On the optimal reachability problem in weighted timed games

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Based on former works with Thomas Brihaye, Kim G. Larsen, Nicolas Markey, etc...
And on recent work with Samy Jaziri and Nicolas Markey

Outline

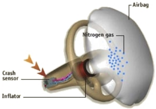
Time-dependent systems

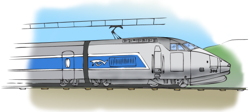
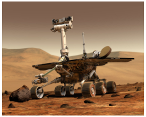
- We are interested in timed systems

Time-dependent systems

- We are interested in timed systems

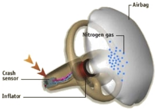


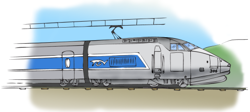


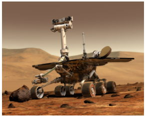




Time-dependent systems

- We are interested in timed systems



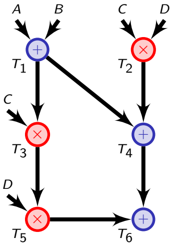




- ... and in their analysis and control

An example: The task graph scheduling problem

Compute $D \times (C \times (A + B)) + (A + B) + (C \times D)$ using two processors:

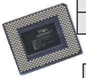


P_1 (fast):

time	
$+$	2 picoseconds
$\times$	3 picoseconds

energy	
idle	10 Watt
in use	90 Watts

P_2 (slow):



time	
$+$	5 picoseconds
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[BFLM10] Bouyer, Fahrenberg, Larsen, Markey. Quantitative Analysis of Real-Time Systems using Priced Timed Automata.

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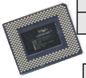

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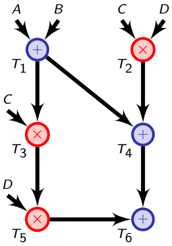

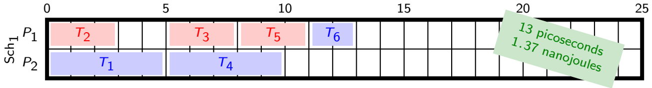

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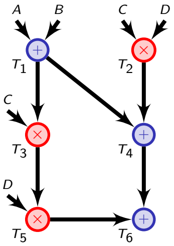

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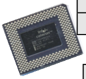

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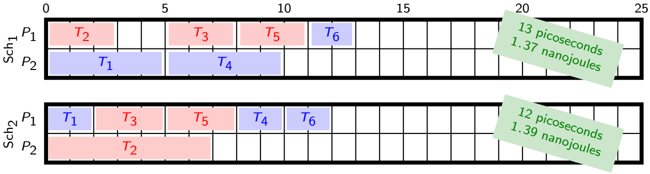

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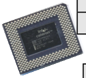

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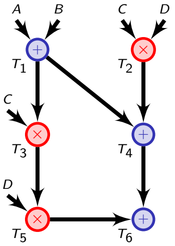

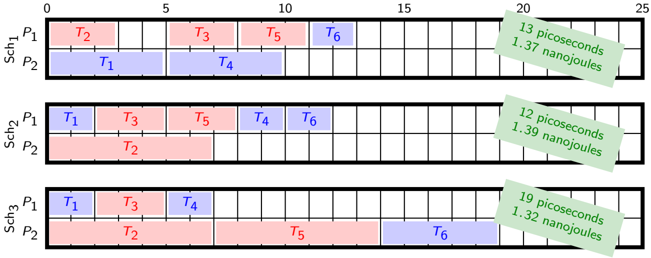

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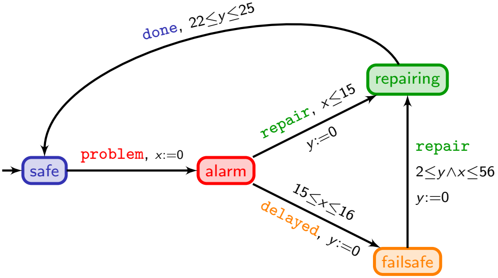
The model of timed automata



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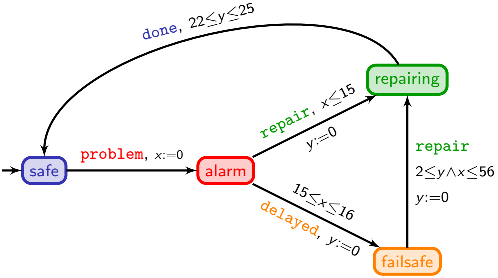


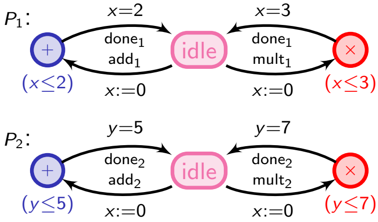
	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe
x	0		23		0		15.6		15.6 ···
y	0		23		23		38.6		0

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\texttt{done}}$	safe
···	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

Modelling the task graph scheduling problem

Modelling the task graph scheduling problem

- Processors



Modelling the task graph scheduling problem

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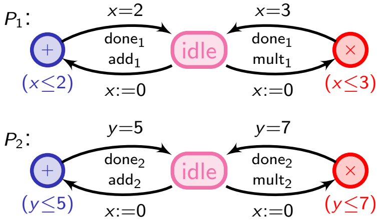
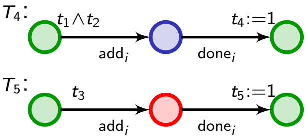
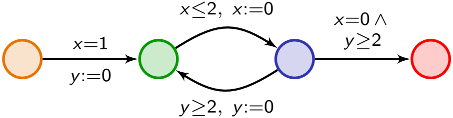


- Tasks



A schedule is a path in the product automaton

Analyzing timed automata



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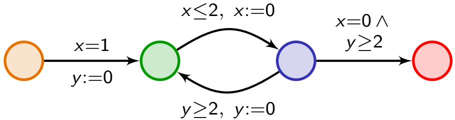

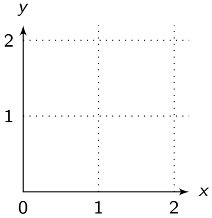

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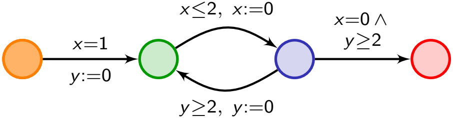

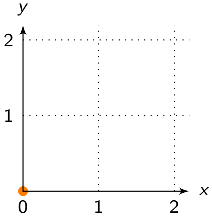

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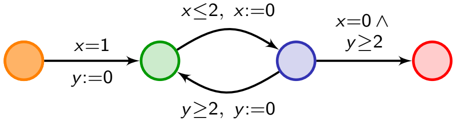

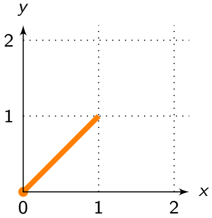

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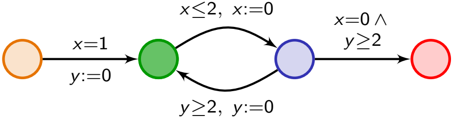

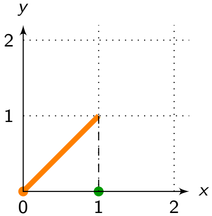

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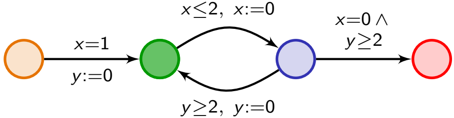

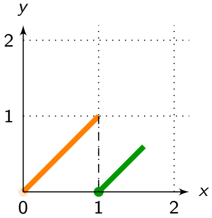

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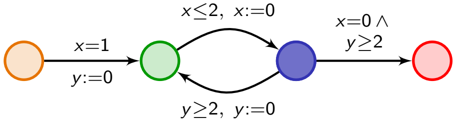

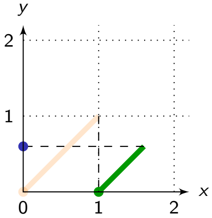

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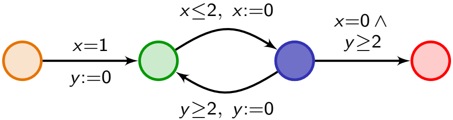

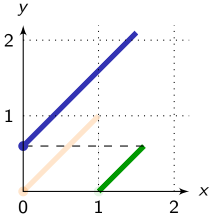

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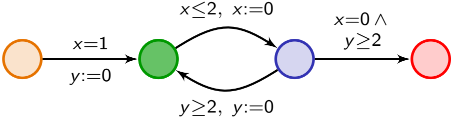

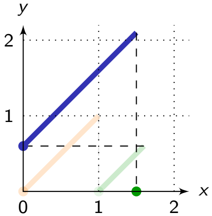

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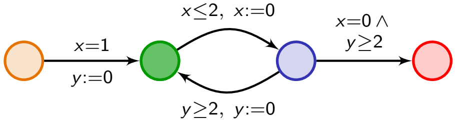

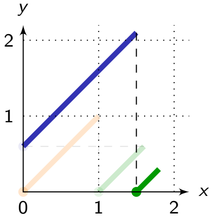

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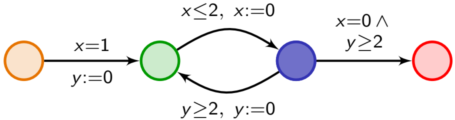

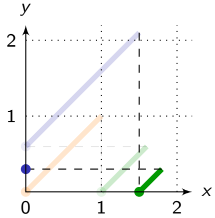

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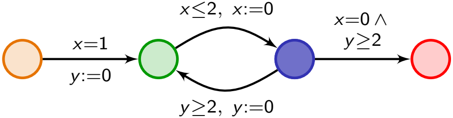

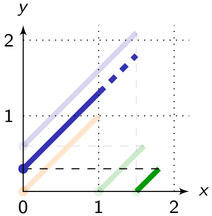

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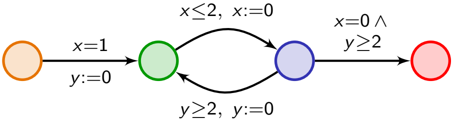

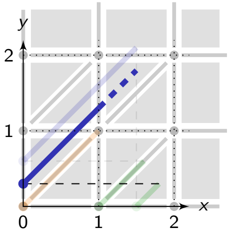

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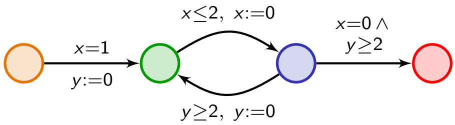
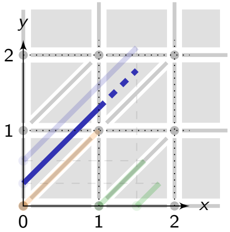


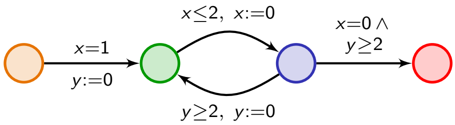


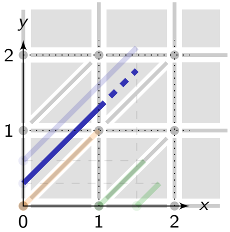
Theorem [AD94]

Reachability in timed automata is decidable (as well as many other important properties). It is PSPACE-complete.

- Technical tool: region abstraction

Analyzing timed automata





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- Efficient symbolic technics based on zones, implemented in tools

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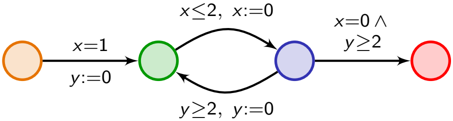

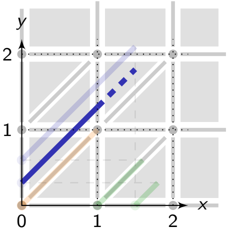

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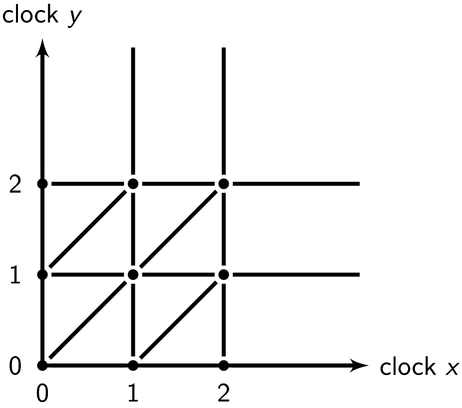
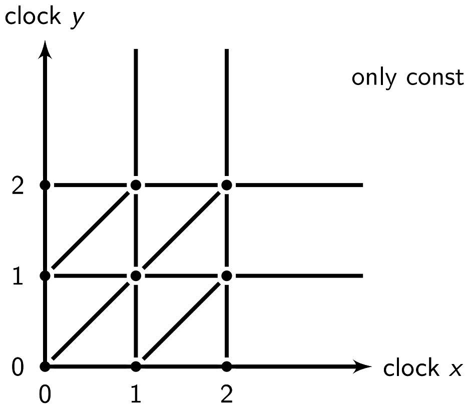
Skip regions

Technical tool: Region abstraction



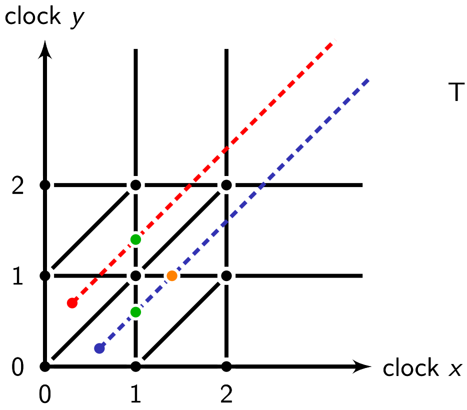
Technical tool: Region abstraction



only constraints: $x \sim c$ with $c \in \{0, 1, 2\}$
$y \sim c$ with $c \in \{0, 1, 2\}$

- "compatibility" between regions and constraints

Technical tool: Region abstraction

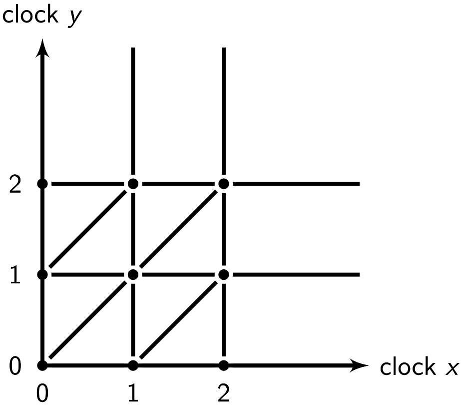


The path 

- can be fired from
- cannot be fired from

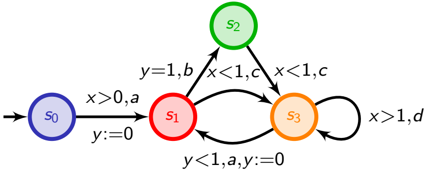
- "compatibility" between regions and constraints
- "compatibility" between regions and time elapsing

Technical tool: Region abstraction



$\rightsquigarrow$ This is a finite time-abstract bisimulation!

Technical tool: Region abstraction – An example [AD94]



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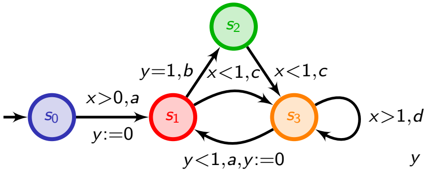

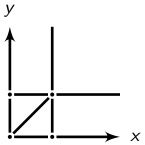

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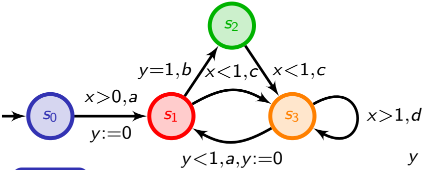

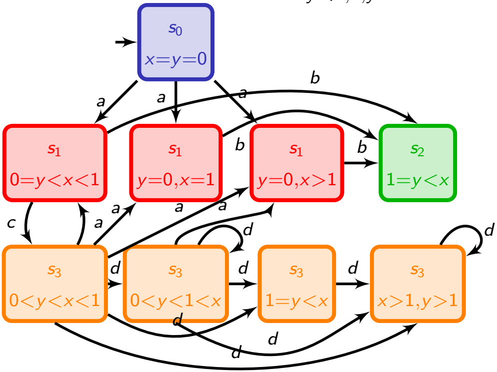

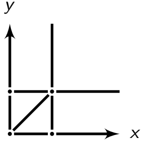

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Modelling resources in timed systems

- System resources might be relevant and even crucial information

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  - energy consumption,
  - memory usage,
  - ...
  - price to pay,
  - bandwidth,

Modelling resources in timed systems

- System resources might be relevant and even crucial information
  - energy consumption,
  - memory usage,
  - ...
  - price to pay,
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$\leadsto$ timed automata are not powerful enough!

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a discrete control (the mode of the system)
$+$ continuous evolution of the variables within a mode

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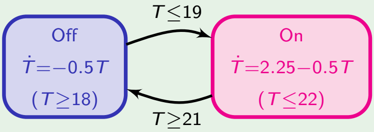
The thermostat example



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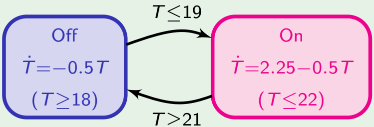

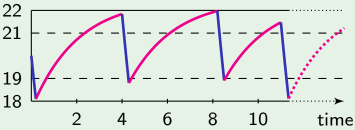

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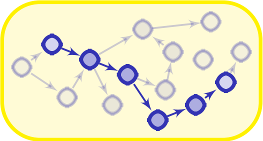

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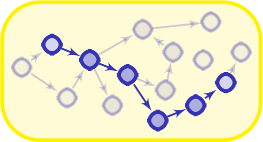
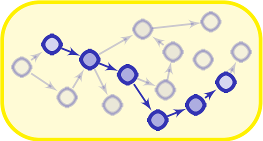


Easy...

Ok...



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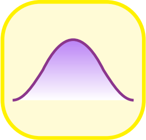

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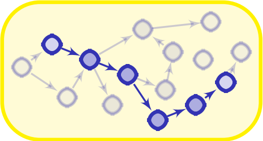

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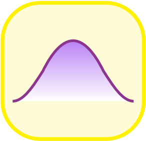

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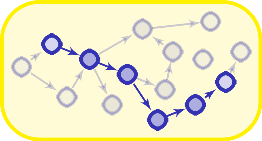
Ok... but?



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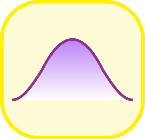

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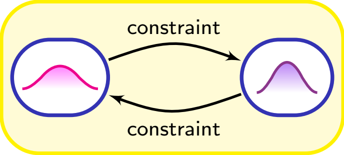

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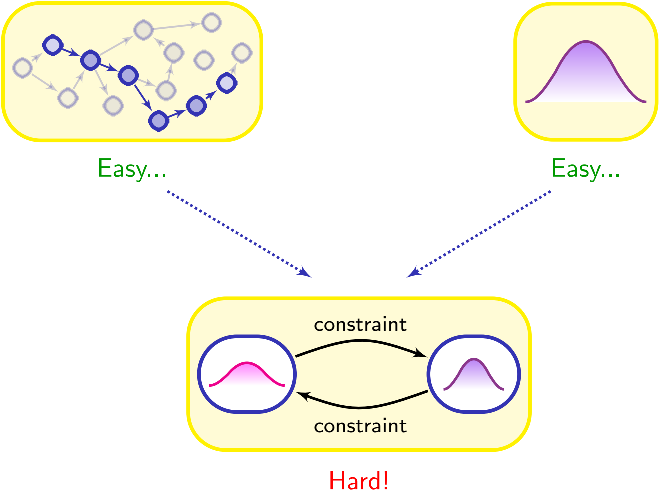

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Theorem [HKPV95]

The reachability problem is undecidable in hybrid automata. Even for the simplest, the so-called stopwatch automata (clocks can be stopped).

[HKPV95] Henzinger, Kopke, Puri, Varaiya. What's decidable wbout hybrid automata? *(SToC'95)*.

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Theorem [HKPV95]

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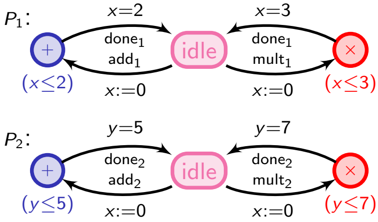
- An alternative: weighted/priced timed automata [ALP01,BFH+01]
  $\leadsto$ hybrid variables do not constrain the system
  hybrid variables are observer variables

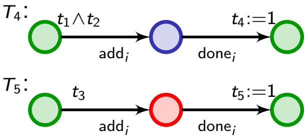
[HKPV95] Henzinger, Kopke, Puri, Varaiya. What's decidable wbout hybrid automata? *(SToC'95)*.[ALP01] Alur, La Torre, Pappas. Optimal paths in weigh
[BFH+01] Behrmann, Fehnker, Hune, Larsen, Pettersson, Romijn, Vaandrager. Minimum-cost reachability in priced timed automata *(HSCC'01)*.

Modelling the task graph scheduling problem

- Processors



- Tasks



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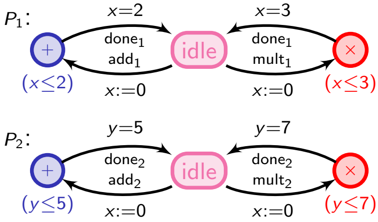

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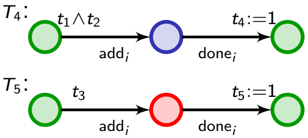
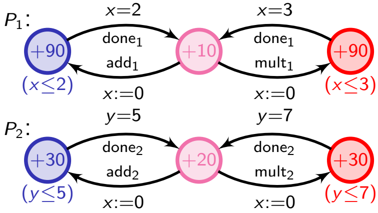
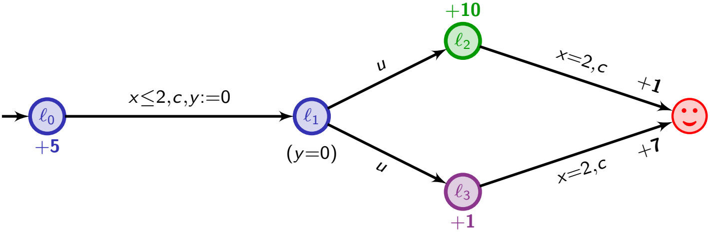


- Modelling energy



A good schedule is a path in the product automaton with a low cost

Weighted/priced timed automata [ALP01,BFH+01]



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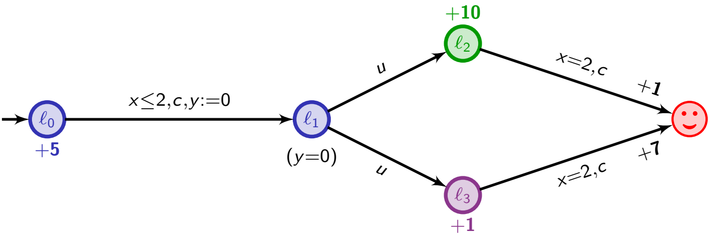

$$\begin{array}{ccccccccccccc} & \ell_0 & \xrightarrow{1.3} & \ell_0 & \xrightarrow{c} & \ell_1 & \xrightarrow{u} & \ell_3 & \xrightarrow{0.7} & \ell_3 & \xrightarrow{c} & ☺ \\ x & 0 & & 1.3 & & 1.3 & & 1.3 & & 2 & & \\ y & 0 & & 1.3 & & 0 & & 0 & & 0.7 & & \end{array}$$

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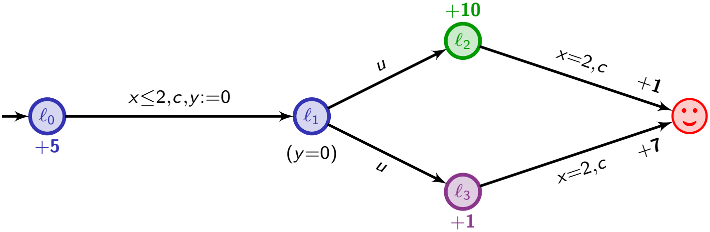

$$
\begin{array}{ccccccccccccc}
 & \ell_0 & \xrightarrow{1.3} & \ell_0 & \xrightarrow{c} & \ell_1 & \xrightarrow{u} & \ell_3 & \xrightarrow{0.7} & \ell_3 & \xrightarrow{c} & ☺ \\
x & 0 & & 1.3 & & 1.3 & & 1.3 & & 2 & & \\
y & 0 & & 1.3 & & 0 & & 0 & & 0.7 & &
\end{array}
$$

cost :

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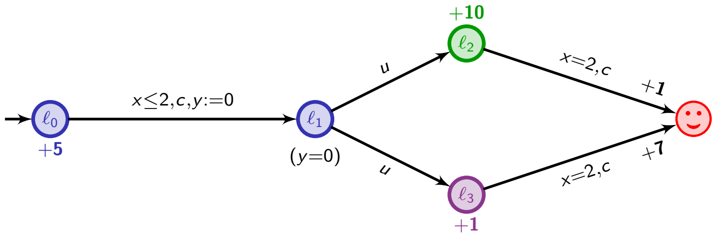

	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : 6.5

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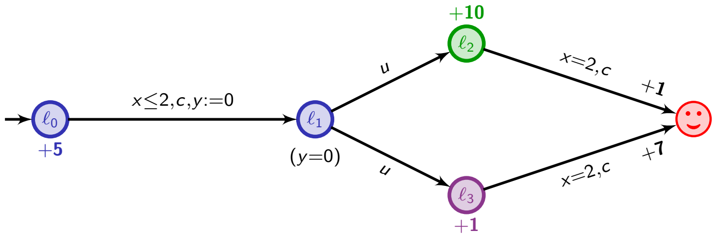

	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : 6.5 + 0

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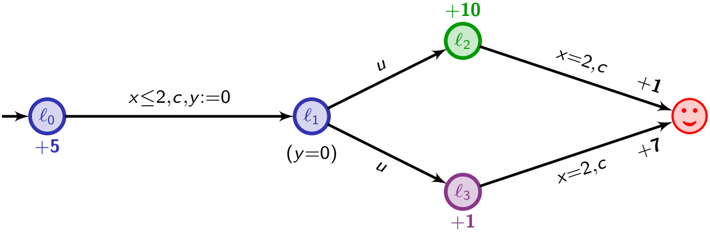

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x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : 6.5 + 0 + 0

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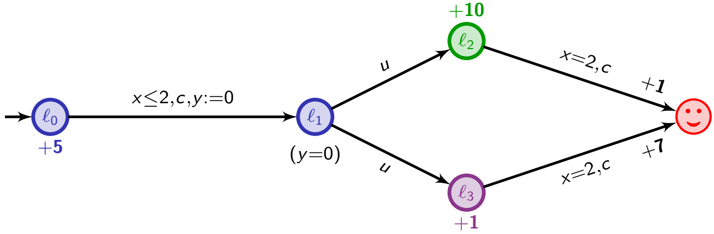

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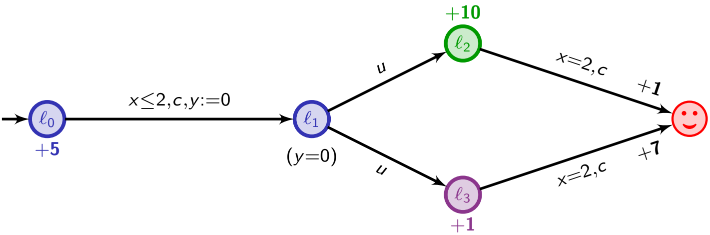

	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : 6.5 + 0 + 0 + 0.7 + 7

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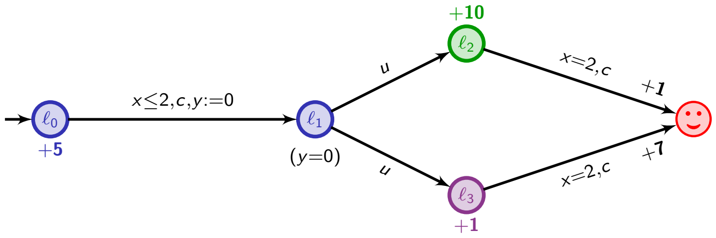

	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : $6.5 + 0 + 0 + 0.7 + 7 = 14.2$

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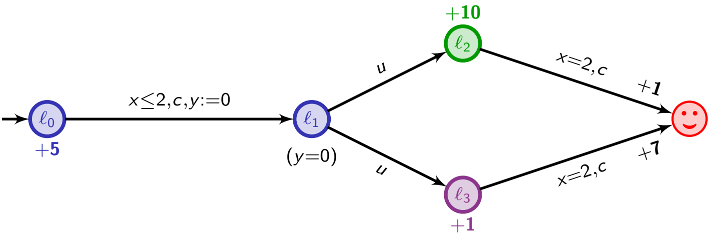

Question: what is the optimal cost for reaching 🙂 ?

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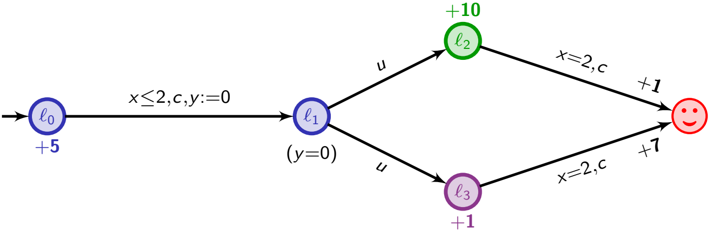

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$$5t + 10(2-t) + 1$$

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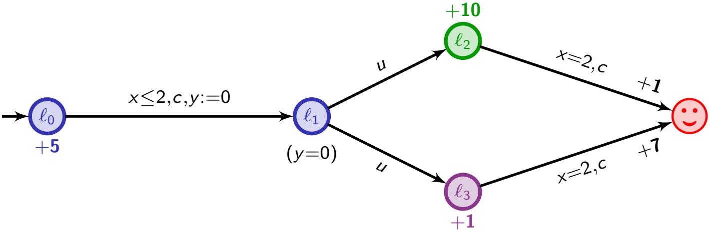

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$$5t + 10(2-t) + 1 \; , \; 5t + (2-t) + 7$$

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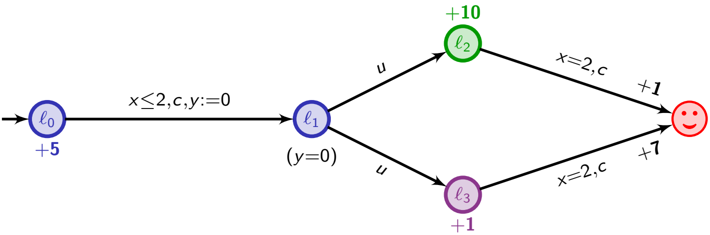
Weighted/priced timed automata [ALP01,BFH+01]



Question: what is the optimal cost for reaching ☺ ?

$$\min\ (\ 5t + 10(2-t) + 1\ ,\ 5t + (2-t) + 7\ )$$

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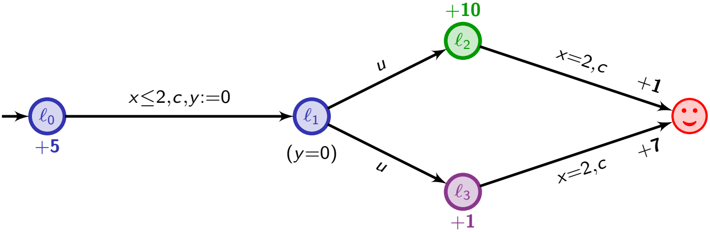

Question: what is the optimal cost for reaching 🙂?

$$\inf_{0 \leq t \leq 2} \min\left(5t + 10(2-t) + 1 \,,\; 5t + (2-t) + 7 \right) = 9$$

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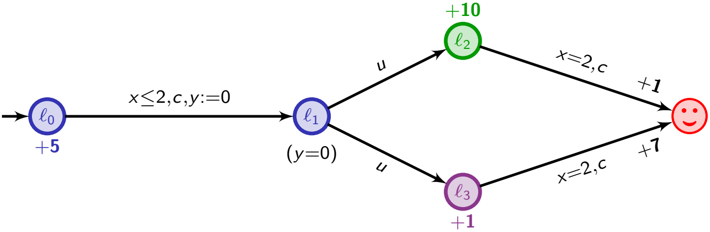

Question: what is the optimal cost for reaching ☺?

$$\inf_{0 \leq t \leq 2} \min\left(5t + 10(2-t) + 1\ ,\ 5t + (2-t) + 7\right) = 9$$

$\rightsquigarrow$ *strategy:* leave immediately ℓ_0, go to ℓ_3, and wait there 2 t.u.

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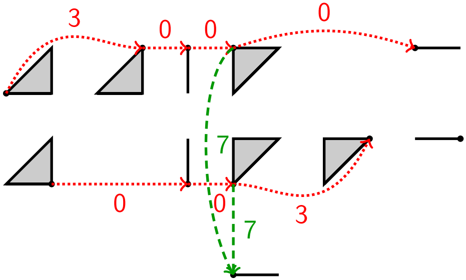
Optimal-cost reachability

Theorem [ALP01,BFH+01,BBBR07]

In weighted timed automata, the optimal cost is an integer and can be computed in PSPACE.

- Technical tool: a refinement of the regions, the corner-point abstraction



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[BFH+01] Behrmann, Fehnker, Hune, Larsen, Pettersson, Romijn, Vaandrager. Minimum-cost reachability in priced timed automata *(HSCC'01)*.
[BBBR07] Bouyer, Brihaye, Bruyère, Raskin. On the optimal reachability problem *(Formal Methods in System Design)*.

Note on the corner-point abstraction

It is a very interesting abstraction, that can be used for many applications:

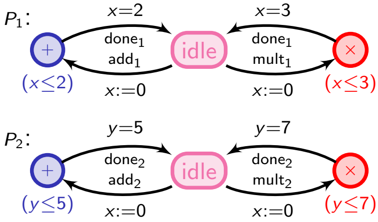
- for mean-cost optimization [BBL04,BBL08]
- for discounted-cost optimization [FL08]
- for all concavely-priced timed automata [JT08]
- for deciding frequency objectives [BBBS11,Sta12]
- ...

[BBL04] Bouyer, Brinksma, Larsen. Staying Alive As Cheaply As Possible *(HSCC'04)*.
[BBL08] Bouyer, Brinksma, Larsen. Optimal infinite scheduling for multi-priced timed automata *(Formal Methods in System Designs)*.
[FL08] Fahrenberg, Larsen. Discount-optimal infinite runs in priced timed automata *(INFINITY'08)*.
[JT08] Judziński, Trivedi. Concavely-priced timed automata *(FORMATS'08)*.
[BBBS11] Bertrand, Bouyer, Brihaye, Stainer. Emptiness and universality problems in timed automata with positive frequency *(ICALP'11)*.
[Sta12] Stainer. Frequencies in forgetful timed automata *(FORMATS'12)*.

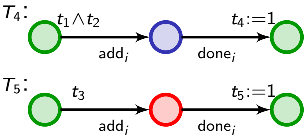
Outline

Modelling the task graph scheduling problem

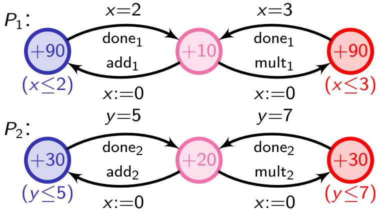
- Processors



- Tasks

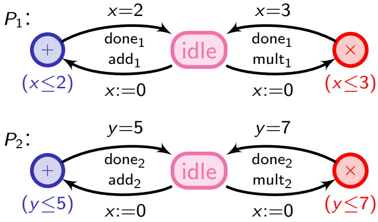


- Modelling energy

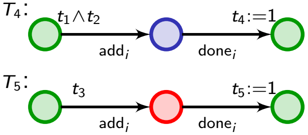


Modelling the task graph scheduling problem

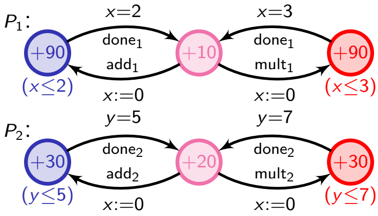
- Processors



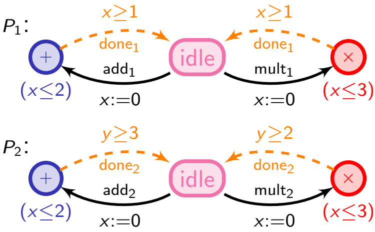
- Tasks



- Modelling energy



- Modelling uncertainty



Modelling the task graph scheduling problem

- Processors

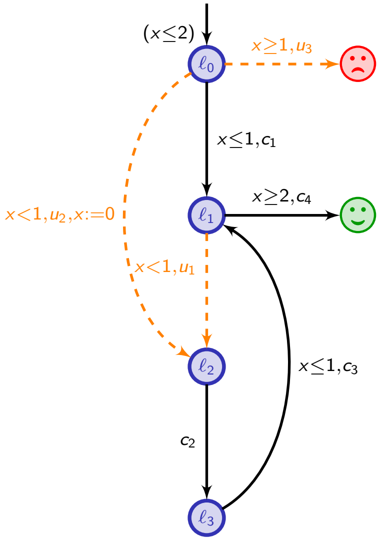
- Tasks

A (good) schedule is a strategy in the product game (with a low cost)

- Modelling energy

- Modelling uncertainty

An example of a timed game



Rule of the game

- Aim: avoid ☹ and reach ☺

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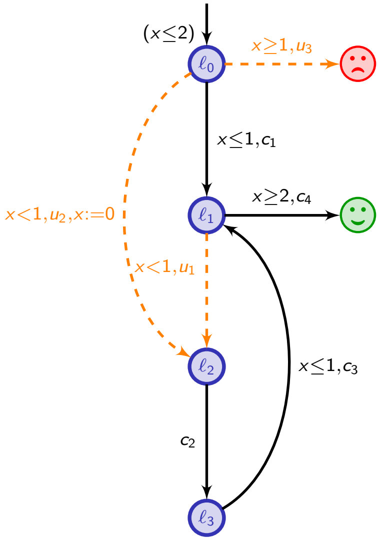

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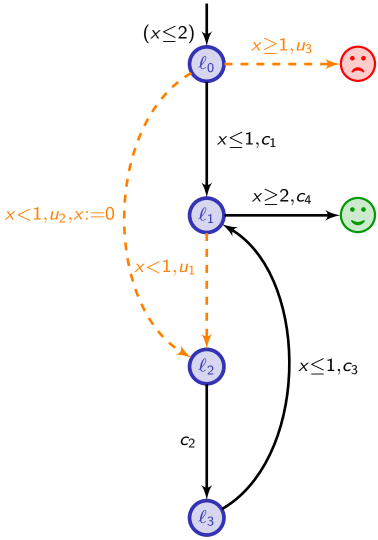

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$$f : \text{history} \mapsto (\text{delay}, \text{cont. transition})$$

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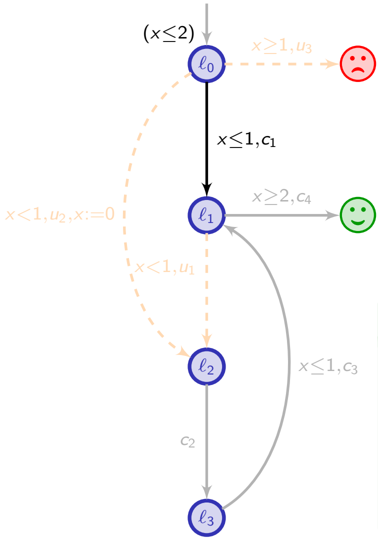

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- from $(\ell_0, 0)$, play $(0.5, c_1)$

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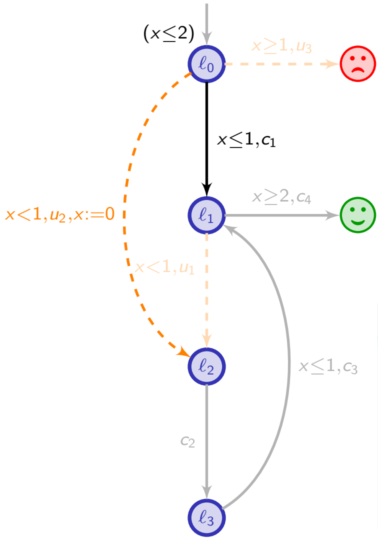

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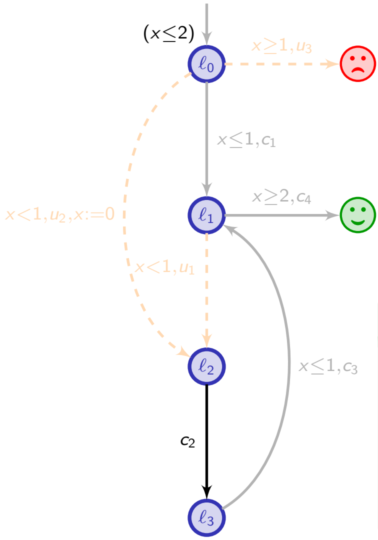

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- from $(\ell_2, \star)$, play $(1-\star, c_2)$

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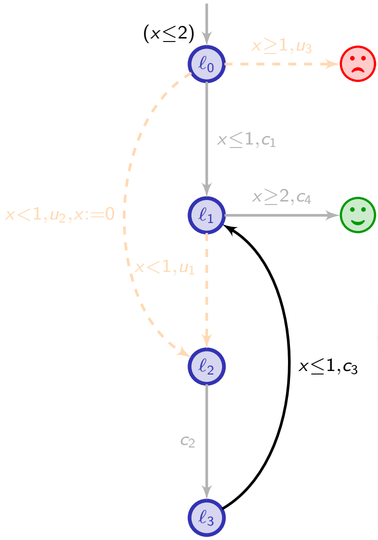

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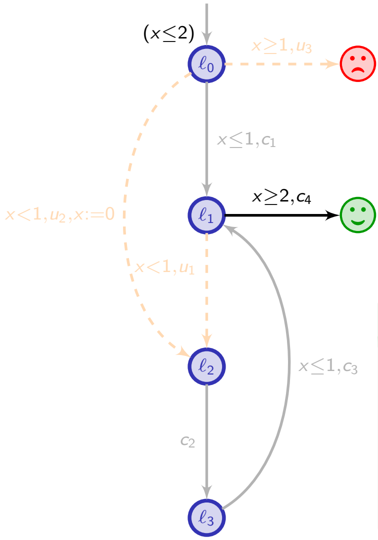

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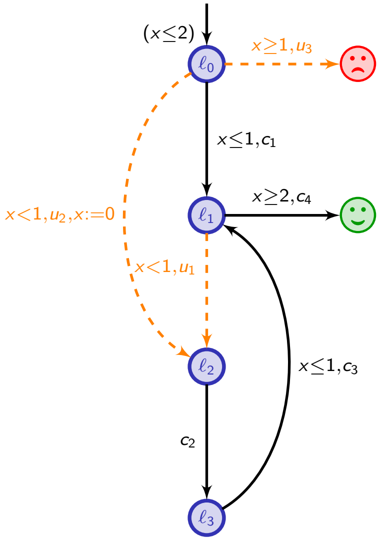
- from $(\ell_1, 1)$, play $(1, c_4)$

An example of a timed game



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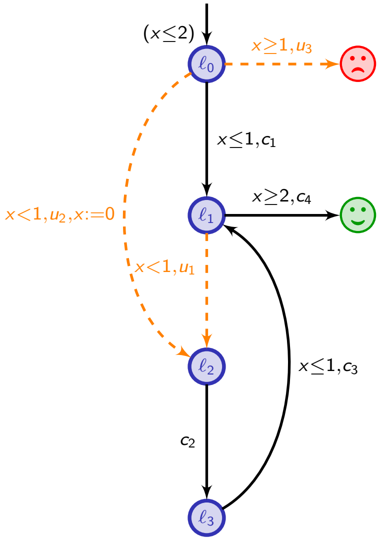
Problems to be considered

An example of a timed game



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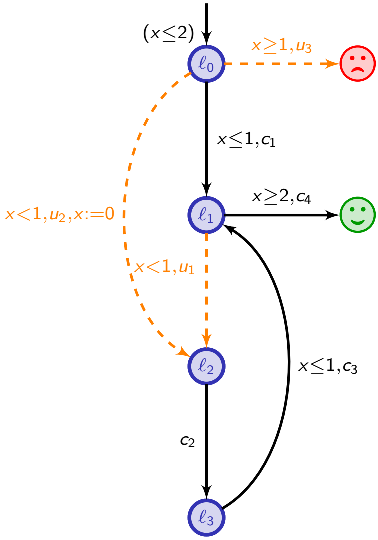

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$$f : \text{history} \mapsto (\text{delay}, \text{cont. transition})$$

Problems to be considered

- Does there exist a winning strategy?
- If yes, compute one (as simple as possible).

Decidability of timed games

Theorem [AMPS98,HK99]

Reachability and safety timed games are decidable and EXPTIME-complete. Furthermore memoryless and "region-based" strategies are sufficient.

[AMPS98] Asarin, Maler, Pnueli, Sifakis. Controller synthesis for timed automata *(SSC'98)*.
[HK99] Henzinger, Kopke. Discrete-time control for rectangular hybrid automata *(Theoretical Computer Science)*.

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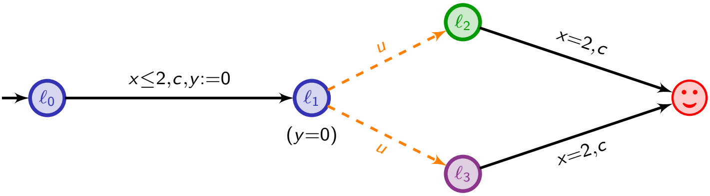
Theorem [AM99,BHPR07,JT07]

Optimal-time reachability timed games are decidable and EXPTIME-complete.

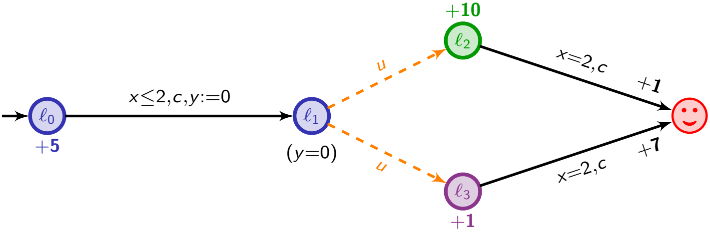
[AM99] Asarin, Maler. As soon as possible: time optimal control for timed automata *(HSCC'99)*.
[BHPR07] Brihaye, Henzinger, Prabhu, Raskin. Minimum-time reachability in timed games *(ICALP'07)*.
[JT07] Jurdziński, Trivedi. Reachability-time games on timed automata *(ICALP'07)*.

Outline

A simple timed game



A simple weighted timed game



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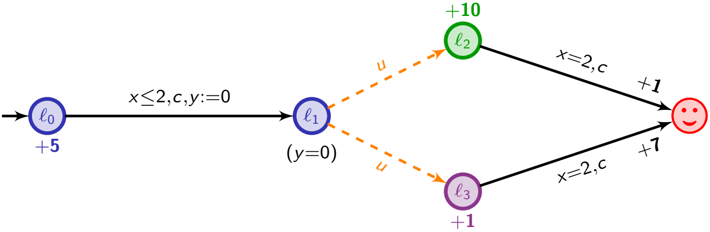

Question: what is the optimal cost we can ensure while reaching 🙂?

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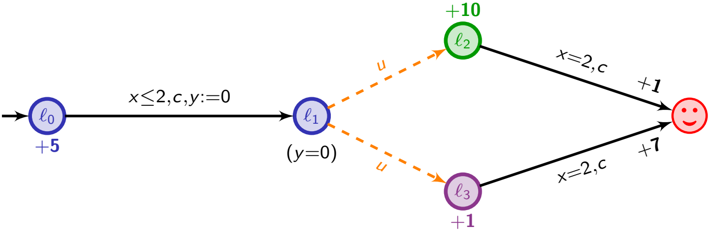

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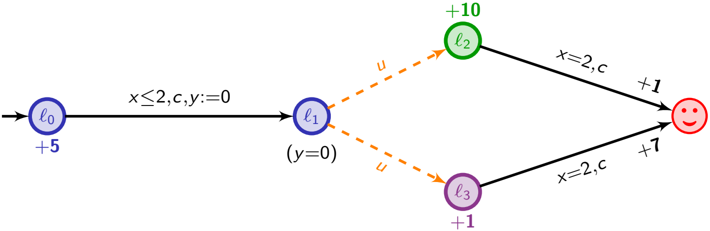
$$5t + 10(2 - t) + 1$$

A simple weighted timed game



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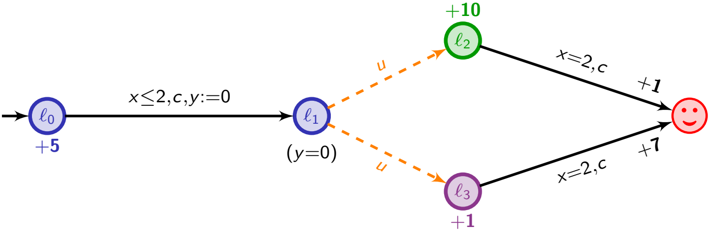
$$5t + 10(2-t) + 1 \ , \ 5t + (2-t) + 7$$

A simple weighted timed game



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$$\max\ (\ 5t + 10(2-t) + 1\ ,\ 5t + (2-t) + 7\ )$$

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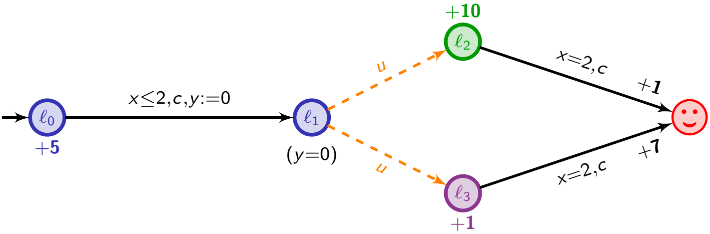
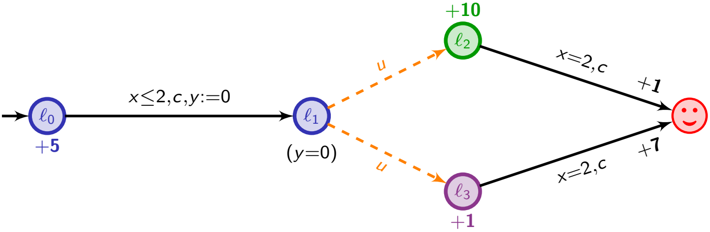


Question: what is the optimal cost we can ensure while reaching ?

$$\inf_{0\leq t\leq 2} \max\left(5t + 10(2-t) + 1 \;,\; 5t + (2-t) + 7 \right) = 14 + \frac{1}{3}$$

A simple weighted timed game



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$$\inf_{0\leq t\leq 2} \max\left(\, 5t+10(2-t)+1 \,,\, 5t+(2-t)+7 \,\right) = 14+\frac{1}{3}$$

$\rightsquigarrow$ *strategy:* wait in ℓ_0, and when $t=\frac{4}{3}$, go to ℓ_1

Optimal reachability in weighted timed games (1)

This topic has been fairly hot these last fifteen years...

[LMM02,ABM04,BCFL04,BBR05,BBM06,BLMR06,Rut11,HIM13,BGK+14]

[LMM02] La Torre, Mukhopadhyay, Murano. Optimal-reachability and control for acyclic weighted timed automata *(TCS@02)*.
[ABM04] Alur, Bernardsky, Madhusudan. Optimal reachability in weighted timed games *(ICALP'04)*.
[BCFL04] Bouyer, Cassez, Fleury, Larsen. Optimal strategies in priced timed game automata *(FSTTCS'04)*.
[BBR05] Brihaye, Bruyère, Raskin. On optimal timed strategies *(FORMATS'05)*.
[BBM06] Bouyer, Brihaye, Markey. Improved undecidability results on weighted timed automata *(Information Processing Letters)*.
[BLMR06] Bouyer, Larsen, Markey, Rasmussen. Almost-optimal strategies in one-clock priced timed automata *(FSTTCS'06)*.
[Rut11] Rutkowski. Two-player reachability-price games on single-clock timed automata *(QAPL'11)*.
[HIM13] Hansen, Ibsen-Jensen, Miltersen. A faster algorithm for solving one-clock priced timed games *(CONCUR'13)*.
[BGK+14] Brihaye, Geeraerts, Krishna, Manasa, Monmege, Trivedi. Adding Negative Prices to Priced Timed Games *(CONCUR'14)*.

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Tree-like weighted timed games can be solved in 2EXPTIME.

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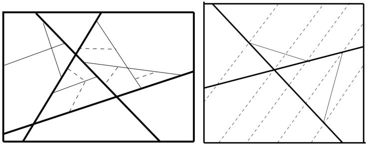
[LMM02]

Tree-like weighted timed games can be solved in 2EXPTIME.

[ABM04,BCFL04]

Depth-k weighted timed games can be solved in EXPTIME. There is a symbolic algorithm to solve weighted timed games **with a strongly non-Zeno cost**.



Optimal reachability in weighted timed games (2)

[BBR05,BBM06]

In weighted timed games, the optimal cost cannot be computed, as soon as games have three clocks or more.

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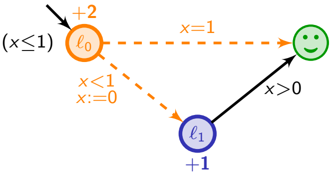
In weighted timed games, the optimal cost cannot be computed, as soon as games have three clocks or more.

[BLMR06,Rut11,HIM13,BGK+14]

Turn-based optimal timed games are decidable in EXPTIME (resp. PTIME) when automata have a single clock (resp. with two rates). They are PTIME-hard.

What is easier with a single clock?

- Memoryless strategies can be non-optimal...



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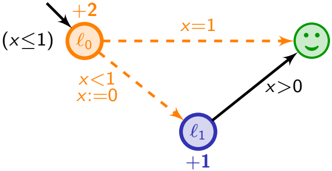

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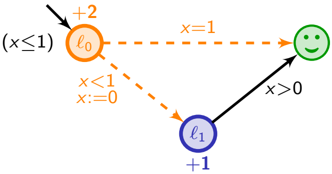

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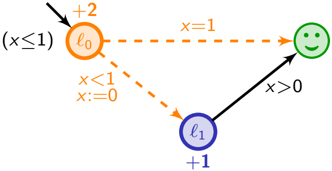

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- By unfolding and removing one by one the locations,we can synthesize memoryless almost-optimal winning strategies.

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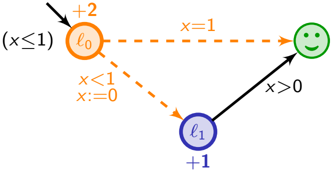

  ... but memoryless almost-optimal strategies will be sufficient.

- Key: resetting the clock somehow resets the history...
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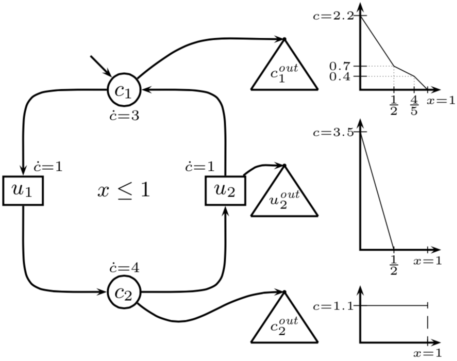
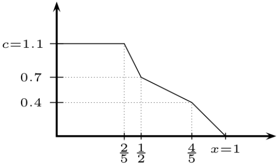
- Rather involved proofs of correctness





$$\sigma(c_2, x) = \begin{cases} c_2^{out} & \text{if } 0 \leq x < 2/5 \\ c_2 & \text{if } 2/5 \leq x < 1/2 \\ u_2 & \text{if } 1/2 \leq x \leq 1 \end{cases}$$

Computing the optimal cost: why is that hard?

Given two clocks x and y, we can check whether $y = 2x$.

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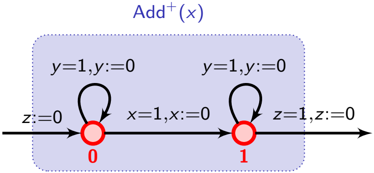
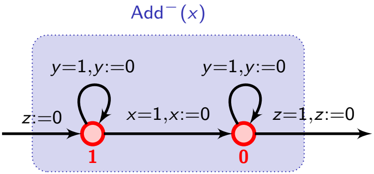


The cost is increased by x_0



The cost is increased by $1-x_0$

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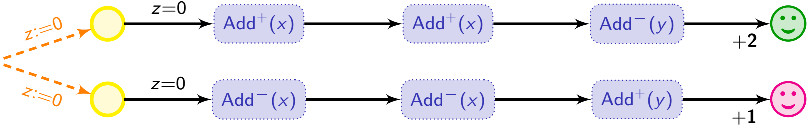

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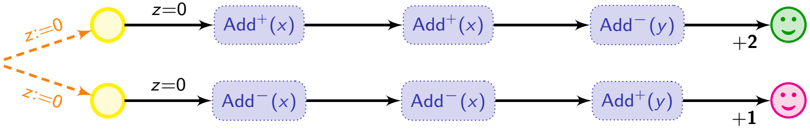

- In ☺, cost $= 2x_0 + (1 - y_0) + 2$

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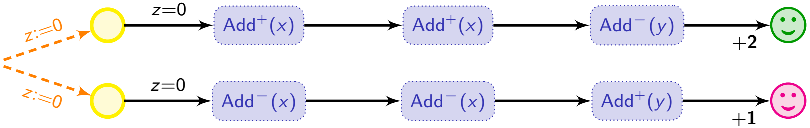

- In , cost $= 2x_0 + (1 - y_0) + 2$
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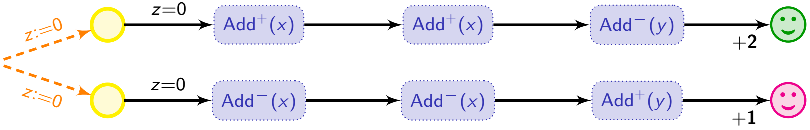

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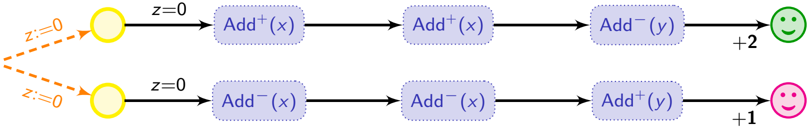

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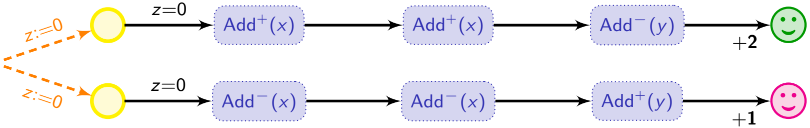

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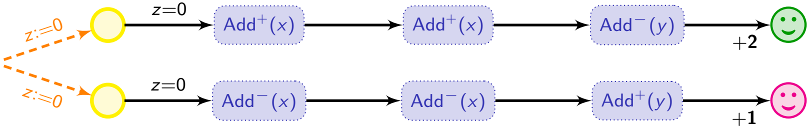

- In 🙂 (green), cost $= 2x_0 + (1 - y_0) + 2$
  In 🙂 (pink), cost $= 2(1 - x_0) + y_0 + 1$
- if $y_0 < 2x_0$, player 2 chooses the first branch: cost > 3
  if $y_0 > 2x_0$, player 2 chooses the second branch: cost > 3
  if $y_0 = 2x_0$, in both branches, cost $= 3$
- Player 1 has a winning strategy with cost ≤ 3 iff $y_0 = 2x_0$

Computing the optimal cost: why is that hard?

Player 1 will simulate a two-counter machine:

- each instruction is encoded as a module;
- the counter values c_1 and c_2 are encoded by two clocks:

$$x = \frac{1}{2^{c_1}} \qquad \text{and} \qquad y = \frac{1}{3^{c_2}}$$

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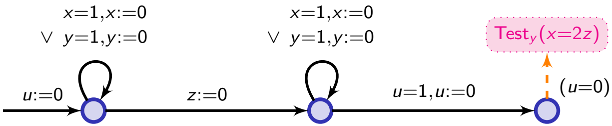
Globally, $(x \leq 1, y \leq 1, u \leq 1)$



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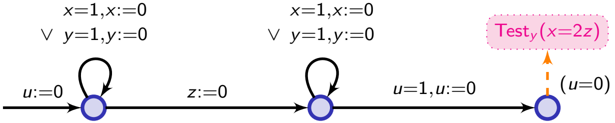

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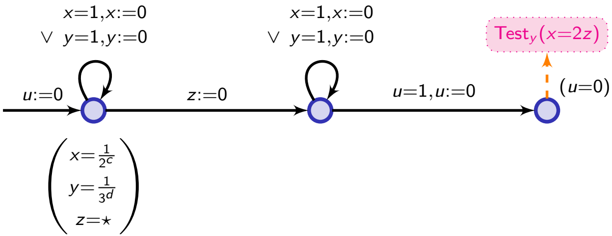

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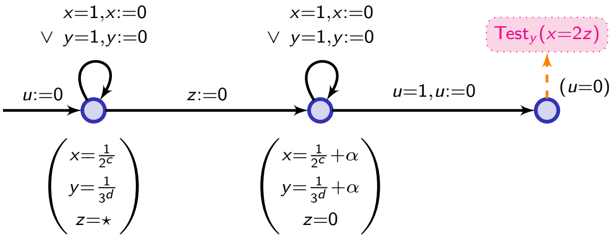

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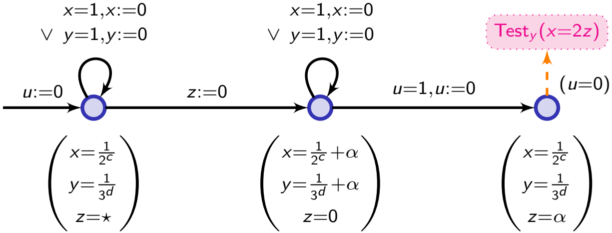

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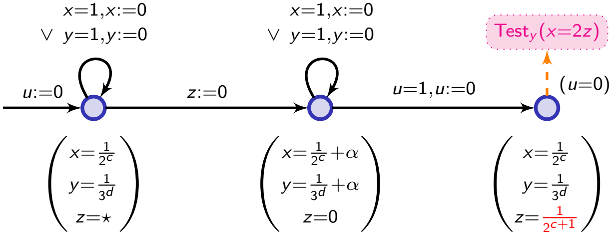

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Note: These problems are distinct...

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2. Propose an approximation algorithm for a large class of weighted timed games (that comprises the class of games used for proving the above undecidability)
   - Almost-optimality in practice should be sufficient
   - Even when we know how to compute the value, we are only able to synthesize almost-optimal strategies...

[BMM14] Bouyer, Markey, Matteplackel. Averaging in LTL *(CONCUR'14)*.

Outline

1. Introduction
2. Overview of “old” results
    - Weighted timed automata
    - Timed games
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3. Some recent developments
    - Undecidability of the value problem
    - Approximation of the optimal cost
4. Conclusion

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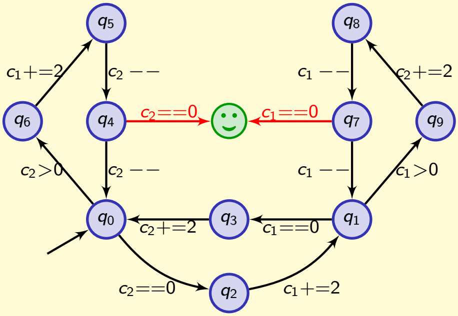

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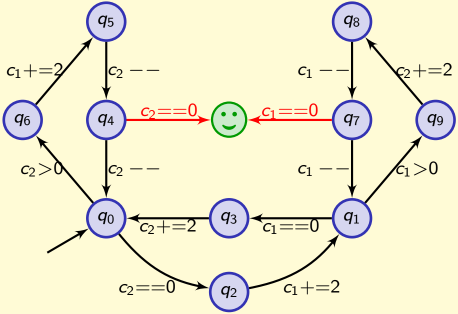

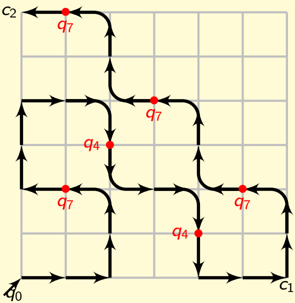

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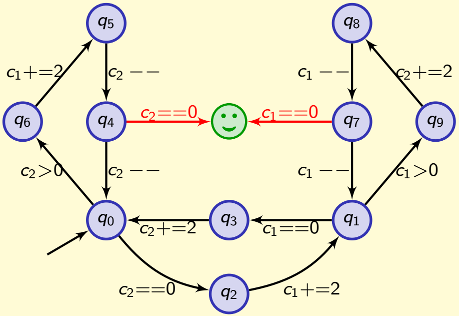

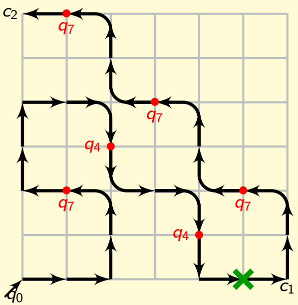

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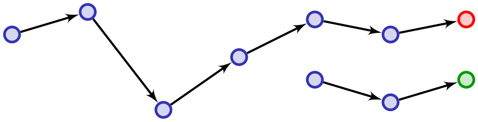

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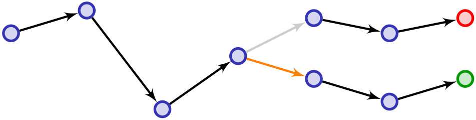

big impact on the cost!

[BMM14] Bouyer, Markey, Matteplackel. Averaging in LTL *(CONCUR'14)*.

The diagonal argument

$$\begin{array}{ll} & B \text{ det. Turing machine can either accept, reject, or not halt} \\ \rightarrow & \mathcal{M}(B) \text{ two-counter machine which simulates } B \text{ on } B \end{array}$$

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Therefore, $\mathcal{H}$ is not computable.

Theorem [BJM15]

The value problem is undecidable in weighted timed games (with four clocks or more).

- Remark on the reduction:
  - Cost 0 within the core of the game
  - The rest of the game is acyclic

[BJM15] Bouyer, Jaziri, Markey. On the value problem in weighted timed games.

Outline

Optimal cost is computable...

... when cost is strongly non-zeno. [AM04,BCFL04]

That is, there exists $\kappa > 0$ such that for every region cycle C, for every real run ϱ read on C,

$$\mathsf{cost}(\varrho) \geq \kappa$$

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Optimal cost is not computable... but is approximable!

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Approximation of the optimal cost

Theorem

Let $\mathcal{G}$ be a weighted timed game, in which the cost is almost-strongly non-zeno. For every $\epsilon > 0$, one can compute:

- two values v_ϵ^- and v_ϵ^+ such that

$$|v_\epsilon^+ - v_\epsilon^-| < \epsilon \quad \text{and} \quad v_\epsilon^- \leq \mathsf{optcost}_{\mathcal{G}} \leq v_\epsilon^+$$

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$\rightsquigarrow$ This is not possible here
There might be runs with prefixes of arbitrary length and cost 0 (e.g. the game of the undecidability proof)

Idea for approximation

Idea

Only partially unfold the game:

- Keep components with cost 0 untouched – we call it the kernel
- Unfold the rest of the game

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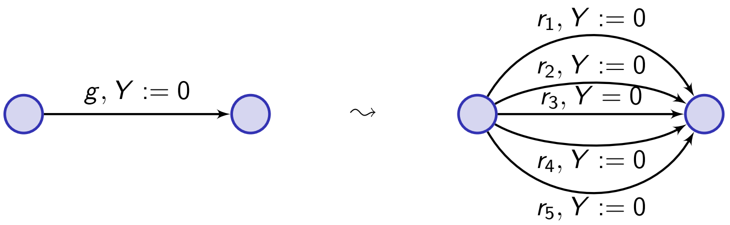
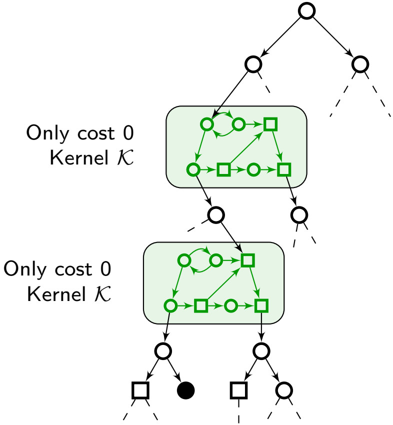
First: split the game along regions!



Semi-unfolding



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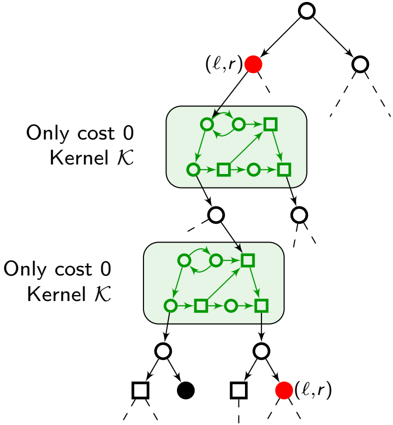

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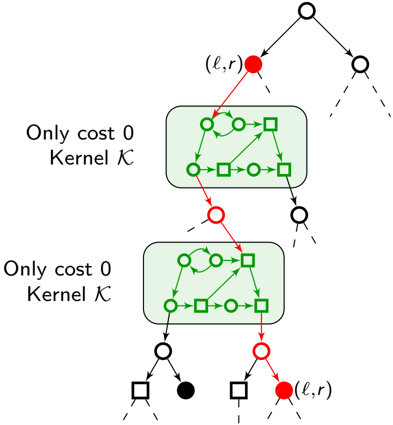

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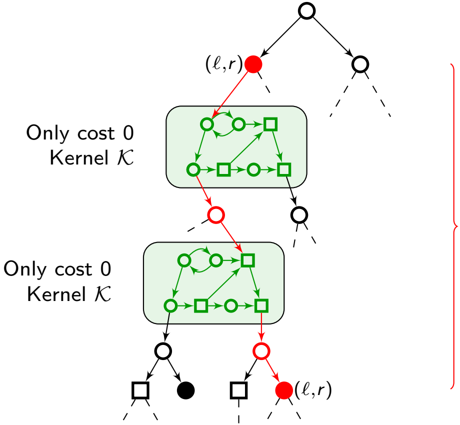

Hypothesis:
cost > 0 implies cost $\geq \kappa$

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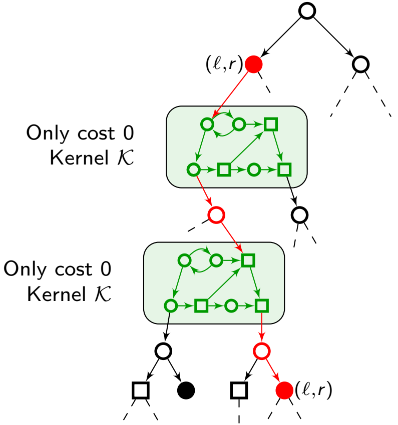

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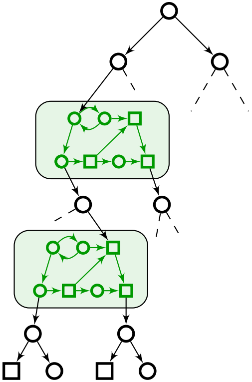
Conclusion: we can stop unfolding the game after N steps
(e.g. $N = (M+2) \cdot |\mathcal{R}(\mathcal{A})|$, where M is a pre-computed bound on $\mathrm{optcost}_{\mathcal{G}}$)

Approximation scheme



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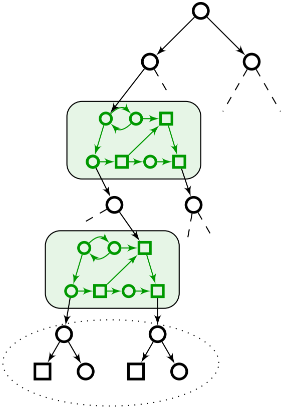

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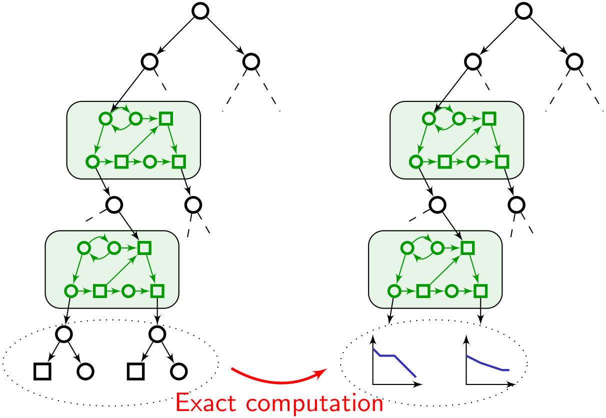

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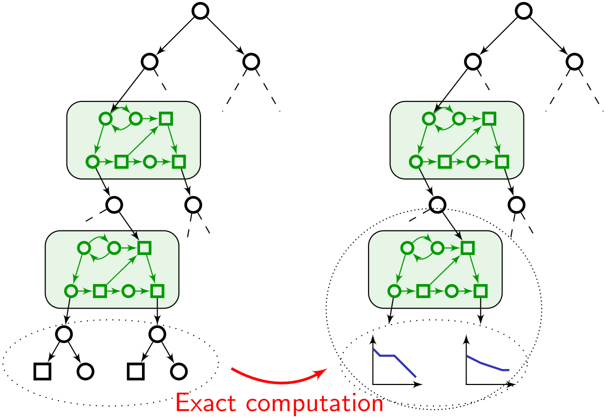

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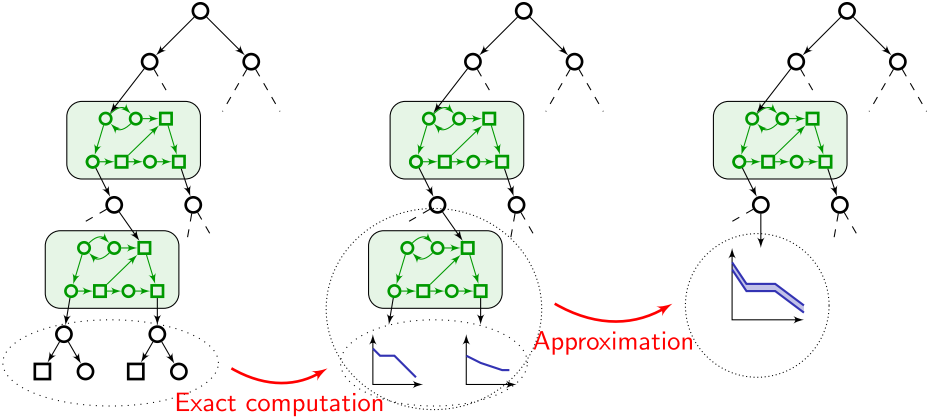

First step: Tree-like parts

$\rightsquigarrow$ Goes back to [LMM02]

[LMM02] La Torre, Mukhopadhyay, Murano. Optimal-reachability and control for acyclic weighted timed automata *(TCS@02)*.

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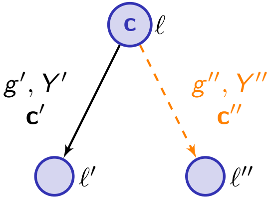

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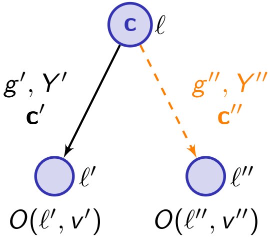

$O(\ell, v) =$

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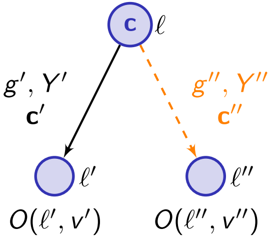

$$O(\ell, v) = \inf_{t' \mid v+t' \models g'}$$

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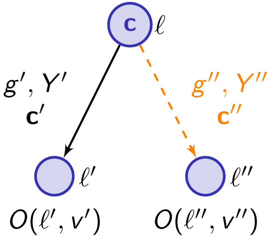

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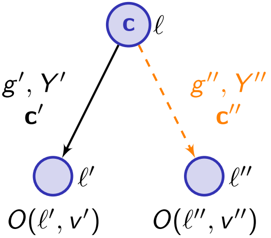

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$v' = [Y' \leftarrow 0](v + t')$

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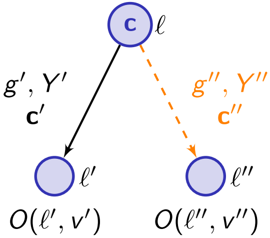

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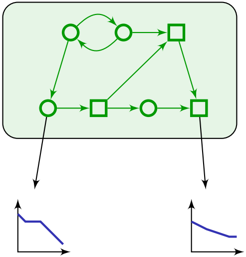

Output cost functions f

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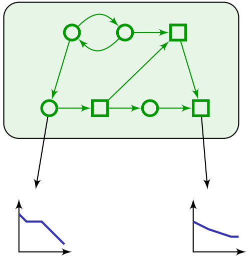
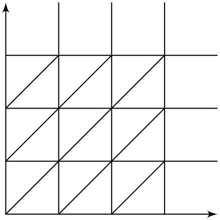


Output cost functions f

1. Refine the regions such that f differs of at most ϵ within a small region



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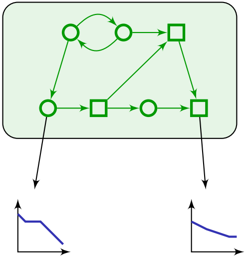

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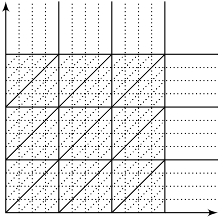
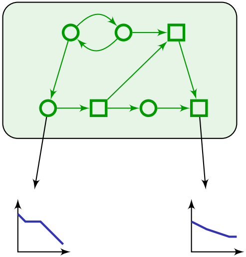


Second step: Kernels



Output cost functions f

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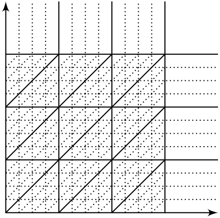

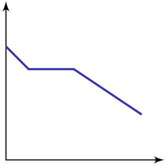

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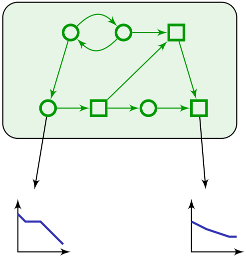

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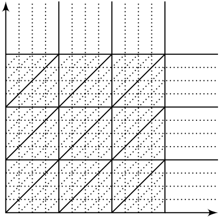
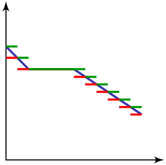
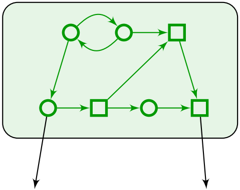


2. Under- and over-approximate by piecewise constant functions f_ϵ^- and f_ϵ^+



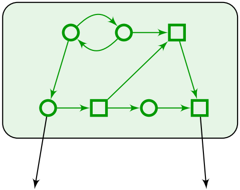
Second step: Kernels

3. Refine/split the kernel along the new small regions and fix f_ϵ^- or f_ϵ^+, write f_ϵ



f_ϵ: constant f_ϵ: constant

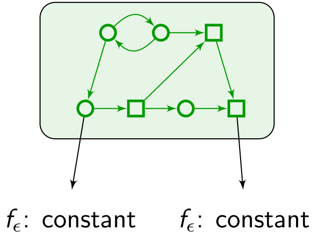
Second step: Kernels



f_ϵ: constant f_ϵ: constant

3. Refine/split the kernel along the new small regions and fix f_ϵ^- or f_ϵ^+, write f_ϵ
4. Since cost is 0 everywhere, the resulting game is nothing more than a reachability timed game with an order on target (output) edges (given by f_ϵ)

Second step: Kernels



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4. Since cost is 0 everywhere, the resulting game is nothing more than a reachability timed game with an order on target (output) edges (given by f_ϵ)
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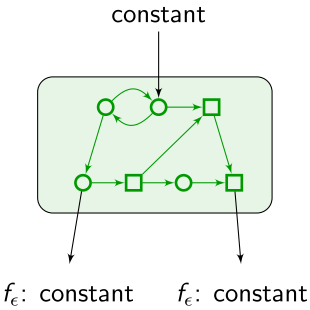

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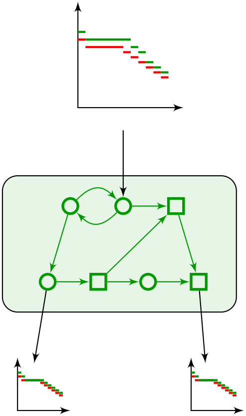

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$\rightsquigarrow$ We have computed ϵ-approximations of the optimal cost, which are constant within small regions. Corresponding strategies can be inferred

Outline

1. Introduction
2. Overview of “old” results
   - Weighted timed automata
   - Timed games
   - Weighted timed games
3. Some recent developments
   - Undecidability of the value problem
   - Approximation of the optimal cost
4. Conclusion

Conclusion

Summary of the talk

- Quick overview of results concerning the optimal reachability problem in weighted timed games
- New insight into the value problem for this model:
  - Undecidability of this problem
  - Approximability of the optimal cost (under some conditions)

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Future work

- Improve the approximation scheme ($2\mathrm{EXP}(|\mathcal{G}|) \cdot \left(1/\epsilon\right)^{|X|}$), and implement it
- Extend to the whole class of weighted timed games, or understand why it is not possible
- Assume stochastic uncertainty