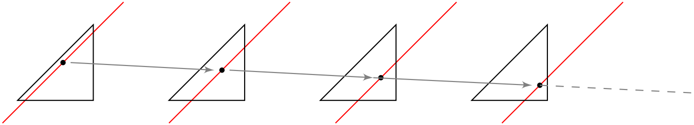
- Some cycles are converging (non-forgetful)



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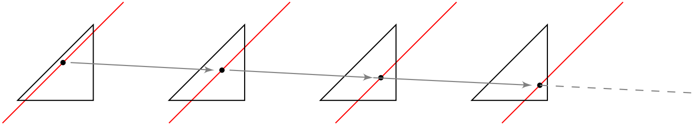

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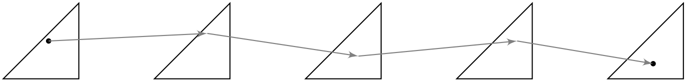

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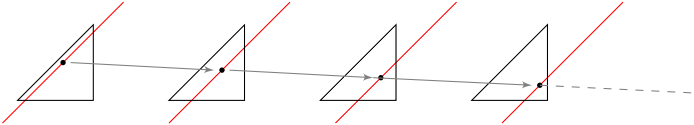

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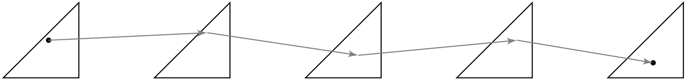


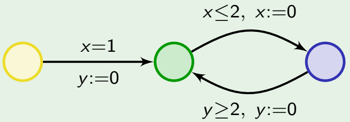
Characterization

There is $\delta > 0$ such that Controller has a δ-robust winning strategy in $\mathcal{G}_\delta(\mathcal{A})$ iff $R(\mathcal{A})$ has a reachable aperiodic[a] non-punctual winning lasso.

[a] That is, all its iterations are forgetful.

Non-forgetful cycle ⇒ deadlock

Example



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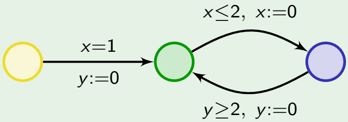
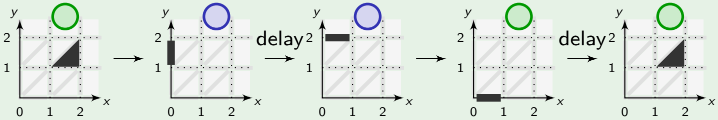


A region cycle:



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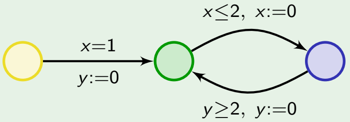

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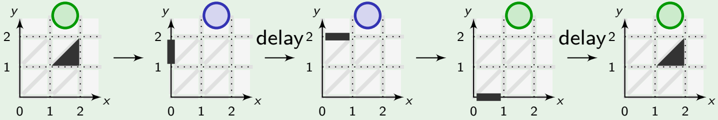
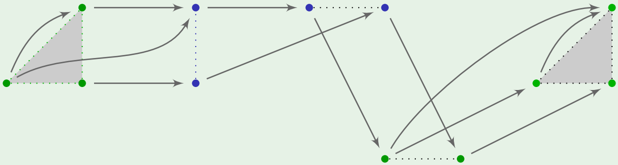


The corresponding orbit graph:



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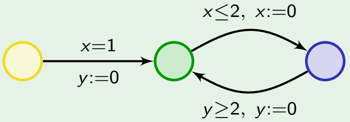

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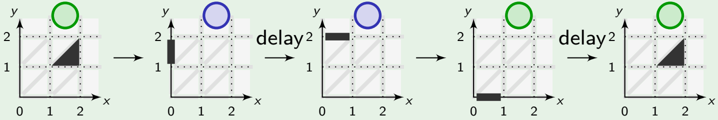
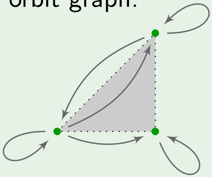


The corresponding (folded) orbit graph:



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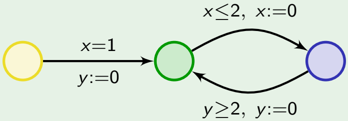

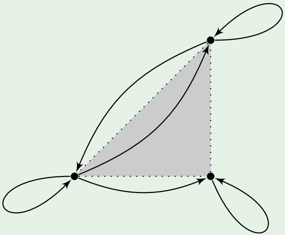

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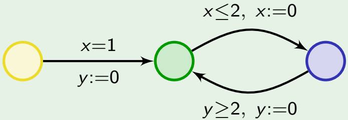
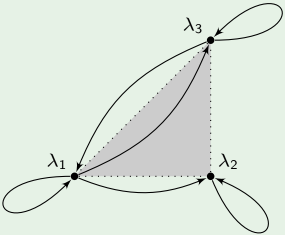
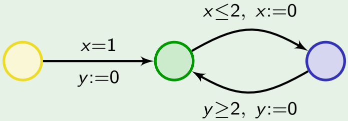
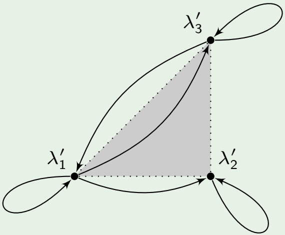




Non-forgetful cycle $\Rightarrow$ deadlock

Example





with
$$\begin{cases} \lambda'_1 = p_1\lambda_1 + p_3\lambda_3 \\ \lambda'_2 = (1 - p_1 - p'_1)\lambda_1 + \lambda_2 \\ \lambda'_3 = p'_1\lambda_1 + (1 - p_3)\lambda_3 \end{cases}$$

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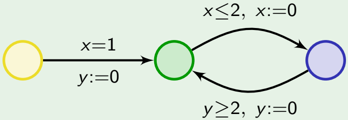
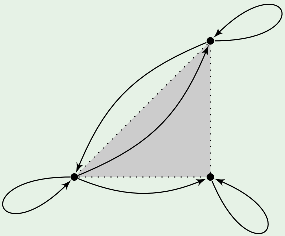


The cycle is not forgetful (that is, not strongly connected)



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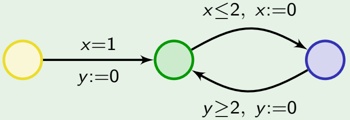
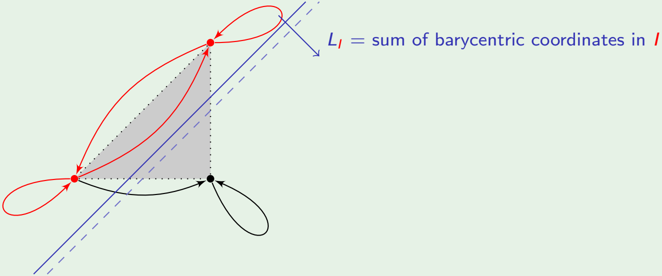


The cycle is not forgetful (that is, not strongly connected): there is an initial component I, and a non-decreasing function L_I.



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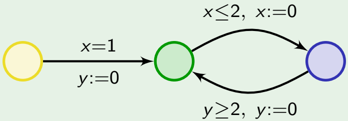
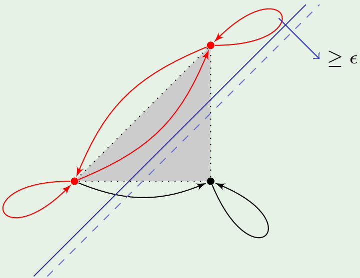


Perturbator can enforce rapid decrease of L_l!



Forgetful cycle $\Rightarrow$ robust strategy

- Idea: target the middle of the regions

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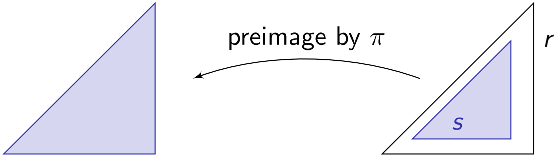
  Let π be a forgetful (in fact complete) cycle.



Forgetful cycle $\Rightarrow$ robust strategy

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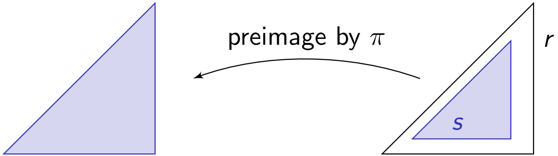

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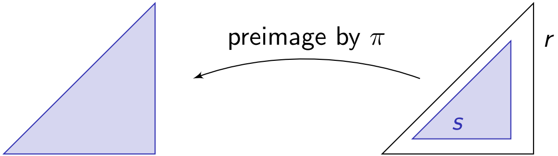

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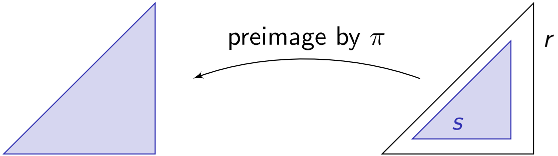

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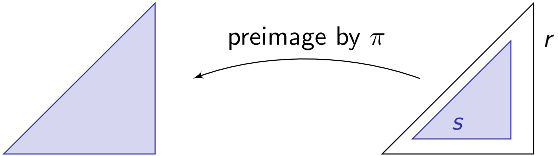

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- Technical tool: shrunk DBMs [BMS12]

[BMS12] Bouyer, Markey, Sankur. Robust reachability in timed automata: A game-based approach. *ICALP'12*.

Conclusion

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Some references:

- PhD thesis of Ocan Sankur: "Robustness in Timed Automata: Analysis, Synthesis, Implementation" (2013)
- Survey at RP'13: "Robustness in Timed Automata" (Bouyer, Markey, Sankur)
- Survey at SiES'11: "Robustness in Real-time Systems" (Markey)