Partial observation of timed systems

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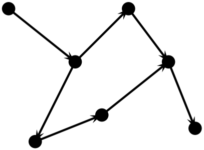
Model-checking

Does the system satisfy the property?

Modelling



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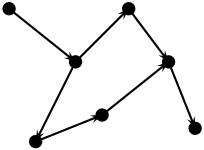

$\models$

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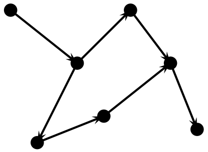
Model-checking Algorithm

Controller synthesis

Can we force the system satisfying the property?

Modelling



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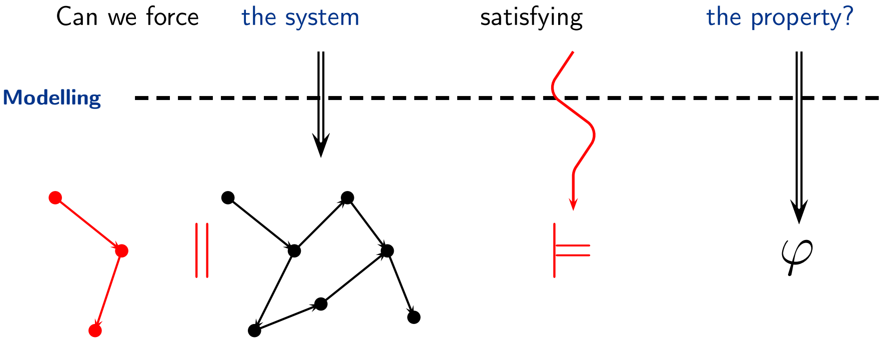


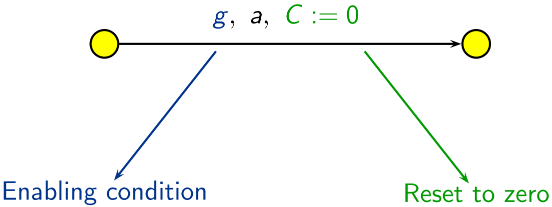
Controller synthesis

Outline

1. Introduction
2. Control synthesis games
3. Control under partial observation
4. Fault diagnosis
5. Conclusion and further developments

Timed automata [Alur & Dill 90's]

- A finite control structure + variables (clocks)
- A transition is of the form:



- An enabling condition (or guard) is:

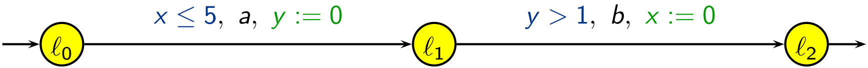
$$g \quad ::= \quad x \sim c \quad | \quad g \wedge g$$

where $\sim \in \{<, \leq, =, \geq, >\}$

- An invariant in each location

Timed automata (example)

x, y : clocks



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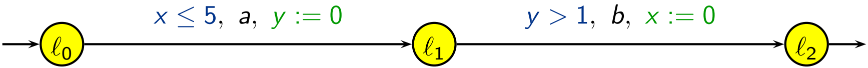

	ℓ_0	$\xrightarrow{\delta(4.1)}$	ℓ_0	$\xrightarrow{a}$	ℓ_1	$\xrightarrow{\delta(1.4)}$	ℓ_1	$\xrightarrow{b}$	ℓ_2
x	0		4.1		4.1		5.5		0
y	0		4.1		0		1.4		1.4

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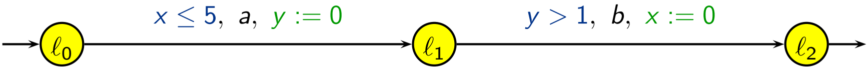

$$
\begin{array}{cccccccccc}
 & \ell_0 & \xrightarrow{\delta(4.1)} & \ell_0 & \xrightarrow{a} & \ell_1 & \xrightarrow{\delta(1.4)} & \ell_1 & \xrightarrow{b} & \ell_2 \\
x & 0 & & 4.1 & & 4.1 & & 5.5 & & 0 \\
y & 0 & & 4.1 & & 0 & & 1.4 & & 1.4
\end{array}
$$

(clock) valuation

Timed automata (example)

x, y : clocks

$$\to \ell_0 \xrightarrow{\quad x \leq 5,\ a,\ y := 0 \quad} \ell_1 \xrightarrow{\quad y > 1,\ b,\ x := 0 \quad} \ell_2 \to$$

$$\begin{array}{cccccccc}
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(clock) valuation

➜ timed word $(a, 4.1)(b, 5.5)$

Fundamental result [Alur & Dill 90's]

Theorem

Emptiness of timed automata is decidable and PSPACE-complete.

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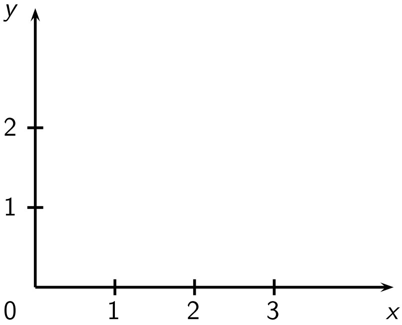
The region abstraction



Equivalence of finite index

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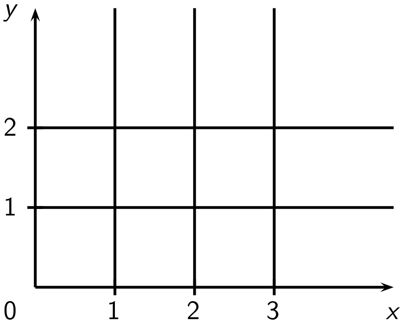

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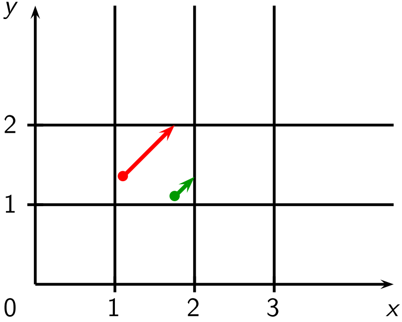

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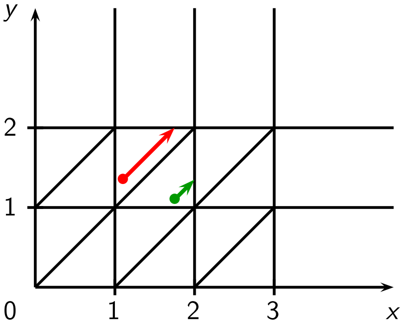

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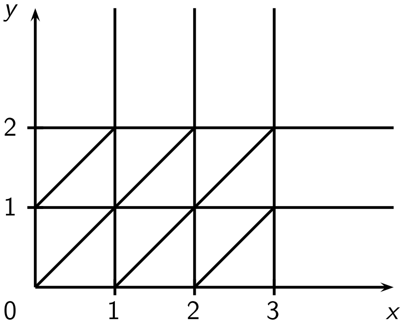

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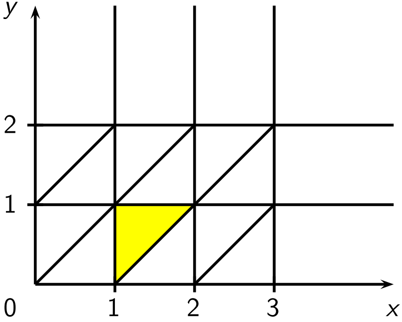

Equivalence of finite index

region defined by

$I_x =]1;2[$, $I_y =]0;1[$

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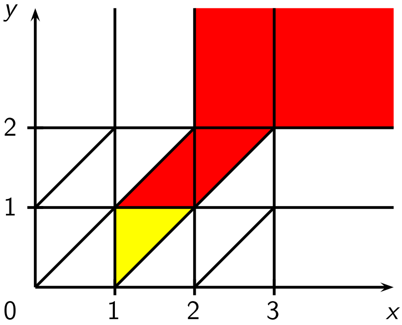

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delay successors

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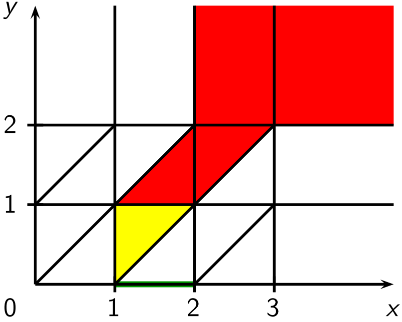

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delay successors

successor by reset

A model not far from undecidability

Properties

- Universality is undecidable [Alur & Dill 90's]
- Inclusion is undecidable [Alur & Dill 90's]
- Determinizability is undecidable [Tripakis 2003]
- Complementability is undecidable [Tripakis 2003]
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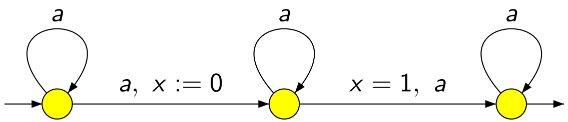
Example

A non-determinizable/non-complementable timed automaton:

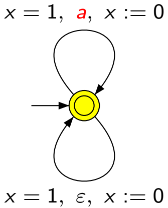


Power of ε-transitions

[Bérard, Diekert, Gastin, Petit 1998]

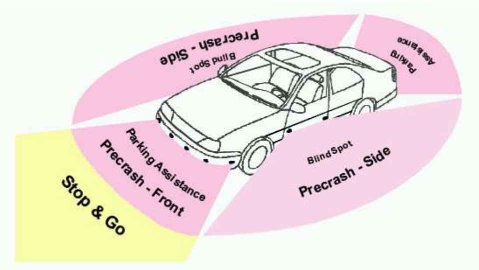
Proposition

- ε-transitions can not be removed in timed automata.
- Timed automata with ε-transitions are strictly more expressive than timed automata without ε-transitions.



Outline

An example, the car periphery supervision



- Embedded system
- Hostile environment
- Sensors
  - distances
  - speeds

Control synthesis games

Environment against controller
(Non-symmetrical game)

- some actions are controllable Σ_c
- some actions are uncontrollable Σ_u
- player "environment" can:
  - interrupt time elapsing,
  - enforce *zeno* behaviours
  - ...
- a **plant** $\mathcal{P}$ is a deterministic timed automaton over alphabet $\Sigma_c \cup \Sigma_u$ (it represents both real system and environment)

Strategies and controllers

- A **strategy** is a partial function

$$f : Runs(\mathcal{P}) \longrightarrow \Sigma_c \cup \{\lambda\} \qquad\qquad \lambda : \text{ time elapsing}$$

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$$f(\rho) = \lambda \implies \exists t > 0,\ \forall 0 \leq t' < t,\ f(\rho \xrightarrow{\delta(t')}) = \lambda$$

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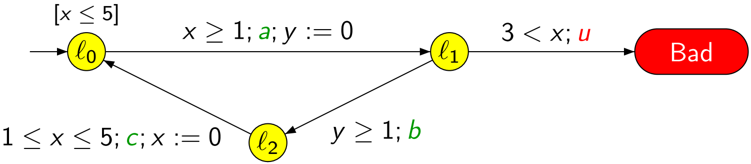
It should not be too powerful!

  - needs to be *non-restricting* for uncontrollable actions
  - needs to be *non-blocking*: if there is no deadlock in the original plant, there will be no deadlock in the controlled system

An example



Aim: control the system in such a way that Bad state is avoided.

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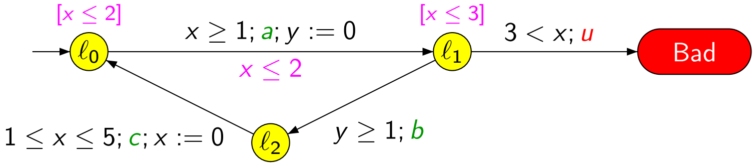

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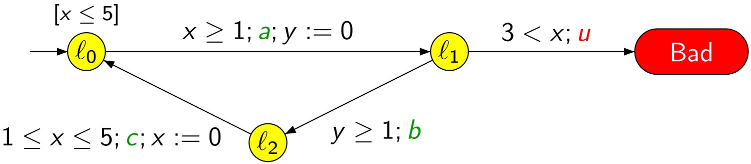

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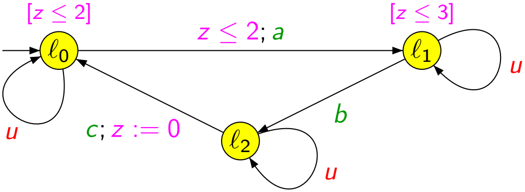

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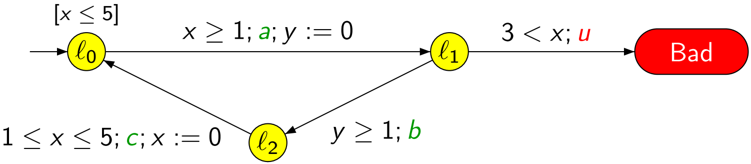

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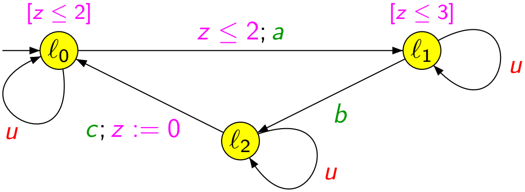

A winning strategy: $\begin{cases} f(\ell_0, x < 1) = \lambda \\ f(\ell_0, x = 1) = a \end{cases}$ $\begin{cases} f(\ell_1, x < 2) = \lambda \\ f(\ell_1, x = 2) = b \\ f(\ell_2, x = 2) = c \end{cases}$

Decidability and complexity

- The **attractor** of a zone-definable set is computable.
- Winning states of safety and reachability games are computable.
- Winning strategies can be computed and are polyhedral.
- Winning strategies can be state-based.

Theorem [Henzinger, Kopke 1999]

Safety and reachability control are decidable and are EXPTIME-complete.

Computing winning states

- controllable and uncontrollable discrete predecessors:

$$\mathrm{cPred}(X) = \bigcup_{c \in \Sigma_c} \mathrm{Pred}^c(X) \qquad \mathrm{uPred}(X) = \bigcup_{u \in \Sigma_u} \mathrm{Pred}^u(X)$$

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- time controllable predecessor of X (Pred_δ):

$$s \xrightarrow{\quad\quad t \quad\quad} s' \in X$$

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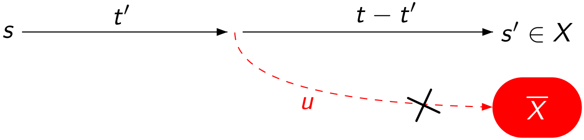

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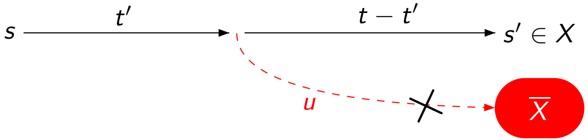


- **winning states:** greatest fixed point of

$$\pi(X) = \mathrm{Pred}_\delta(X \cap \mathrm{cPred}(X), \mathrm{uPred}(\overline{X}))$$

Further objectives

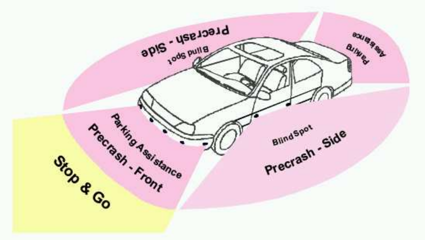
- TCTL objectives **[Faella, La Torre, Murano 2002]**
- CTL, LTL objectives **[Faella, La Torre, Murano 2002]**
- general symmetric parity games **[de Alfaro, Faella, Henzinger, Majumdar, Stoelinga 2003]**
- external specifications given by timed automata **[D'Souza, Madhusudan 2002]**

➜ use theory of classical untimed games

Outline

Why partial observation?

Example (The car periphery supervision)



Environment is seen through **sensors**.

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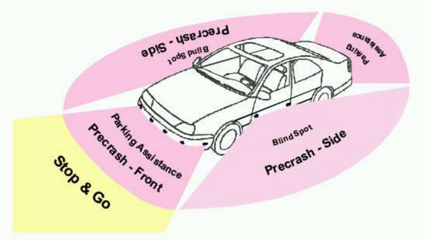

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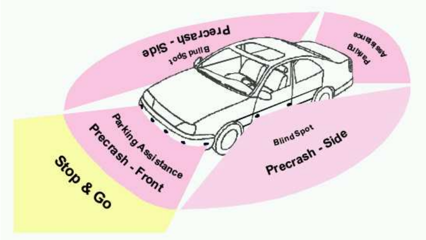

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[Partial observation]

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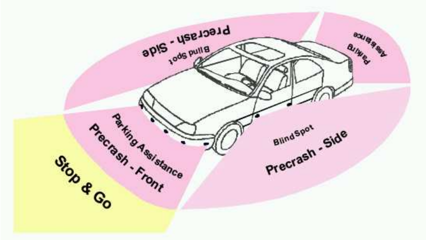

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[Partial observation]

Stumbling blocks:

- ε-transitions can not be removed from timed automata
- timed automata can not be determinized

Control under partial observation

Theorem [Bouyer, D'Souza, Madhusudan, Petit 2003]

Safety and reachability control under partial observation is undecidable.

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➜ by reduction of universality problem for timed automata

Take $\mathcal{A}$ a (complete) timed automaton. Construct $\mathcal{P}$ as follows.

$\ell \xrightarrow{g,a,C:=0} \ell'$ is replaced by $\ell \xrightarrow{(\ell,g,a,C:=0,\ell'),\, z:=0} \bullet \xrightarrow{g \wedge z=0,\, a,\, C:=0} \ell'$

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Thus,

- $\mathcal{P}$ is a *deterministic* timed automaton, thus a plant
- $(\delta_0, t_0)(a_0, t'_0)(\delta_1, t_1)(a_1, t'_1)\ldots$ is accepted by $\mathcal{P}$ iff $t_i = t'_i$ for every i and $(a_0, t_0)(a_1, t_1)\ldots$ is accepted by $\mathcal{A}$ along the path $\delta_0\delta_1\ldots$

We note $\Delta = \{(\ell, g, a, C := 0, \ell')$ transition of $\mathcal{A}\}$ and make all actions from Δ non-observable.

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NB: this undecidability result seems robust...

Fixing resources

[Bouyer, D'Souza, Madhusudan, Petit 2003]

Resources: $\mu = (X, m, \max)$

$$x \sim c \quad \Longrightarrow \quad c \in \frac{\mathbb{Z}}{m} \quad \text{and} \quad |c| \leq \max$$

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With fixed resources, control of simple winning objectives becomes decidable (and 2EXPTIME-complete).

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With fixed resources, control of simple winning objectives becomes decidable (and 2EXPTIME-complete).

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Fixing resources

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Control under partial observation
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➜ We focus on a simpler problem, where partial observation is crucial

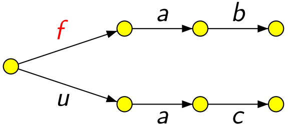
Outline

Principle of fault diagnosis

[Sampath, Sengupta, Lafortune, Sinnamohideen, Teneketzis 1995]

Principle: "observe the behavior of a plant, and tell if something wrong has happened"

System:



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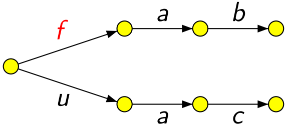

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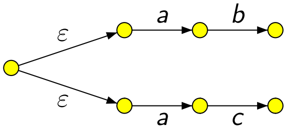

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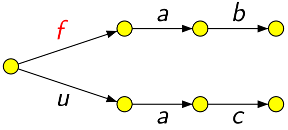

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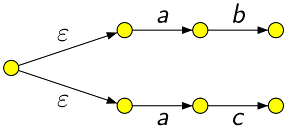

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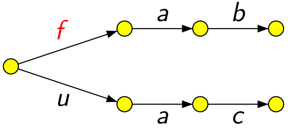

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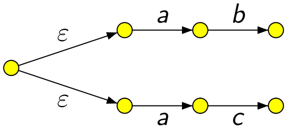

Observation: «ab» or «ac»

Did a fault occur?

The timed framework

- Plant = timed automaton
- Σ_o observable events, and Σ_u unobservable events

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Example: $\Sigma_o = \{a, b\}$ $\Sigma_u = \{f\}$

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- Observation: $\pi(w) = (a, 1)(b, 4.5)$

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1-diagnoser:	has to announce fault on $\pi(w)$
2-diagnoser:	can announce fault on $\pi(w)$
	may announce nothing on $\pi(w)$

Δ-diagnosis

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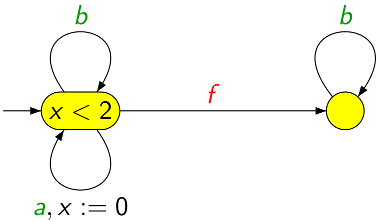

This system is 2-diagnosable... but not 1-diagnosable because $(f,0)(b,1)$ and $(b,1)$ raise the same observation.

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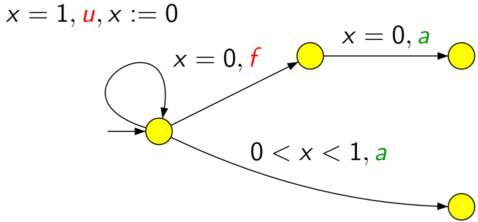
$$L_{\Delta f}(\mathcal{P}) \subseteq L(\mathcal{O}) \subseteq L_{\neg f}(\mathcal{P})^c$$

Diagnosis with deterministic timed automata

- less general than previous diagnosis



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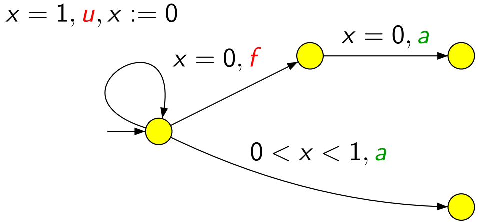

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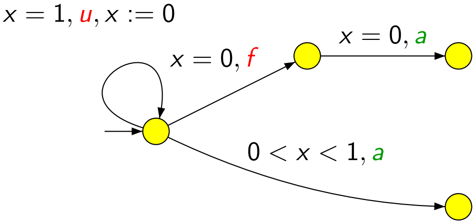

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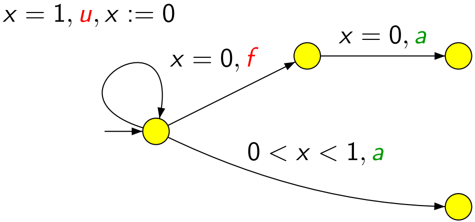

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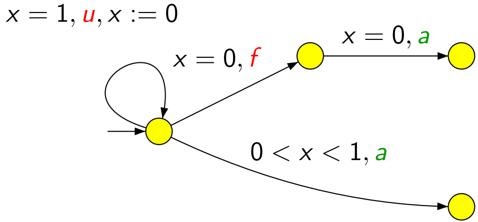

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Theorem **[Bouyer, Chevalier, D'Souza 2005]**

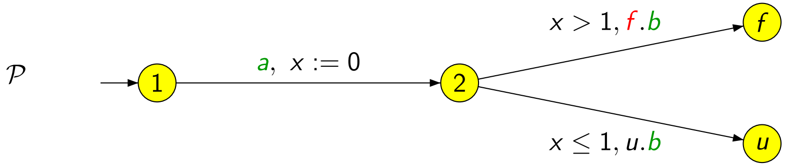
Δ-diagnosis of timed systems with DTA_μ is 2EXPTIME-complete.

Observation as a game

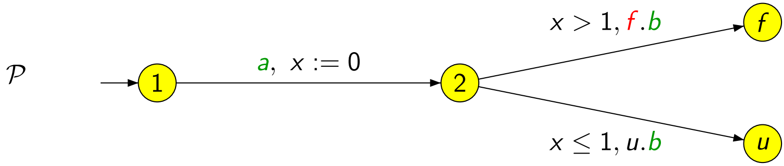
We will transform the diagnosis problem into a two-player safety game:

- one player is the observer $\square$
- the other player is the environment $\bigcirc$

The plant is Δ-DTA$_\mu$-diagnosable iff $\square$ has a winning strategy



Is there an observer for the plant with one clock and constants 0 and 1?



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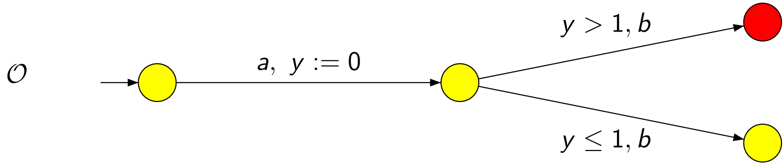
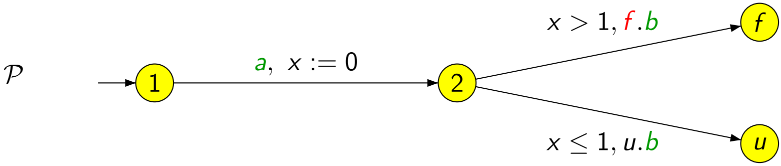
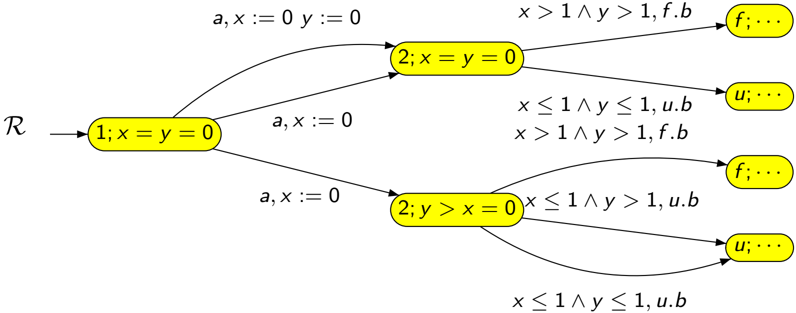
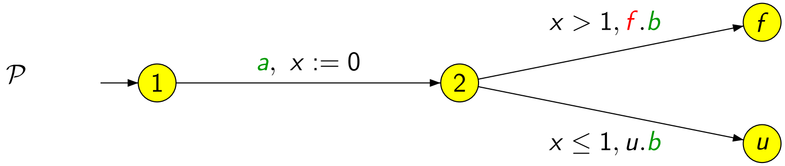
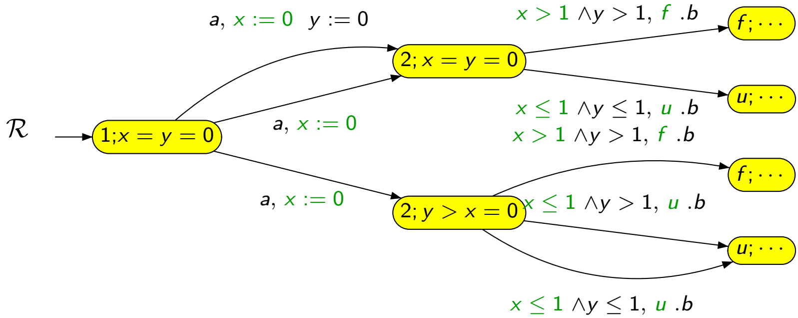



$\mathcal{P}$
1
$a, \ x := 0$
2
$x > 1, f.b$
$x \leq 1, u.b$
f
u


$\mathcal{R}$
$1; x = y = 0$
$a, x := 0 \ y := 0$
$a, x := 0$
$a, x := 0$
$2; x = y = 0$
$2; y > x = 0$
$x > 1 \wedge y > 1, f.b$
$x \leq 1 \wedge y \leq 1, u.b$
$x > 1 \wedge y > 1, f.b$
$x \leq 1 \wedge y > 1, u.b$
$x \leq 1 \wedge y \leq 1, u.b$
$f; \cdots$
$u; \cdots$
$f; \cdots$
$u; \cdots$


$\mathcal{P}$
1
$a,\ x := 0$
2
$x > 1, f.b$
f
$x \leq 1, u.b$
u


$\mathcal{R}$
$1; x = y = 0$
$a,\ x := 0\ \ y := 0$
$a,\ x := 0$
$a,\ x := 0$
$2; x = y = 0$
$2; y > x = 0$
$x > 1 \wedge y > 1,\ f\ .b$
$f; \cdots$
$x \leq 1 \wedge y \leq 1,\ u\ .b$
$u; \cdots$
$x > 1 \wedge y > 1,\ f\ .b$
$f; \cdots$
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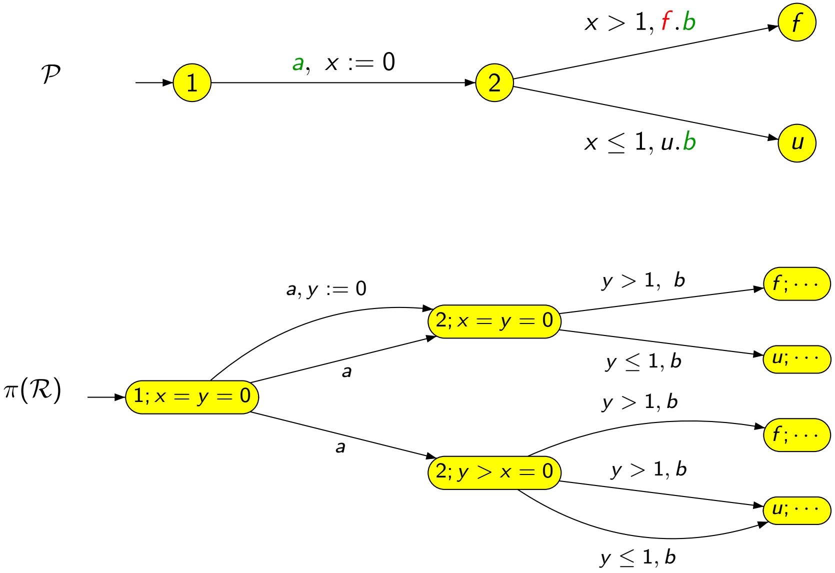
$\mathcal{P}$
1
$a,\ x := 0$
2
$x > 1, f.b$
f
$x \leq 1, u.b$
u
$\pi(\mathcal{R})$
$1; x = y = 0$
$a, y := 0$
a
a
$2; x = y = 0$
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$y > 1,\ b$
$y \leq 1, b$
$y > 1, b$
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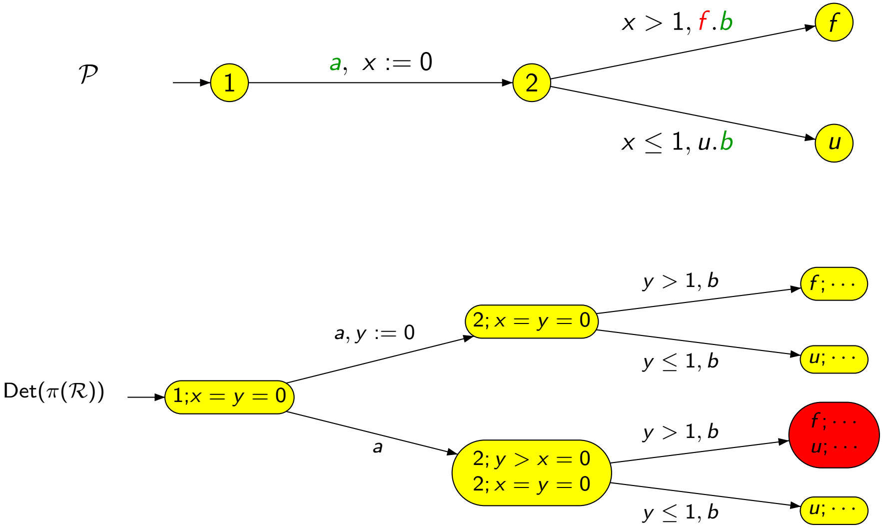
$\mathcal{P}$
1
$a, \ x := 0$
2
$x > 1, f.b$
f
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u
$\mathrm{Det}(\pi(\mathcal{R}))$
$1; x = y = 0$
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$u; \cdots$
a
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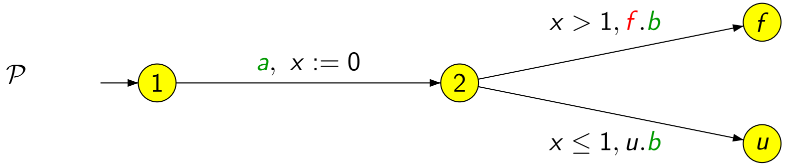
$\mathcal{P}$
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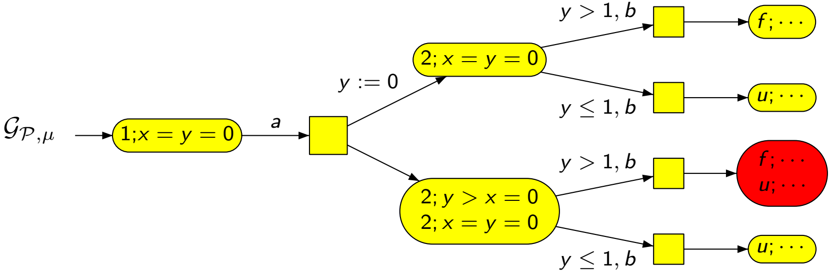
f
u


$\mathcal{G}_{\mathcal{P},\mu}$
1;$x = y = 0$
a
$y := 0$
2; $x = y = 0$
2; $y > x = 0$
2; $x = y = 0$
$y > 1, b$
$y \leq 1, b$
$y > 1, b$
$y \leq 1, b$
$f; \cdots$
$u; \cdots$
$f; \cdots$
$u; \cdots$
$u; \cdots$

Diagnosis by DTA$_\mu$

Proposition

$\square$ has a winning strategy in $\mathcal{G}_{\mathcal{P},\mu}$ iff there is a diagnoser for $\mathcal{P}$ in DTA$_\mu$.

➜ Δ-DTA$_\mu$-diagnosability is in 2EXPTIME

Δ-DTA$_\mu$-observability if 2EXPTIME-hard

$\longrightarrow$ By reduction of the acceptance of
an Alternating Turing Machine using exponential space

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$\rightarrow$ By reduction of the acceptance of
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- The plant plays "a"'s.
- The diagnoser reads these "a"'s and plays a sequence of configurations.
- The plant verifies that this sequence is correct.

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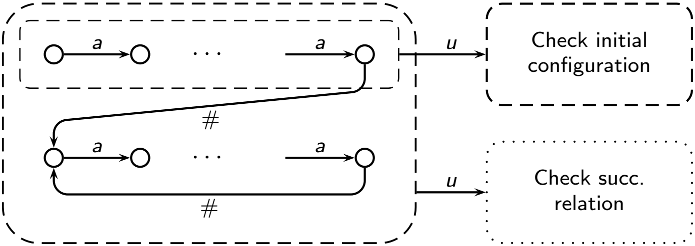
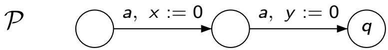
NB: the plant non-deterministically chooses one test

Shape of the plant



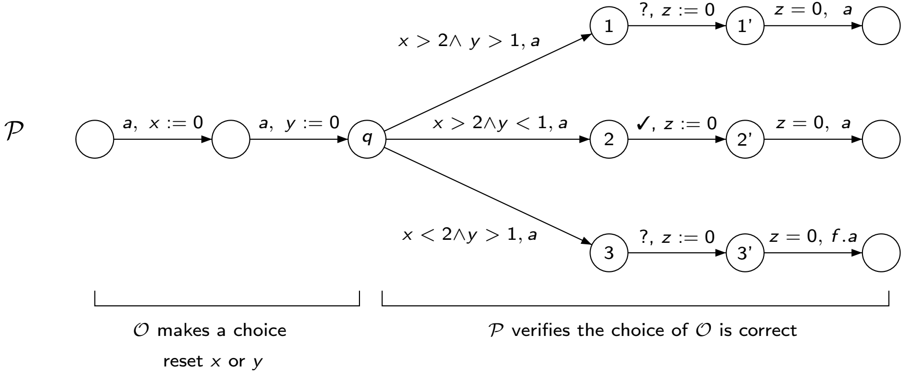
$\mathcal{O}$ has 1 clock.



$\mathcal{O}$ makes a choice

reset x or y

$\mathcal{O}$ has 1 clock.



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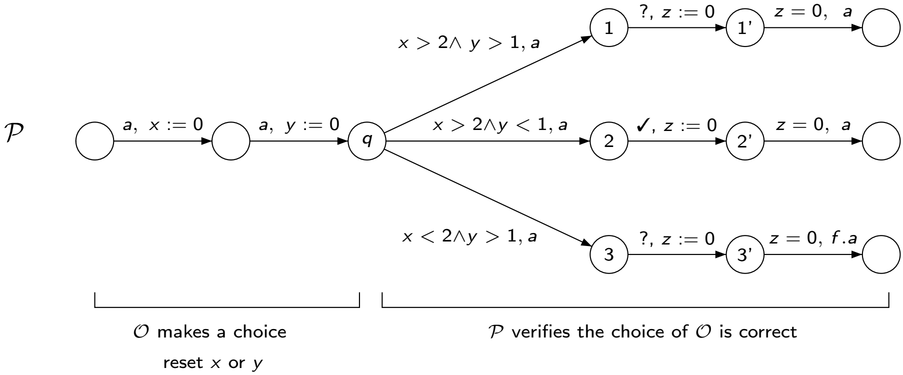

$\mathcal{P}$ can force $\mathcal{O}$ "remember" x:

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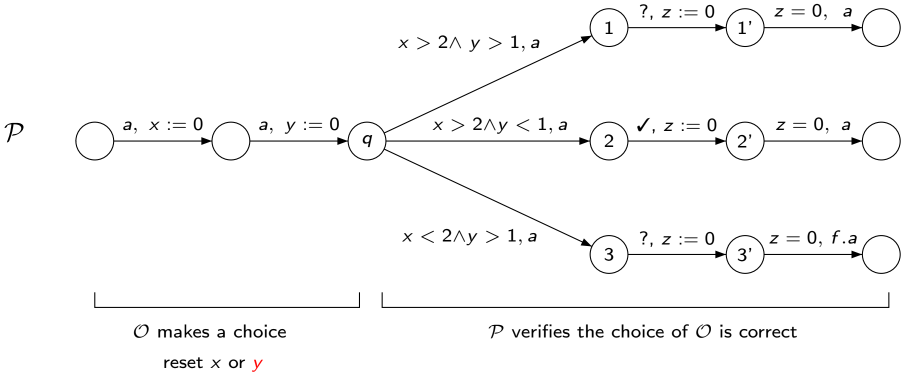

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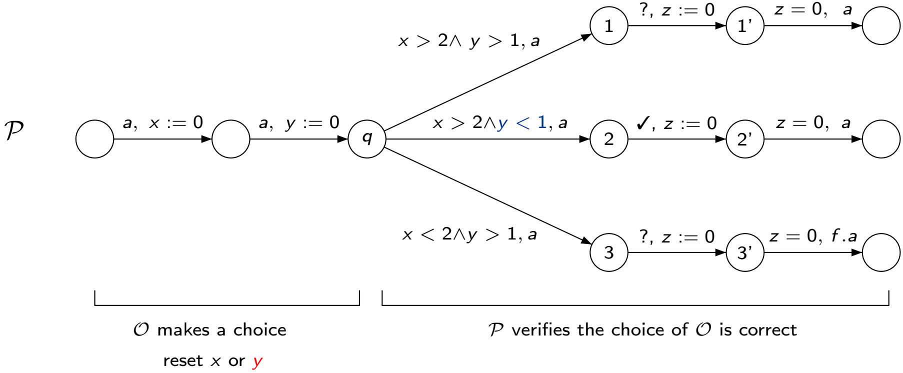

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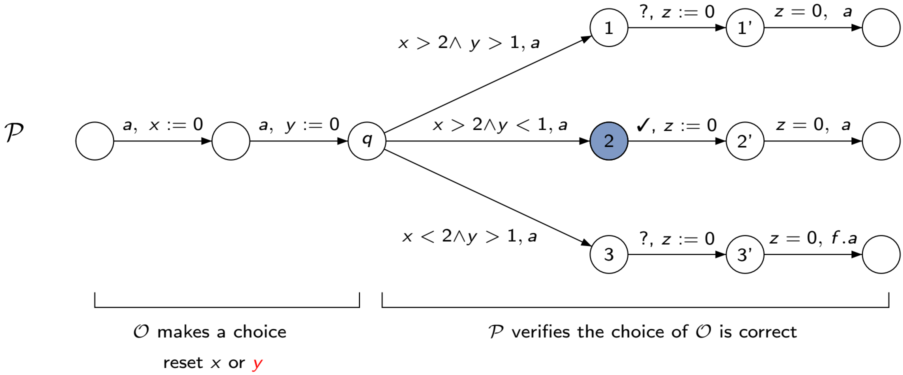

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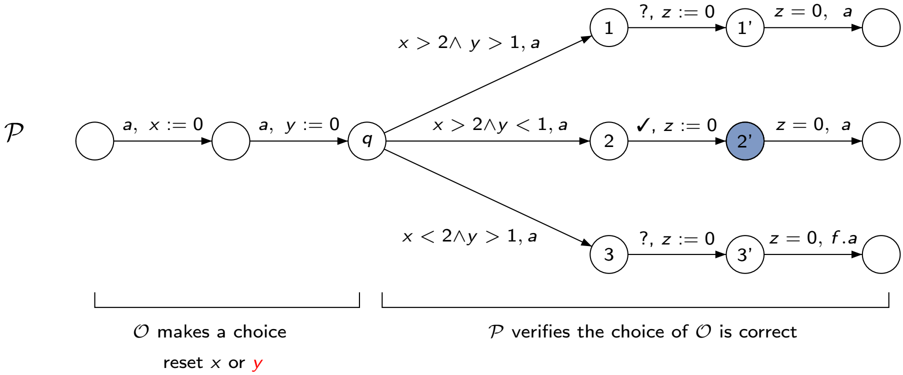

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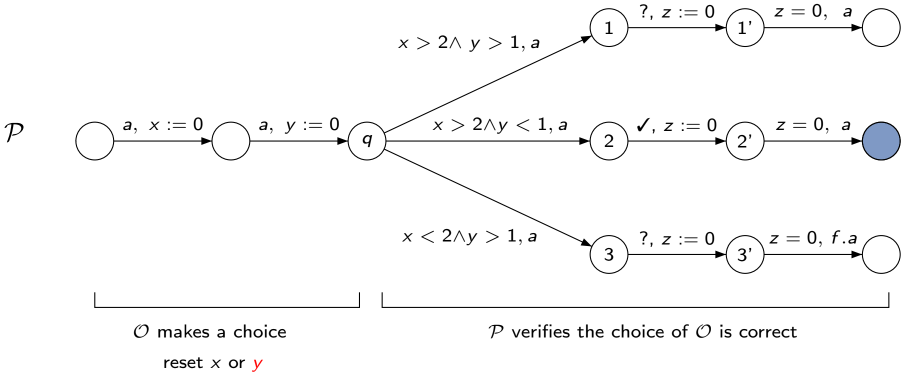

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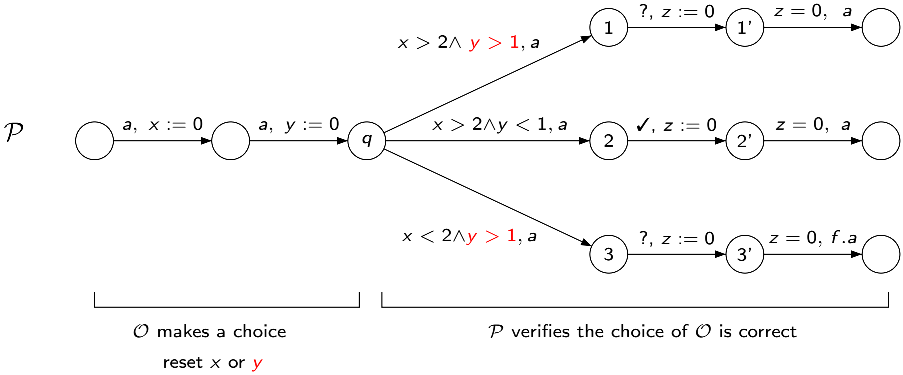

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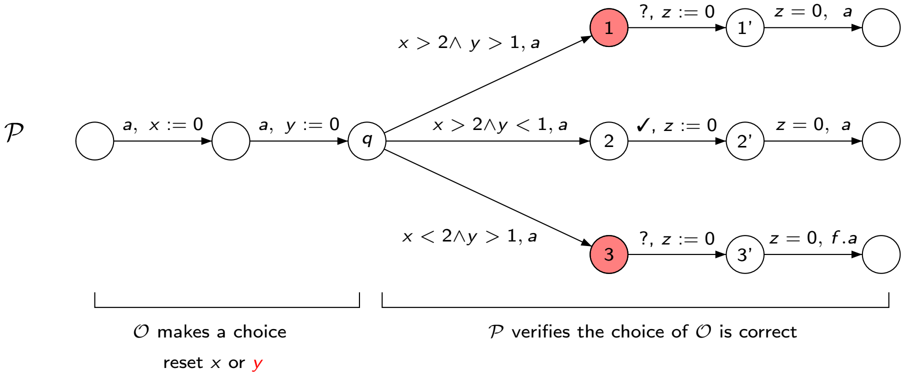

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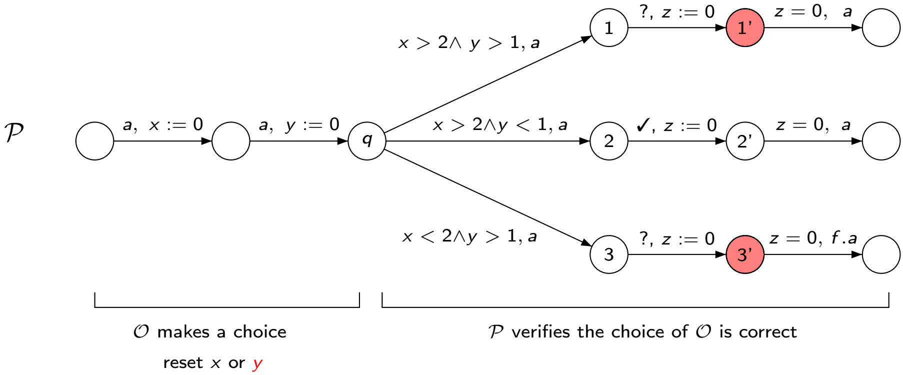

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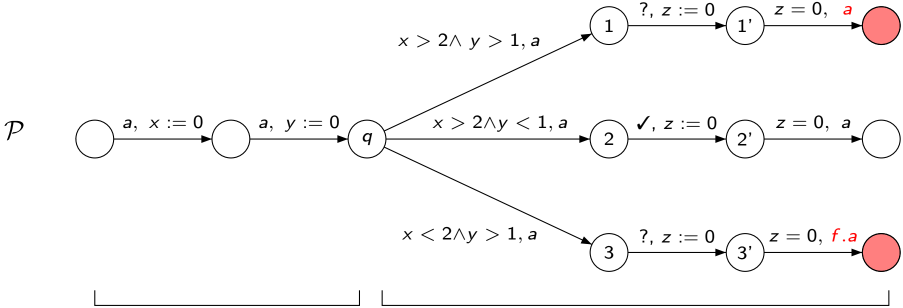

$\mathcal{O}$ makes a choice

reset x or y

$\mathcal{P}$ verifies the choice of $\mathcal{O}$ is correct

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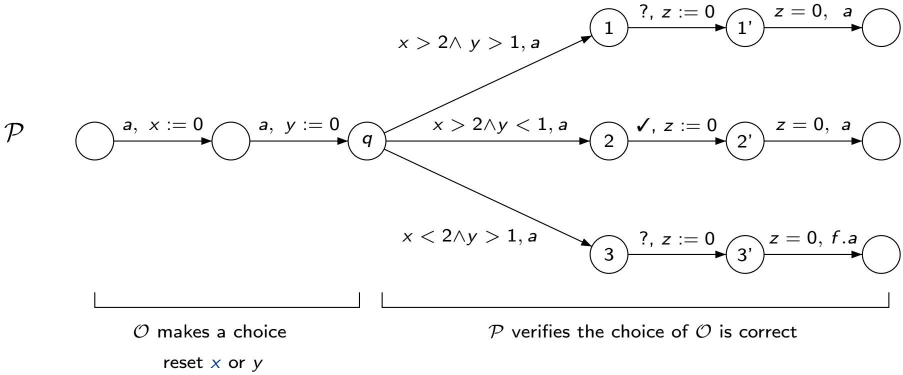

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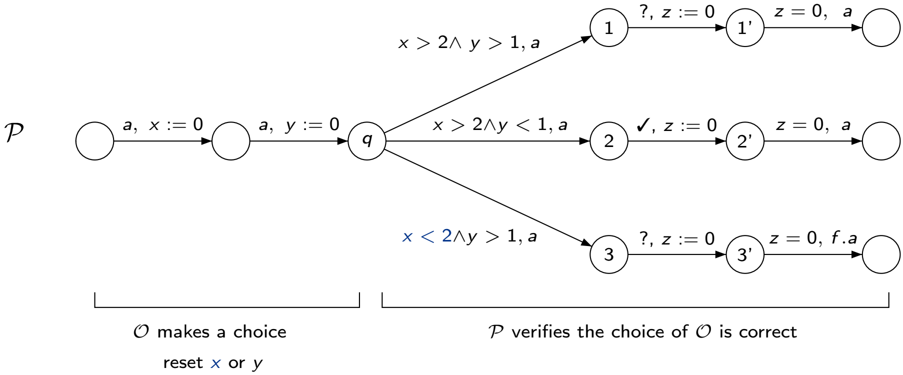

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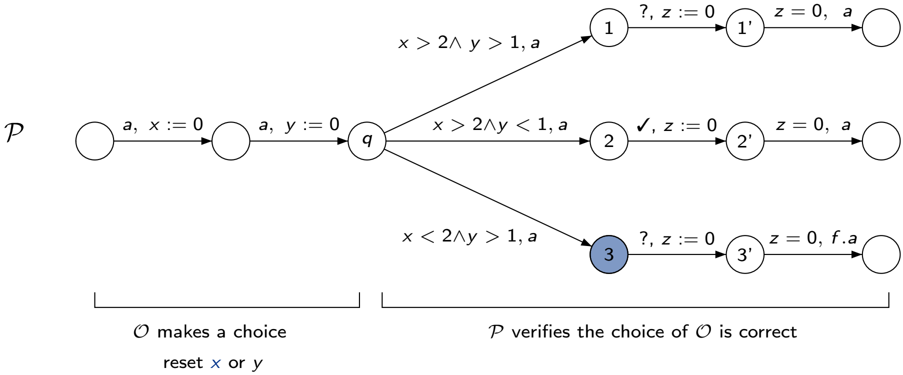

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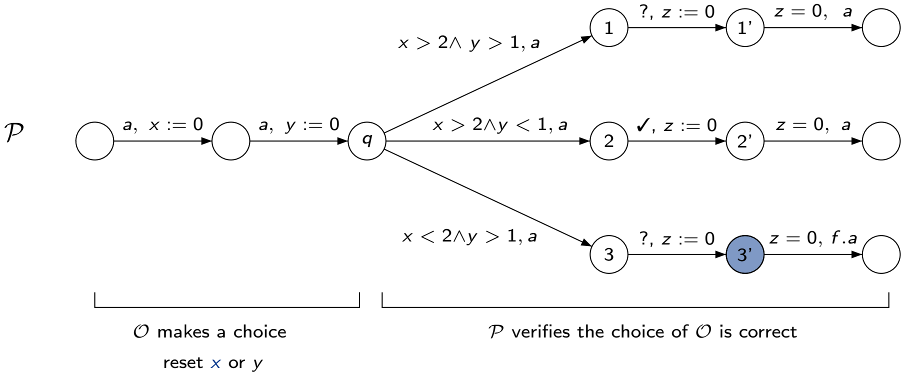

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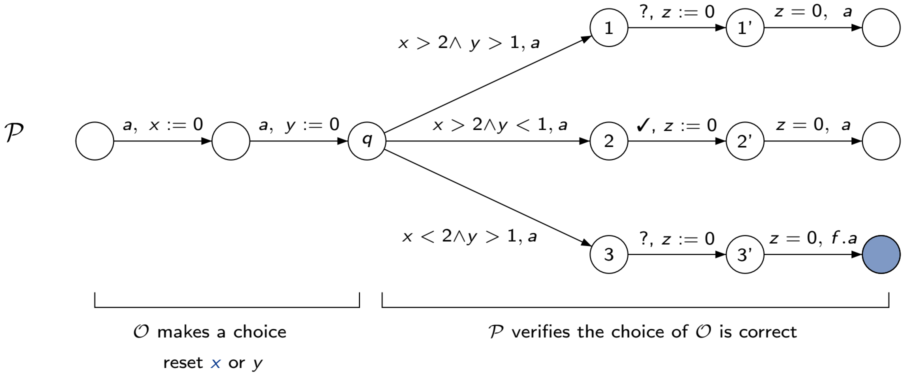

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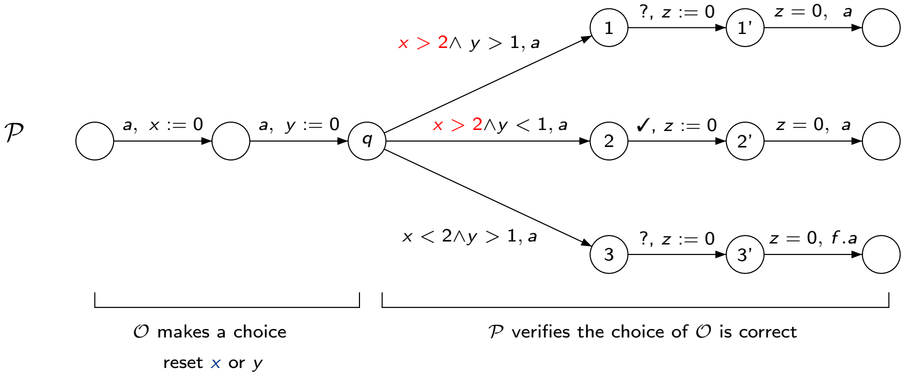

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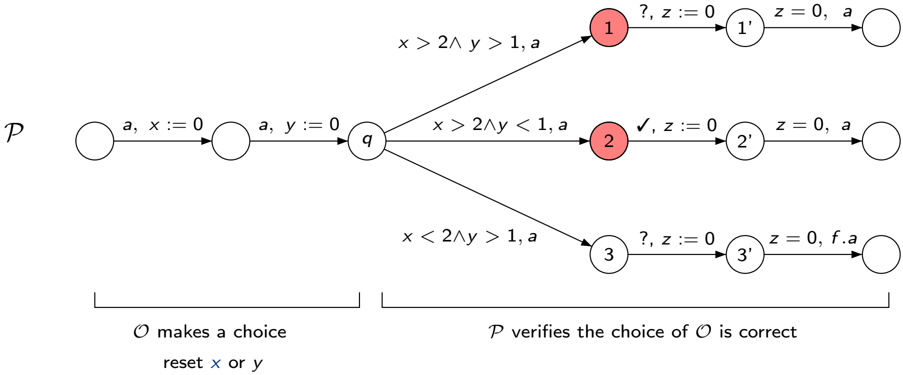

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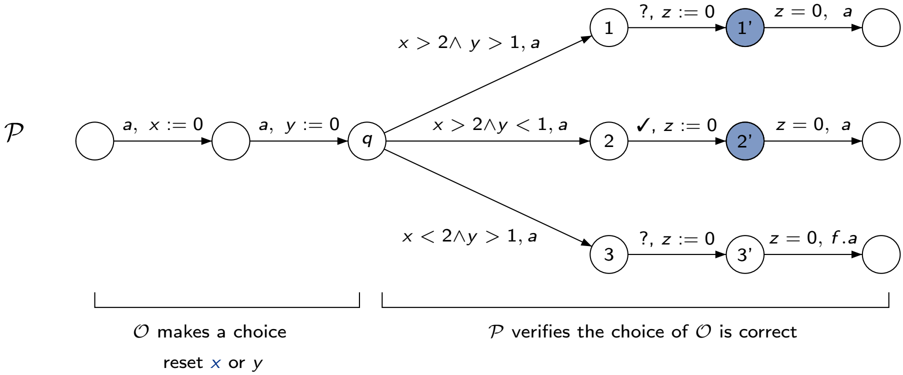

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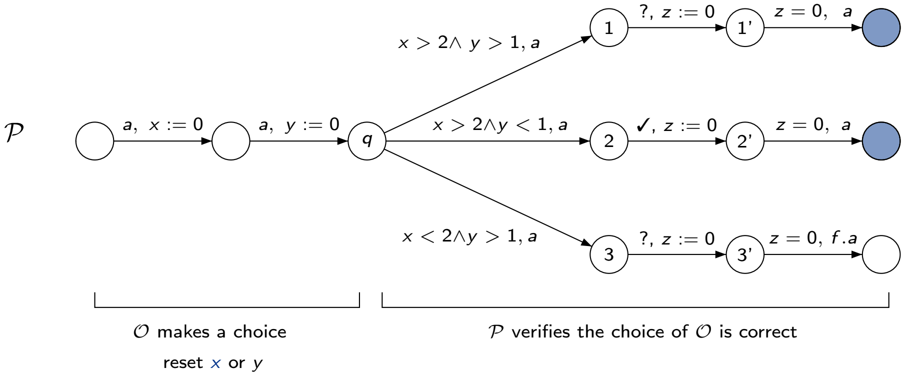

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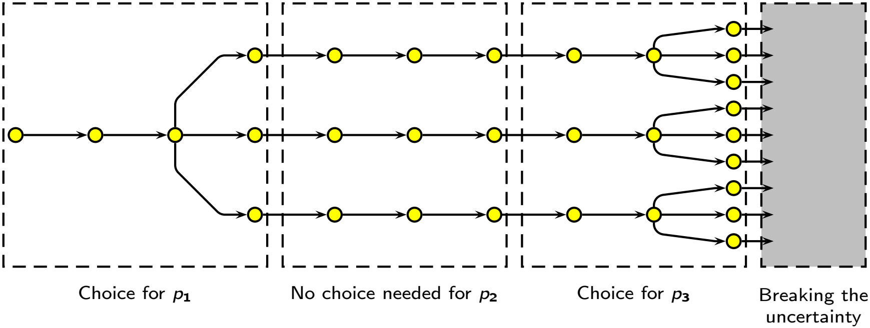
An example of encoding for a 3SAT formula

Formula $p_1 \vee \neg p_3$**:**



Diagnosis by event-recording timed automata

- one clock x_a per event a
- clock x_a is reset when a occurs

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➜ Diagnosis (with bounded resources) becomes PSPACE-complete
[BCD05]

Outline

1. Introduction
2. Control synthesis games
3. Control under partial observation
4. Fault diagnosis
5. **Conclusion and further developments**

Conclusion & further developments

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- Partial observation adds complexity to control problems
- Even fault diagnosis is difficult
- **Related domains:** conformance testing, monitoring. . .

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Further developments

- Algorithms for control under partial observation
  e.g. forward zone-based algorithm (*cf* Emmanuel's talk)
- Fault diagnosis with DTA/ERA
- Get rid of some resources or the Δ parameter
- Control under partial observation for other classes of systems
  (*e.g.* o-minimal hybrid games)

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