Managing resources in timed systems

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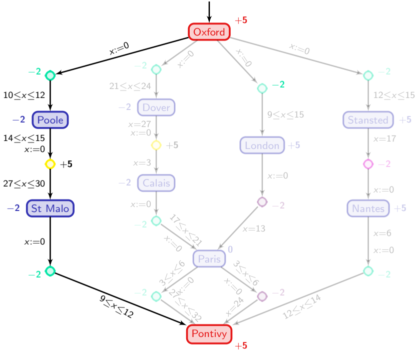
LSV, CNRS & ENS Cachan, France Aalborg University, Denmark

Dagstuhl seminar – January 2010

Can I work with my computer all the way?



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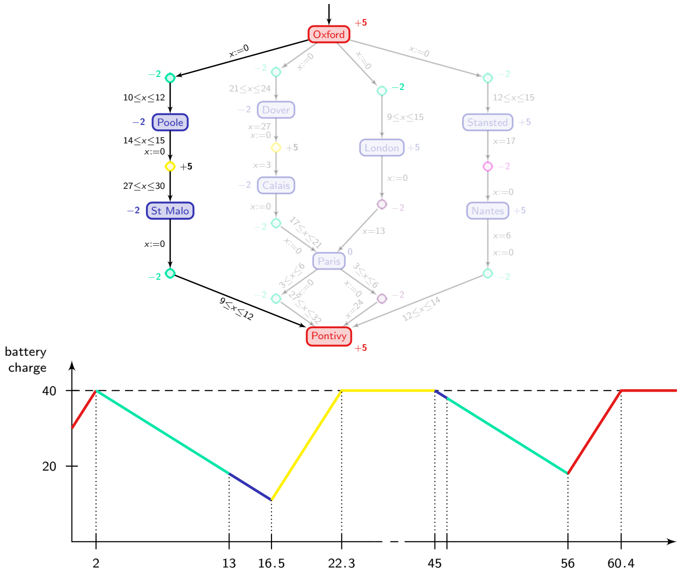
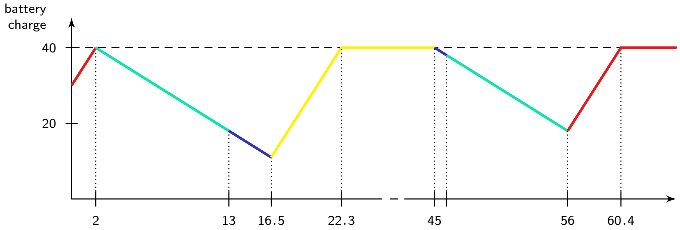


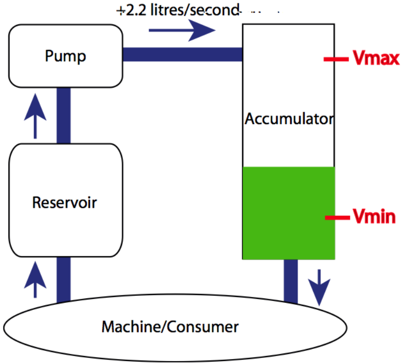
Can I work with my computer all the way?

Energy is not only consumed, but can be regained.

$\leadsto$ the aim is to continuously satisfy some energy constraints.



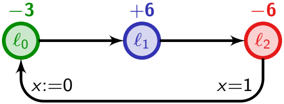
An oil pump control system



[CJL+09] Cassez, Jessen, Larsen, Raskin, Reynier. Automatic synthesis of robust and optimal controllers - An industrial case study *(HSCC'09)*.

An example of resource management

Globally $(x \leq 1)$



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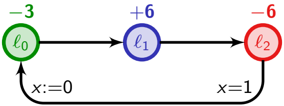

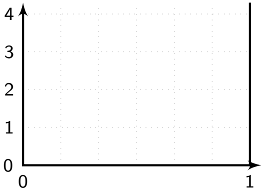

- Lower-bound problem: can we stay above 0?

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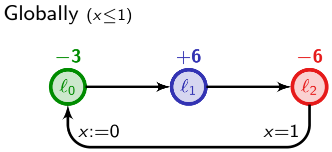

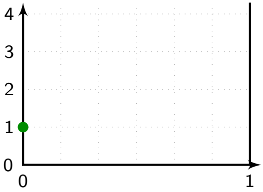

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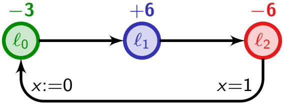

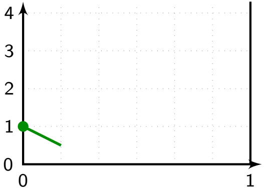

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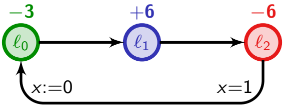

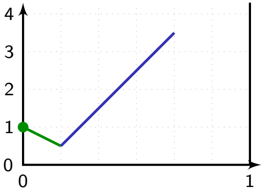

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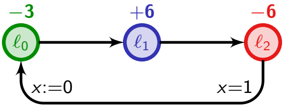

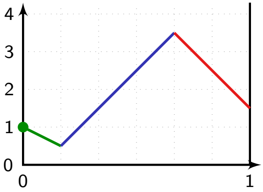

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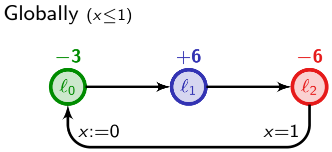

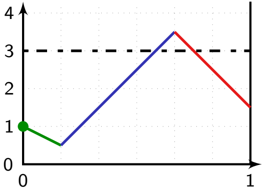

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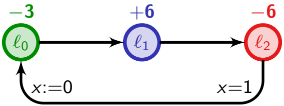

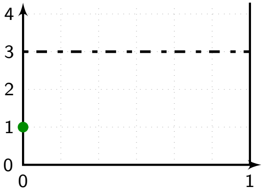

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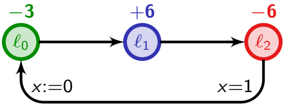

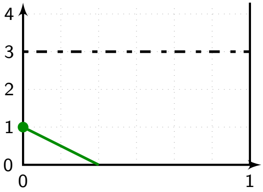

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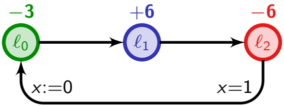

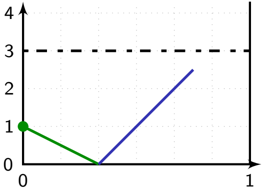

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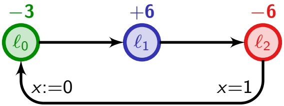

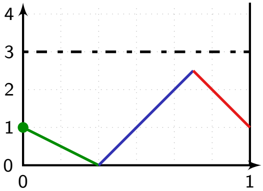

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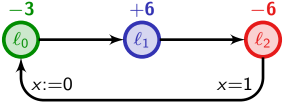

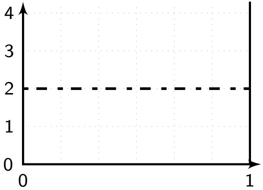

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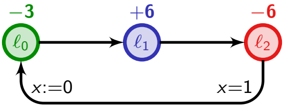

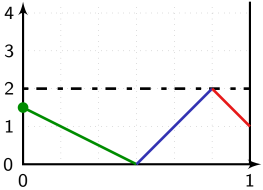

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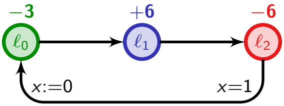

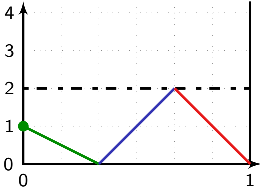

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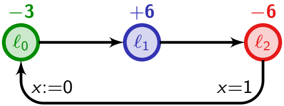

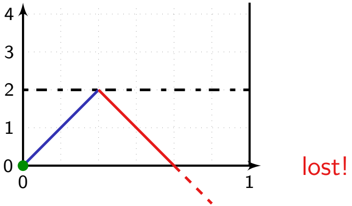

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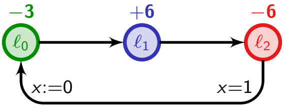

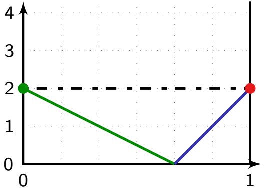

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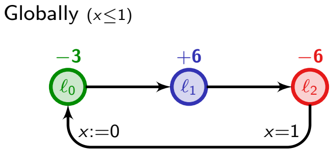

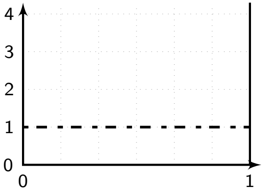

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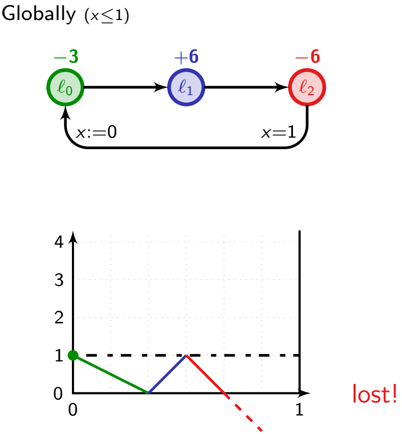

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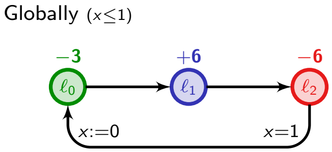
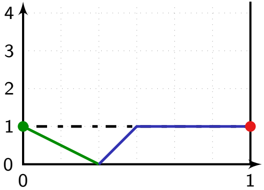
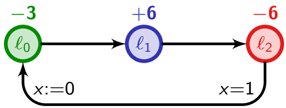


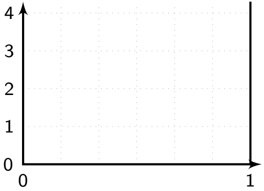


- Lower-bound problem
- Lower-upper-bound problem
- Lower-weak-upper-bound problem: can we "weakly" stay within bounds?

An example of resource management

Globally $(x \leq 1)$





- Lower–bound problem $\rightsquigarrow$ **L**
- Lower-upper-bound problem $\rightsquigarrow$ **L+U**
- Lower-weak-upper-bound problem $\rightsquigarrow$ **L+W**

Only partial results so far

0 clock!	**exist. problem**	**univ. problem**	**games**
L	$\in$ PTIME	$\in$ PTIME	$\in$ UP $\cap$ co-UP PTIME-hard
L+W	$\in$ PTIME	$\in$ PTIME	$\in$ NP $\cap$ co-NP PTIME-hard
L+U	$\in$ PSPACE NP-hard	$\in$ PTIME	EXPTIME-c.

Only partial results so far

1 clock	**exist. problem**	**univ. problem**	**games**
L	$\in$ PTIME	$\in$ PTIME	?
L+W	$\in$ PTIME	$\in$ PTIME	?
L+U	?	?	undecidable

Only partial results so far

n clocks	**exist. problem**	**univ. problem**	**games**
L	?	?	?
L+W	?	?	?
L+U	?	?	undecidable

Back to 0 clock

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$\leadsto$ PTIME: Bellman-Ford algorithm

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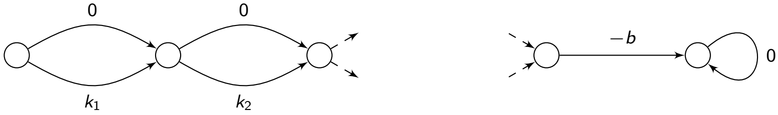
$\rightsquigarrow$ PSPACE: guess an infinite path in the graph augmented with the energy level

$\rightsquigarrow$ NP-hardness: encode SUBSET-SUM:



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$\rightsquigarrow$ EXPTIME: play the game in the graph augmented with the energy level

$\rightsquigarrow$ EXPTIME-hardness: encode COUNTDOWN-GAME [JLS07]

[JLS07] Jurdziński, Laroussinie, Sproston. Model checking probabilistic timed automata with one or two clocks *(TACAS'07)*.

0 clock: Relation with mean-payoff games

Definition

Mean-payoff games: in a weighted game graph, does there exists a strategy s.t. the mean-cost of any play is nonnegative?

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L-games and **L**$+$**W**-games are determined, and memoryless strategies are sufficient to win.

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- from mean-payoff games to **L**-games or **L**+**W**-games: play in the same game graph G with initial credit $-M \geq 0$ (where M is the sum of negative costs in G).

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- from **L**-games to mean-payoff games: transform the game as follows:

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Corollary

Mean-payoff games (and hence parity games) and **L**-games have the same complexity (log-space reducibility).

$\leadsto$ a way to improve complexity of mean-payoff games [DGR09]

[DGR09] Doyen, Gentilini, Raskin. Faster Pseudo-Polynomial Algorithms for Mean-Payoff Games.

What about 1 clock?

1 clock	**exist. problem**	**univ. problem**	**games**
L	$\in$ PTIME	$\in$ PTIME	?
L+W	$\in$ PTIME	$\in$ PTIME	?
L+U	?	?	undecidable

1 clock: **L**- and **L+W**-cases

Idea: delay in the most profitable location

$\rightsquigarrow$ the corner-point abstraction

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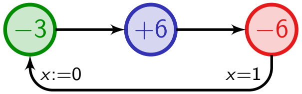

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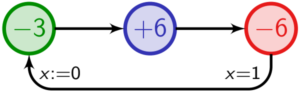

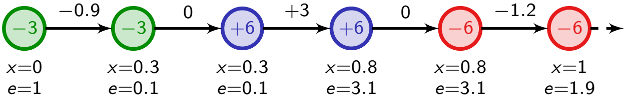

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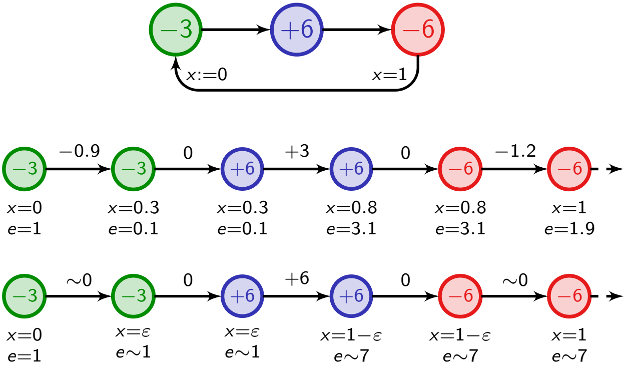

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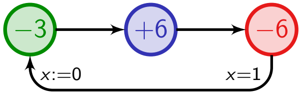

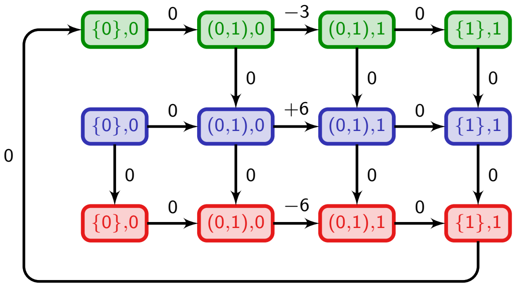

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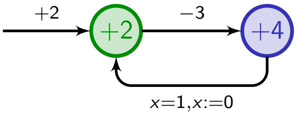
Remark

The corner-point abstraction is not correct with discrete costs.



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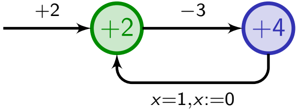

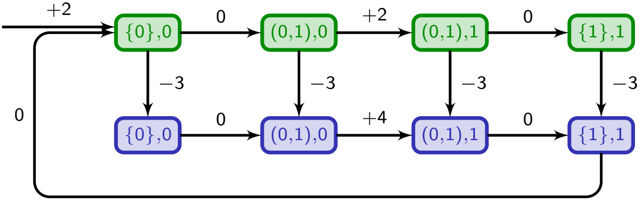

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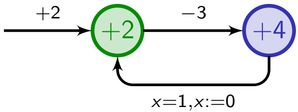

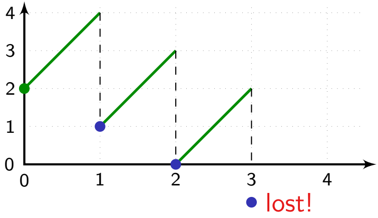

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Idea: delay in the most profitable location

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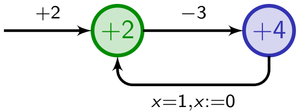

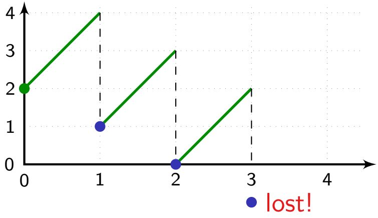

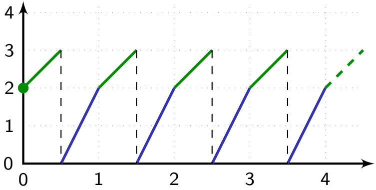

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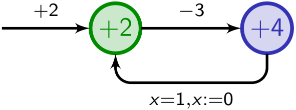

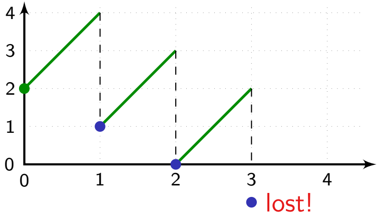

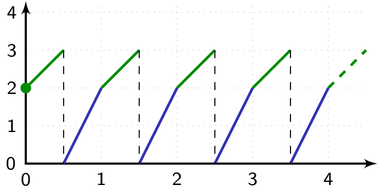

$\rightsquigarrow$ next talk by Nicolas Markey

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The backward fixpoint computation, which is correct in the limit, does not terminate in general.

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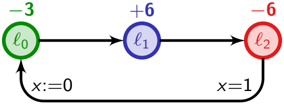

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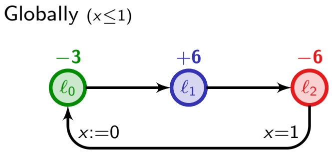

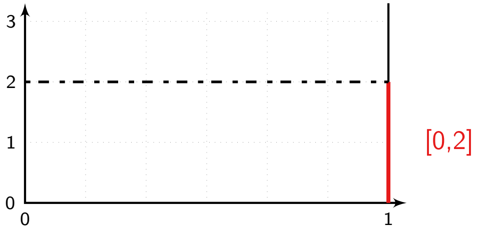

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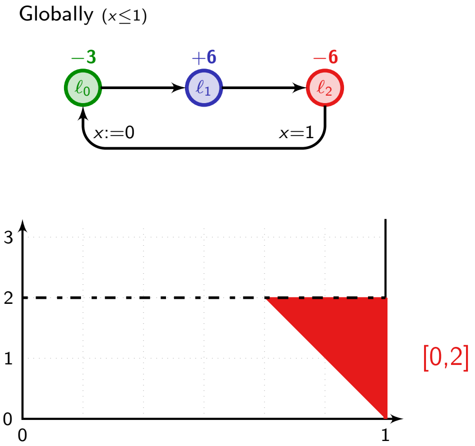

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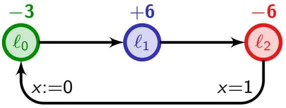

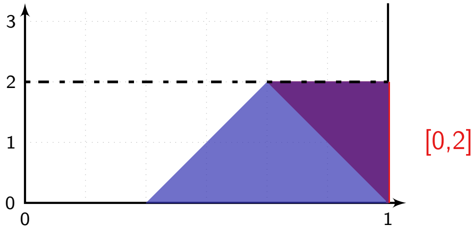

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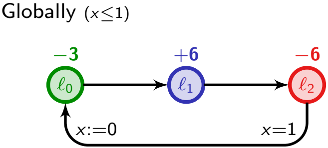

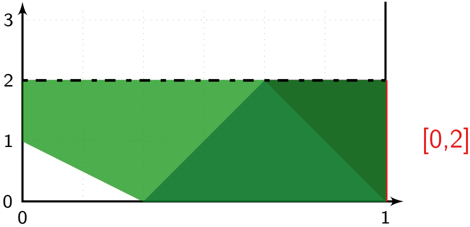

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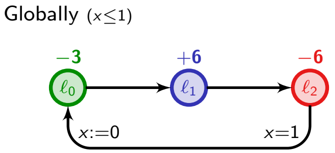

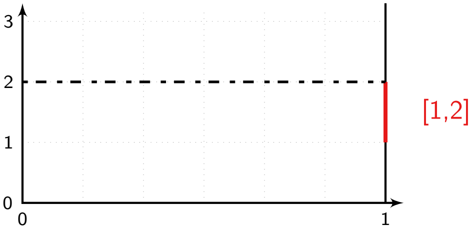

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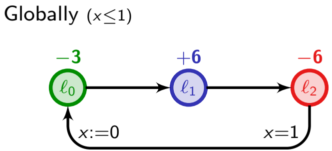

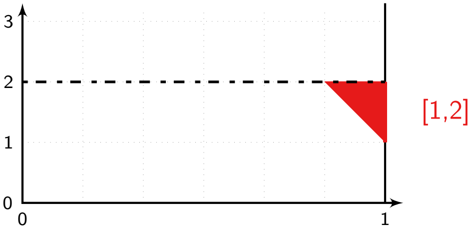

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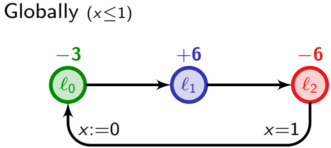

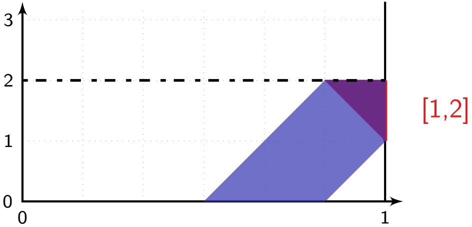

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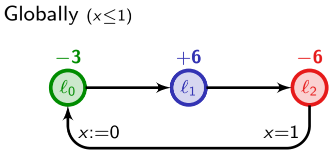

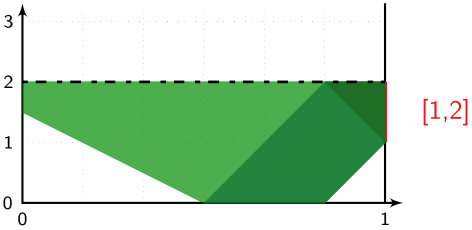

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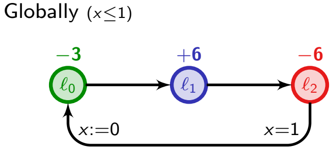

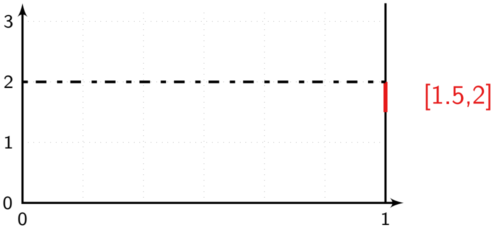

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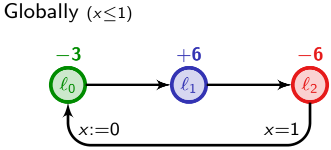

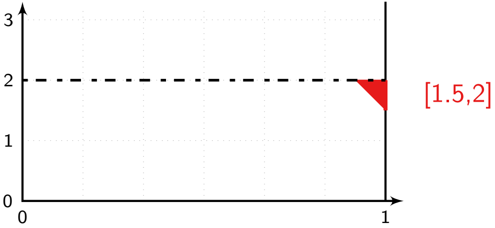

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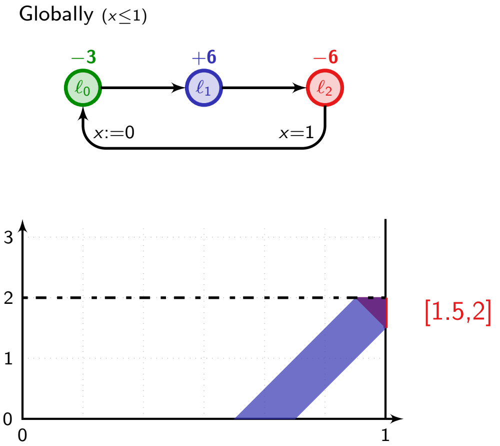

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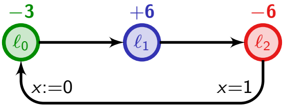

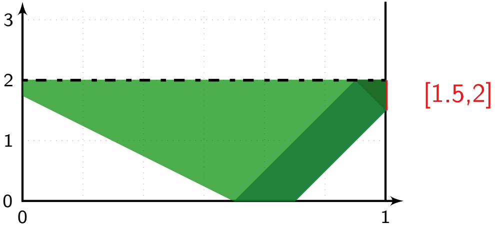

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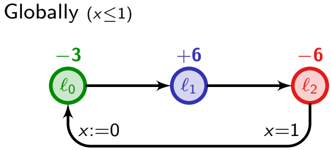

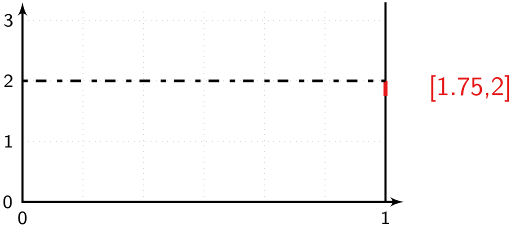

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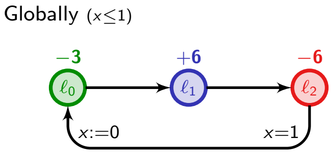

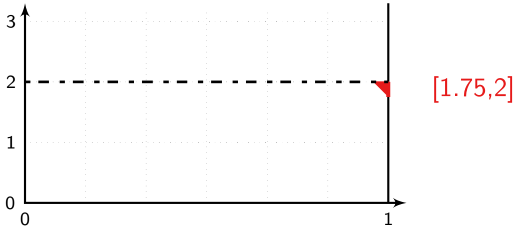

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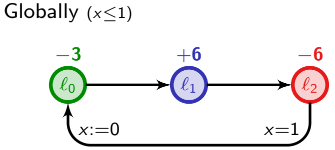

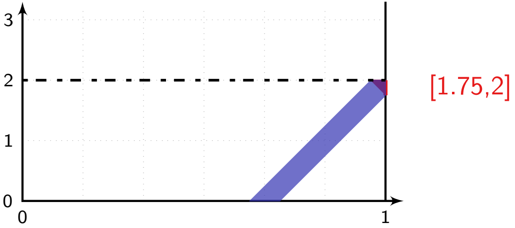

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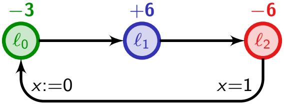

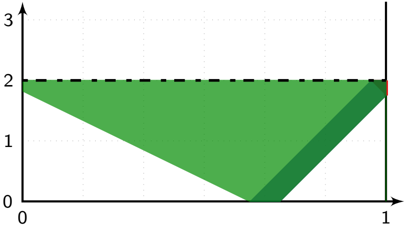

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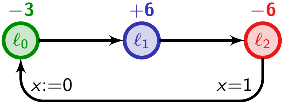

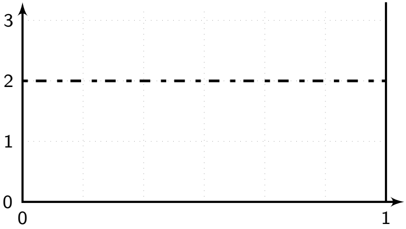

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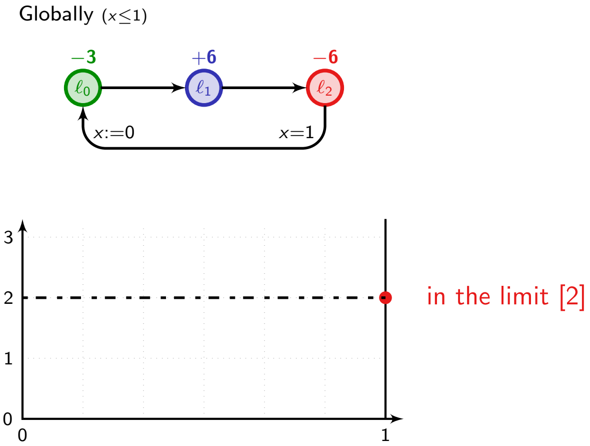

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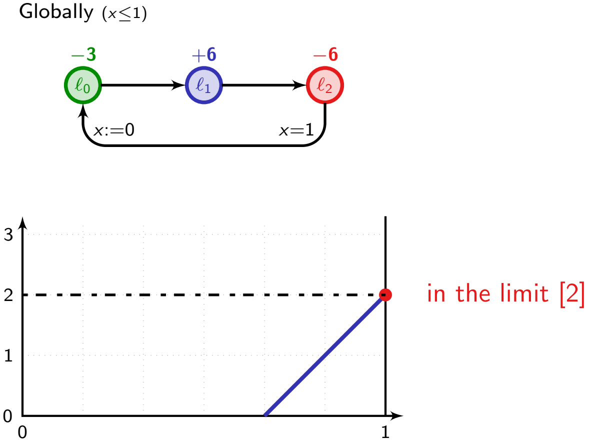

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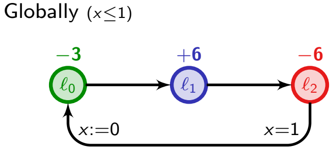

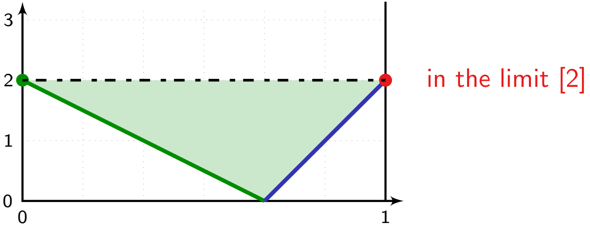

1 clock: **L+U**-games

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We encode the behaviour of a two-counter machine:

- each instruction is encoded as a module;
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$$e = 5 - \frac{1}{2^{c_1} \cdot 3^{c_2}}$$

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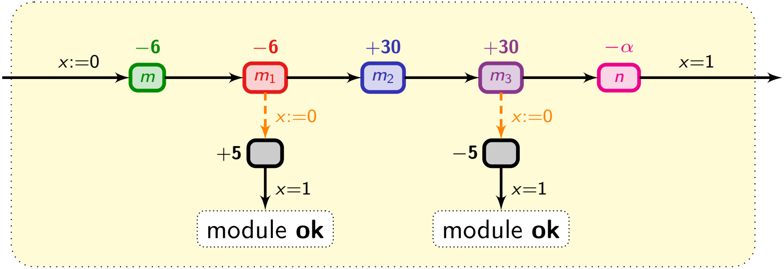
$\rightsquigarrow$ We present a generic construction for incrementing/decrementing the counters.

Generic module for incrementing/decrementing



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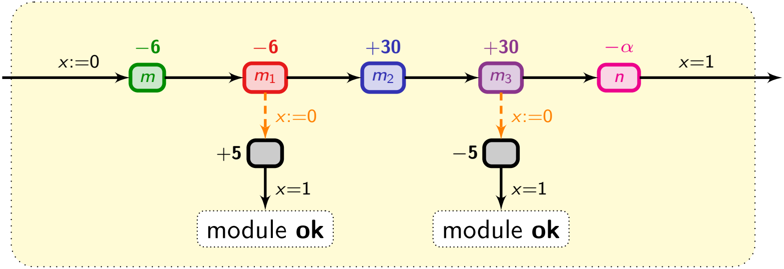

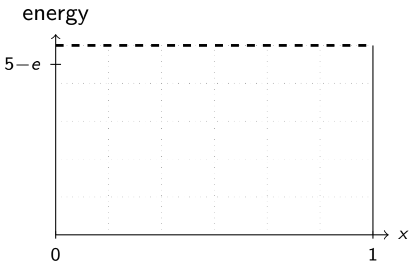

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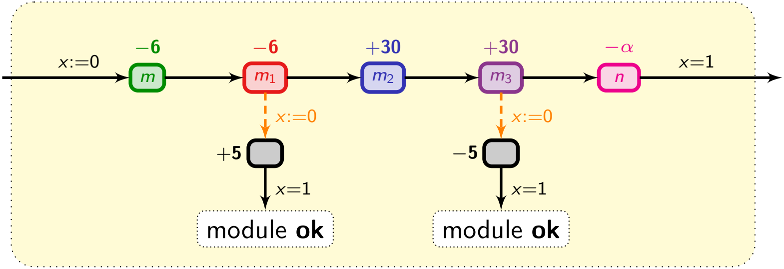

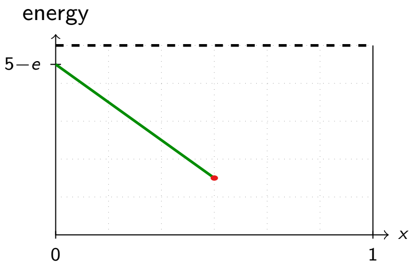

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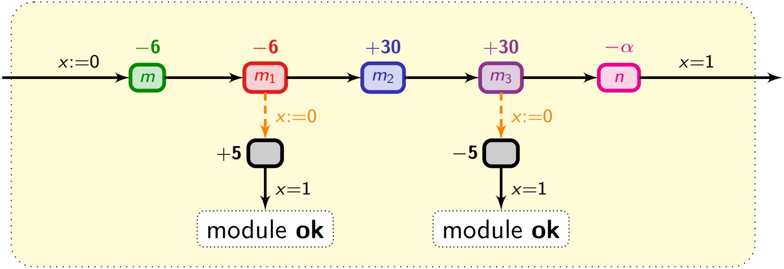

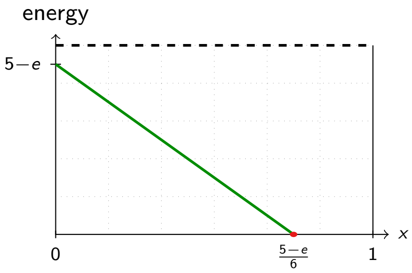

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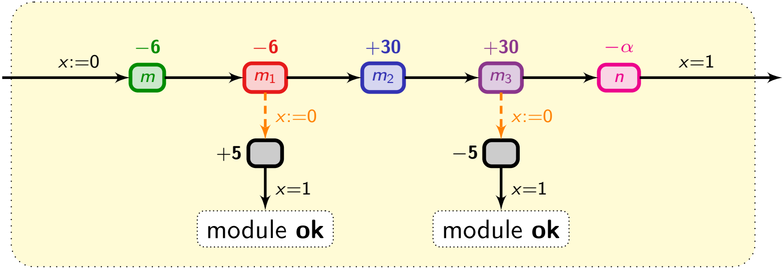

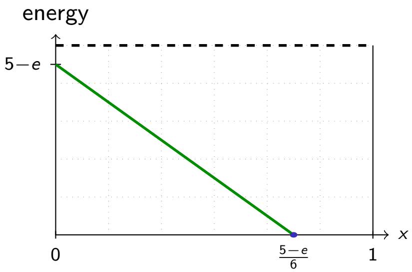

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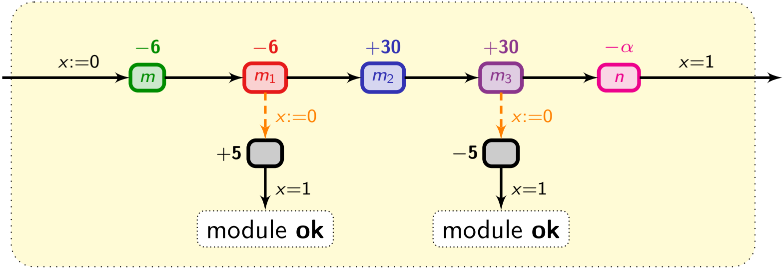

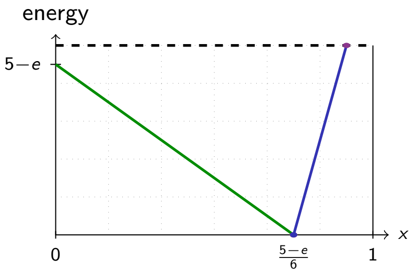

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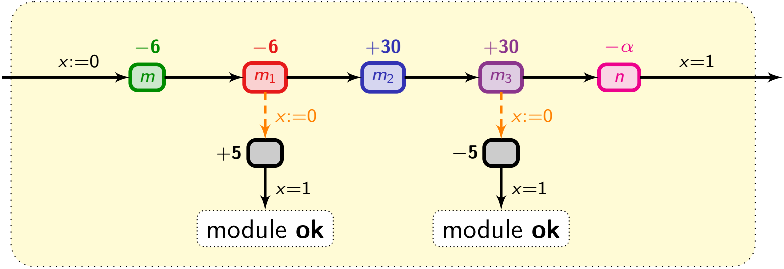

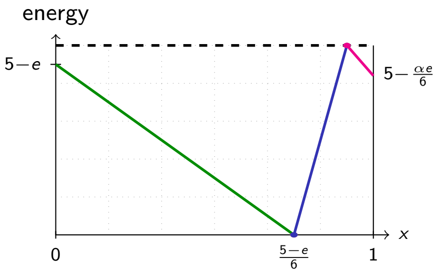

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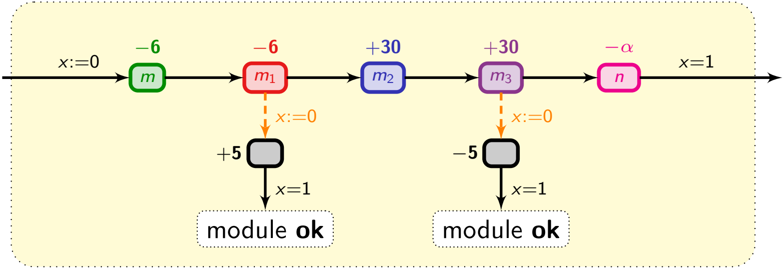

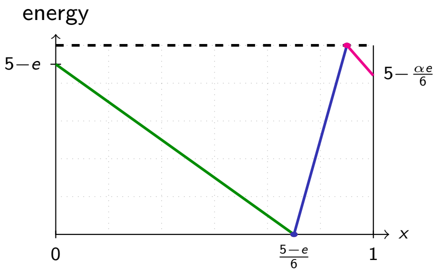

- α=3: increment c_1
- α=2: increment c_2
- α=12: decrement c_1
- α=18: decrement c_2

Conclusion & ongoing/future work

- Extension of weighted/priced (timed) automata with negative costs:
  - three natural problems related to the management of resources;
  - reasonable complexity in the untimed case;
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  - reasonable complexity in the untimed case;
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- Many open problems:
  - in the untimed case:
    - existential problem with interval constraints: the problem is related to reachability in 2-clock timed automata;
    - games with lower-bound constraint: the problem is equivalent to the mean-payoff game problem.
  - in the timed case:
    - many open questions for the 1-clock case, and no results for the general case.

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- Many open problems:
  - in the untimed case:
    - existential problem with interval constraints: the problem is related to reachability in 2-clock timed automata;
    - games with lower-bound constraint: the problem is equivalent to the mean-payoff game problem.
  - in the timed case:
    - many open questions for the 1-clock case, and no results for the general case.

- Other cost functions? $\leadsto$ see next talk