Timed Models for Concurrent Systems

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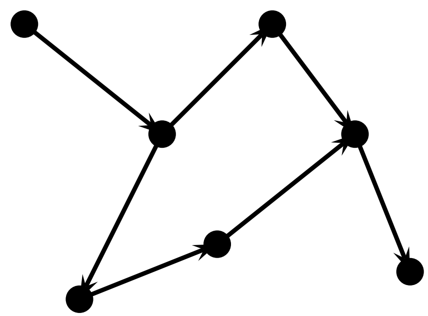
Model-Checking

Does the system satisfy the property?

Modelization



φ

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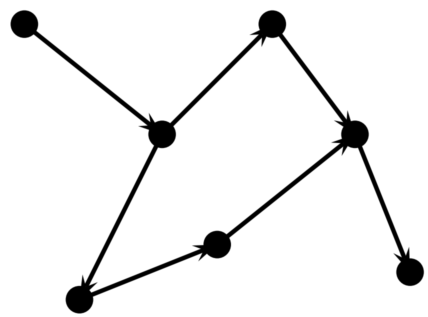


$\models$ φ

Model-checking algorithm

Time!

Context: verification of embedded critical systems

Time

- ✔ naturally appears in real systems
- ✔ appears in properties (for ex. bounded response time)

➜ Need of models and specification languages integrating timing aspects

➜ Challenge: Integrate time in concurrent models

Roadmap

- ✔ About time semantics
- ✔ Timed specification languages
- ✔ Some possible timed models
- ✔ Timed automata
- ✔ Networks of TA, discussion
- ✔ Verification methods
- ✔ Conclusion remarks

About Time Semantics

[Alur's PhD Thesis 1991]

Adding Timing Informations

Which semantics?

- ✔ **Untimed case:** sequence of observable events

  a: send message b: receive message

  $$a\ b\ a\ b\ a\ b\ a\ b\ a\ b\ \cdots = (a\ b)^\omega$$

- ✔ **Timed case:** sequence of dated observable events

  $$(a, d_1)\ (b, d_2)\ (a, d_3)\ (b, d_4)\ (a, d_5)\ (b, d_6)\ \cdots$$

  d_1: date at which the first a occurs
  d_2: date at which the first b occurs
  ...

Process: set of such (un)timed sequences

Three Propositions

- ✔ **Discrete-time semantics:**

  dates are taken in $\mathbb{N}$, the set of integers

  Ex: (a, 1).(b, 3).(c, 4).(a, 6)

- ✔ **Dense-time semantics:**

  dates are taken in $\mathbb{Q}^+$, the set of positive rationals,
  or in $\mathbb{R}^+$, the set of positive reals

  Ex: (a, 1.28).(b, 3.1).(c, 3.98)(a, 6.13)

- ✔ **Fictitious-clock semantics:**

  "tick" action denoting each unit of time

  Ex: tick.a.tick.tick.b.c.tick.tick.tick.a
  or alternatively (a, 1).(b, 3).(c, 3).(a, 6)

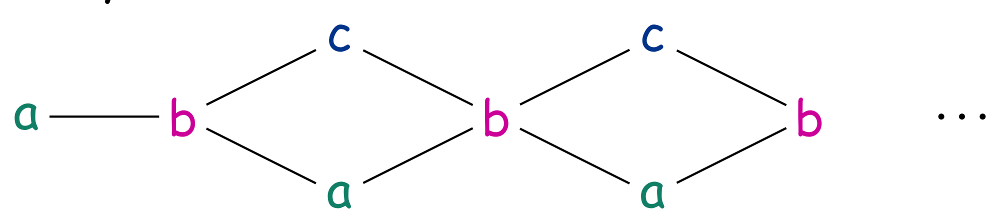
Synchronization of Processes

- ✔ **Untimed case:** Synchronization on common events, interleaving of causally independent events

Example: P = ({a, b}, (a b)$^\omega$) || Q = ({b, c}, (b c)$^\omega$)

a b c a b {a, c} b a c b

Can be represented by:



- ✔ **Timed case:** No interleaving possible; time orders events

> **Hyp:** All components are driven by a common clock

The Discrete-Time Semantics

- ✔ the simplest one
- ✔ equivalent to the untimed semantics (if no action, say action $\emptyset$)

Ex: the timed sequence

$$(a,1)\,.\,(b,2)\,.\,(\{a,b\},4)\,.\,(b,5)\ \ldots$$

is represented by the untimed sequence

$$\{a\}\,.\,\{b\}\,.\,\emptyset\,.\,\{a,b\}\,.\,\{b\}\ \ldots$$

➜ no really new technique needed

The Dense-Time Semantics

- ✔ **a more realistic model:** causally independent events may appear arbitrarly close to each other

  Ex: (a, 1) . (b, 2) . (c, 3.93) . (a, 3.98) . (b, 5) . (c, 6.02)

- ✔ a system and its **environment**: no constraint on the timing of signals from the environment

- ✔ if strange behaviours are not wished (e.g. zeno behaviours), one can simply avoid them

➜ new techniques needed

The Fictitous-Clock Model

Much different from the two previous models, more uncertainty.

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$$(a, d_1) . (b, d_2) . (c, d_3) . (a, d_4)$$

where $1 \leq d_1 < 2$, $3 \leq d_2 \leq d_3 < 4$ and $6 \leq d_4 < 7$.

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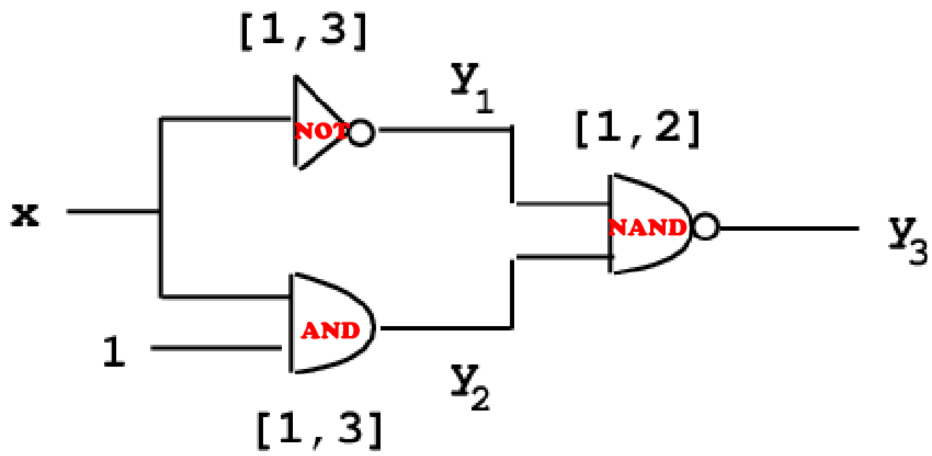
- Can also be viewed as an approximation of the dense-time semantics
- **Pb:** no precise timing informations (if k ticks in between two actions, it means that these two actions are separated by some delay in $[k-1, k+1[$)

A Case for Dense-Time [Alur 1991]

- ✔ **Correctness:** discussion in the context of reachability problems for asynchronous digital circuits **[Brzozowski, Seger 1991]**

A Digital Circuit [BS91]



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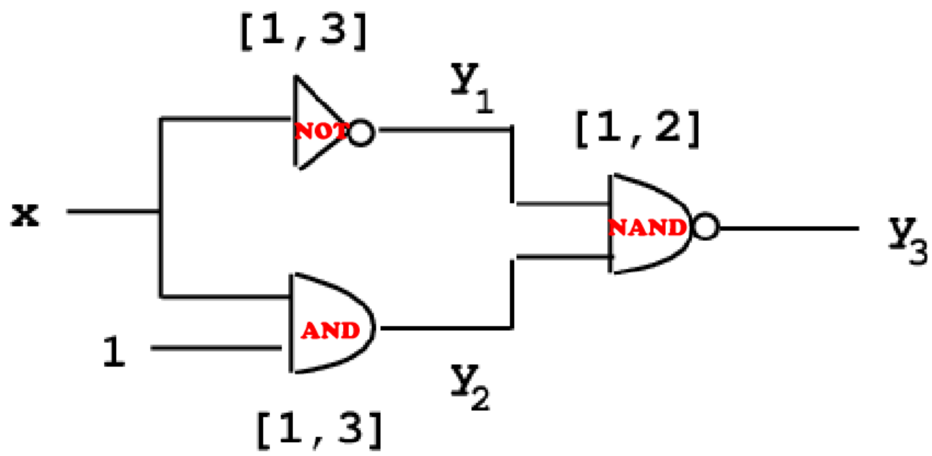

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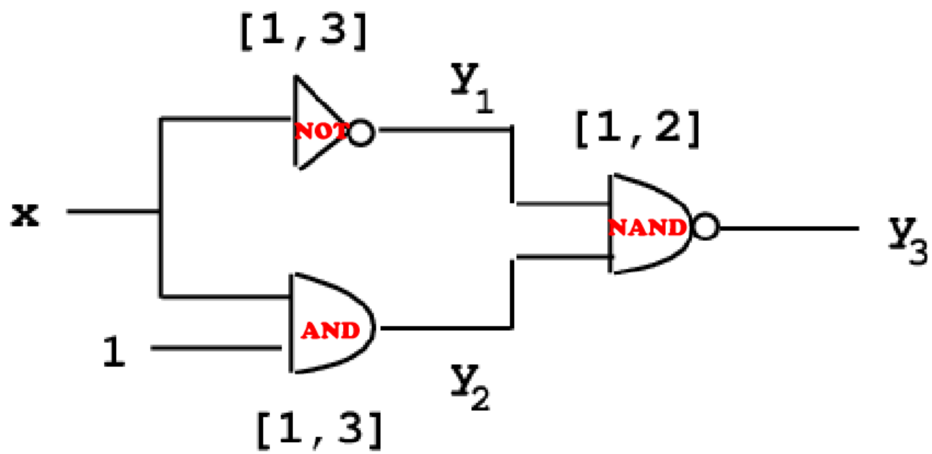

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The input x changes to 1. The corresponding stable state is y=[011]

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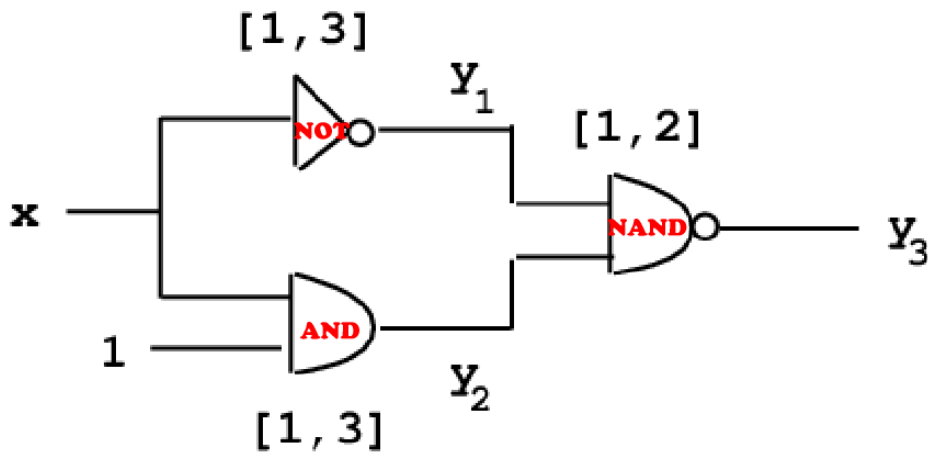

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However, many possible behaviours, e.g.

$$[101] \xrightarrow[1.2]{y_2} [111] \xrightarrow[2.5]{y_3} [110] \xrightarrow[2.8]{y_1} [010] \xrightarrow[4.5]{y_3} [011]$$

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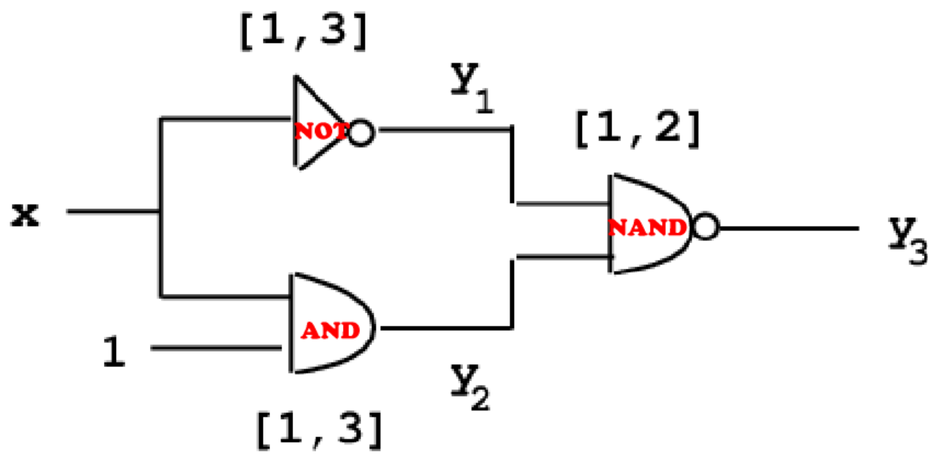

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Reachable configurations: {[101], [111], [110], [010], [011], [001]}

Discretizing is Not Sufficient

[Brzozowski Seger 1991]

Theorem: for every $k \geq 1$, there exists a digital circuit such that the reachability set of states in dense-time is strictly larger than the one in discrete time (with granularity $\frac{1}{k}$).

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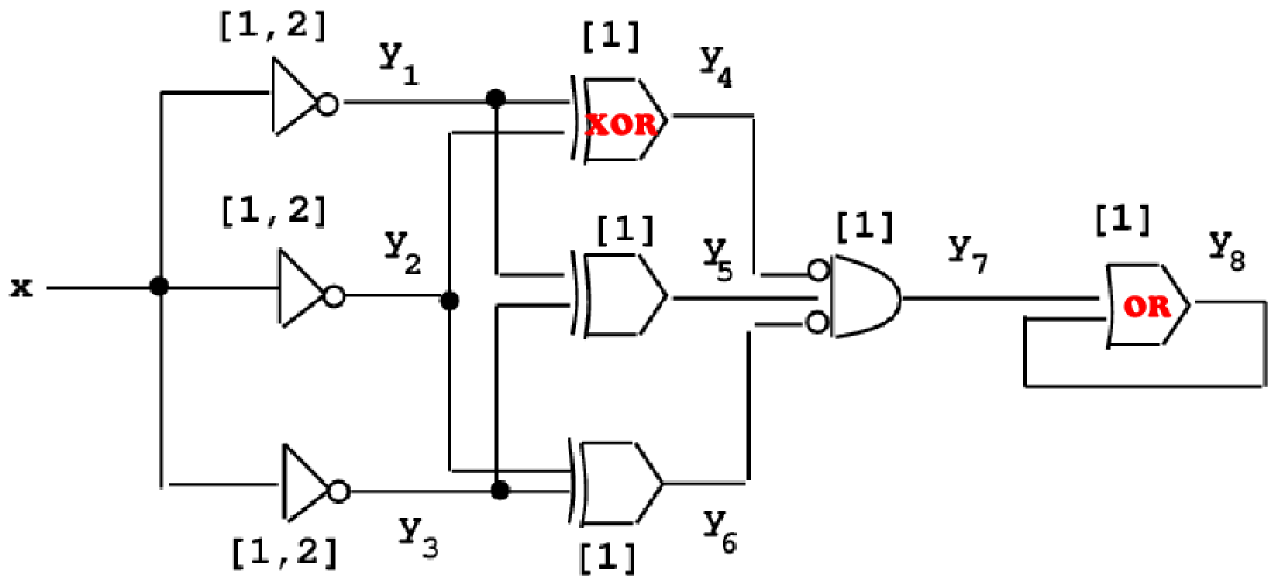

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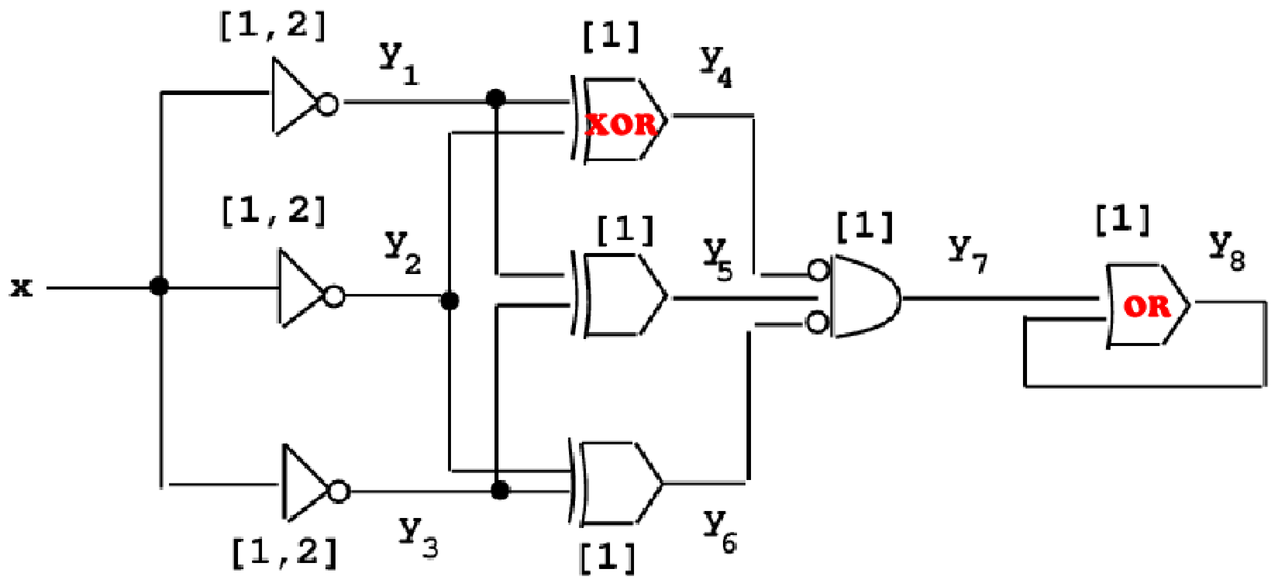

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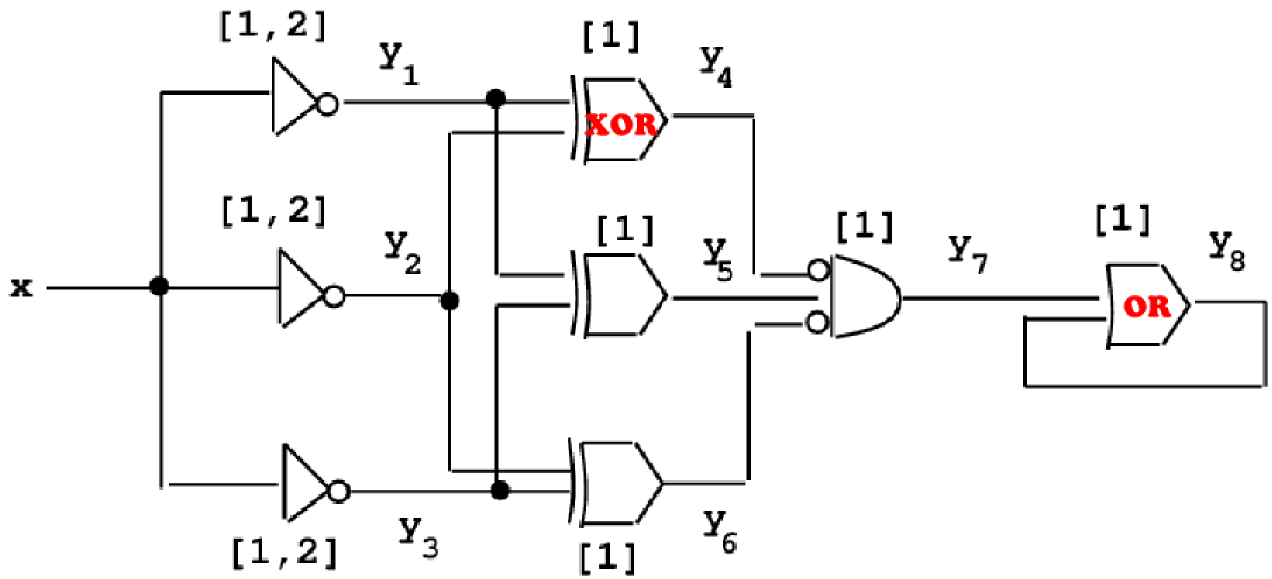

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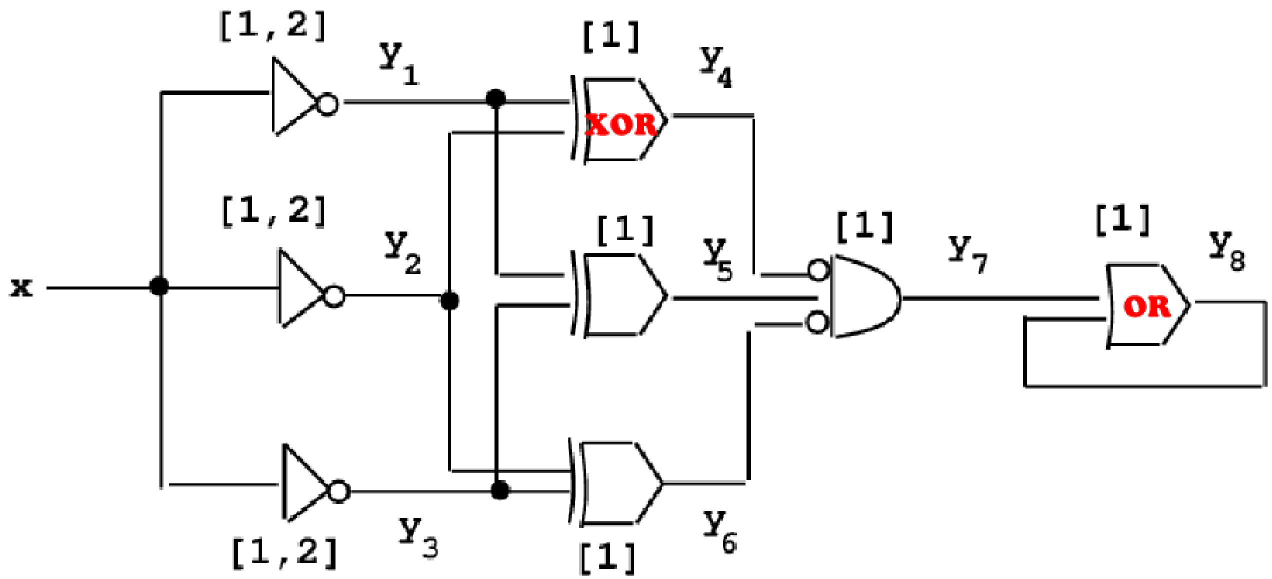

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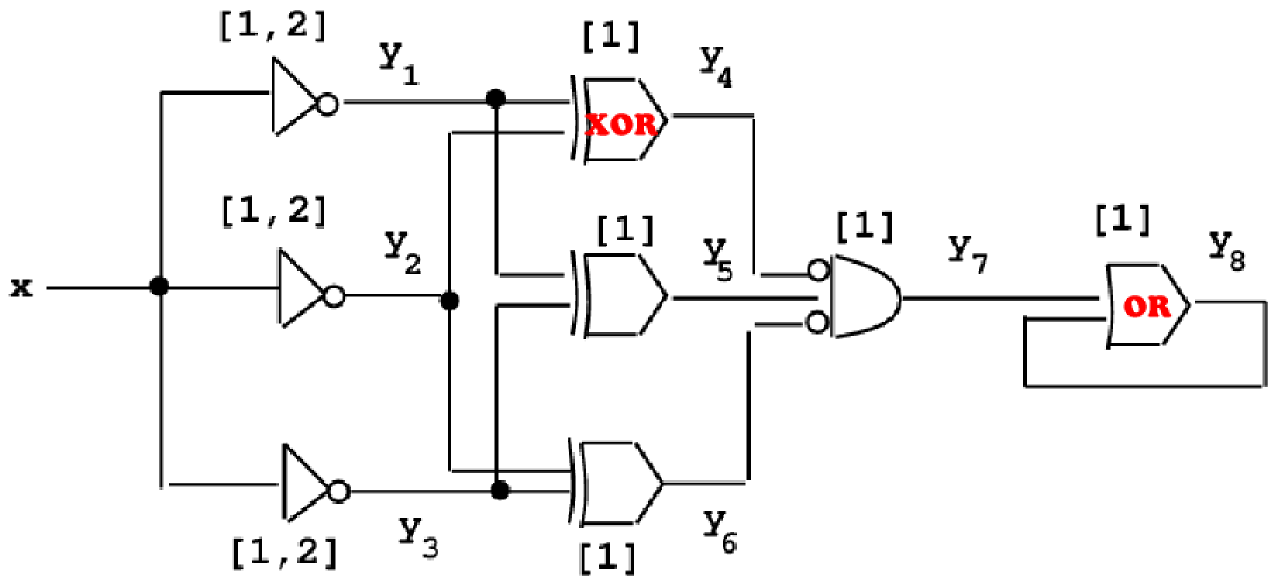

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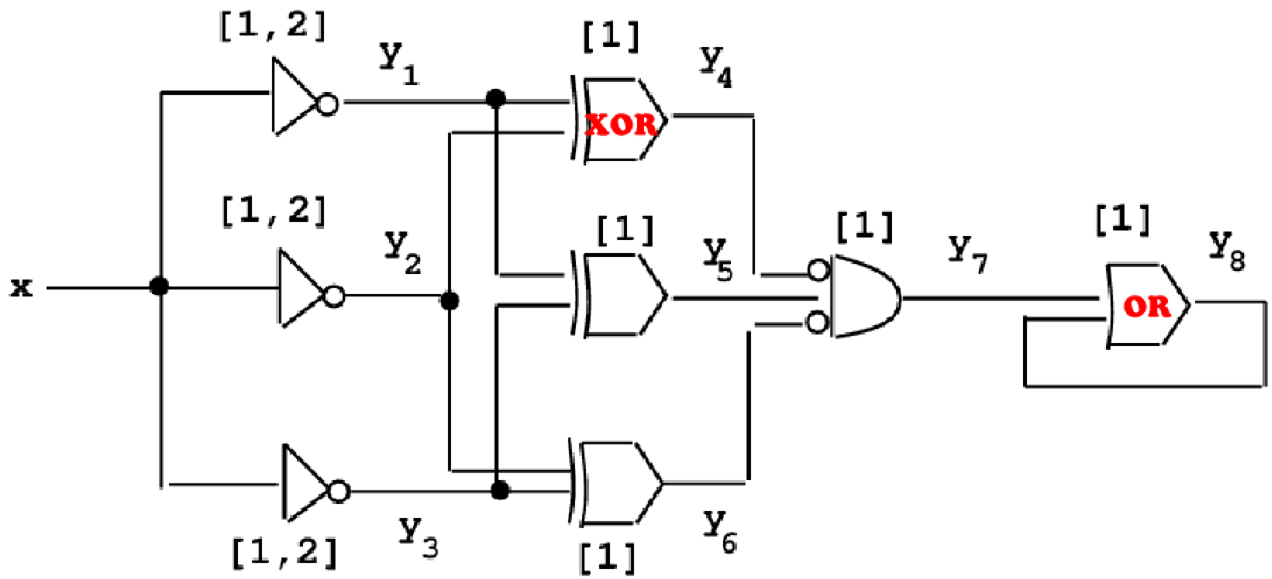

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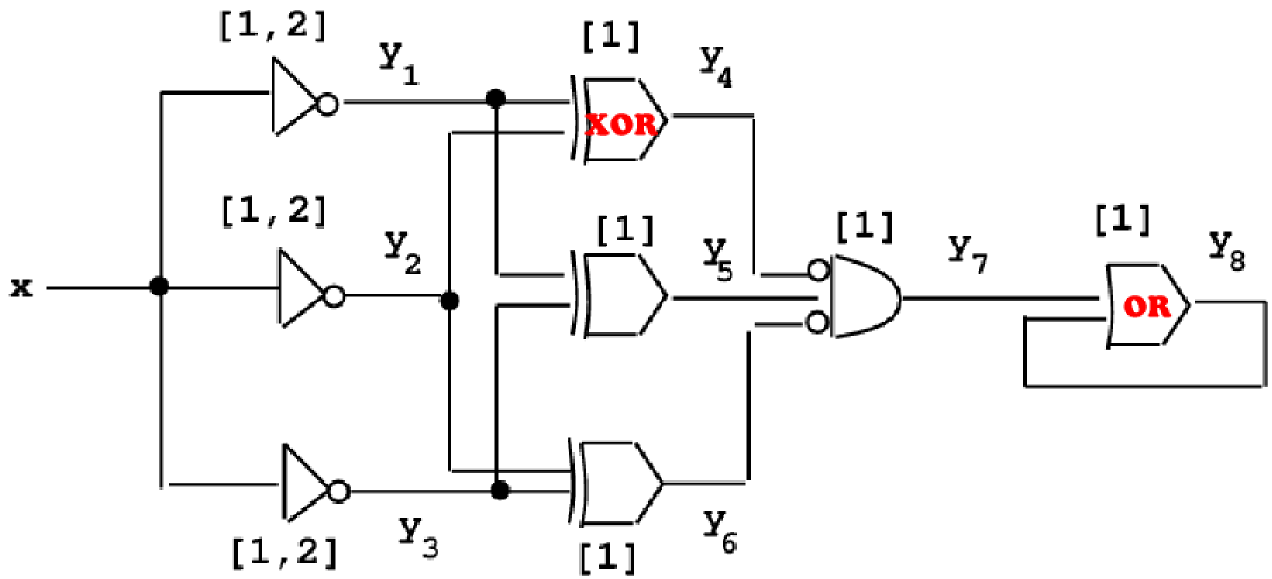

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Theorem: for every $k \geq 1$, there exists a digital circuit such that the reachability set of states in dense-time is strictly larger than the one in discrete time (with granularity $\frac{1}{k}$).

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Claim: finding a correct granularity is as difficult as computing the set of reachable states in dense-time

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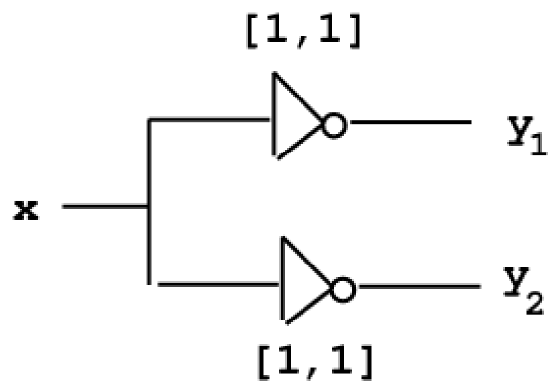
Further counter-example: there exist systems for which no granularity exists

(see later)

Fictitious-Clock Model: Too Large



- ✔ Dense-time: {[11], [00]}
- ✔ Fictitious-clock: {[11], [10], [01], [00]}

$$(tick.y_1) \parallel (tick.y_2) \quad = \quad \{tick.y_1.y_2, tick.y_2.y_1\}$$

➜ over-approximation of the set of reachable states

A Case for Dense-Time

- ✔ **Correctness**
- ✔ **Expressiveness:** discrete-time and fictitious-clock models can be expressed by dense-time models

A Case for Dense-Time

- ✔ **Correctness**
- ✔ **Expressiveness**
- ✔ **Compositionality:** the semantics of one component depends on the granularity of the whole system and of the property we want to check

Ex: P: process such that a and b strictly alternate and each b is exactly one unit of time later than a
Q process such that a and b strictly alternate, each b is exactly one unit of time later than a, and each a is at least one unit of time later than each b

- If the granularity is 1,

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  - If the granularity is 1, T(P)=T(Q)
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➜ Dense-time: a good alternative to have a compositional semantics.

A Case for Dense-Time

- ✔ **Correctness**
- ✔ **Expressiveness**
- ✔ **Compositionality**
- ✔ **Complexity:** dense-time more complex than the two other semantics (ex: inclusion)

However: refining the granularity increases the complexity...

A Case for Dense-Time

- ✔ **Correctness**
- ✔ **Expressiveness**
- ✔ **Compositionality**
- ✔ **Complexity**

In the following we choose the **dense-time** semantics

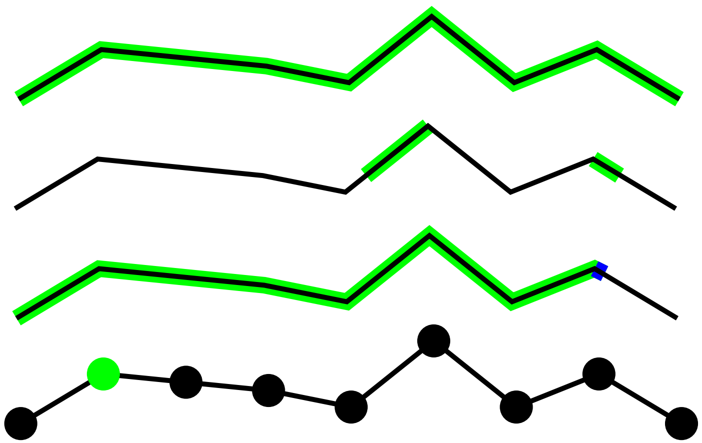
Timed Specification Languages

Classical Verification Problems

- reachability of a control state
- $\mathcal{S} \sim \mathcal{S}'$: bisimulation, etc...
- $L(\mathcal{S}) \subseteq L(\mathcal{S}')$: language inclusion
- $\mathcal{S} \models \varphi$ for some formula φ: model-checking
- $\mathcal{S} \parallel A_T$ + reachability: testing automata
- ...

Classical Temporal Logics

Path formulas:



$G\varphi$ « Always »

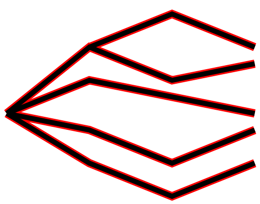
$F\varphi$ « Eventually »

$\varphi U \varphi'$ « Until »

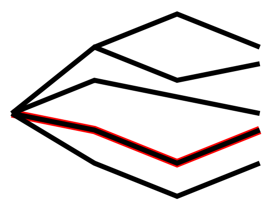
$X\varphi$ « Next »

State formulas:

$A\psi$



$E\psi$



➜ LTL: Linear Temporal Logic **[Pnueli 1977]**,
CTL: Computation Tree Logic **[Emerson, Clarke 1982]**

Adding Time to Temporal Logics

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✔ Temporal logics with **subscripts**.

ex: CTL + $\left|\begin{array}{l} E\varphi U_{\sim k}\psi \\ A\varphi U_{\sim k}\psi \end{array}\right.$

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- ✔ Temporal logics with **clocks**.

$$AG(\text{problem} \Rightarrow (x\ \mathbf{in}\ AF(x \leq 20 \wedge \text{alarm})))$$

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→ **TCTL**: Timed CTL [ACD90,ACD93,HNSY94]

An Other Specification Language

Timed modal logics or timed μ-calculus with **clocks** e.g. [LLW95]

- ✔ prop, boolean combinators
- ✔ $< a > \varphi$, $[a]\,\varphi$
- ✔ $\exists\!\!\exists\ \varphi$, $\forall\!\!\forall\ \varphi$
- ✔ min(X, φ), max(X, φ)
- ✔ $x \leq cte$, x in φ

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Examples: "$AG\,\varphi$": $\max(X, \varphi \wedge \bigwedge_a [a]X \wedge \forall \varphi)$

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"$A(\varphi W \psi)$": $\max(X, \psi \vee (\varphi \wedge \bigwedge_a [a]X \wedge \forall\!\!\!\forall X))$

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"$A(\varphi W \psi)$": $\max(X, \psi \vee (\varphi \wedge \bigwedge_a [a]X \wedge \forall X))$

"non-zenoness (action and time)": x in $\max(X, x \leq 1 \wedge \forall X \wedge \bigwedge_a [a]X)$

An Other Specification Language

Timed modal logics or timed μ-calculus with **clocks** e.g. [LLW95]

- ✔ prop, boolean combinators
- ✔ $<a>\varphi$, $[a]\varphi$
- ✔ $\exists\!\!\!\!\!\exists\, \varphi$, $\forall\!\!\!\!\forall\, \varphi$
- ✔ $min(X, \varphi)$, $max(X, \varphi)$
- ✔ $x \leq cte$, x in φ

Examples: "$AG\,\varphi$": $max(X, \varphi \wedge \bigwedge_a [a]X \wedge \forall\!\!\!\!\forall \varphi)$

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"non-zenoness (action and time)": x in $max(X, x \leq 1 \wedge \forall\!\!\!\!\forall X \wedge \bigwedge_a [a]X)$

$$\Phi = \text{problem} \Rightarrow x \text{ in } max(Z, alarme \vee (x \leq 20 \wedge \bigwedge_a [a]Z \wedge \forall\!\!\!\!\forall Z))$$

$$max(Y, \Phi \wedge \bigwedge_a [a]Y \wedge \forall\!\!\!\!\forall Y)$$

An Other Example

- the bell rings every 15 minutes

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- the bell rings every 15 minutes

$$\text{bell} \wedge \qquad\qquad AG(\text{bell} \Rightarrow AF_{=15}\ \text{bell})$$

An Other Example

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$$\text{bell} \wedge AG_{0<.<15} \neg\text{bell} \wedge AG(\text{bell} \Rightarrow AF_{=15}\ \text{bell}) \wedge AG(\neg\text{bell} \Rightarrow AG_{=15} \neg\text{bell})$$

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$\text{bell}\wedge$ $(0 < x < 15 \wedge \neg\text{bell}) \vee ((x = 15 \vee x = 0) \wedge \text{bell})$

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$$\text{bell} \wedge x \text{ in } \max(X, x \leq 15 \Rightarrow ((0 < x < 15 \wedge \neg\text{bell}) \vee ((x = 15 \vee x = 0) \wedge \text{bell} \wedge x \text{ in } (\forall X \wedge \bigwedge_a [a]X)$$

Some Possible Timed Models

- Time Petri nets
- Timed process algebra
- Timed MSCs
- Graphs with durations
- Timed automata
- ...

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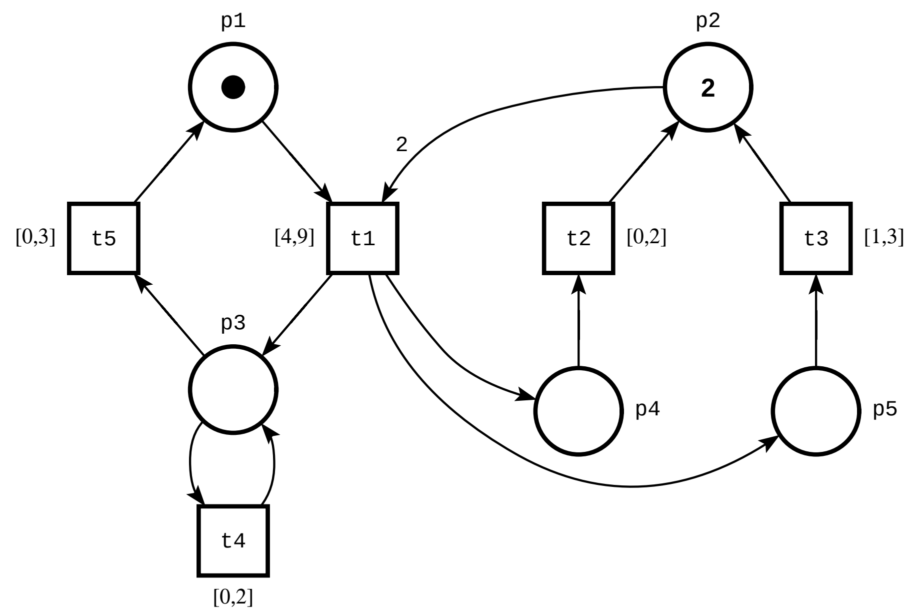
Time Petri Nets

[Merlin 1974, Berthomieu & Diaz 1991]



Initial marking: $(p_1, p_2(2))$

Only t_1 can be fired: $4 \leq t_1 \leq 9$

➜ t_1 is fired after θ_1

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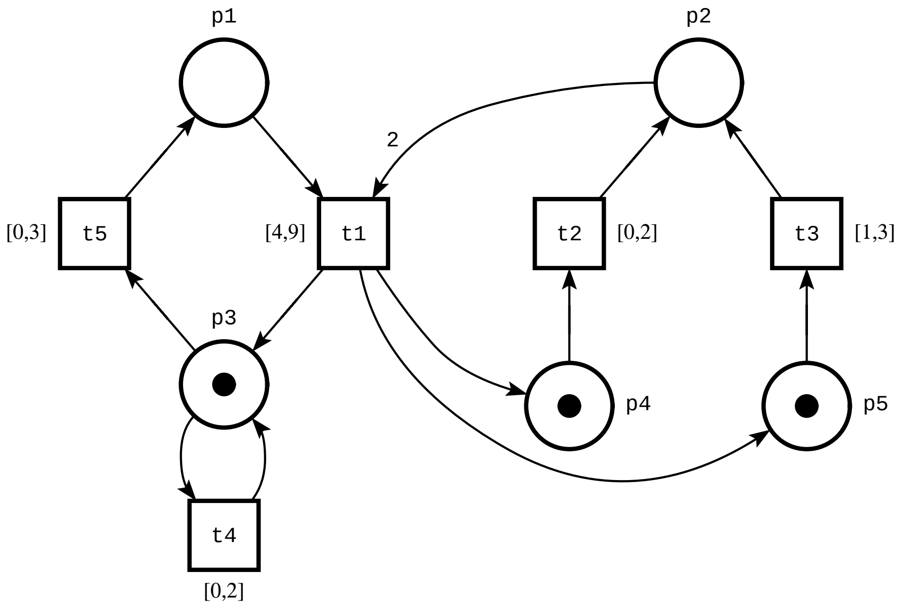

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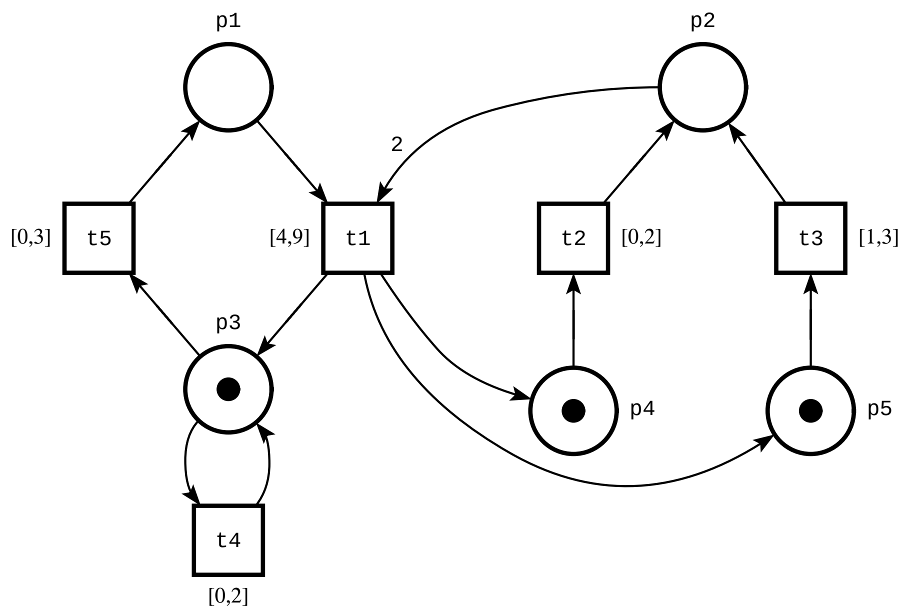

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$\rightarrow$ t_2 is fired after θ_2

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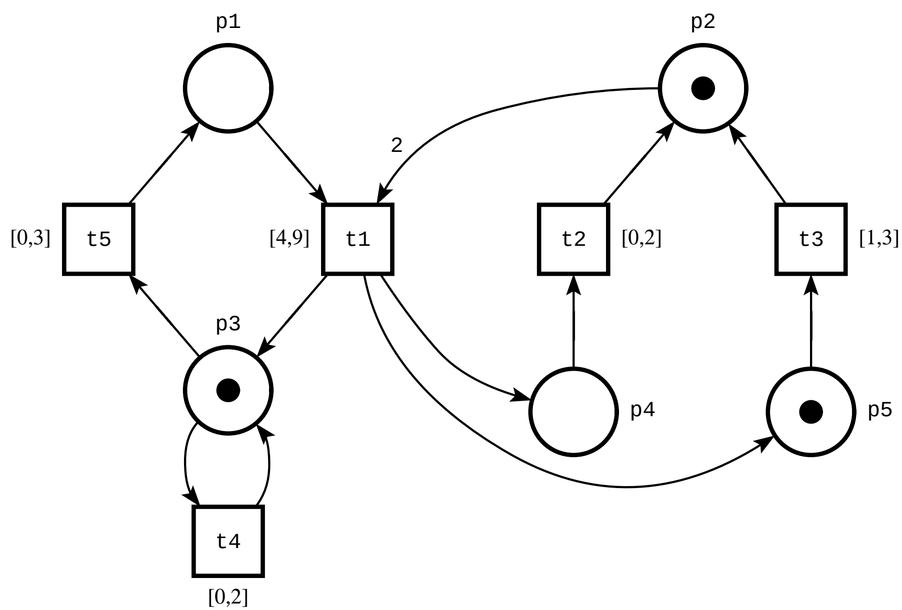

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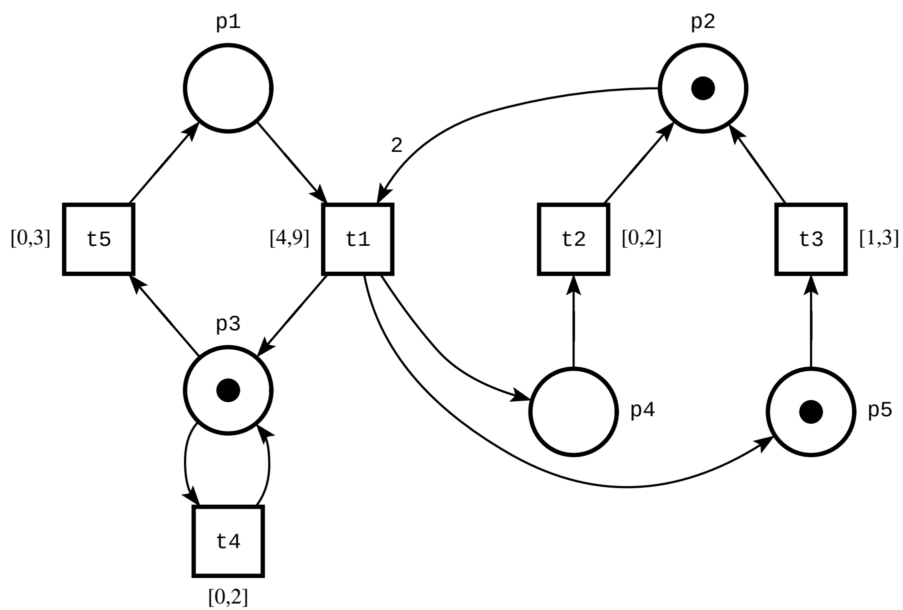

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t_3, t_4 and t_5 can be fired:

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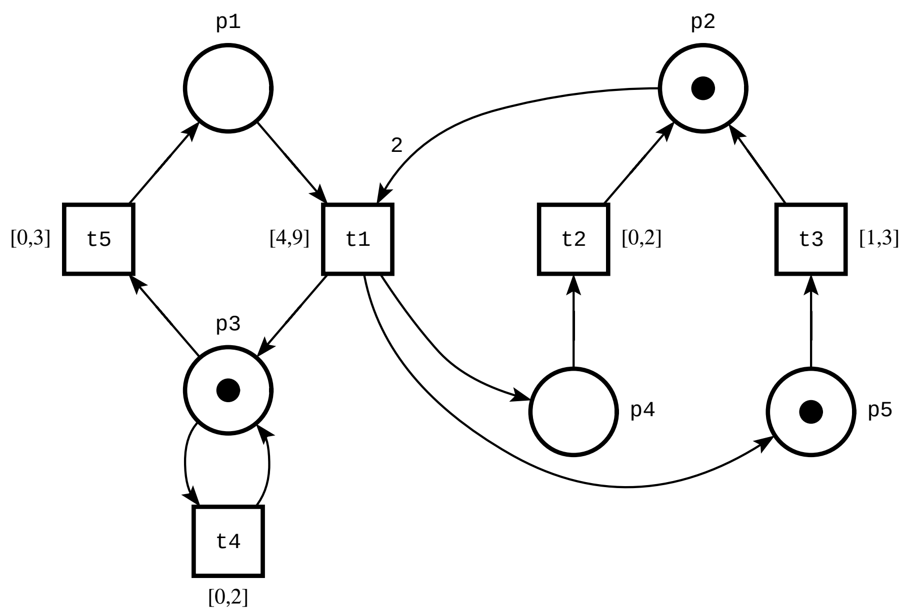

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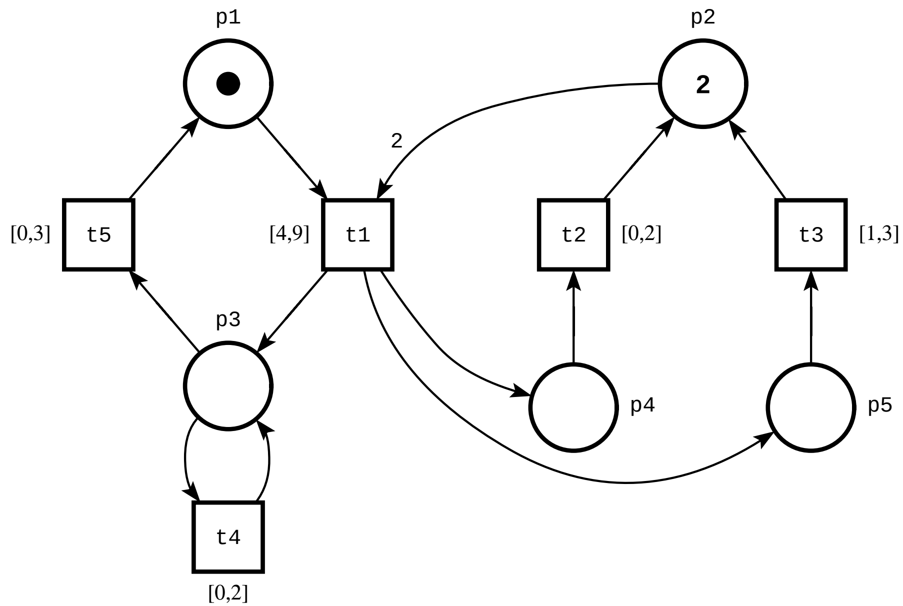
The scheduling $(t_1.t_2, \theta_1 = 5.\theta_2 = 0)$ is realizable

Time Petri Nets - Symbolic Analysis

Initial marking: $(p_1, p_2(2))$

Initial class: $4 \leq t_1 \leq 9$

Graph of State Classes



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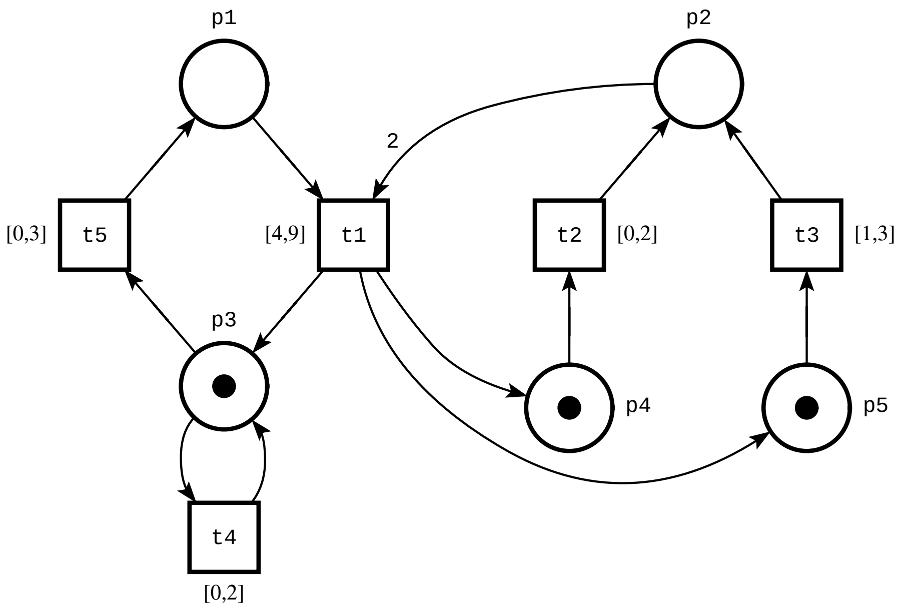

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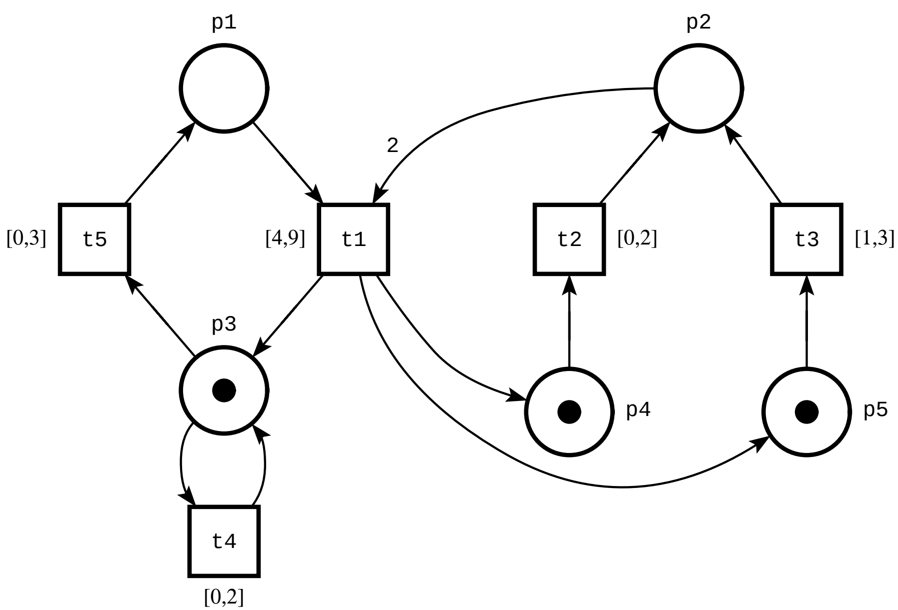

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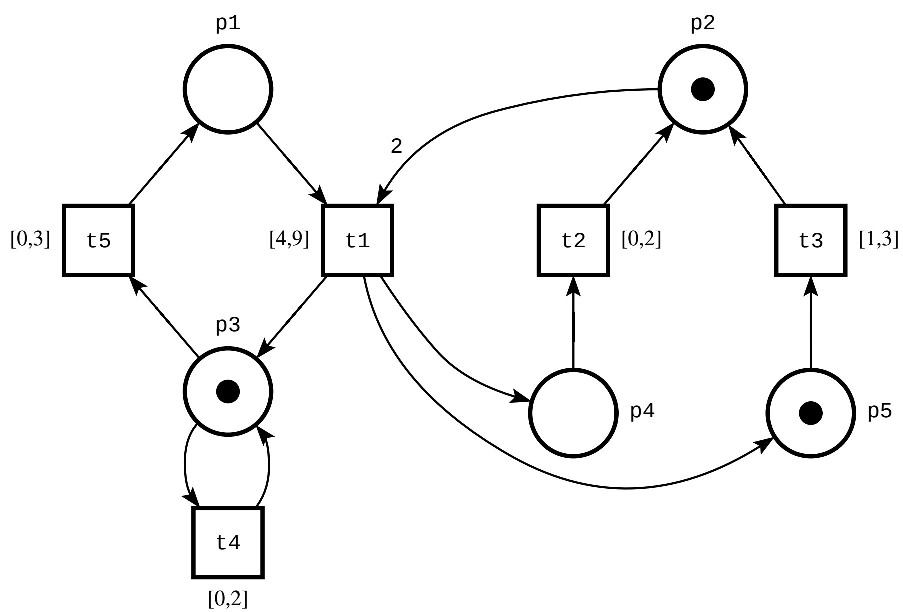

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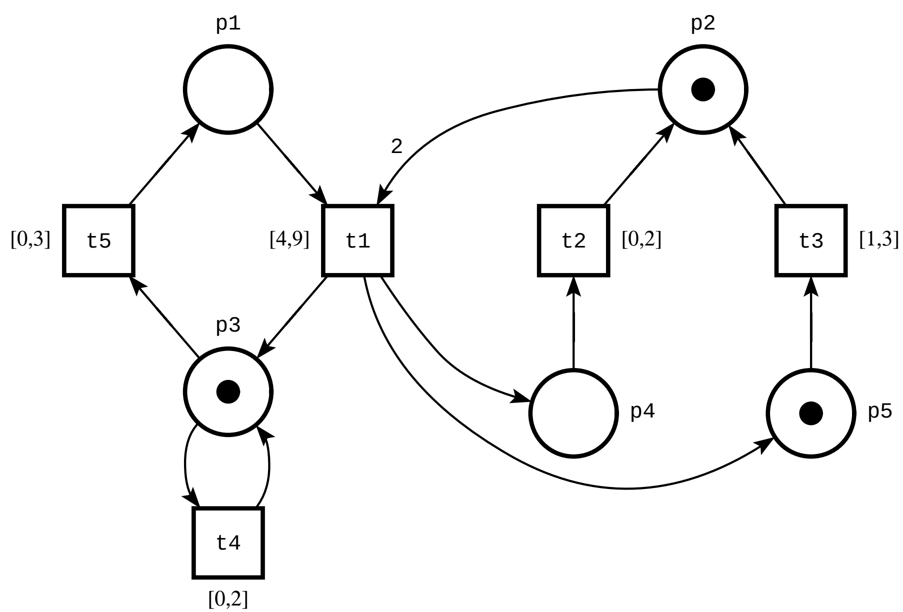
and so on...

Time Petri Nets - Symbolic Analysis

Graph of State Classes



potentially infinite graph...

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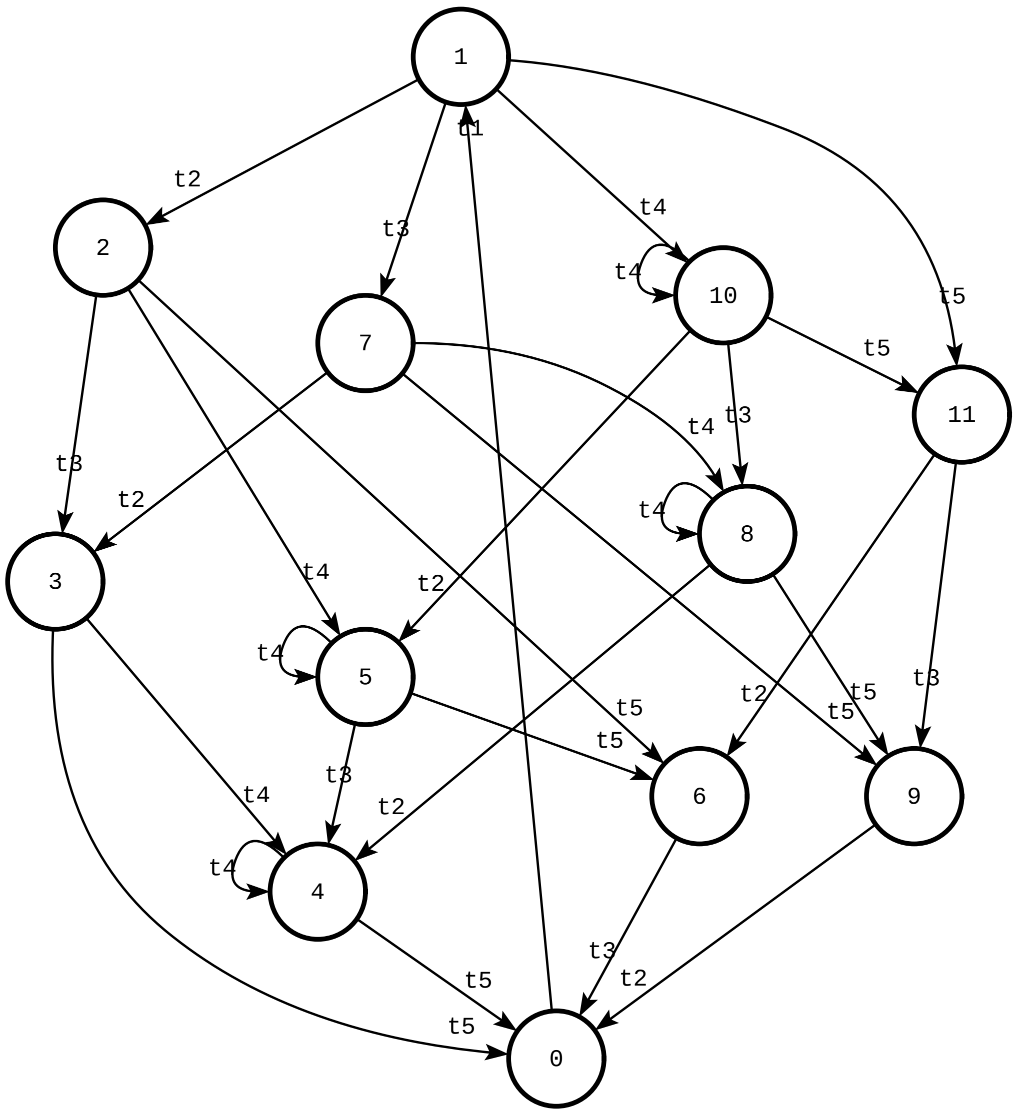
Graph of State Classes for the Example



Time Petri Nets - Properties

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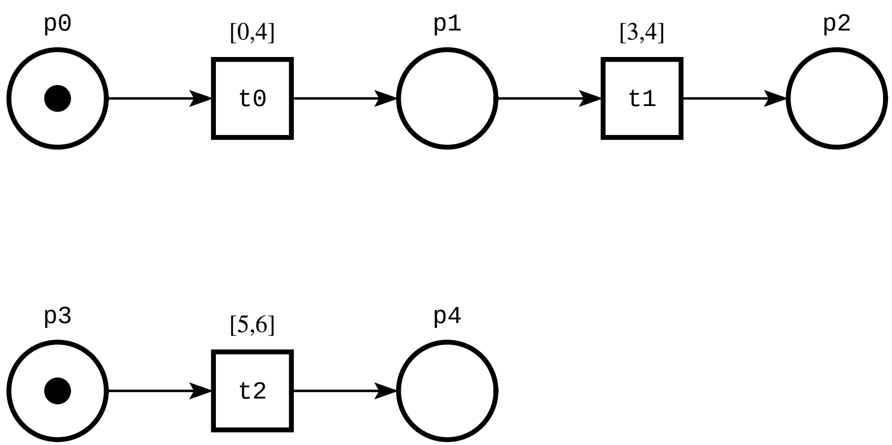
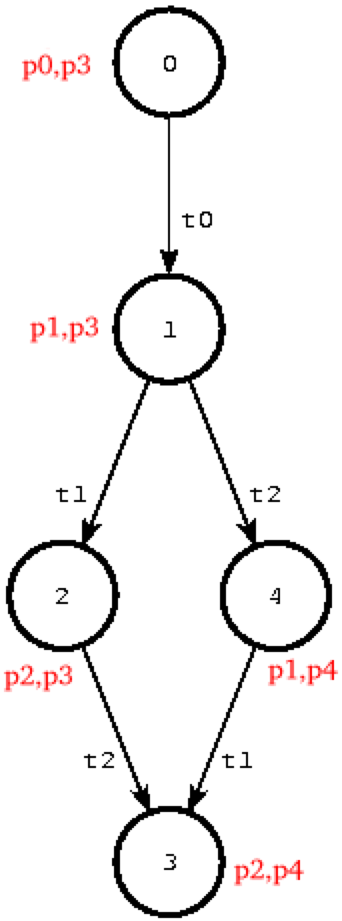
→ formal proof of the decidability of TCTL for bounded TPNs

[Gardey, Roux & Roux 2003] Zone-based algorithm for checking reachability

Problem with Branching Time





CTL formula: $EF(p1 \wedge p3 \wedge AF(p2 \wedge p3))$

Graphs with Durations

✔ $\ell \xrightarrow{[n,m]} \ell'$: "moving from ℓ to ℓ' takes some duration d in $[n, m]$"

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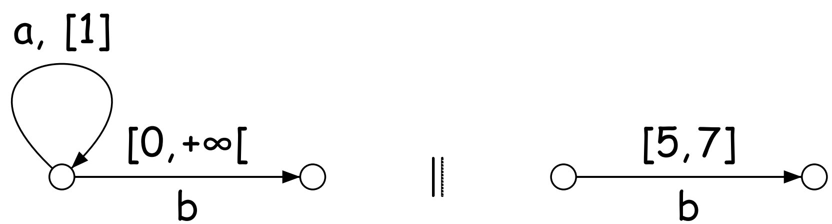
- ✔ **Problem with parallel composition**



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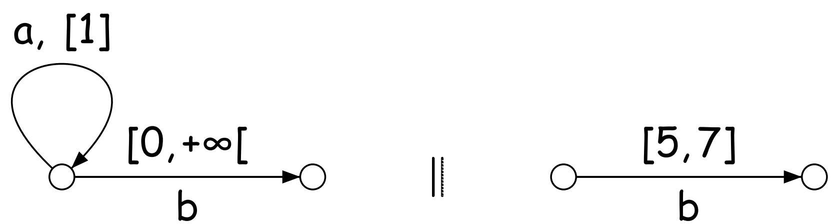

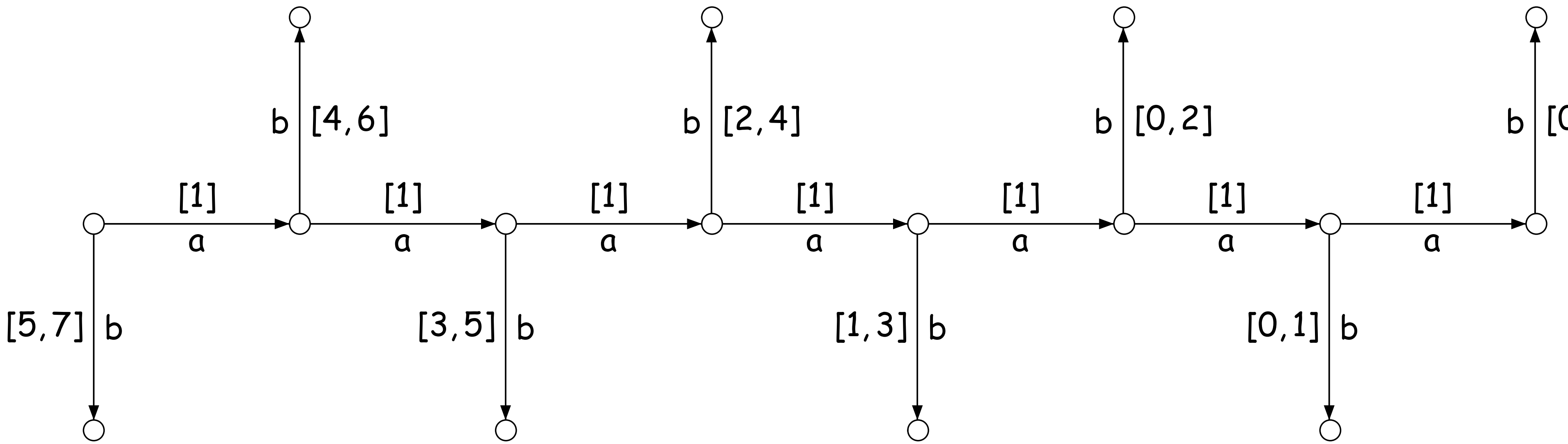

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- ✔ **Problem with parallel composition**

  ➜ Even if low complexity bounds, not convenient for modelling concurrency

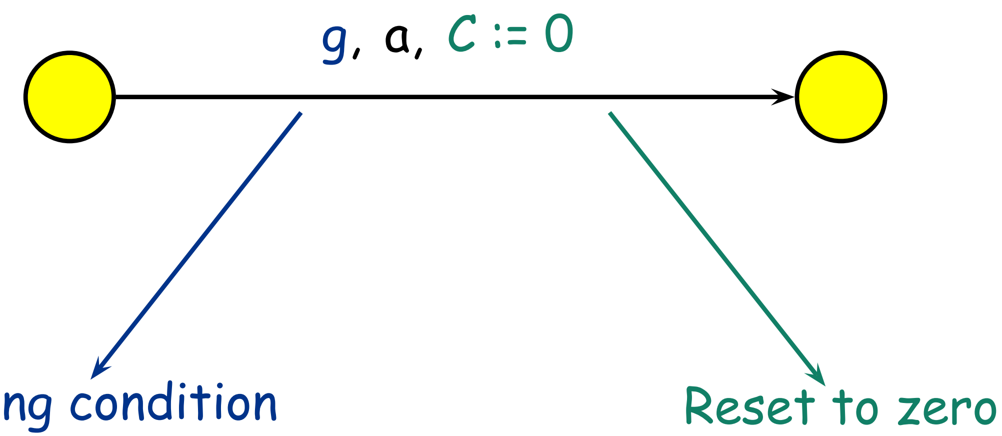
Some Comments

- ✔ the most-accepted timed model is **timed automata**
- ✔ the techniques used for analyzing TPNs and timed automata are very similar
  - state class graph $\leftrightarrow$ zone automaton
  - strong state class graph **[Berthomieu & Vernadat 2003]**
    $\leftrightarrow$ minimal graph **[Bouajjani, Fernandez, Halbwachs & Raymond 1992]**

Timed Automata

Timed Automata [Alur & Dill 90's]

- ✔ A finite control structure + variables (clocks)
- ✔ A transition is of the form:



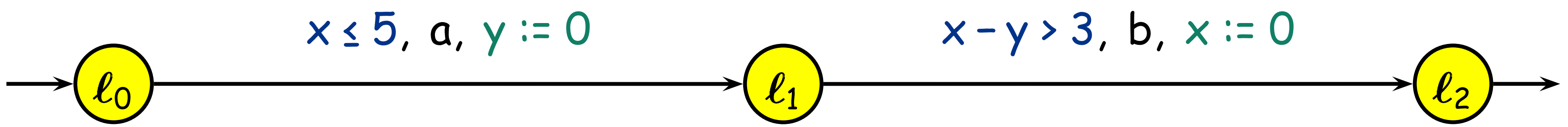
- ✔ An enabling condition (or guard) is:

$$g \ ::= \ x \sim c \ \mid \ x - y \sim c \ \mid \ g \wedge g$$

where $\sim \in \{<, \leq, =, \geq, >\}$

Timed Automata, an Example

x, y : clocks



Timed Automata, an Example

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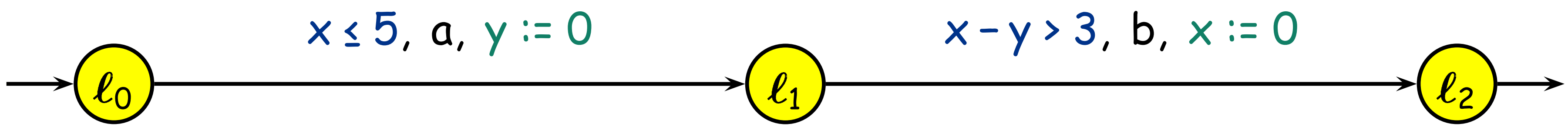
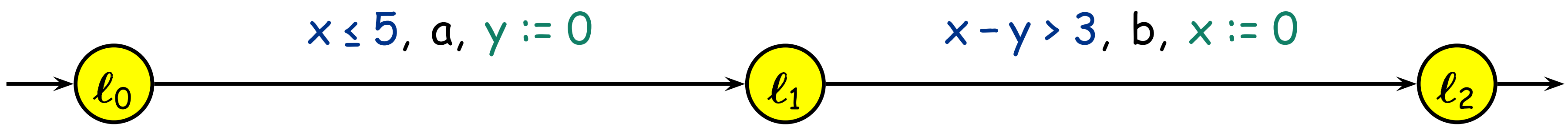


	ℓ_0	$\xrightarrow{\delta(4.1)}$	ℓ_0	$\xrightarrow{a}$	ℓ_1	$\xrightarrow{\delta(1.4)}$	ℓ_1	$\xrightarrow{b}$	ℓ_2
x	0		4.1		4.1		5.5		0
y	0		4.1		0		1.4		1.4

Timed Automata, an Example

x, y : clocks



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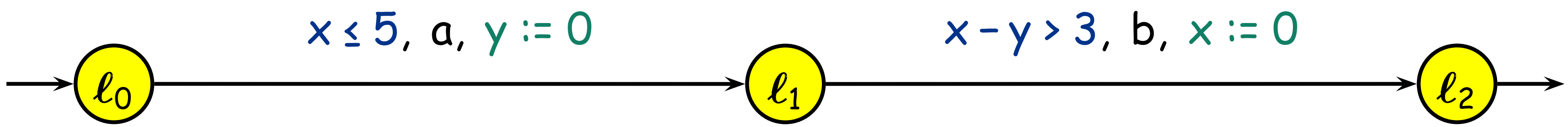

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(clock) valuation

→ timed word $(a, 4.1)(b, 5.5)$

TA Semantics

- $\mathcal{A} = (\Sigma, L, X, \longrightarrow)$ is a TA
- **Configurations:** $(\ell, v) \in L \times T^X$ where T is the time domain
- **Timed Transition System:**
  - **action transition:** $(\ell, v) \xrightarrow{a} (\ell', v')$ if $\exists \ell \xrightarrow{g,a,r} \ell' \in \mathcal{A}$ s.t. $\begin{array}{l} v \models g \\ v' = v[r \leftarrow 0] \end{array}$
  - **delay transition:** $(q, v) \xrightarrow{\delta(d)} (q, v + d)$ if $d \in T$

Some Exercices

What do the following TA recognize?

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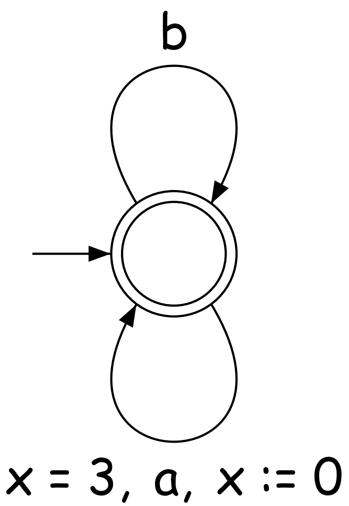

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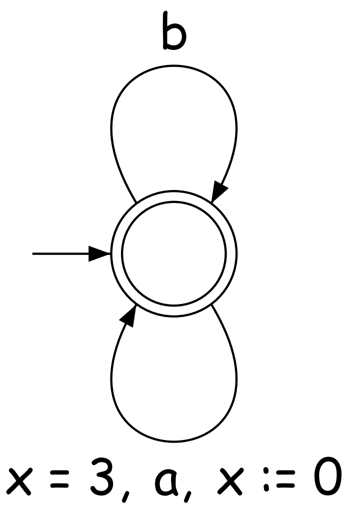

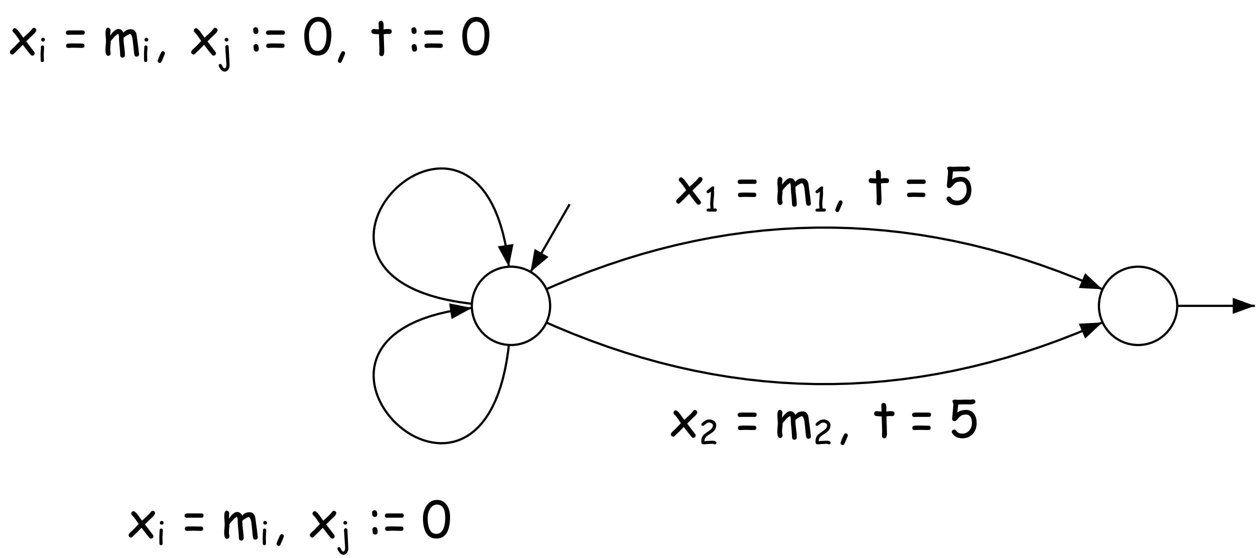

Composition of TA

To model concurrent systems: several communicating components

➜ n-ary synchronization function

(combine synchronization rules and interleaving rules)

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➜ n-ary synchronization function

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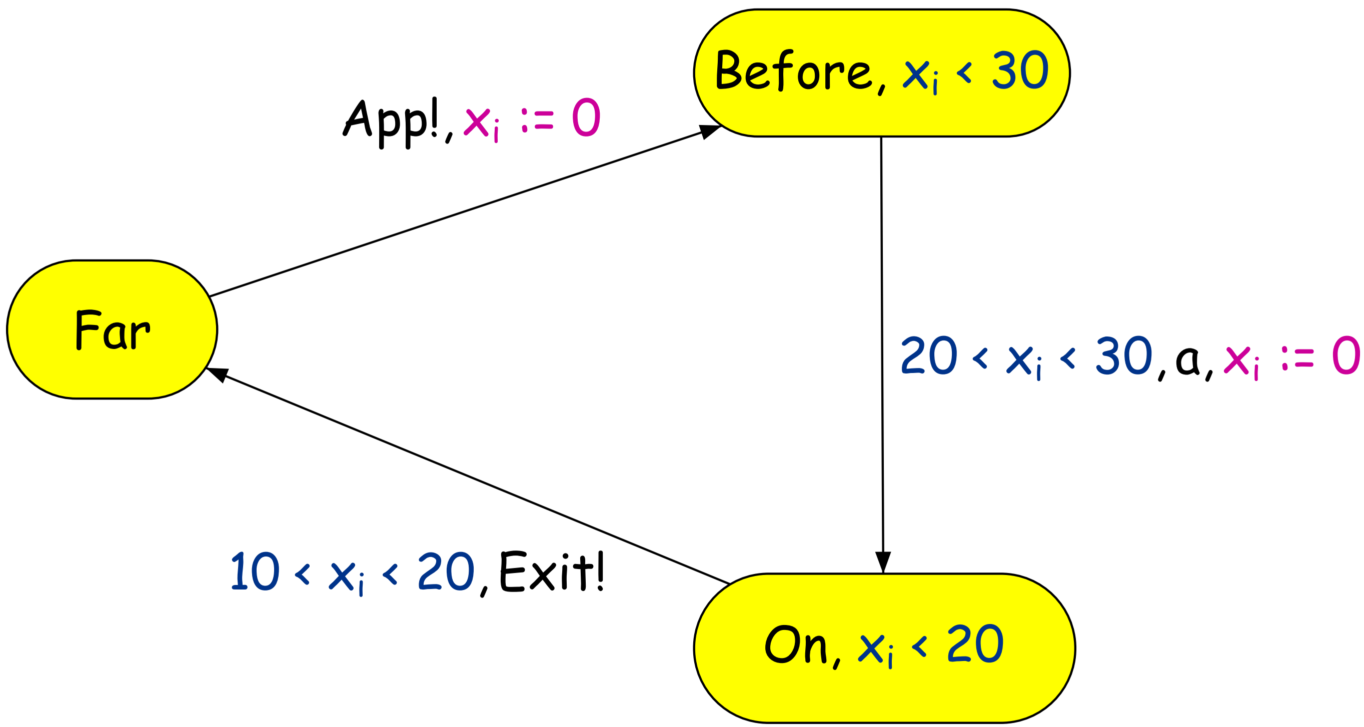
Remark: concurrent timed automata **[Lanotte, Maggiolo-Schettini & Tini 2003]**

- notion of private/shared clocks
- relative conciseness

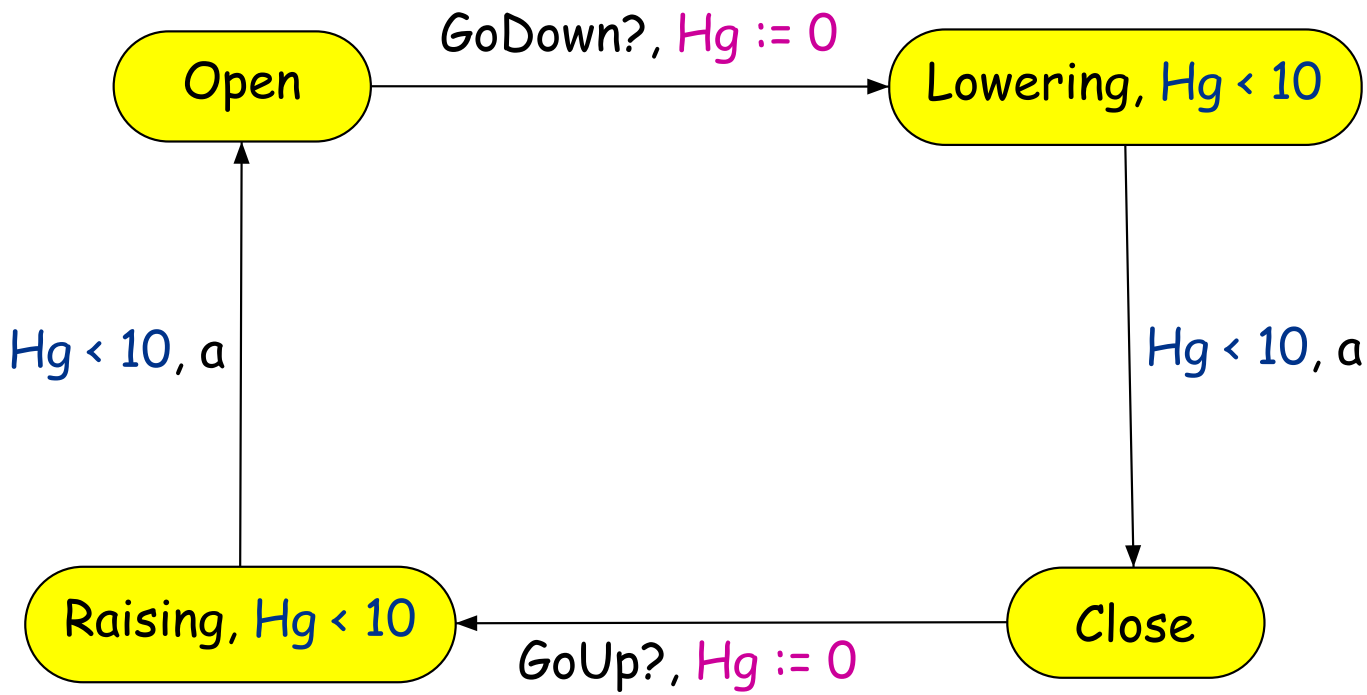
The Train Crossing Example (1)

Train$_i$ with $i = 1, 2, \ldots$



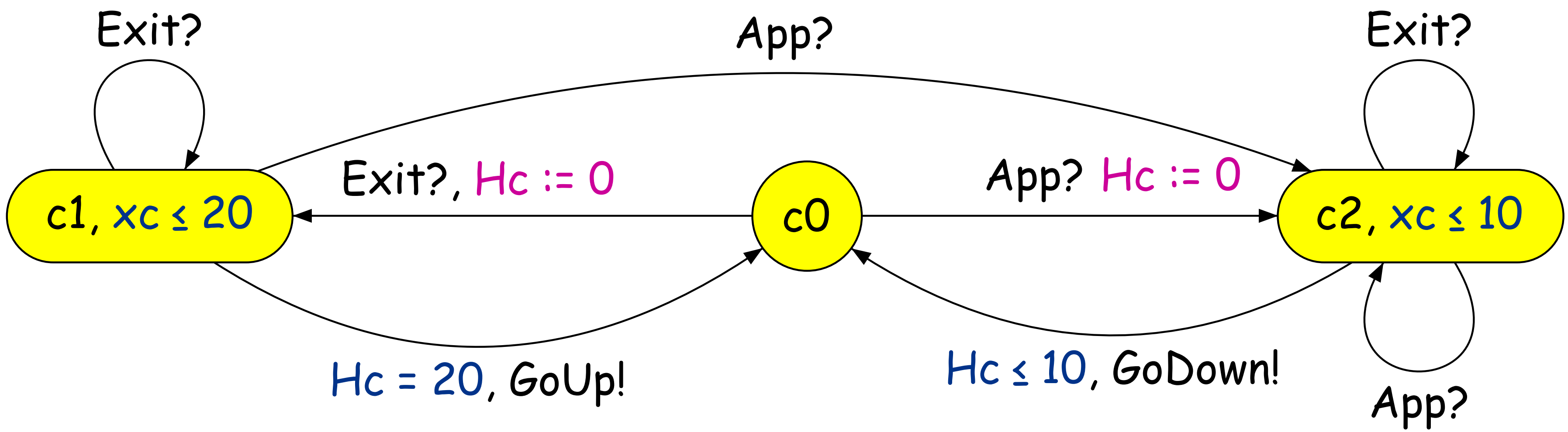
The Train Crossing Example (2)

The gate:



The Train Crossing Example (3)

The controller:



The Train Crossing Example (4)

We use the synchronization function f:

$Train_1$	$Train_2$	Gate	Controller	
App!	.	.	App?	App
.	App!	.	App?	App
Exit!	.	.	Exit?	Exit
.	Exit!	.	Exit?	Exit
a	.	.	.	a
.	a	.	.	a
.	.	a	.	a
.	.	GoUp?	GoUp!	GoUp
.	.	GoDown?	GoDown!	GoDown

to define the parallel composition $(Train_1 \parallel Train_2 \parallel Gate \parallel Controller)$

NB: the parallel composition does not add expressive power !

The Train Crossing Example (5)

Some properties one could check:

- ✔ Is the gate closed when a train crosses the road?

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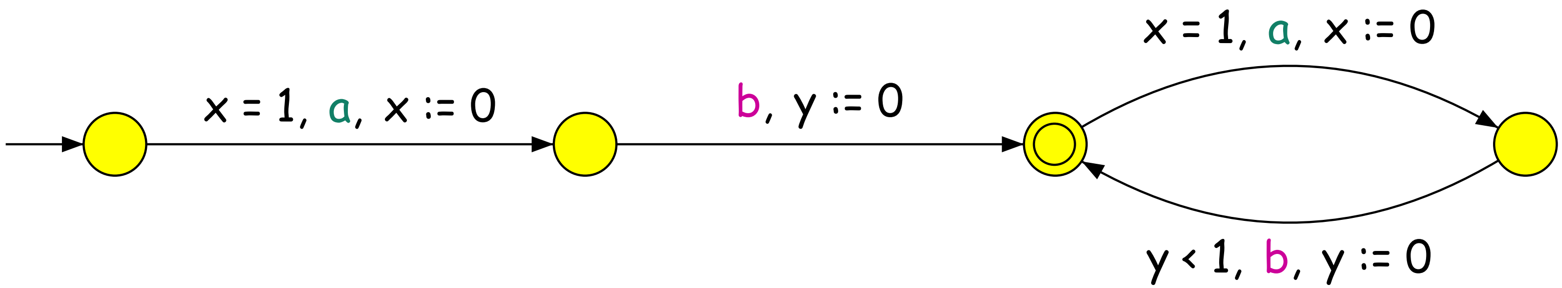
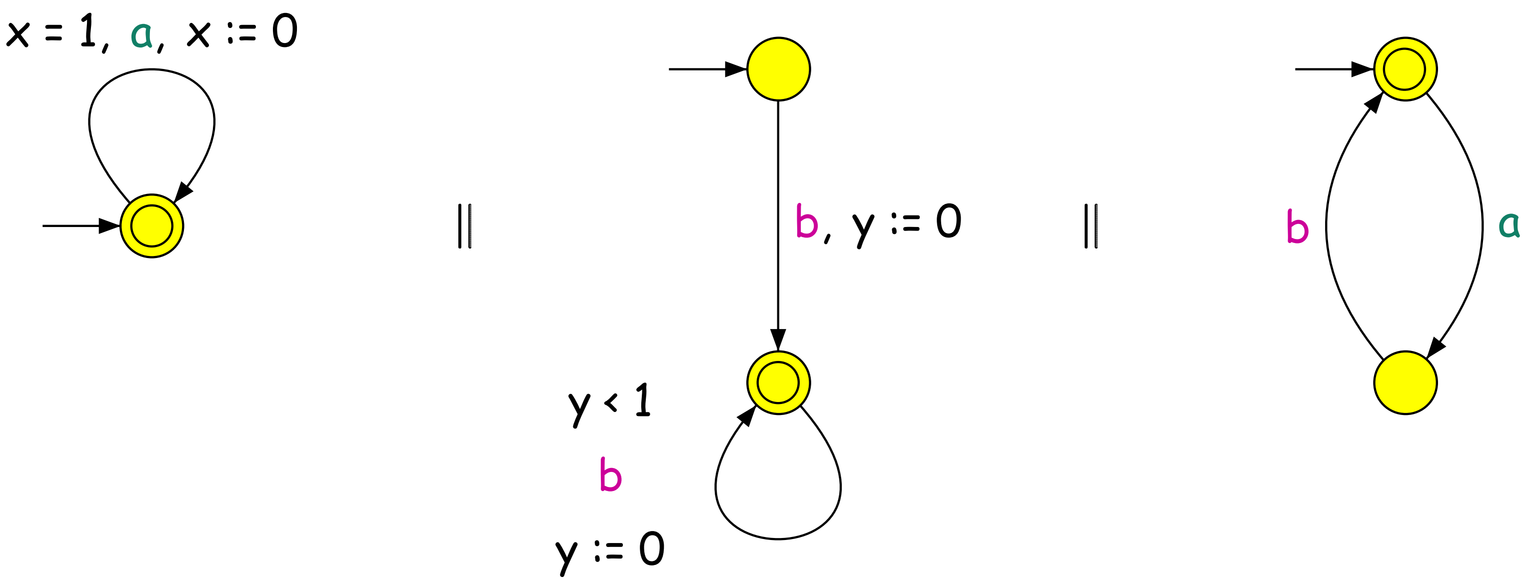
$$\neg EF(\text{gate.Close} \wedge (\text{gate.Close}\ U_{>5\ \text{min}}\ \neg\text{gate.Close}))$$

Discrete vs Dense-Time Semantics



- ✔ Dense-time: $L_{dense} = \{((ab)^\omega, \tau) \mid \forall i,\ \tau_{2i-1} = i \text{ and } \tau_{2i} - \tau_{2i-1} > \tau_{2i+2} - \tau_{2i+1}\}$
- ✔ Discrete-time: $L_{discrete} = \emptyset$



Verification of TA

Problem: the set of configurations is infinite

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Positive key point: variables (clocks) have the same speed

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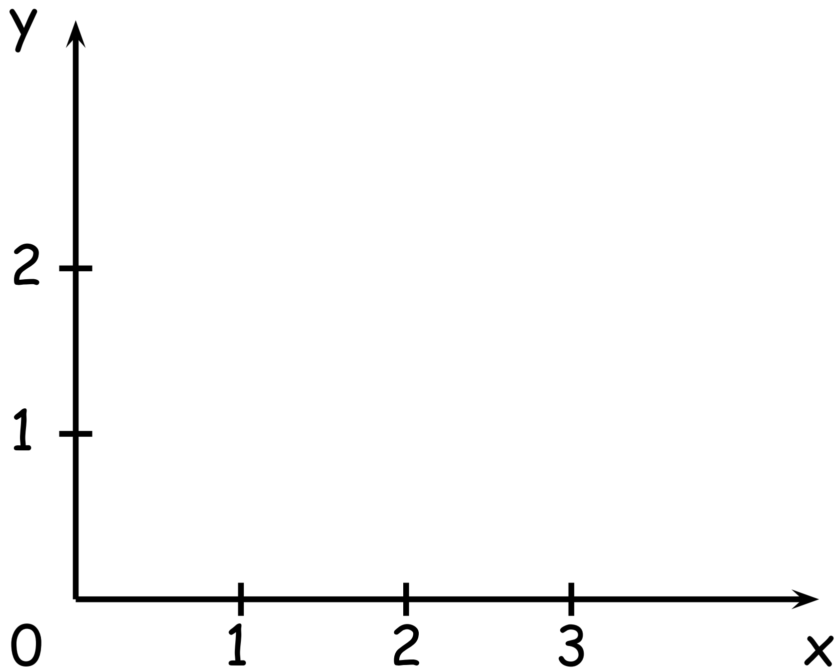
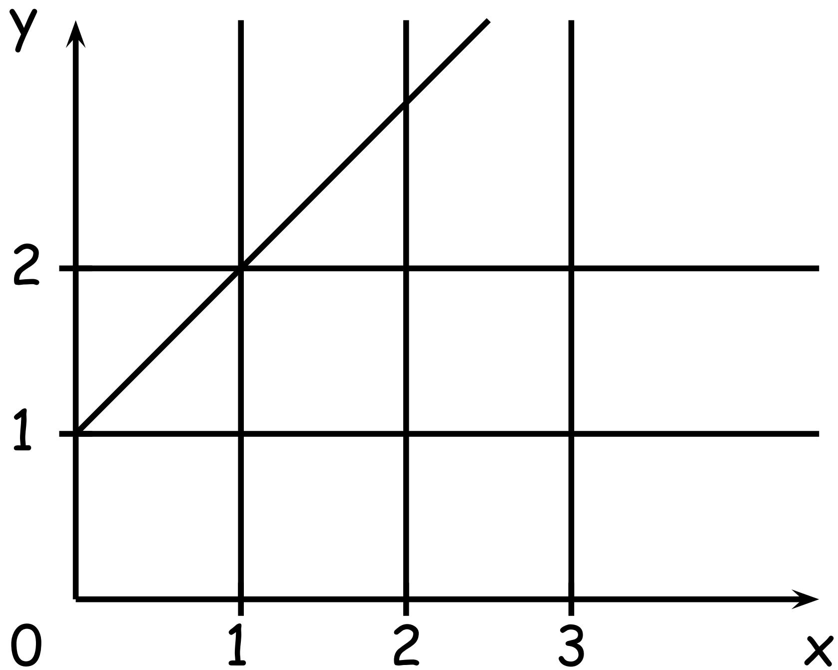
Aim: construct a finite abstraction

The Region Abstraction



Equivalence of finite index

The Region Abstraction



Equivalence of finite index

- ✔ “compatibility” between regions and constraints

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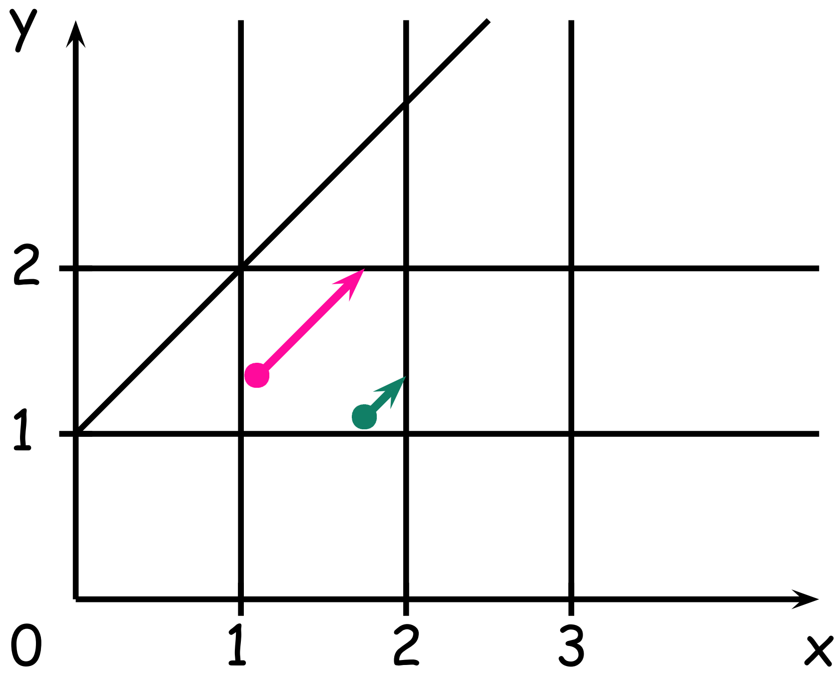

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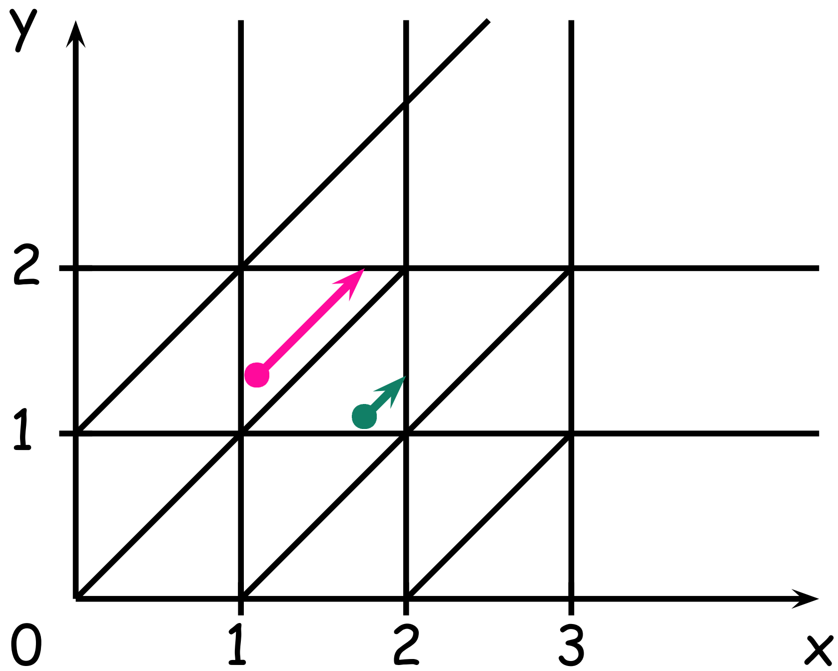

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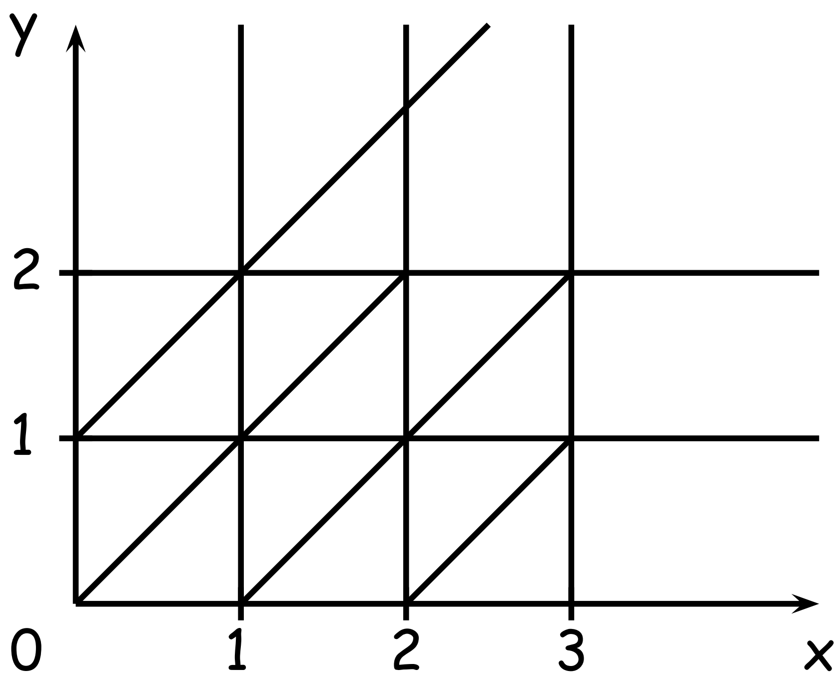

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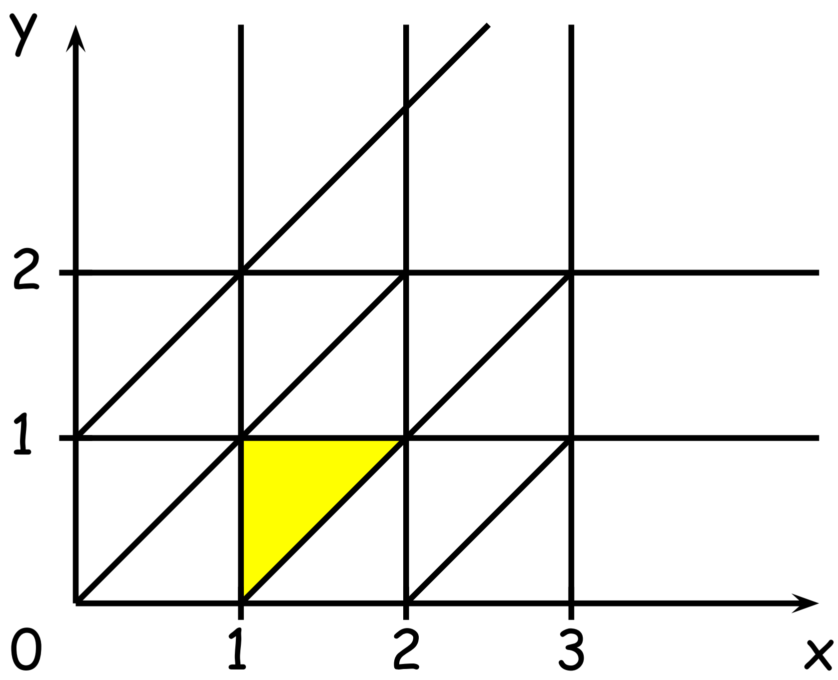

Equivalence of finite index

region defined by
$I_x =]1;2[$, $I_y =]0;1[$
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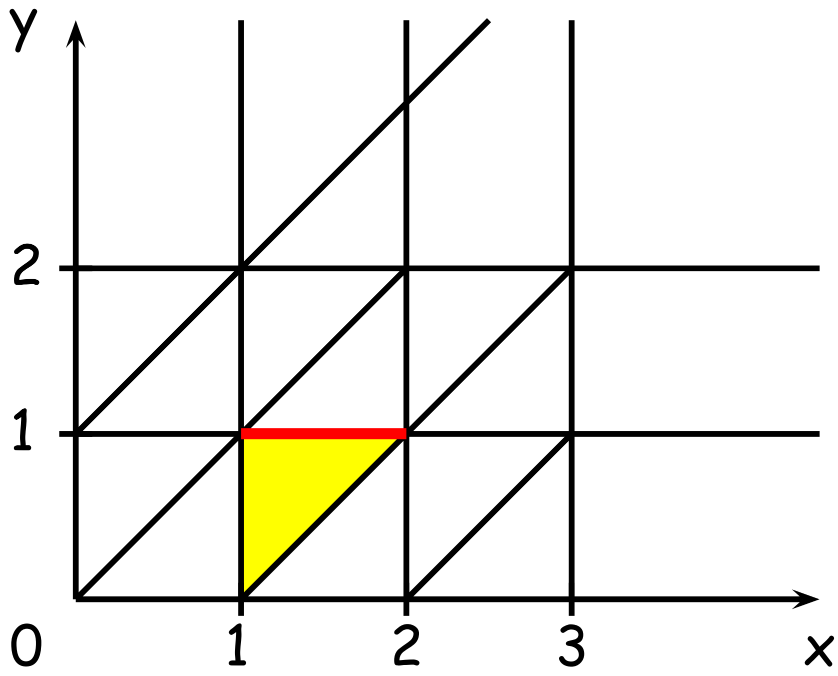

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successor region
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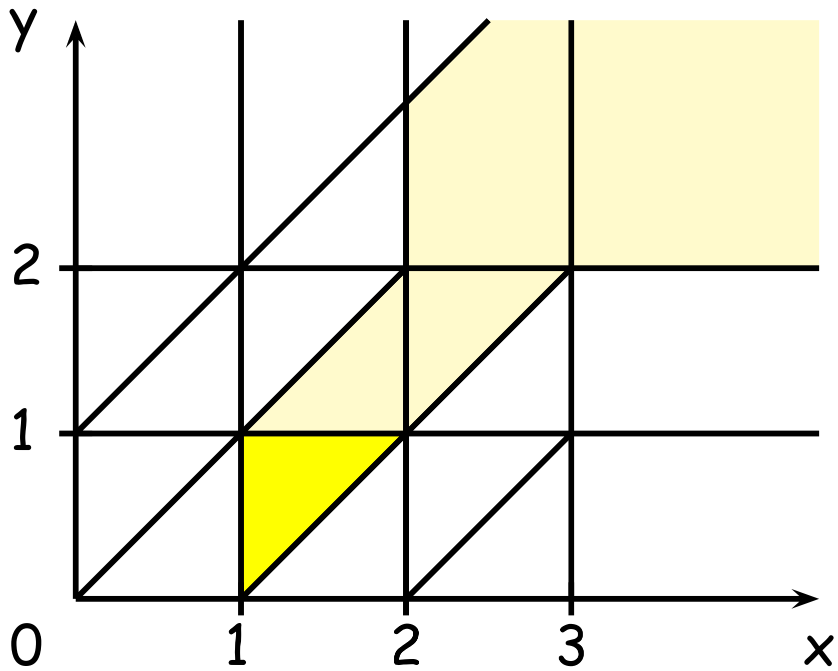

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The Region Automaton

timed automaton $\otimes$ region abstraction

$\ell \xrightarrow{g,a,C:=0} \ell'$ is transformed into:

$(\ell, R) \xrightarrow{a} (\ell', R')$ if there exists $R'' \in \mathrm{Succ}_t^*(R)$ s.t.

- ✔ $R'' \subseteq g$
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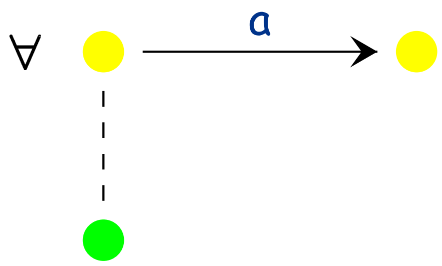
Time-Abstract Bisimulation



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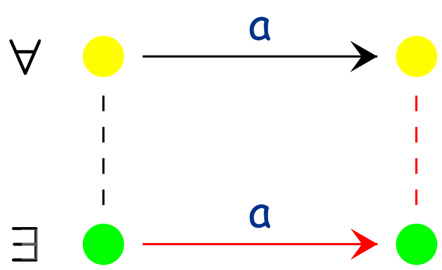

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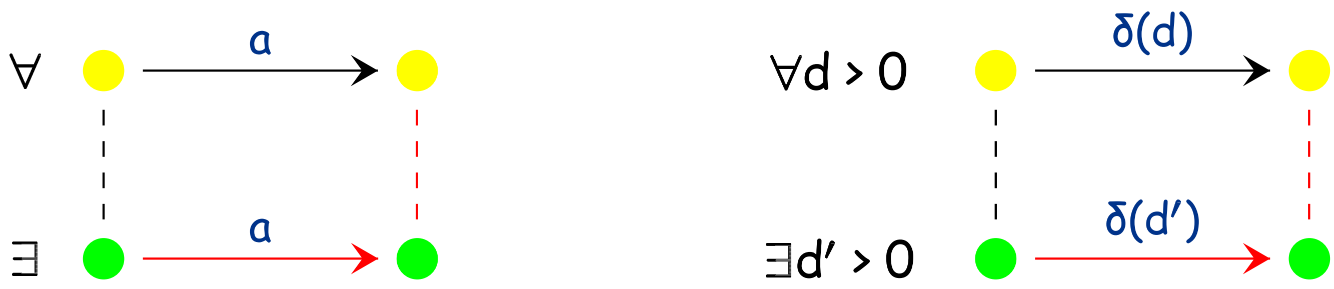

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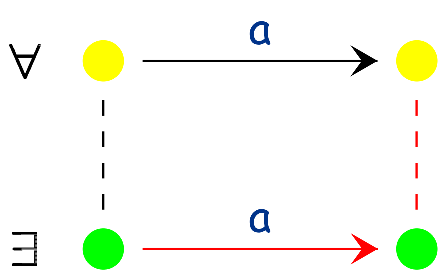

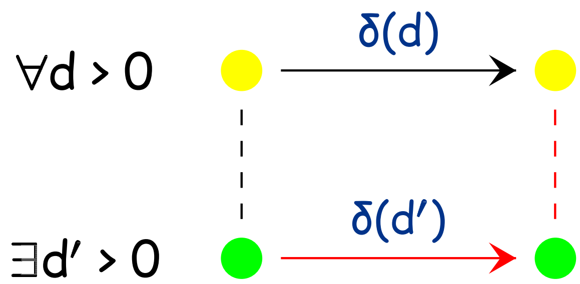

$$(\ell_0, v_0) \xrightarrow{a_1, t_1} (\ell_1, v_1) \xrightarrow{a_2, t_2} (\ell_2, v_2) \xrightarrow{a_3, t_3} \ldots$$

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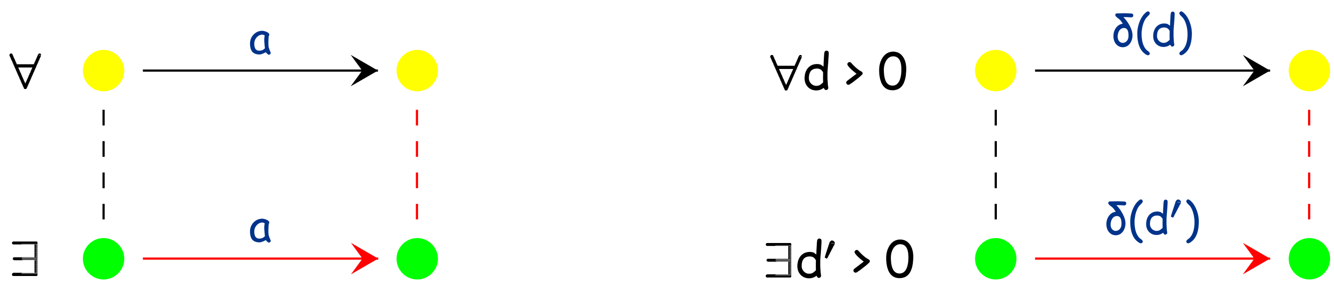

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\downarrow & & \downarrow & & \downarrow & & \\
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\end{array}
$$

with $v_i \in R_i$ for all i.

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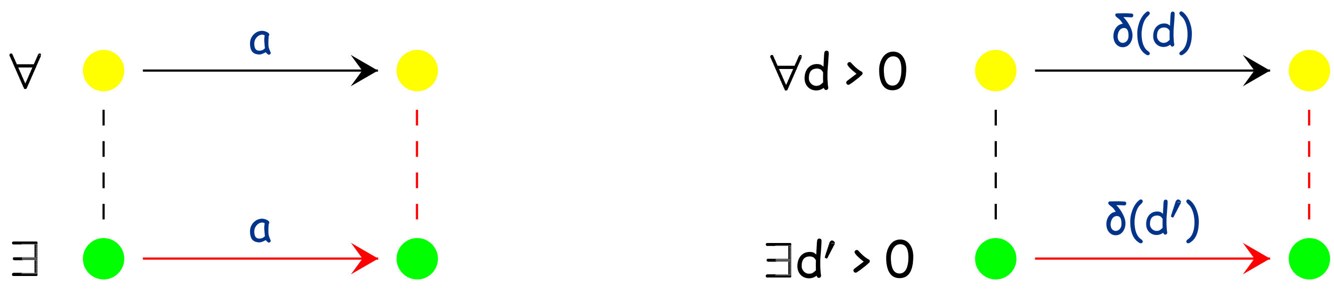

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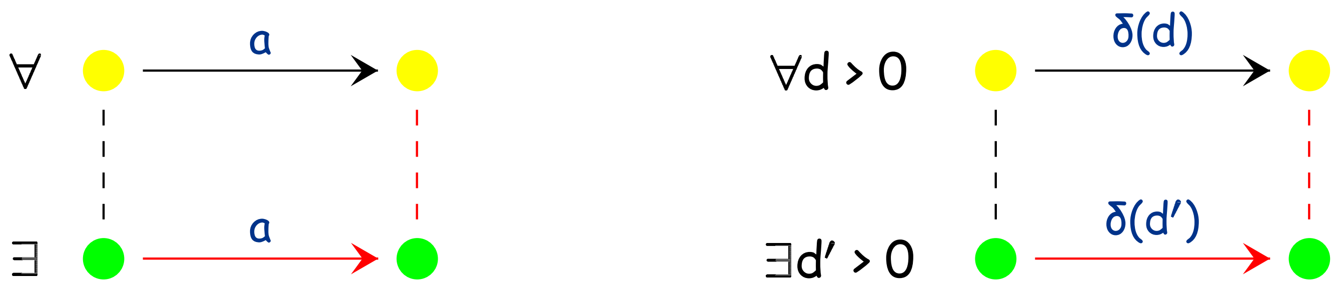

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Remark: We can not check real-time properties with a time-abstract bisimulation.
We need to add clocks for the formula we want to check.

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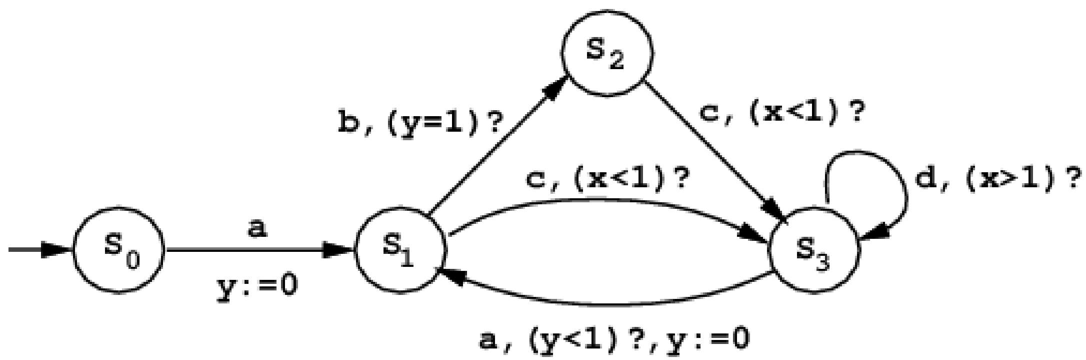
$\mathcal{L}$(reg. aut.) = UNTIME($\mathcal{L}$(timed aut.))

where $\mathrm{UNTIME}((a_1, t_1)(a_2, t_2)\ldots) = a_1 a_2 \ldots$

An Example [Alur & Dill 90's]



Questions:

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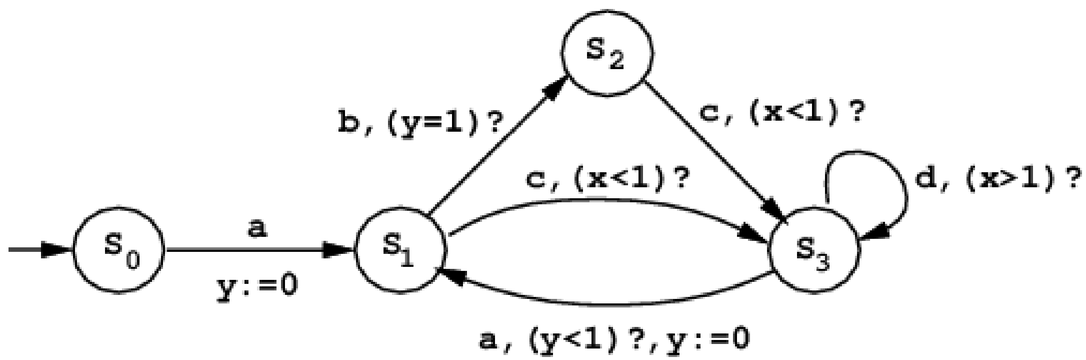

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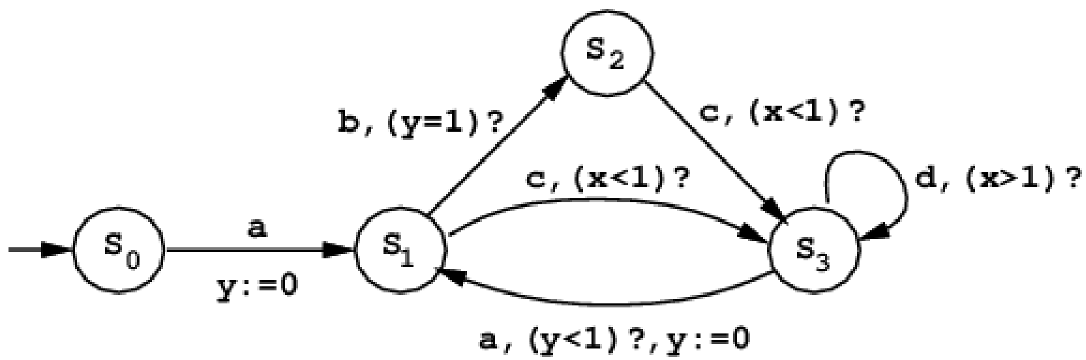

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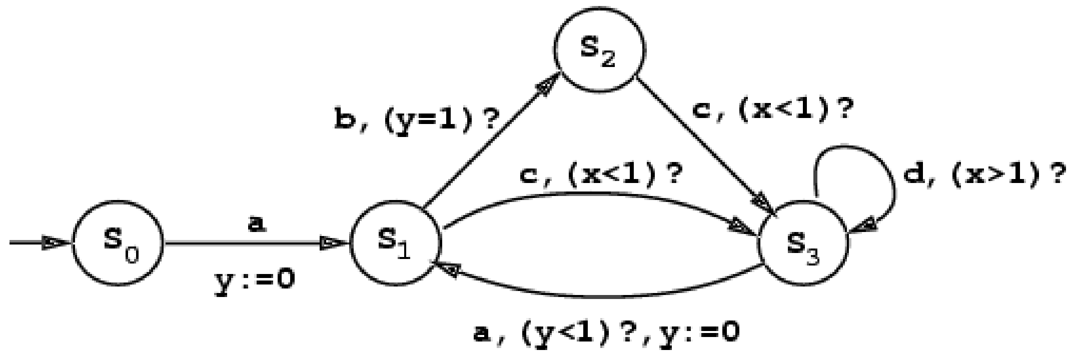

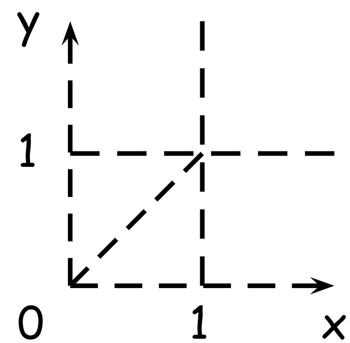

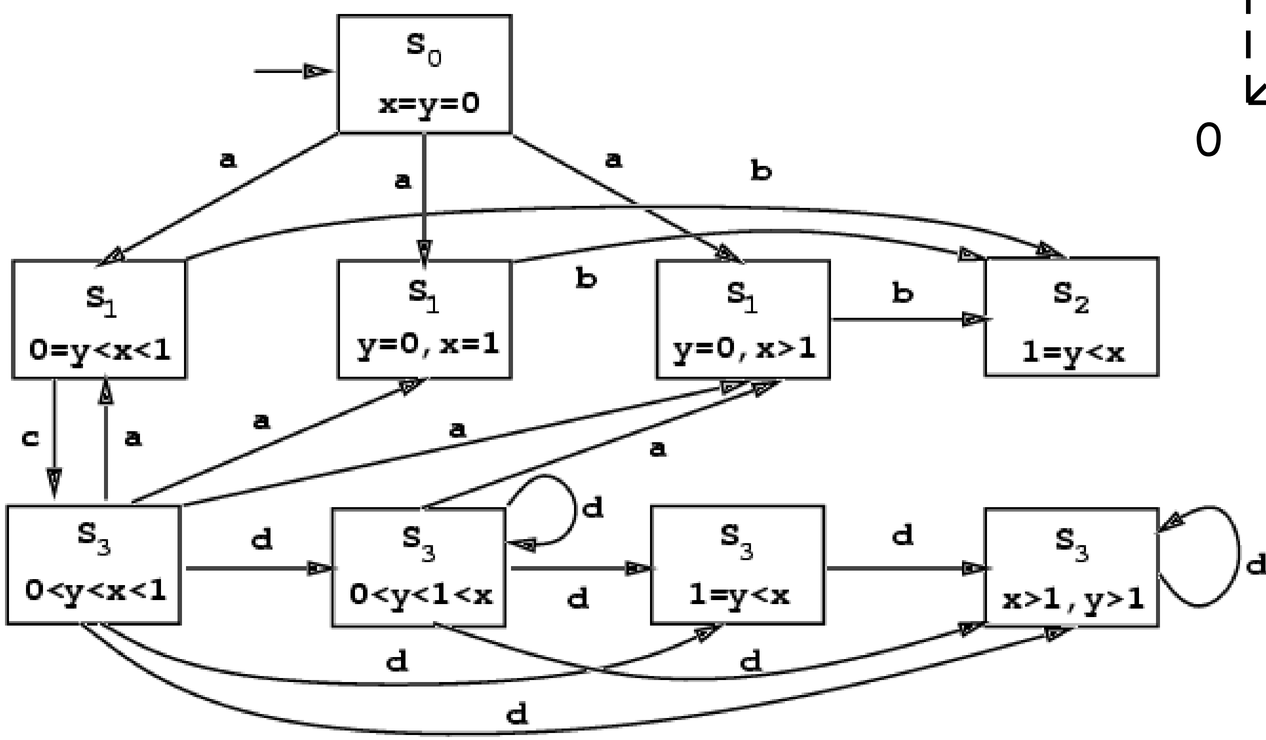

Main Basis Result

Theorem **[Alur & Dill 90's]** Reachability is decidable for TA.

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¡ The size of the region graph is in $\mathcal{O}(|X|!.2^{|X|})$!

Theorem **[Alur & Dill 90's]** Reachability is decidable for TA.
It is even PSPACE-complete.

PSPACE-Easyness

✔ **One configuration:** a discrete location + a region

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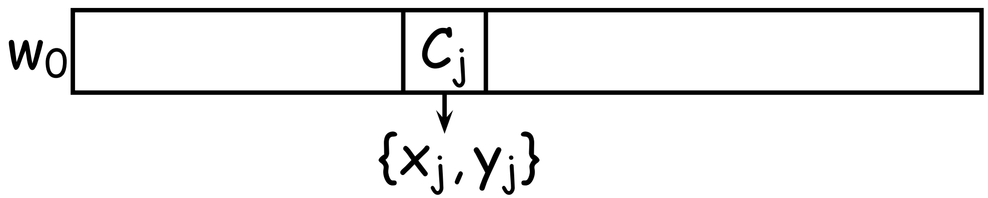
➜ in NPSPACE, thus in PSPACE

PSPACE-Hardness

$\left.\begin{array}{c}\mathcal{M} \text{ LBTM} \\ w_0 \in \{a,b\}^*\end{array}\right\} \rightsquigarrow A_{\mathcal{M},w_0}$ s.t. $\mathcal{M}$ accepts w_0 iff the final state of $A_{\mathcal{M},w_0}$ is reachable



C_j contains a "a" iff $x_j = y_j$

C_j contains a "b" iff $x_j < y_j$

(these conditions are invariant by time elapsing)

➜ proof taken in **[Aceto & Laroussinie 2002]**

PSPACE-Hardness (cont.)

If $q \xrightarrow{\alpha,\alpha',\delta} q'$ is a transition of $\mathcal{M}$, then for each position i of the tape, we have a transition

$$(q,i) \xrightarrow{g,r:=0} (q',i')$$

where:

- g is $x_i = y_i$ (resp. $x_i < y_i$) if $\alpha = a$ (resp. $\alpha = b$)
- $r = \{x_i, y_i\}$ (resp. $r = \{x_i\}$) if $\alpha = a$ (resp. $\alpha = b$)
- $i' = i + 1$ (resp. $i' = i - 1$) if δ is right and $i < n$ (resp. left)

Enforcing time elapsing: on each transition, add the condition $t = 1$ and clock t is reset.

Initialization: $\text{init} \xrightarrow{t=1, r_0:=0} (q_0, 1)$ where $r_0 = \{x_i \mid w_0[i] = b\} \cup \{t\}$

Termination: $(q_f, i) \longrightarrow \text{end}$

Tighter Results

- ✔ Reachability in TA is PSPACE-complete even if the time is discrete!
  [Alur & Dill 90's]

- ✔ Reachability in TA with integer constants in $\{1, 2\}$ is PSPACE-complete.
  [Courcoubetis & Yannakakis 1992]

- ✔ Reachability in TA with 3 clocks is PSPACE-complete.
  [Courcoubetis & Yannakakis 1992]

- ✔ Reachability in TA with 1 clock is NLOGSPACE-complete.
  [Laroussinie, Markey & Schnoebelen 2004]

- ✔ Reachability in TA with 2 clocks is NP-hard.
  [Laroussinie, Markey & Schnoebelen 2004]

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Theorem [Alur, Courcoubetis & Dill 1990]
Model-checking of TCTL is PSPACE-complete for TA.

A Model Not Far From Undecidability

- ✔ Universality is undecidable **[Alur & Dill 90's]**
- ✔ Inclusion is undecidable **[Alur & Dill 90's]**
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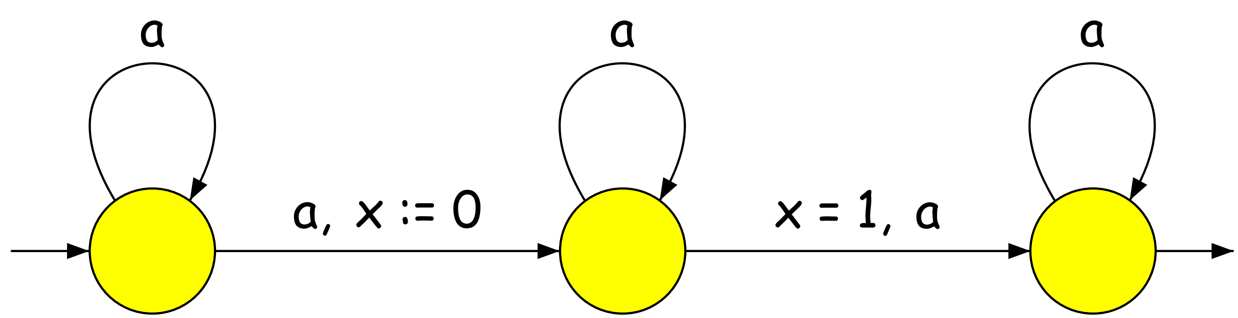
An example of non-deterministic TA:



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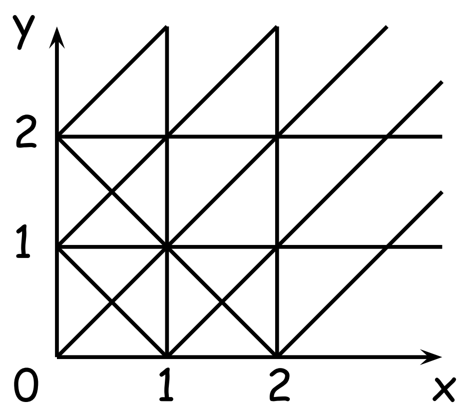
  [Bouyer, Dufourd, Fleury, Petit 2000]
- ✔ more general variables, e.g. hybrid systems Undecidable!
  [Alur, Courcoubetis, Henzinger, Ho 1993]
  [Henzinger 1996] [Henzinger, Kopke, Puri, Varaiya 1998]
  ...

Adding Constraints of the Form $x + y \sim c$

$x + y \sim c$ and $x \sim c$ [Bérard,Dufourd 2000]

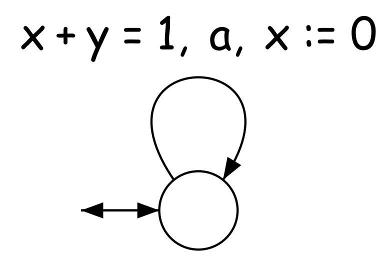
✔ **Decidability:** - for two clocks, decidable using the abstraction



- for four clocks (or more), undecidable!

✔ **Expressiveness:** more expressive! (even using two clocks)

$$\{(a^n, t_1 \ldots t_n) \mid n \geq 1 \text{ and } t_i = 1 - \tfrac{1}{2^i}\}$$



The Two-Counter Machine

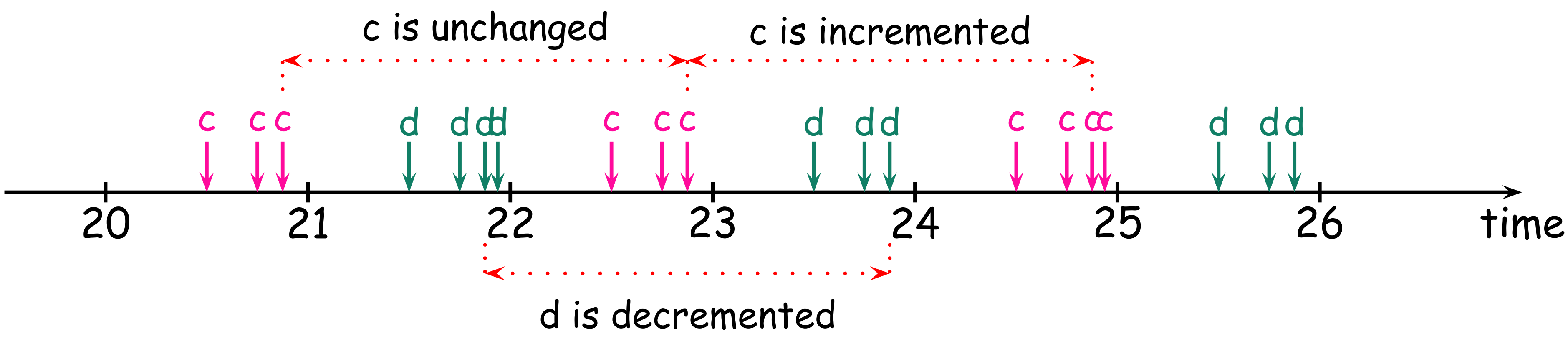
Definition. A two-counter machine is a finite set of instructions over two counters (x and y):

- Incrementation:
  (p): $x := x + 1$; **goto (q)**

- Decrementation:
  (p): if $x > 0$ **then** $x := x - 1$; **goto (q) else goto (r)**

Theorem. [Minsky 67] The emptiness problem for two counter machines is undecidable.

Undecidability Proof



➜ simulation of
- decrement of d
- increment of c

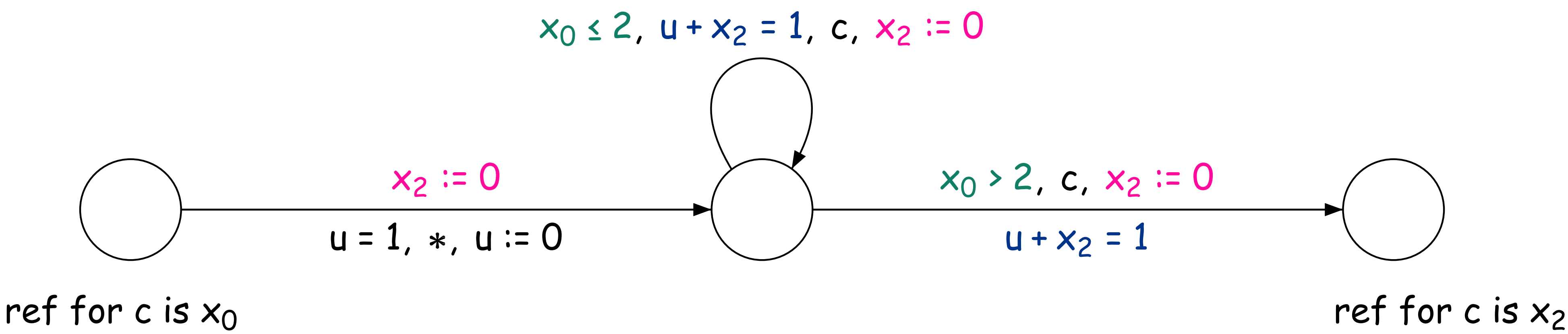
We will use 4 clocks:
- u, "tic" clock (each time unit)
- x_0, x_1, x_2: reference clocks for the two counters

"x_i reference for c" $\equiv$ "the last time x_i has been reset is the last time action c has been performed"

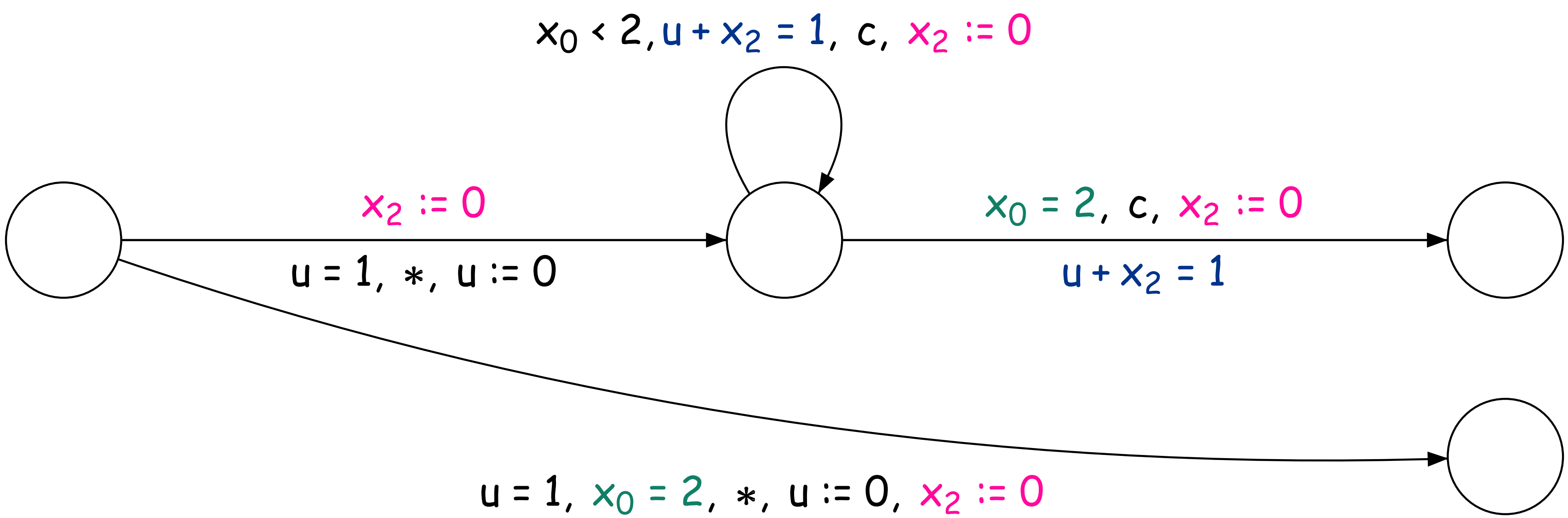
[Bérard, Dufourd 2000]

Undecidability Proof (cont.)

- ✔ **Increment of counter** c:

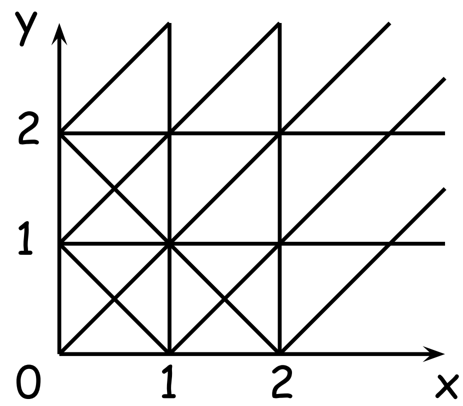


- ✔ **Decrement of counter** c:



Adding Constraints of the Form $x + y \sim c$

- Two clocks: decidable! using the abstraction



- Four clocks (or more): undecidable!

Adding Constraints of the Form $x + y \sim c$

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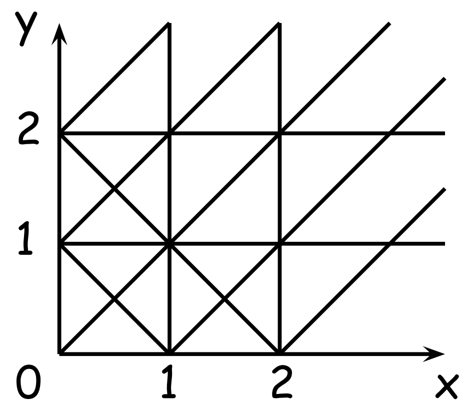

- ✔ Three clocks: **open question**

- ✔ Four clocks (or more): undecidable!

Networks of TA, discussion

Complexity of Model-Checking

	Kripke structures S	Timed automata A
Reachability	NLOGSPACE-complete	
CTL/TCTL	P-complete	
AF-μ-calc./$L_{\mu,\nu}$	P-complete	
full μ-calc./$L^{+}_{\mu,\nu}$	UP $\cap$ co-UP	

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Timing constraints induce a complexity blowup !

[Alur 1991, Alur Henzinger 1994, Alur Courcoubetis Dill 1993, Aceto Laroussinie 1999]

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- ✔ due to parallel composition
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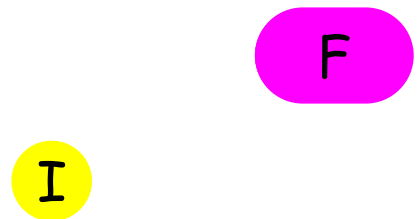
Verification Methods

- ✔ on-the-fly backward algorithms
- ✔ on-the-fly forward algorithms
- ✔ compositional algorithms

Reachability Analysis

✔ **forward analysis algorithm:**
compute the successors of initial configurations



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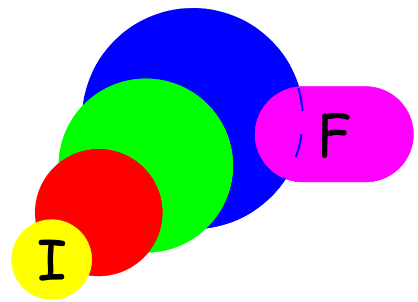

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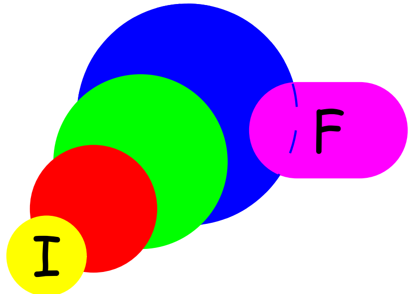

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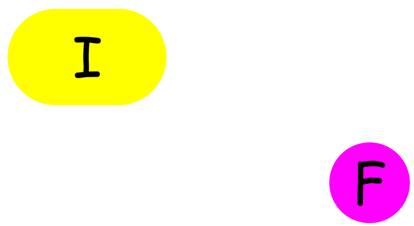

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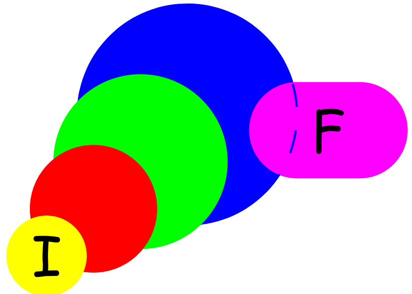

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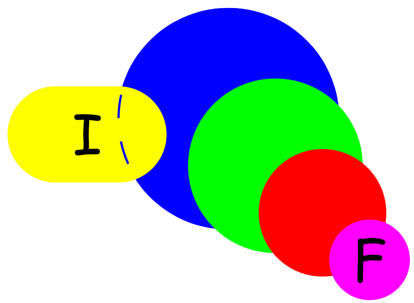
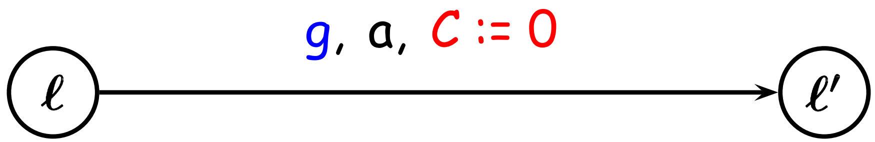


Note on the Backward Analysis



$\overleftarrow{[C \leftarrow 0]^{-1}(Z \cap (C = 0)) \cap g}$ $\qquad$ Z

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$$\ell \xrightarrow{g,\ a,\ C := 0} \ell'$$

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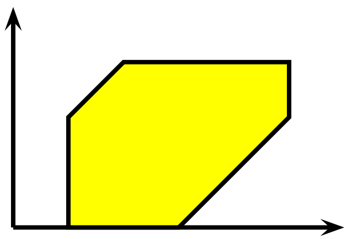

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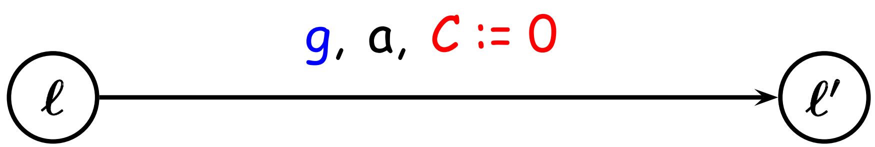

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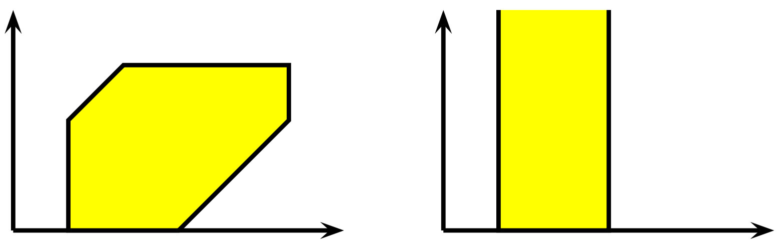

Z $[C \leftarrow 0]^{-1}(Z \cap (C = 0))$

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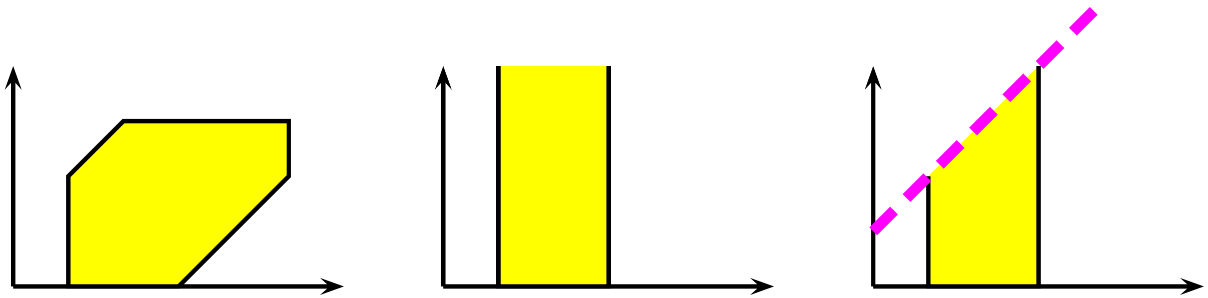
$$[C \leftarrow 0]^{-1}(Z \cap (C = 0)) \cap g \longleftarrow Z$$



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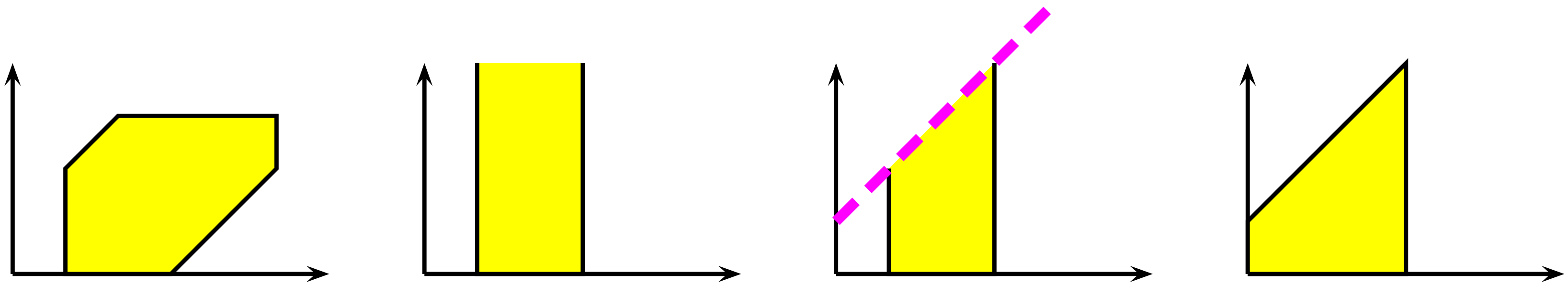

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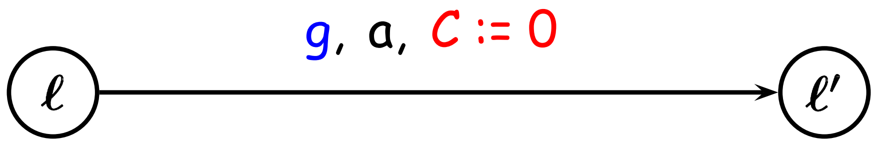

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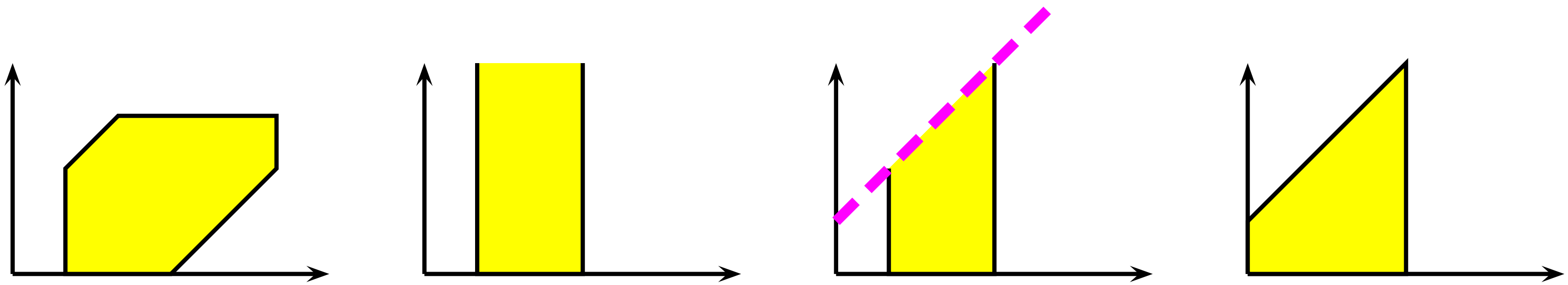

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The exact backward computation terminates and is correct!

Note on the Backward Analysis (cont.)

If $\mathcal{A}$ is a timed automaton, we construct its corresponding set of regions.

Because of the bisimulation property, we get that:

"Every set of valuations which is computed along the backward computation is a finite union of regions"

Note on the Backward Analysis (cont.)

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Because of the bisimulation property, we get that:

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Let R be a region. Assume:

- ✔ $v \in \overleftarrow{R}$ (for ex. $v + t \in R$)
- ✔ $v' \equiv_{reg.} v$

There exists t' s.t. $v' + t' \equiv_{reg.} v + t$, which implies that $v' + t' \in R$ and thus $v' \in \overleftarrow{R}$.

Note on the Backward Analysis (cont.)

If $\mathcal{A}$ is a timed automaton, we construct its corresponding set of regions.

Because of the bisimulation property, we get that:

"Every set of valuations which is computed along the backward computation is a finite union of regions"

But, the backward computation is not so nice, when also dealing with integer variables...

$$i := j.k + \ell.m$$

Remark: Verification of TCTL

For checking $\mathcal{S} \models \varphi$:

- for all subformulas ψ of φ, compute the states $[\psi]$ satisfying ψ
- can be done using backward computations, f.ex.

$$\mathrm{Pre}[\psi](\varphi) = \{v \mid \exists \delta \text{ s.t. } v + \delta \in [\varphi] \wedge \forall 0 \leq \delta' \leq \delta,\ v + \delta' \in [\psi]\}$$

- as previously, everything computed is a finite union of regions...

[Henzinger, Nicollin, Sifakis & Yovine 1994] [Yovine 1998]

Forward Analysis of TA

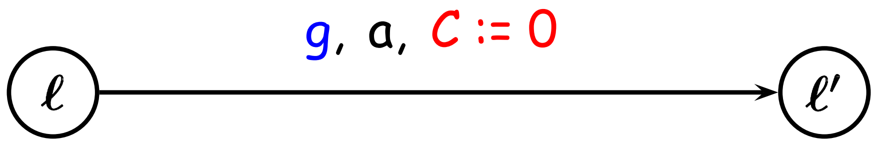
$$\ell \xrightarrow{\quad g,\ a,\ C := 0 \quad} \ell'$$

zones	Z	$[C \leftarrow 0](\overrightarrow{Z} \cap g)$

A zone is a set of valuations defined by a clock constraint

$$\varphi \ ::= \ x \sim c \ \mid \ x - y \sim c \ \mid \ \varphi \wedge \varphi$$

Forward Analysis of TA



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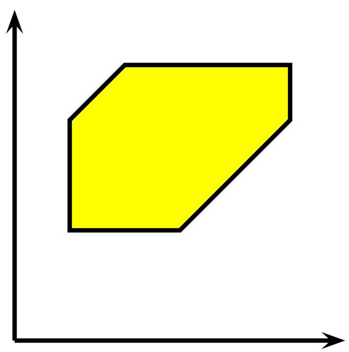

Z

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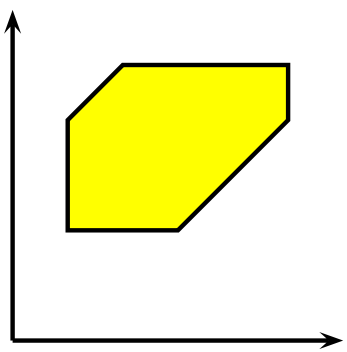

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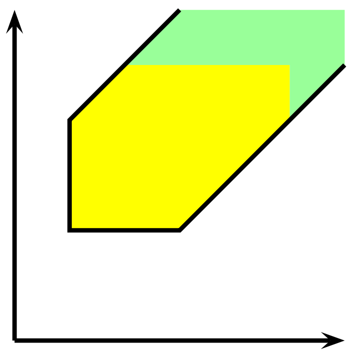

$\overrightarrow{Z}$

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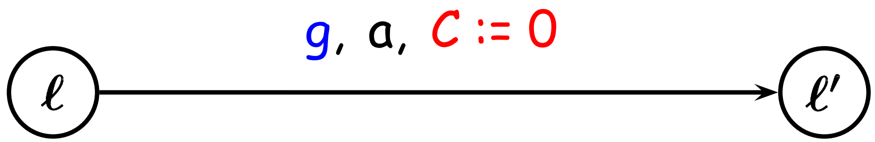

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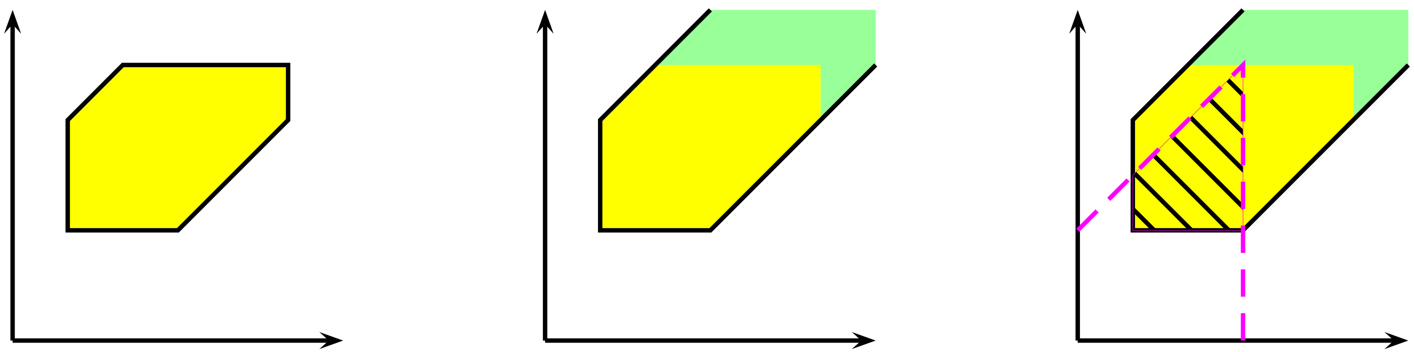

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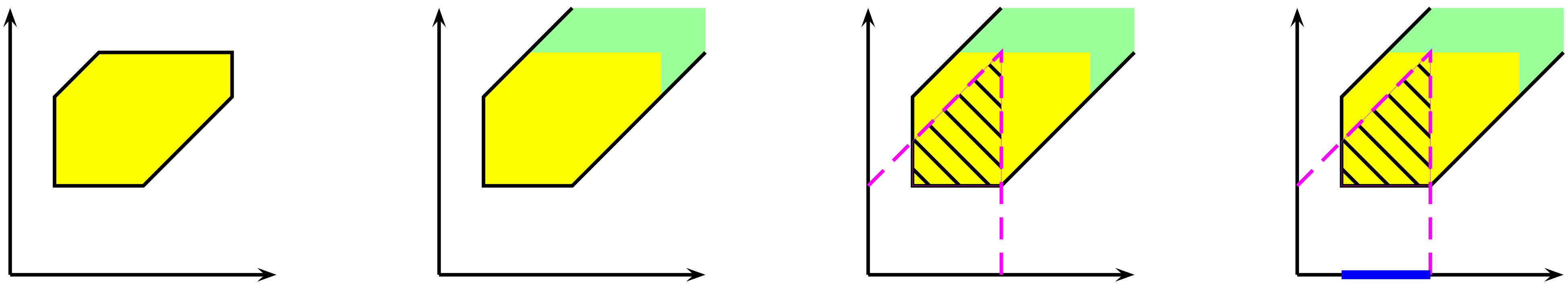

$Z \qquad \overrightarrow{Z} \qquad \overrightarrow{Z} \cap g \qquad [y \leftarrow 0](\overrightarrow{Z} \cap g)$

Forward Analysis of TA

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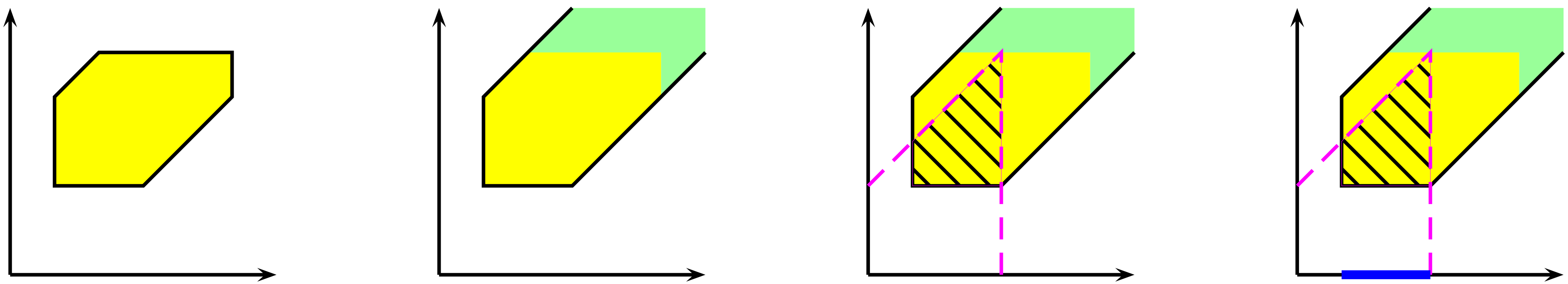
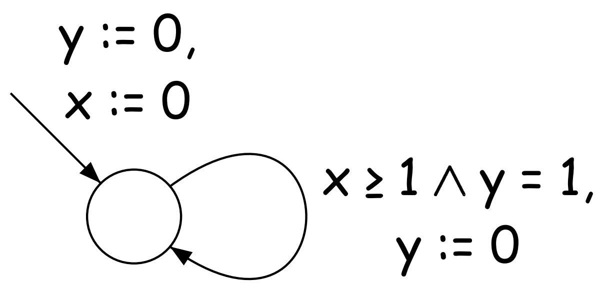
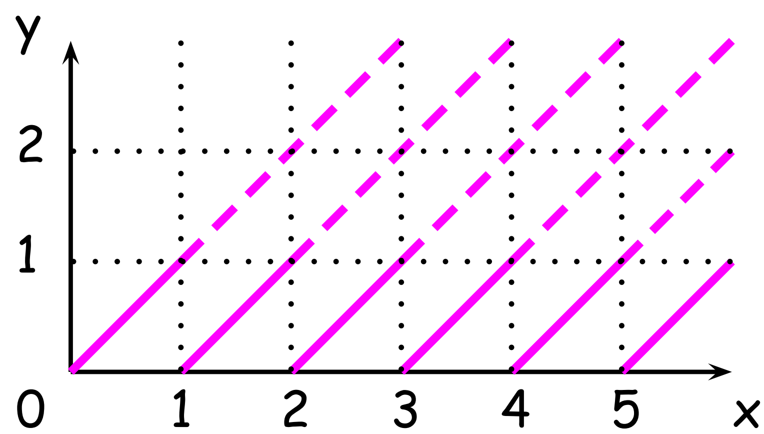


Z $\overrightarrow{Z}$ $\overrightarrow{Z} \cap g$ $[y \leftarrow 0](\overrightarrow{Z} \cap g)$

➜ a termination problem

Non Termination of the Forward Analysis





➜ an infinite number of steps...

"Solutions" to this Problem

(f.ex. in [Larsen,Pettersson,Yi 1997] or in [Daws,Tripakis 1998])

✔ **inclusion checking**: if $Z \subseteq Z'$ and Z' still handled, then we don't need to handle Z

→ correct w.r.t. reachability

...

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- ✔ **inclusion checking**: if $Z \subseteq Z'$ and Z' still handled, then we don't need to handle Z

  ➜ correct w.r.t. reachability

- ✔ **activity**: eliminate redundant clocks [Daws,Yovine 1996]

  ➜ correct w.r.t. reachability

$$q \xrightarrow{g,a,C:=0} q' \quad\Longrightarrow\quad Act(q) = clocks(g) \cup (Act(q') \setminus C)$$

...

"Solutions" to this Problem (cont.)

✔ **convex-hull approximation**: if Z and Z' are computed then we overapproximate using "$Z \sqcup Z'$".

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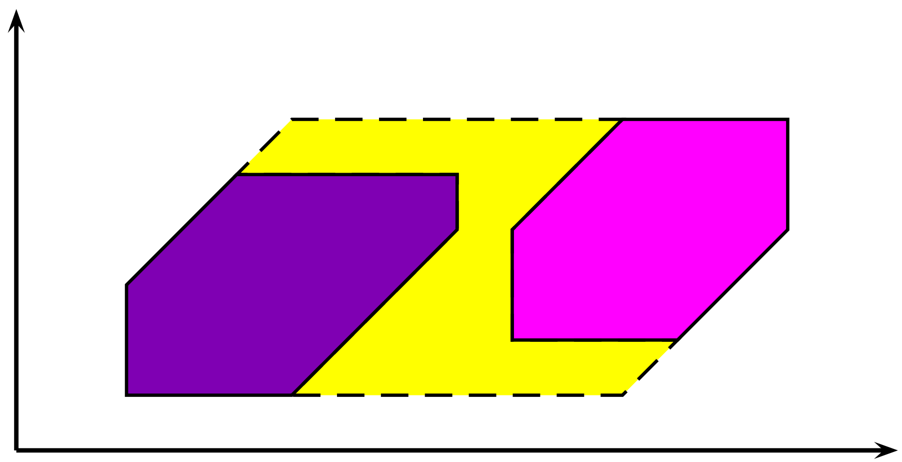

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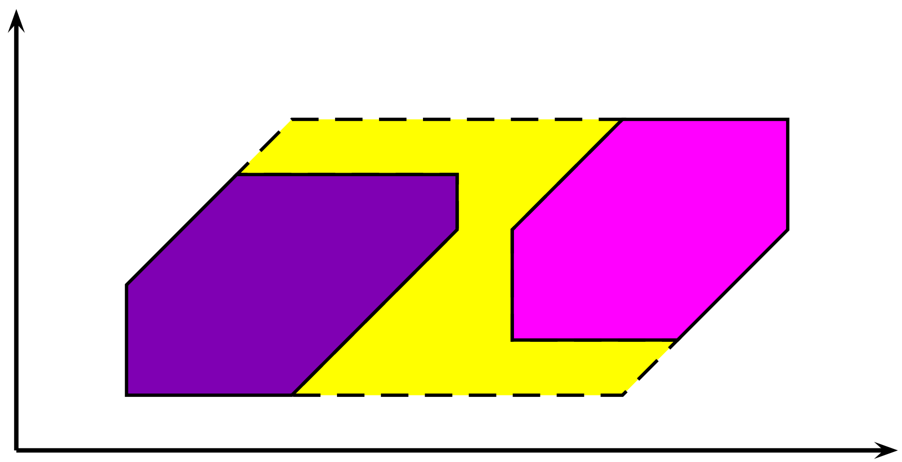

- ✔ **extrapolation**, a widening operator on zones

The DBM Data Structure

DBM (Difference Bounded Matrice) data structure [Dill 1989]

$(x_1 \geq 3) \wedge (x_2 \leq 5) \wedge (x_1 - x_2 \leq 4)$

$$\begin{array}{c c} & \begin{array}{ccc} x_0 & x_1 & x_2 \end{array} \\ \begin{array}{c} x_0 \\ x_1 \\ x_2 \end{array} & \begin{bmatrix} +\infty & -3 & +\infty \\ +\infty & +\infty & 4 \\ 5 & +\infty & +\infty \end{bmatrix} \end{array}$$

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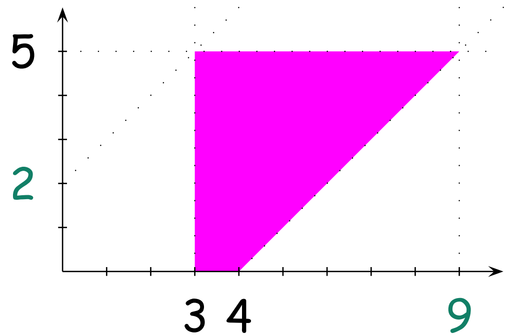

$$\begin{bmatrix} 0 & \mathbf{-3} & 0 \\ 9 & 0 & \mathbf{4} \\ \mathbf{5} & 2 & 0 \end{bmatrix}$$

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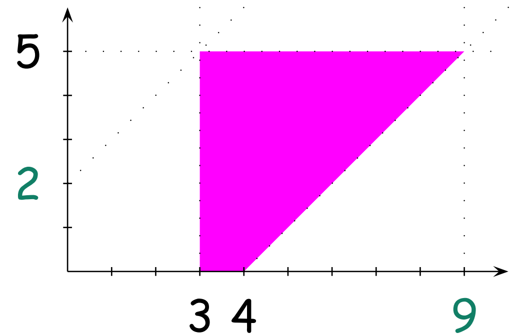

$$\begin{bmatrix} 0 & -3 & 0 \\ 9 & 0 & 4 \\ 5 & 2 & 0 \end{bmatrix}$$

- ✔ All previous operations on zones can be computed using DBMs

The Extrapolation Operator

Fix an integer k (“$*$” represents an integer between $-k$ and $+k$)

$$\begin{bmatrix} * & \boxed{>k} & * \\ * & * & * \\ \boxed{<-k} & * & * \end{bmatrix} \rightsquigarrow \begin{bmatrix} * & \boxed{+\infty} & * \\ * & * & * \\ \boxed{-k} & * & * \end{bmatrix}$$

- ✔ “intuitively”, erase non-relevant constraints

➜ *ensures termination*

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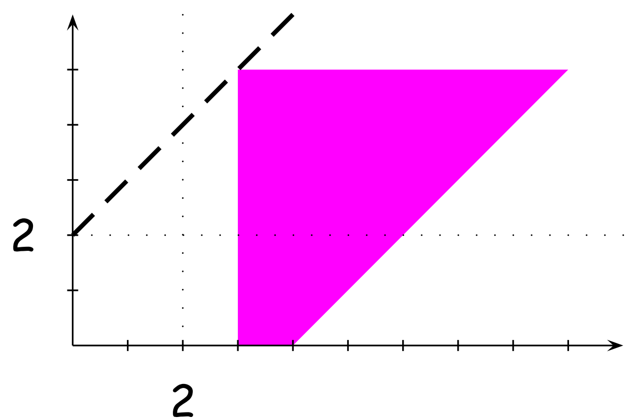

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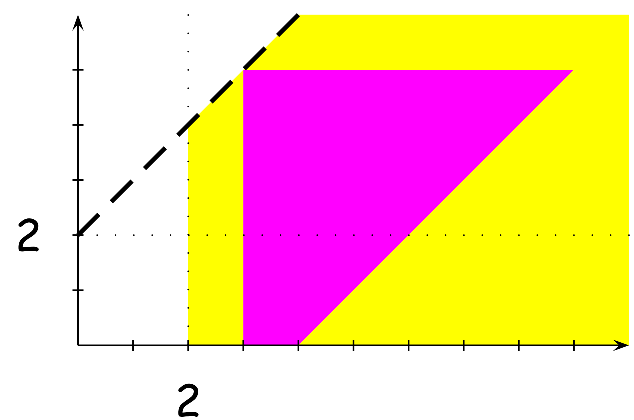

➜ ensures termination

Challenge

Propose a **good** constant for the extrapolation:

- ✔ keep the correctness of the forward computation

Solution by the past: maximal constant appearing in the automaton

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- ✔ Implemented in tools like UPPAAL, KRONOS, RT-SPIN...
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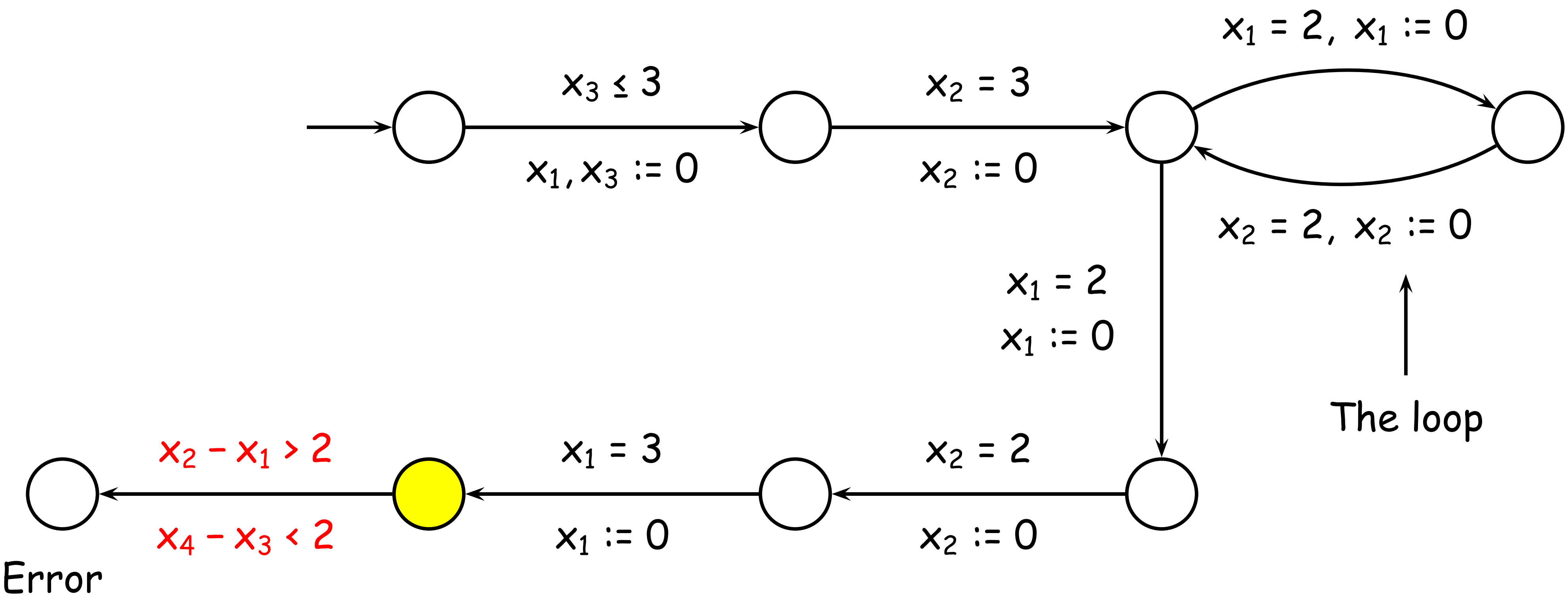
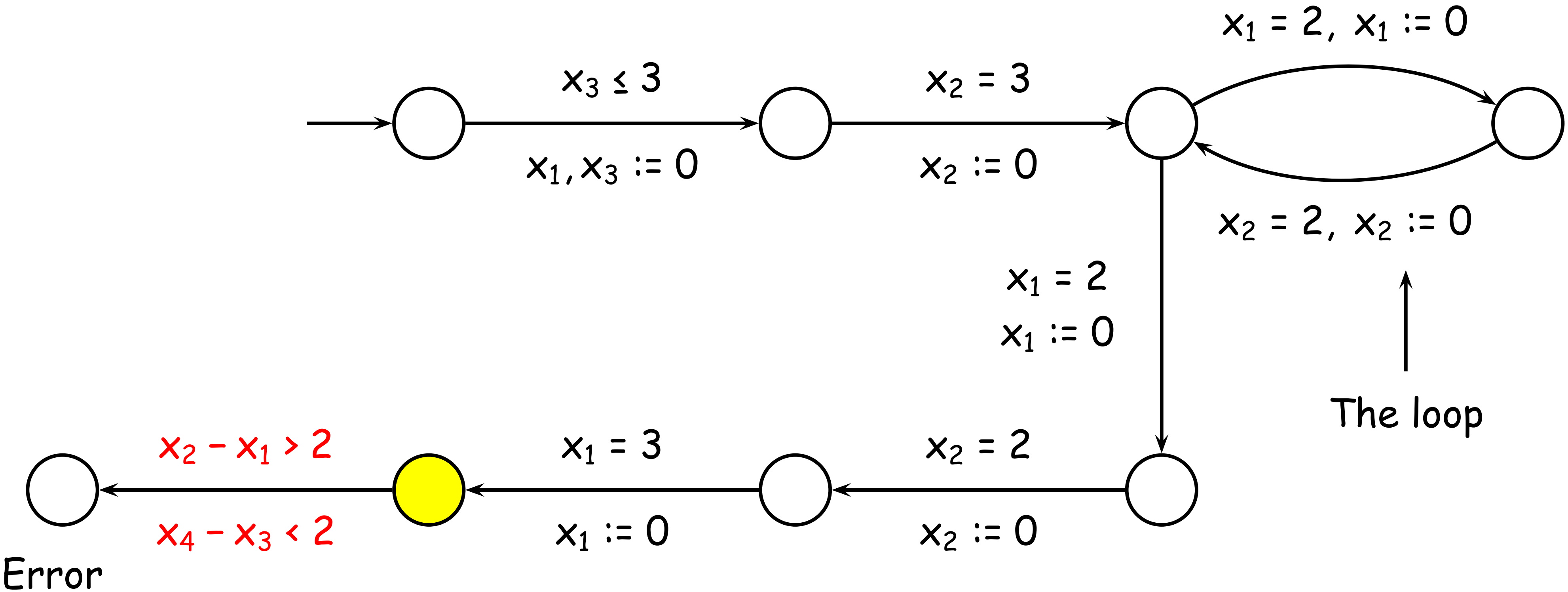
However...

A Problematic Automaton



A Problematic Automaton



$v(x_1) = 0$
$v(x_2) = d$
$v(x_3) = 2a + 5$
$v(x_4) = 2a + 5 + d$

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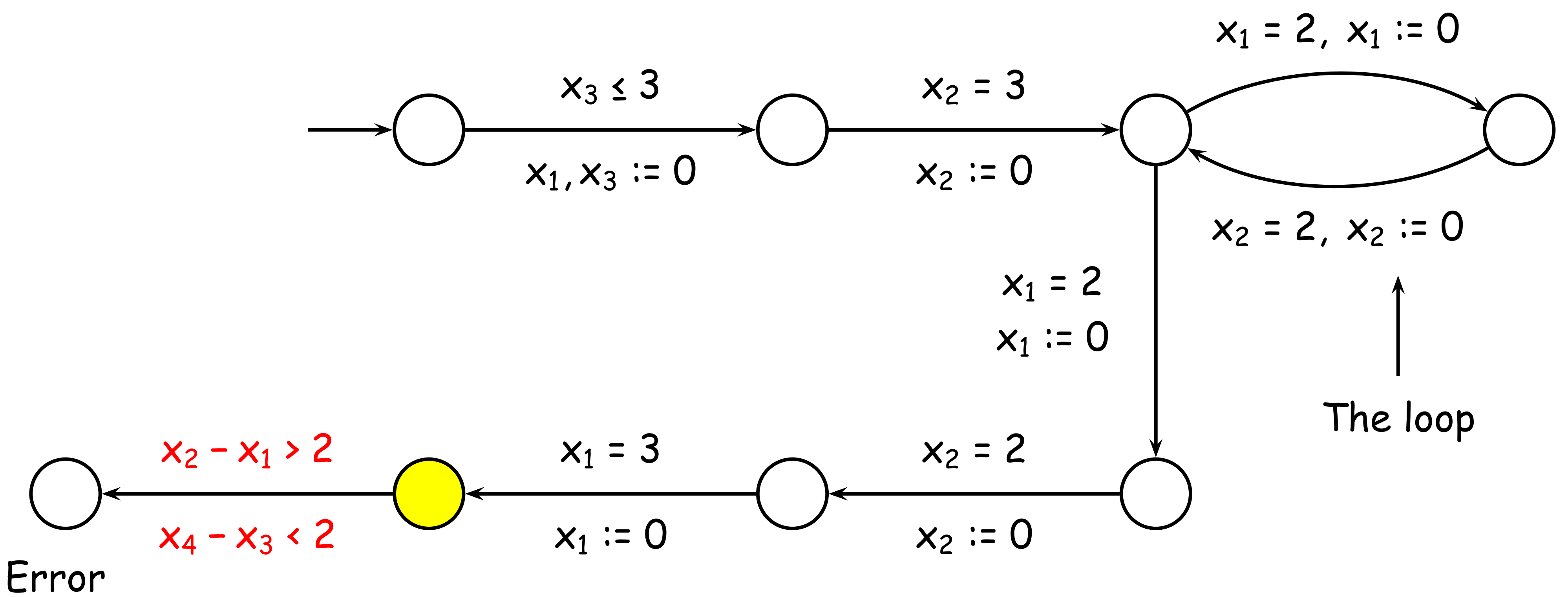

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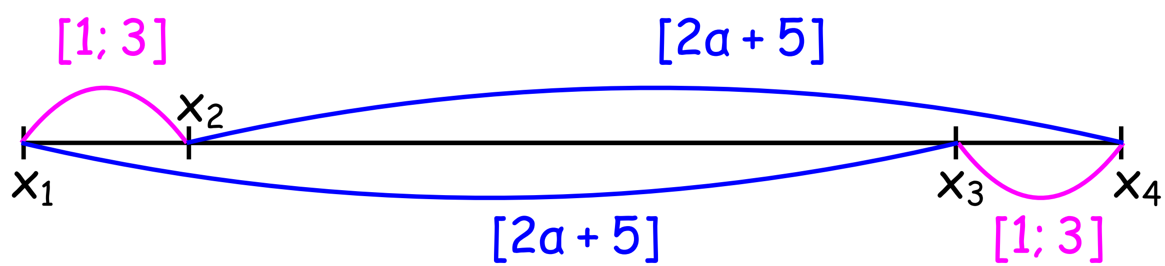
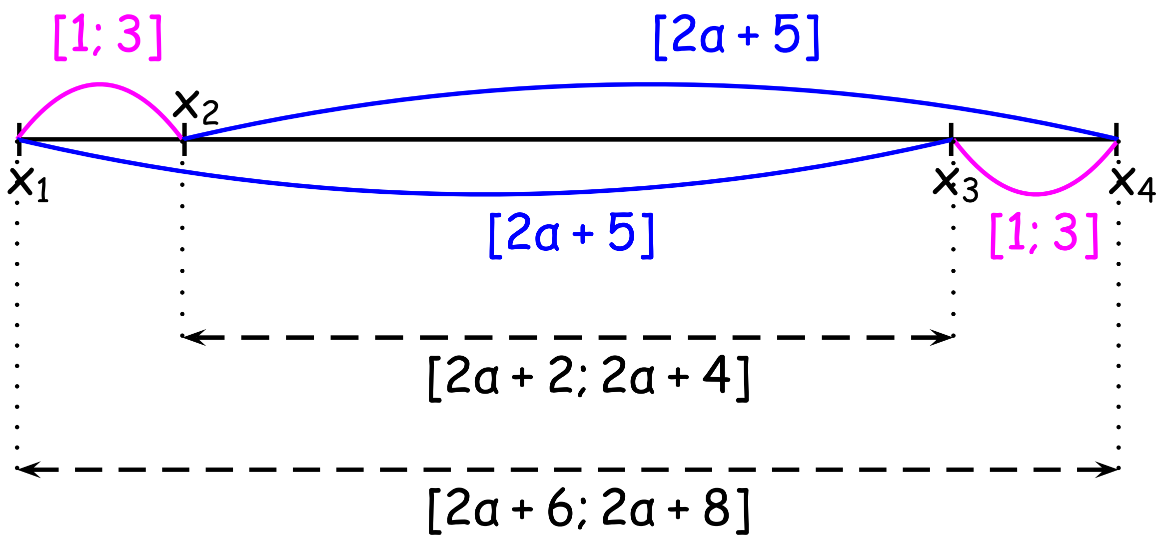


The Problematic Zone



implies $x_1 - x_2 = x_3 - x_4$.

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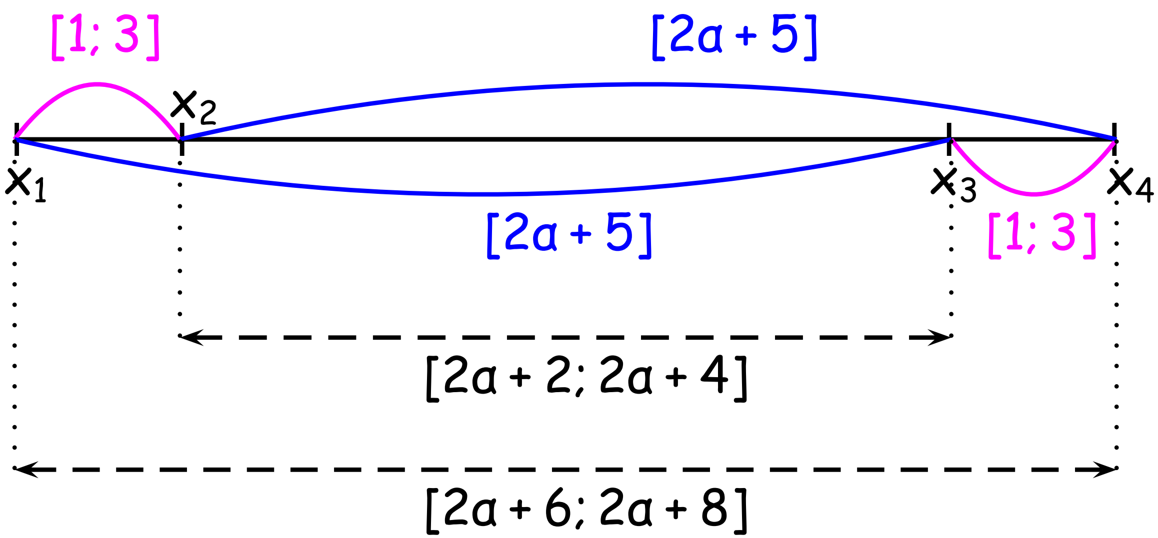
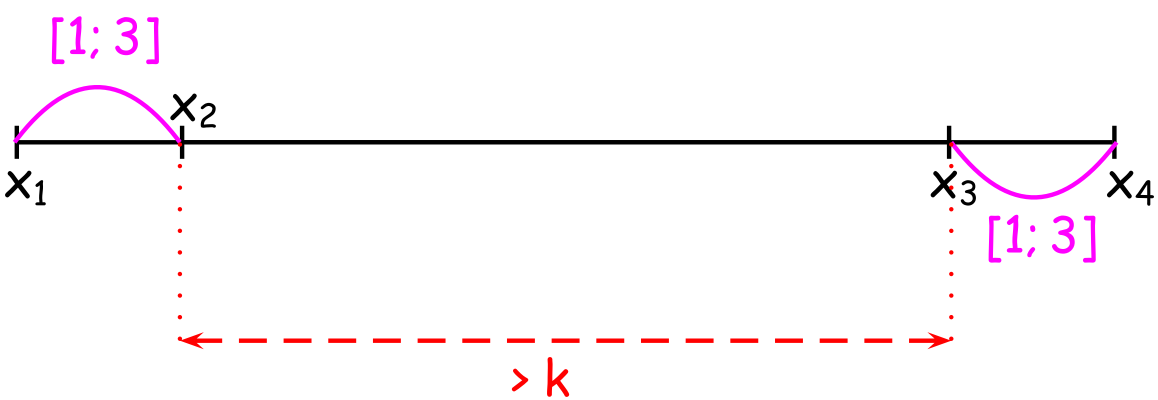


implies $x_1 - x_2 = x_3 - x_4$.

If a is sufficiently large, after extrapolation:



does not imply $x_1 - x_2 = x_3 - x_4$.

General Abstractions

Criteria for a good abstraction operator Abs:

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For the previous automaton,

no abstraction operator can satisfy all these criteria!

Why That?

Assume there is a "nice" operator Abs.

The set {M DBM representing a zone Abs(Z)} is finite.

➜ k the max. constant defining one of the previous DBMs

We get that, for every zone Z,

$$Z \subseteq \mathrm{Extra}_k(Z) \subseteq \mathrm{Abs}(Z)$$

Problem!

Open questions:

- which conditions can be made weaker?
- find a clever termination criterium?
- use an other data structure than zones/DBMs?
- ?

What Can We Cling To?

Diagonal-free: only guards $x \sim c$
(no guard $x - y \sim c$)

Theorem: the classical algorithm is correct for diagonal-free timed automata.

[Bouyer 2003]

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Theorem: the classical algorithm is correct for diagonal-free timed automata.

General: both guards $x \sim c$ and $x - y \sim c$

Proposition: the classical algorithm is correct for timed automata that use **less than 3 clocks.**

(the constant used is bigger than the maximal constant...)

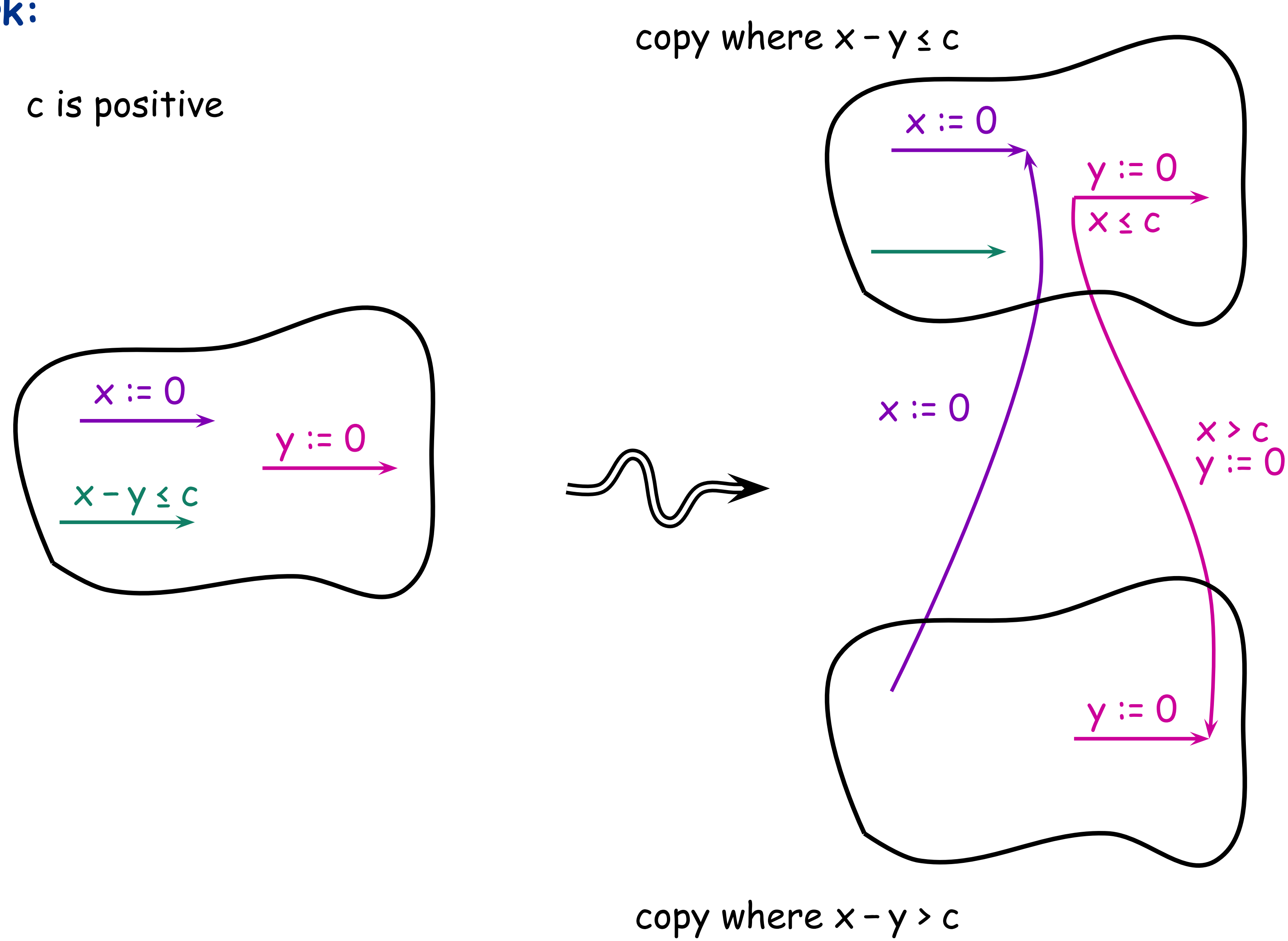
[Bouyer 2003]

How to Deal with Diagonals?

→ proof in **[Bérard, Diekert, Gastin & Petit 1998]**

Remark:

c is positive



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Actual work: counter-example refinement hope ☺

A Note on Compositional Methods

Basic idea:

untimed: **[Andersen 1995]**

timed: **[Laroussinie, Larsen 1995]**

$$(A_1 \parallel \cdots \parallel A_n) \models \varphi \quad \Leftrightarrow \quad (A_1 \parallel \cdots \parallel A_{n-1}) \models \varphi / A_n$$

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- ✔ simplification rules

Bad news: for those logics, nil model-checking is as difficult as simple m.-c.

[Aceto, Laroussinie 2002]

Existing Tools

- ✔ **Uppaal**: made in Uppsala (Sweden) & Aalborg (Denmark)
  - reachability, deadlock, a simple fragment of TCTL
  - forward analysis

  `http://www.uppaal.com`

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- ✔ **Kronos**: made in Grenoble (France)
  - full TCTL
  - forward and backward analysis

  `http://www-verimag.imag.fr/TEMPORISE/kronos/`

Conclusion Remarks

Actual Challenges

Deal with both discrete and time explosions!

untimed systems	time information
BBD-like techniques	more and more optimizations
	static analysis of TA
	[BBFL03,BBLP04...]

Some attempts for the data-structures:

- ✔ the CDD data-structure [Larsen, Pearson, Weise & Yi 1999]
- ✔ the data-structure of RED [Wang since 2000]

Some attempts for the techniques:

- ✔ partial-order reduction [Bengtsson, Jonsson, Lilius & Yi 1998]
- ✔ partial-order semantics approach [Lugiez, Niebert & Zennou 2004]

Actual Challenges (cont.)

Intermediate challenges

- better understand **geometry** of reachable state spaces
  (in particular, find a satisfactory solution for dealing with diagonals)
- data-structures for both **discrete and dense** parts
  (up to now: time is not really integrated, it is only added as a feature)
- propose **true concurrent models**?
- and then use techniques from concurrency theory?

Other challenges

- controller synthesis,
- implementability issues (program synthesis)

Thanks to F. Laroussinie, F. Cassez, O.-H. Roux and J.-F. Raskin

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