Nash equilibria in games on graphs with a public signal monitoring

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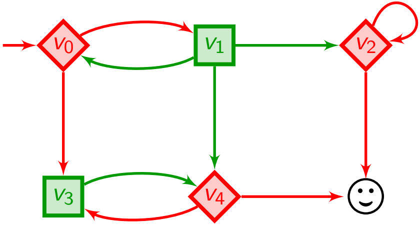
What this talk is about

- pure Nash equilibria in game graphs
- imperfect information monitoring
- public signals
- epistemic abstraction
- computability issues

Two-player turn-based zero-sum games



- Game graph $G = (V, E)$
- V partitioned into $V_\Diamond$ and $V_\Box$

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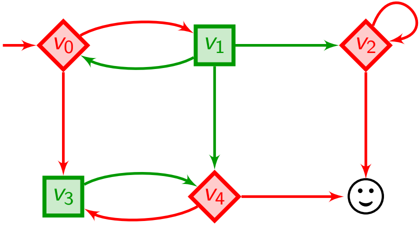

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$$\sigma_i : V^* V_i \to V$$

- out($\sigma_\diamond$): all paths compatible with $\sigma_\diamond$

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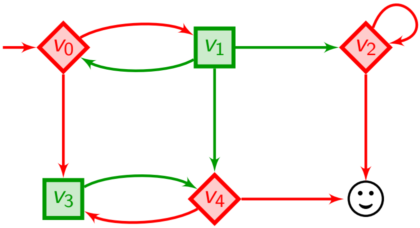

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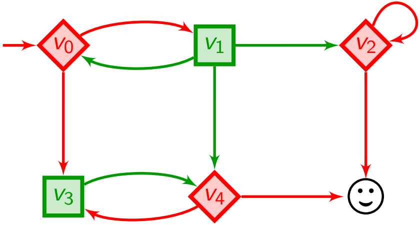
- $\mathrm{out}(\sigma_\diamond)$: all paths compatible with $\sigma_\diamond$

- Objective for $\diamond$: $\Omega \subseteq V^\omega$
- $\sigma_\diamond$ winning strat. if $\mathrm{out}(\sigma_\diamond) \subseteq \Omega$

Two-player turn-based zero-sum games



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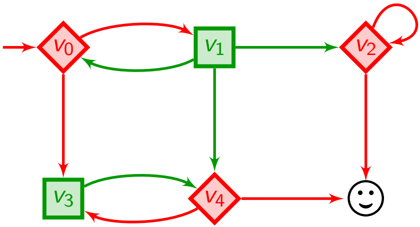

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- Game graph $G = (V, E)$
- V partitioned into $V_\diamond$ and $V_\square$

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- Objective for $\diamond$: $\Omega \subseteq V^\omega$
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- Determinacy: Either $\diamond$ has a winning strategy for Ω, or $\square$ has a winning strategy for $V^\omega \setminus Omega$

Non-zero-sum multiplayer games

- Several players $\text{Agt} = \{A_1, \ldots, A_N\}$

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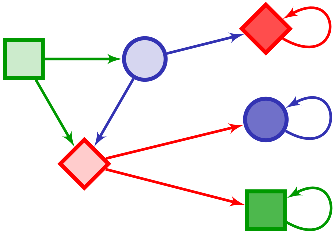
- The simplest: Nash equilibria

Nash equilibria in turn-based games

Nash equilibrium

A strategy profile $(\sigma_A)_{A \in \mathsf{Agt}}$ is a Nash equilibrium if no player can improve her payoff by unilaterally changing her strategy.



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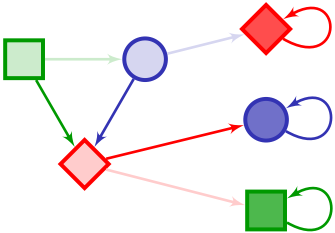

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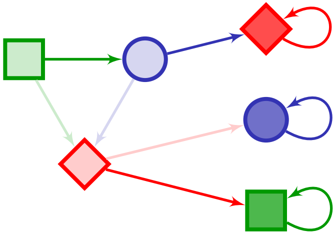

is not a Nash equilibrium

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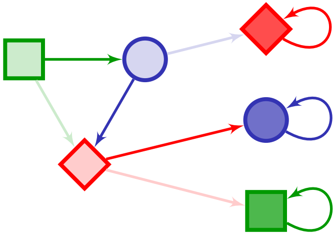

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Boolean Nash equilibria in turn-based games

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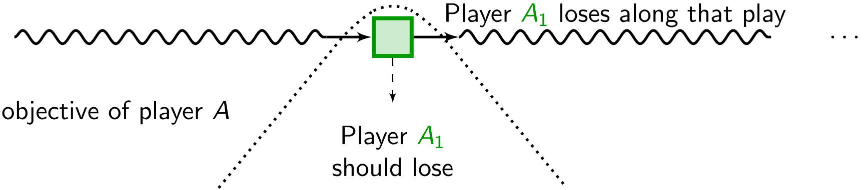

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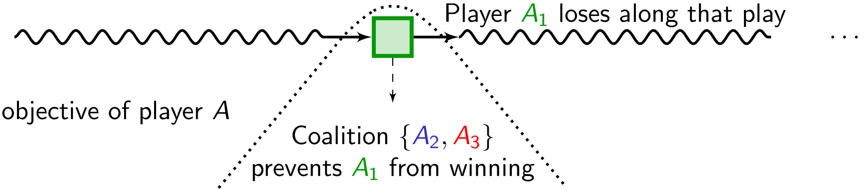

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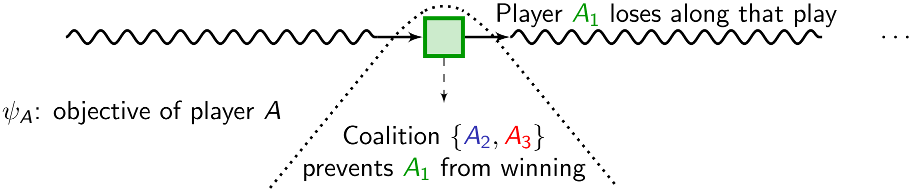

Recipe

- for every $A \in \mathsf{Agt}$, compute the set of winning states W_A
- find a path witness for the formula:

$$\Phi_{\mathsf{NE}} = \bigwedge_{A \in \mathsf{Agt}} \left(\neg\psi_A \Rightarrow \mathbf{G}\neg W_A\right)$$

(valid for tail or reachability objectives)

Existing results in the framework of turn-based games

[UW11,Umm11]

- There always exists a Nash equilibrium for Boolean ω-regular objectives

[UW11] Ummels, Wojtczak. The Complexity of Nash Equilibria in Stochastic Multiplayer Games *(LMCS)*
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$\leadsto$ this is why we restrict to pure equilibria

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What about concurrent games?

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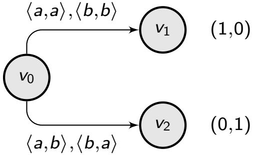
The matching-penny game:



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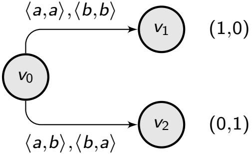

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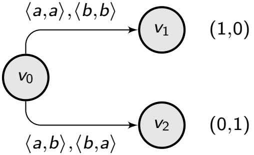

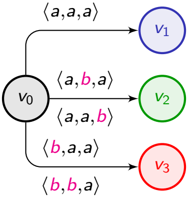

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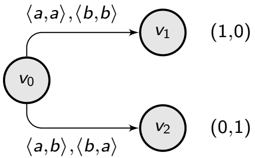

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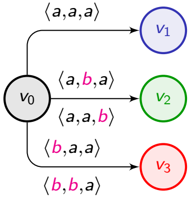

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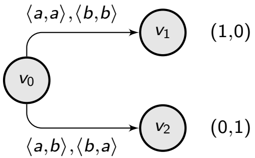

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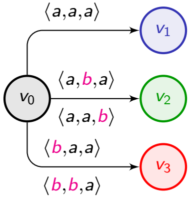

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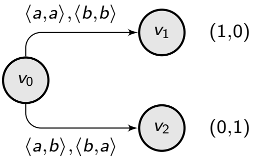

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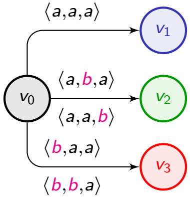

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Solution via the suspect game abstraction,
a structure to track suspect players

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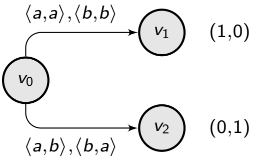

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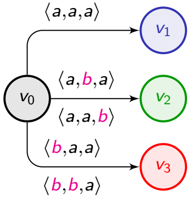

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Can we add more partial information to that framework?

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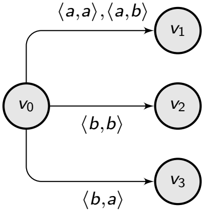
Concurrent games with signals



[Tom98] Tomala. Pure equilibria of repeated games with public observation *(International Journal of Game Theory)*

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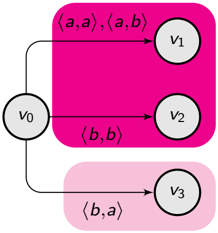

- Signal for player A_1: ● and ●

- On playing a, player A_1 will receive ●
- On playing b, player A_1 will receive either ● or ●

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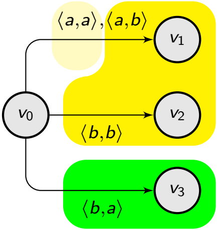

- Signal for player A_1:  and
- Signal for player A_2: , and 

- On playing a, player A_1 will receive 
- On playing b, player A_1 will receive either  or
- On playing a, player A_2 will receive either or 
- On playing b, player A_2 will receive

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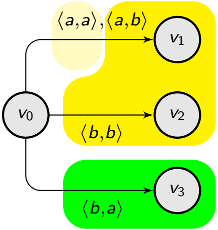

- Signal for player A_1: ● and ●
- Signal for player A_2: ●, ● and ●

- On playing a, player A_1 will receive ●
- On playing b, player A_1 will receive either ● or ●
- On playing a, player A_2 will receive either ● or ●
- On playing b, player A_2 will receive ●

Public signal

Same signal to every player!

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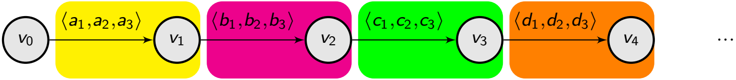
Our specific framework



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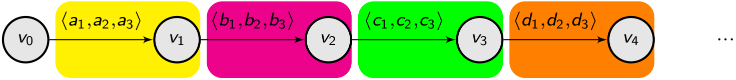
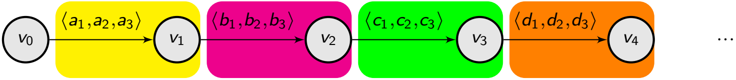


- What player A_i sees:

a_i ● b_i ● c_i ● d_i ● $\dots$

Our specific framework



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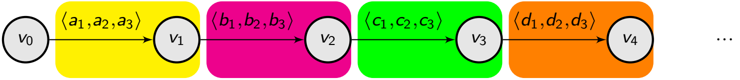

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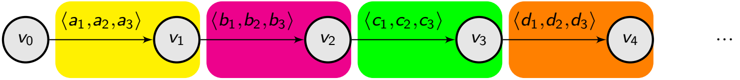

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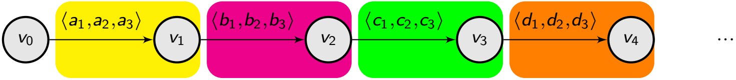

- What player A_i sees:

$$a_i \; \text{●} \; b_i \; \text{●} \; c_i \; \text{●} \; d_i \; \text{●} \; \cdots$$

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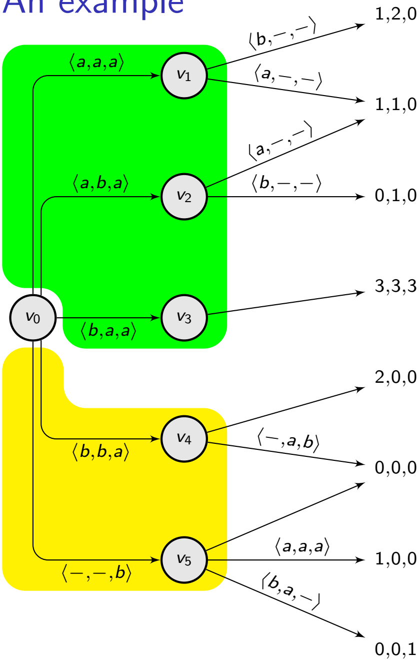
- Publicly visible payoff: based on sequences of colors

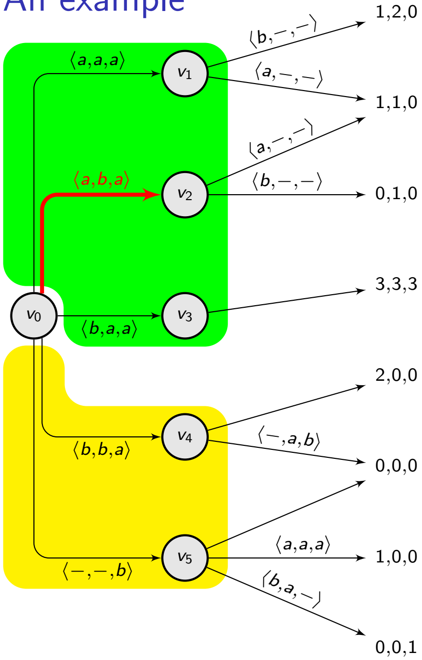
$$\text{●} \; \text{●} \; \text{●} \; \text{●} \; \ldots$$

An example



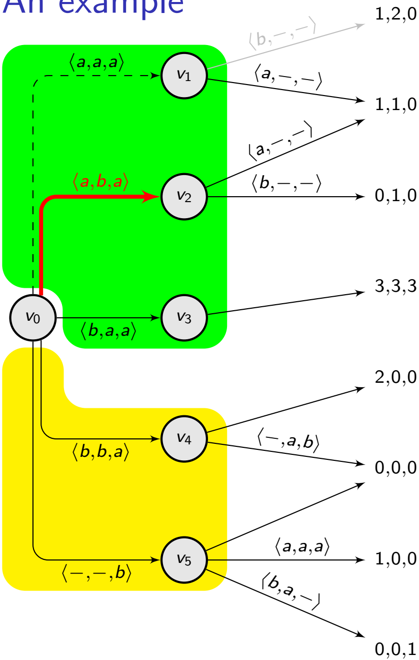
- Three players concurrent game with public signal

An example



- Three players concurrent game with public signal
- Consider the (partial) strategy profile σ_{Agt}. Can we complete it into a Nash equilibrium?

An example



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- This is an A_2-deviation, which is invisible to both A_1 and A_3. A_1 has to play a.

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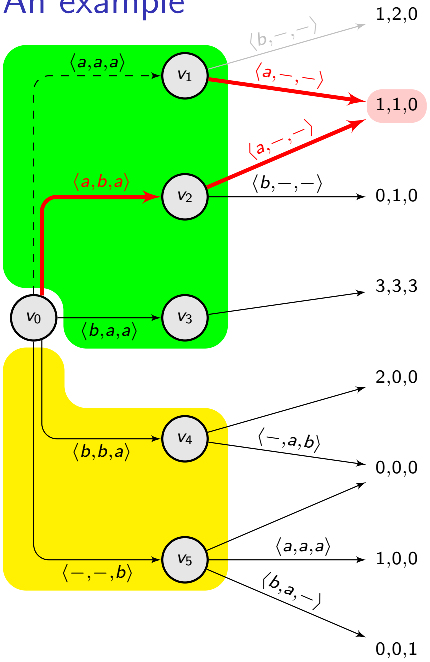

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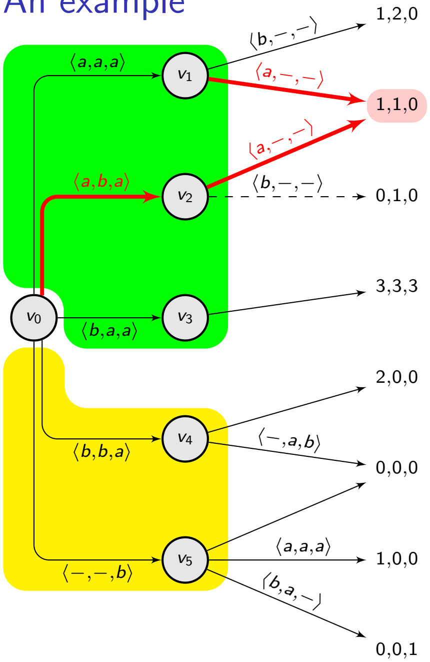

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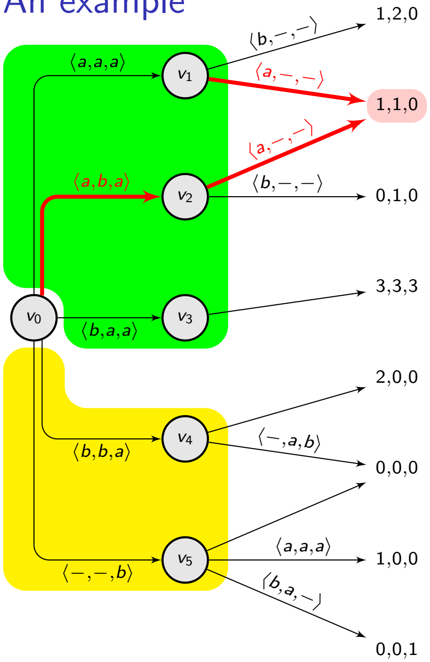
- This is a non-profitable A_1-deviation.

An example



- Three players concurrent game with public signal
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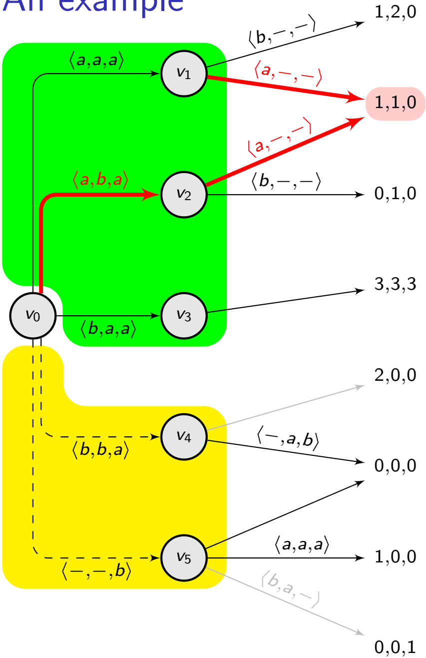

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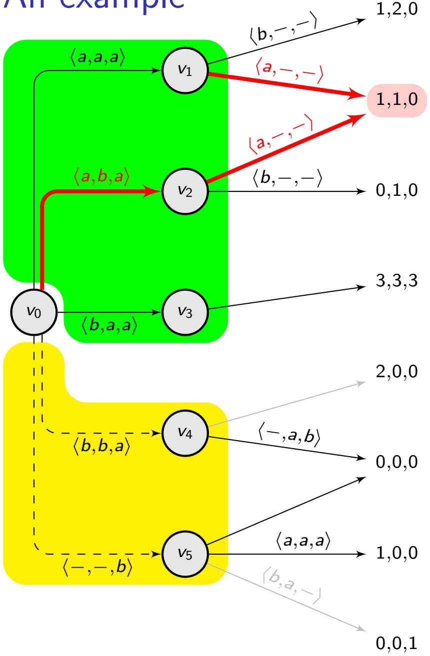

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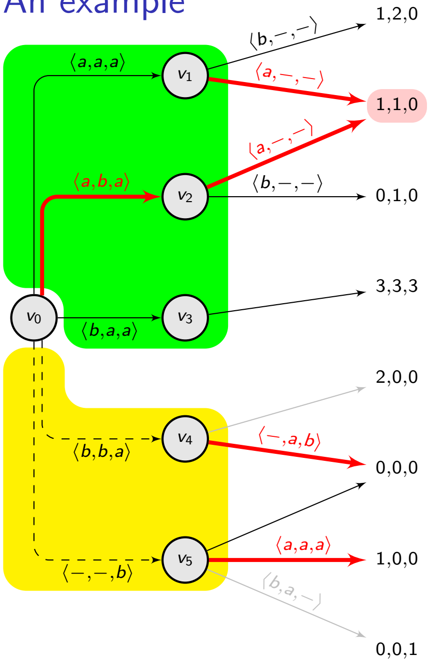

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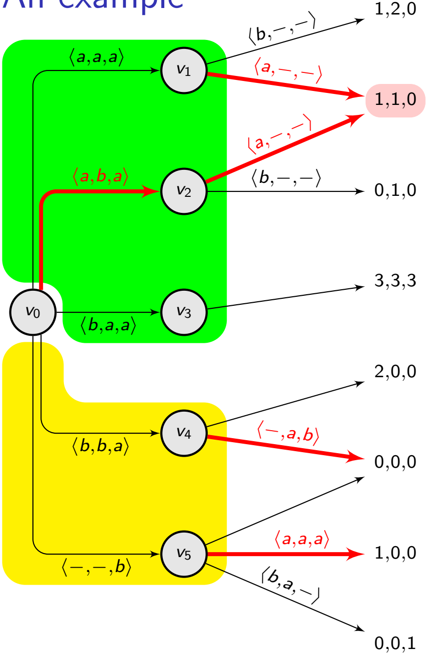

1,2,0
⟨b,−,−⟩
⟨a,a,a⟩
v_1
⟨a,−,−⟩
1,1,0
⟨a,−,−⟩
⟨a,b,a⟩
v_2
⟨b,−,−⟩
0,1,0
3,3,3
v_0
v_3
⟨b,a,a⟩
2,0,0
v_4
⟨−,a,b⟩
⟨b,b,a⟩
0,0,0
⟨a,a,a⟩
v_5
1,0,0
⟨−,−,b⟩
⟨b,a,−⟩
0,0,1

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- How to systematically track all individual deviations and uncertainty induced by imperfect information monitoring?
- Is that always possible?
- Can we build a finite epistemic structure?

The epistemic game abstraction

Inspired by:

- the standard powerset construction [Rei84]
- the epistemic unfolding for coordination/distributed synthesis [BKP11]
- the suspect game [BBMU15]
- the deviator game [Bre16]

[Rei84] Reif. The complexity of two-player games of incomplete information *(J. Comp. and Syst. Sc.)*
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The idea is to track all possible undistinguishable behaviours, including the single-player deviations

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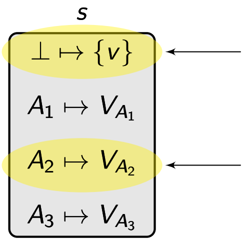
Epistemic states



vertex the game is in
if no deviation has occurred

vertices the game might be in
if A_2 has deviated (invisible deviation)

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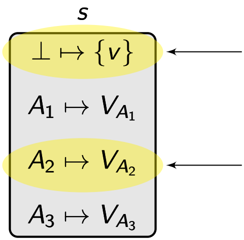

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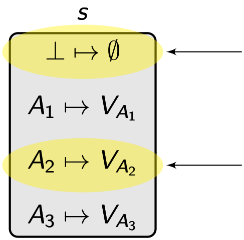

a visible deviation has for sure occurred

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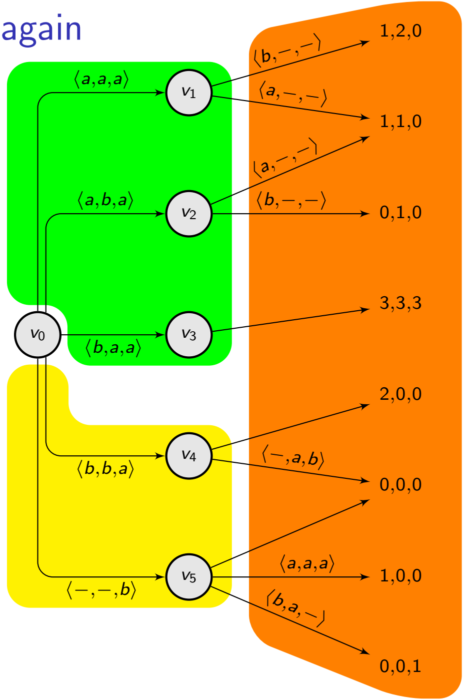
The example again



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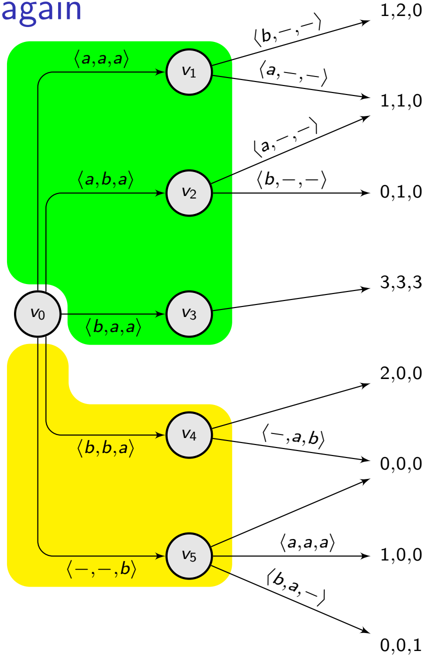
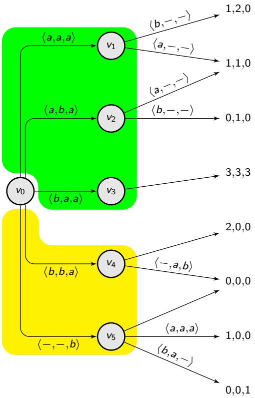


Example of construction



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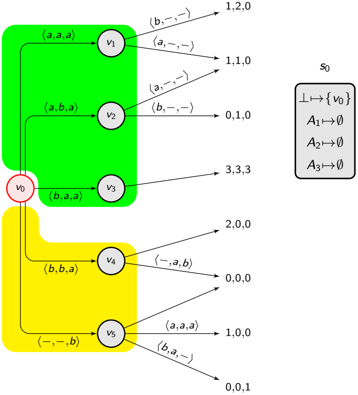

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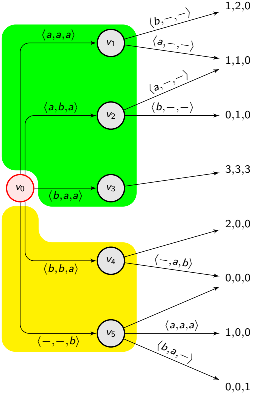

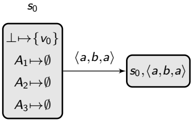

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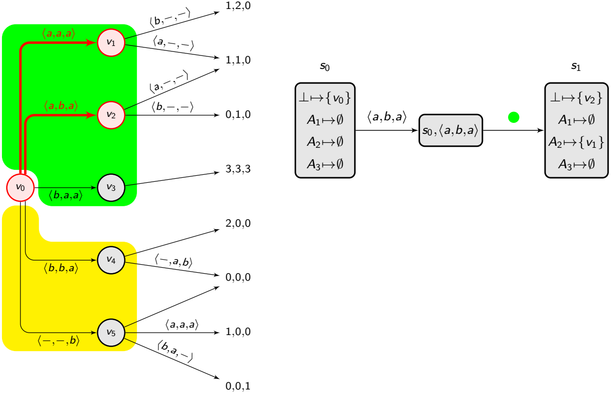

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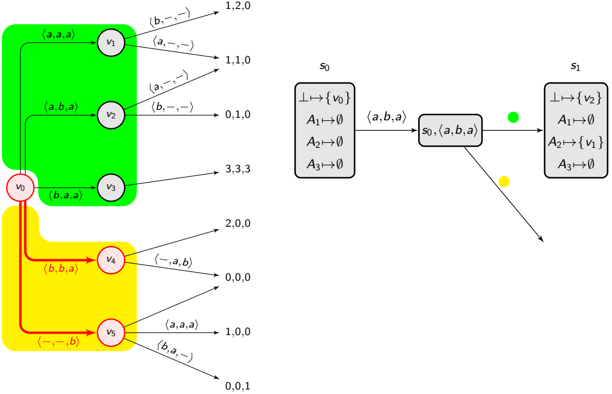

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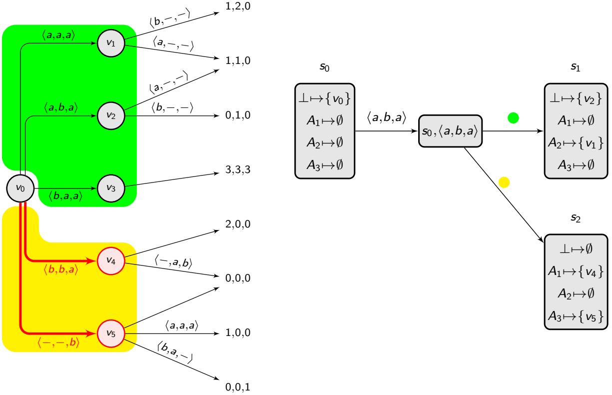

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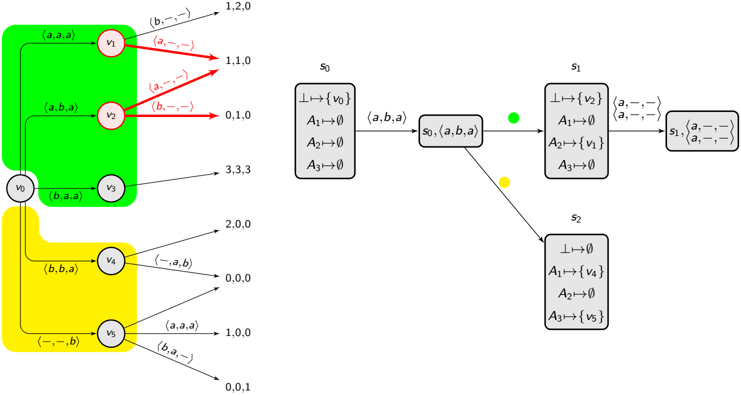

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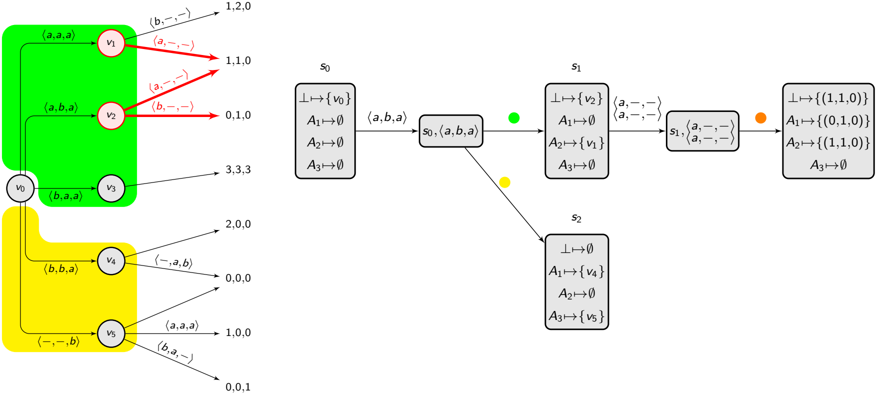

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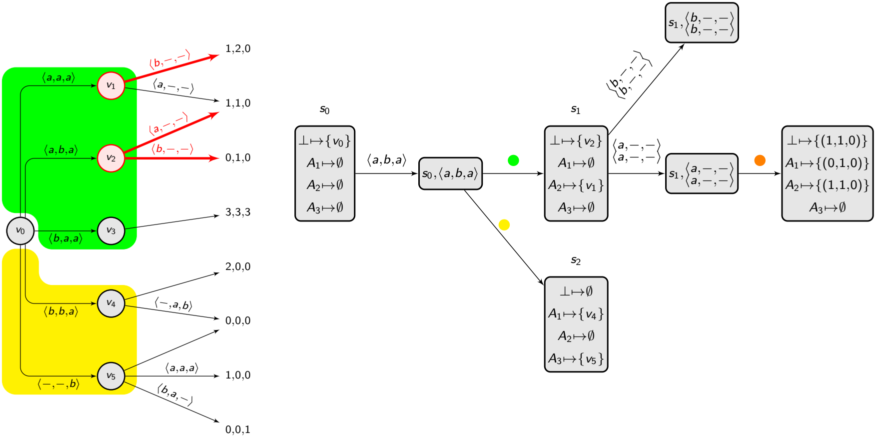

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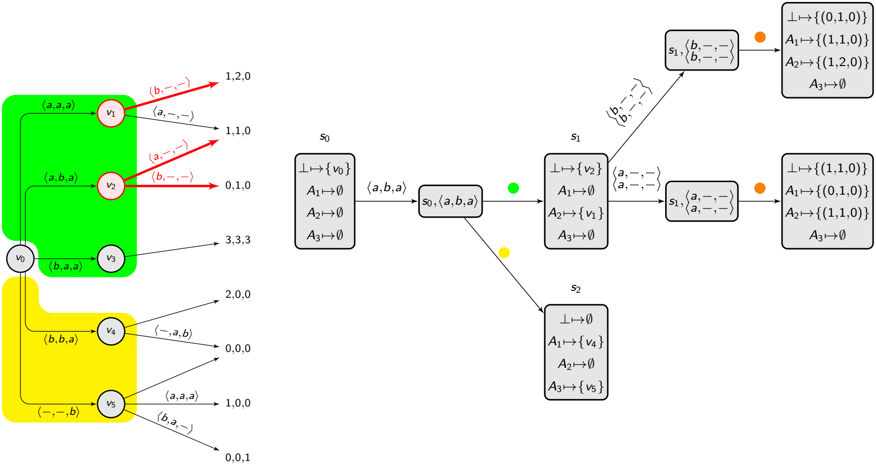

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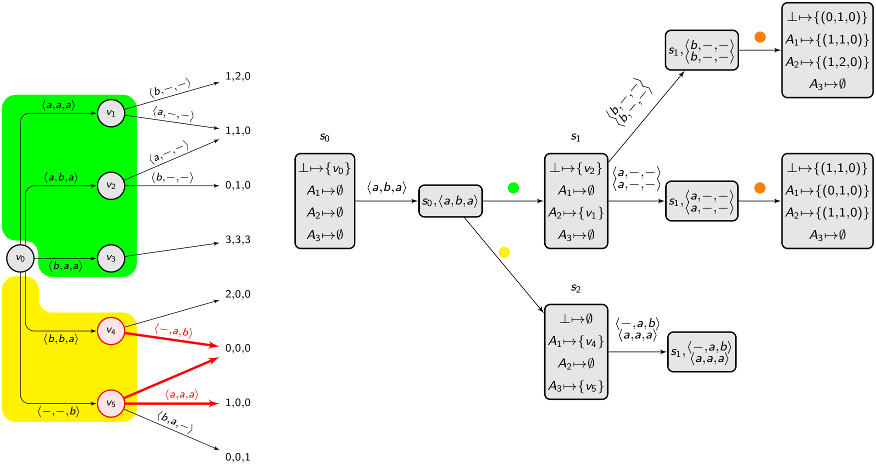

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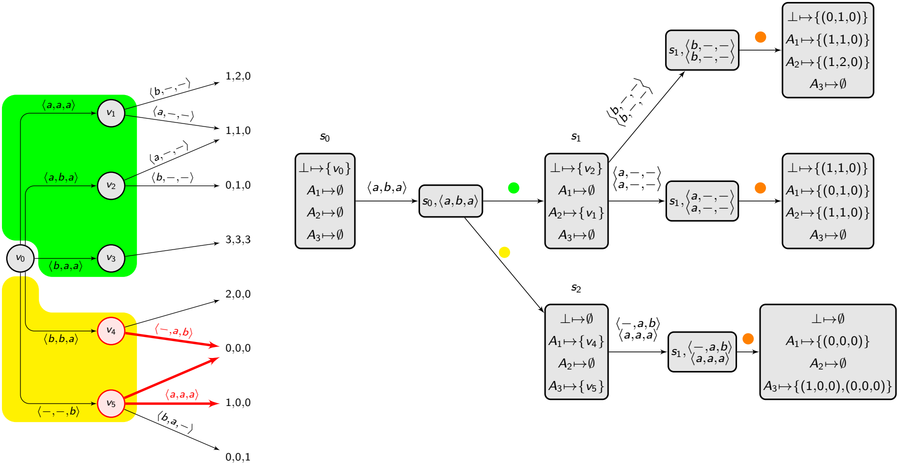

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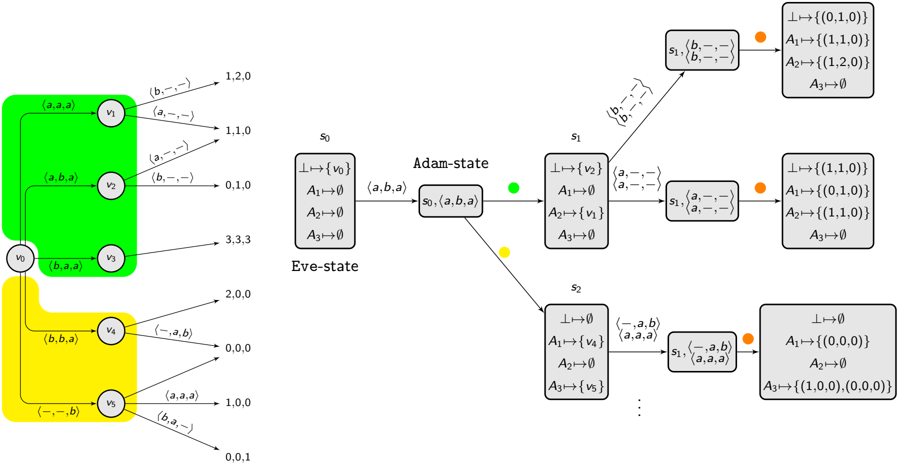

Properties of the epistemic game

- To every history H in the epistemic game, one can associate sets
  - $\mathit{concrete}_{\perp}(H)$: at most one concrete real history (if no deviation)
  - $\mathit{concrete}_{A}(H)$: all possible A-deviations
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H history in the epistemic game. For every $h_1 \neq h_2 \in \mathit{concrete}(H)$,

$$h_1 \sim_A h_2 \qquad \text{iff} \qquad h_1, h_2 \notin \mathit{concrete}_A(H)$$

Properties of the epistemic game (cont'd)

Winning condition for Eve

A strategy $\sigma_{\texttt{Eve}}$ is said winning for payoff $p \in \mathbb{R}^{\mathrm{Agt}}$ from s_0 whenever $\mathrm{payoff}(\mathit{concrete}_{\perp}(\mathrm{out}_{\perp}(\sigma_{\texttt{Eve}}, s_0))) = p$, and for every $R \in \mathrm{out}(\sigma_{\texttt{Eve}}, s_0)$, for every $A \in \mathrm{Agt}$, for every $\rho \in \mathit{concrete}_A(R)$, $\mathrm{payoff}_A(\rho) \leq p_A$.

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Winning condition for Eve (publicly visible payoffs)

A strategy $\sigma_{\mathtt{Eve}}$ is said winning for p from s_0 whenever $\mathsf{payoff}'(\mathsf{out}_\perp(\sigma_{\mathtt{Eve}}, s_0)) = p$, and for every $R \in \mathsf{out}(\sigma_{\mathtt{Eve}}, s_0)$, for every $A \in \mathsf{susp}(R)$, $\mathsf{payoff}'_A(R) \leq p_A$.

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Proposition

There is a Nash equilibrium in $\mathcal{G}$ with payoff p from v_0 if and only if Eve has a winning strategy for p in $\mathcal{E}_\mathcal{G}$ from s_0.

Application to ω-regular objectives

ψ_A: objective of player A

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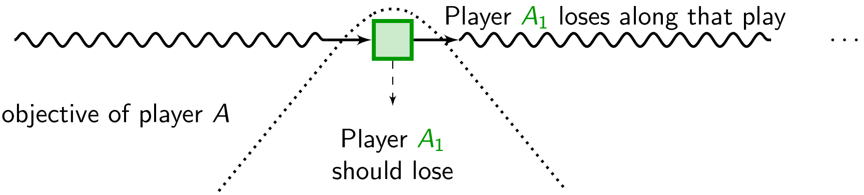

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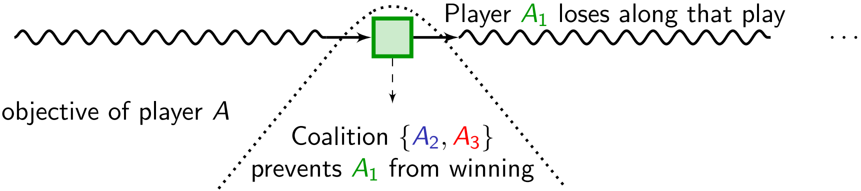

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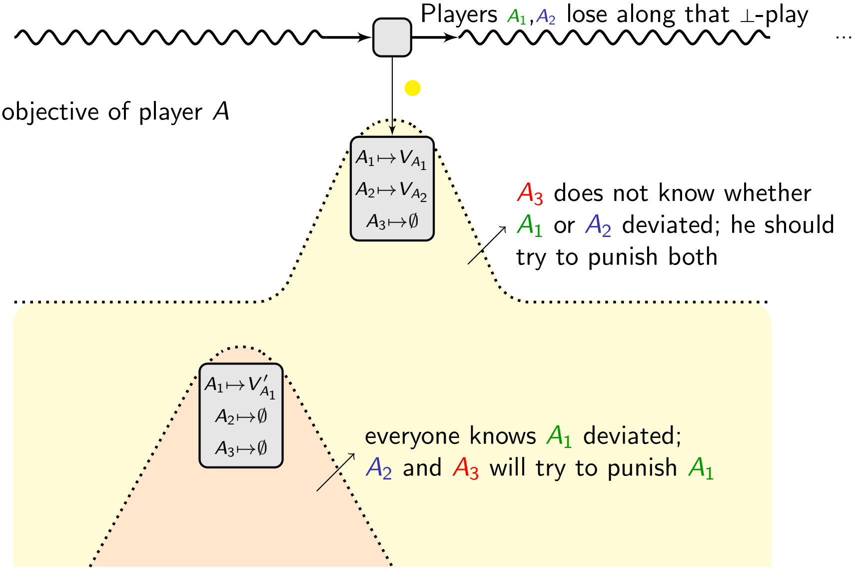

Application to ω-regular objectives (cont'd)

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- By reduction from the distributed synthesis problem (proof of [BK10]):

Theorem

One cannot decide the existence of a Nash equilibrium in a game with private signals and publicly visible ω-regular payoff functions. Even for three players.

[BK10] Berwanger, Kaiser. Information Tracking in Games on Graphs *(Journal of Logic, Language and Information)*

Application to mean-payoff functions

- Using results on the polyhedron problem [BR15]:

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One can decide the (constrained) existence of a Nash equilibrium in a game with public signal and publicly visible mean-payoff functions, in NP, with a coNEXPTIME oracle. This in particular can be solved in EXPSPACE. It is EXPTIME-hard.

[BR15] Brenguier, Raskin. Pareto curves of multidimensional mean-payoff games *(CAV'15)*

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- By reduction from blind mean-payoff games (proven undecidable in [DDG+10])

Theorem

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[BR15] Brenguier, Raskin. Pareto curves of multidimensional mean-payoff games *(CAV'15)*
[DDG+10] Degorre, Doyen, Gentilini, Raskin, Toruńczyk. Energy and Mean-Payoff Games with Imperfect Information *(CSL'10)*

Conclusion

We have:

- proposed a framework for games over graphs with a public signal monitoring Note: framework inspired by [Tom98]
- proposed an abstraction called the epistemic game abstraction, which allows to detect deviators and tocharacterize Nash equilibria in the original game
- used it to show several decidability results.

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We want to:

- work out the precise complexities
- understand whether one can extend the approach to other communication architectures ([RT98]??)
- understand whether other multiagent frameworks (like fragments of Strategy Logic) can be handled under the assumption of public signal

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