On the optimal reachability problem in weighted timed automata and games

Patricia Bouyer-Decitre

LSV, CNRS & ENS Cachan, France

Time-dependent systems

- We are interested in timed systems

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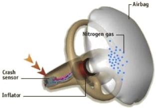

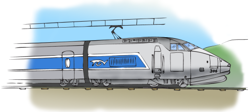

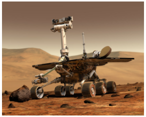

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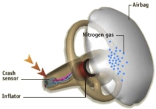
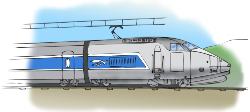
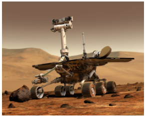






- ... and in their analysis and control

An example: The task graph scheduling problem

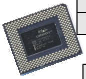
Compute $D \times (C \times (A + B)) + (A + B) + (C \times D)$ using two processors:

P_1 (fast):

time	
$+$	2 picoseconds
$\times$	3 picoseconds

energy	
idle	10 Watt
in use	90 Watts

P_2 (slow):



time	
$+$	5 picoseconds
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energy	
idle	20 Watts
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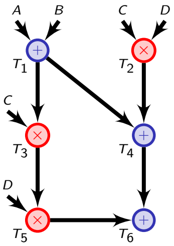

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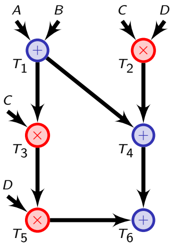

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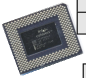

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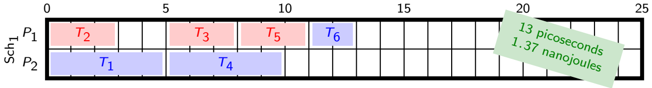

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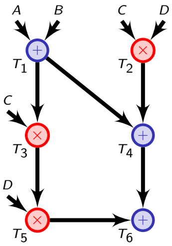

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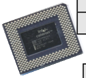

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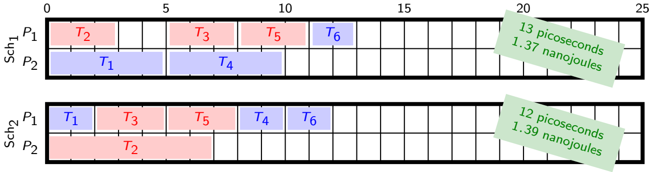

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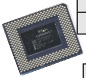

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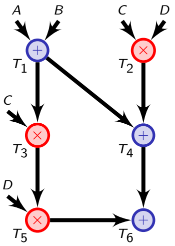

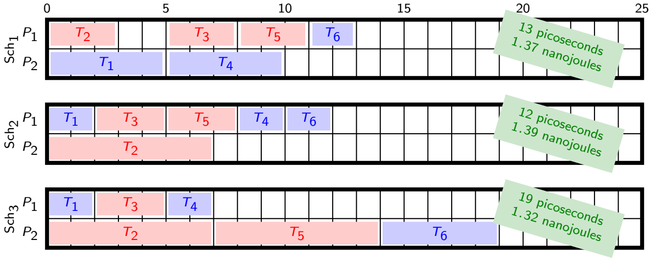

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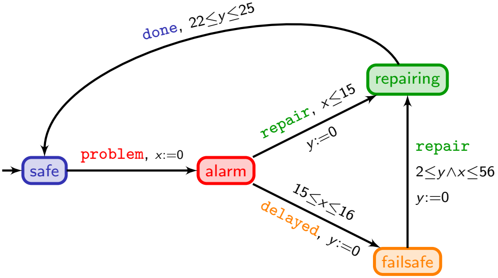
Outline

The model of timed automata



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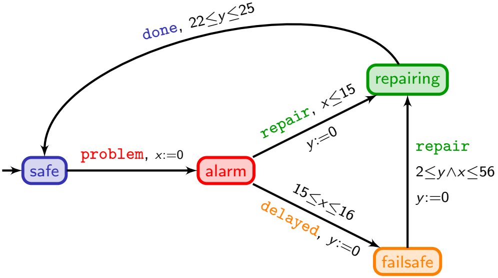

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	$\cdots$
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\texttt{done}}$	safe
$\cdots$	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

Modelling the task graph scheduling problem

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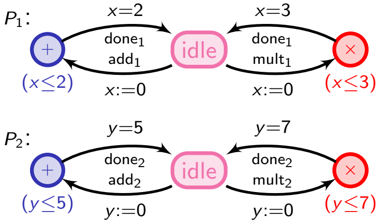
- Processors



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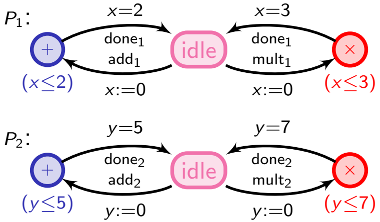
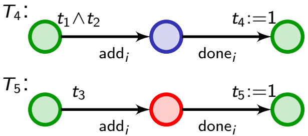


- Tasks



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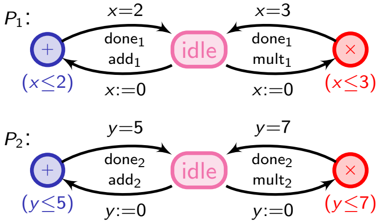

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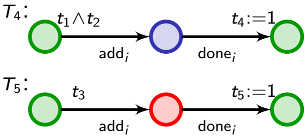

$\rightsquigarrow$ build the synchronized product of all these automata

$$(P_1 \parallel P_2) \parallel_s (T_1 \parallel T_2 \parallel \cdots \parallel T_6)$$

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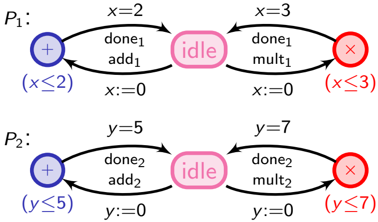

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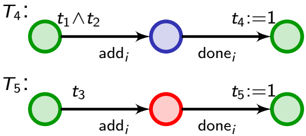

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A schedule: a path in the global system which reaches $t_1 \wedge \cdots \wedge t_6$

Modelling the task graph scheduling problem

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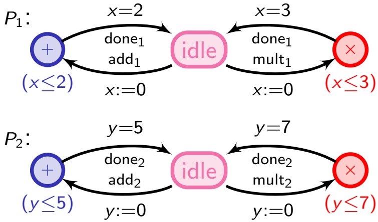

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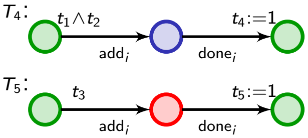

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Questions one can ask

- Can the computation be made in no more than 10 time units?
- Is there a scheduling along which no processor is ever idle?
- ...

Discrete-time semantics

...because computers are digital!

[Alur91] Alur. Techniques for automatic verification of real-time systems. *PhD thesis*, 1991.

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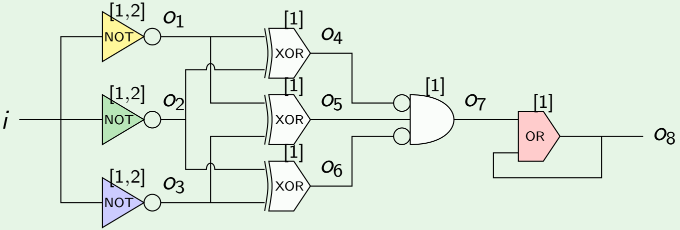

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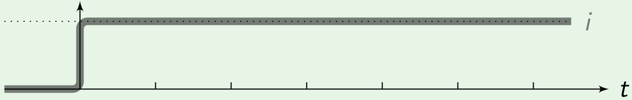

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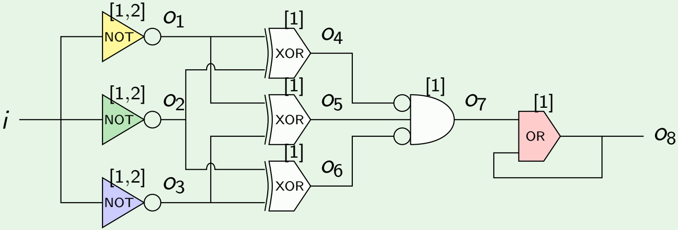

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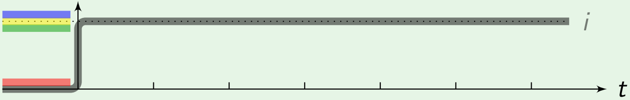

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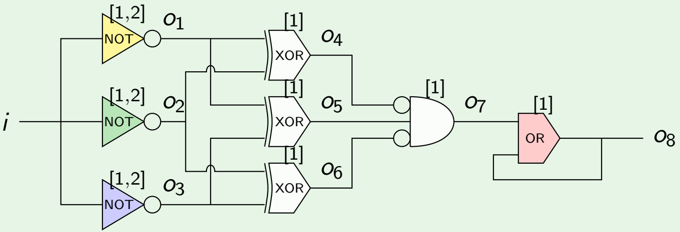

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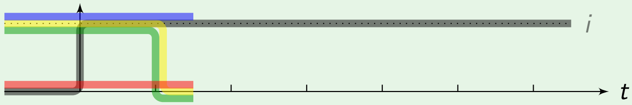

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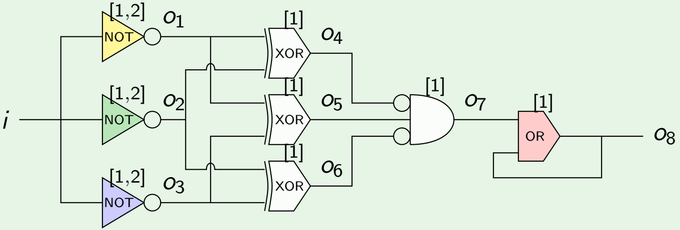

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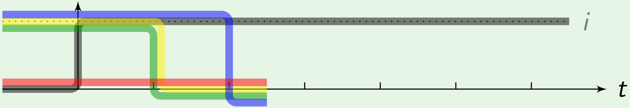

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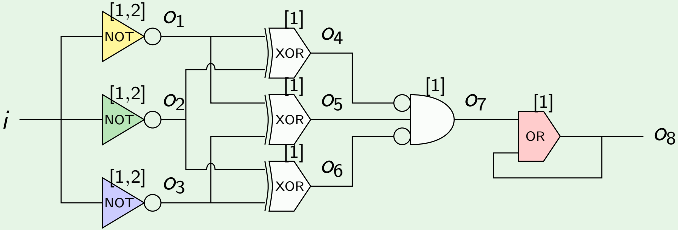

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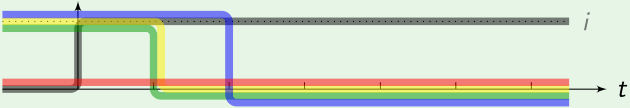

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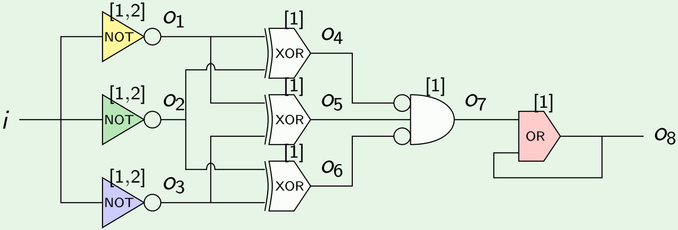
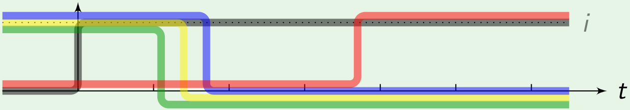


- under continuous-time, the output can be 1:



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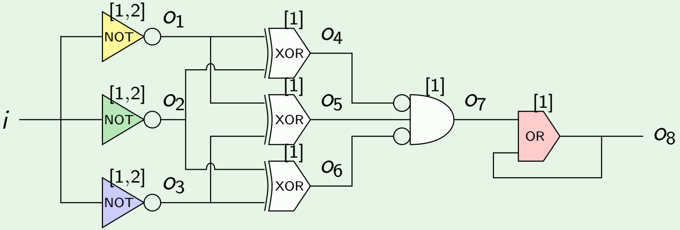

Finding the correct granularity (if one exists) is hard!

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...real-time models for real-time systems!

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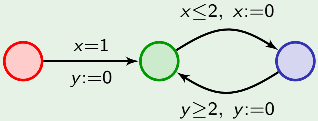

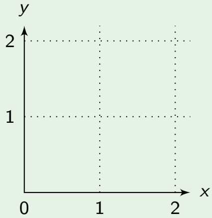

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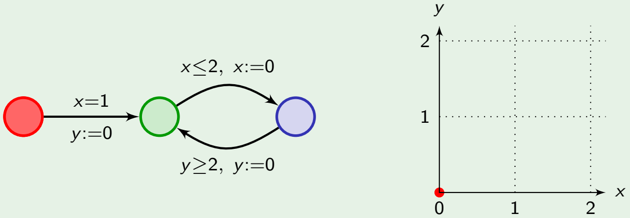

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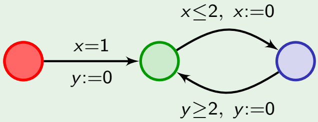

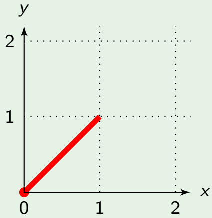

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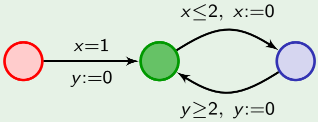

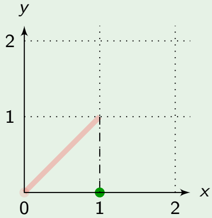

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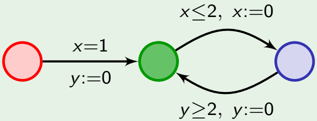

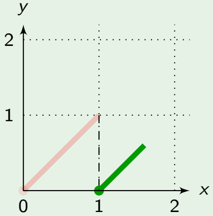

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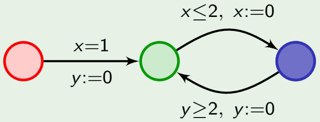

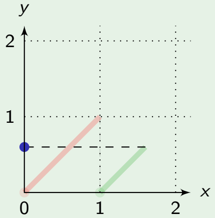

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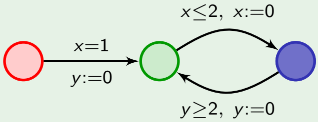

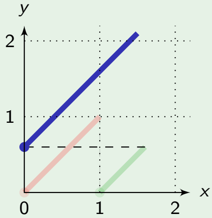

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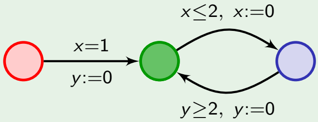

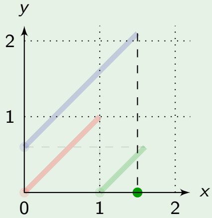

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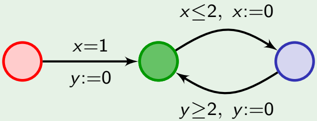

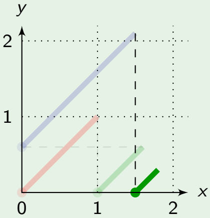

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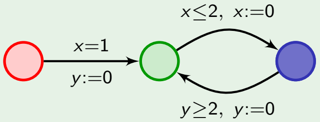

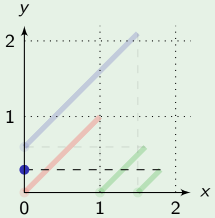

Continuous-time semantics

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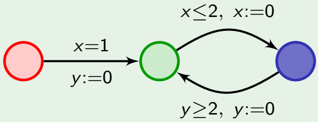

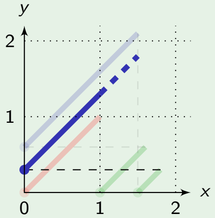

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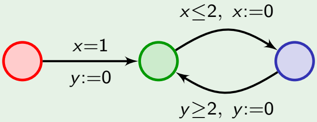
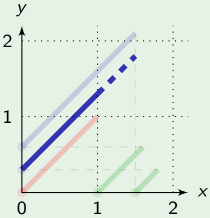
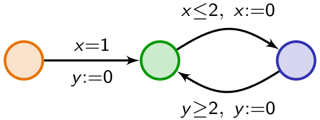
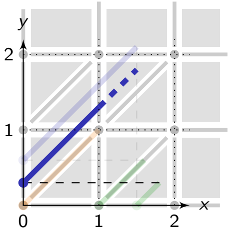




We will focus on the continuous-time semantics

Analyzing timed automata





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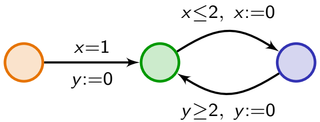

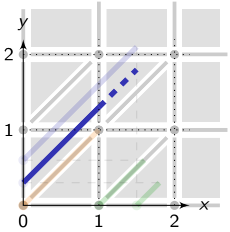

Theorem [AD94]

Reachability in timed automata is decidable (as well as many other important properties). It is PSPACE-complete.

- Technical tool: region abstraction

[AD94] Alur, Dill. A Theory of Timed Automata *(Theoretical Computer Science)*.

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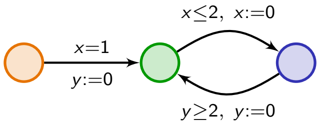

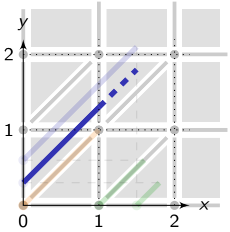

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- Technical tool: region abstraction
- Efficient symbolic technics based on zones, implemented in tools

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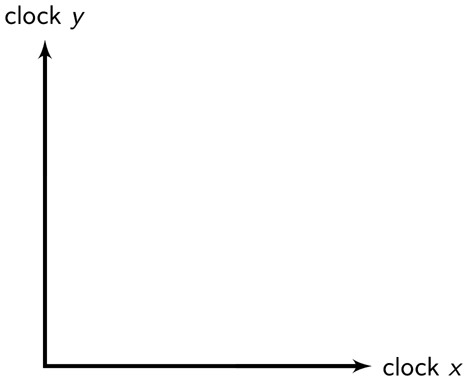
Technical tool: Region abstraction



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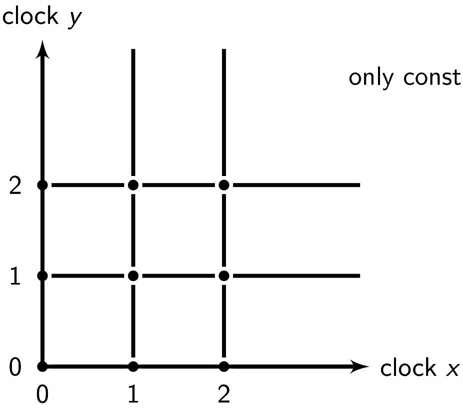
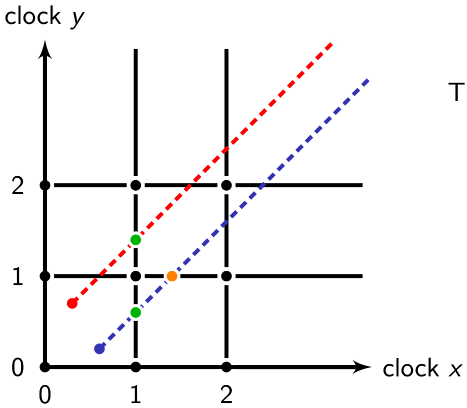


only constraints: $x \sim c$ with $c \in \{0, 1, 2\}$
$y \sim c$ with $c \in \{0, 1, 2\}$

- "compatibility" between regions and constraints

Technical tool: Region abstraction



The path 

- can be fired from
- cannot be fired from

- "compatibility" between regions and constraints
- "compatibility" between regions and time elapsing

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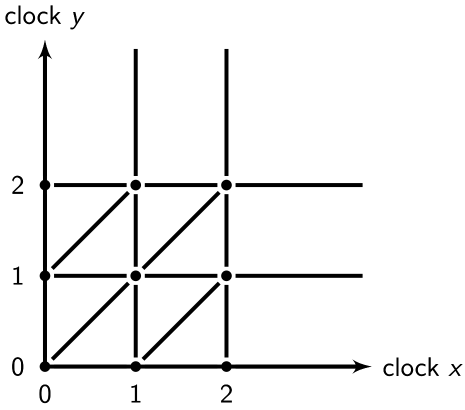

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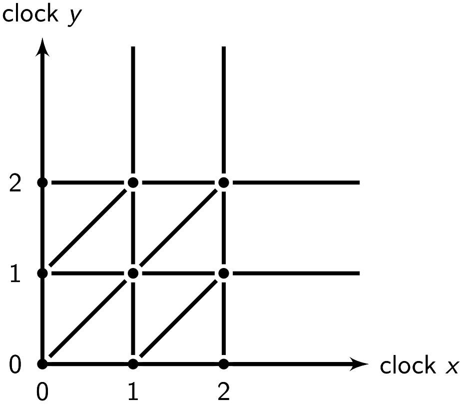

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$\rightsquigarrow$ This is a finite time-abstract bisimulation!

Time-abstract bisimulation

This is a relation between • and • such that:

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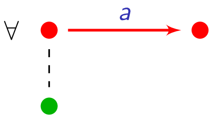

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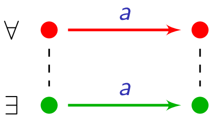

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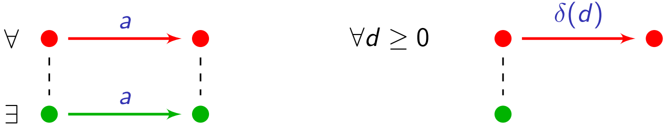

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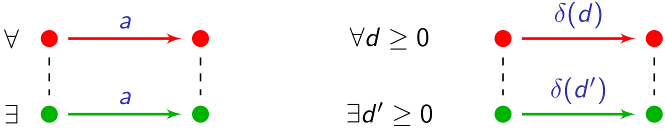

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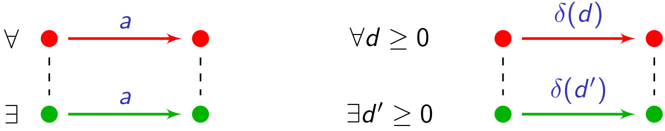

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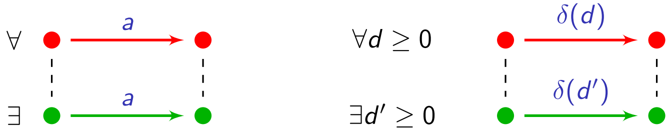

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Consequence

$$\forall \quad (\ell_1, v_1) \xrightarrow{d_1, a_1} (\ell_2, v_2) \xrightarrow{d_2, a_2} (\ell_3, v_3) \xrightarrow{d_3, a_3} \quad \cdots$$

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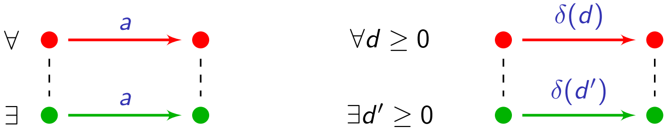

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$$\begin{tikzcd}
\forall & (\ell_1, v_1) \arrow[r, "d_1, a_1"] \arrow[d] & (\ell_2, v_2) \arrow[r, "d_2, a_2"] \arrow[d] & (\ell_3, v_3) \arrow[r, "d_3, a_3"] \arrow[d] & \cdots \\
& (\ell_1, R_1) \arrow[r, "a_1"] & (\ell_2, R_2) \arrow[r, "a_2"] & (\ell_3, R_3) \arrow[r, "a_3"] & \cdots & \text{with } v_i \in R_i
\end{tikzcd}$$

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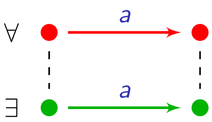

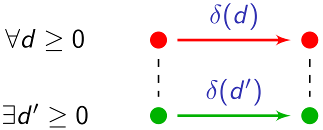

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$\forall v_1' \in R_1$

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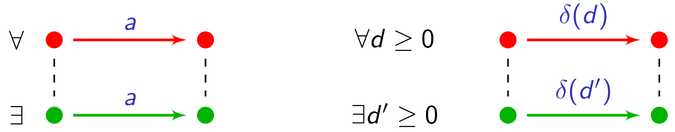

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\forall v'_1 \in R_1 \ \exists & (\ell_1, v'_1) \arrow[r, "d'_1{,}a_1"] & (\ell_2, v'_2) \arrow[r, "d'_2{,}a_2"] & (\ell_3, v'_3) \arrow[r, "d'_3{,}a_3"] & \cdots & \text{with } v'_i \in R_i
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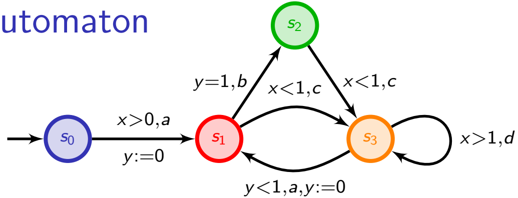
The region automaton



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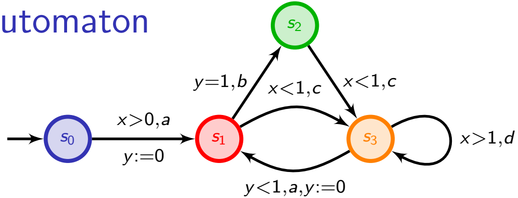

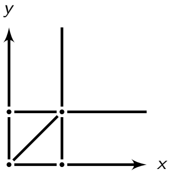

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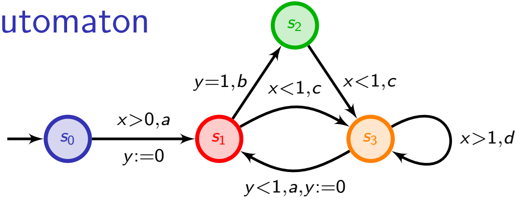

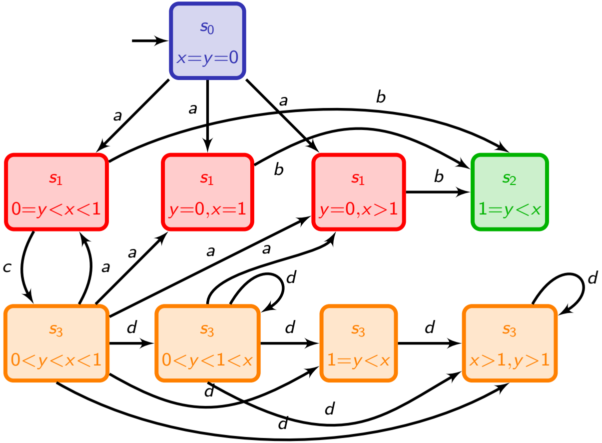

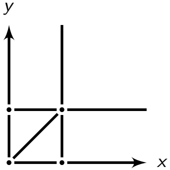

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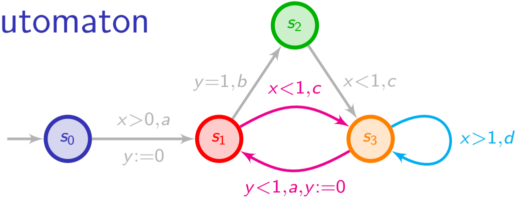
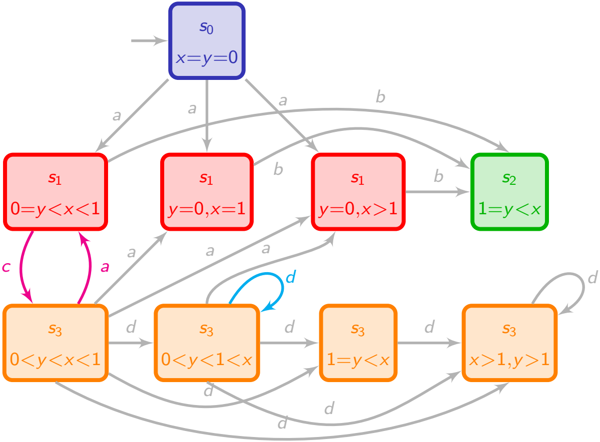
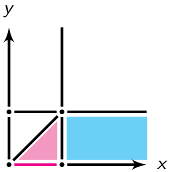
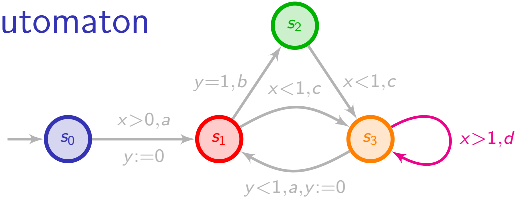


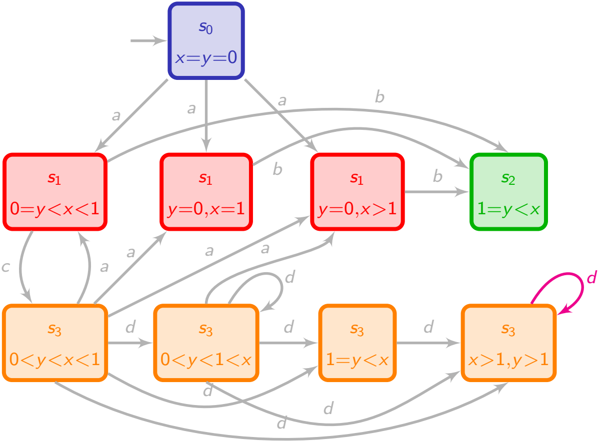


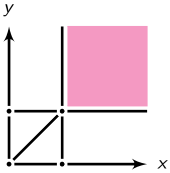


Zeno cycles

The region automaton







Cycles with non-Zeno behaviours

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- the region automaton is never computed
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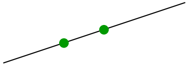

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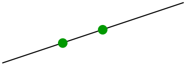

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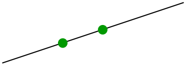

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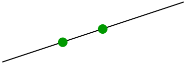

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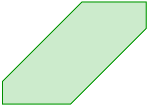

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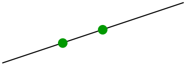

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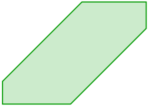

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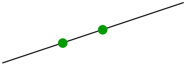

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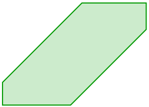
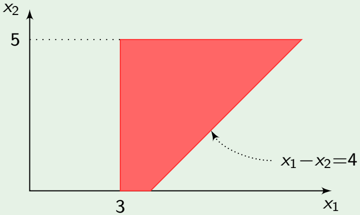


- Need of abstractions, heuristics, etc...

Zones: A symbolic representation for timed systems

Example of a zone and its DBM representation

$$Z \;=\; (x_1 \geq 3) \;\wedge\; (x_2 \leq 5) \;\wedge\; (x_1 - x_2 \leq 4)$$



$$\begin{array}{c c} & \begin{array}{ccc} x_0 & x_1 & x_2 \end{array} \\ \begin{array}{c} x_0 \\ x_1 \\ x_2 \end{array} & \begin{pmatrix} \infty & -3 & \infty \\ \infty & \infty & 4 \\ 5 & \infty & \infty \end{pmatrix} \end{array}$$

DBM: Difference Bound Matrice [BM83,Dill89]

[BM83] Berthomieu, Menasche. An enumerative approach for analyzing time Petri nets *World Comupter Congress*.
[Dill89] Dill. Timing assumptions and verification of finite-state concurrent systems *(Automatic Verification Methods for Finite State Systems)*.

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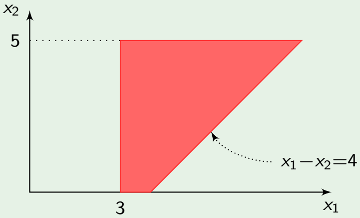

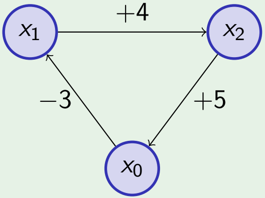

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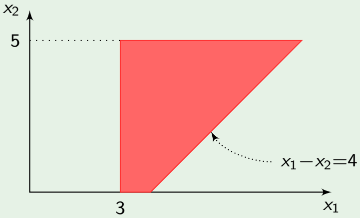

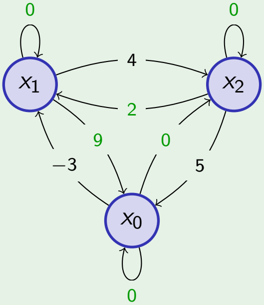

DBM: Difference Bound Matrice [BM83,Dill89]

[BM83] Berthomieu, Menasche. An enumerative approach for analyzing time Petri nets *World Comupter Congress*.
[Dill89] Dill. Timing assumptions and verification of finite-state concurrent systems *(Automatic Verification Methods for Finite State Systems)*.

Zones: A symbolic representation for timed systems

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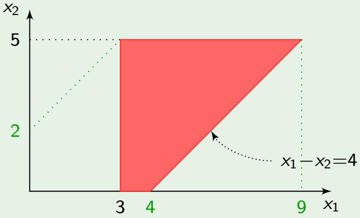

$$\begin{array}{c|ccc} & x_0 & x_1 & x_2 \\ \hline x_0 & 0 & -3 & 0 \\ x_1 & 9 & 0 & 4 \\ x_2 & 5 & 2 & 0 \end{array}$$

"normal form"

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Backward computation

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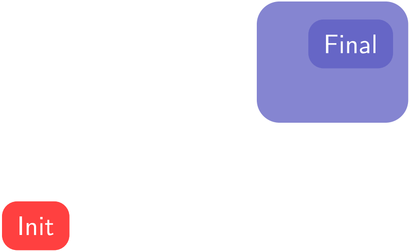

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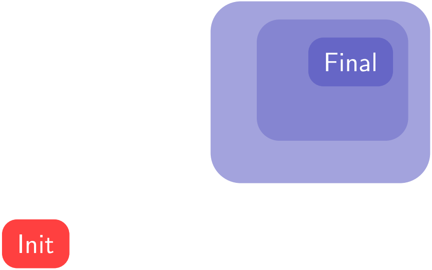

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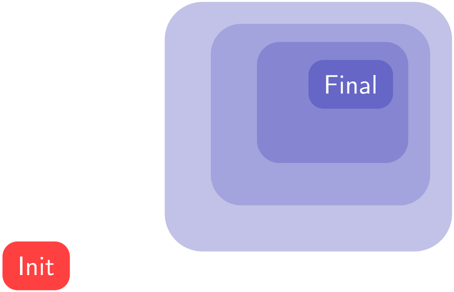
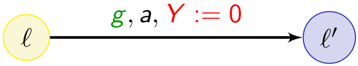


Backward computation

Note on the backward analysis of timed automata



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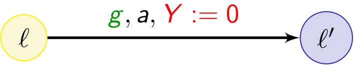

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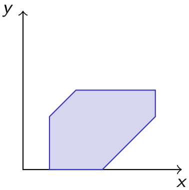

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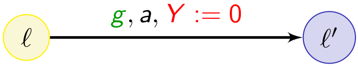

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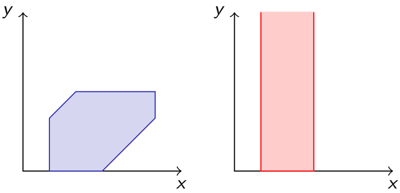

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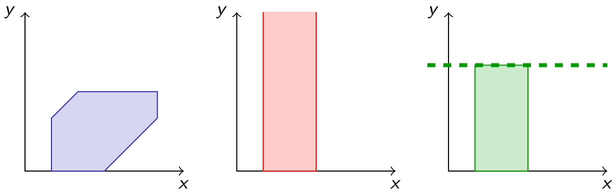

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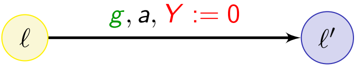

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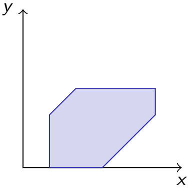

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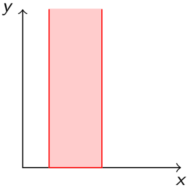

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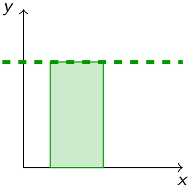

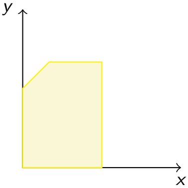

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Note on the backward analysis (cont.)

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Because of the bisimulation property of the region abstraction:

"Every set of valuations which is computed along the backward computation is a finite union of regions"

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Let R be a region. Assume:

- $v \in \overleftarrow{R}$ (for ex. $v + t \in R$)
- $v' \equiv_{\text{reg.}} v$

There exists t' s.t. $v' + t' \equiv_{\text{reg.}} v + t$, which implies that $v' + t' \in R$ and thus $v' \in \overleftarrow{R}$.

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However the backward computation is not appropriate to manipulate other variables (think for instance of assignment $i := j.k + l$)

Forward computation

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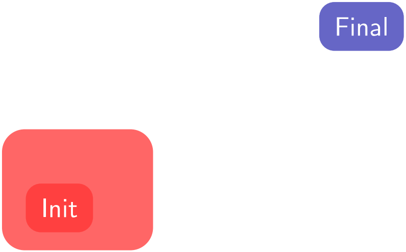

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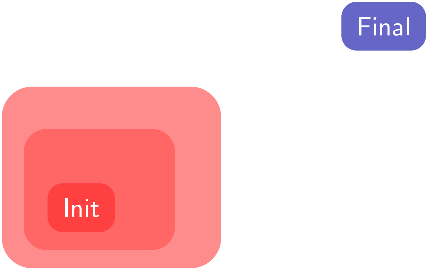

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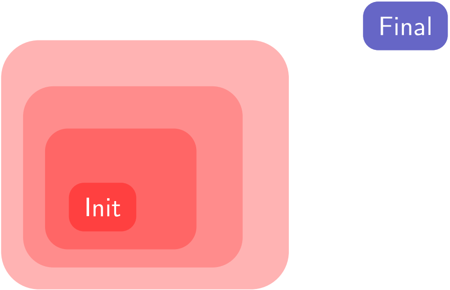

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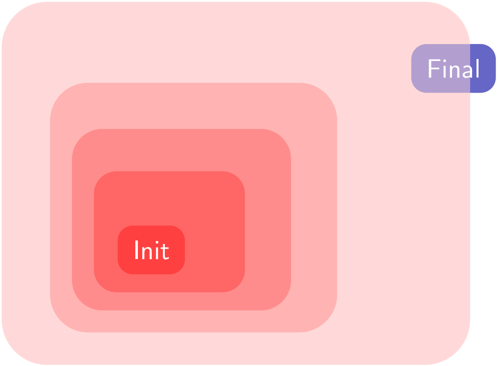
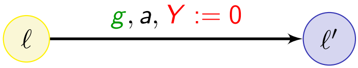


Forward analysis of timed automata



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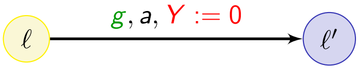

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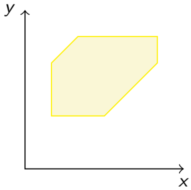

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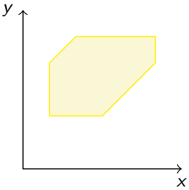

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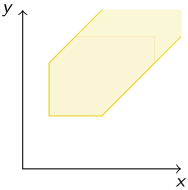

$\overrightarrow{Z}$

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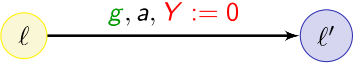

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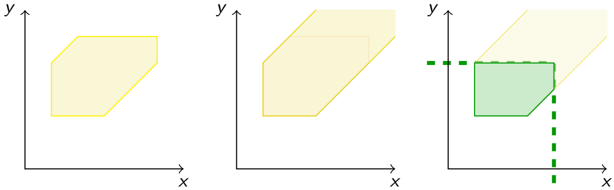

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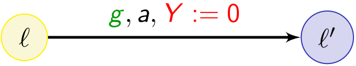

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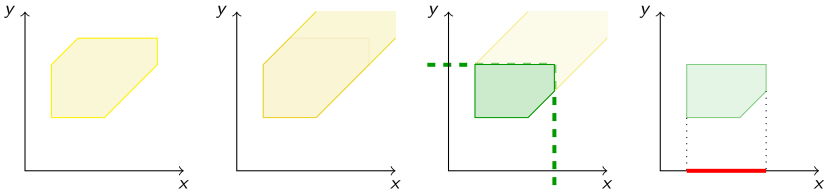

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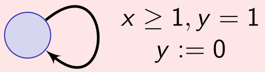

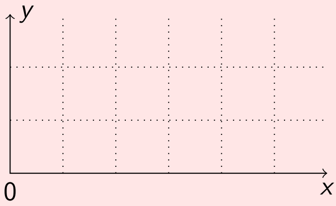

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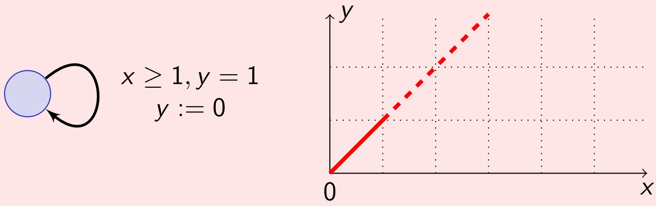

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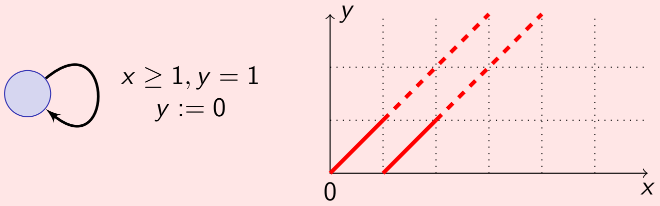

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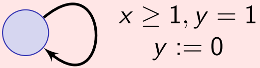

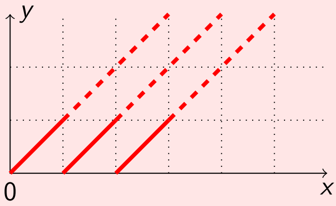

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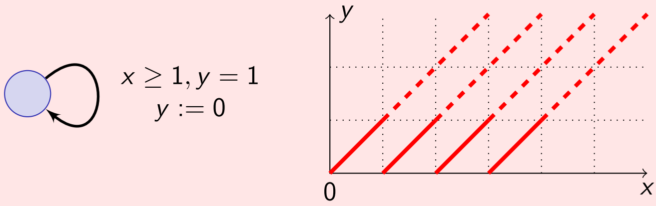

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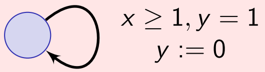

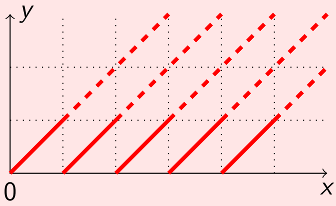

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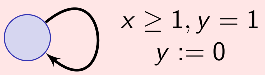

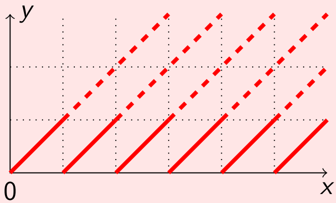

$\rightsquigarrow$ an infinite number of steps...

Forward reachability algorithm

Parameters: Abstraction abs and inclusion test $\preceq$

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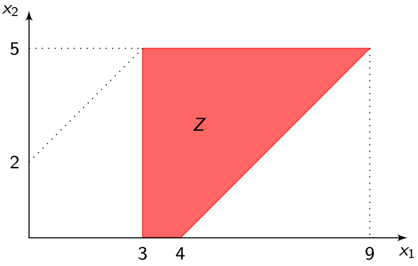
- $\texttt{Passed} \leftarrow \emptyset$ and $\texttt{Waiting} \leftarrow \{(\ell_0, Z_0)\}$
- While $\texttt{Waiting} \neq \emptyset$
    - select (ℓ, Z) from Waiting
    - If ℓ is final, then return "Reachable!"
    - If forall $(\ell, Z') \in \texttt{Passed}$, $Z \not\preceq Z'$, then add $\mathsf{abs}(\ell, Z)$ to Passed and add $\mathsf{Post}(\mathsf{abs}(\ell, Z))$ to Waiting
- Return "Not reachable!"

Standard solution: the extrapolation operator

$\mathsf{Extra}_2(Z)$: "the smallest zone containing Z that is defined only with constants no more than 2"



$$\begin{pmatrix} 0 & -3 & 0 \\ 9 & 0 & 4 \\ 5 & 2 & 0 \end{pmatrix}$$

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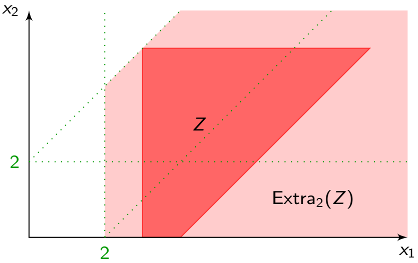

$$\begin{pmatrix} 0 & -3 & 0 \\ 9 & 0 & 4 \\ 5 & 2 & 0 \end{pmatrix} \xRightarrow{\mathsf{Extra}_2} \begin{pmatrix} 0 & -2 & 0 \\ \infty & 0 & \infty \\ \infty & 2 & 0 \end{pmatrix}$$

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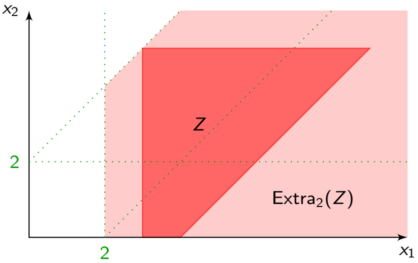

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$\rightsquigarrow$ The extrapolation operator ensures termination of the computation!

The extrapolation: correctness

Theorem [Bou04]

The forward algorithm with $\mathsf{abs} = \mathsf{Extra}_M$ and $\preceq \, = \, \subseteq$ is correct for timed automata.

[Bou04] Bouyer. Forward analysis of updatable timed automata *(Formal Methods in System Design)*.

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- the extrapolation operator can be made coarser:
  - use local extrapolation constants [BBFL03];
  - distinguish between lower- and upper-bounded contraints [BBLP04,BBLP06]
  - use non-convex (but optimal!) abstractions [HSW12]
  - compute constants dynamically [HSW13]

[Bou04] Bouyer. Forward analysis of updatable timed automata *(Formal Methods in System Design)*.
[BBFL03] Behrmann, Bouyer, Fleury, Larsen. Static Guard Analysis in Timed Automata Verification *(TACAS'03)*.
[BBLP04] Behrmann, Bouyer, Larsen, Pelánek. Lower and Upper Bounds in Zone Based Abstractions of Timed Automata *(TACAS'04)*.
[BBLP06] Behrmann, Bouyer, Larsen, Pelánek. Lower and Upper Bounds in Zone-Based Abstractions of Timed Automata *(International Journal on Software Tools for Technology Transfer)*.
[HSW12] Herbreteau, Srivathsan, Walukiewicz. Better abstractions for timed automata *(LICS'12)*.
[HSW13] Herbreteau, Srivathsan, Walukiewicz. Lazy abstractions for timed automata *(CAV'13)*.

Alternative: Improved inclusion test

Develop an inclusion test $\sqsubseteq_{\mathsf{abs}}$ such that:

$$Z \sqsubseteq_{\mathsf{abs}} Z' \quad \text{iff} \quad Z \subseteq \mathsf{abs}(Z')$$

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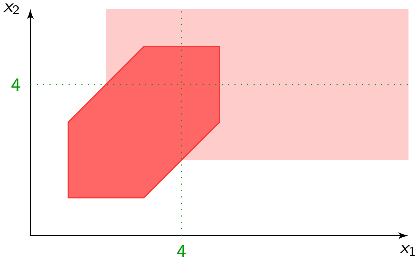
Simple case:

$M = L = U = 4$



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Theorem

The forward algorithm with $\mathsf{abs} = \mathsf{Id}$ and $\preceq = \sqsubseteq_{\mathsf{abs}_{LU}}$ is correct for timed automata.

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$\rightsquigarrow$ see description and demo later

Outline

Modelling resources in timed systems

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$$\begin{array}{ll} & \text{a discrete control (the mode of the system)} \\ + & \text{continuous evolution of the variables within a mode} \end{array}$$

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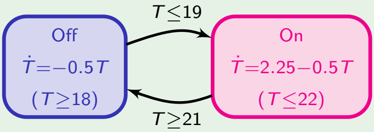
The thermostat example



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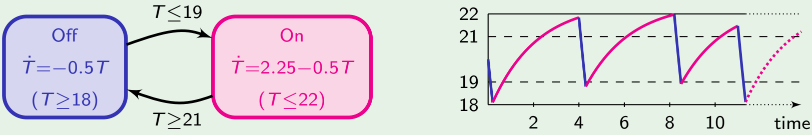
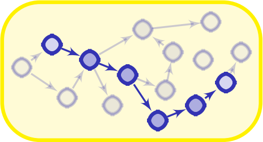


Ok...



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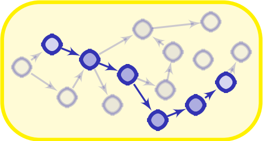
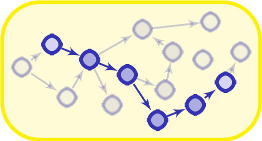


Easy...

Ok...



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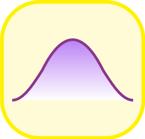

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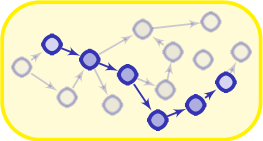

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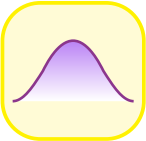

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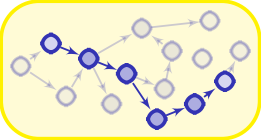
Ok... but?



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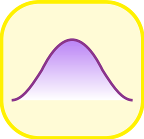

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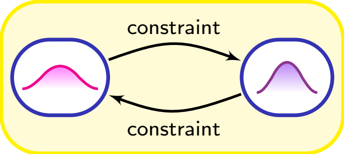

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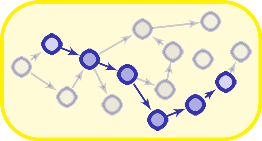

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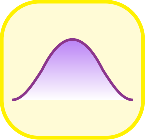

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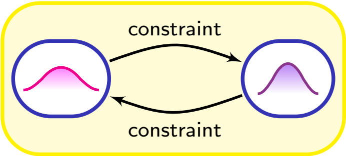

Hard!

Modelling resources in timed systems

- System resources might be relevant and even crucial information
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Theorem [HKPV95]

The reachability problem is undecidable in hybrid automata. Even for the simplest, the so-called stopwatch automata (clocks can be stopped).

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- An alternative: weighted/priced timed automata [ALP01,BFH+01]
  $\rightsquigarrow$ hybrid variables do not constrain the system
  hybrid variables are observer variables

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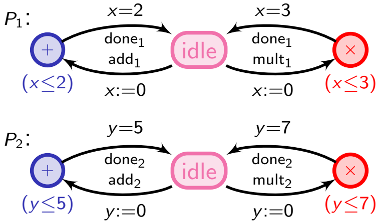
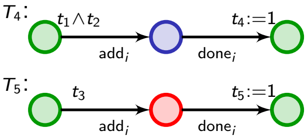
Modelling the task graph scheduling problem

- Processors



- Tasks



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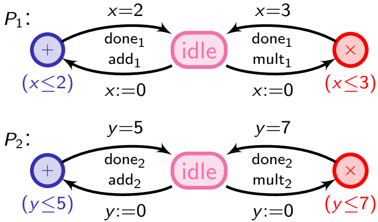

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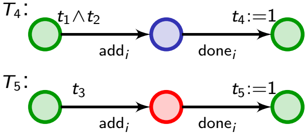
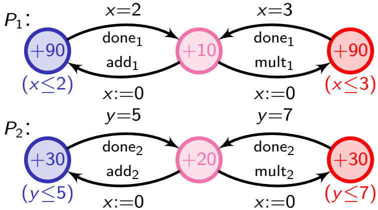
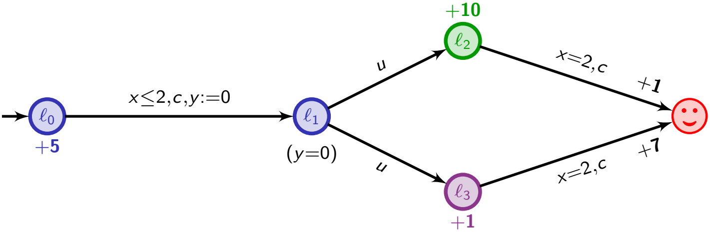


- Modelling energy



A good schedule is a path in the product automaton with a low cost

Weighted/priced timed automata [ALP01,BFH+01]



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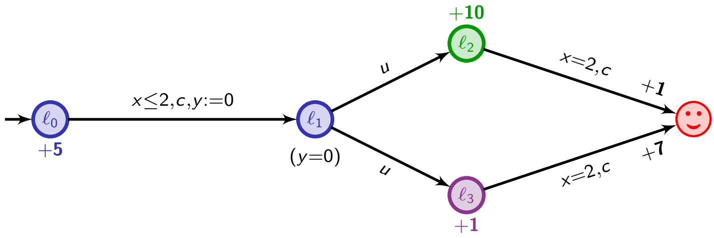

$$
\begin{array}{ccccccccccccc}
 & \ell_0 & \xrightarrow{1.3} & \ell_0 & \xrightarrow{c} & \ell_1 & \xrightarrow{u} & \ell_3 & \xrightarrow{0.7} & \ell_3 & \xrightarrow{c} & ☺ \\
x & 0 & & 1.3 & & 1.3 & & 1.3 & & 2 & & \\
y & 0 & & 1.3 & & 0 & & 0 & & 0.7 & &
\end{array}
$$

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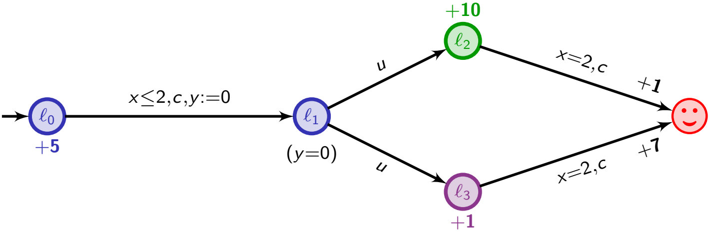

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cost :

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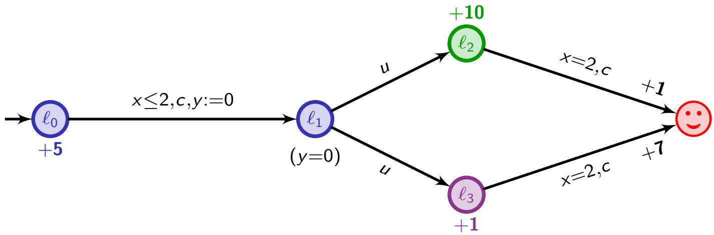

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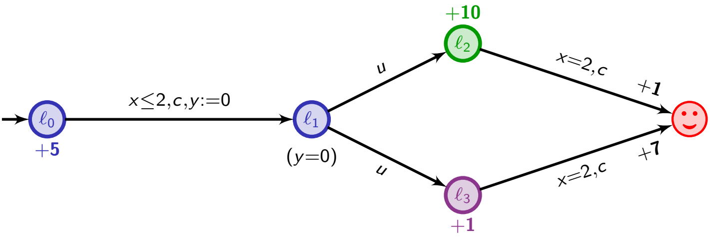

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cost : 6.5 + 0

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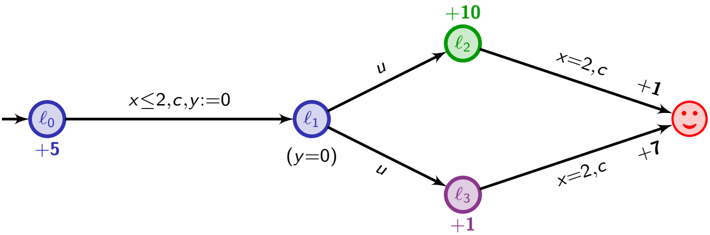

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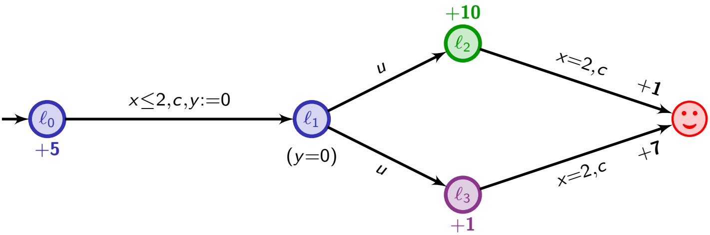

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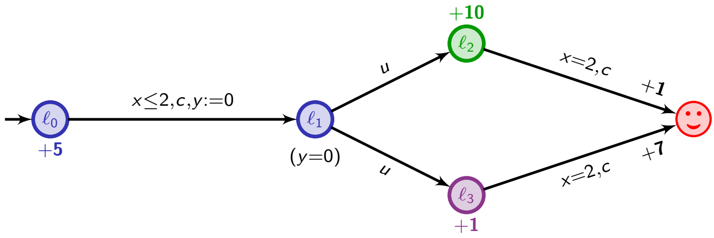

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x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : 6.5 + 0 + 0 + 0.7 + 7

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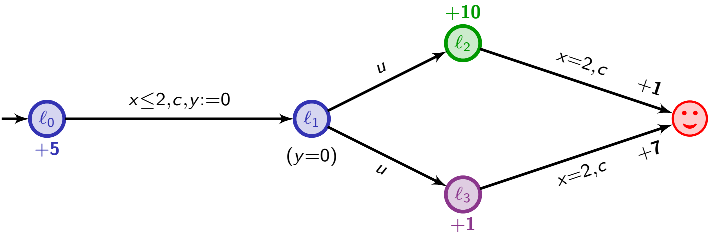

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y & 0 & & 1.3 & & 0 & & 0 & & 0.7 & &
\end{array}
$$

cost : 6.5 + 0 + 0 + 0.7 + 7 = 14.2

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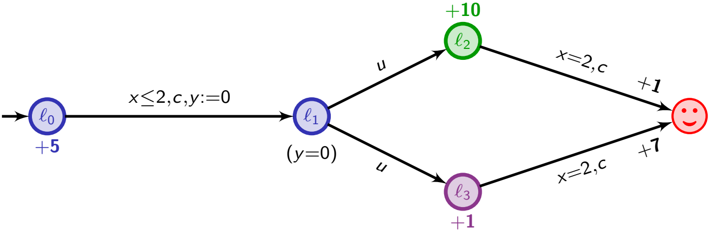

Question: what is the optimal cost for reaching ☺?

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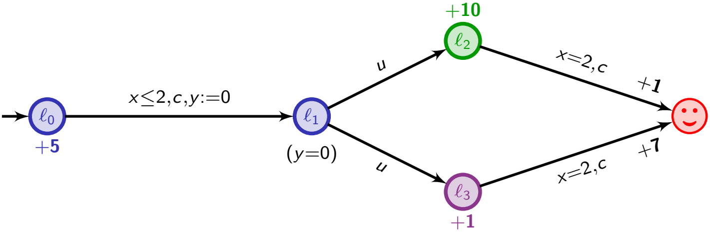

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$$5t + 10(2-t) + 1$$

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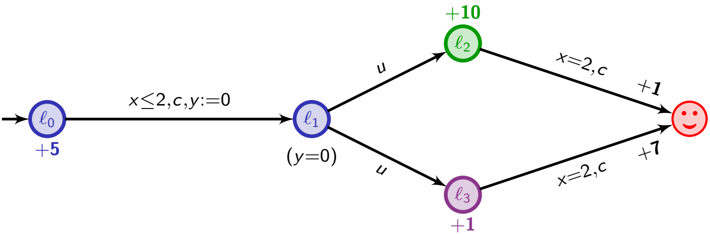

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$$5t + 10(2-t) + 1 \;,\; 5t + (2-t) + 7$$

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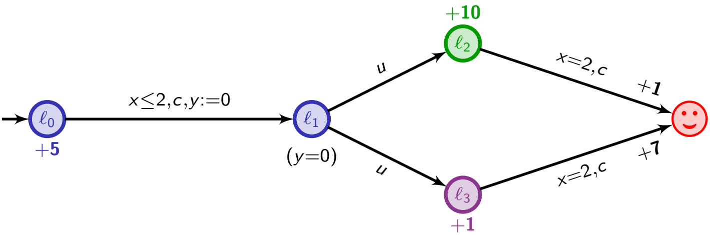
[BFH+01] Behrmann, Fehnker, Hune, Larsen, Pettersson, Romijn, Vaandrager. Minimum-cost reachability in priced timed automata *(HSCC'01)*.

Weighted/priced timed automata [ALP01,BFH+01]



Question: what is the optimal cost for reaching ☺ ?

$$\min\ (\ 5t + 10(2-t) + 1\ ,\ 5t + (2-t) + 7\ )$$

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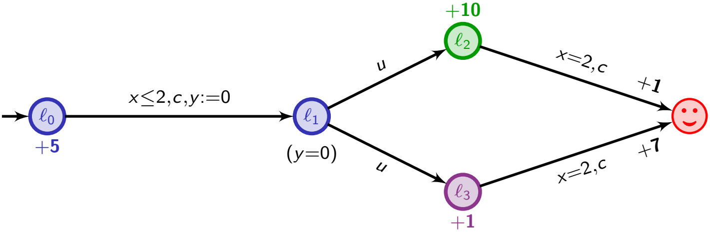

Question: what is the optimal cost for reaching ☺?

$$\inf_{0\leq t\leq 2} \min\left(\ 5t+10(2-t)+1\ ,\ 5t+(2-t)+7\ \right) = 9$$

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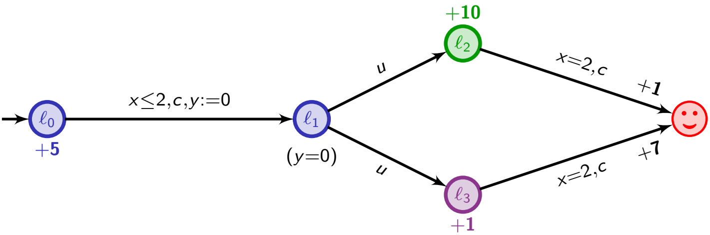

Question: what is the optimal cost for reaching ☺ ?

$$\inf_{0\leq t\leq 2} \min\left(\, 5t+10(2-t)+1 \,,\, 5t+(2-t)+7 \,\right) = 9$$

$\rightsquigarrow$ *strategy:* leave immediately ℓ_0, go to ℓ_3, and wait there 2 t.u.

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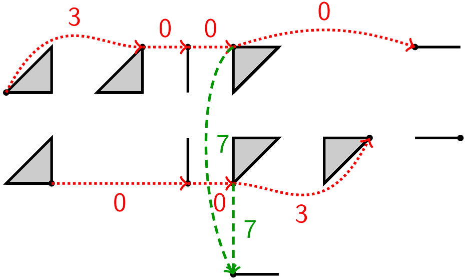
Optimal-cost reachability

Theorem [ALP01,BFH+01,BBBR07]

In weighted timed automata, the optimal cost is an integer and can be computed in PSPACE.

- Technical tool: a refinement of the regions, the corner-point abstraction



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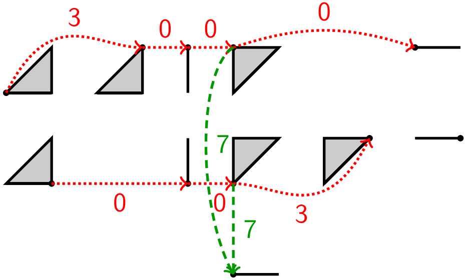

- Symbolic technics based on priced zones

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
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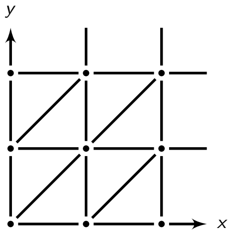
Technical tool: the corner-point abstraction



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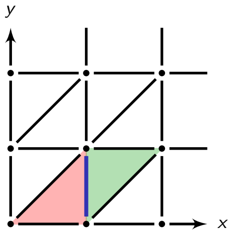

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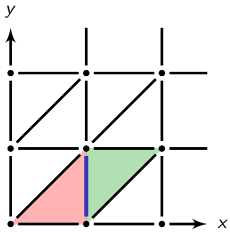
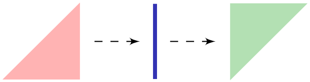


Abstract time successors:



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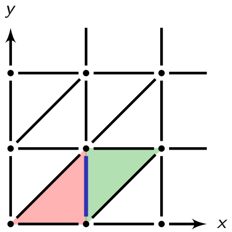

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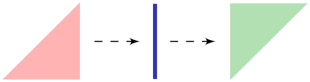

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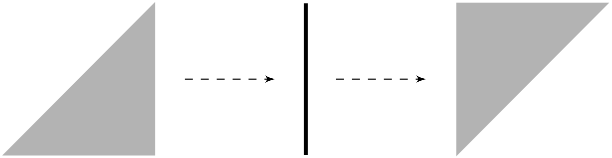

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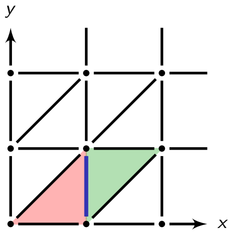

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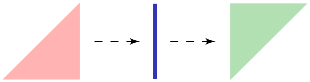

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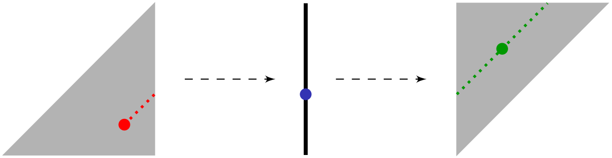

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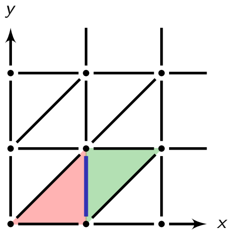

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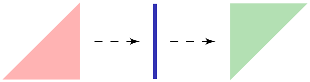

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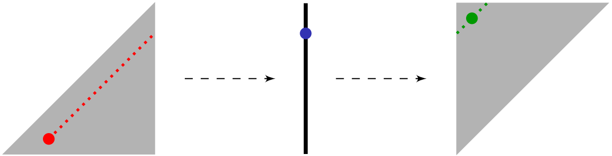

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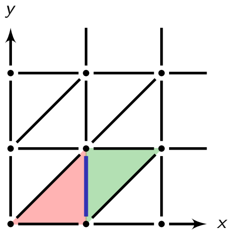

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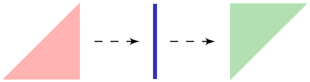

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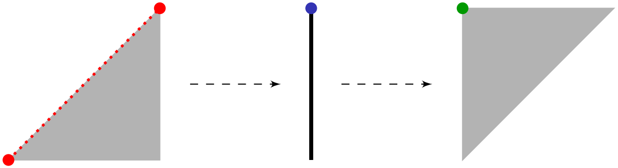

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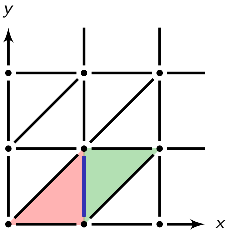

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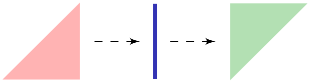

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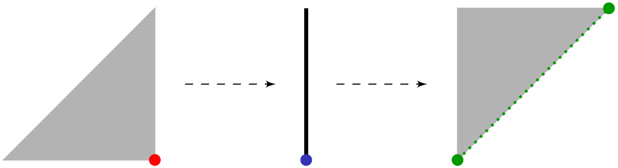

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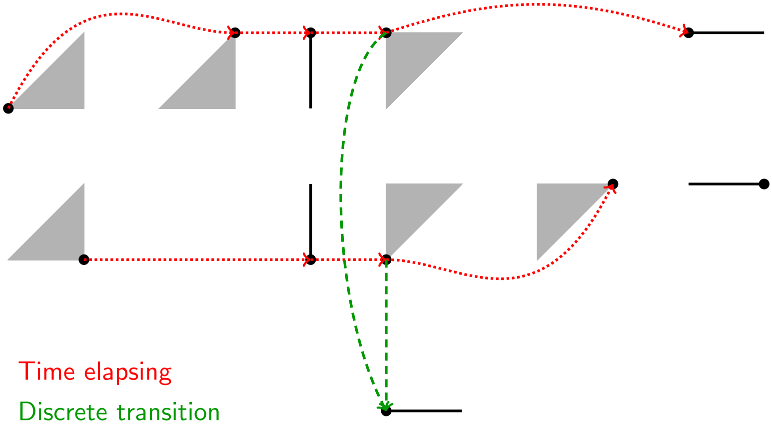

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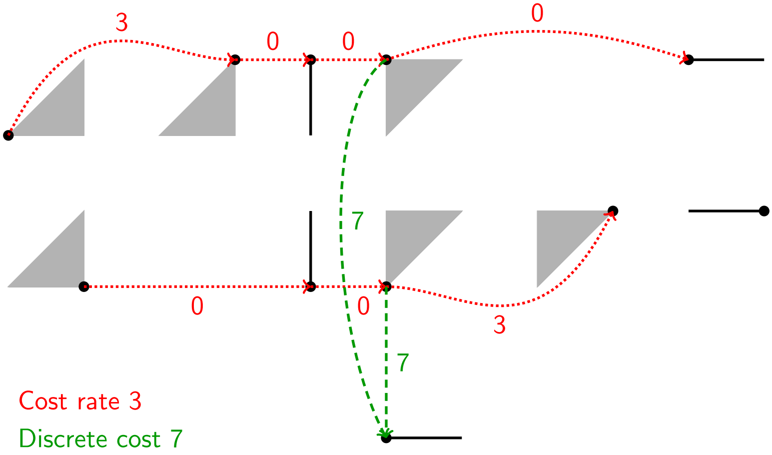

From timed to discrete behaviours

Optimal reachability as a linear programming problem

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$\circ \xrightarrow{t_1} \circ \xrightarrow{t_2} \circ \xrightarrow{t_3} \circ \xrightarrow{t_4} \circ \xrightarrow{t_5} \circ \quad \cdots$$

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$\circ \xrightarrow{t_1} \circ \xrightarrow[x\leq 2]{t_2} \circ \xrightarrow{t_3} \circ \xrightarrow{t_4} \circ \xrightarrow{t_5} \circ \quad \cdots \quad \left\{ \begin{array}{l} t_1+t_2\leq 2 \\ \\ \end{array} \right.$$

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$\circ \xrightarrow[y:=0]{t_1} \circ \xrightarrow[x\leq 2]{t_2} \circ \xrightarrow{t_3} \circ \xrightarrow[y\geq 5]{t_4} \circ \xrightarrow{t_5} \circ \quad \cdots \quad \begin{cases} t_1+t_2\leq 2 \\ t_2+t_3+t_4\geq 5 \end{cases}$$

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$$\begin{array}{ccccc} T_1 & T_2 & T_3 & T_4 & T_5 \end{array}$$

$$\circ \xrightarrow[y:=0]{t_1} \circ \xrightarrow[x\leq 2]{t_2} \circ \xrightarrow{t_3} \circ \xrightarrow[y\geq 5]{t_4} \circ \xrightarrow{t_5} \circ \quad \cdots \quad \begin{cases} t_1+t_2\leq 2 & T_2\leq 2 \\ t_2+t_3+t_4\geq 5 & T_4-T_1\geq 5 \end{cases}$$

From timed to discrete behaviours

Optimal reachability as a linear programming problem

$T_1 \quad T_2 \quad T_3 \quad T_4 \quad T_5$

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Lemma

Let Z be a bounded zone and f be a function

$$f : (T_1, ..., T_n) \mapsto \sum_{i=1}^{n} c_i T_i + c$$

well-defined on $\overline{Z}$. Then $inf_Z f$ is obtained on the border of $\overline{Z}$ with integer coordinates.

From timed to discrete behaviours

Optimal reachability as a linear programming problem

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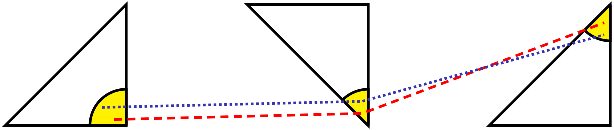
$\leadsto$ for every finite path π in $\mathcal{A}$, there exists a path Π in $\mathcal{A}_{\mathrm{cp}}$ such that

$$\mathrm{cost}(\Pi) \leq \mathrm{cost}(\pi)$$

[Π is a “corner-point projection” of π]

From discrete to timed behaviours

Approximation of abstract paths:



For any path Π of $\mathcal{A}_{\mathsf{cp}}$,

From discrete to timed behaviours

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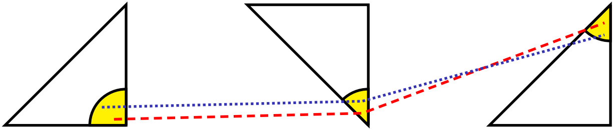

For any path Π of $\mathcal{A}_{\mathsf{cp}}$, for any $\varepsilon > 0$,

From discrete to timed behaviours

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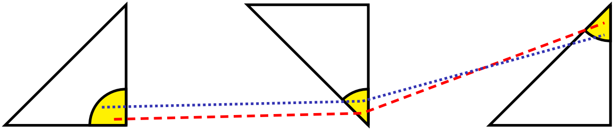

For any path Π of $\mathcal{A}_{\mathsf{cp}}$, for any $\varepsilon > 0$, there exists a path π_ε of $\mathcal{A}$ s.t.

$$\|\Pi - \pi_\varepsilon\|_\infty < \varepsilon$$

From discrete to timed behaviours

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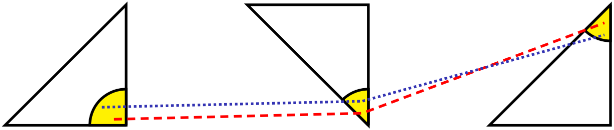


For any path Π of $\mathcal{A}_{\mathsf{cp}}$, for any $\varepsilon > 0$, there exists a path π_ε of $\mathcal{A}$ s.t.

$$\|\Pi - \pi_\varepsilon\|_\infty < \varepsilon$$

For every $\eta > 0$, there exists $\varepsilon > 0$ s.t.

$$\|\Pi - \pi_\varepsilon\|_\infty < \varepsilon \Rightarrow |\mathsf{cost}(\Pi) - \mathsf{cost}(\pi_\varepsilon)| < \eta$$

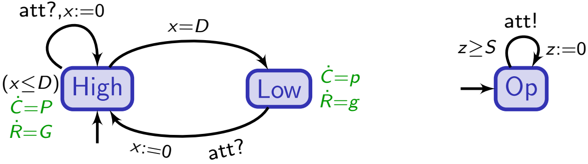
Use of the corner-point abstraction

It is a very interesting abstraction, that can be used in several other contexts:

- for mean-cost optimization [BBL04,BBL08]
- for discounted-cost optimization [FL08]
- for all concavely-priced timed automata [JT08]
- for deciding frequency objectives [BBBS11,Sta12]
- ...

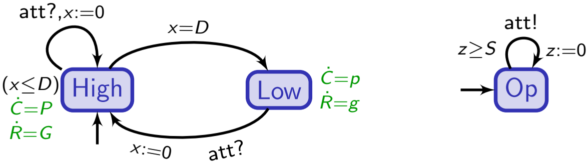
[BBL04] Bouyer, Brinksma, Larsen. Staying Alive As Cheaply As Possible *(HSCC'04)*.
[BBL08] Bouyer, Brinksma, Larsen. Optimal infinite scheduling for multi-priced timed automata *(Formal Methods in System Designs)*.
[FL08] Fahrenberg, Larsen. Discount-optimal infinite runs in priced timed automata *(INFINITY'08)*.
[JT08] Judziński, Trivedi. Concavely-priced timed automata *(FORMATS'08)*.
[BBBS11] Bertrand, Bouyer, Brihaye, Stainer. Emptiness and universality problems in timed automata with positive frequency *(ICALP'11)*.
[Sta12] Stainer. Frequencies in forgetful timed automata *(FORMATS'12)*.

Going further 1: mean-cost optimization



[BBL08] Bouyer, Brinksma, Larsen. Optimal infinite scheduling for multi-priced timed automata *(Formal Methods in System Designs)*.

Going further 1: mean-cost optimization



$\rightsquigarrow$ compute optimal infinite schedules that minimize

$$\text{mean-cost}(\pi) = \limsup_{n \to +\infty} \frac{\text{cost}(\pi_n)}{\text{reward}(\pi_n)}$$

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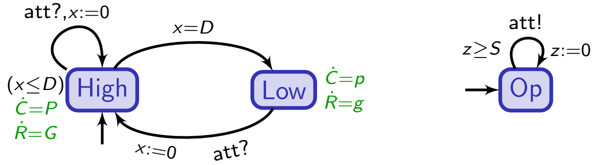

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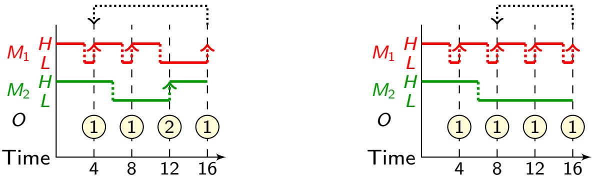

Schedule with ratio ≈ 1.455 Schedule with ratio ≈ 1.478

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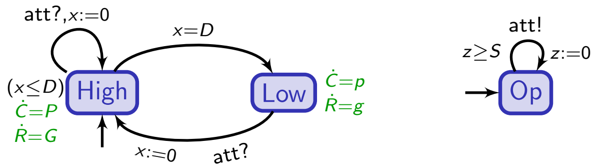
Going further 1: mean-cost optimization



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Theorem [BBL08]

In weighted timed automata, the optimal mean-cost can be compute in PSPACE.

$\rightsquigarrow$ the corner-point abstraction can be used

[BBL08] Bouyer, Brinksma, Larsen. Optimal infinite scheduling for multi-priced timed automata *(Formal Methods in System Designs)*.

From timed to discrete behaviours

- **Finite behaviours:** based on the following property

Lemma

Let Z be a bounded zone and f be a function

$$f : (t_1, ..., t_n) \mapsto \frac{\sum_{i=1}^{n} c_i t_i + c}{\sum_{i=1}^{n} r_i t_i + r}$$

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$\leadsto$ for every finite path π in $\mathcal{A}$, there exists a path Π in $\mathcal{A}_{\mathsf{cp}}$ s.t.
mean-cost$(\Pi) \leq$ mean-cost(π)

From timed to discrete behaviours

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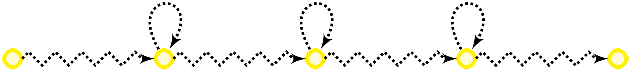
- **Infinite behaviours:** decompose each sufficiently long projection into cycles:



The (acyclic) linear part will be negligible!

From timed to discrete behaviours

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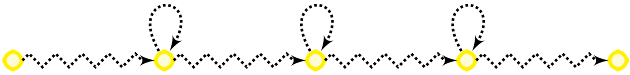
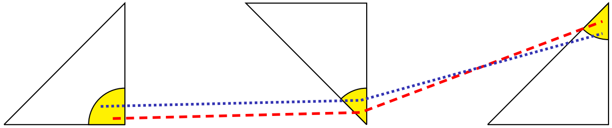


The (acyclic) linear part will be negligible!

$\rightsquigarrow$ the optimal cycle of $\mathcal{A}_{\text{cp}}$ is better than any infinite path of $\mathcal{A}$!

From discrete to timed behaviours

Approximation of abstract paths:



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From discrete to timed behaviours

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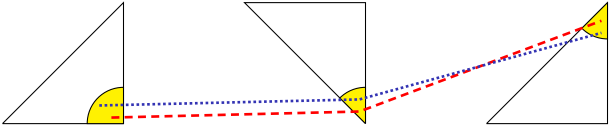
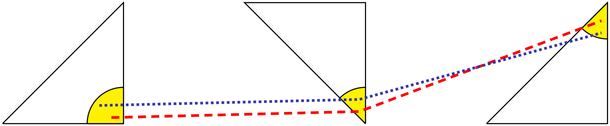


For any path Π of $\mathcal{A}_{\mathsf{cp}}$, for any $\varepsilon > 0$,

From discrete to timed behaviours

Approximation of abstract paths:



For any path Π of $\mathcal{A}_{\mathsf{cp}}$, for any $\varepsilon > 0$, there exists a path π_ε of $\mathcal{A}$ s.t.

$$\|\Pi - \pi_\varepsilon\|_\infty < \varepsilon$$

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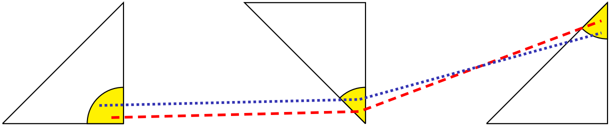

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For every $\eta > 0$, there exists $\varepsilon > 0$ s.t.

$$\|\Pi - \pi_\varepsilon\|_\infty < \varepsilon \Rightarrow |\text{mean-cost}(\Pi) - \text{mean-cost}(\pi_\varepsilon)| < \eta$$

Going further 2: concavely-priced cost functions

$\rightsquigarrow$ A general abstract framework for quantitative timed systems

Theorem [JT08]

In concavely-priced timed automata, optimal cost is computable, if we restrict to quasi-concave cost functions. For the following cost functions, the (decision) problem is even PSPACE-complete:

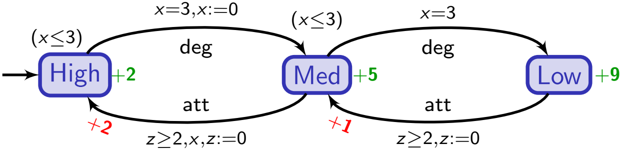
- optimal-time and optimal-cost reachability;
- optimal discrete discounted cost;
- optimal mean-cost.

$\rightsquigarrow$ the corner-point abstraction can be used

[JT08] Judziński, Trivedi. Concavely-priced timed automata *(FORMATS'08)*.

Going further 3: discounted-time cost optimization

Globally, $(z \le 8)$



[FL08] Fahrenberg, Larsen. Discount-optimal infinite runs in priced timed automata *(INFINITY'08)*.

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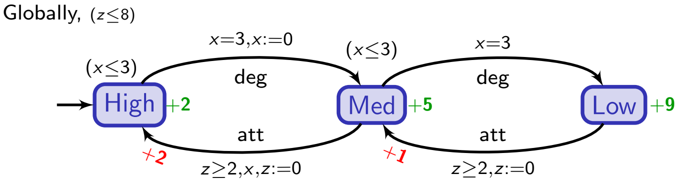

$\rightsquigarrow$ compute optimal infinite schedules that minimize
discounted cost over time

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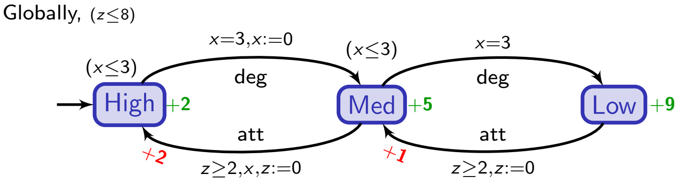
Going further 3: discounted-time cost optimization



$\rightsquigarrow$ compute optimal infinite schedules that minimize

$$\text{discounted-cost}_\lambda(\pi) = \sum_{n \geq 0} \lambda^{T_n} \int_{t=0}^{\tau_{n+1}} \lambda^t \text{cost}(\ell_n)\,\mathrm{d}t + \lambda^{T_{n+1}} \text{cost}(\ell_n \xrightarrow{a_{n+1}} \ell_{n+1})$$

if $\pi = (\ell_0, v_0) \xrightarrow{\tau_1, a_1} (\ell_1, v_1) \xrightarrow{\tau_2, a_2} \cdots$ and $T_n = \sum_{i \leq n} \tau_i$

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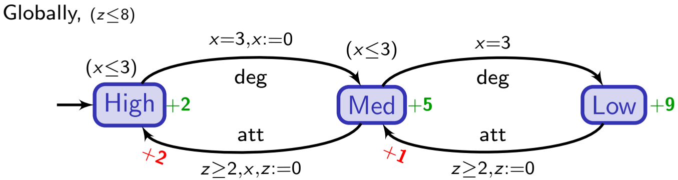

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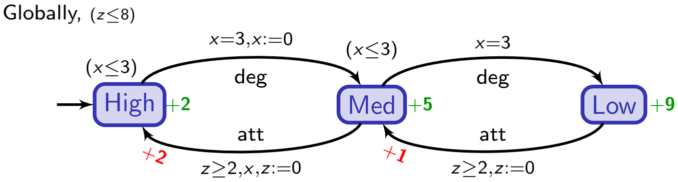

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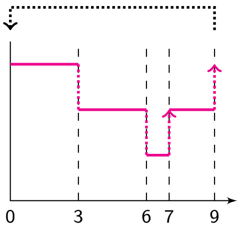

if $\lambda = e^{-1}$, the discounted cost of that infinite schedule is ≈ 2.16

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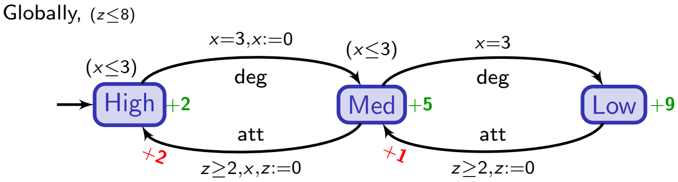
Going further 3: discounted-time cost optimization



$\rightsquigarrow$ compute optimal infinite schedules that minimize
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Theorem [FL08]

In weighted timed automata, the optimal discounted cost is computable in EXPTIME.

$\rightsquigarrow$ the corner-point abstraction can be used

[FL08] Fahrenberg, Larsen. Discount-optimal infinite runs in priced timed automata *(INFINITY'08)*.

And symbolically?

- Only for optimal reachability

[LBB+01] Larsen, Behrmann, Brinksma, Fehnker, Hune, Pettersson, Romijn. As cheap as possible: Efficient cost- optimal reachability for priced timed automata *(CAV'01)*.

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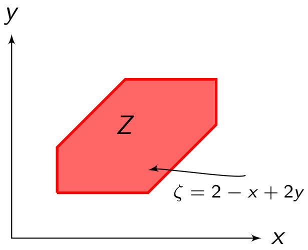
Priced zones

priced zone = zone + affine cost function



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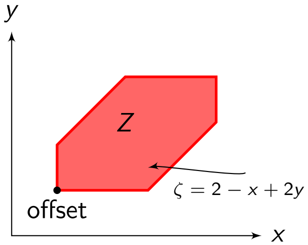
☺ efficient representation: DBM + offset cost + affine coefficient for each clock



Represented by: zone Z
offset cost: $+4$
rate for x: -1
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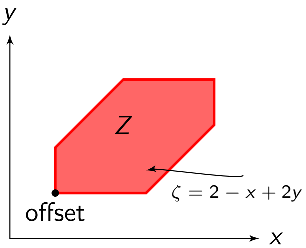
☹ the successor of a priced zone is a *union* of priced zones



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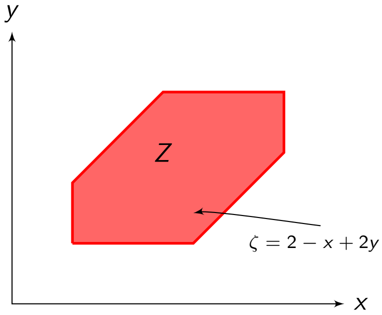
Computing the (time) successor of a priced zone



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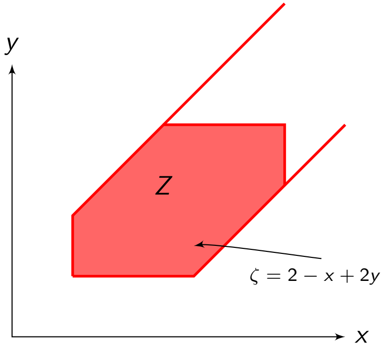

Cost rate in current location: $+3$

We want (Z', ζ') with
$\zeta'(v') = \min_{v'-\delta \in Z} \zeta(v' - \delta) + 3\delta$

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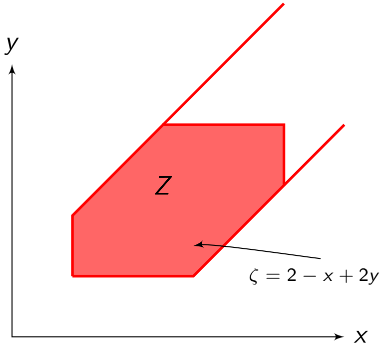

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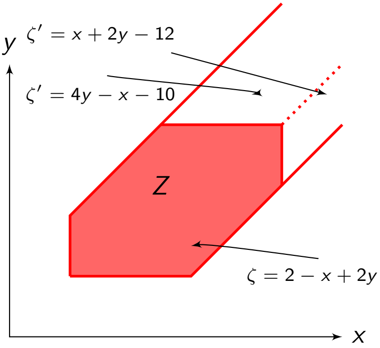

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- otherwise, depends on the facet

Forward optimal reachability algorithm

Parameters: Abstraction abs and inclusion test $\preceq$

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- cost $\leftarrow +\infty$, $\texttt{Passed} \leftarrow \emptyset$ and $\texttt{Waiting} \leftarrow \{(\ell_0, \mathcal{Z}_0)\}$
- While $\texttt{Waiting} \neq \emptyset$
  - select $(\ell, \mathcal{Z})$ from Waiting
  - If ℓ is final and $\mathit{minCost}(\mathcal{Z}) <$ cost, then set $\mathit{minCost}(\mathcal{Z})$ to cost
  - If forall $(\ell, \mathcal{Z}') \in \texttt{Passed}$, $\mathcal{Z} \not\preceq \mathcal{Z}'$, then add abs$(\ell, \mathcal{Z})$ to Passed and add Post(abs$(\ell, \mathcal{Z})$) to Waiting
- Return cost

Results

Theorem [LBB+01,RLS06]

The forward algorithm with abs $=$ Id and $\preceq = \subseteq$ is correct and terminates for **bounded** timed automata with non-negative costs.

Termination: well-quasi-order on priced zones

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The forward algorithm with abs = Id and $\preceq = \sqsubseteq_M$ is correct and terminates for timed automata with some conditions on the cost. It is always better than $\preceq = \subseteq$ for bounded timed automata.

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Tools

- Uppaal-Cora, developed in Aalborg (Denmark) between 2001-2005

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$\rightsquigarrow$ no more maintained

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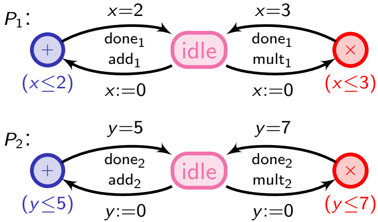
  $\leadsto$ no more maintained

- Our new tool TiAMo

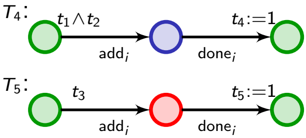
Outline

Modelling the task graph scheduling problem

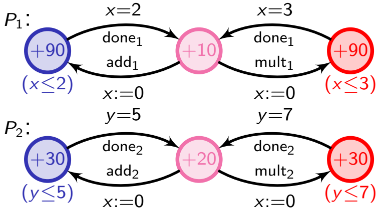
- Processors



- Tasks



- Modelling energy



Modelling the task graph scheduling problem

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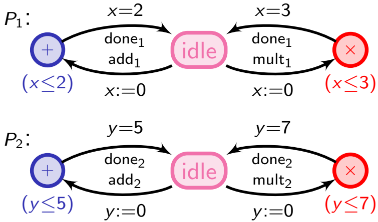

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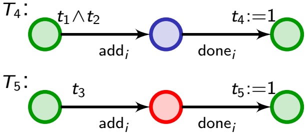

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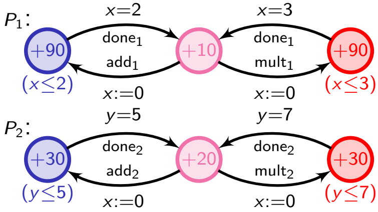
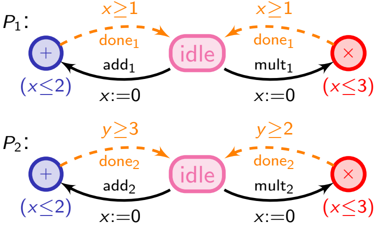


- Modelling uncertainty



Modelling the task graph scheduling problem

- Processors

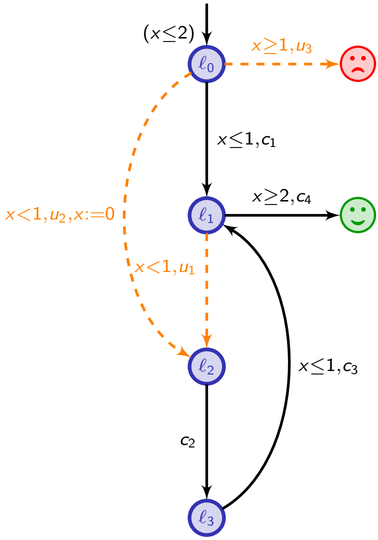
- Tasks

A (good) schedule is a strategy in the product game (with a low cost)

- Modelling energy

- Modelling uncertainty

An example of a timed game



Rule of the game

- Aim: avoid ☹ and reach ☺

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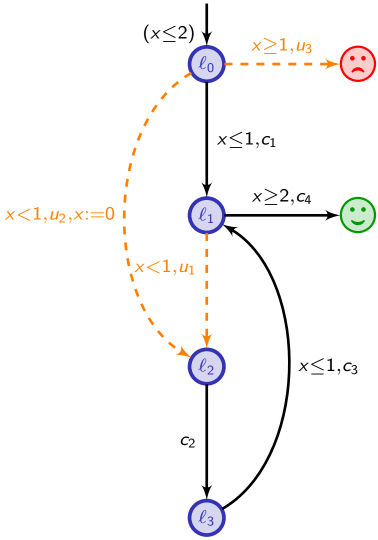

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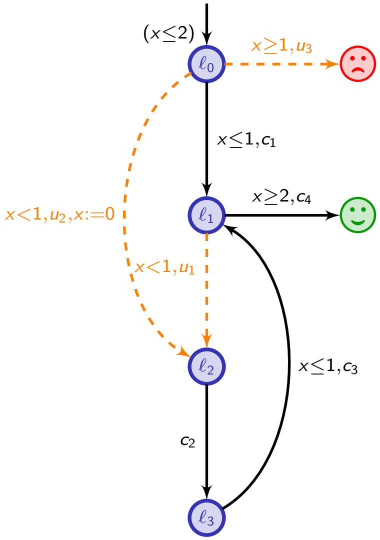

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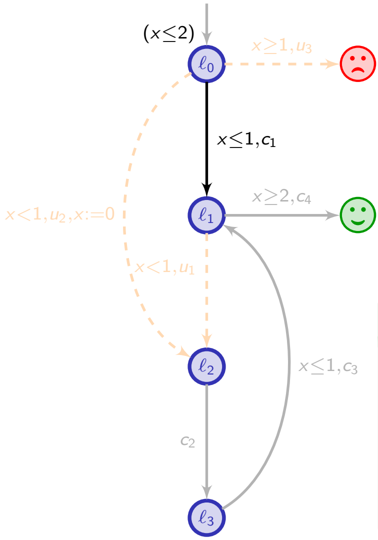

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A (memoryless) winning strategy

- from $(\ell_0, 0)$, play $(0.5, c_1)$

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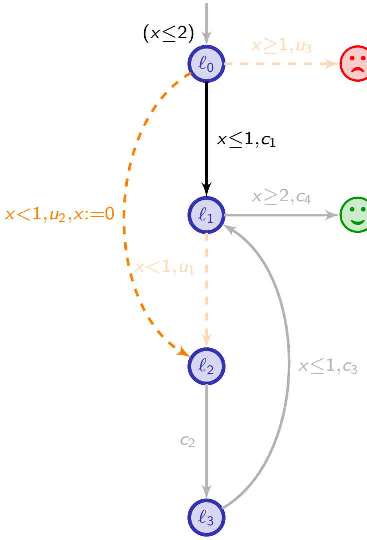

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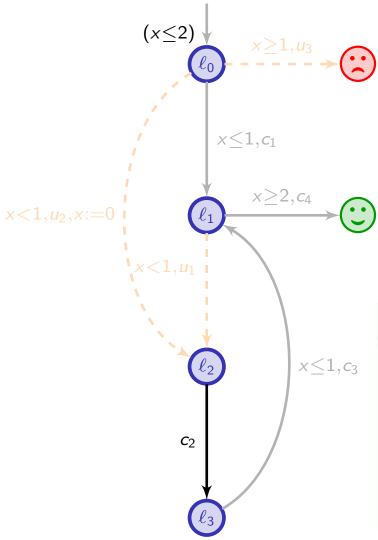

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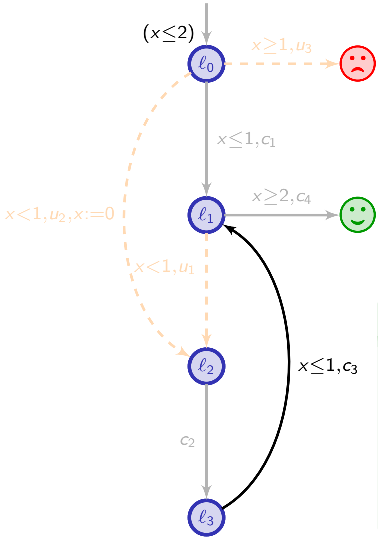

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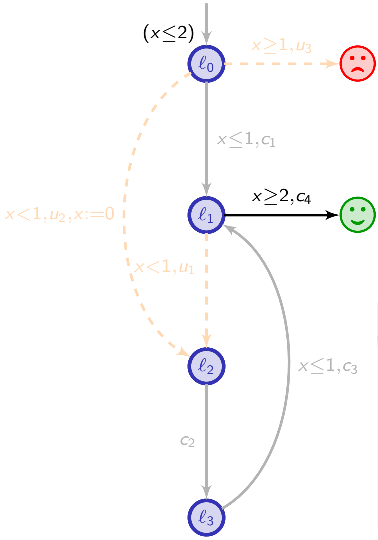

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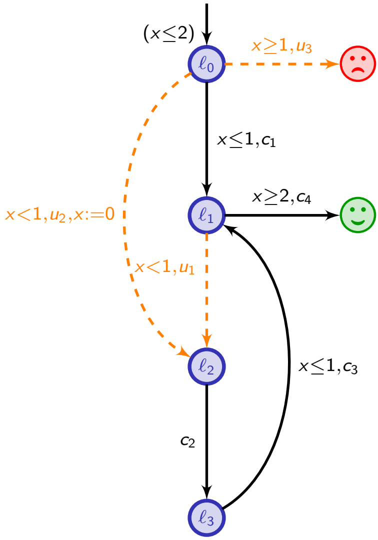

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Problems to be considered

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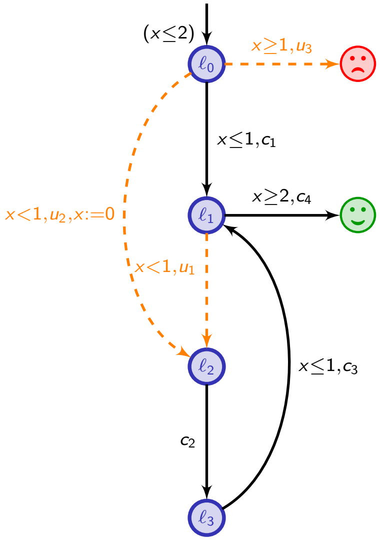

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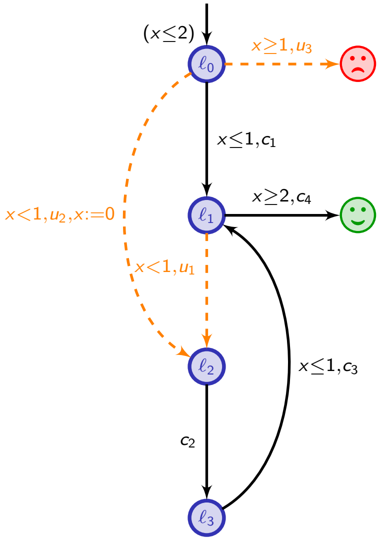

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Problems to be considered

- Does there exist a winning strategy?
- If yes, compute one (as simple as possible).

Decidability of timed games

Theorem [AMPS98,HK99]

Reachability and safety timed games are decidable and EXPTIME-complete. Furthermore memoryless and "region-based" strategies are sufficient.

[AMPS98] Asarin, Maler, Pnueli, Sifakis. Controller synthesis for timed automata *(SSC'98)*.
[HK99] Henzinger, Kopke. Discrete-time control for rectangular hybrid automata *(Theoretical Computer Science)*.

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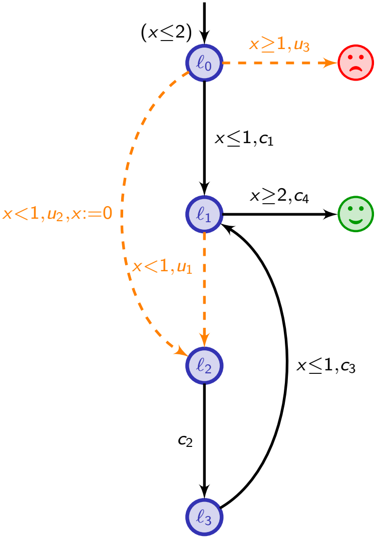
Theorem [AM99,BHPR07,JT07]

Optimal-time reachability timed games are decidable and EXPTIME-complete.

[AM99] Asarin, Maler. As soon as possible: time optimal control for timed automata *(HSCC'99)*.
[BHPR07] Brihaye, Henzinger, Prabhu, Raskin. Minimum-time reachability in timed games *(ICALP'07)*.
[JT07] Jurdziński, Trivedi. Reachability-time games on timed automata *(ICALP'07)*.

Back to the example: computing winning states



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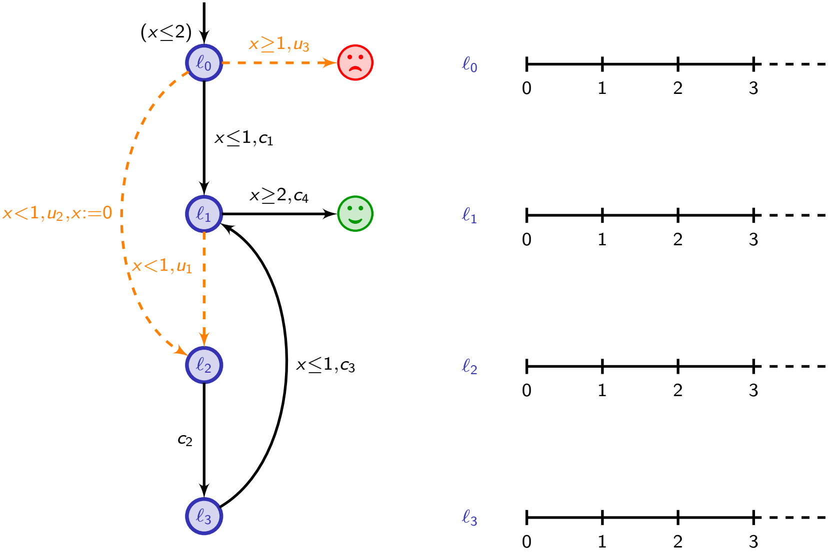

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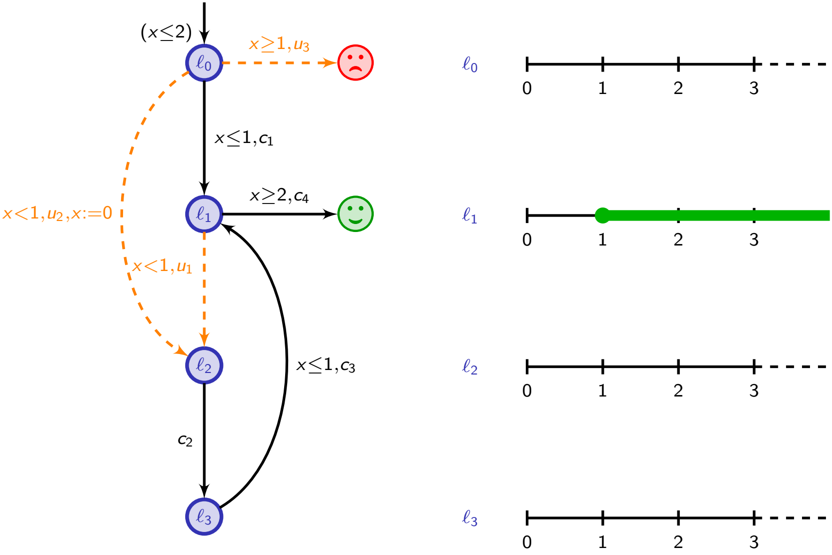

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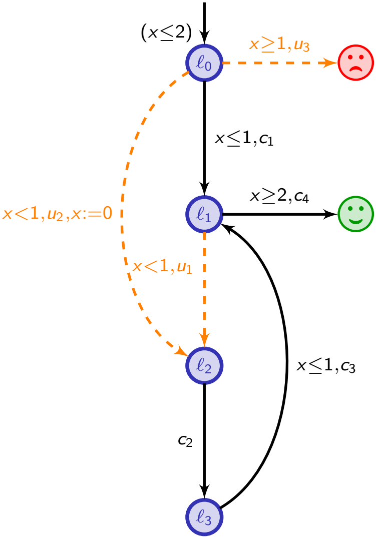

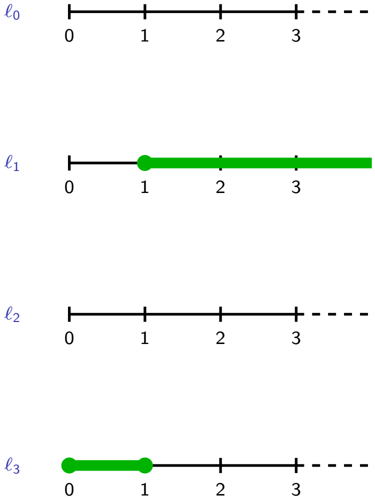

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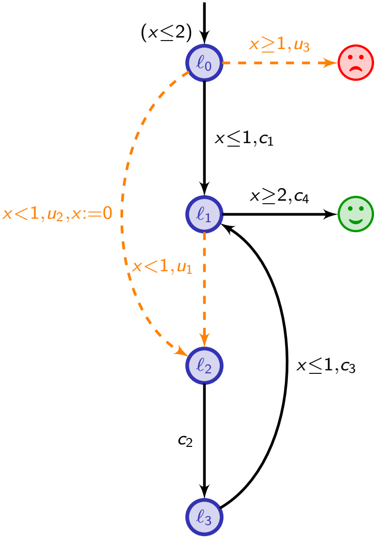

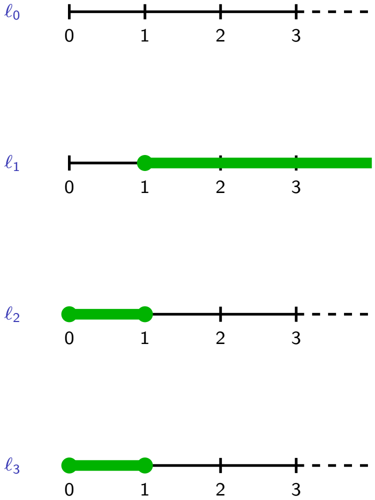

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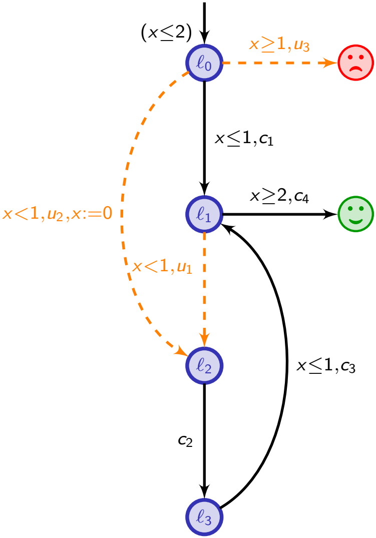

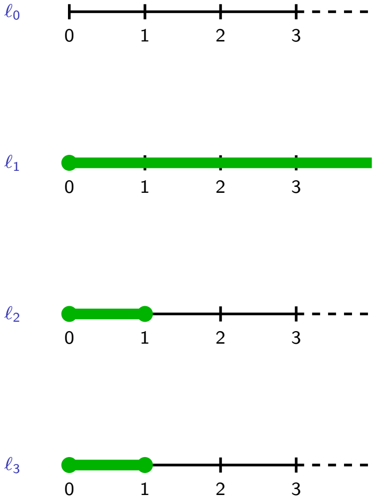

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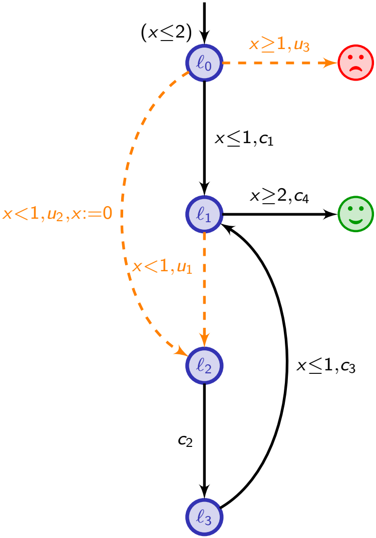

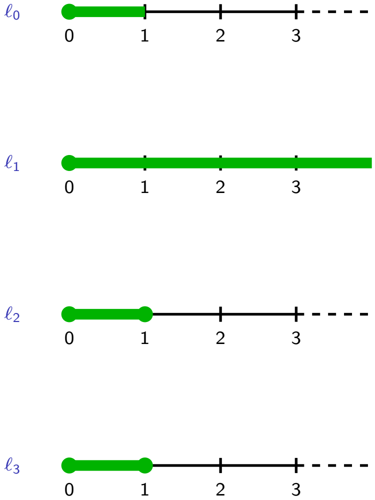

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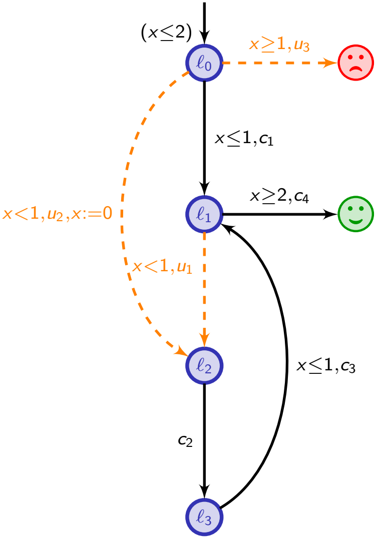

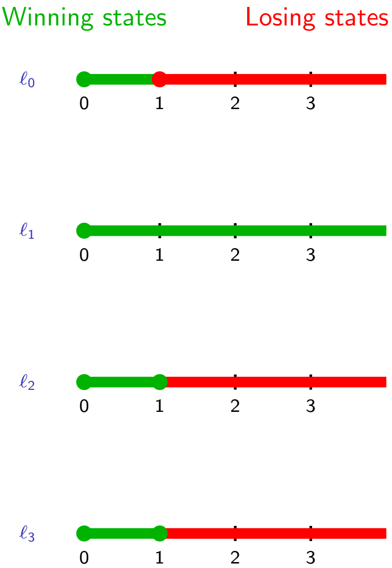

Decidability *via* attractors

Skip attractors

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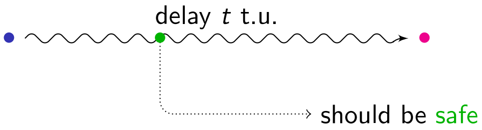

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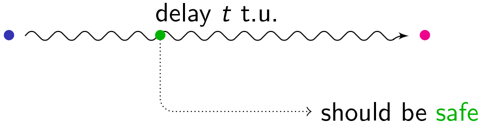

$$\mathrm{Pred}_\delta(X, \mathrm{Safe}) = \{\bullet \mid \exists t \geq 0,\ \bullet \xrightarrow{\delta(t)} \bullet$$
$$\text{and } \forall 0 \leq t' \leq t,\ \bullet \xrightarrow{\delta(t')} \bullet \in \mathrm{Safe}\}$$

Timed games with a reachability objective

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Stability w.r.t. regions

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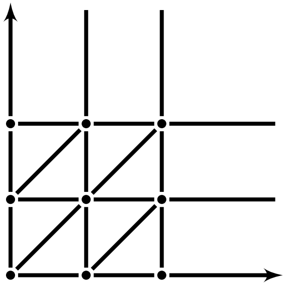

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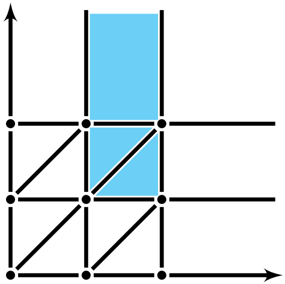

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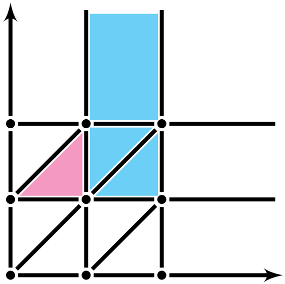

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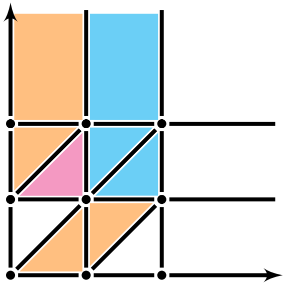

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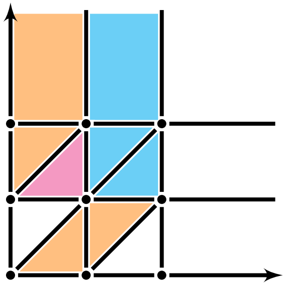

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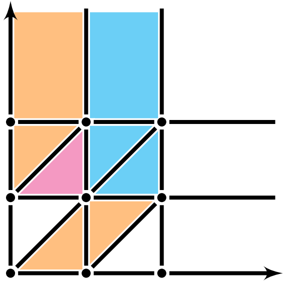

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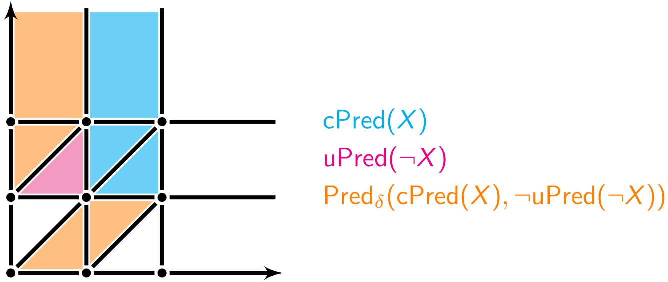

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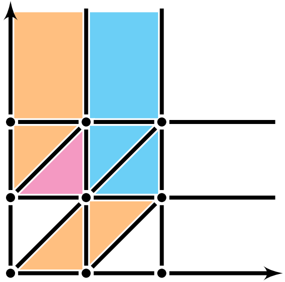

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$\leadsto$ the computation of $\pi^*(☺)$ terminates!

... and is correct

And in practice?

- A zone-based forward algorithm with backtracking [CDF+05,BCD+07]

[CDF+05] Cassez, David, Fleury, Larsen, Lime. Efficient On-the-Fly Algorithms for the Analysis of Timed Games *(CONCUR'05)*.
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And in practice?

- A zone-based forward algorithm with backtracking [CDF+05,BCD+07]
- A tool: Uppaal-TiGa, developed in Aalborg (Denmark) since 2005

  `http://people.cs.aau.dk/~adavid/tiga/`

[CDF+05] Cassez, David, Fleury, Larsen, Lime. Efficient On-the-Fly Algorithms for the Analysis of Timed Games *(CONCUR'05)*.
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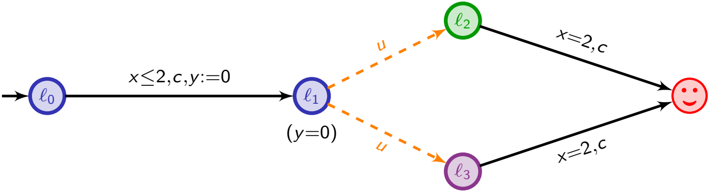
Outline

A simple timed game



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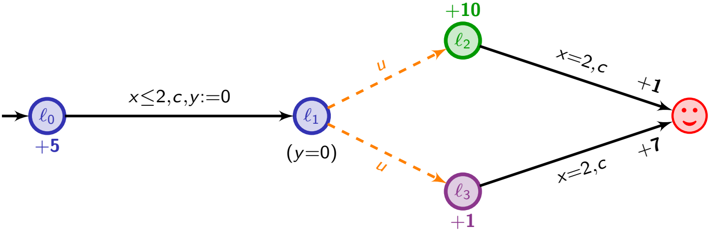

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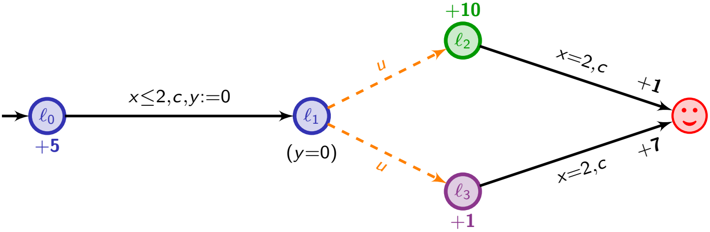

Question: what is the optimal cost we can ensure while reaching 🙂?

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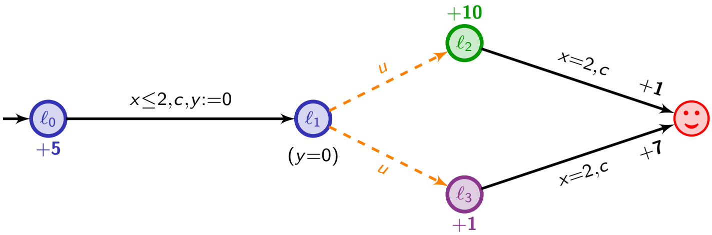

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$$5t + 10(2-t) + 1$$

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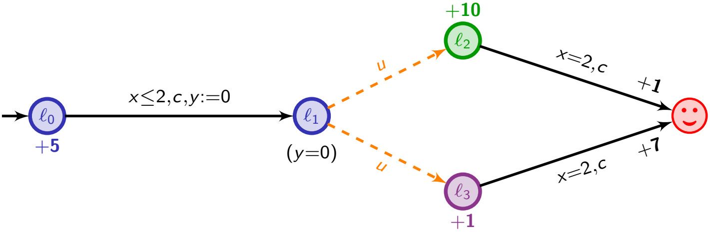

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$$5t + 10(2-t) + 1\ ,\ 5t + (2-t) + 7$$

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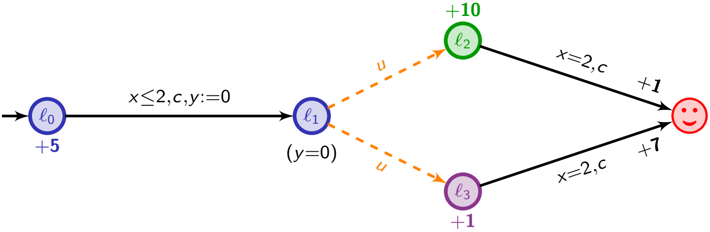

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$$\max\ (\ 5t + 10(2-t) + 1\ ,\ 5t + (2-t) + 7\ )$$

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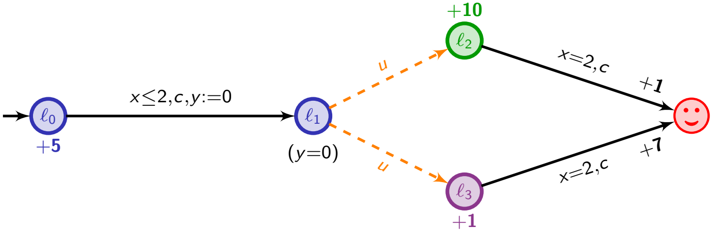

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$$\inf_{0\leq t\leq 2} \max\left(\ 5t+10(2-t)+1\ ,\ 5t+(2-t)+7\ \right) = 14+\frac{1}{3}$$

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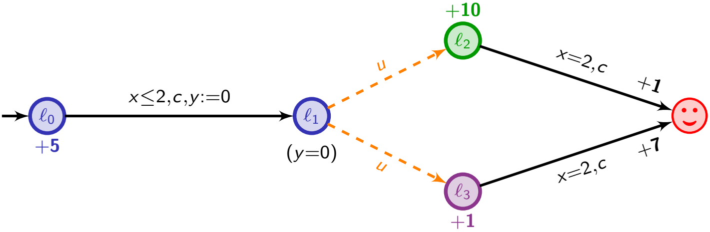

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$$\inf_{0\leq t\leq 2} \max\left(5t+10(2-t)+1 \,,\, 5t+(2-t)+7 \right) = 14+\frac{1}{3}$$

$\rightsquigarrow$ *strategy:* wait in ℓ_0, and when $t=\frac{4}{3}$, go to ℓ_1

Optimal reachability in weighted timed games (1)

This topic has been fairly hot these last fifteen years...

[LMM02,ABM04,BCFL04,BBR05,BBM06,BLMR06,Rut11,HIM13,BGK+14]

[LMM02] La Torre, Mukhopadhyay, Murano. Optimal-reachability and control for acyclic weighted timed automata *(TCS@02)*.
[ABM04] Alur, Bernardsky, Madhusudan. Optimal reachability in weighted timed games *(ICALP'04)*.
[BCFL04] Bouyer, Cassez, Fleury, Larsen. Optimal strategies in priced timed game automata *(FSTTCS'04)*.
[BBR05] Brihaye, Bruyère, Raskin. On optimal timed strategies *(FORMATS'05)*.
[BBM06] Bouyer, Brihaye, Markey. Improved undecidability results on weighted timed automata *(Information Processing Letters)*.
[BLMR06] Bouyer, Larsen, Markey, Rasmussen. Almost-optimal strategies in one-clock priced timed automata *(FSTTCS'06)*.
[Rut11] Rutkowski. Two-player reachability-price games on single-clock timed automata *(QAPL'11)*.
[HIM13] Hansen, Ibsen-Jensen, Miltersen. A faster algorithm for solving one-clock priced timed games *(CONCUR'13)*.
[BGK+14] Brihaye, Geeraerts, Krishna, Manasa, Monmege, Trivedi. Adding Negative Prices to Priced Timed Games *(CONCUR'14)*.

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Tree-like weighted timed games can be solved in 2EXPTIME.

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[LMM02]

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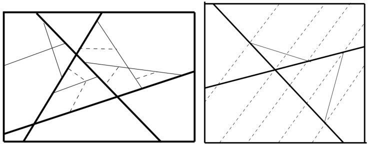
[ABM04,BCFL04]

Depth-k weighted timed games can be solved in EXPTIME. There is a symbolic algorithm to solve weighted timed games **with a strongly non-Zeno cost**.



Optimal reachability in weighted timed games (2)

[BBR05,BBM06,BJM15]

In weighted timed games, the optimal cost (and the value) cannot be computed, as soon as games have three clocks or more.

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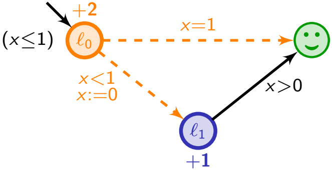
[BLMR06,Rut11,HIM13,BGK+14]

Turn-based optimal timed games are decidable in EXPTIME (resp. PTIME) when automata have a single clock (resp. with two rates). They are PTIME-hard.

What is easier with a single clock?

- Memoryless strategies can be non-optimal...



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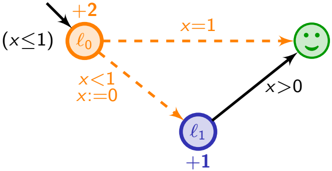

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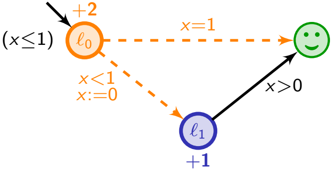

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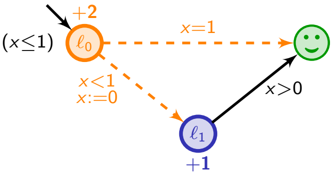

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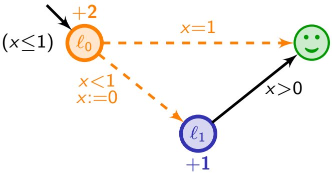

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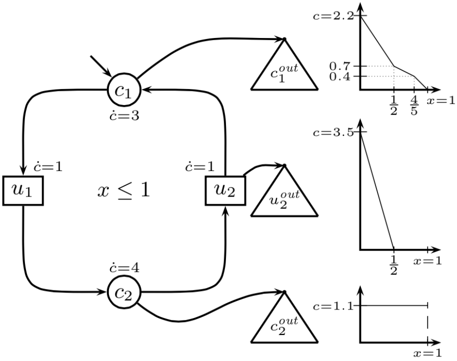
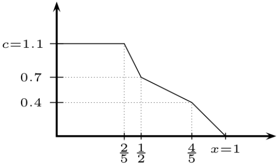
- Rather involved proofs of correctness





$$\sigma(c_2, x) = \begin{cases} c_2^{out} & \text{if } 0 \leq x < 2/5 \\ c_2 & \text{if } 2/5 \leq x < 1/2 \\ u_2 & \text{if } 1/2 \leq x \leq 1 \end{cases}$$

Computing the optimal cost: why is that hard?

Given two clocks x and y, we can check whether $y = 2x$.

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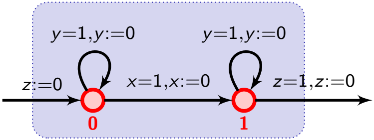
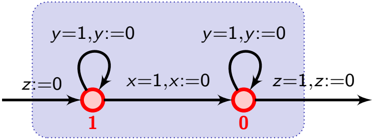
$\mathsf{Add}^+(x)$



The cost is increased by x_0

$\mathsf{Add}^-(x)$



The cost is increased by $1-x_0$

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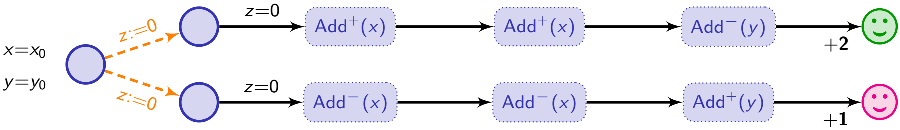

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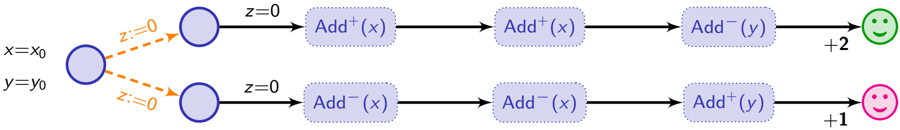

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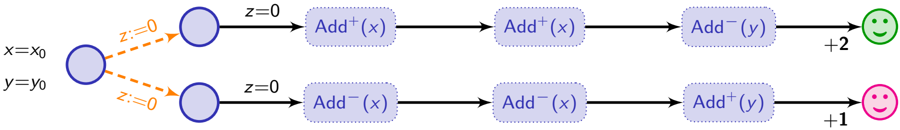

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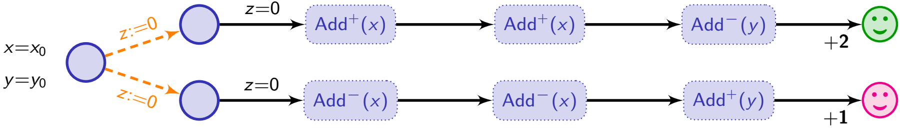

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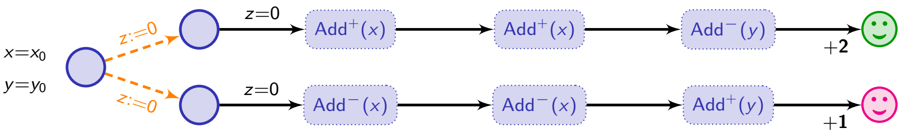

- In 🙂 (green), cost $= 2x_0 + (1 - y_0) + 2$
  In 🙂 (pink), cost $= 2(1 - x_0) + y_0 + 1$
- if $y_0 < 2x_0$, player 2 chooses the first branch: cost > 3
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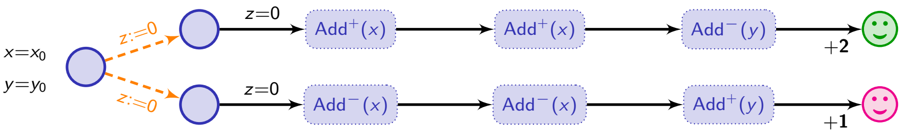

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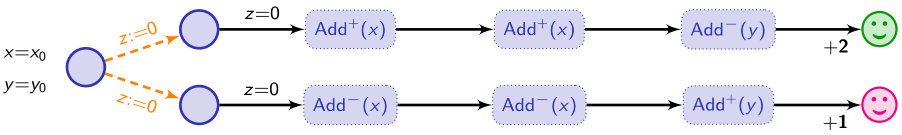

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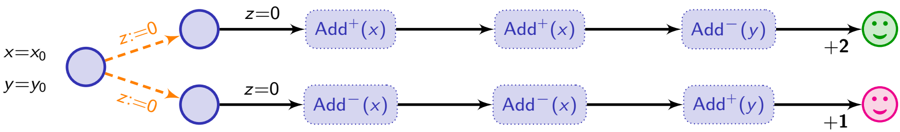

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  $\rightsquigarrow$ player 2 can enforce cost $3 + |y_0 - 2x_0|$
- Player 1 has a winning strategy with cost ≤ 3 iff $y_0 = 2x_0$

Computing the optimal cost: why is that hard?

Player 1 will simulate a two-counter machine:

- each instruction is encoded as a module;
- the counter values c_1 and c_2 are encoded by two clocks:

$$x = \frac{1}{2^{c_1}} \qquad \text{and} \qquad y = \frac{1}{2^{c_2}}$$

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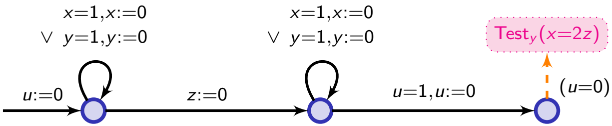
Globally, $(x \leq 1, y \leq 1, u \leq 1)$



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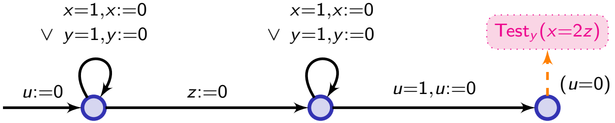

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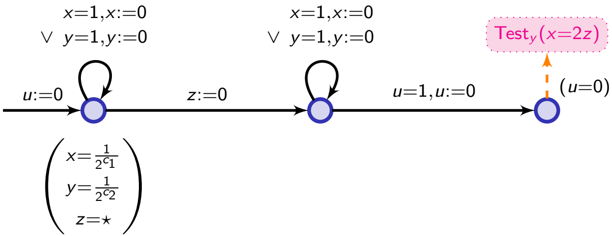

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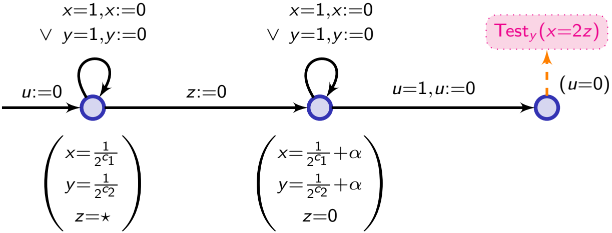

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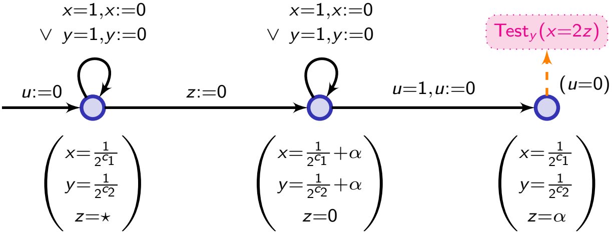

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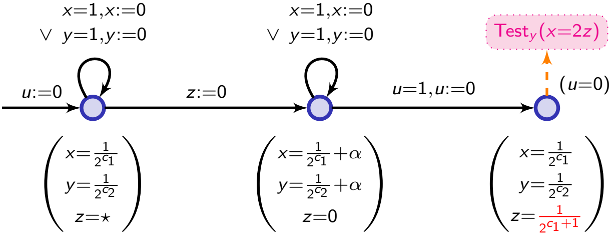
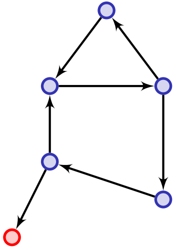


Shape of the reduction



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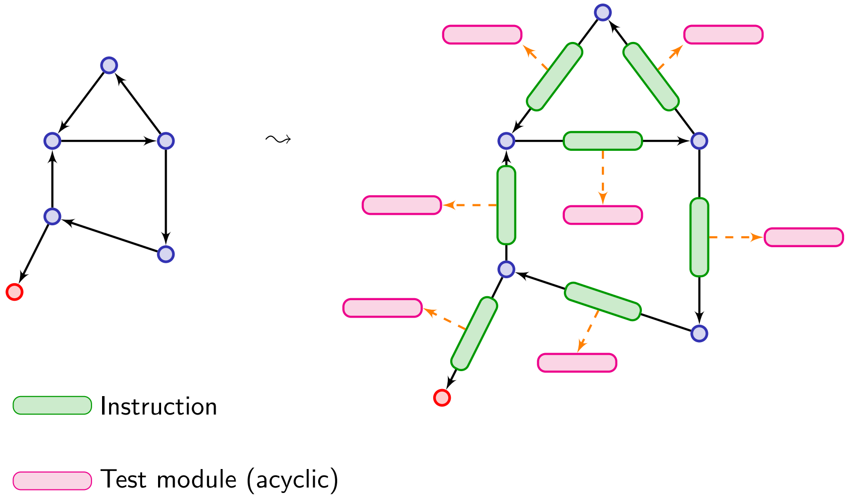

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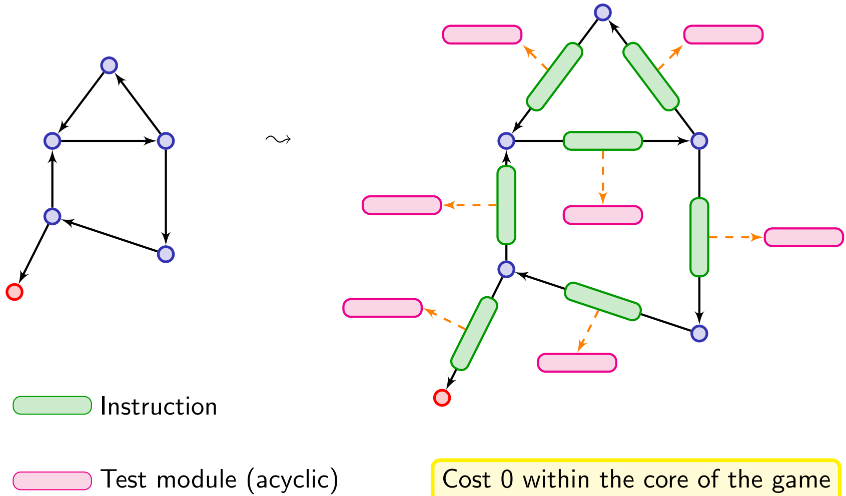

Cost 0 within the core of the game

Are we done?

Are we done? No!

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Optimal cost is computable...

... when cost is strongly non-zeno. [AM04,BCFL04]

There is $\kappa > 0$ s.t. for every region cycle C, for every real run ϱ read on C,

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Note: In both cases, we can assume $\kappa = 1$.

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Approximation of the optimal cost

Theorem

Let $\mathcal{G}$ be a weighted timed game, in which the cost is almost-strongly non-zeno. For every $\epsilon > 0$, one can compute:

- two values v_ϵ^- and v_ϵ^+ such that

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Skip approximation scheme

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$\rightsquigarrow$ This is not possible here
There might be runs with prefixes of arbitrary length and cost 0 (e.g. the game of the undecidability proof)

Idea for approximation

Idea

Only partially unfold the game:

- Keep components with cost 0 untouched – we call it the kernel
- Unfold the rest of the game

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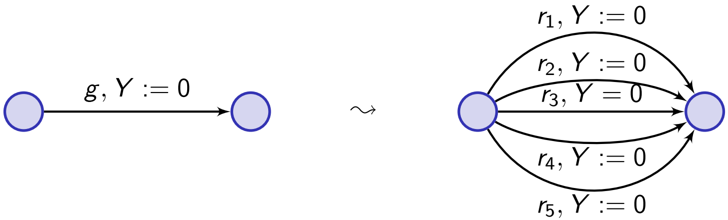
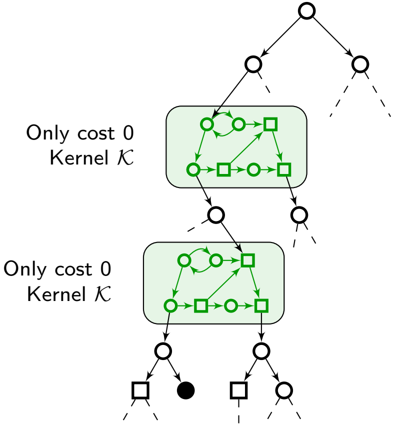
First: split the game along regions!



Semi-unfolding



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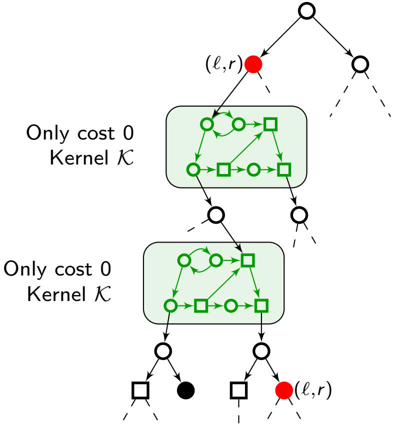

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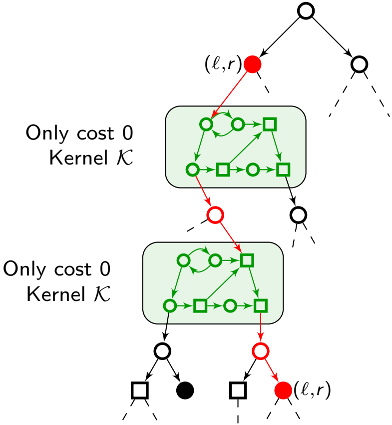

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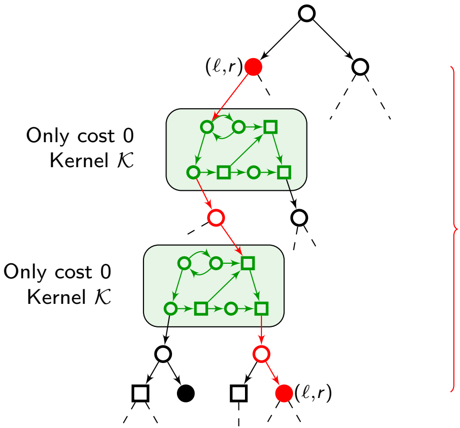
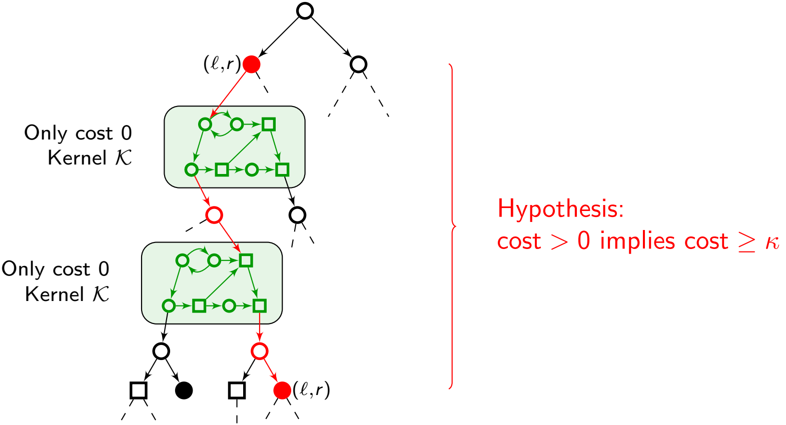


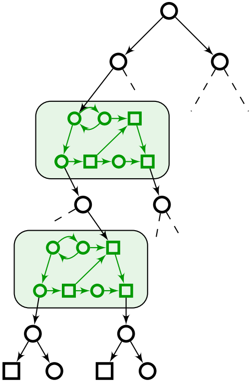
Hypothesis:
cost > 0 implies cost $\geq \kappa$

Semi-unfolding



Conclusion: we can stop unfolding the game after N steps
(e.g. $N = (M+2) \cdot |\mathcal{R}(\mathcal{A})|$, where M is a pre-computed bound on $\mathrm{optcost}_{\mathcal{G}}$)

Approximation scheme



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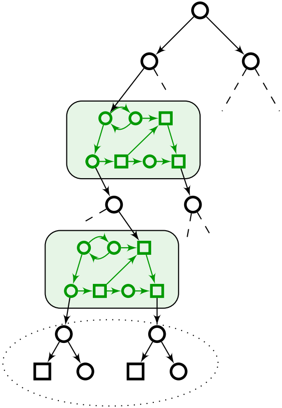

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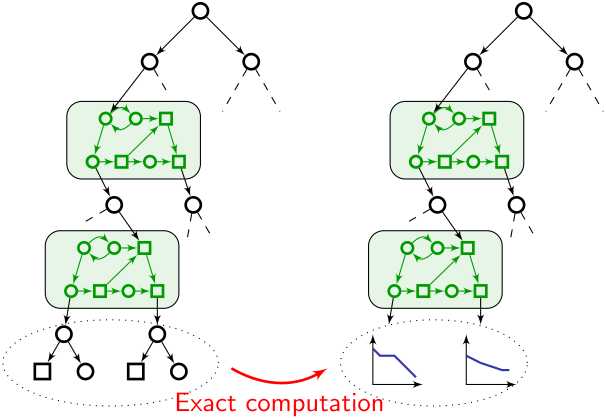

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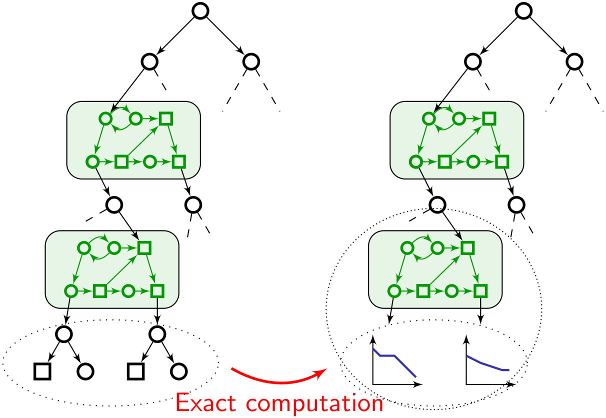

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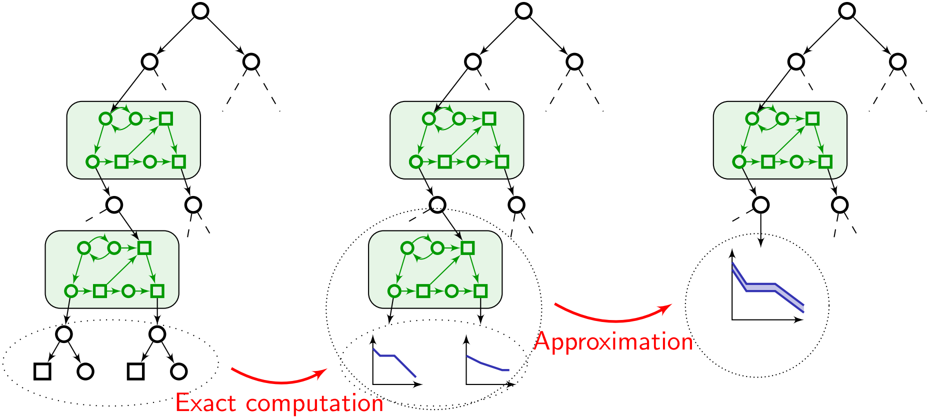

First step: Tree-like parts

$\rightsquigarrow$ Goes back to [LMM02]

[LMM02] La Torre, Mukhopadhyay, Murano. Optimal-reachability and control for acyclic weighted timed automata *(TCS@02)*.

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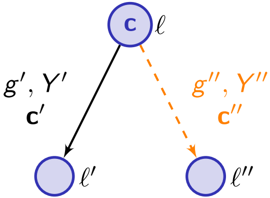

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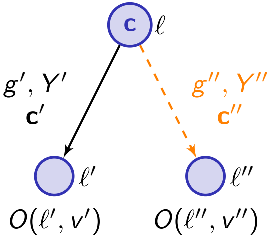

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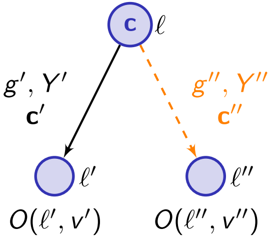

$$O(\ell, v) = \inf_{t' \mid v+t' \models g'}$$

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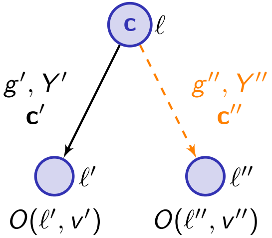

$$O(\ell, v) = \inf_{t' \mid v+t' \models g'} \max(\quad , \quad)$$

[LMM02] La Torre, Mukhopadhyay, Murano. Optimal-reachability and control for acyclic weighted timed automata *(TCS@02)*.

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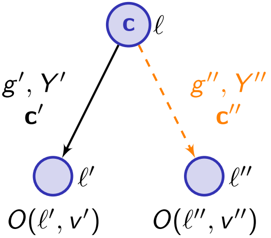

$$O(\ell, v) = \inf_{t' \mid v+t' \models g'} \max\big((\alpha), \quad \big)$$

$$(\alpha) = t'\mathbf{c} + \mathbf{c}' + O(\ell', v')$$

$v' = [Y' \leftarrow 0](v + t')$

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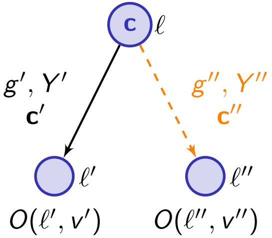

$$O(\ell, v) = \inf_{t' \mid v+t' \models g'} \max\big((\alpha), (\beta)\big)$$

$$(\alpha) = t'\mathbf{c} + \mathbf{c}' + O(\ell', v')$$

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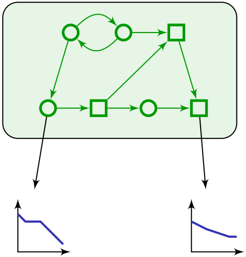
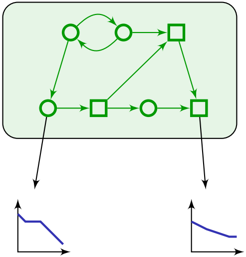
Second step: Kernels



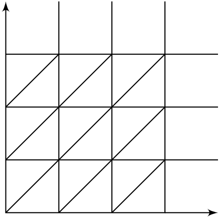
Output cost functions f

Second step: Kernels



Output cost functions f

1. Refine the regions such that f differs of at most ϵ within a small region



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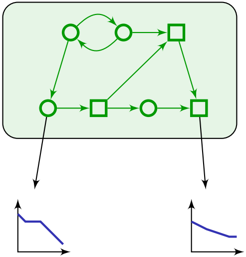

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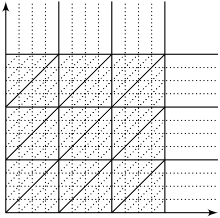

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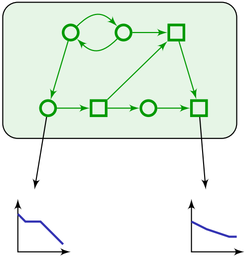

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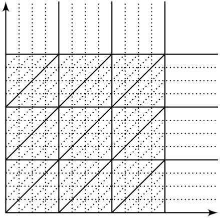

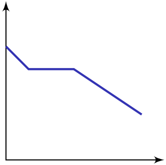

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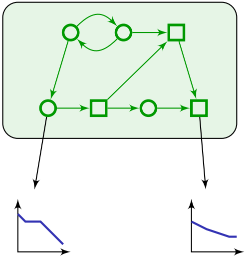

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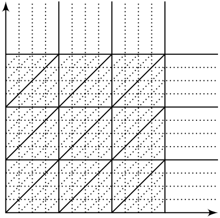
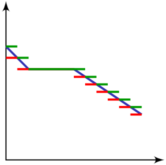
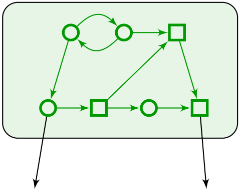


2. Under- and over-approximate by piecewise constant functions f_ϵ^- and f_ϵ^+



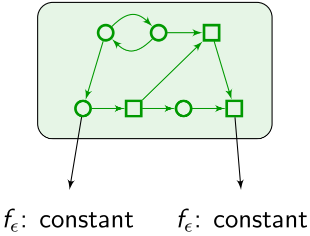
Second step: Kernels

3. Refine/split the kernel along the new small regions and fix f_ϵ^- or f_ϵ^+, write f_ϵ



f_ϵ: constant f_ϵ: constant

Second step: Kernels



3. Refine/split the kernel along the new small regions and fix f_ϵ^- or f_ϵ^+, write f_ϵ
4. Since cost is 0 everywhere, the resulting game is nothing more than a reachability timed game with an order on target (output) edges (given by f_ϵ)

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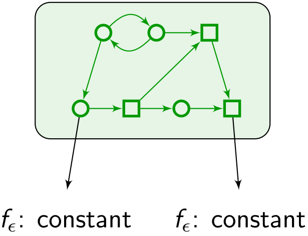

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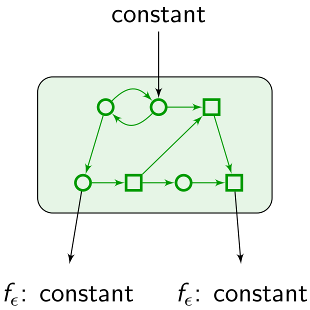

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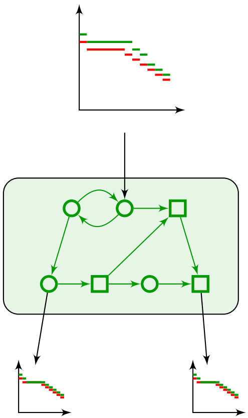

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$\rightsquigarrow$ We have computed ϵ-approximations of the optimal cost, which are constant within small regions. Corresponding strategies can be inferred

Outline

TiAMo = Timed Automata Model-checker

- Development started in September 2015
- Main developer: Maximilien Colange (LSV)
- Uses some previous code by Ocan Sankur (IRISA)

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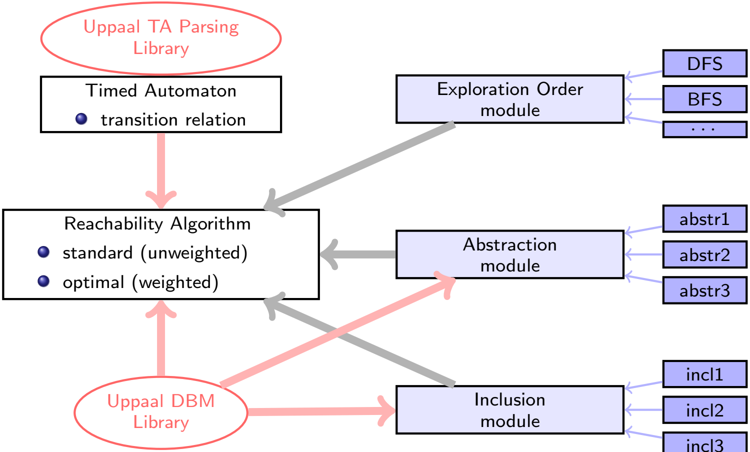
What TiAMo targets

- Be a platform for experiments (open source!)
- Assert and compare algorithms

`https://git.lsv.fr/colange/tiamo`

TiAMo architecture



What is implemented

Exploration strategies

- BFS, DFS
- best cost first (for weighted models)
- preference-based (use a special "preference" variable in the model)
- "smart" BFS: inspired by [HT15]

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- identity (i.e. no abstraction), to be used with abstract inclusion tests
- LU-abstraction [BBLP06]

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Inclusions

- set inclusion
- weighted set inclusion [RLS06]
- abstract inclusion [HSW12]
- abstract weighted inclusion [BCM16]

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- Various metrics
  - wall-clock time
  - number of symbolic states explored
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- The Energy-optimal Task-graph Scheduling (ETS)
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Experimental results

- for mentioned weighted models
- -P (no pruning) / +P (pruning)
- $\subseteq$ (standard inclusion – no guarantee of term.) / $\sqsubseteq$ (abstract inclusion – guarantee of term.)

			# Waiting	# Passed	# stored	# tests	# succ. tests	time (s.)
ASL	+P	$\sqsubseteq$	11,820	4,785	9,324	3.7×10^{05}	13,676	0.3
		$\subseteq$	32,322	13,036	26,555	2.9×10^{06}	32,263	0.7
	-P	$\sqsubseteq$	1.7×10^{06}	1.5×10^{06}	6.9×10^{05}	8.1×10^{08}	1.2×10^{07}	312.7
		$\subseteq$	TO	TO	TO	TO	TO	TO
ETS	+P	$\sqsubseteq$	107	84	83	174	66	0.0
		$\subseteq$	664	606	590	17,684	455	0.0
VRPTW	+P	$\sqsubseteq$	6.0×10^{05}	4.8×10^{05}	5.6×10^{05}	6.2×10^{06}	1.7×10^{05}	11.3
		$\subseteq$	1.5×10^{06}	1.3×10^{06}	1.4×10^{06}	9.1×10^{07}	7.0×10^{05}	27.5
	-P	$\sqsubseteq$	1.3×10^{06}	1.3×10^{06}	1.3×10^{06}	2.5×10^{07}	7.0×10^{05}	23.9
		$\subseteq$	5.8×10^{06}	5.8×10^{06}	5.4×10^{06}	1.1×10^{09}	1.9×10^{06}	111.2
unbound.	+P	$\sqsubseteq$	14	13	14	135	3	0.0
		$\subseteq$	TO	TO	TO	TO	TO	TO
	-P	$\sqsubseteq$	14	14	14	135	3	0.0
		$\subseteq$	TO	TO	TO	TO	TO	TO

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Summary of the talk

- Overview of results concerning the optimal reachability problem in weighted timed automata and games
- Various (un)decidability + symbolic technics
- Our new tool TiAMo

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  - Apply further the idea of approximation
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- Our new tool TiAMo

Future work

- Various theoretical issues
  - Apply further the idea of approximation
  - Stochastic uncertainty
- Continue working on TiAMo
  - Implementation of (weighted) timed games
  - More applications (e.g. motion planning problems using the funnel automata approach [BMPS15])

[BMPS15] Bouyer, Markey, Perrin, Schlehuber-Caissier. Timed-Automata Abstraction of Switched Dynamical Systems Using Control Funnels *(FORMATS'15)*.