Soutenance d'habilitation

"From Qualitative to Quantitative Analysis of Timed Systems"

Patricia Bouyer

January 12, 2009

Outline

Context: verification of timed systems

- What are timed systems?

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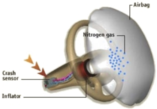

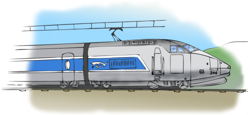

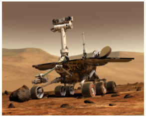

Context: verification of timed systems

- What are timed systems?
- What we want to avoid

Context: verification of timed systems

- What are timed systems?
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Radiothérapie : nouvel incident au CHU de Toulouse

Jeudi 24 mai 2007

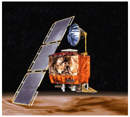
Après le centre hospitalier Jean Monnet d'Epinal, c'est au tour du CHU de Toulouse-Rangueil d'avouer la survenue d'une surexposition lors d'un traitement par radiothérapie. L'appareil de radiothérapie impliqué a été installé en avril 2006. L'erreur de calibration de l'appareil a été détectée lors du contrôle technique systématique appliqué à tous les appareils de radiothérapie.

Radiotherapy in Toulouse, 2007

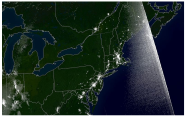
Ariane 5, 1996

Atlanta airport, 2008

Bug Pentium, 1994



Mars climate obs., 1998

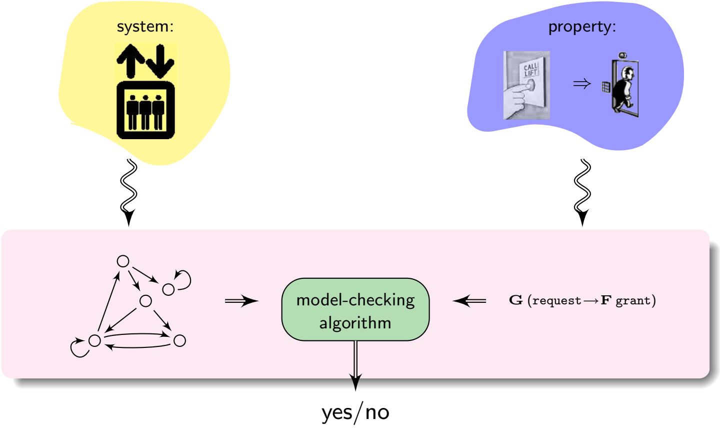


Blackout, 2003

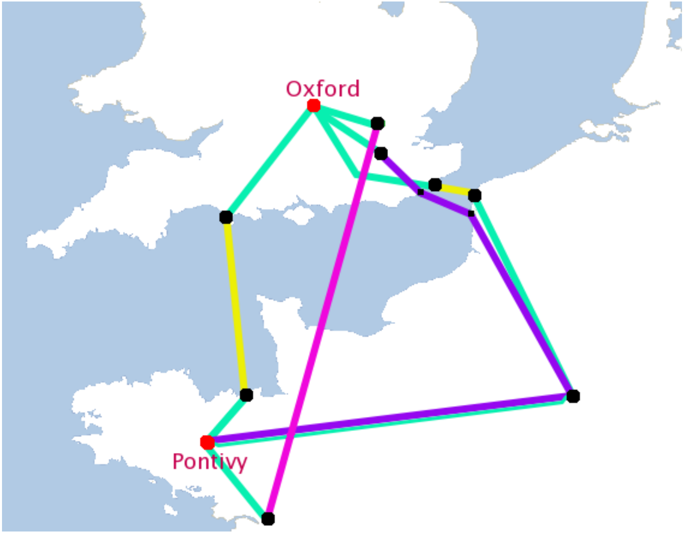
Context: verification of timed systems

- What are timed systems?
- What we want to avoid

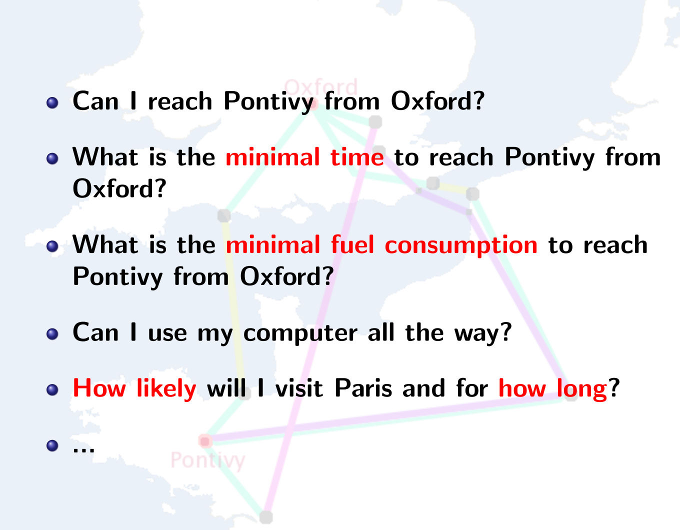
⇝ Verification by model-checking



A running example

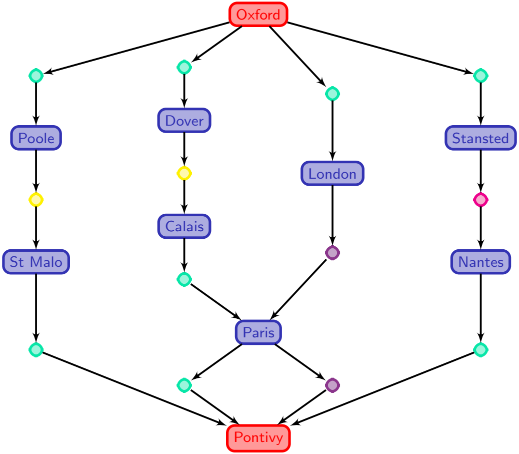


A running example: natural questions

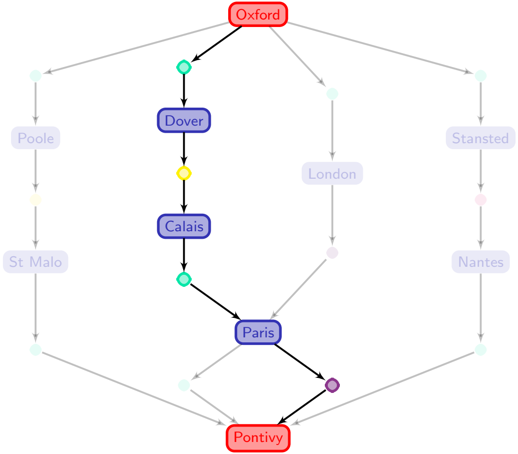


- **Can I reach Pontivy from Oxford?**
- **What is the minimal time to reach Pontivy from Oxford?**
- **What is the minimal fuel consumption to reach Pontivy from Oxford?**
- **Can I use my computer all the way?**
- **How likely will I visit Paris and for how long?**
- **...**

A first model of the system

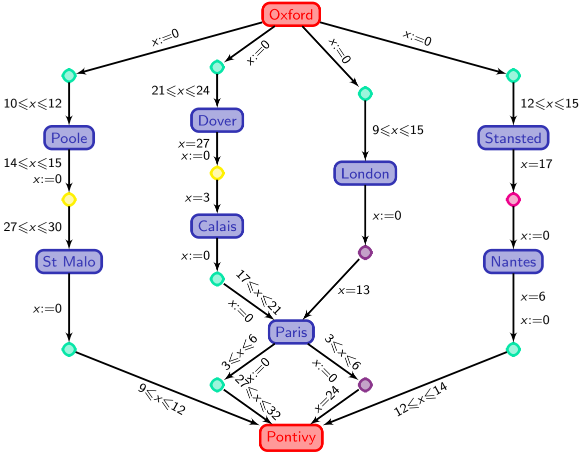


Can I reach Pontivy from Oxford?

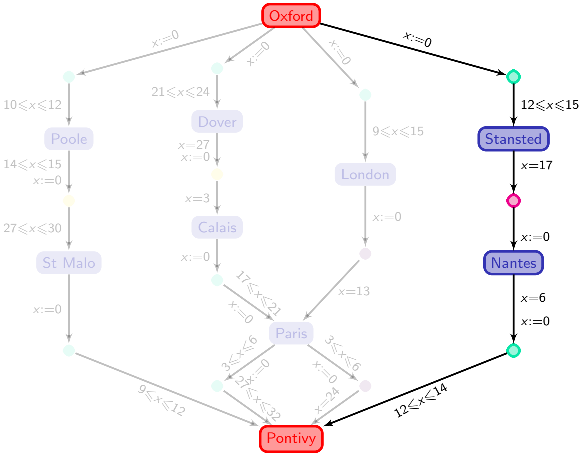


This is a reachability question in a finite graph: Yes, I can!

A second model of the system

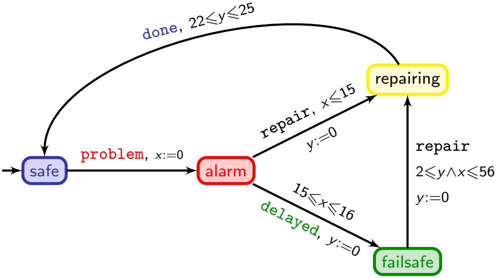


How long will that take?



It is a reachability (and optimization) question
in a timed automaton: at least 350mn = 5h50mn!

The timed automaton model: an example



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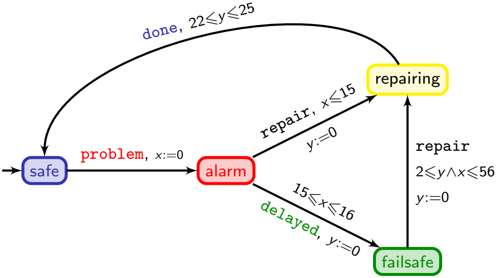

	safe
x	0
y	0

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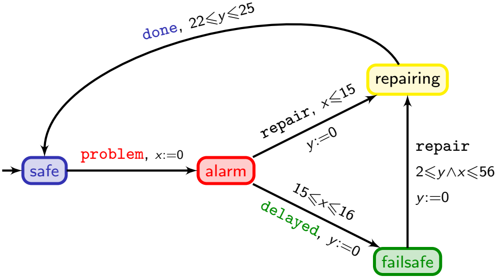

	safe	$\xrightarrow{23}$	safe
x	0		23
y	0		23

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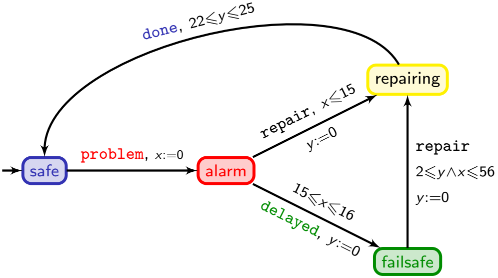

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\text{problem}}$	alarm
x	0		23		0
y	0		23		23

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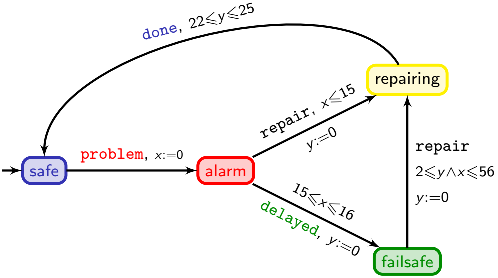
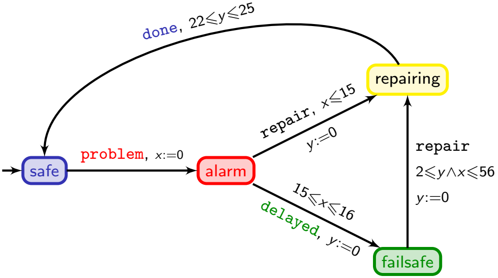


	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm
x	0		23		0		15.6
y	0		23		23		38.6

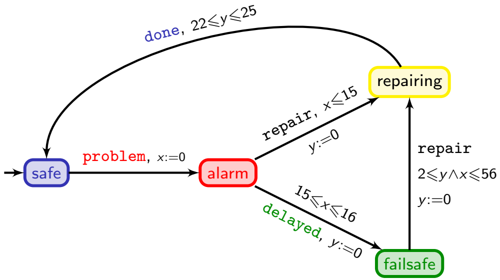
The timed automaton model: an example



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe
⋯	15.6
	0

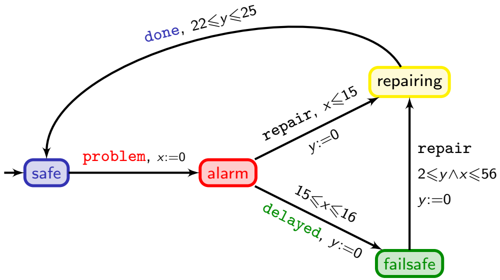
The timed automaton model: an example



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	$\cdots$
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe
$\cdots$	15.6		17.9
	0		2.3

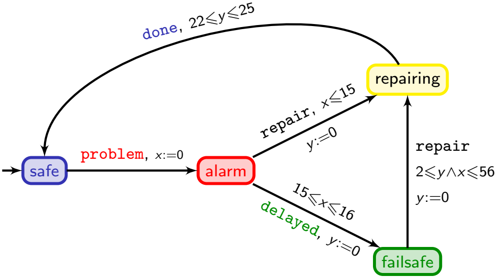
The timed automaton model: an example



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\text{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\text{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	···
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\text{repair}}$	repairing
···	15.6		17.9		17.9
	0		2.3		0

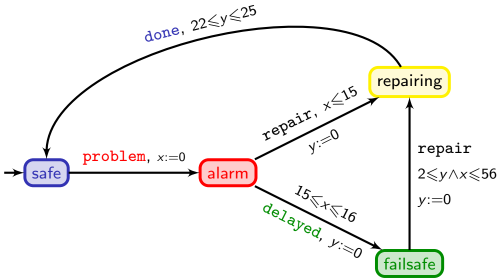
The timed automaton model: an example



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	$\cdots$
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing
$\cdots$	15.6		17.9		17.9		40
	0		2.3		0		22.1

The timed automaton model: an example



	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	···
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\texttt{done}}$	safe
···	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

Basics of timed automata

Theorem [AD90,AD94]

The reachability problem is decidable (and PSPACE-complete) in timed automata.

[AD90] Alur, Dill. Automata for modeling real-time systems *(ICALP'90)*.
[AD94] Alur, Dill. A theory of timed automata *(Theoretical Computer Science)*.

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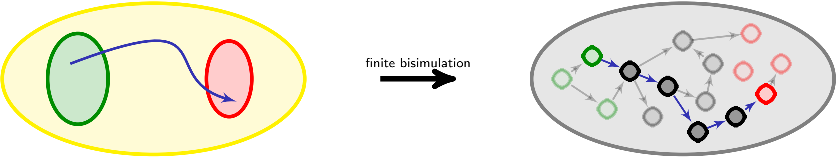

timed automaton

large (but finite) automaton
(region automaton)

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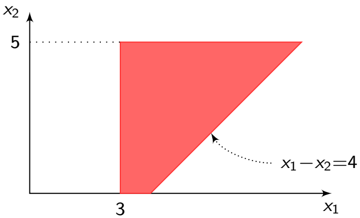
What about the practice?

- the region automaton is never computed
- instead, symbolic computations are performed using zones

Example of a zone

$$Z = (x_1 \geqslant 3) \wedge (x_2 \leqslant 5) \wedge (x_1 - x_2 \leqslant 4)$$



$$\begin{array}{c} \\ x_0 \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{array}{ccc} x_0 & x_1 & x_2 \end{array} \\ \begin{pmatrix} \infty & -3 & \infty \\ \infty & \infty & 4 \\ 5 & \infty & \infty \end{pmatrix} \end{array}$$

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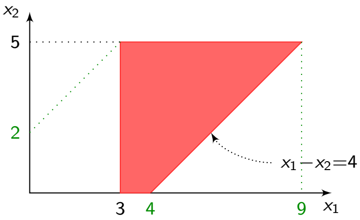
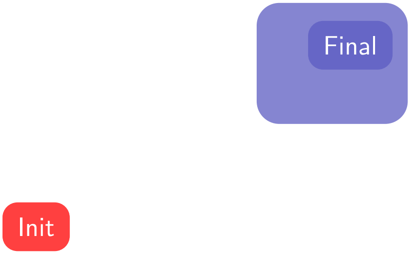


$$\begin{array}{c} \\ x_0 \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{array}{ccc} x_0 & x_1 & x_2 \end{array} \\ \begin{pmatrix} 0 & -3 & 0 \\ 9 & 0 & 4 \\ 5 & 2 & 0 \end{pmatrix} \end{array}$$

Backward computation

Backward computation



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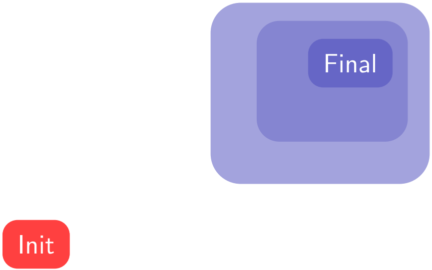

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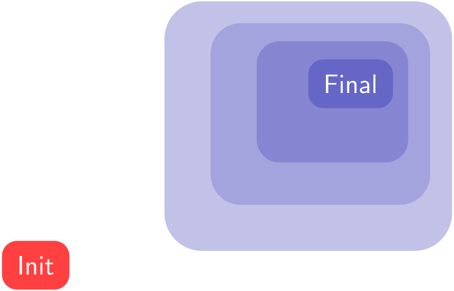


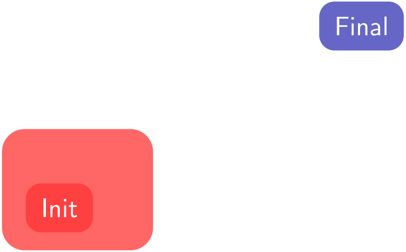
Backward computation

Backward computation

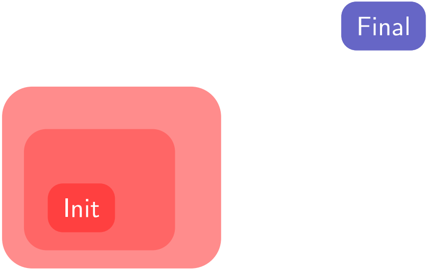
☺ the backward computation always terminates!

Forward computation

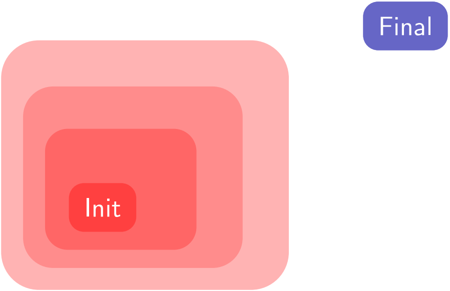
Forward computation



Forward computation



Forward computation



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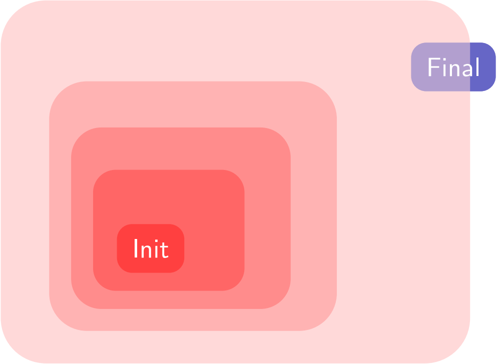

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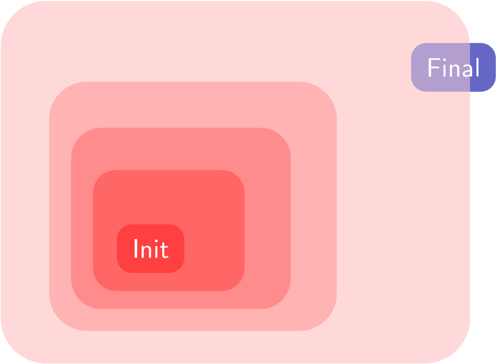
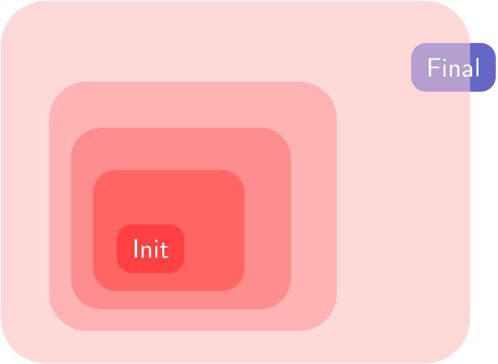


☹ the forward computation may not terminate...

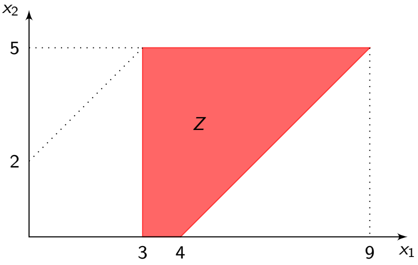
Forward computation



☹ the forward computation may not terminate...

$\rightsquigarrow$ abstractions need to be used, that ensure termination...

An abstraction: the extrapolation operator



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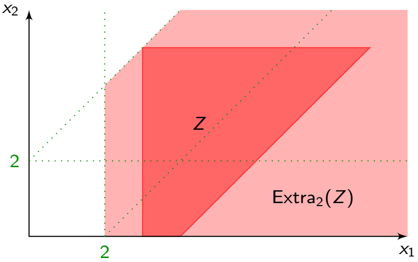
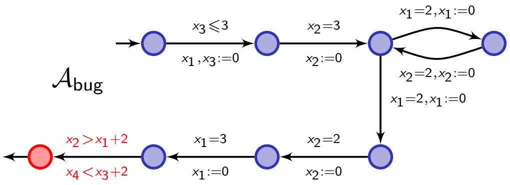
An abstraction: the extrapolation operator



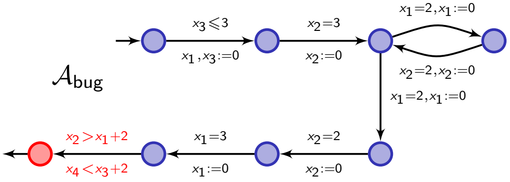
$$\begin{pmatrix} 0 & -3 & 0 \\ 9 & 0 & 4 \\ 5 & 2 & 0 \end{pmatrix} \xRightarrow{\mathsf{Extra}_2} \begin{pmatrix} 0 & -2 & 0 \\ \infty & 0 & \infty \\ \infty & 2 & 0 \end{pmatrix}$$

Results



[Bou03] Bouyer. Untameable Timed Automata! *(STACS'03)*.
[Bou04] Bouyer. Forward Analysis of Updatable Timed Automata *(Formal Methods in System Design)*.

Results



Theorem [Bou03,Bou04]

- In $\mathcal{A}_{\text{bug}}$, any extrapolation operator is incorrect!

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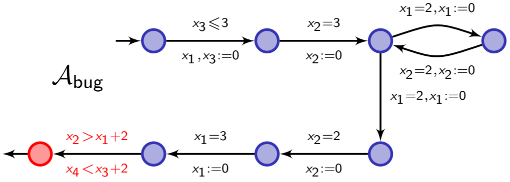

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- In $\mathcal{A}_{\text{bug}}$, any extrapolation operator is incorrect!
- The extrapolation operator is correct in *diagonal-free* timed automata.

[Bou03] Bouyer. Untameable Timed Automata! *(STACS'03)*.
[Bou04] Bouyer. Forward Analysis of Updatable Timed Automata *(Formal Methods in System Design)*.

Improving further

- the extrapolation operator can be made coarser:
  - local extrapolation constants [BBFL03];
  - distinguish between lower- and upper-bounded contraints [BBLP03,BBLP06]

$\rightsquigarrow$ has leaded to a practical improvement in UPPAAL of up to 20%!

[BBFL03] Behrmann, Bouyer, Fleury, Larsen. Static Guard Analysis in Timed Automata Verification *(TACAS'03)*.
[BBLP04] Behrmann, Bouyer, Larsen, Pelánek. Lower and Upper Bounds in Zone Based Abstractions of Timed Automata *(TACAS'04)*.
[BBLP06] Behrmann, Bouyer, Larsen, Pelánek. Lower and Upper Bounds in Zone-Based Abstractions of Timed Automata *(International Journal on Software Tools for Technology Transfer)*.

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Since then...

- further improvement due to better data structure manipulations...
- ... but no further algorithmic improvement!

[BBFL03] Behrmann, Bouyer, Fleury, Larsen. Static Guard Analysis in Timed Automata Verification *(TACAS'03)*.
[BBLP04] Behrmann, Bouyer, Larsen, Pelánek. Lower and Upper Bounds in Zone Based Abstractions of Timed Automata *(TACAS'04)*.
[BBLP06] Behrmann, Bouyer, Larsen, Pelánek. Lower and Upper Bounds in Zone-Based Abstractions of Timed Automata *(International Journal on Software Tools for Technology Transfer)*.

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Motivation

- Checking reachability properties may not be enough:

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  - basically only safety properties can be verified [ABBL98,ABBL03]

[ABBL98] Aceto, Bouyer, Burgueño, Larsen. The power of reachability testing for timed automata *(FSTTCS'98)*.
[ABBL03] Aceto, Bouyer, Burgueño, Larsen. The power of reachability testing for timed automata *(Theoretical Computer Science)*.

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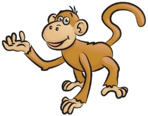
  - what about liveness properties?



"the monkey will eventually write the complete works of Shakespeare"

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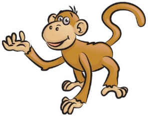

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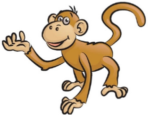

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"the machine produces 56 items per day until it needs to be repaired"

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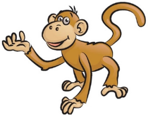

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- Need for specification languages expressing timing constraints...

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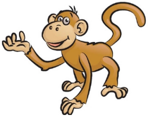

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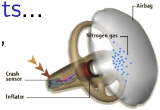
"the machine produces 56 items per day until it needs to be repaired"

- Need for specification languages expressing timing constraints...

"the airbag inflates no more than 56ms after the car crashes"

$\rightsquigarrow$ critical property



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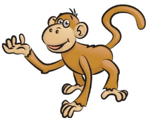

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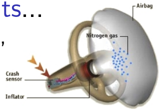

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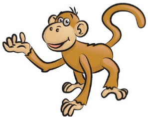

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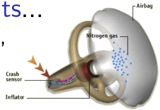

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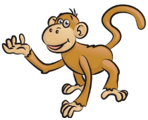

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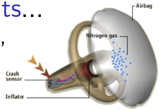

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We will focus on timed extensions of LTL [Pnu77]

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Two classical timed extensions of LTL: MTL and TPTL

MTL (Metric Temporal Logic) [Koy90]

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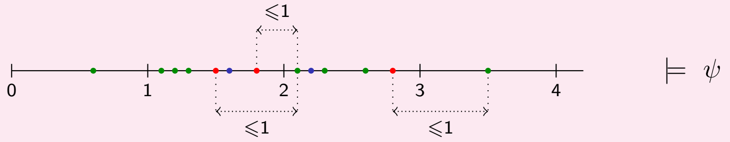

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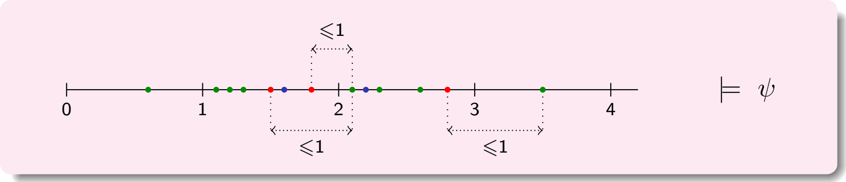

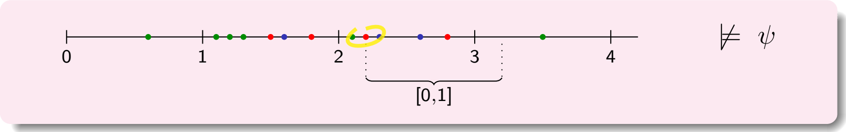

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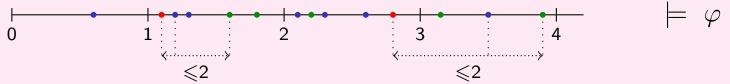

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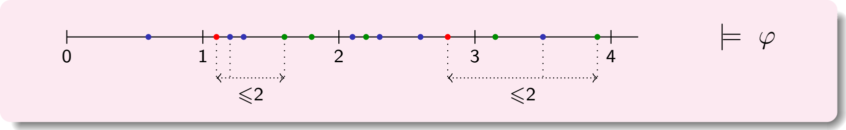

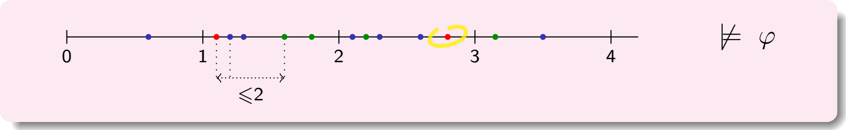

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Expressiveness of these logics

Conjecture (Alur & Henzinger, since 1990)

TPTL is strictly more expressive than MTL, and the TPTL formula

$$\varphi = \mathbf{G}\,(\bullet \rightarrow x \cdot \mathbf{F}\,(\bullet \wedge \mathbf{F}\,(\bullet \wedge x \leqslant 2)))$$

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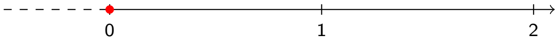

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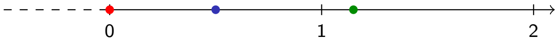

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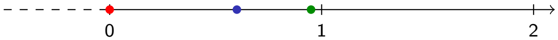

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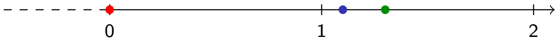

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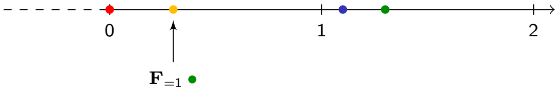

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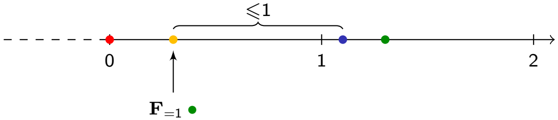

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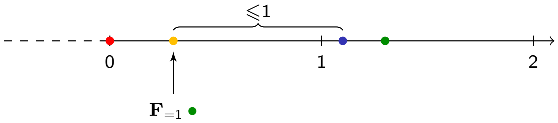

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Theorem [BCM05]

The conjecture is correct: TPTL is strictly more expressive than MTL.

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Recall: LTL+Past and LTL are equally expressive [Kam68,GPSS80].

[Kam68] Kamp. Tense logic and the theory of linear order *(PhD UCLA)*.
[GPSS80] Gabbay, Pnueli, Shelah, Stavi. On the temporal analysis of fairness *(POPL'80)*.
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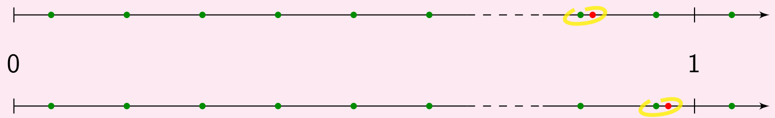

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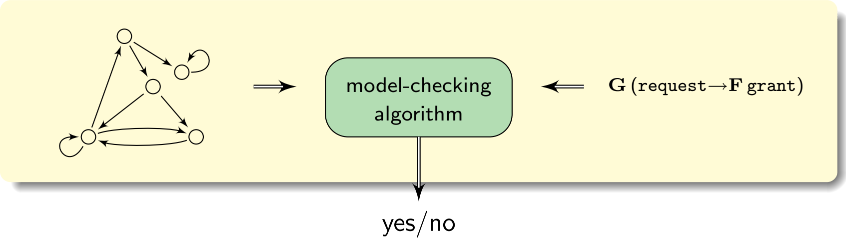

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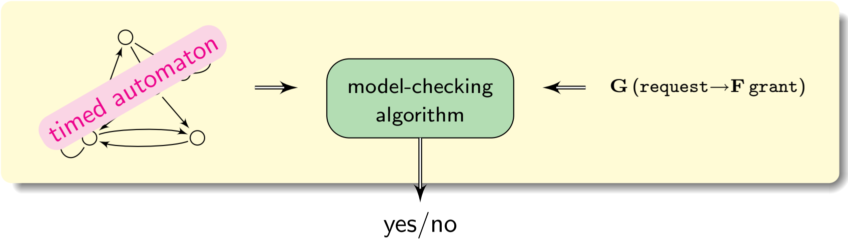

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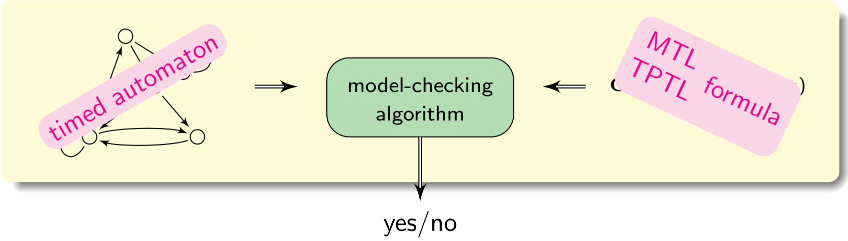

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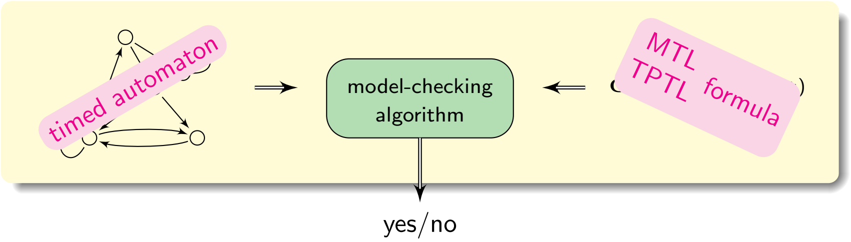

Theorem [AH94,AFH96,OW06...]

The model-checking problem is (mostly) undecidable...

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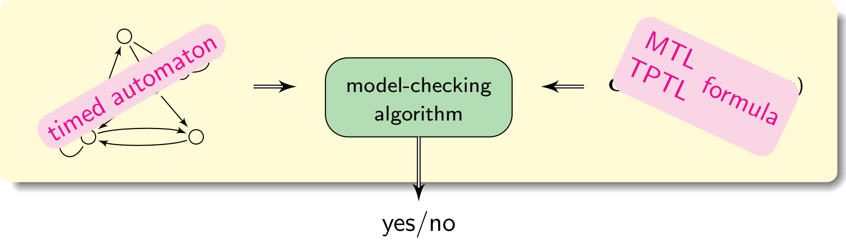

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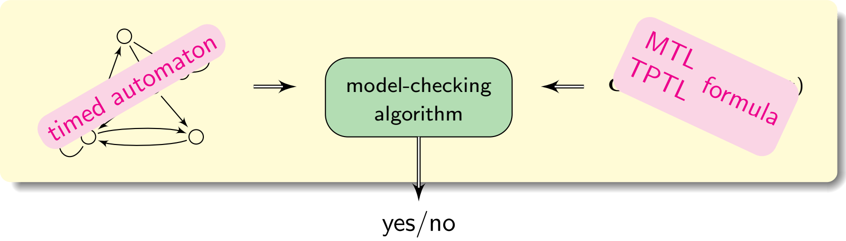

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- ... and more tractable fragments need be defined!

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The quest for tractable fragments of MTL

MTL

LTL

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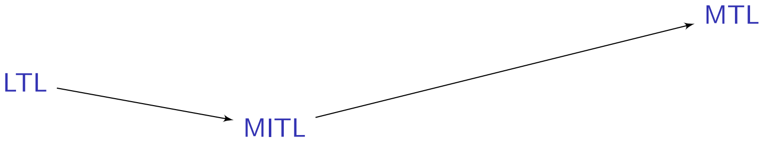

- MITL [AFH96]: ban punctuality
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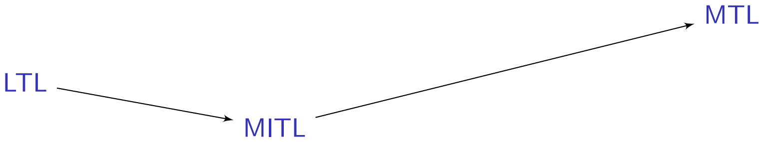

- MITL [AFH96]: ban punctuality
  - timed regularity
  - verification in exponential space

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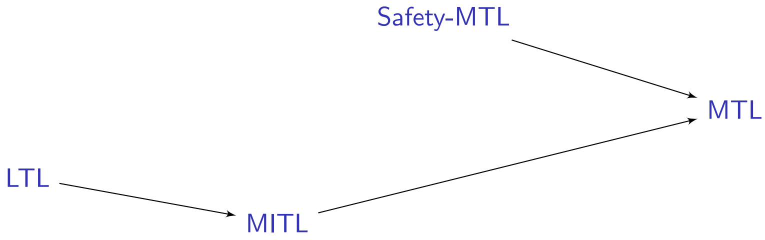

- Safety-MTL [OW05]: restrict to safety properties
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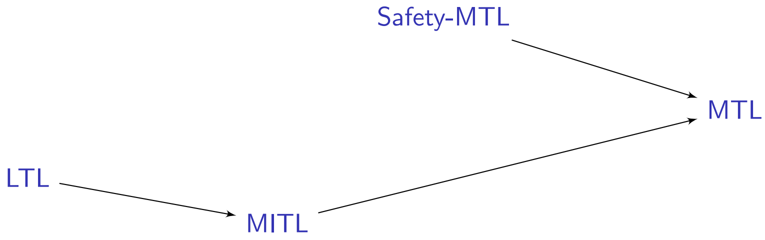
  - $\mathbf{G}\,(\bullet \rightarrow \mathbf{F}\,\bullet)$ is not in Safety-MTL

[OW05] Ouaknine, Worrell. On the decidability of Metric Temporal Logic *(LICS'05)*.

The quest for tractable fragments of MTL



- Safety-MTL [OW05]: restrict to safety properties
  - only finite counter-examples need be looked for
  - verification using alternating timed automata
  - decidable, but non-primitive recursive [OW08]

[OW05] Ouaknine, Worrell. On the decidability of Metric Temporal Logic *(LICS'05)*.
[OW08] Ouaknine, Worrell. Some recent results in Metric Temporal Logic *(FORMATS'08)*.

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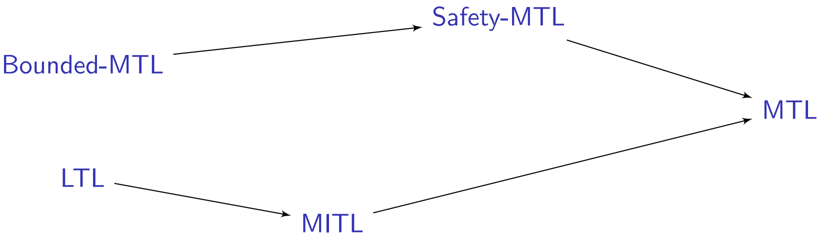

- Bounded-MTL [BMOW07]: restrict to bounded future
  - $\mathbf{G}_{\leqslant 5}\,({\color{red}\bullet} \rightarrow \mathbf{F}_{\leqslant 2}\,{\color{green}\bullet})$ is in Bounded-MTL
  - $\mathbf{G}\,({\color{red}\bullet} \rightarrow \mathbf{F}_{\leqslant 2}\,{\color{green}\bullet})$ is not in Bounded-MTL

[BMOW07] Bouyer, Markey, Ouaknine, Worrell. The cost of punctuality *(LICS'07)*.

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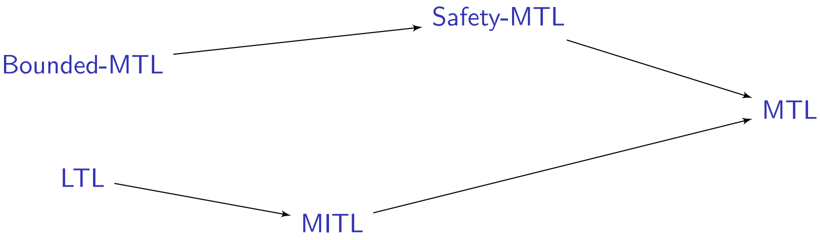

- Bounded-MTL [BMOW07]: restrict to bounded future
  - expresses non-regular properties
  - only time-bounded prefixes need to be verified
  - verification in exponential space

[BMOW07] Bouyer, Markey, Ouaknine, Worrell. The cost of punctuality *(LICS'07)*.

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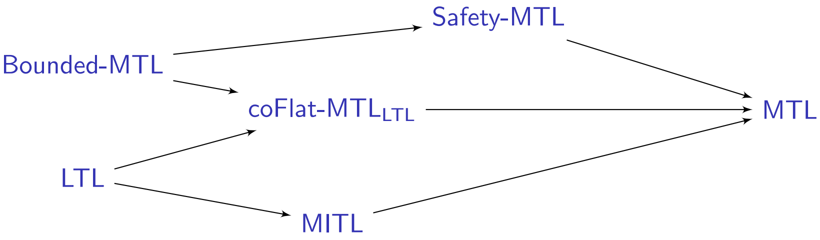

- coFlat-MTL$_{\text{LTL}}$ [BMOW07]: a flatness condition on the formula
  - $\mathbf{G}\,(\bullet \rightarrow \mathbf{F}_{=1}\,\bullet)$ is in coFlat-MTL$_{\text{LTL}}$
  - $\mathbf{F}\,\mathbf{G}_{\leqslant 1}\,\bullet$ is not in coFlat-MTL$_{\text{LTL}}$

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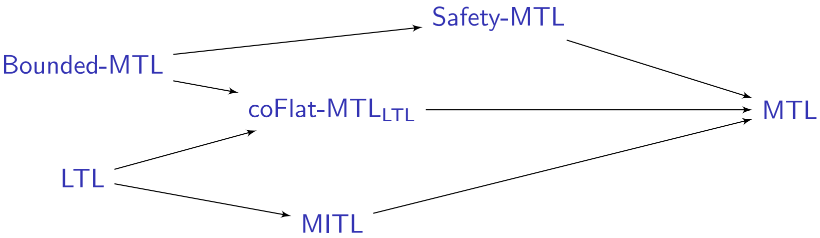
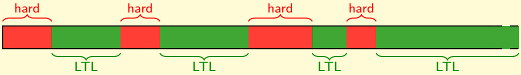


- coFlat-MTL$_{\text{LTL}}$ [BMOW07]: a flatness condition on the formula
  - expresses non-regular properties
  - only counter-examples of the following form need to be looked for:



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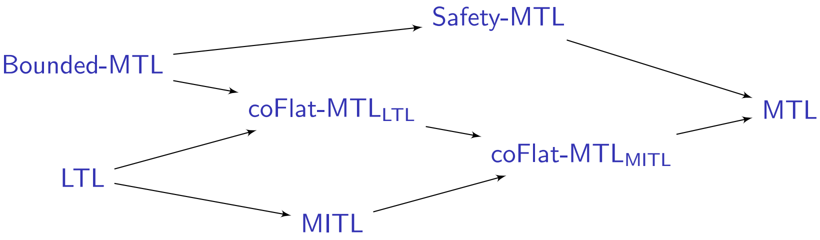

- coFlat-MTL_{MITL} [BMOW08]: a flatness condition on the formula
  - $\mathbf{F}\,\mathbf{G}_{\leqslant 1}\,\bullet$ is in coFlat-MTL_{MITL}
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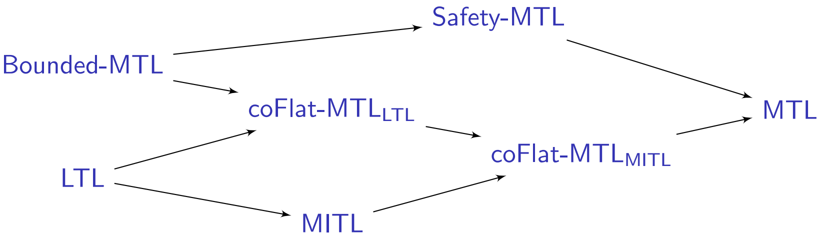
[BMOW08] Bouyer, Markey, Ouaknine, Worrell. On expressiveness and complexity in real-time model checking *(ICALP'08)*.

The quest for tractable fragments of MTL



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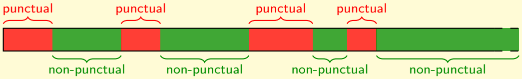

  - the largest (known) fragment for which...
  - ... verification in exponential space!

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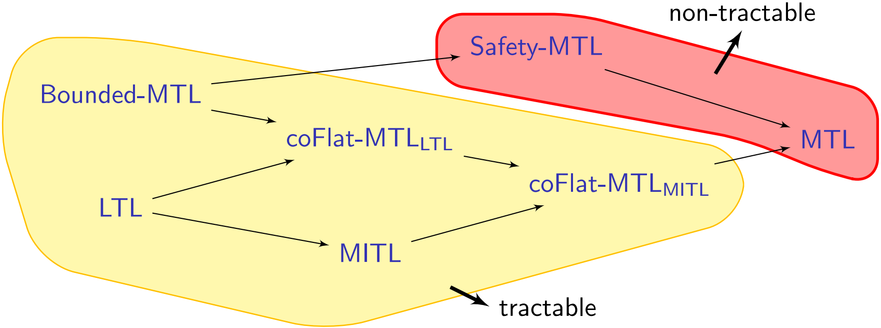
The quest for tractable fragments of MTL



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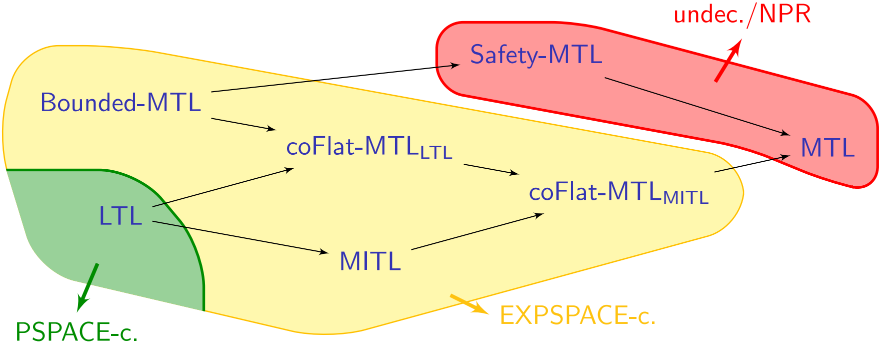

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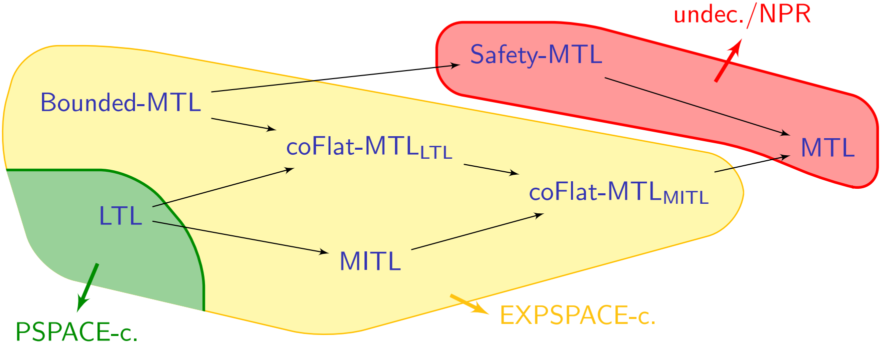

Algorithmics needs now to be worked out!

Outline

Motivation

- System resources might be relevant and even crucial information

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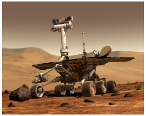
  - ...



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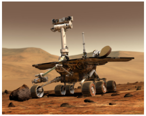

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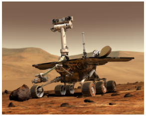

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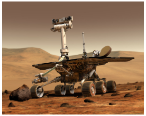

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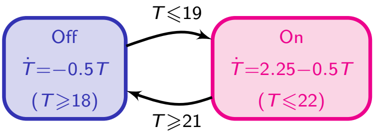
The thermostat example



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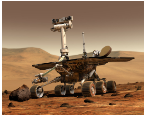

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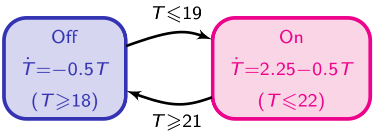

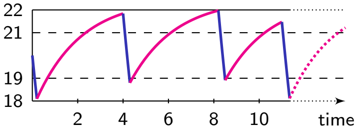

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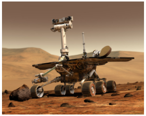

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Theorem [HKPV95]

The reachability problem is undecidable in hybrid automata.

[HKPV95] Henzinger, Kopke, Puri, Varaiya. What's decidable wbout hybrid automata? *(SToC'95)*.

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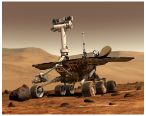

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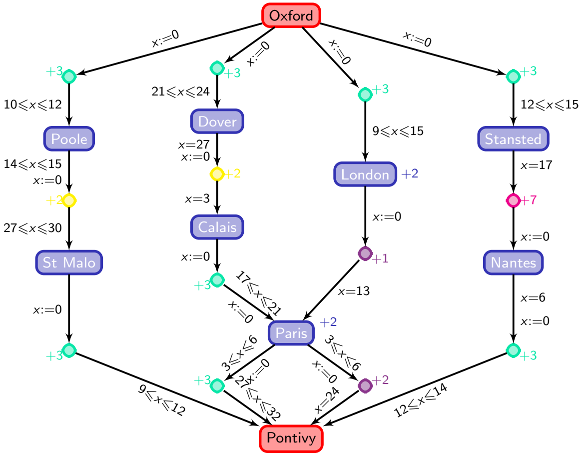
Theorem [HKPV95]

The reachability problem is undecidable in hybrid automata.

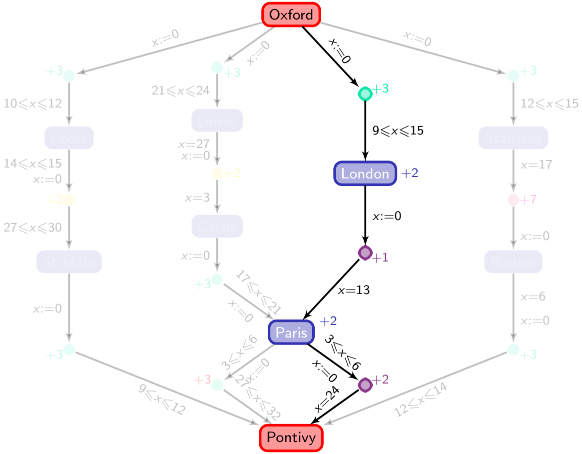
- An alternative: weighted/priced timed automata [ALP01,BFH+01]
  $\rightsquigarrow$ hybrid variables do not constrain the system

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
[BFH+01] Behrmann, Fehnker, Hune, Larsen, Pettersson, Romijn, Vaandrager. Minimum-cost reachability in priced timed automata *(HSCC'01)*.

A third model of the system

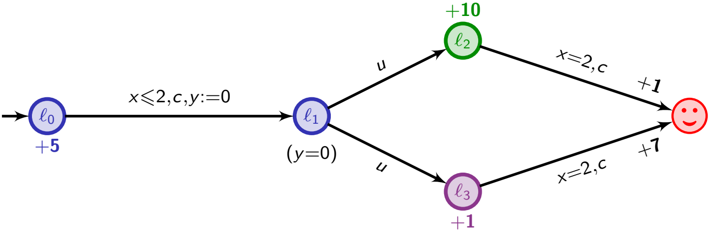


How much fuel will I use?



It is a quantitative (optimization) question
in a weighted timed automaton: at least 68 anti-planet units!

Weighted/priced timed automata [ALP01,BFH+01]



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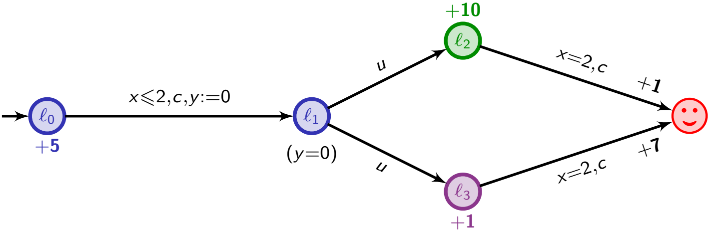

	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

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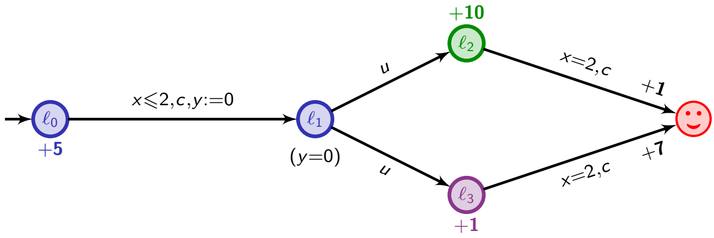

	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
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cost :

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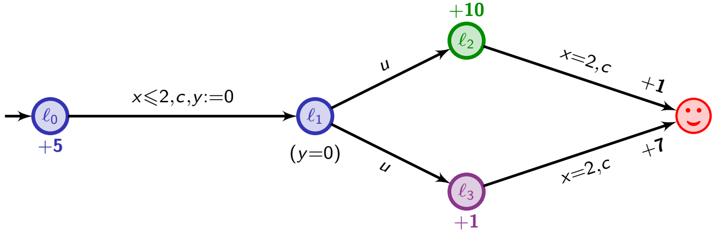

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cost : 6.5

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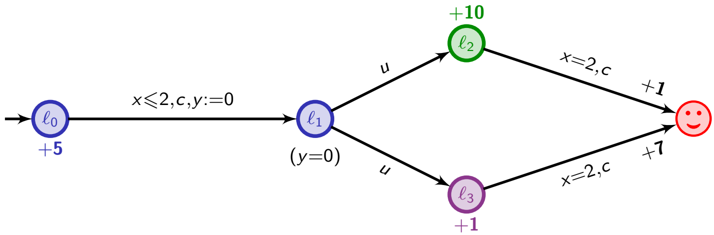

	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : 6.5 + 0

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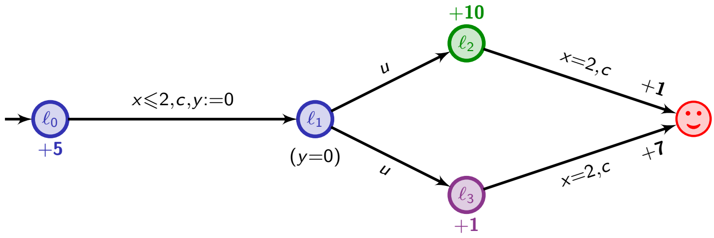

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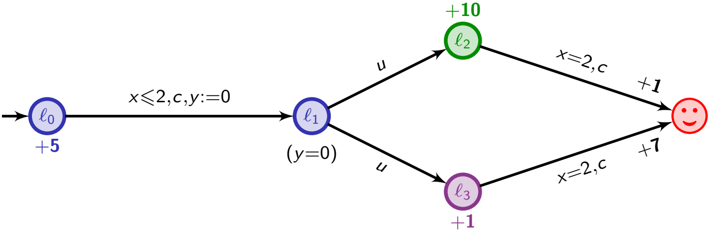

$$\begin{array}{lcccccccccccc} & \ell_0 & \xrightarrow{1.3} & \ell_0 & \xrightarrow{c} & \ell_1 & \xrightarrow{u} & \ell_3 & \xrightarrow{0.7} & \ell_3 & \xrightarrow{c} & ☺ \\ x & 0 & & 1.3 & & 1.3 & & 1.3 & & 2 & & \\ y & 0 & & 1.3 & & 0 & & 0 & & 0.7 & & \end{array}$$

cost : 6.5 + 0 + 0 + 0.7

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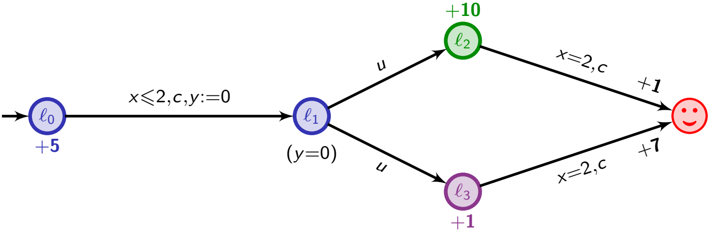

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x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		
cost :		6.5	+	0	+	0	+	0.7	+	7	

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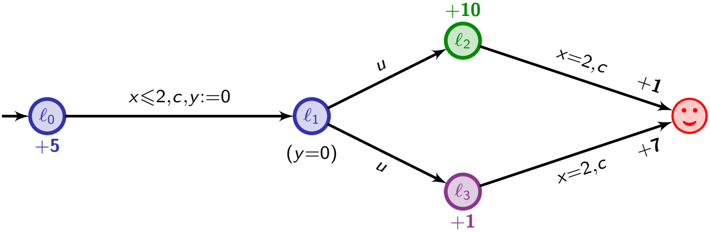

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y	0		1.3		0		0		0.7				
cost :		6.5	+	0	+	0	+	0.7	+	7		=	14.2

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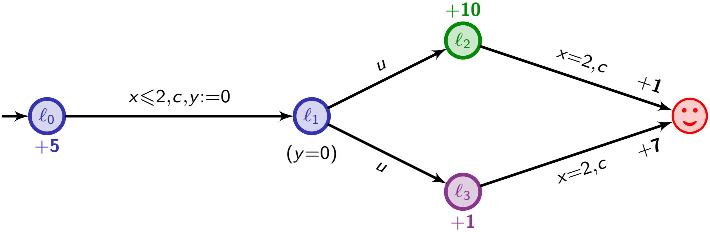

Question: what is the optimal cost for reaching 🙂 ?

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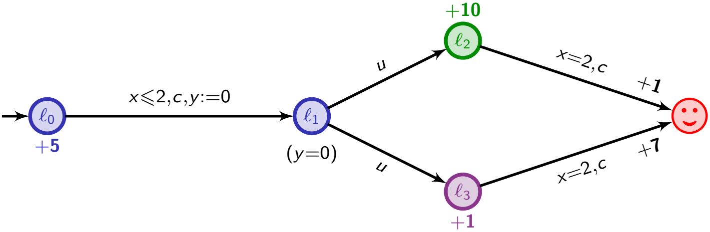

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$$5t + 10(2 - t) + 1$$

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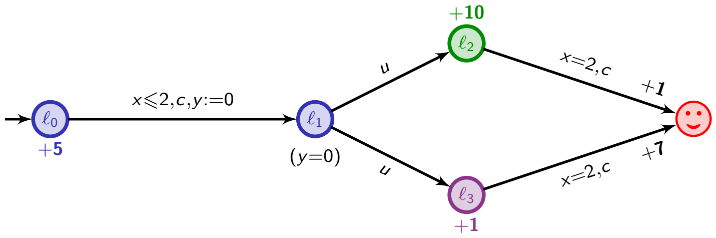

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$$5t + 10(2 - t) + 1 \ , \ 5t + (2 - t) + 7$$

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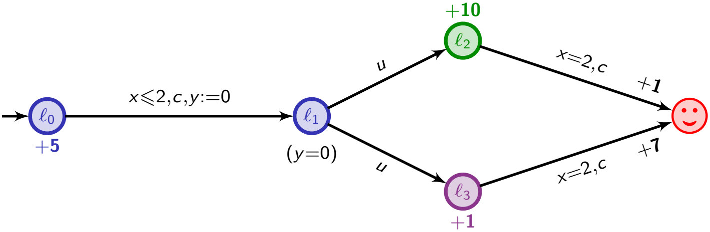

Question: what is the optimal cost for reaching ☺ ?

$$\min\left(\, 5t + 10(2-t) + 1 \,,\; 5t + (2-t) + 7 \,\right)$$

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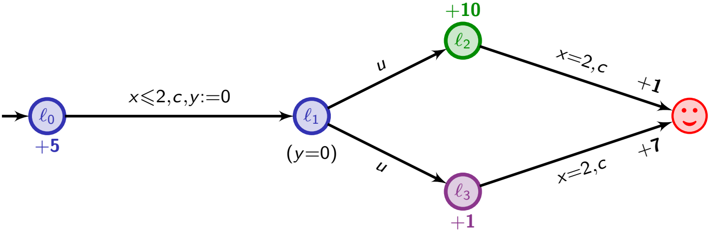
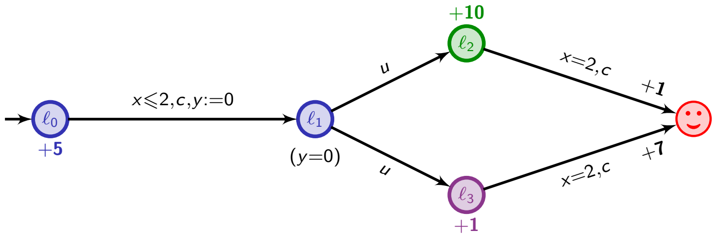


Question: what is the optimal cost for reaching 🙂 ?

$$\inf_{0\leqslant t\leqslant 2} \min\left(5t + 10(2-t) + 1 \,,\; 5t + (2-t) + 7 \right) = 9$$

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
[BFH+01] Behrmann, Fehnker, Hune, Larsen, Pettersson, Romijn, Vaandrager. Minimum-cost reachability in priced timed automata *(HSCC'01)*.

Weighted/priced timed automata [ALP01,BFH+01]



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$$\inf_{0 \leqslant t \leqslant 2} \min\left(5t + 10(2-t) + 1 \,,\, 5t + (2-t) + 7 \right) = 9$$

$\rightsquigarrow$ *strategy:* leave immediately ℓ_0, go to ℓ_3, and wait there 2 t.u.

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
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Optimization problems in weighted timed automata

Theorem [ALP01,BFH+01,BBBR07]

The optimal reachability problem is decidable (and PSPACE-complete) in (weighted) timed automata.

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[BBBR07] Bouyer, Brihaye, Bruyère, Raskin. On the optimal reachability problem *(Formal Methods in System Design)*.

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The optimal mean-cost problem is decidable (and PSPACE-complete) in (weighted) timed automata.

$\rightsquigarrow$ In both cases, the corner-point abstraction can be used (a refinement of the region automaton)

[BBL04] Bouyer, Brinksma, Larsen. Staying alive as cheaply as possible *(HSCC'04)*.
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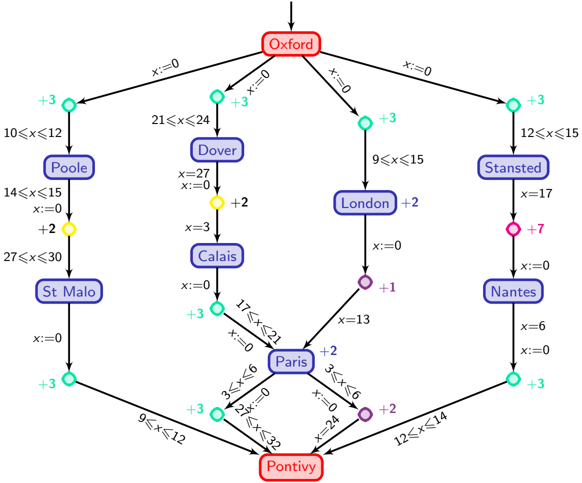
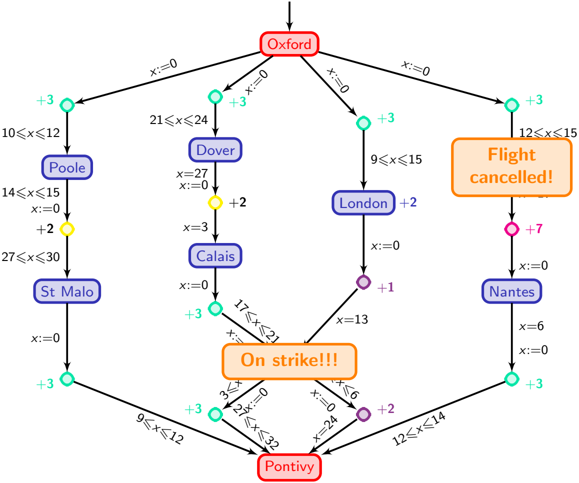
What if an unexpected event happens?



What if an unexpected event happens?



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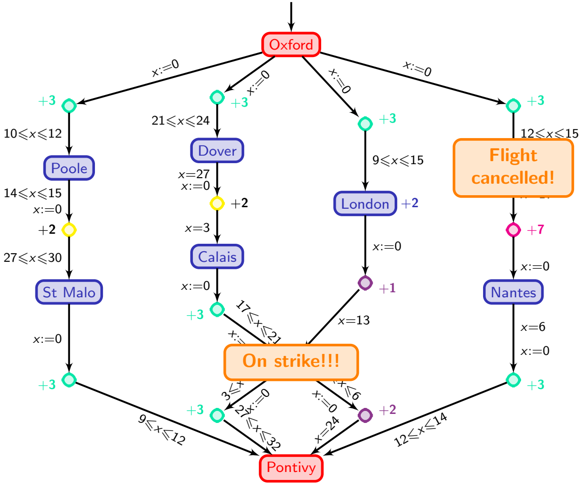
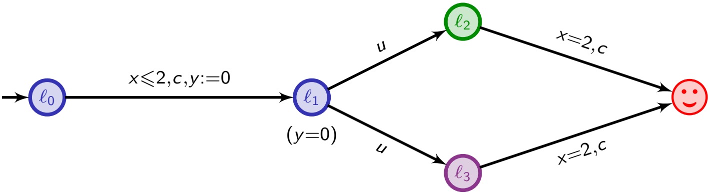
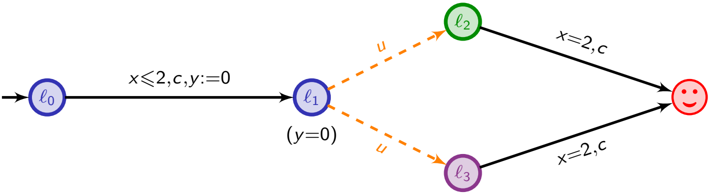


⤳ modelled as timed games

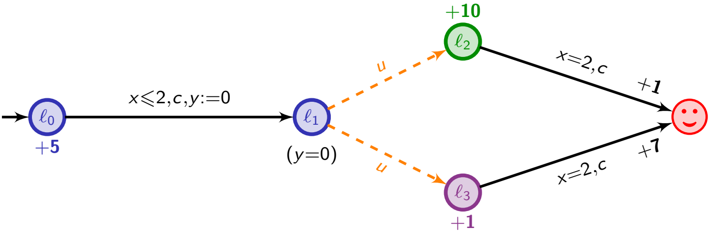
Weighted timed games



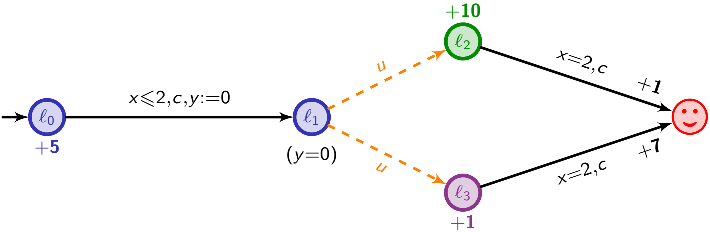
Weighted timed games



Weighted timed games

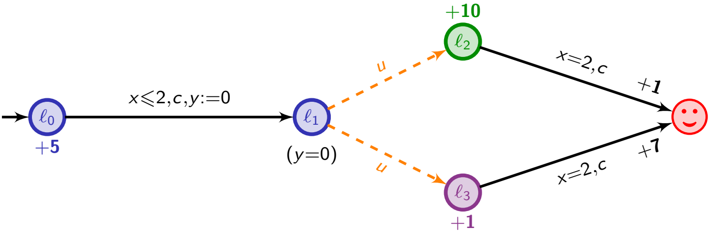


Weighted timed games



Question: what is the optimal cost we can ensure while reaching 🙂?

Weighted timed games



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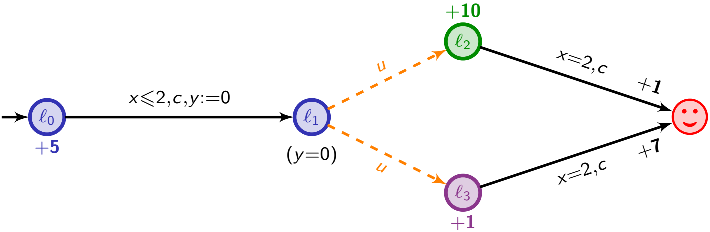
$5t + 10(2 - t) + 1$

Weighted timed games



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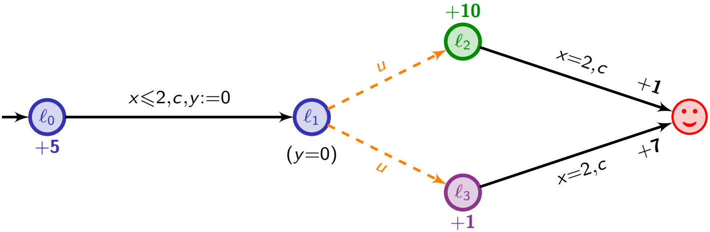
$$5t + 10(2-t) + 1\ ,\ 5t + (2-t) + 7$$

Weighted timed games



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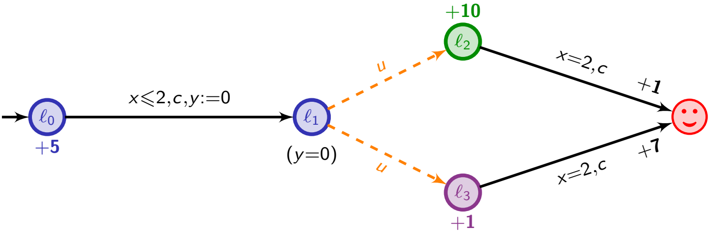
$$\max\ (\ 5t + 10(2-t) + 1\ ,\ 5t + (2-t) + 7\ )$$

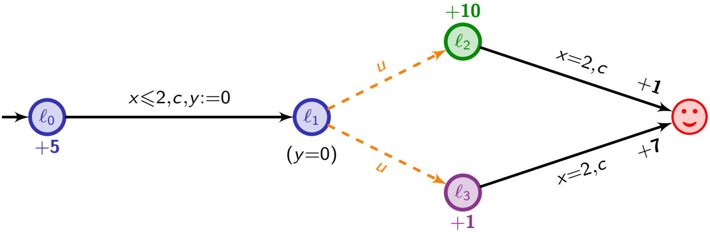
Weighted timed games



Question: what is the optimal cost we can ensure while reaching 🙂?

$$\inf_{0 \leqslant t \leqslant 2} \max \left(5t + 10(2-t) + 1 \,,\, 5t + (2-t) + 7 \right) = 14 + \frac{1}{3}$$

Weighted timed games



Question: what is the optimal cost we can ensure while reaching ☺ ?

$$\inf_{0\leqslant t\leqslant 2} \max\left(5t + 10(2-t) + 1 \,,\, 5t + (2-t) + 7 \right) = 14 + \frac{1}{3}$$

$\leadsto$ *strategy:* wait in ℓ_0, and when $t = \frac{4}{3}$, go to ℓ_1

Optimal reachability in weighted timed games

This topic has been fairly hot these last couple of years...

e.g. [LMM02,ABM04,BCFL04]

[LMM02] La Torre, Mukhopadhyay, Murano. Optimal-reachability and control for acyclic weighted timed automata *(TCS@02)*.
[ABM04] Alur, Bernardsky, Madhusudan. Optimal reachability in weighted timed games *(ICALP'04)*.
[BCFL04] Bouyer, Cassez, Fleury, Larsen. Optimal strategies in priced timed game automata *(FSTTCS'04)*.

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Theorem [BBR05,BBM06]

Optimal timed games are undecidable, as soon as automata have three clocks or more.

[BBR05] Brihaye, Bruyère, Raskin. On optimal timed strategies *(FORMATS'05)*.
[BBM06] Bouyer, Brihaye, Markey. Improved undecidability results on weighted timed automata *(Information Processing Letters)*.

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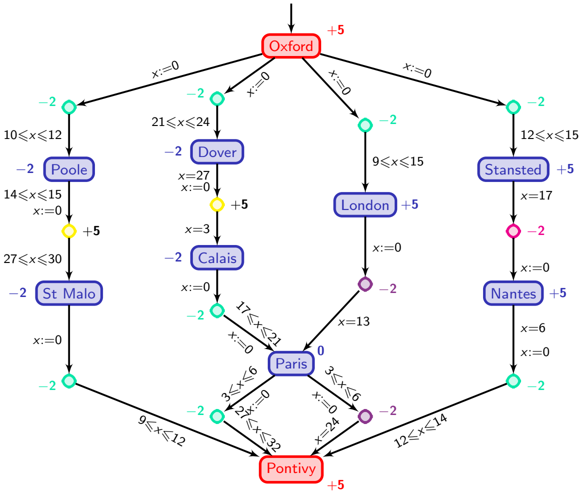
Theorem [BLMR06]

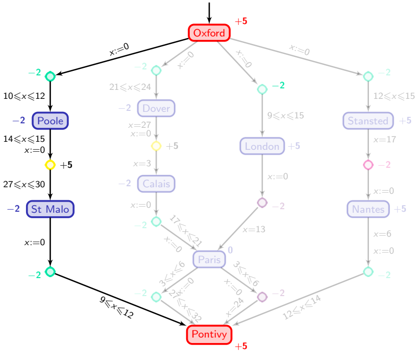
Turn-based optimal timed games are decidable in 3EXPTIME when automata have a single clock. They are PTIME-hard.

[BBR05] Brihaye, Bruyère, Raskin. On optimal timed strategies *(FORMATS'05)*.
[BBM06] Bouyer, Brihaye, Markey. Improved undecidability results on weighted timed automata *(Information Processing Letters)*.
[BLMR06] Bouyer, Larsen, Markey, Rasmussen. Almost-optimal strategies in one-clock priced timed automata *(FSTTCS'06)*.

A fourth model of the system



Can I work with my computer all the way?



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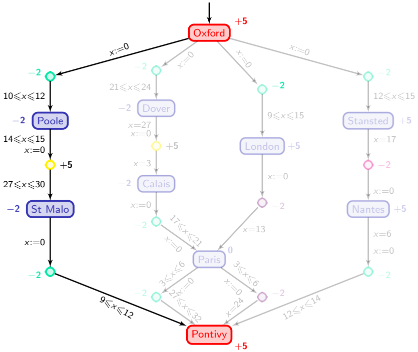
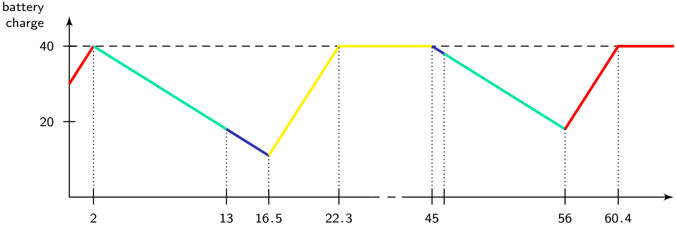
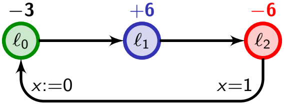




An example of resource management

Globally $(x \leqslant 1)$



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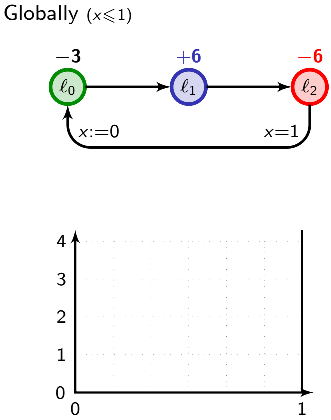
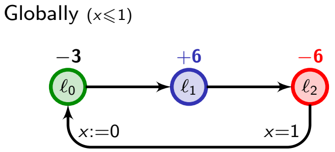
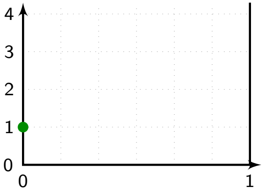


- Lower-bound problem: can we stay above 0?

An example of resource management





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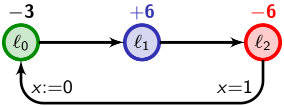

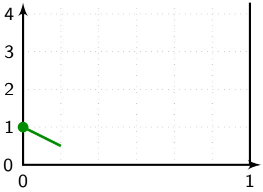

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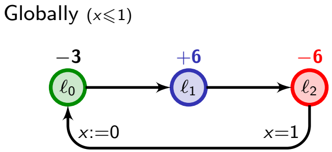

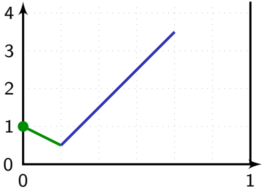

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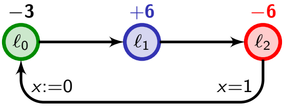

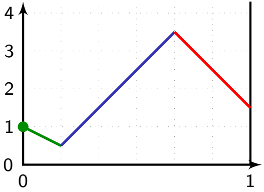

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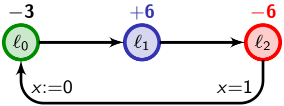

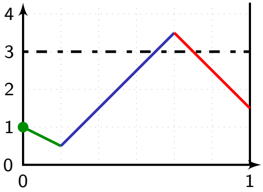

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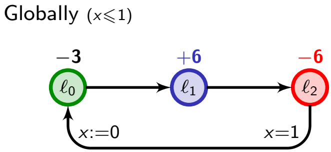
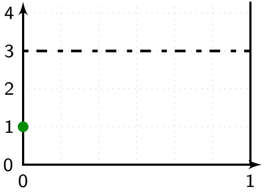
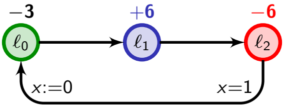


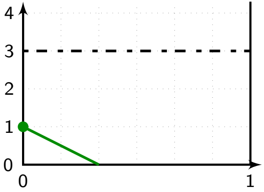


- Lower-bound problem
- Lower-upper-bound problem: can we stay within bounds?

An example of resource management

Globally $(x \leqslant 1)$





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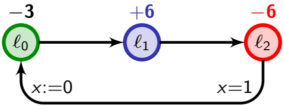

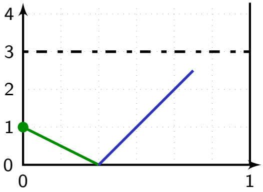

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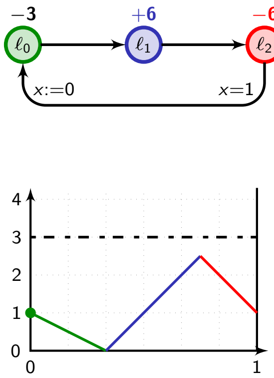

- Lower-bound problem
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An example of resource management

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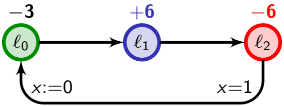

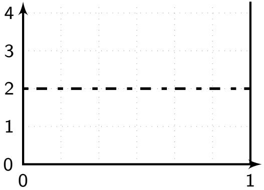

- Lower-bound problem
- Lower-upper-bound problem: can we stay within bounds?

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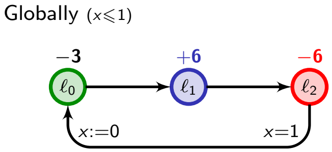
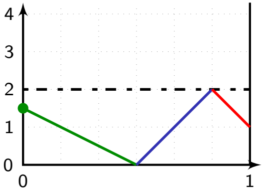
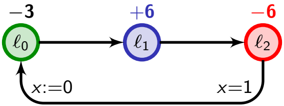


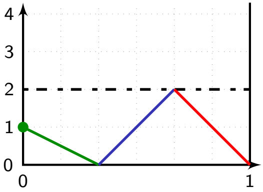


- Lower-bound problem
- Lower-upper-bound problem: can we stay within bounds?

An example of resource management

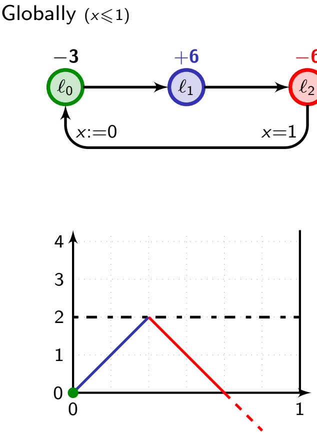
Globally $(x \leqslant 1)$





- Lower-bound problem
- Lower-upper-bound problem: can we stay within bounds?

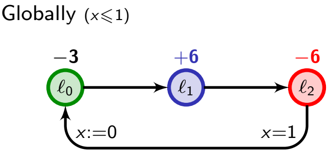
An example of resource management

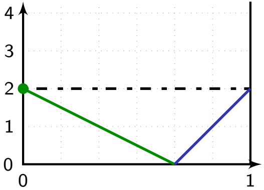


lost!

- Lower-bound problem
- Lower-upper-bound problem: can we stay within bounds?

An example of resource management

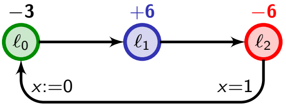


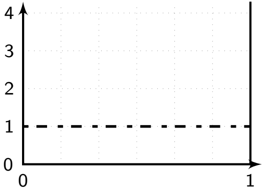


- Lower-bound problem
- Lower-upper-bound problem: can we stay within bounds?

An example of resource management

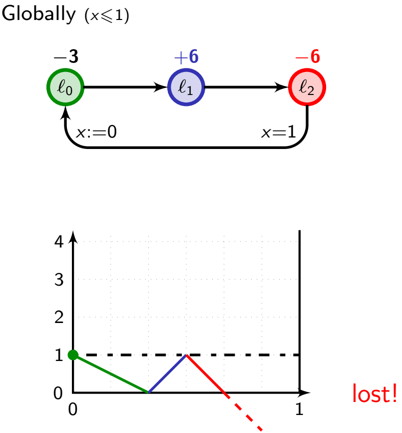
Globally $(x \leqslant 1)$





- Lower-bound problem
- Lower-upper-bound problem: can we stay within bounds?

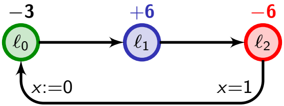
An example of resource management

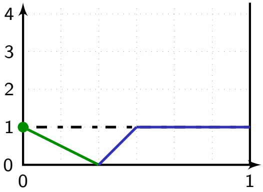


- Lower-bound problem
- Lower-upper-bound problem: can we stay within bounds?

An example of resource management

Globally $(x \leqslant 1)$

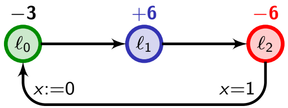


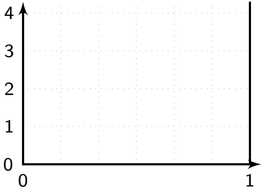


- Lower-bound problem
- Lower-upper-bound problem
- Lower-weak-upper-bound problem: can we "weakly" stay within bounds?

An example of resource management

Globally $(x \leqslant 1)$





- Lower–bound problem $\rightsquigarrow$ **L**
- Lower-upper-bound problem $\rightsquigarrow$ **L+U**
- Lower-weak-upper-bound problem $\rightsquigarrow$ **L+W**

Only partial results so far [BFLMS08]

0 clock!	**exist. problem**	**univ. problem**	**games**
L	$\in$ PTIME	$\in$ PTIME	$\in$ UP $\cap$ co-UP PTIME-hard
L+W	$\in$ PTIME	$\in$ PTIME	$\in$ NP $\cap$ co-NP PTIME-hard
L+U	$\in$ PSPACE NP-hard	$\in$ PTIME	EXPTIME-c.

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Only partial results so far [BFLMS08]

1 clock	**exist. problem**	**univ. problem**	**games**
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- each instruction is encoded as a module;
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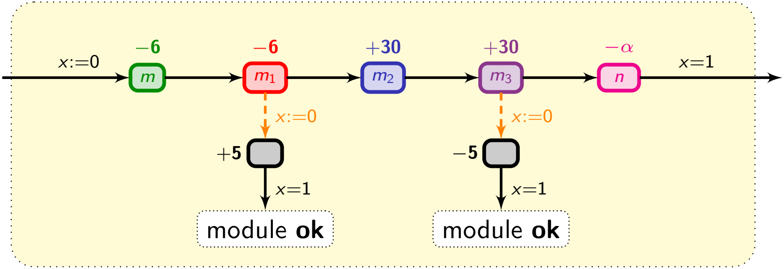
$\rightsquigarrow$ We present a generic construction for incrementing/decrementing the counters.

Generic module for incrementing/decrementing



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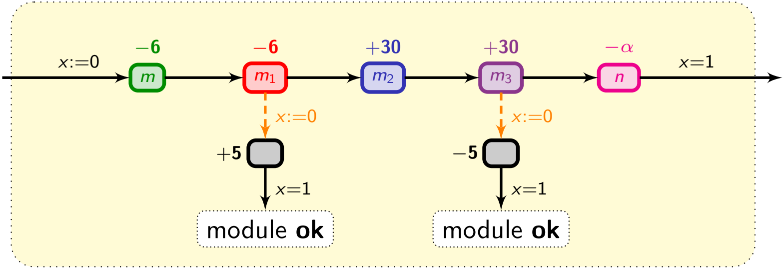

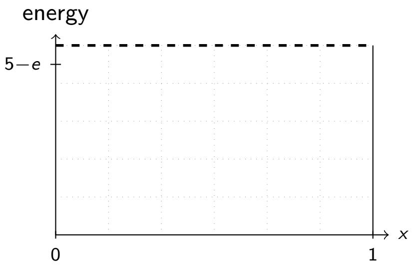

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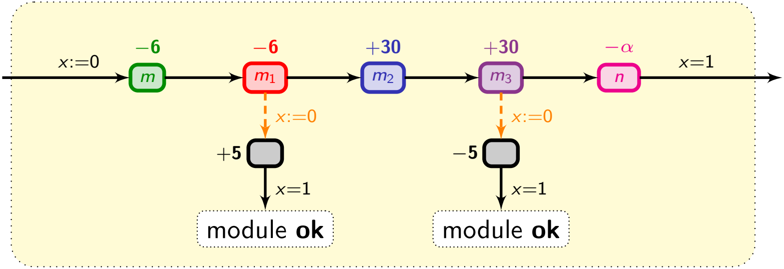

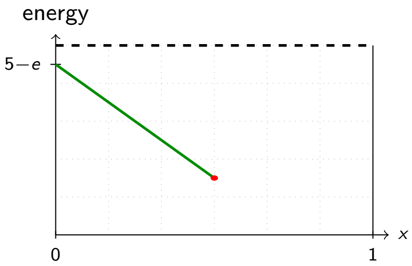

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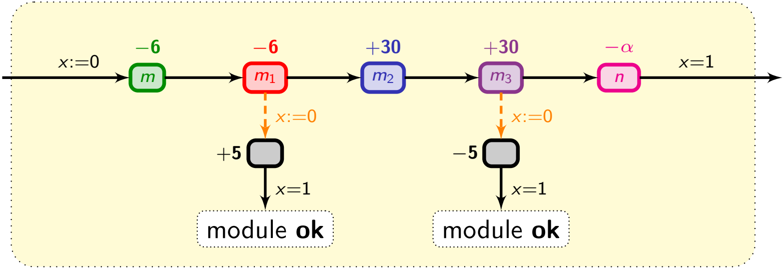

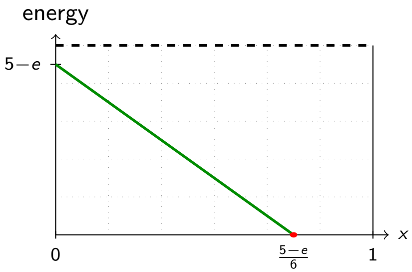

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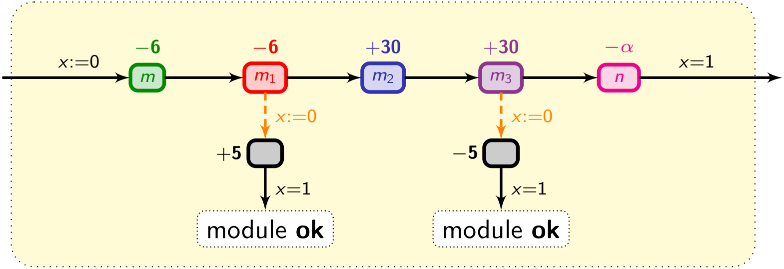

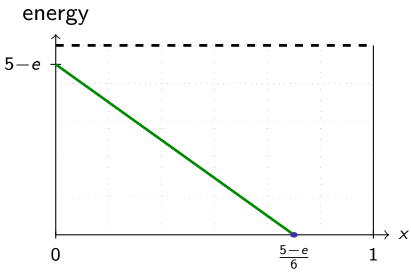

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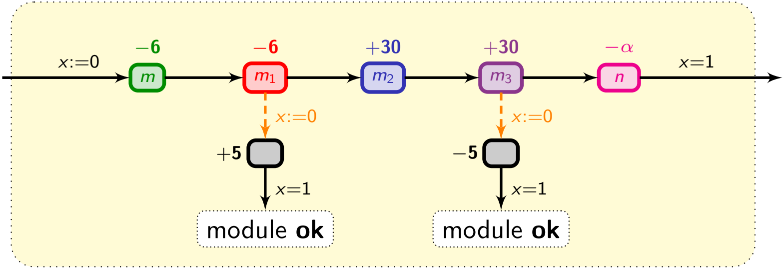

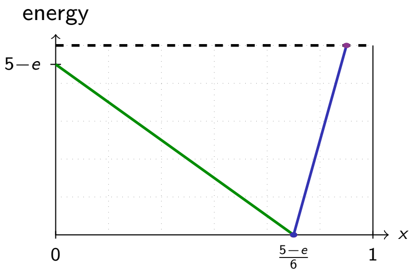

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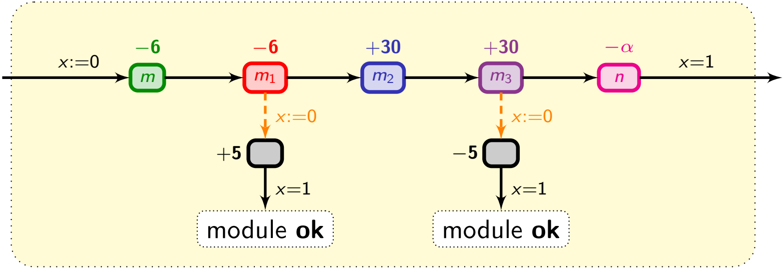

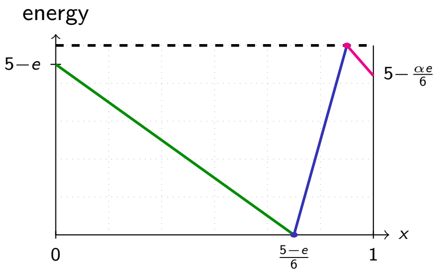

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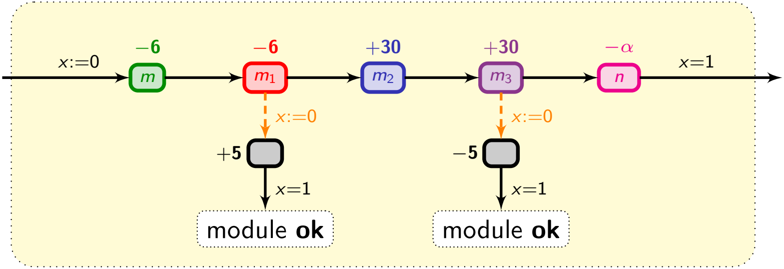

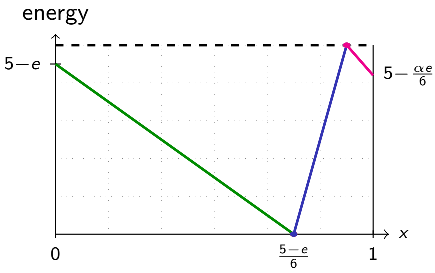

- α=3: increment c_1
- α=2: increment c_2
- α=12: decrement c_1
- α=18: decrement c_2

Partial conclusion and perspectives

Weighted timed automata, an interesting model for representing resources in timed systems

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- Compute equilibria in weighted timed games
$\leadsto$ towards a theory of timed games

Outline

Motivation

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- A new timed and probabilistic model
  - models a purely probabilistic environment
  - related models include continuous-time Markov chains, and probabilistic timed automata

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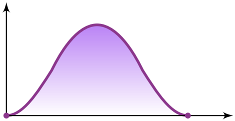

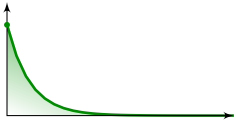

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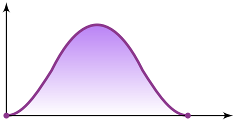

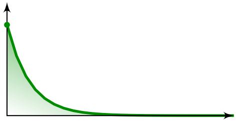

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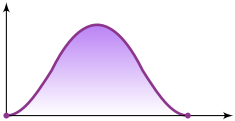

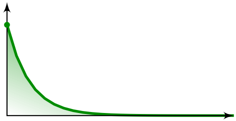

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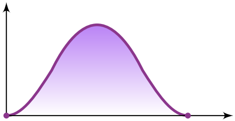
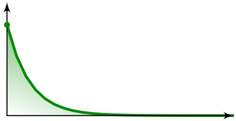

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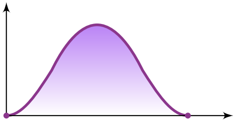

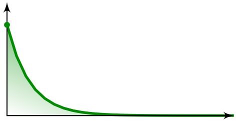

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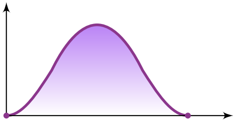

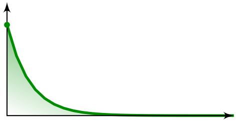

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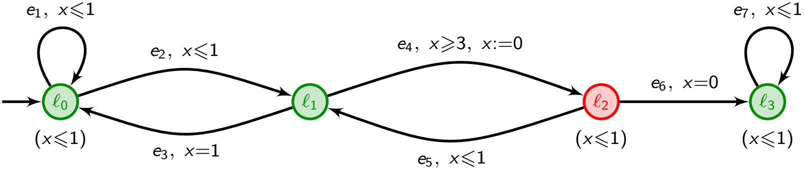
An example


$e_1,\ x \leqslant 1$
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ℓ_0
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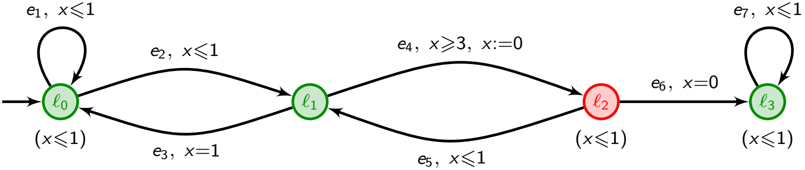

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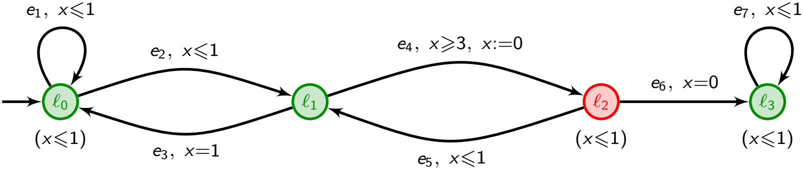

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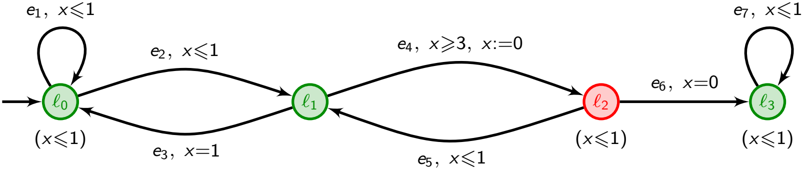

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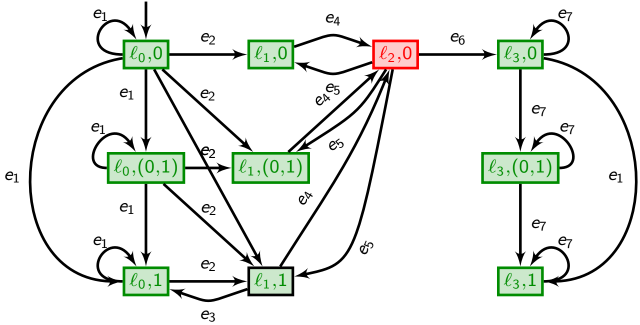
Indeed, almost surely, paths are of the form $e_1^* e_2 \big(e_4 e_5\big)^\omega$

The classical region automaton



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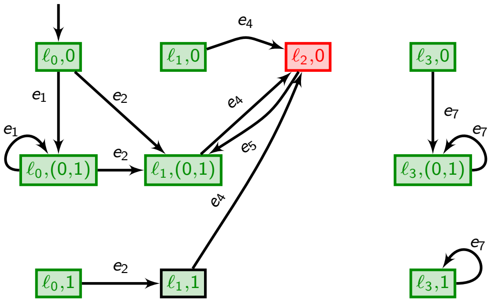

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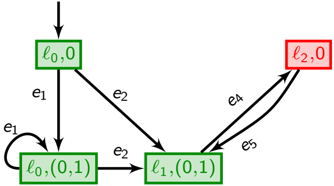

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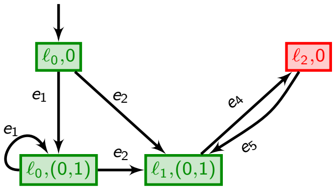

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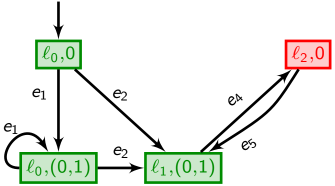

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Proposition

For single-clock timed automata,

$$\mathbb{P}(\mathcal{A} \models \varphi) = 1 \quad \text{iff} \quad \mathbb{P}(MC(\mathcal{A}) \models \varphi) = 1$$

(this is independent of the choice of the distributions...)

Probabilistic model-checking

Theorem [BBBBG08]

For single-clock timed automata, the almost-sure model-checking

- of LTL is PSPACE-complete;
- of ω-regular properties is NLOGSPACE-complete;
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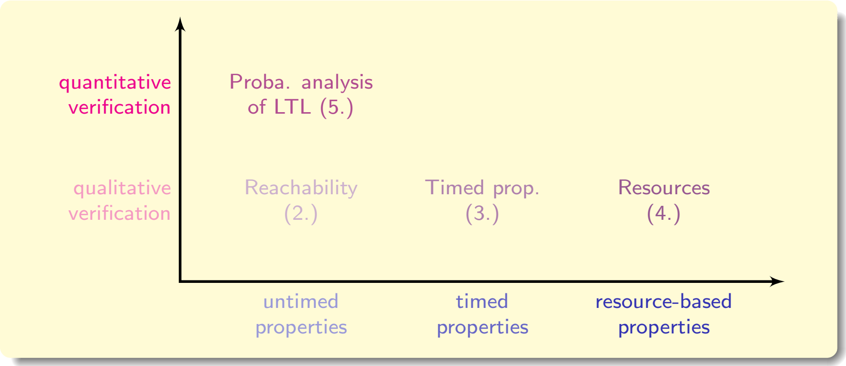
Summary

A progressive lift from qualitative to quantitative considerations:



Perspectives

Modelling resources in timed systems

- study further the resource management problem
- develop approximation schemes
  (undecidability relies on the infinite precision of the system)
- compute equilibria

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Probabilities and timed automata

- investigate further the $\frac{1}{2}$-player model
  - model with several clocks
    (*hint:* restrict the allowed distributions)
  - quantitative properties (time and resources)
  - performance analysis: expected time, *etc.*
- add non-determinism and interaction ($1\frac{1}{2}$- and $2\frac{1}{2}$-player models)
  (preliminary results: undecidability rather far away...)
- design an irrefutable example

Perspectives

Adequation of timed models to timed systems

Two main approaches: relaxed satisfaction
robust satisfaction

[BMR08] Bouyer, Markey, Reynier. Robust analysis of timed automata *via* channel machines *(FoSSaCS'08)*.

Perspectives

Adequation of timed models to timed systems

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- Relaxed satisfaction: *cf* probabilities and timed automata

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Perspectives

Adequation of timed models to timed systems

Two main approaches: relaxed satisfaction
robust satisfaction

- Relaxed satisfaction: *cf* probabilities and timed automata
- Robust satisfaction:
  - develop further the purely channel-machine approach of [BMR08]
  - synthesize systems that are robustly correct by construction
  - further think of other notions of robustness

[BMR08] Bouyer, Markey, Reynier. Robust analysis of timed automata *via* channel machines *(FoSSaCS'08)*.