Robustness in Timed Systems

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LSV, CNRS & ENS Cachan, France

Acknowledgment to Nicolas Markey and Ocan Sankur for slides

Outline

Time-dependent systems

- We are interested in timed systems

Time-dependent systems

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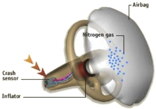
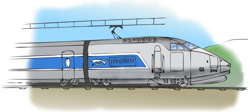
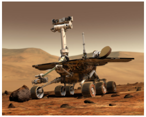
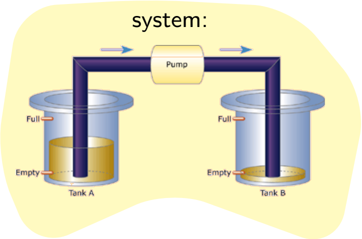
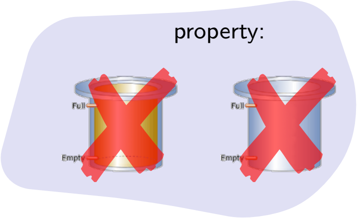


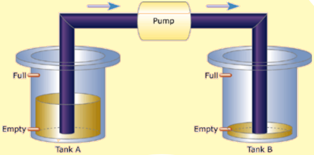
Model-checking and control



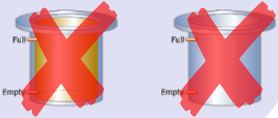


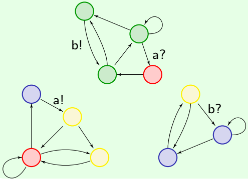
Model-checking and control

system:



property:



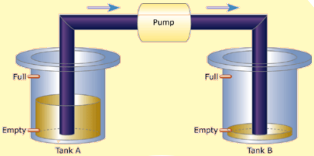


$\mathbf{AG}(\neg\texttt{B.overfull}$

$\quad\wedge\ \neg\texttt{B.dried\_up})$

Model-checking and control

system:



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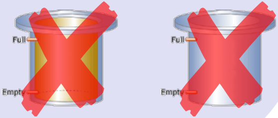

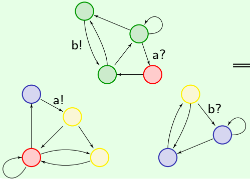

$\Longrightarrow$ algorithm $\Longleftarrow$

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$\wedge\ \neg\texttt{B.dried\_up})$

Model-checking and control

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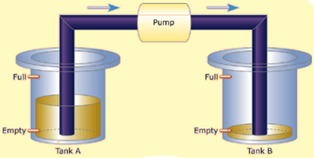

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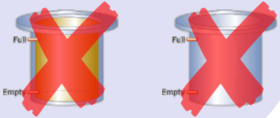
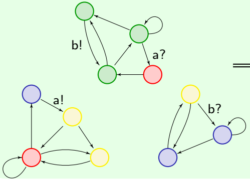




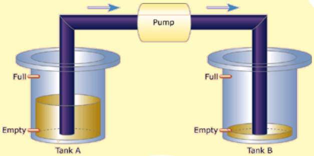
model-checking algorithm

$$\mathbf{AG}(\neg\texttt{B.overfull} \wedge \neg\texttt{B.dried\_up})$$

yes/no

Model-checking and control

system:



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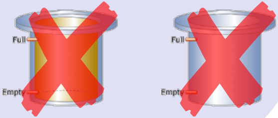
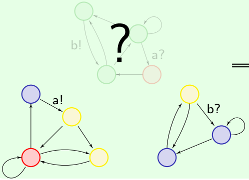
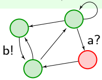




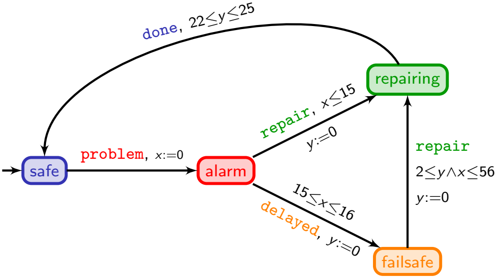
control/synthesis algorithm

AG($\neg$`B.overfull`

$\wedge$ $\neg$`B.dried_up`)



The model of timed automata



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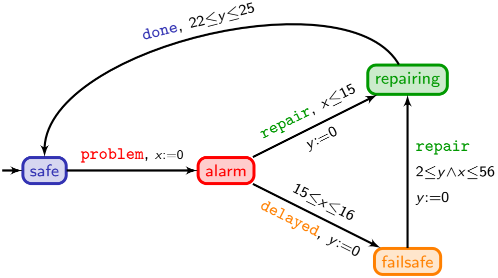

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	$\cdots$
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\texttt{done}}$	safe
$\cdots$	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

Discrete-time semantics

...because computers are digital!

[Alur91] Techniques for automatic verification of real-time systems *(PhD thesis, 1991)*.

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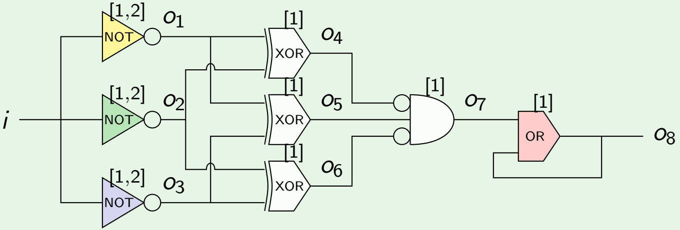
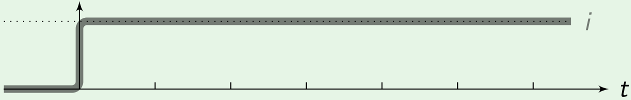
Example [Alur91]



- under discrete-time, the output is always 0:



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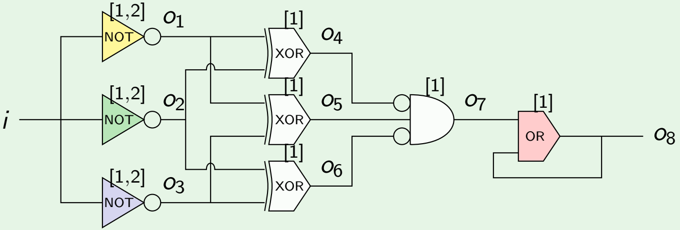

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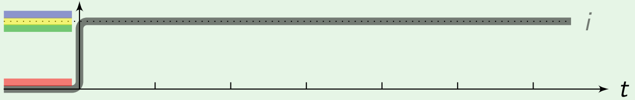

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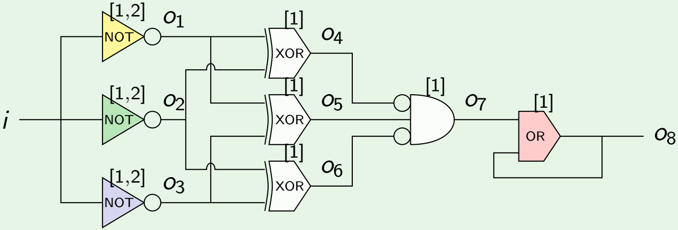

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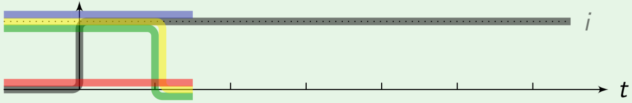

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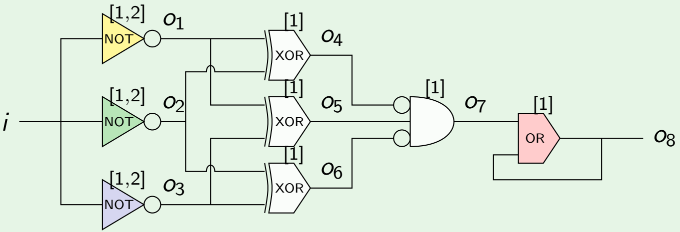

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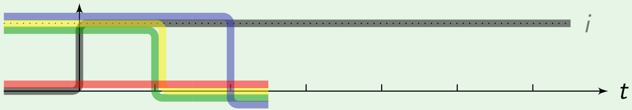

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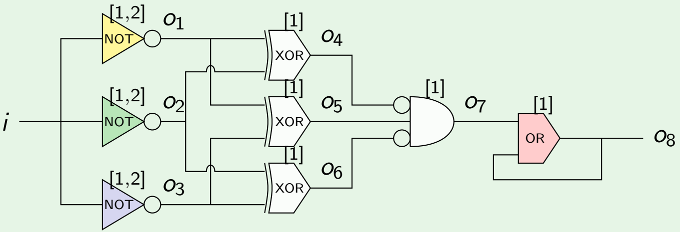

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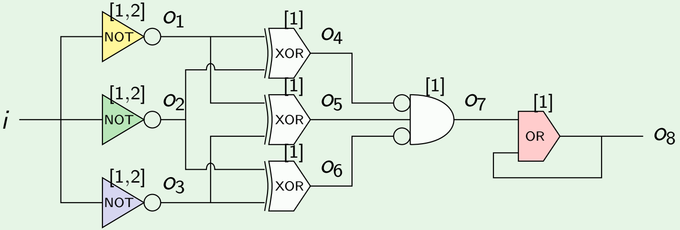
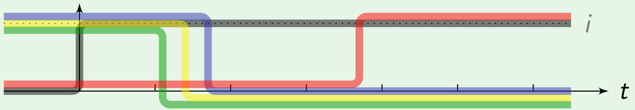


- under continuous-time, the output can be 1:



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...real-time models for real-time systems!

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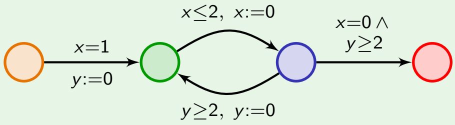

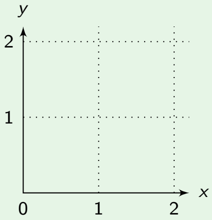

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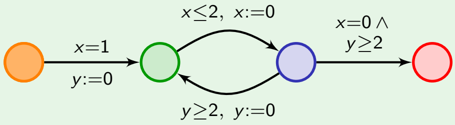

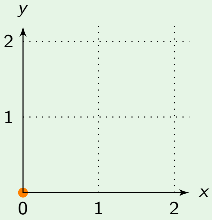

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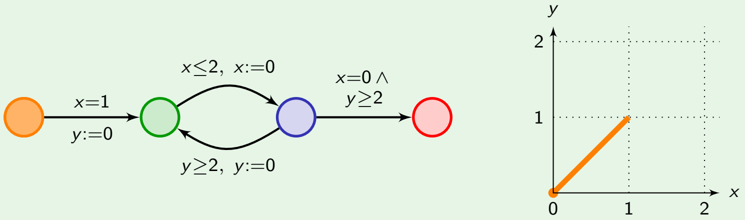

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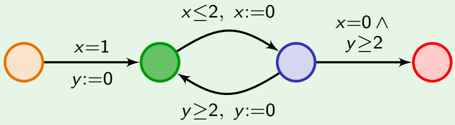

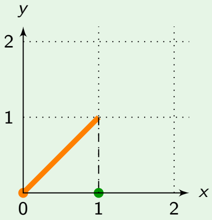

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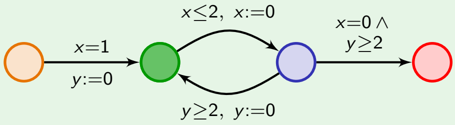

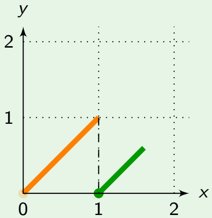

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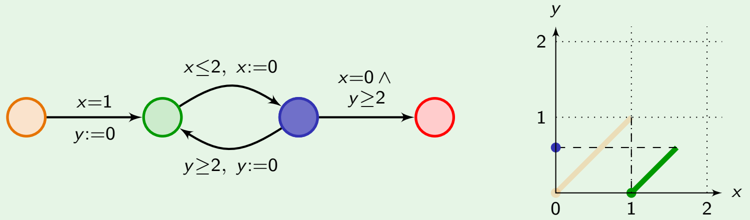

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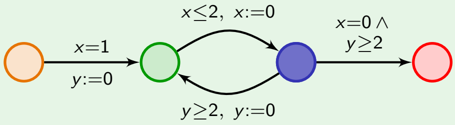

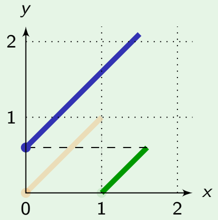

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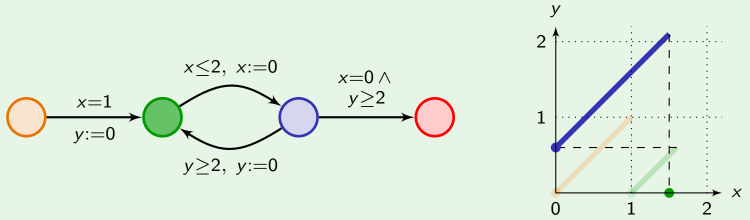

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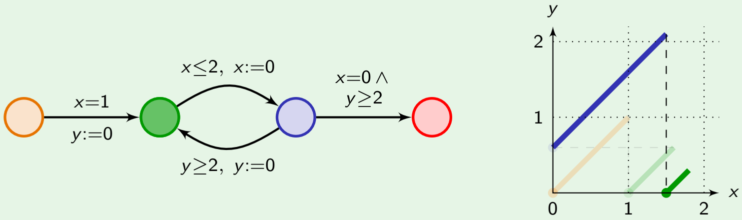

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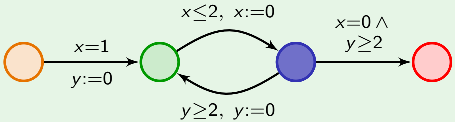

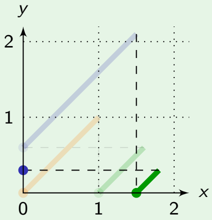

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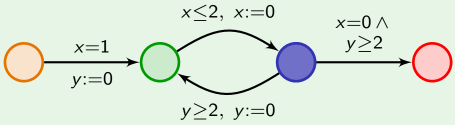

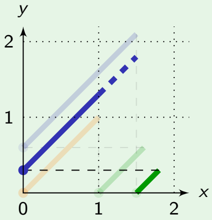

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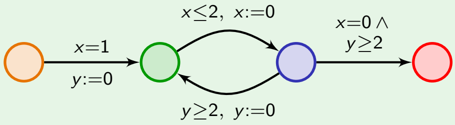

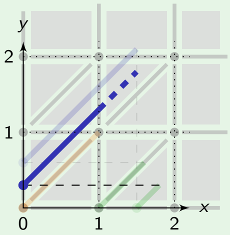

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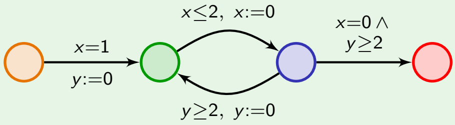

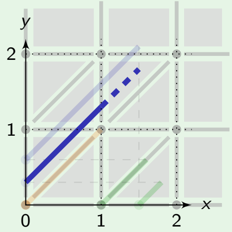

Theorem [AD94]

Reachability in timed automata is decidable (as well as many other important properties). It is PSPACE-complete.

- Technical tool: region abstraction

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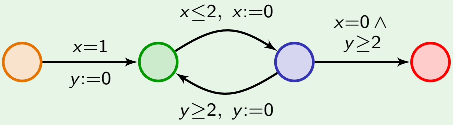
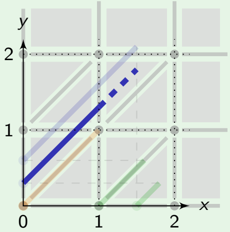


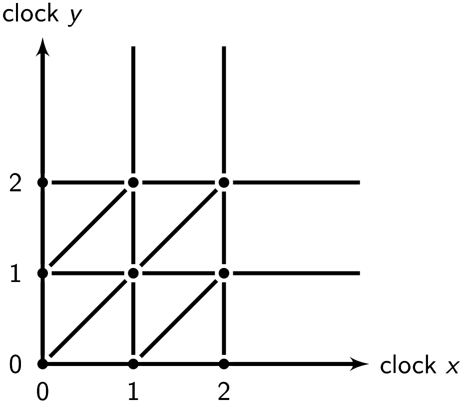


Theorem [AD94]

Reachability in timed automata is decidable (as well as many other important properties). It is PSPACE-complete.

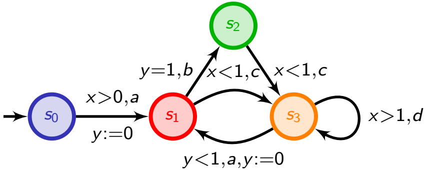
- Technical tool: region abstraction
- Efficient symbolic technics based on zones, implemented in tools

Technical tool: Region abstraction



$\rightsquigarrow$ This is a finite time-abstract bisimulation!

Technical tool: Region abstraction – An example [AD94]



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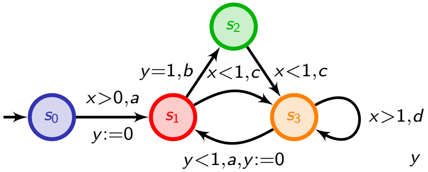

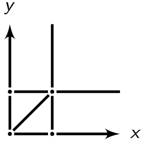

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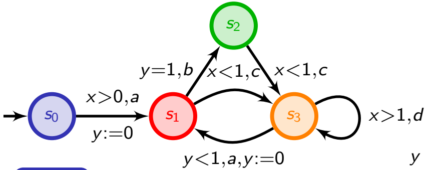
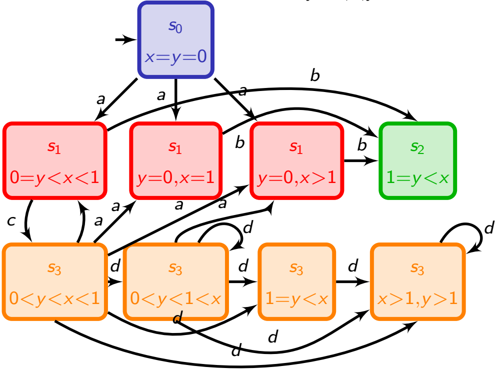
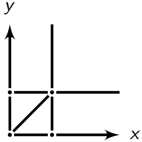
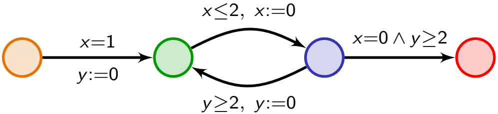






Technical tool: Region abstraction – Another example



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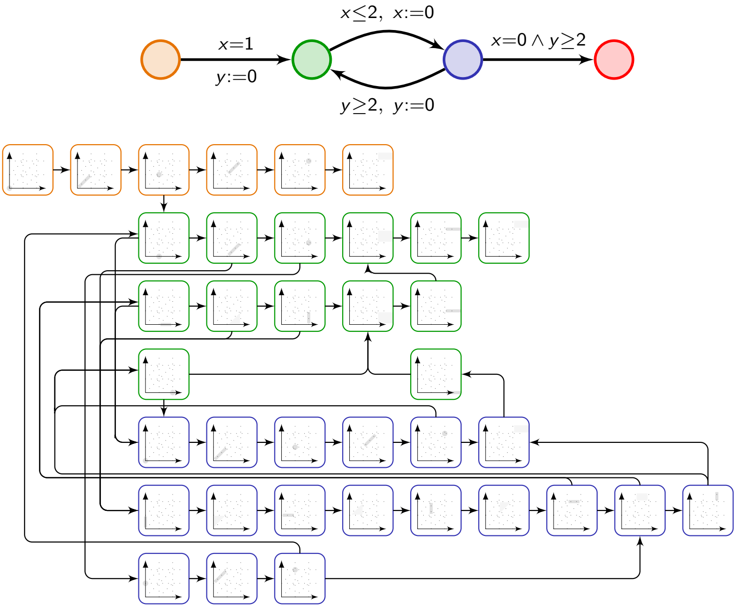

Technical tool: Zones and DBMs

DBM = Difference Bound Matrix

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Zones, or DBMs...

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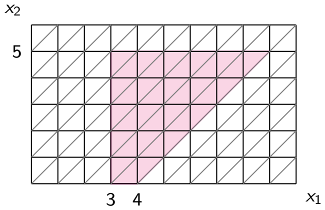

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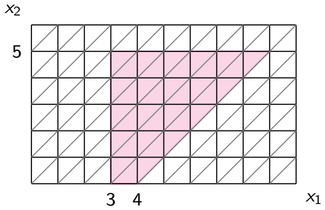

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$$|||$$

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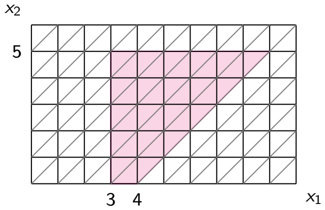

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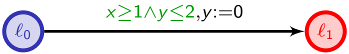
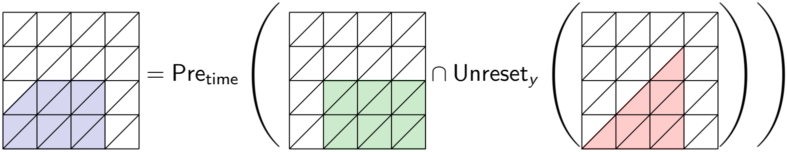
They can be used to compute sets of states in timed automata





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25 February 1991, during Gulf war.
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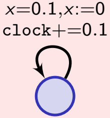

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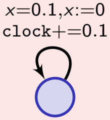

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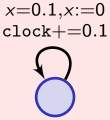

After 100 hours, the total drift was 0.34 seconds.
The incoming missile could not be destroyed.

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  - it assumes zero-delay transitions
  - it assumes infinite precision of the clocks
  - it assumes immediate communication between systems
  - it assumes infinite frequency

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- It might not be proper for implementation:
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- It does not exclude non-realizable behaviours:
  - not only Zeno behaviours
  - many convergence phenomena are hidden

$\rightsquigarrow$ this requires infinite precision and might not be realizable

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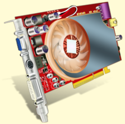
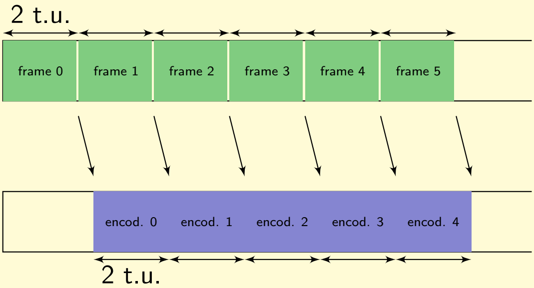
Important questions

- Is the real system correct when it is proven correct on the model?
- Does actual work transfer to real-world systems? To what extent?

Example 1: Imprecision on clock values

Frame capture [ACS10]





[ACS10] Abdellatif, Combaz, Sifakis. Model-based implementation of real-time applications *(Int. Conf. Embedded Software, 2010)*.

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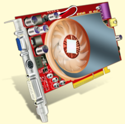
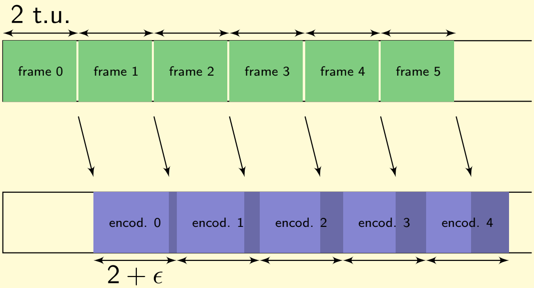


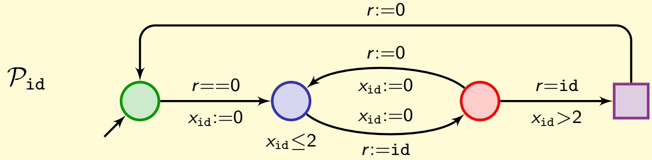


$\rightsquigarrow$ A frame will eventually be skipped

[ACS10] Abdellatif, Combaz, Sifakis. Model-based implementation of real-time applications *(Int. Conf. Embedded Software, 2010).*

Example 2: Strict timing constraints

Mutual exclusion protocol [KLL+97]

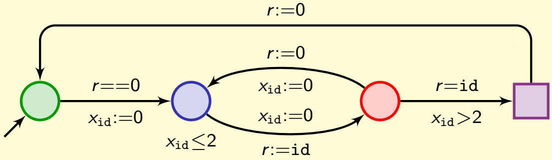


[KLL+97] Kristoffersen, Laroussinie, Larsen, Pettersson, Yi. A compositional proof of a real-time mutual exclusion protocol *(TAPSOFT'97)*.

Example 2: Strict timing constraints

Mutual exclusion protocol [KLL+97]

$\mathcal{P}_{\mathtt{id}}$



- When $\mathcal{P}_1$ and $\mathcal{P}_2$ run in parallel (sharing variable r), the state where both of them are in ☐ is not reachable.

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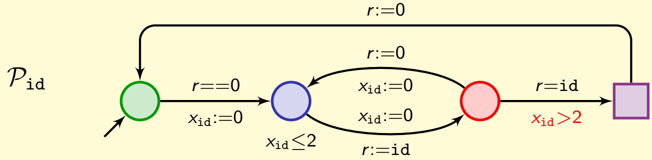

- When $\mathcal{P}_1$ and $\mathcal{P}_2$ run in parallel (sharing variable r), the state where both of them are in □ is not reachable.
- This property is lost when $x_{\texttt{id}} > 2$ is replaced with $x_{\texttt{id}} \geq 2$.

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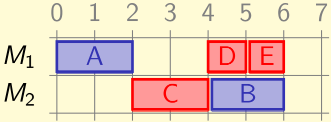
Example 3: Scheduling and timing anomaly

- Scheduling analysis with timed automata [AAM06]
- **Goal:** analyze a *work-conserving* scheduling policy on given scenarios (no machine is idle if a task is waiting for execution)

Example of a scenario



with the dependency constraints: $A \to B$ and $C \to D, E$.

1. A, D, E must be scheduled on machine M_1
2. B, C must be scheduled on machine M_2
3. C starts no sooner than 2 time units

[AAM06] Abdeddaim, Asarin, Maler. Scheduling with timed automata *(Theor. Comp. Science, 2006)*.

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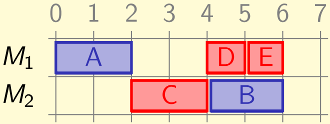

$\rightsquigarrow$ Schedulable in 6 time units

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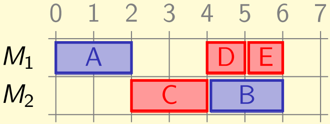

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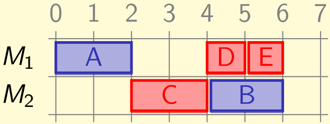

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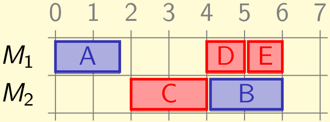

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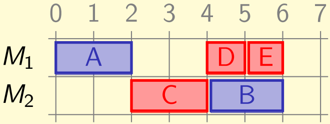

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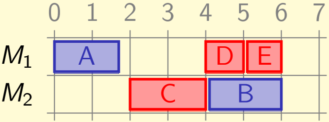

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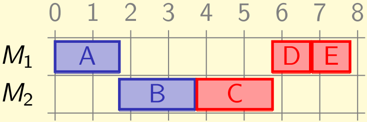

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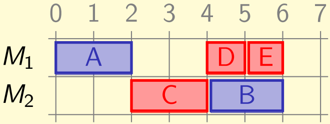

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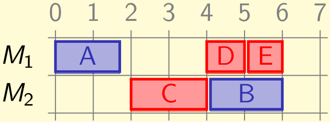

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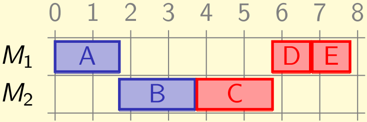

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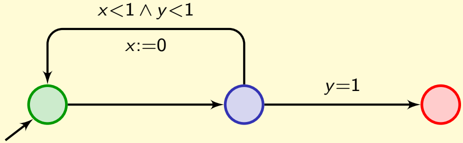
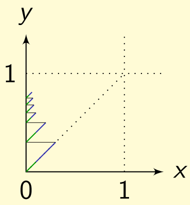
$\rightsquigarrow$ Standard analysis does not capture this **timing anomaly**

Example 4: Zeno behaviours





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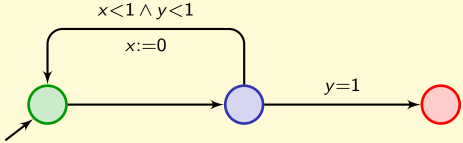

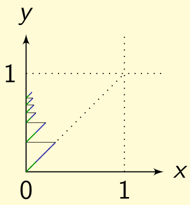

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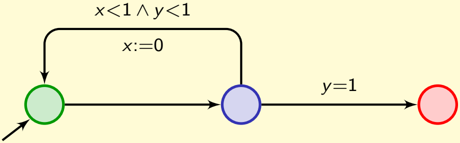

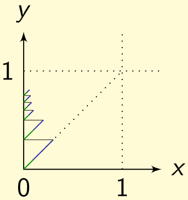

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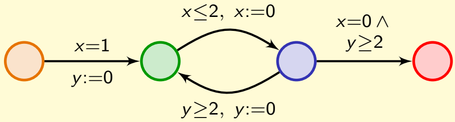

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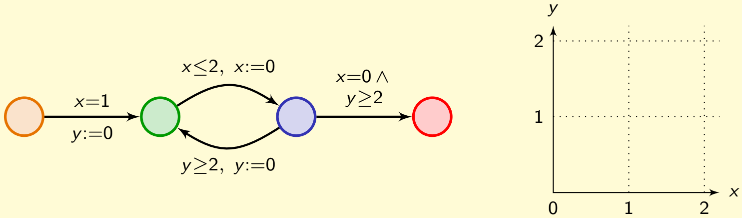

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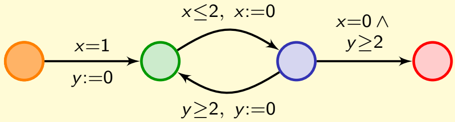

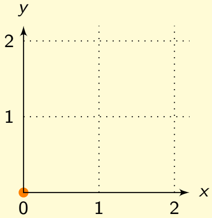

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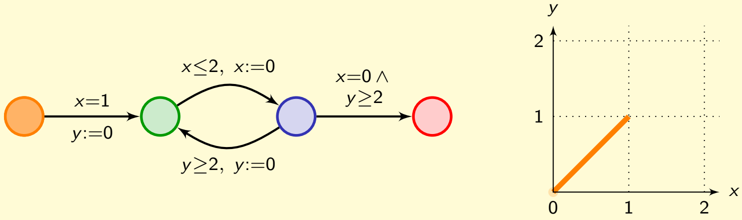

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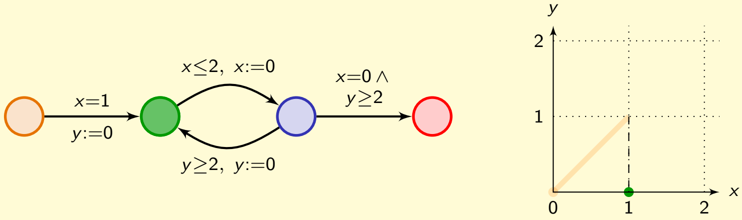

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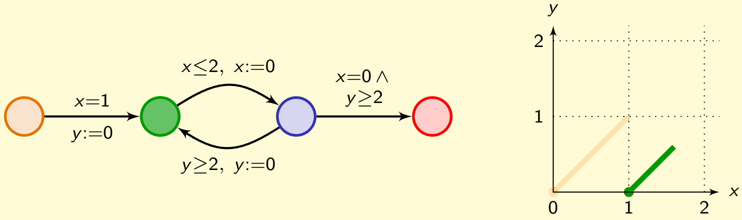

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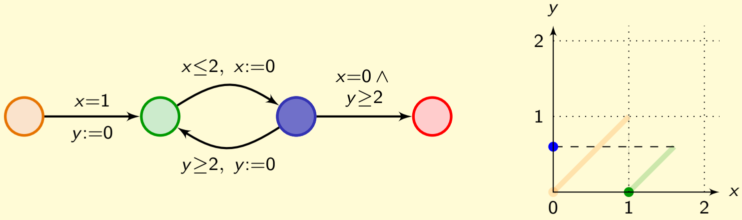

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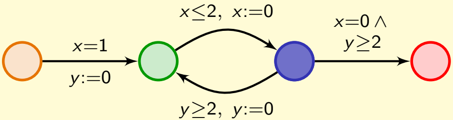

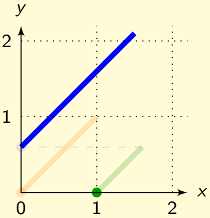

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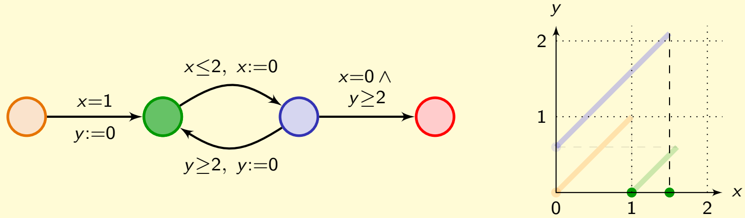

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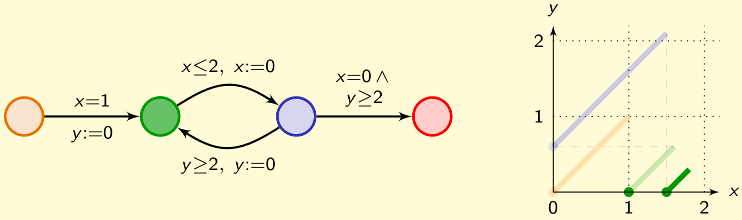

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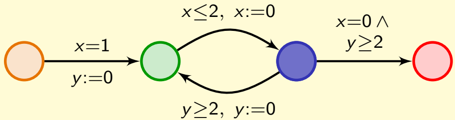

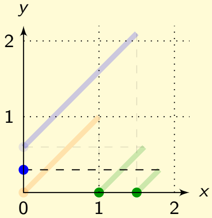

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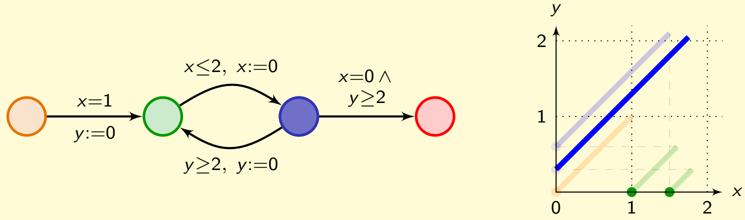

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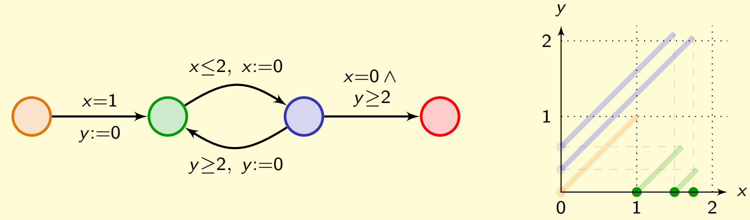

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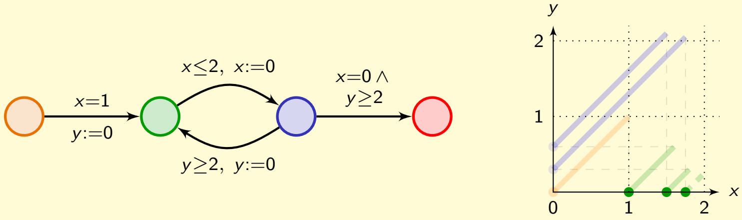

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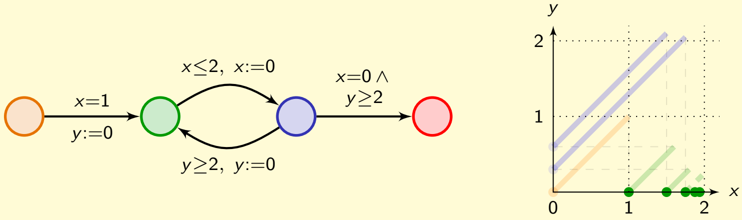

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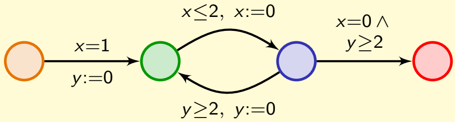

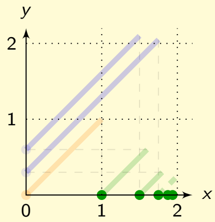

$\rightsquigarrow$ Value of clock x when hitting is converging,
even though global time diverges

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Add robustness to the theory of timed automata

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- Understand the real system behind the mathematical model

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- Describe frameworks and provide tools to develop robustly correct systems

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Add robustness to the theory of timed automata

- Understand the real system behind the mathematical model
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Rest of the talk

- We present a couple of frameworks that have been developed recently
- We focus on the tolerance to slight timing perturbations, that is, to perturbations on time measurements and jitter

Outline

Robust model-checking approach

Idea

Capture any real (or approximate) behaviours (e.g. the implementation) in the verification process

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$\leadsto$ We aim at proposing frameworks in which the correctness of the real system will be ensured once the model is verified

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We describe two such frameworks:

1. either we implement $\mathcal{A}$ and we prove:

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2. or we build $\mathcal{A}$ and implement $\mathcal{B}$, and we prove:

$$\text{correctness of } \mathcal{A} \quad \Rightarrow \quad \text{“robust” correctness of } \mathcal{B}$$
$$\Rightarrow \quad \text{correctness of } \mathcal{B}_{\texttt{real}}$$

Outline

1. Introduction
2. Robust model-checking
   - Parameterized enlarged semantics
   - Automatic generation of an implementation
   - Implementation by shrinking
3. Robust realisability and control
   - Excess semantics
   - Strict semantics
4. Conclusion

Parameterized enlarged semantics for timed automata

A transition can be taken at any time in $[t - \delta; t + \delta]$

Parameterized enlarged semantics for timed automata

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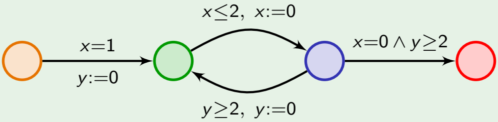
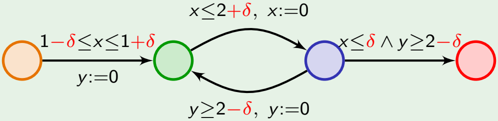
Example

Given a parameter δ,



is transformed into



Parameterized model $\mathcal{A}_\delta$

Parameterized enlarged semantics – Discussion

What is the relevance of this semantics?

- This is a worst-case approach
- This captures approximate behaviours of the system
- One can define program semantics such that for every $\epsilon > 0$:

$$\mathcal{A} \subseteq \mathtt{program}_{\epsilon}(\mathcal{A}) \subseteq \mathcal{A}_{f(\epsilon)}$$

ϵ: parameters of the semantics

[DDR04] De Wulf, Doyen, Raskin. Almost ASAP semantics: From timed models to timed implementations *(HSCC'04)*.
[SBM11] Sankur, Bouyer, Markey. Shrinking Timed Automata *(FSTTCS'11)*.

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$\rightsquigarrow$ This is a good approach for designing systems with simple timing constraints (e.g. equalities).

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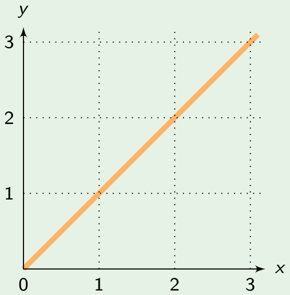

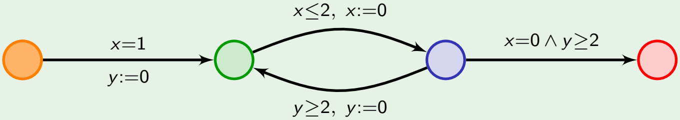

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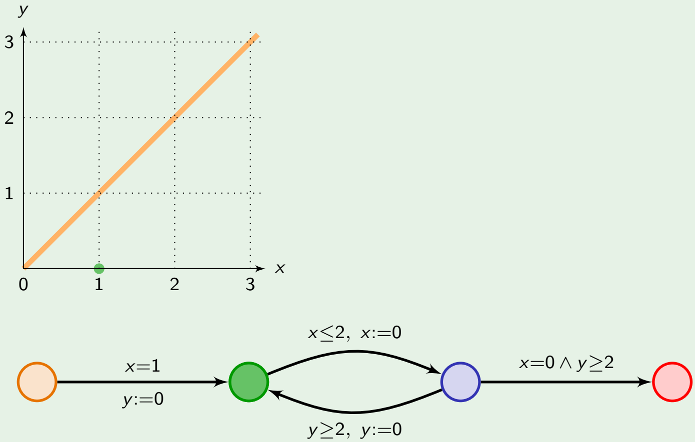

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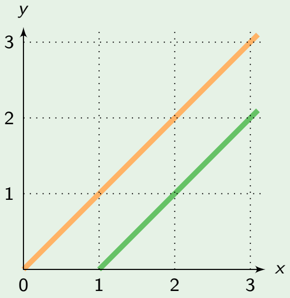

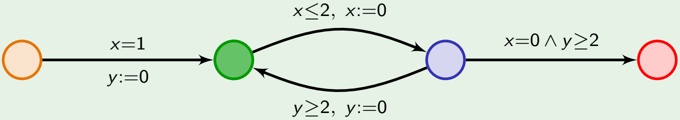

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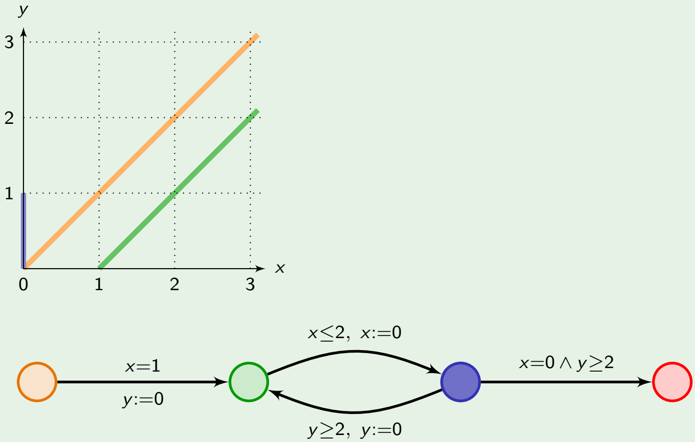

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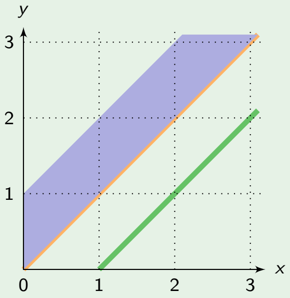

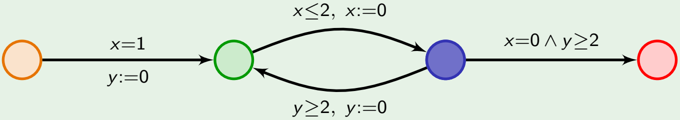

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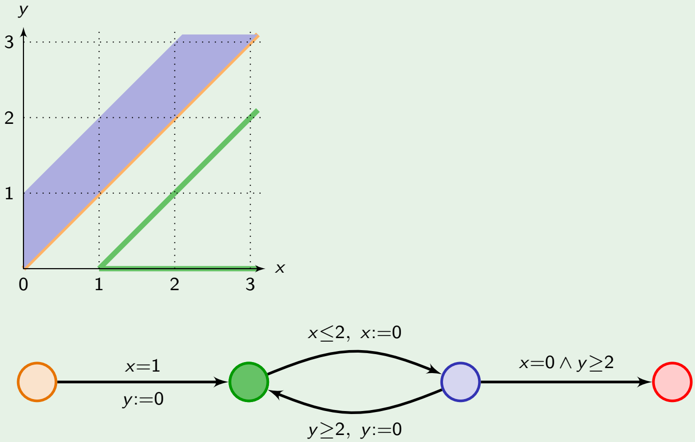

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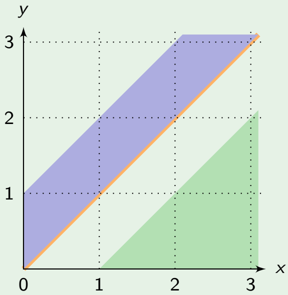

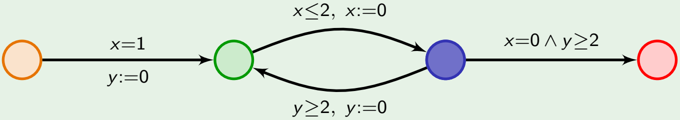

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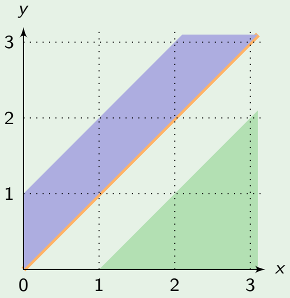

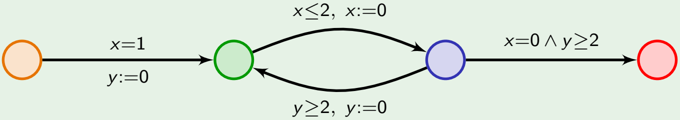

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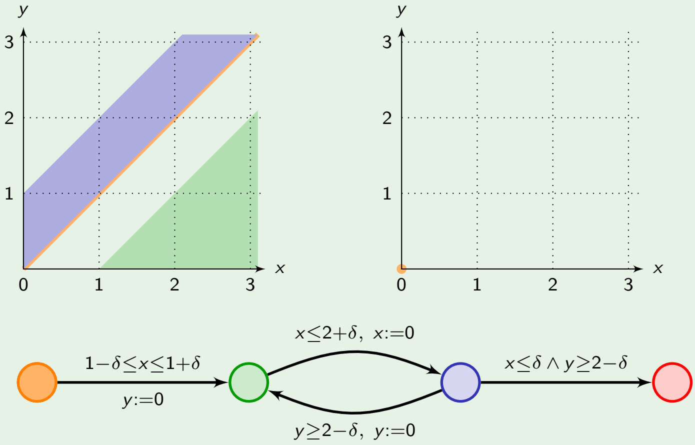

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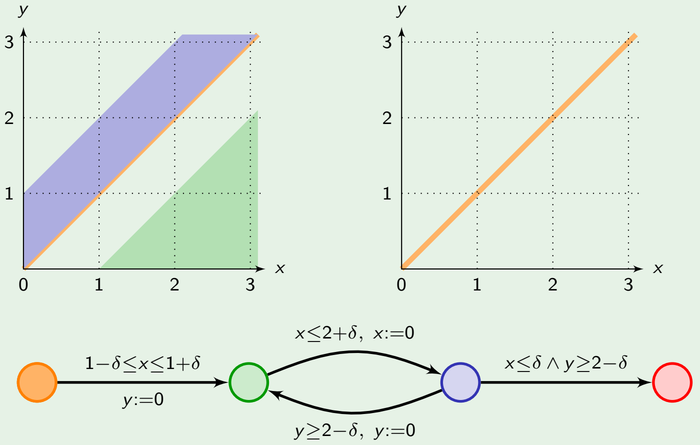

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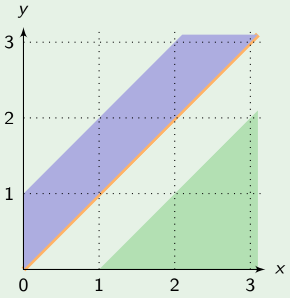

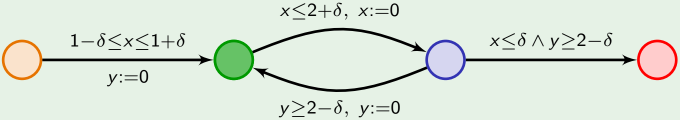

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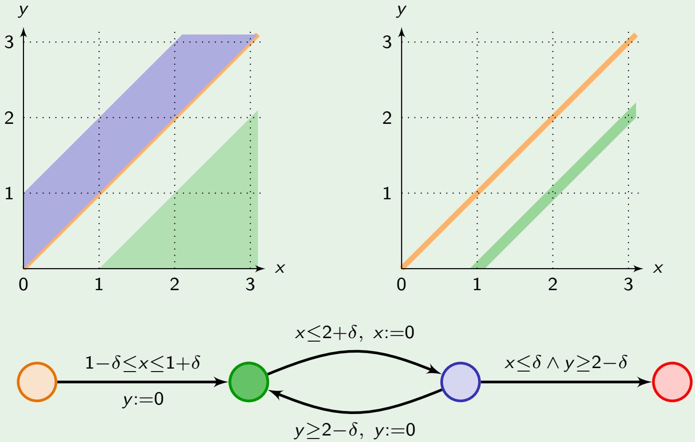

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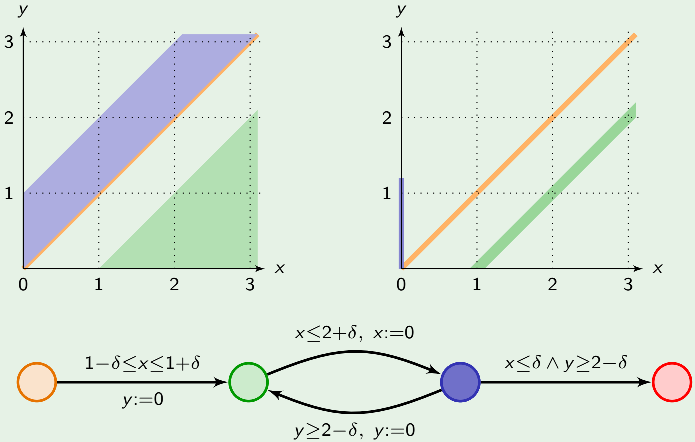

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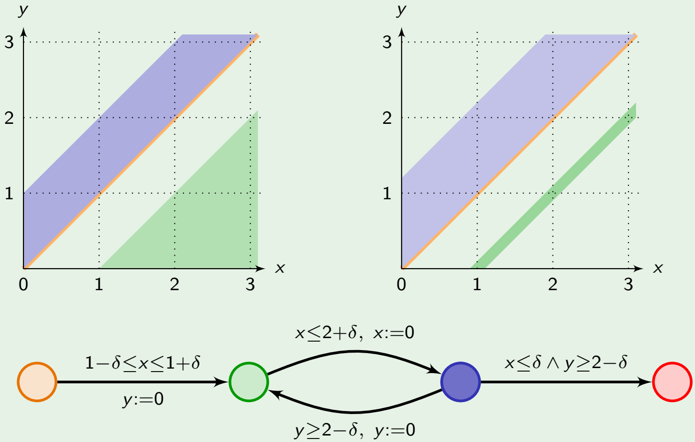

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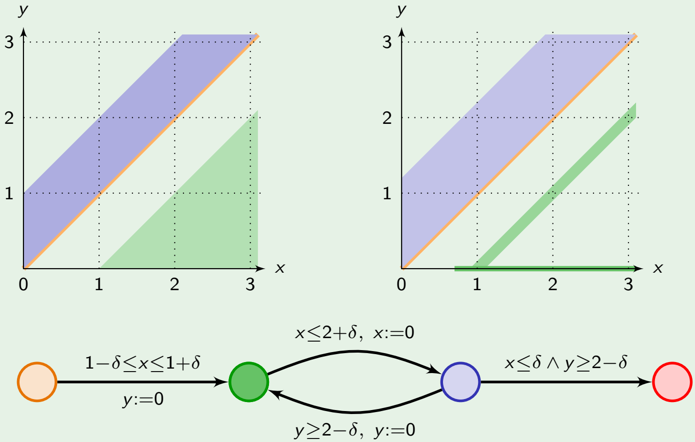

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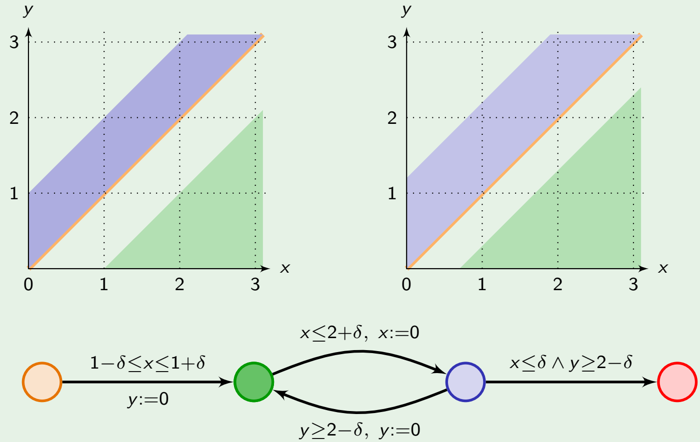

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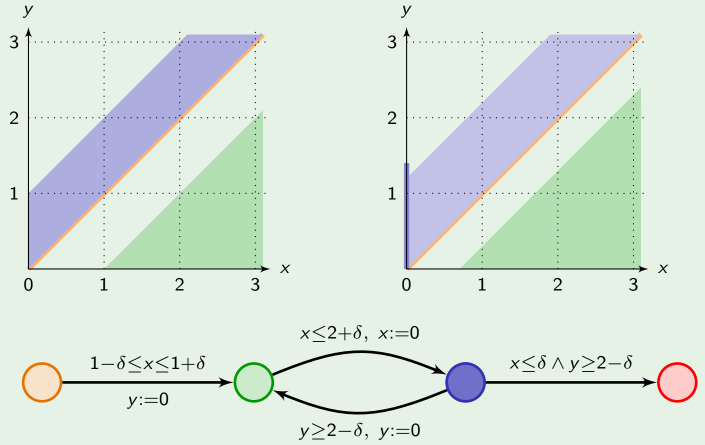

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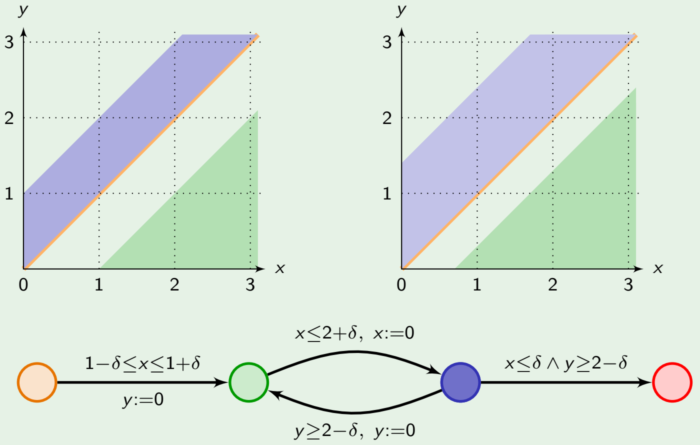

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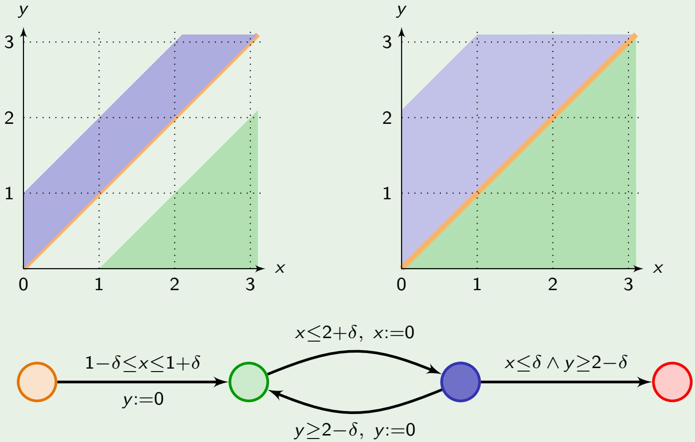

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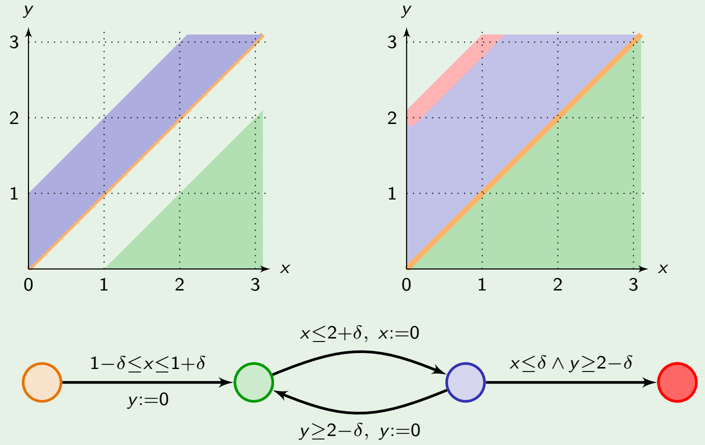

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It asks whether there is $\delta_0 > 0$ s.t. for every $0 \leq \delta \leq \delta_0$, $\mathcal{A}_\delta \models \varphi$.

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[Mil00] Miller. Decidability and Complexity Results for Timed Automata and Semi-linear Hybrid Automata *(HSCC'00)*.

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$$\begin{array}{rcl} \delta_1 \leq \delta_2 & \Rightarrow & \text{Reach}(\mathcal{A}_{\delta_1}) \subseteq \text{Reach}(\mathcal{A}_{\delta_2}) \\ & \Rightarrow & (\mathcal{A}_{\delta_2} \text{ safe} \Rightarrow \mathcal{A}_{\delta_1} \text{ safe}) \end{array}$$

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Theorem

Robust model-checking of safety, LTL, CoflatMTL properties is decidable.
Complexities are those of standard non robust model-checking problems.

[Puri00] Puri. Dynamical properties of timed automata *(Disc. Event Dyn. Syst., 2000)*.
[DDMR08] De Wulf, Doyen, Markey, Raskin. Robust safety of timed automata *(Form. Meth. Syst. Design, 2008)*.
[BMR06] Bouyer, Markey, Reynier. Robust model-checking of timed automata *(LATIN'06)*.
[BMR08] Bouyer, Markey, Reynier. Robust analysis of timed automata via channel machines *(FoSSaCS'08)*.
[BMS11] Bouyer, Markey, Sankur. Robust Model-Checking of Timed Automata via Pumping in Channel Machines *(FORMATS'11)*.

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(An exponential bound on δ_0 is proven)

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Technical tool: extended region automaton

Extended region automaton

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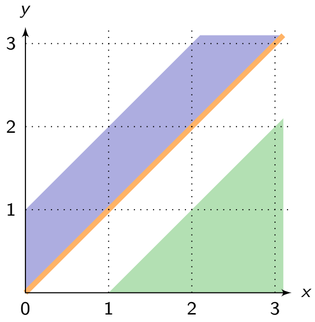

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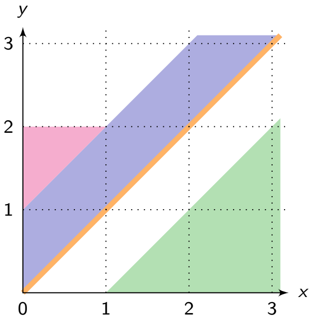

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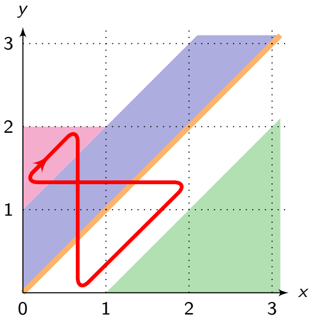

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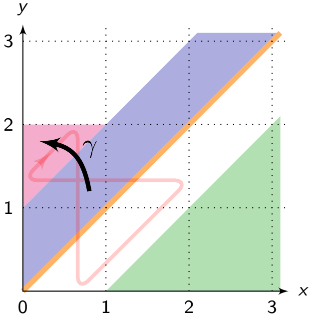

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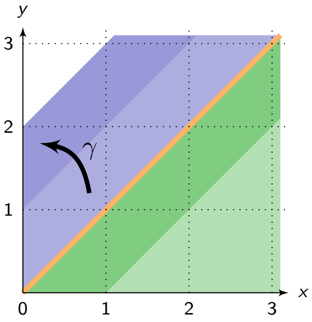

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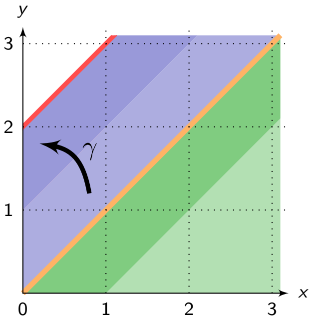
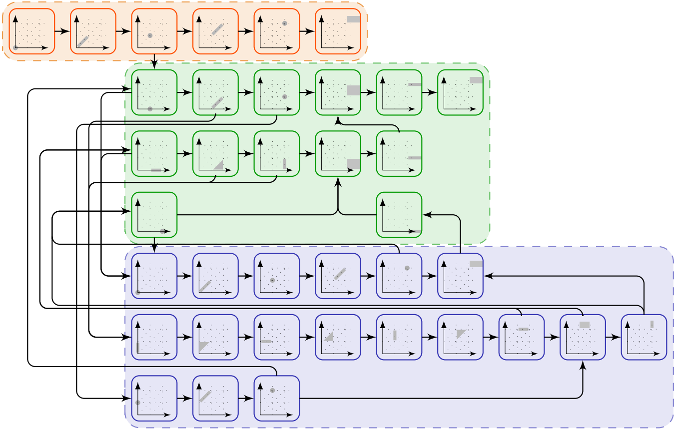


Parameterized enlarged semantics – An example



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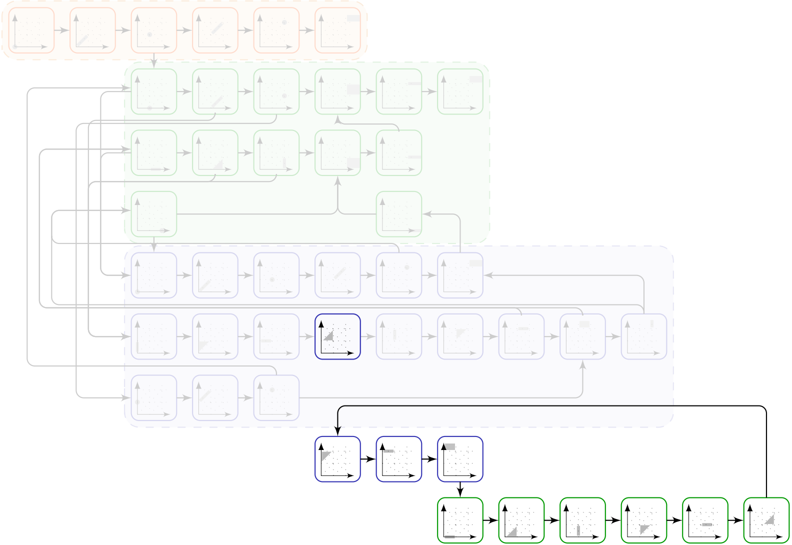
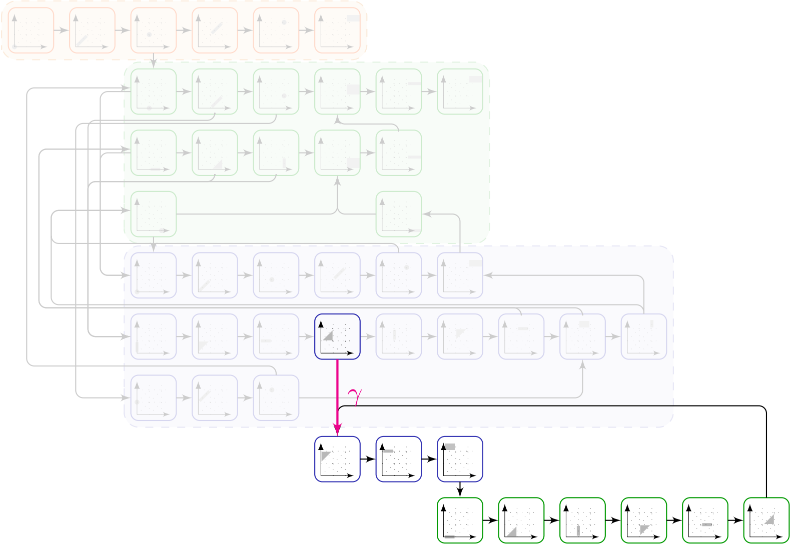


Parameterized enlarged semantics – An example



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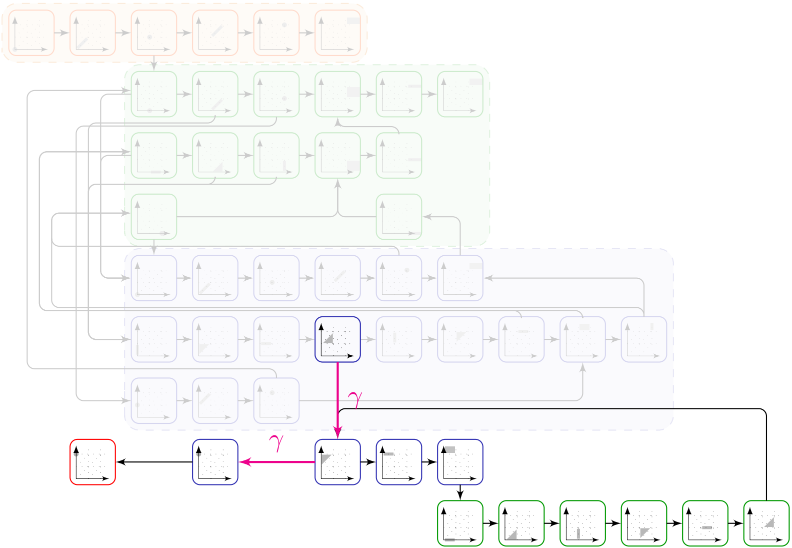

Outline

1. Introduction
2. Robust model-checking
   - Parameterized enlarged semantics
   - Automatic generation of an implementation
   - Implementation by shrinking
3. Robust realisability and control
   - Excess semantics
   - Strict semantics
4. Conclusion

Automatic generation of an implementation

The (approx.) implementation synthesis problem

Given $\mathcal{A}$, build $\mathcal{A}'$ such that:

- $\mathcal{A}'$ 'identical' (e.g. bisimilar) to $\mathcal{A}$
- $\mathcal{A}'$ is 'robust' (that is, good enough for implementation)

[BLM+11] Bouyer, Larsen, Markey, Sankur, Thrane. Timed automata can always be made implementable *(CONCUR'11)*.

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Theorem

All timed automata are approximately implementable!
(for approx. bisimulation)

- Technical tool: region construction

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Automatic generation of an implementation

Methodology

- Design and verify $\mathcal{A}$
- Implement $\mathcal{A}'$ (automatically generated)

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- ☺ Separates design and implementation
- ☹ $\mathcal{A}'$ is much bigger than $\mathcal{A}$

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Parameterized shrunk semantics for timed automata

A constraint $[a, b]$ is shrunk to $[a + \delta; b - \delta']$

[SBM11] Sankur, Bouyer, Markey. Shrinking Timed Automata *(FSTTCS'11)*.

Parameterized shrunk semantics for timed automata

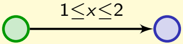
A constraint $[a, b]$ is shrunk to $[a + \delta; b - \delta']$

Why should we do that?

Abstract model

Impl. model

Models



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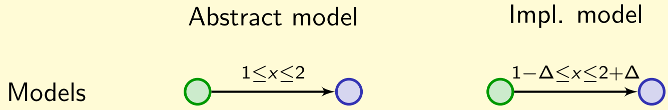

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	Abstract model	Impl. model
Models	$\circ \xrightarrow{1 \leq x \leq 2} \circ$	$\circ \xrightarrow{1-\Delta \leq x \leq 2+\Delta} \circ$
Real-world	$\circ \xrightarrow{1+\delta \leq x \leq 2-\delta'} \circ$	

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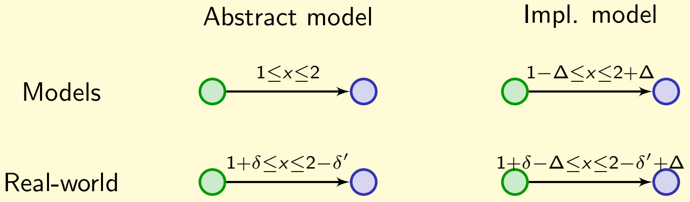

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It is fine as soon as $[1 + \delta - \Delta; 2 - \delta' + \Delta] \subseteq [1; 2]$,
which is the case when $\delta, \delta' \geq \Delta$.

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Parameterized shrunk semantics for timed automata

A constraint $[a, b]$ is shrunk to $[a + \delta; b - \delta']$

Summary of the approach

$\rightsquigarrow$ Shrink the clock constraints in the model, to prevent additional behaviours in the implementation

- If $\mathcal{B} = \mathcal{A}_{-\mathbf{k}\delta}$, then

$$\mathcal{B} \subseteq \mathtt{program}_\epsilon(\mathcal{B}) \subseteq \mathcal{B}_{f(\epsilon)} = \mathcal{A}_{-\mathbf{k}\delta + f(\epsilon)} \subseteq \mathcal{A}$$

[SBM11] Sankur, Bouyer, Markey. Shrinking Timed Automata *(FSTTCS'11)*.

Parameterized shrunk semantics – Discussion

What is the relevance of that approach?

Anticipate imprecisions to prevent additional behaviours in the real-world

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- Design and verify $\mathcal{A}$
- Implement $\mathcal{A}_{-\mathbf{k}\delta}$ (parameters are $\mathbf{k}$ and δ)

$\rightsquigarrow$ This is a good approach for designing systems with strong/hard timing constraints

Make sure that no important behaviours are lost in $\mathcal{A}_{-\mathbf{k}\delta}$!!

Parameterized shrunk semantics – Algorithmics

The (parameterized) shrinkability problem

Find parameters **k** and δ such that:

- $\mathcal{A} \sqsubseteq_{\texttt{t.a.}} \mathcal{A}_{-\mathbf{k}\delta}$ (or $\mathcal{F} \sqsubseteq_{\texttt{t.a.}} \mathcal{A}_{-\mathbf{k}\delta}$ for some finite automaton $\mathcal{F}$)
  [shrinkability w.r.t. untimed simulation]
- $\mathcal{A}_{-\mathbf{k}\delta}$ is non-blocking whenever $\mathcal{A}$ is non-blocking
  [shrinkability w.r.t. non-blockingness]

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  [shrinkability w.r.t. non-blockingness]

Theorem

Parameterized shrinkability can be decided (in exponential time).

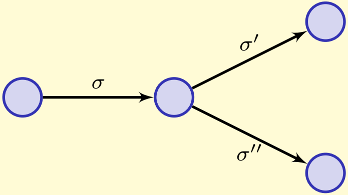
- Challenge: take care of the accumulation of perturbations
- Technical tools: parameterized shrunk DBM, max-plus equations
- Tool Shrinktech developed by Ocan Sankur [San13]

  http://www.lsv.ens-cachan.fr/Software/shrinktech/

[San13] Sankur. Shrinktech: A tool for the robustness analysis of timed automata *(CAV'13)*.

The case of non-blockingness

Non-blockingness

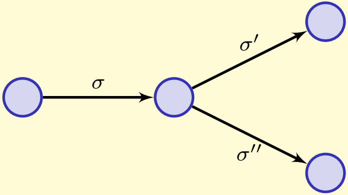


$$\begin{cases} G_\sigma \text{ guard of } \sigma \\ R_\sigma \text{ reset of } \sigma \end{cases}$$

Whenever σ is taken, either σ' or σ'' is eventually firable.

The case of non-blockingness

Non-blockingness



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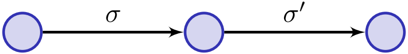
Whenever σ is taken, either σ' or σ'' is eventually firable.

Fix-point characterization

Let G_σ denote the **guards** of the timed automaton. It is non-blocking iff,

$$\forall \sigma, \quad [\![G_\sigma]\!] \subseteq \bigcup_{l_1 \xrightarrow{\sigma} l_2 \xrightarrow{\sigma'} l_3} \mathsf{Unreset}_{R_\sigma}(\mathsf{Pre}_{\mathsf{time}}([\![G_{\sigma'}]\!])).$$

Technical tools: shrunk DBMs...



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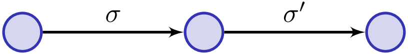

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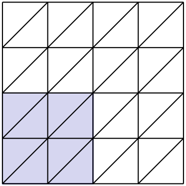
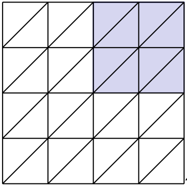


$\subseteq \mathsf{Unreset}_y \left(\mathsf{Pre}_{\mathsf{time}} \left(\right. \right.$



$\left. \left. \right) \right)$

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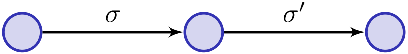

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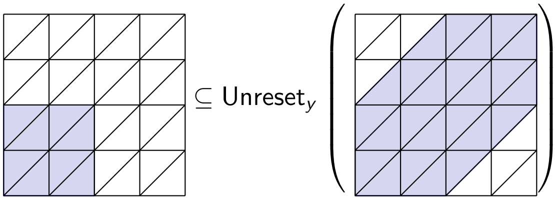

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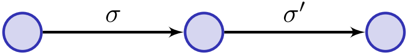

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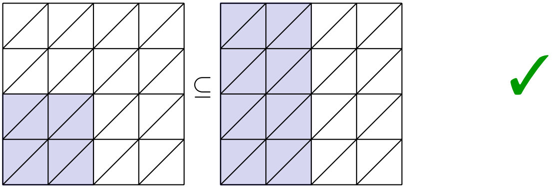

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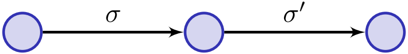
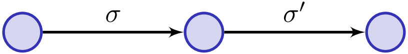


$$[\![\langle G_\sigma \rangle_{-\vec{k}\delta}]\!] \subseteq \mathsf{Unreset}_{R_\sigma}(\mathsf{Pre}_{\mathsf{time}}([\![\langle G_{\sigma'} \rangle_{-\vec{k}\delta}]\!])) \qquad ?$$

Determine $\vec{k}$

Technical tools: shrunk DBMs...



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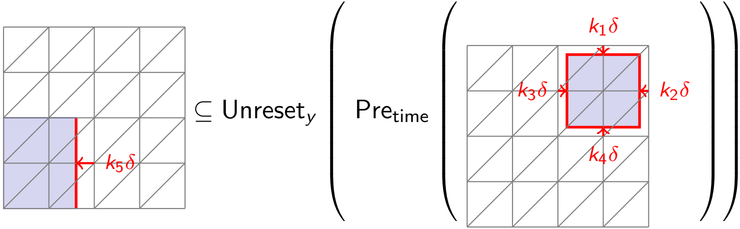

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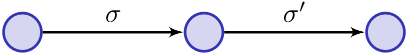

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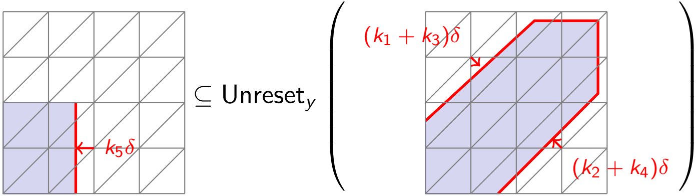

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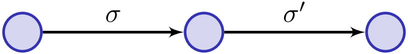

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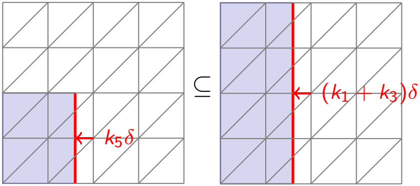
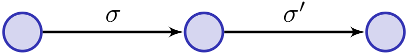


Technical tools: shrunk DBMs...and max-plus equations



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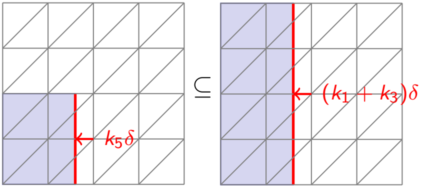

Then, $\vec{k}$ should satisfy

$$k_5 \geq k_1 + k_3 \qquad \text{that is,} \qquad k_5 = \max(k_5, k_1 + k_3)$$

In this case, the above inclusion equation holds for small enough δ's

$$[\![\langle G_\sigma \rangle_{-\vec{k}\delta}]\!] \subseteq \mathsf{Unreset}_{R_\sigma}(\mathsf{Pre}_{\mathsf{time}}([\![\langle G_{\sigma'} \rangle_{-\vec{k}\delta}]\!]))$$
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Key Theorem

Let $\vec{M} = f(\vec{M})$ be a **fixpoint equation on zones**, and $\vec{M}$ a solution.
f uses $\mathrm{Pre}_{\mathrm{time}}()$, $\cap$, $\mathrm{Unreset}_{\cdot}()$.
For any $\vec{k} \in \mathbb{N}^n_{>0}$,

$$\langle\vec{M}\rangle_{-\vec{k}\delta} = f(\langle\vec{M}\rangle_{-\vec{k}\delta}) \qquad \forall \text{ small } \delta > 0$$
$$\Leftrightarrow$$
$$\vec{k} = \varphi(\vec{k}),$$

where φ is a **max-plus expression**.

$$[\![\langle G_\sigma \rangle_{-\vec{k}\delta}]\!] \subseteq \mathrm{Unreset}_{R_\sigma}(\mathrm{Pre}_{\mathrm{time}}([\![\langle G_{\sigma'} \rangle_{-\vec{k}\delta}]\!]))$$
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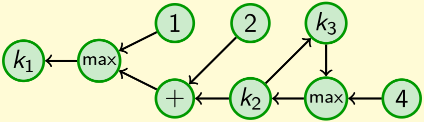
where φ is a **max-plus expression**.

$\rightsquigarrow$ Max-plus algebra: the above fixpoint equations can be solved in polynomial time

Solving max-plus equations

Max-plus graph

$k_1 \geq \max(1, 2 + k_2) \ \wedge \ k_3 \geq k_2 \ \wedge \ k_2 \geq \max(4, k_3)$



Solving max-plus equations

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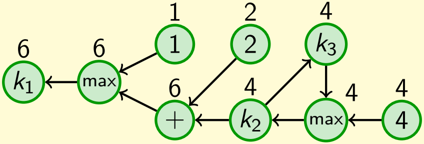

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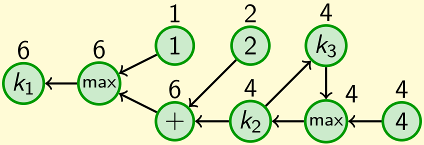

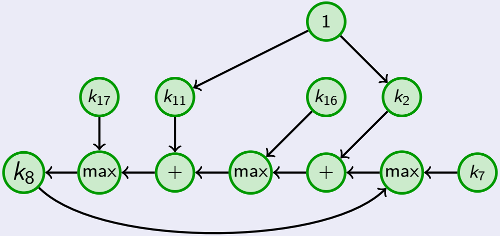

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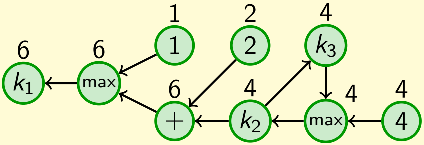
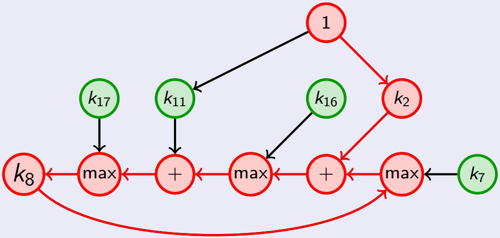




$\rightsquigarrow$ No solution!

Summary of shrinkability

Deciding shrinkability

Apply theorem to following fix-point equations:

- Non-blockingness:

$$\forall\sigma,\quad [\![G_\sigma]\!] \subseteq \bigcup_{l_1 \xrightarrow{\sigma} l_2 \xrightarrow{\sigma'} l_3} \mathsf{Unreset}_{R_\sigma}(\mathsf{Pre}_{\mathsf{time}}([\![G_{\sigma'}]\!])).$$

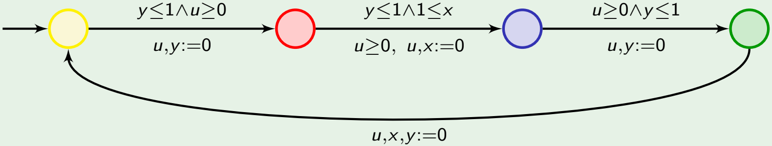
  (Do technical work to remove the union)

- Time-abstract simulation ($\mathcal{A} \sqsubseteq_{\text{t.a.}} \mathcal{A}_{-\delta\vec{k}}$):

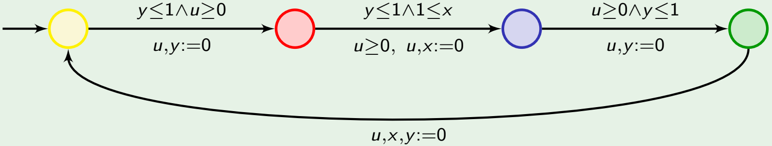
$$[\![M_{l,r}]\!] = \bigcap_{\sigma\in\Sigma} \bigcap_{(l,r)\xrightarrow{\sigma}(l',r')} \mathsf{Pre}_{\mathsf{time}}(\mathsf{Unreset}_{R_\sigma}([\![M_{l',r'}]\!]) \cap [\![G_\sigma]\!]),$$

  where $M_{l,r}$ is the time-abstract simulator set of the region (l, r).

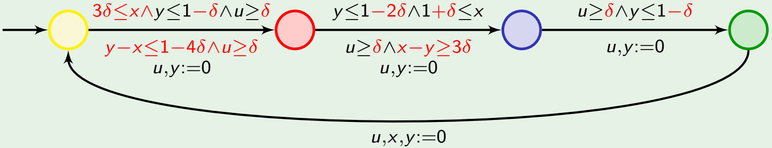
Example


$y \leq 1 \wedge u \geq 0$
$u,y{:=}0$
$y \leq 1 \wedge 1 \leq x$
$u \geq 0,\ u,x{:=}0$
$u \geq 0 \wedge y \leq 1$
$u,y{:=}0$
$u,x,y{:=}0$

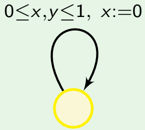
Example



The largest shrunk automaton which is correct w.r.t. untimed simulation and non-blockingness (for all $\delta \in [0, \frac{1}{4}]$) is:



Counter-example



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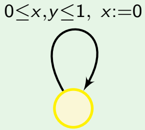

There is no shrunk automaton which is correct w.r.t. non-blockingness.
Indeed, the max-plus equations we obtain are:

$$\begin{cases} \dots \\ k_8 = \max(k_{17}, k_{11} + \max(k_{16}, k_2 + \max(k_7, k_8))) \\ k_{11} = \max(1, k_{11}) \end{cases}$$

which has no solution!
(remember the max-plus graph with no solution)

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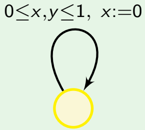

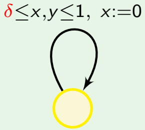

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- Same complexities as standard model-checking!
- Technical tools:
  - Extended region automaton
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  - And also characterization of reachability relations in timed automata (hidden in this presentation)
- What is missing:
  - A symbolic approach
  - A tool support
    - Shinktech is a prototype for the shrinking approach

      `http://www.lsv.ens-cachan.fr/Software/shrinktech/`
  - Stochastic approach (see later)

Outline

1. Introduction

2. Robust model-checking
   - Parameterized enlarged semantics
   - Automatic generation of an implementation
   - Implementation by shrinking

3. Robust realisability and control
   - Excess semantics
   - Strict semantics

4. Conclusion

Robust realisability

Here, a strategy in a timed automaton is a way to resolve (time and action) non-determinism

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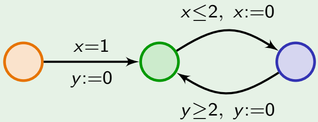

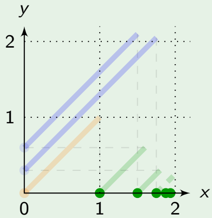

Strategy: in location ◯ with value x, delay $\frac{2-x}{2}$

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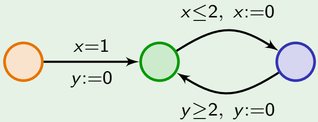

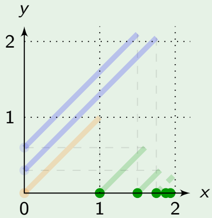

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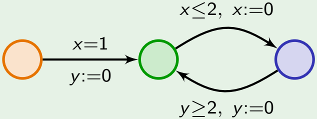

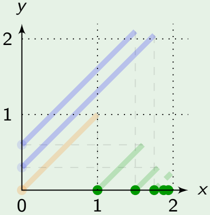

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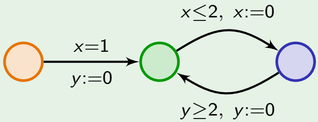

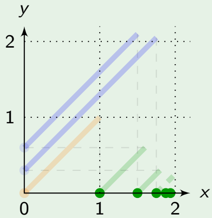

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- In practice, when x is close to 2, no additional delay is supported: the run is theoretically infinite, but it is actually blocking
- And that is unavoidable

Robust realisability

Here, a strategy in a timed automaton is a way to resolve (time and action) non-determinism

Idea of robust realisability

Synthesize strategies that realise some property, even under perturbations: strategies should adapt to previous imprecisions

$\rightsquigarrow$ develop a theory of robust strategies that tolerate errors/imprecisions and avoid convergence

Game semantics of a timed automaton

Game semantics $\mathcal{G}_\delta(\mathcal{A})$ of timed automaton $\mathcal{A}$...

... between Controller and Perturbator:

- from (ℓ, v), Controller suggests a delay $d \geq \delta$ and a next edge $e = (\ell \xrightarrow{g,Y} \ell')$ that is available after delay d
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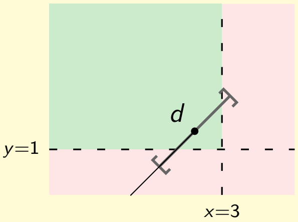
A δ-robust strategy for Controller is then a strategy that satisfies the expected property, whatever plays Perturbator.

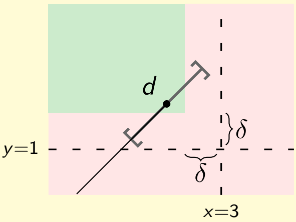
Two possible semantics

Consider a transition with guard $x \leq 3 \wedge y \geq 1$:

excess semantics



strict semantics



Outline

The excess game semantics

Constraints may not be satisfied after the perturbation
only $v + d$ should satisfy g

[BMS12] Bouyer, Markey, Sankur. Robust reachability in timed automata: A game-based approach *(ICALP'12)*.

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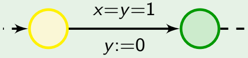

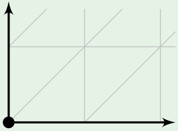

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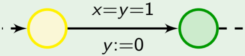

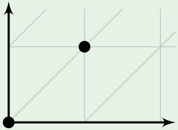

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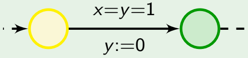

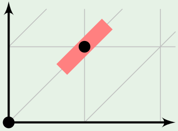

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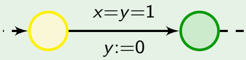

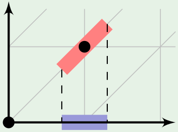

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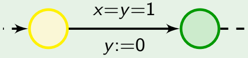

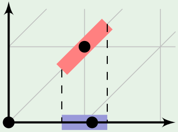

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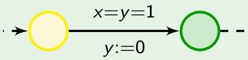

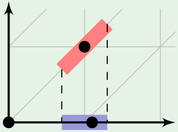

$\rightsquigarrow$ Allows simple design of constraints, ensures divergence of time, avoids convergence phenomena

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The excess game semantics – Algorithmics

The (parameterized) synthesis problem

Synthesize $\delta > 0$ and a δ-robust strategy that achieves a given goal.

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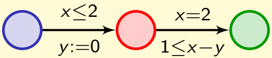

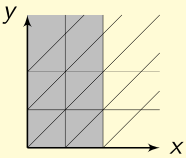

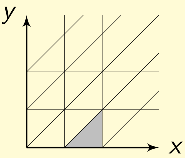

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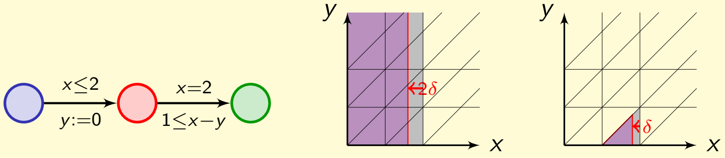

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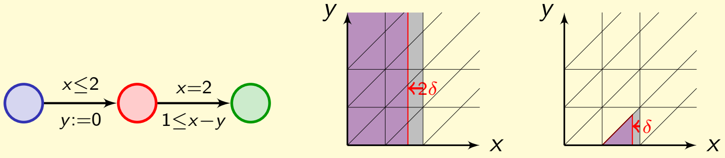
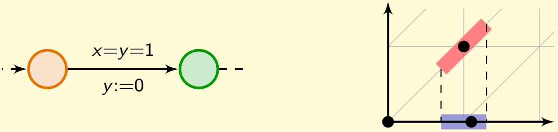


2. New regions become reachable



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The parameterized synthesis problem for reachability properties is decidable and EXPTIME-complete. Furthermore, uniform winning strategies (w.r.t. δ) can be computed.

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- Technical tool: a region-based refined game abstraction, shrunk DBMs
- ☺ Extends to two-player games (i.e. to real control problems)
- ☹ Only valid for reachability properties

The excess game semantics – Algorithm overview

1. (Forward) Construct an equivalent finite turn-based game $\mathbf{F}(\mathcal{A})$ (based on regions)
2. Solve it
3. (Backward) Construct winning states in $\mathcal{G}_\delta(\mathcal{A})$, and deduce δ_0

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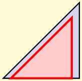
Winning states will be described by shrinkings of regions:



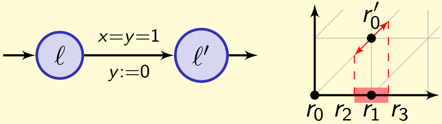
$r - \delta P$

One can win from a region r in $\mathbf{F}(\mathcal{A})$

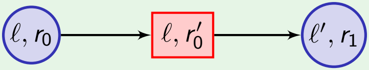
$\Updownarrow$

one can win from a shrinking of r in $\mathcal{G}_\delta(\mathcal{A})$

Construction of the finite turn-based game



region automaton:



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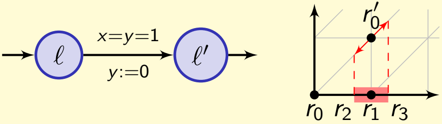
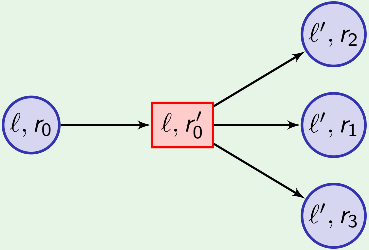
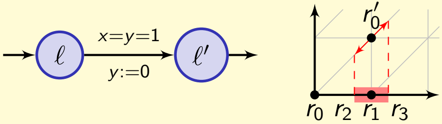


Extended region automaton:



Idea: We win from *some* shrinking of r_0, if, and only if we win from *some* shrinkings of r_1, r_2, r_3.

Construction of the finite turn-based game



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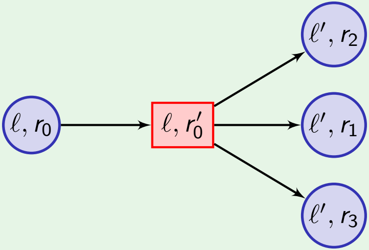

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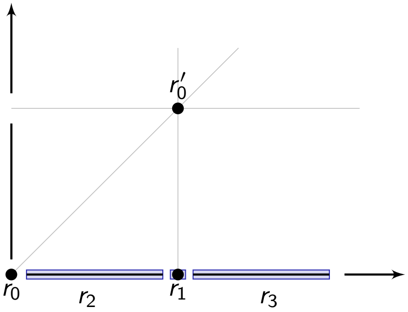
Assume that we win from **some** shrinkings of r_1, r_2, r_3.



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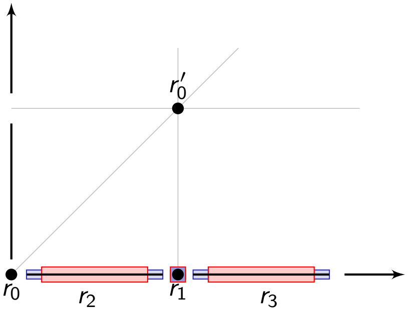

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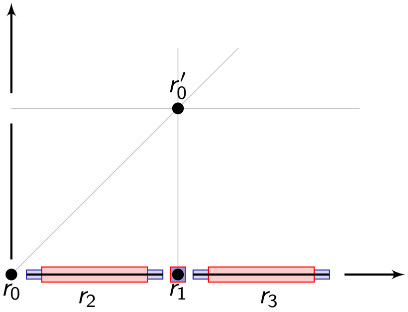

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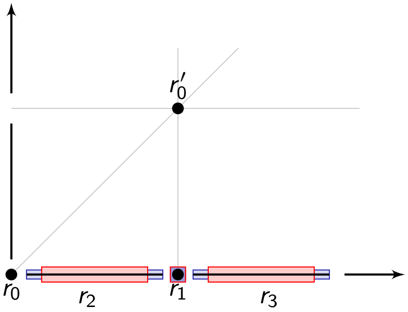
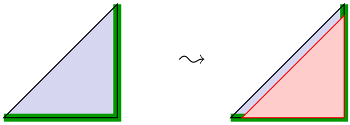


Can these be combined to a winning strategy from r_0?

No: we don't have a strategy for valuations around r_1.

Solution: Look for a shrinking of some regions with constraints.

A constrained region is a region with some marked facets.
A shrinking of a constrained region does not shrink from marked facets.



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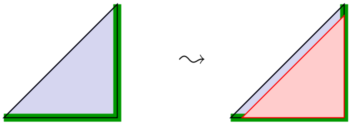
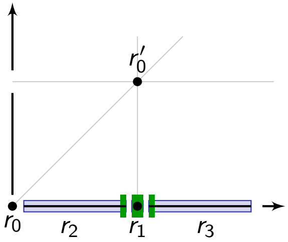


We win from r_0 iff we win from constrained shrinkings of r_1, r_2, r_3.



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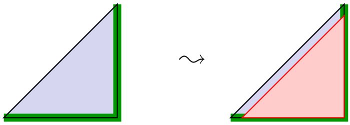
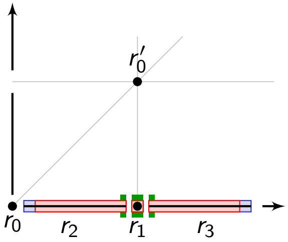


We win from r_0 iff we win from constrained shrinkings of r_1, r_2, r_3.



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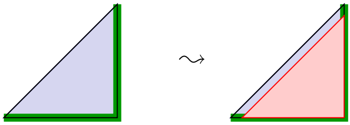

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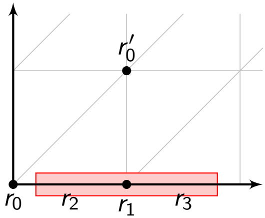

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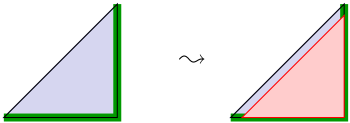

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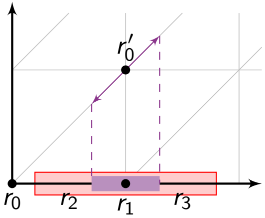
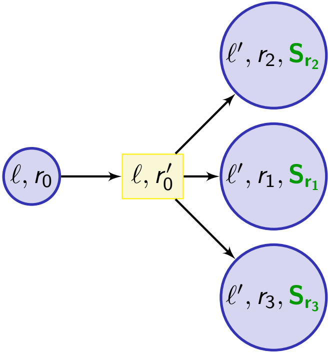


OK, we have a strategy for all the points in the violet area.

Finite game $\mathbf{F}(\mathcal{A})$

Shrinking constraint for region r is represented by a boolean matrix $\mathbf{S_r}$.



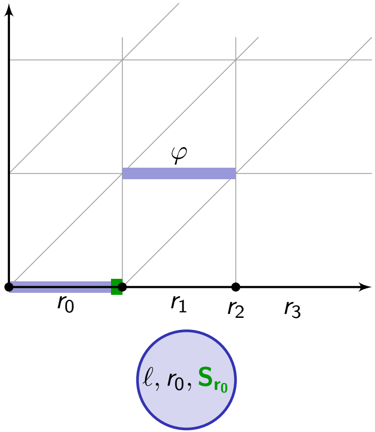
Theorem

Controller wins in $\mathcal{G}_\delta(\mathcal{A})$ for all $\delta \in [0, \delta_0]$ for some $\delta_0 > 0$

$\Updownarrow$

Controller wins in $\mathbf{F}(\mathcal{A})$.

Details on the definition of $\mathbf{F}(\mathcal{A})$



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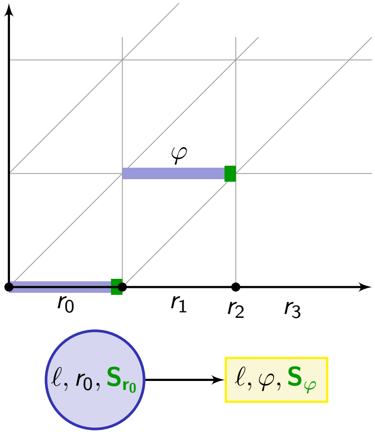

$\mathbf{S}_\varphi$ is defined such that:

Controller wins from *some* shrinking of $(\varphi, \mathbf{S}_\varphi)$ **iff**
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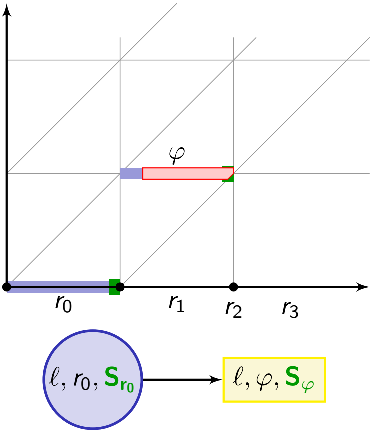

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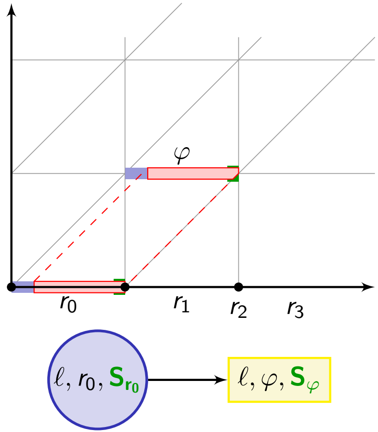

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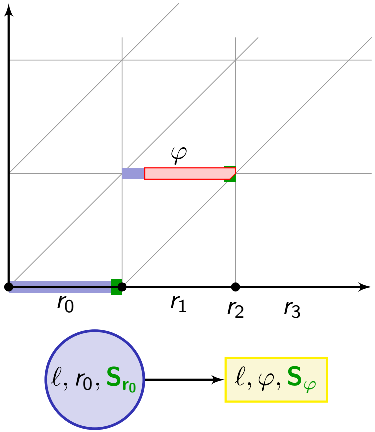

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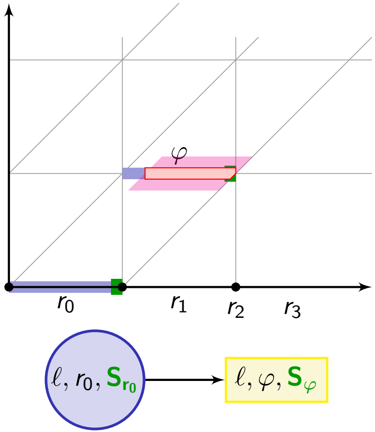

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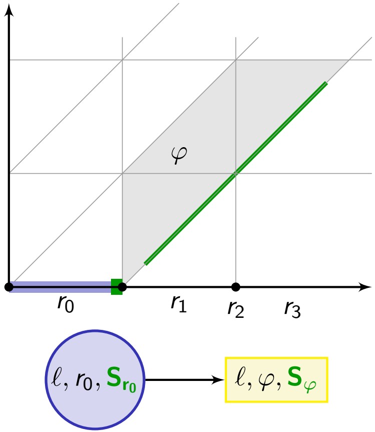

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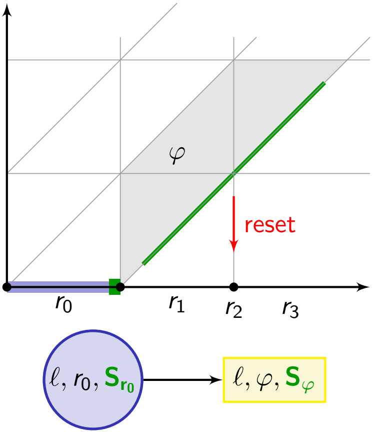

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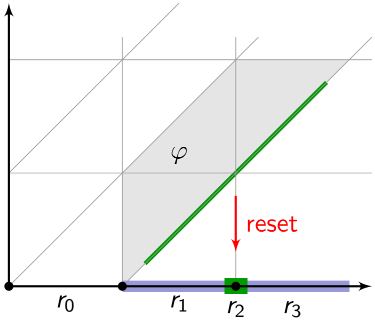
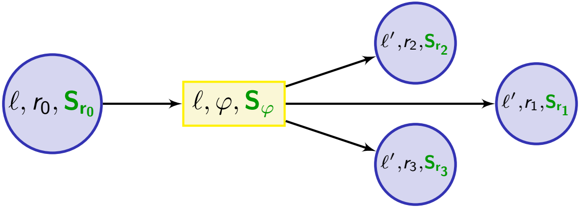
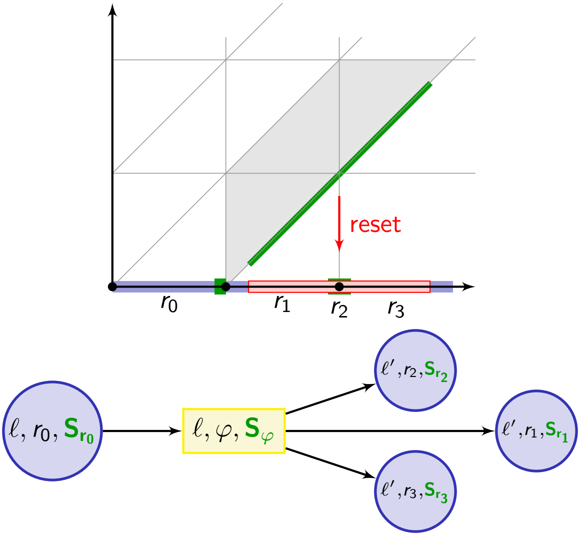




Constructing a winning strategy from $\mathbf{F}(\mathcal{A})$



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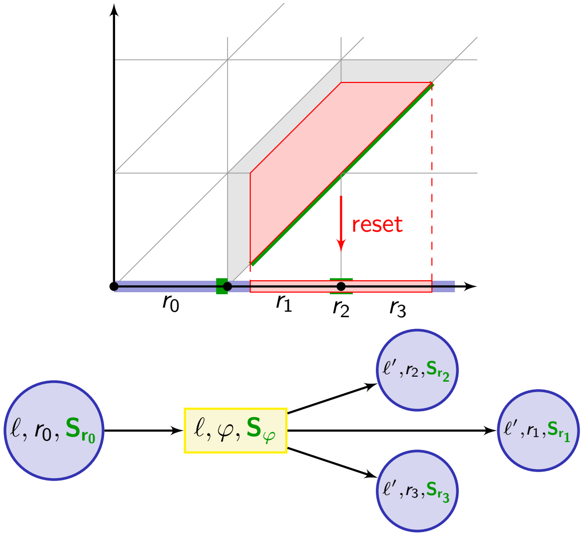

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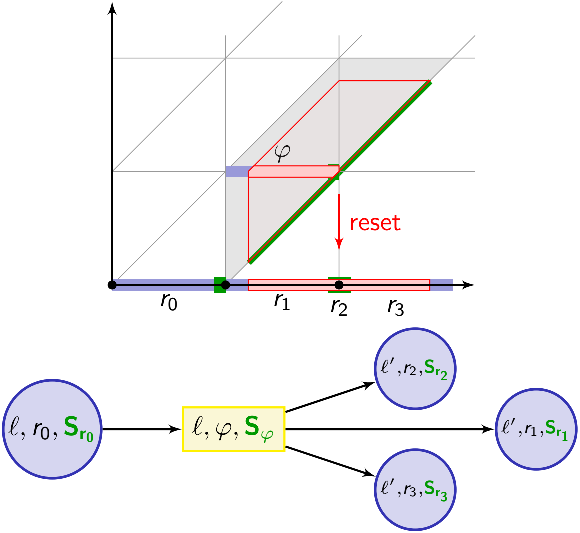

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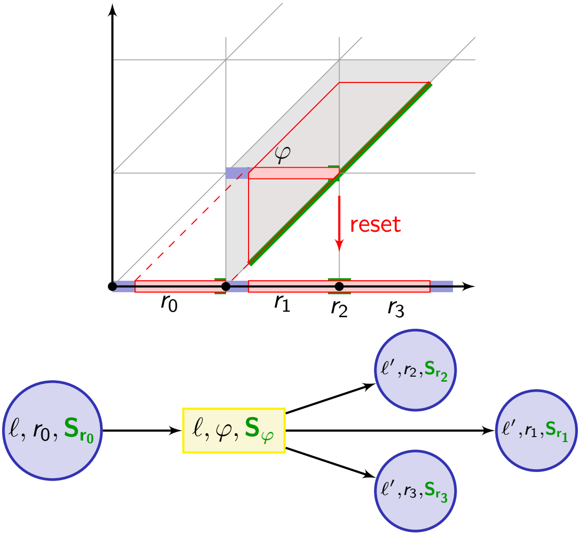

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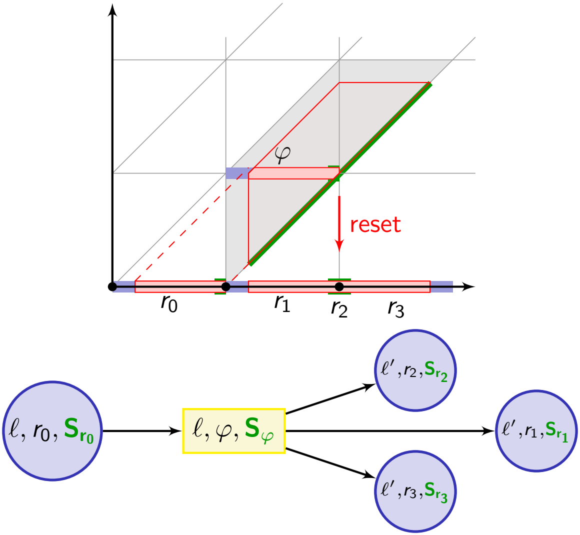

- Each step of the backward propagation gives an upper bound on δ.

EXPTIME-hardness

Usual semantics in timed automata can encode reachability in linearly bounded Turing machines (PSPACE-complete).

Robust semantics in timed automata can encode reachability in alternating linearly bounded Turing machines (EXPTIME-complete).

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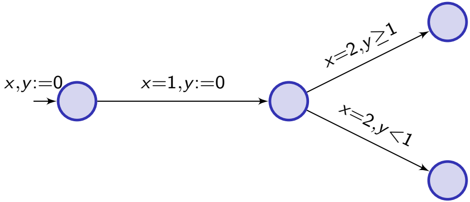

Perturbator has a strategy to choose between any of the two branches.

- Top branch: make the first transition earlier
- Bottom branch: delay the first transition

Outline

The strict game semantics

Constraints have to be satisfied after the perturbation:
$v + d + \epsilon$ should satisfy g for every $\epsilon \in [-\delta; +\delta]$

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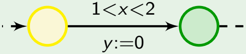

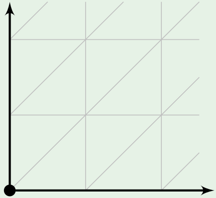

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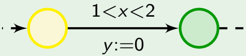

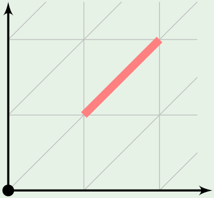

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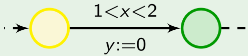

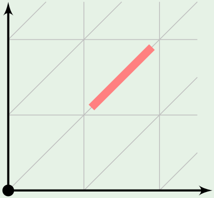

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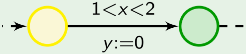

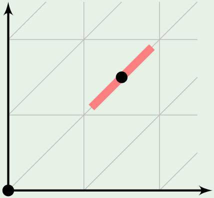

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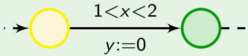

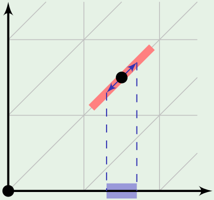

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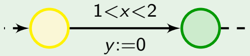

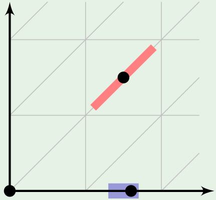

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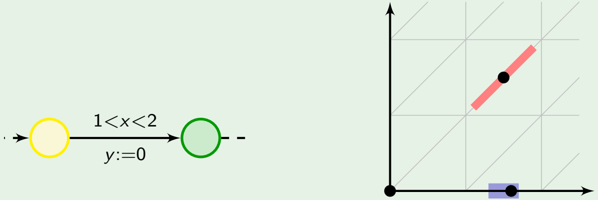

$\rightsquigarrow$ Strongly ensures timing constraints, ensures divergence of time, prevents converging phenomena

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The strict game semantics – Algorithmics

The (parameterized) synthesis problem

Synthesize $\delta > 0$ and a δ-robust strategy that achieves a given goal.

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Synthesize $\delta > 0$ and a δ-robust strategy that achieves a given goal.

Theorem

The synthesis problem for Büchi properties is decidable and PSPACE-complete. Furthermore, δ is at most doubly-exponential, and uniform winning strategies (w.r.t. δ) can be computed.

The problem consists in finding cycles that do not become blocked.

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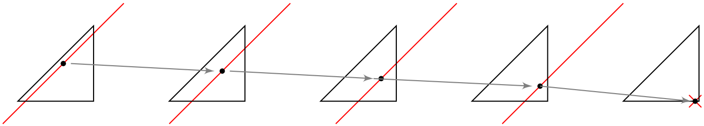
- A converging phenomena:



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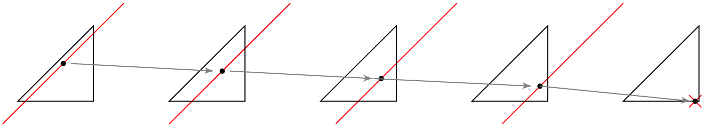

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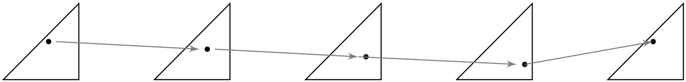

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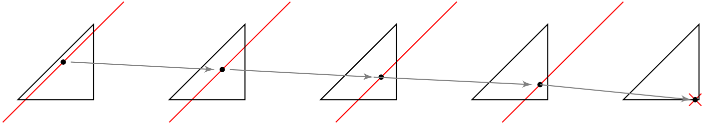

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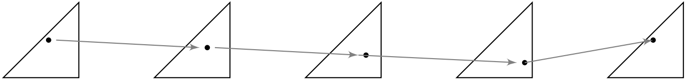

Tools for solving the synthesis problem

- Orbit graphs, forgetful cycles [AB11]
- Forgetful orbit graph $\Leftrightarrow$ no convergence phenomena
  $\leadsto$ strong relation with thick automata.

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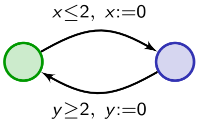
Technical tool: the (folded) orbit graph



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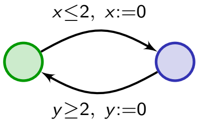
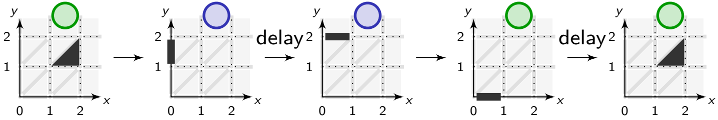


A region cycle:



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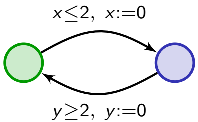

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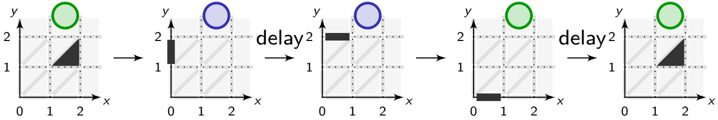
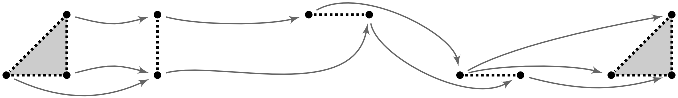


The corresponding orbit graph:



$\rightsquigarrow$ stores the reachability relation between vertices of the regions

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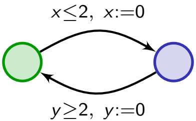

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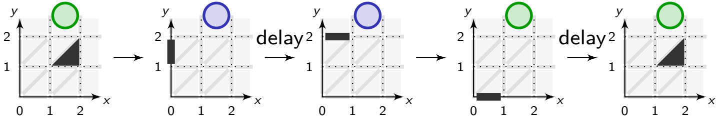

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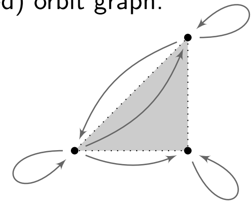
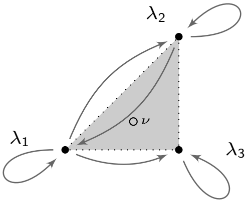


Understanding the folded orbit graph



$\nu = \vec{\lambda} \cdot \vec{v}$ (convex combination of the vertices)

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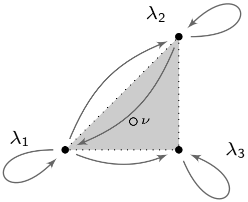

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Reachability relation [Pur00]

Given a region cycle ρ, and valuation $\nu = \vec{\lambda} \cdot \vec{v}$,

$\vec{\lambda} \cdot \vec{v} \xrightarrow{\rho} \vec{\lambda}' \vec{v} \quad \Leftrightarrow$ $\vec{\lambda}'$ is computed by distributing each λ_v to its successors following a probability distribution

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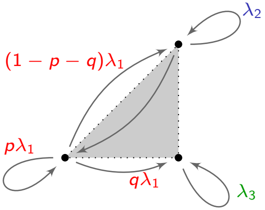

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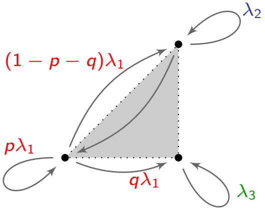

$$\begin{cases} \lambda_1' = p\lambda_1 \\ \\ \end{cases}$$

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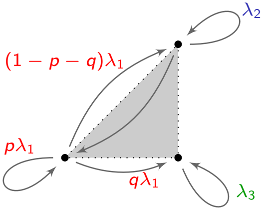

$$\begin{cases} \lambda_1' = p\lambda_1 \\ \lambda_2' = (1-p-q)\lambda_1 + \lambda_2 \\ \end{cases}$$

Reachability relation [Pur00]

Given a region cycle ρ, and valuation $\nu = \vec{\lambda} \cdot \vec{v}$,

$\vec{\lambda} \cdot \vec{v} \xrightarrow{\rho} \vec{\lambda}' \vec{v} \quad \Leftrightarrow$ $\vec{\lambda}'$ is computed by distributing each λ_v to its successors following a probability distribution

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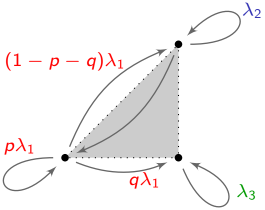

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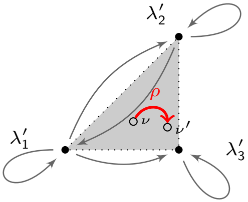

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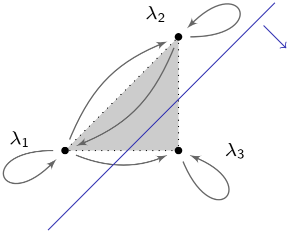

$\lambda_1 + \lambda_2$ is non-increasing and λ_3 is non-decreasing

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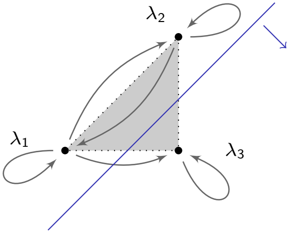

Generalization

- The reachability relation along one cycle is complete iff its folded orbit graph is complete. [Pur00]

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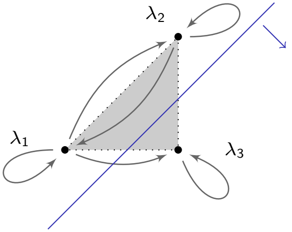

Generalization

- The reachability relation along one cycle is complete iff its folded orbit graph is complete. [Pur00]
- If the folded orbit graph is connected but not strongly connected, then there is some convergence phenomenon in the direction of the hyperplane $\sum_{v \in I} \lambda_v$ [AB11]

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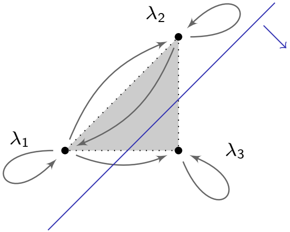

Classification of cycles

A cycle is aperiodic if all its iterations are strongly connected.

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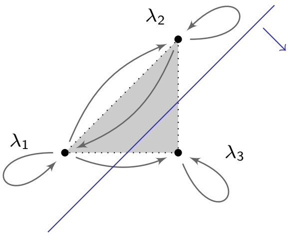

Classification of cycles

A cycle is aperiodic if all its iterations are strongly connected. Then:

- aperiodic cycle: no convergence phenomenon
  (some iterate is complete)
- non-aperiodic cycle: convergence phenomenon
  (convergence phenomenon from the non strongly connected iterate)

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Back to robustness

Characterization

There exists $\delta > 0$ such that Controller has a δ-robust strategy ensuring a Büchi condition in $\mathcal{G}_\delta(\mathcal{A})$ if, and only if there is a reachable aperiodic cycle in $\mathcal{A}$ which satisfies the Büchi condition.

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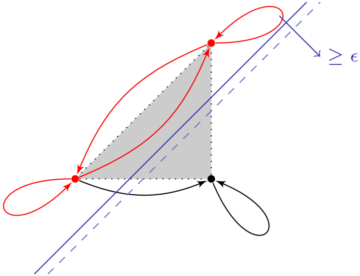

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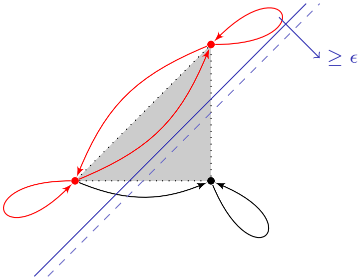

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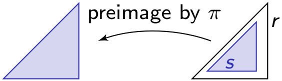

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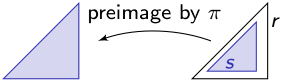

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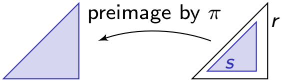

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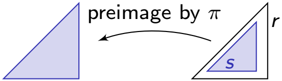

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$\Rightarrow$ Robust strategy: enforce s at each cycle

Going further [ORS14]

Extension to two-player games

- New rules: Controller chooses a delay and an action, and Perturbator perturbs the delay and resolves the non-determinism, if any
- Robustness under strict semantics can be solved in this case as well (EXPTIME)

[ORS14] Oualhadj, Reynier, Sankur. Probabilistic robust timed games *(CONCUR'14)*.

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- Assume perturbations are randomized!
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Beyond worst-case robustness

- Assume perturbations are randomized!
  (uniform distributions over $[d - \delta; d + \delta]$)
- Existence of an almost-sure winning strategy for Controller can be decided in EXPTIME. Furthermore there is a dichotomy:
  - either Controller wins almost-surely
  - or Perturbator wins almost-surely

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Partial conclusion

- We have presented a possible approach to the robust realizability and control problems
  - There are two natural semantics (excess or strict)
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  - Interesting relation between non-convergent cycles and robust cycles
  - Interesting complexities as well!
- Teachnical tools:
  - Regions
  - Shrunk DBMs
  - Orbit graphs
- What is missing:
  - A symbolic approach
  - A tool support
  - Stochastic approach at the beginning only

Outline

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- This list of possible approaches is not exhaustive:
  - tube acceptance [GHJ97]
  - sampling approach [KP05,BLM+11]
  - probabilistic approach [BBB+08,BBJM12]
  - ...