Laboratoire Méthodes Formelles

Zone-based verification of timed automata: Extrapolations, simulations and what next?

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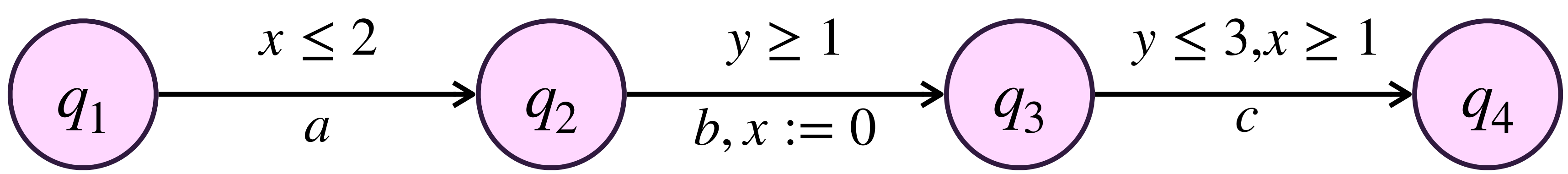
Based on a survey paper written with Paul Gastin, Frédéric Herbreteau, Ocan Sankur and B. Srivathsan

Partly supported by ANR projet Ticktac

Timed Automata

[AD90,AD94]



[AD90] Alur, Dill: Automata For Modeling Real-Time Systems (ICALP'90)
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Timed Automata

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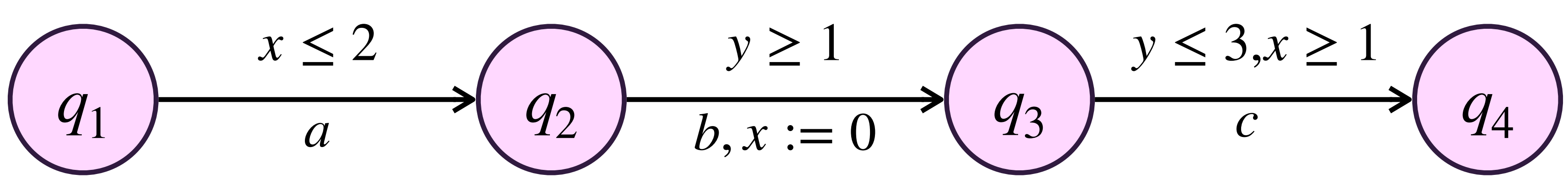

- Infinitely many configurations!
- Decidability proven using regions
- Reachability is PSPACE-complete

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Enumerative approach: not possible
Region construction: not feasible in general
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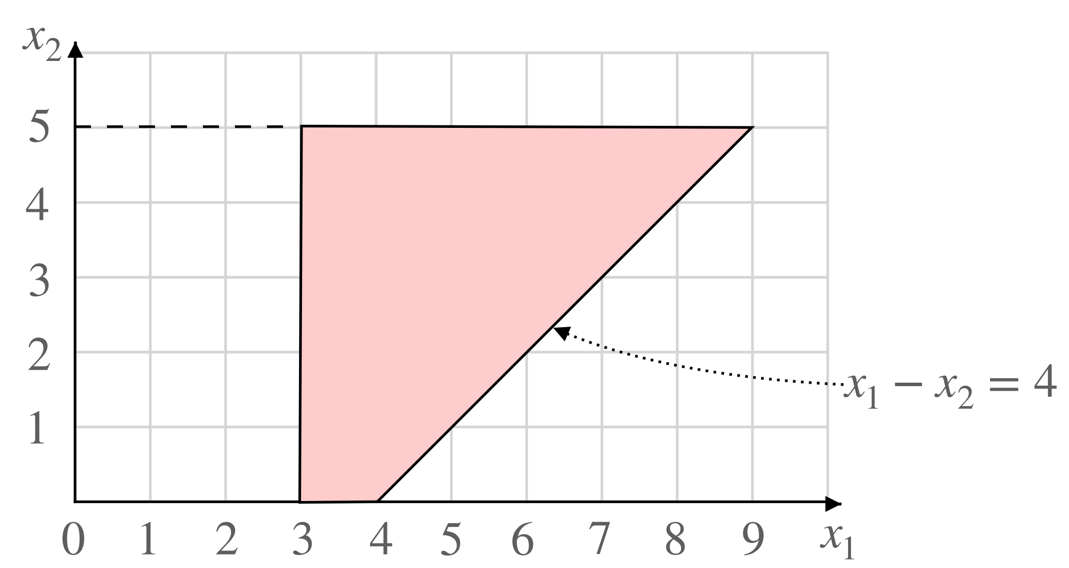

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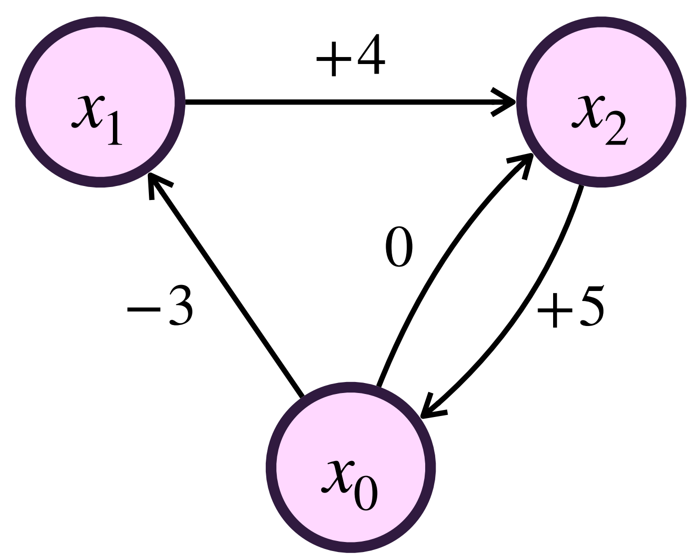

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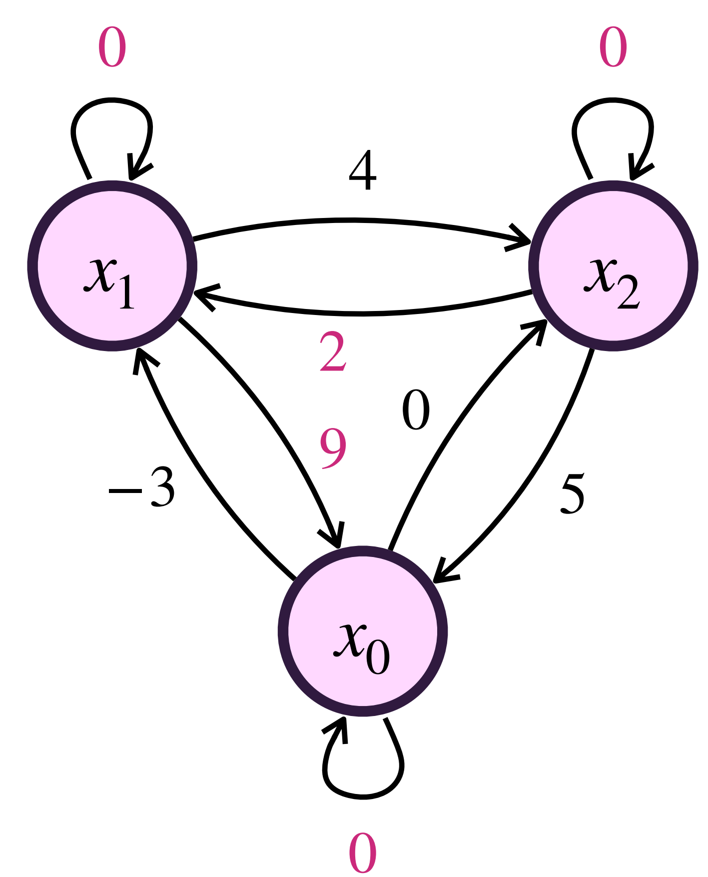

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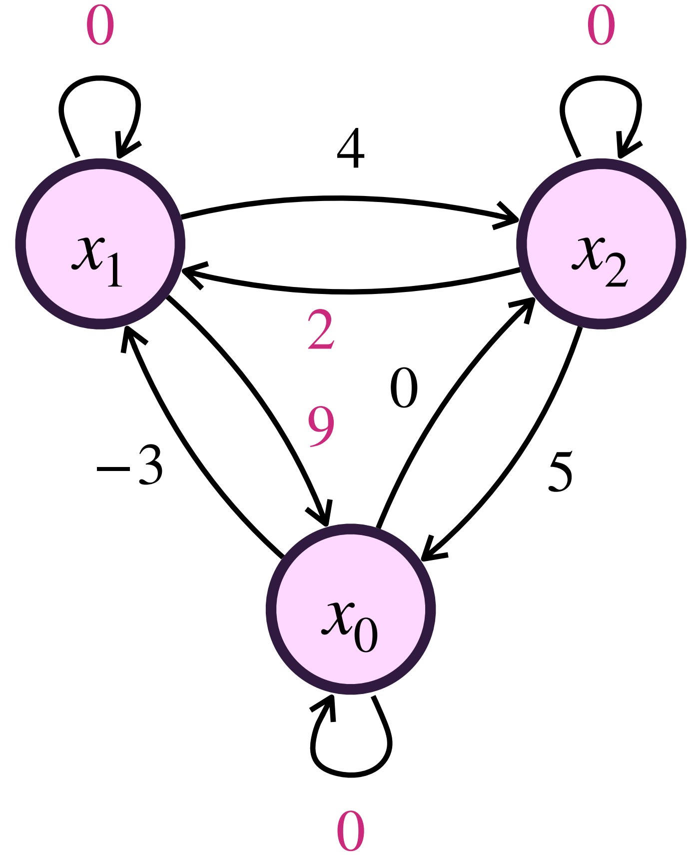
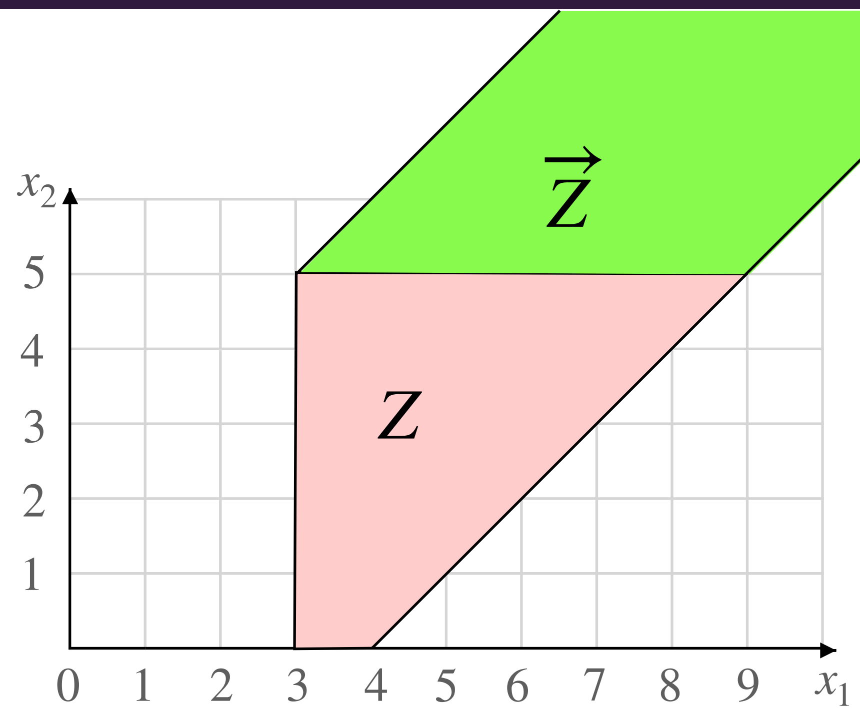
Normal form



Operations on zones



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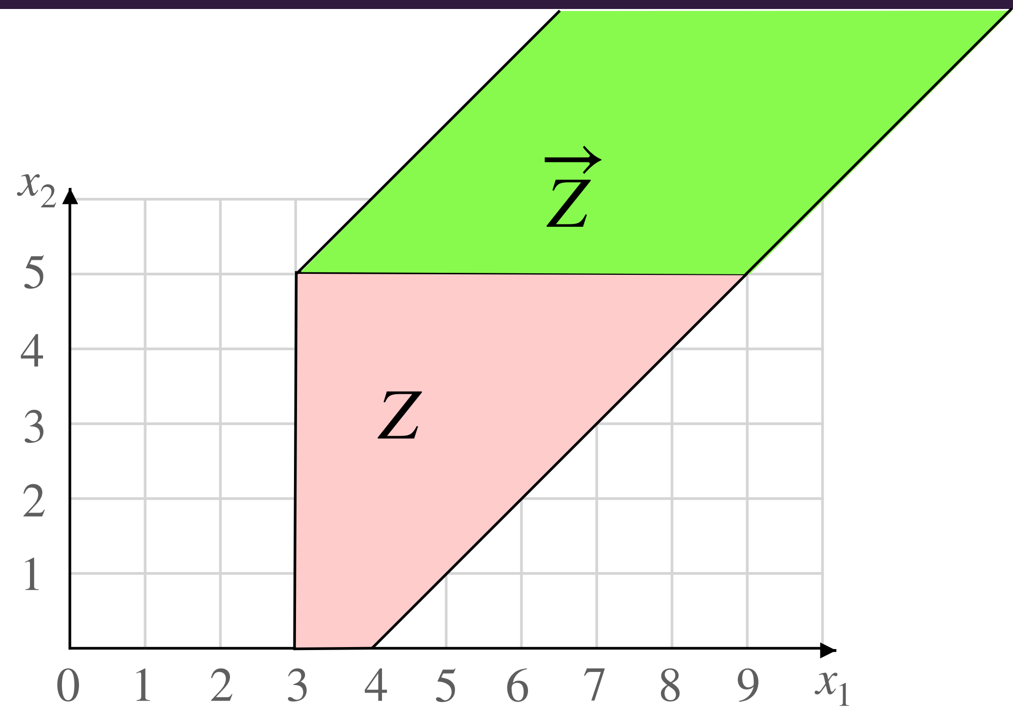

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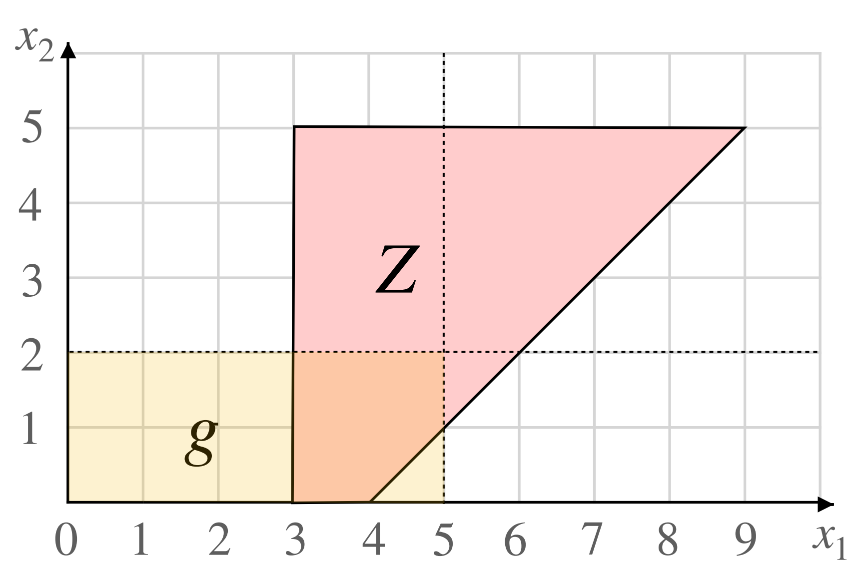

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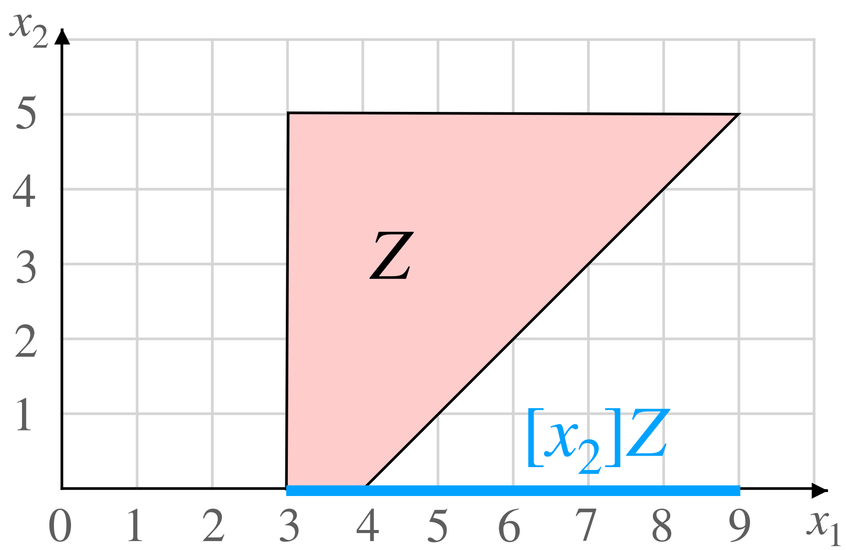

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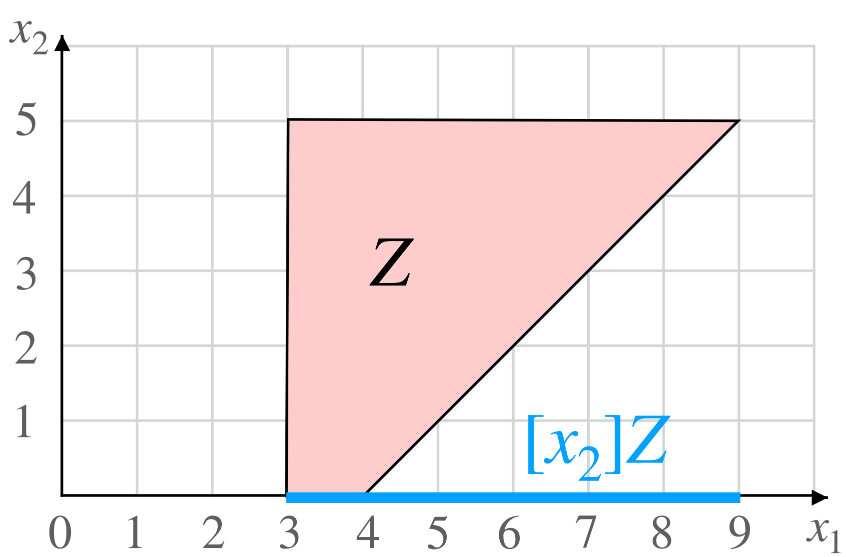

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If Z is a zone, then $Z' = \overrightarrow{[Y](Z \cap g)}$ is a zone

The computation can be made in $\mathcal{O}(|X|^2 \cdot |g|)$

Standard forward computation

- Initialize $\mathcal{S}$ with $(q_0, \vec{0})$
- Repeat until saturation:
  - If $(q, Z) \in \mathcal{S}$, then add (q', Z') to $\mathcal{S}$, where $Z' = \overrightarrow{[Y](Z \cap g)}$ is the successor via $q \xrightarrow{g,Y} q'$ unless there is $(q', Z'') \in \mathcal{S}$ s.t. $Z' \subseteq Z''$

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  - If $(q, Z$

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The computation does not terminate in general

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Two approaches

- Extrapolation
- Simulation

The extrapolation approach

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Inclusion can be decided in $\mathcal{O}(|X|^2)$

Operator extra defined s.t.

- Termination is ensured (extra has finite range)
- Completeness is obvious
- *Soundness is challenging*

Extrapolation

Remove « irrelevant » constants w.r.t. the automaton
⇝ syntactic on the DBM

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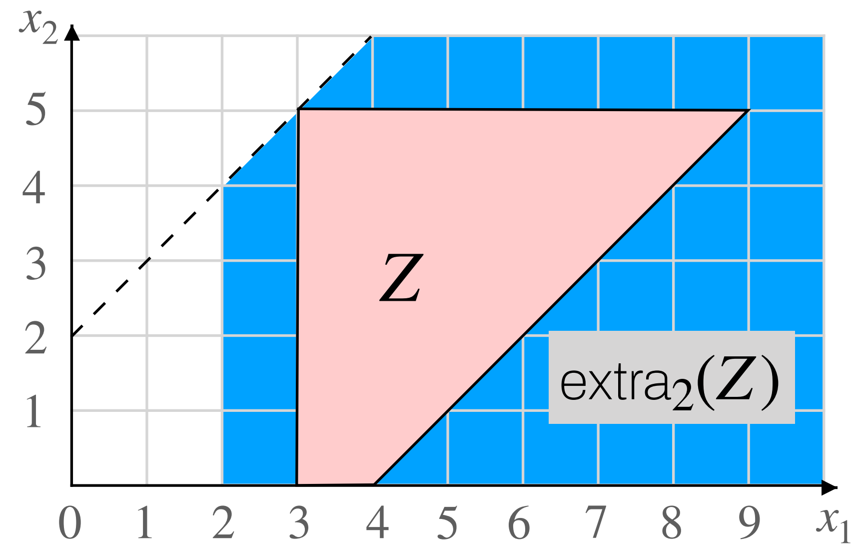

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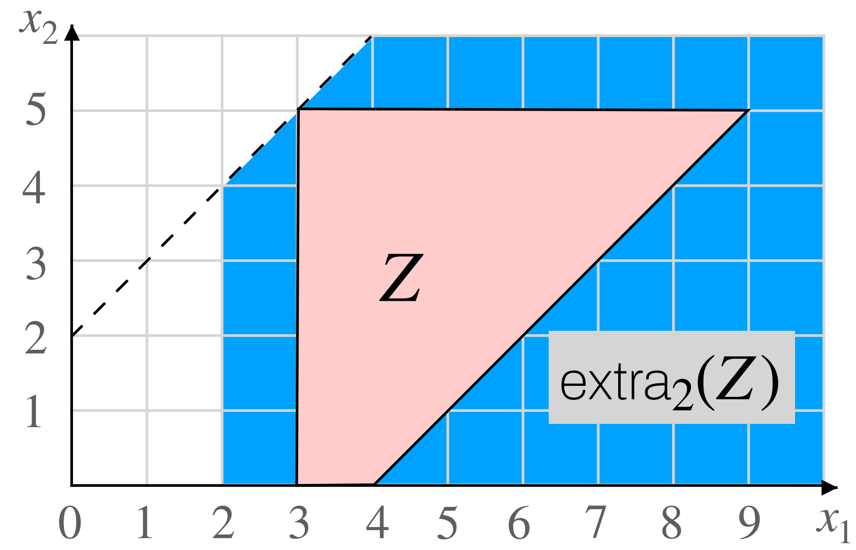

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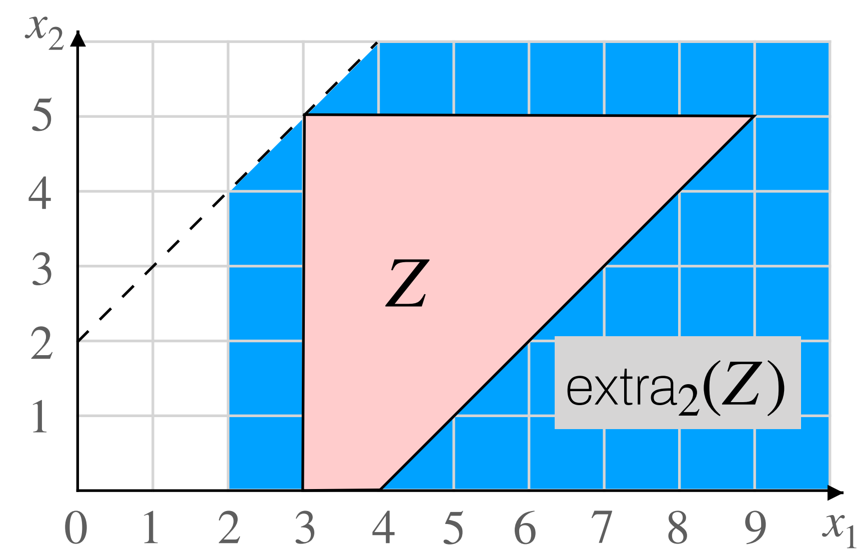

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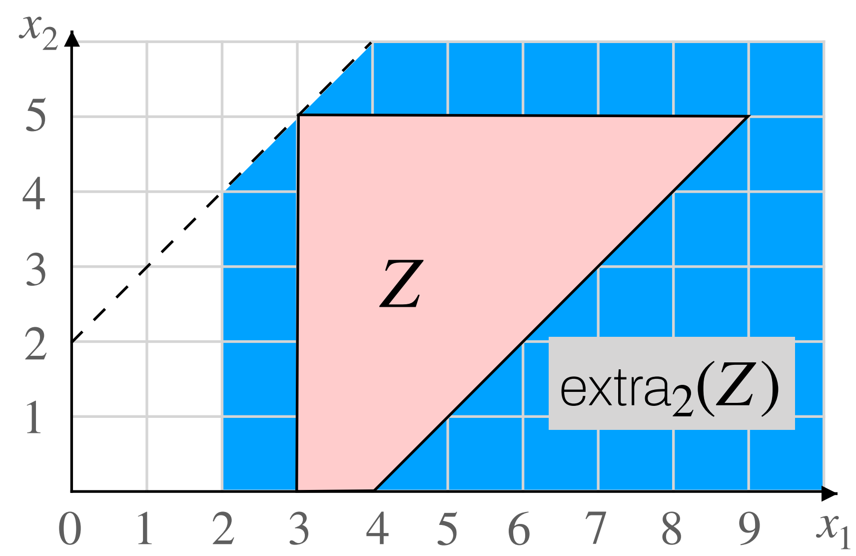

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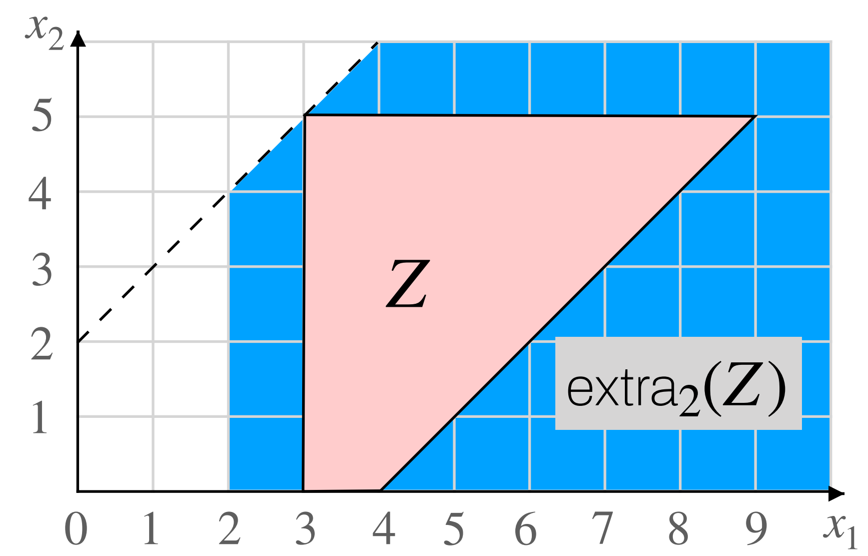

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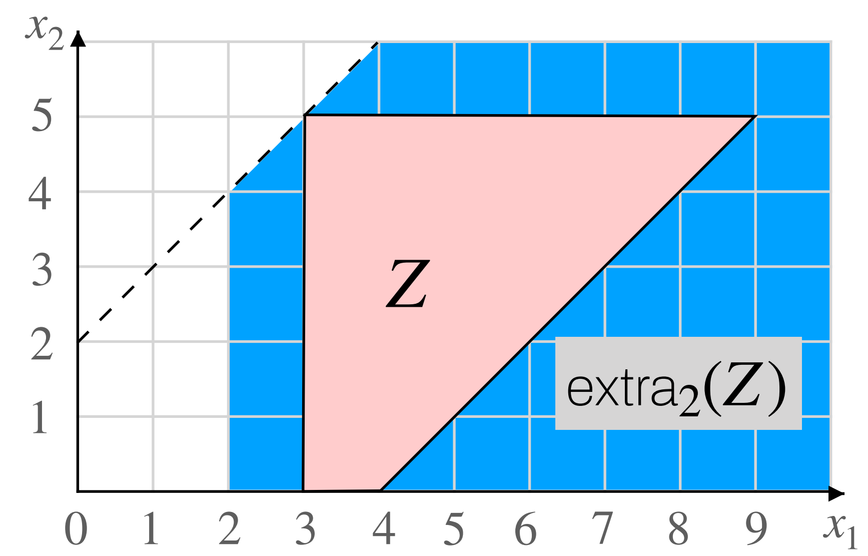

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[DT98] Daws, Tripakis: Model-checking of real-time reachability properties using abstractions (TACAS'98)
[Bou03] Bouyer: Untameable timed automata! (STACS'03)
[Bou04] Bouyer: Forward analysis of updatable timed automata (FMSD)
[BBFL03] Behrmann, Bouyer, Fleury, Larsen: Static guard analysis in timed automata verification (TACAS'03)
[BBLP04] Behrmann, Bouyer, Larsen, Pelánek: Lower and upper bounds in zone based abstractions of timed automata (TACAS'04)
[BBLP06] Behrmann, Bouyer, Larsen, Pelánek: Zone-based abstractions for timed automata exploiting lower and upper bounds (STTT)

Limits of the extrapolation approach

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Limits of the extrapolation approach

- ➡ The extrapolation is required to transform a zone into a zone
- ➡ It does not benefit from the coarsest abstractions of zones [HKSW11]
  - The region closure would in principle be sound, but it is not convex
  - The LU-abstraction $\mathfrak{a}_{LU}(Z) = \{v' \mid \exists v \in Z \text{ s.t. } v' \preceq_{LU} v\}$ would in principle be sound [BBLP06], but it is not convex

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➡ The approach does not apply to timed automata with diagonal constraints [Bou03]

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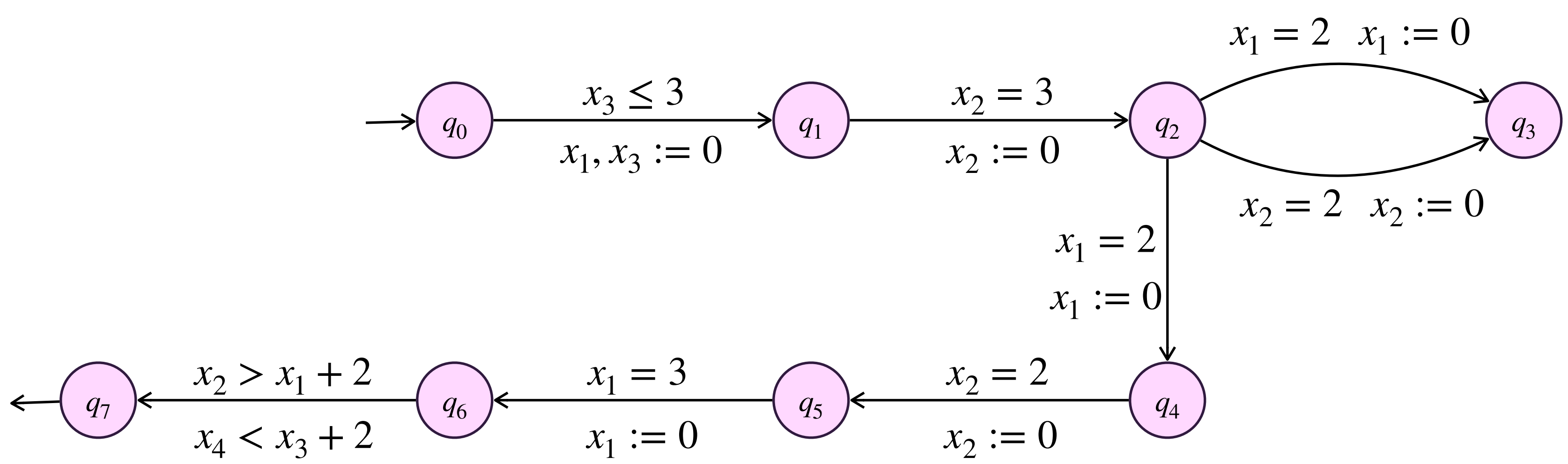
The buggy automaton



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[Bou04] Bouyer. Forward analysis of updatable timed automata (Formal Methods in System Design).

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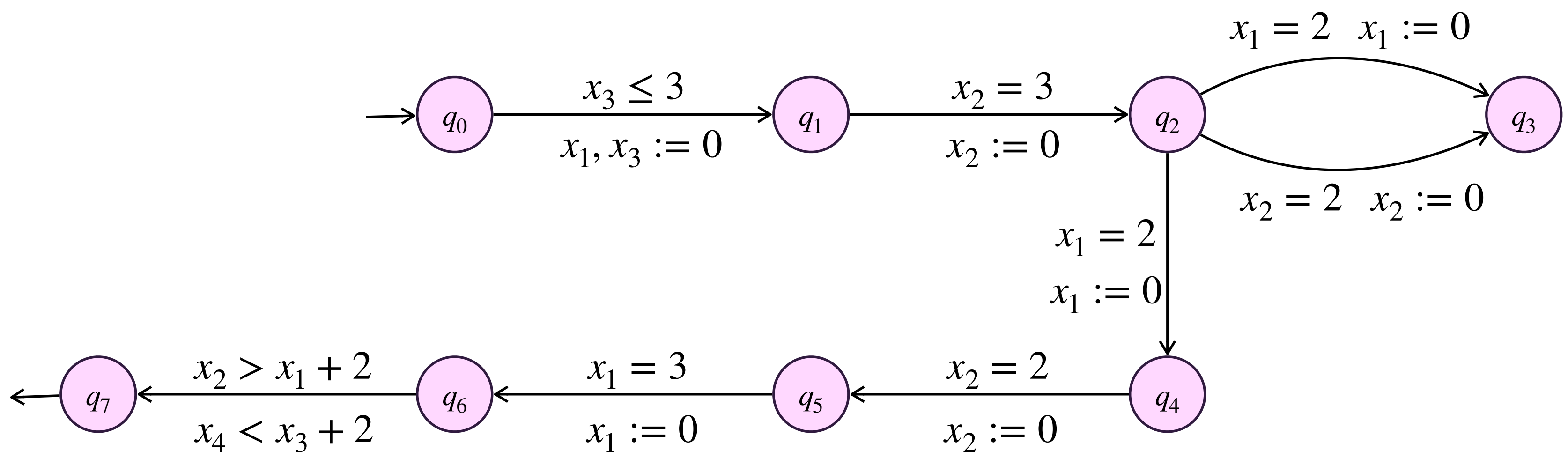

After α loops, the zone which is reached at q_6 is

$$Z_\alpha \ := \ (1 \leq x_2 - x_1 \leq 3) \wedge (1 \leq x_4 - x_3 \leq 3) \wedge (x_4 - x_2 = x_3 - x_1 = 2\alpha + 5)$$

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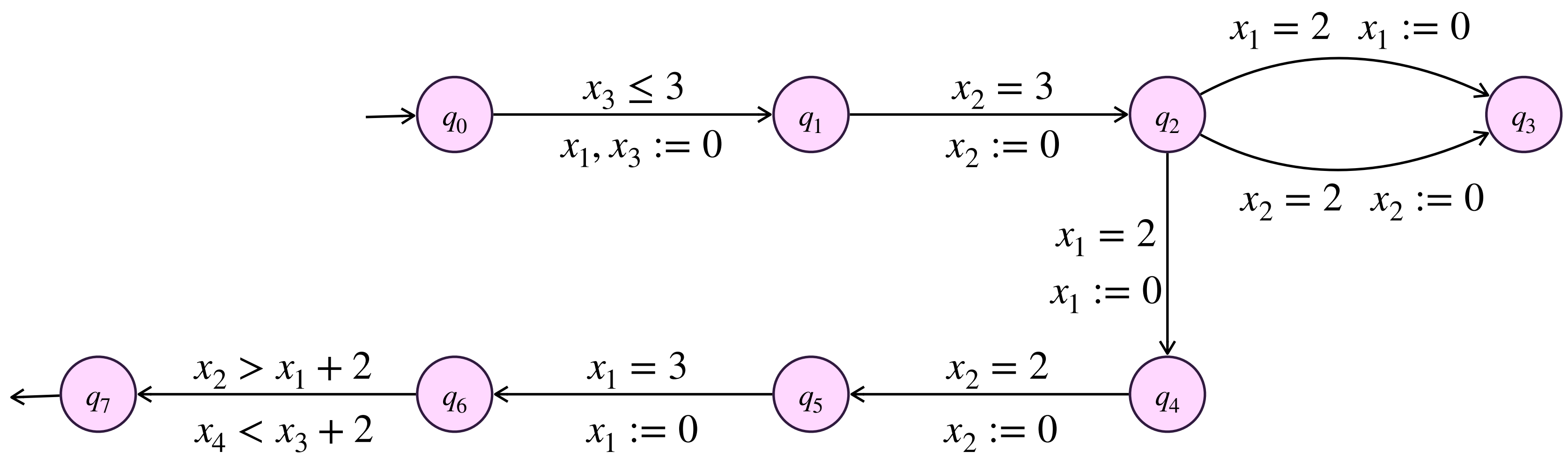

After α loops, the zone which is reached at q_6 is

$$Z_\alpha \;:=\; (1 \leq x_2 - x_1 \leq 3) \wedge (1 \leq x_4 - x_3 \leq 3) \wedge (x_4 - x_2 = x_3 - x_1 = 2\alpha + 5)$$

- There is no extrapolation, which preserves zones, which is sound and finite for this timed automaton with diagonal constraints.

[Bou03] Bouyer. Untameable timed automata! (STACS'03).
[Bou04] Bouyer. Forward analysis of updatable timed automata (Formal Methods in System Design).

The simulation approach

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Should be computationally efficient!

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- Note: $(q, Z_1) \preceq (q, Z_2)$ iff $Z_1 \subseteq$ Closure$_{\preceq}(Z_2)$
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The corresponding inclusion « up-to » can be decided in $\mathcal{O}(|X|^2)$

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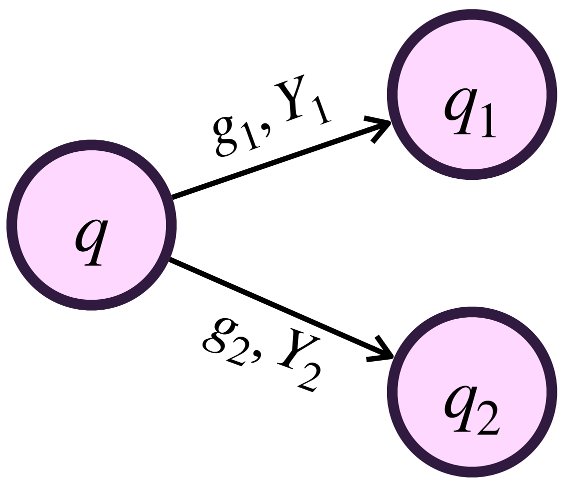
[HKSW11] Herbreteau, Kini, Srivathsan, Walukiewicz: Using non-convex approximations for efficient analysis of timed automata (FSTTCS'11)
[HSW12] Herbreteau, Srivathsan, Walukiewicz: Better Abstractions for Timed Automata (LICS'12)
[GMS18] Gastin, Mukherjee, Srivathsan: Reachability in Timed Automata with Diagonal Constraints (CONCUR'18)
[GMS19] Gastin, Mukherjee, Srivathsan: Fast Algorithms for Handling Diagonal Constraints in Timed Automata (CAV'19)
[GMS20] Gastin, Mukherjee, Srivathsan: Reachability for Updatable Timed Automata Made Faster and More Effective (FSTTCS'20)

Constraints relevant at q: $\mathscr{G}(q)$



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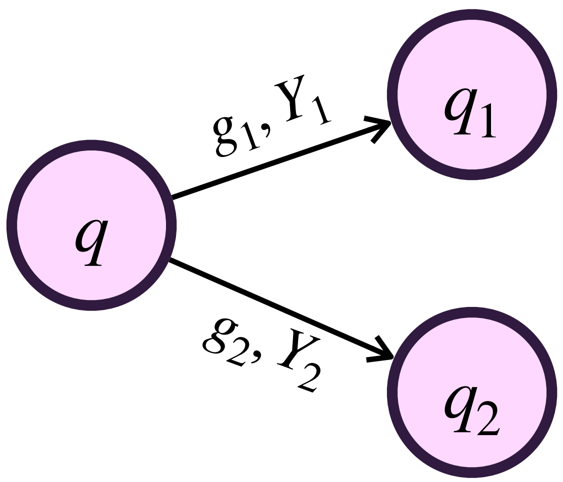

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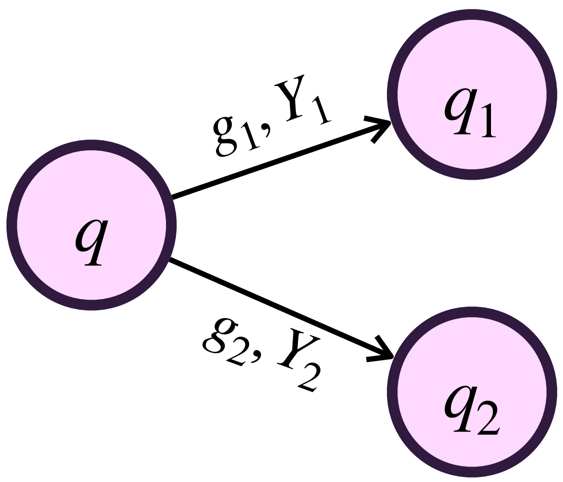

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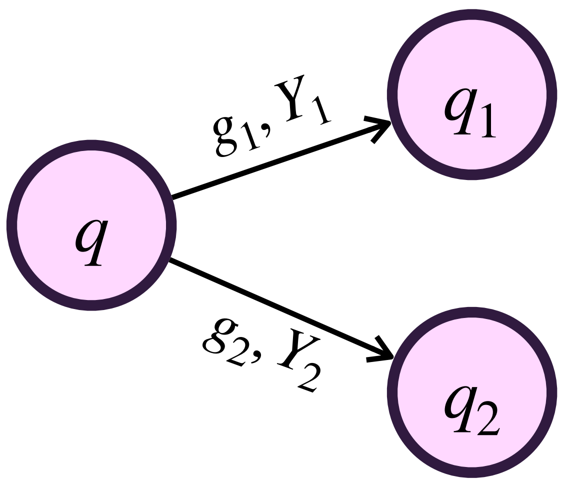

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- Fixpoint computation terminates for timed automata; it also terminates for known decidable classes of updatable timed automata

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Let $\mathcal{G}$ be the previous mapping

- We say that $(q, v) \preceq_{\mathcal{G}} (q, v')$ whenever for every $\varphi \in \mathcal{G}$, for every $\delta \geq 0$, $v + \delta \models \varphi$ implies $v' + \delta \models \varphi$

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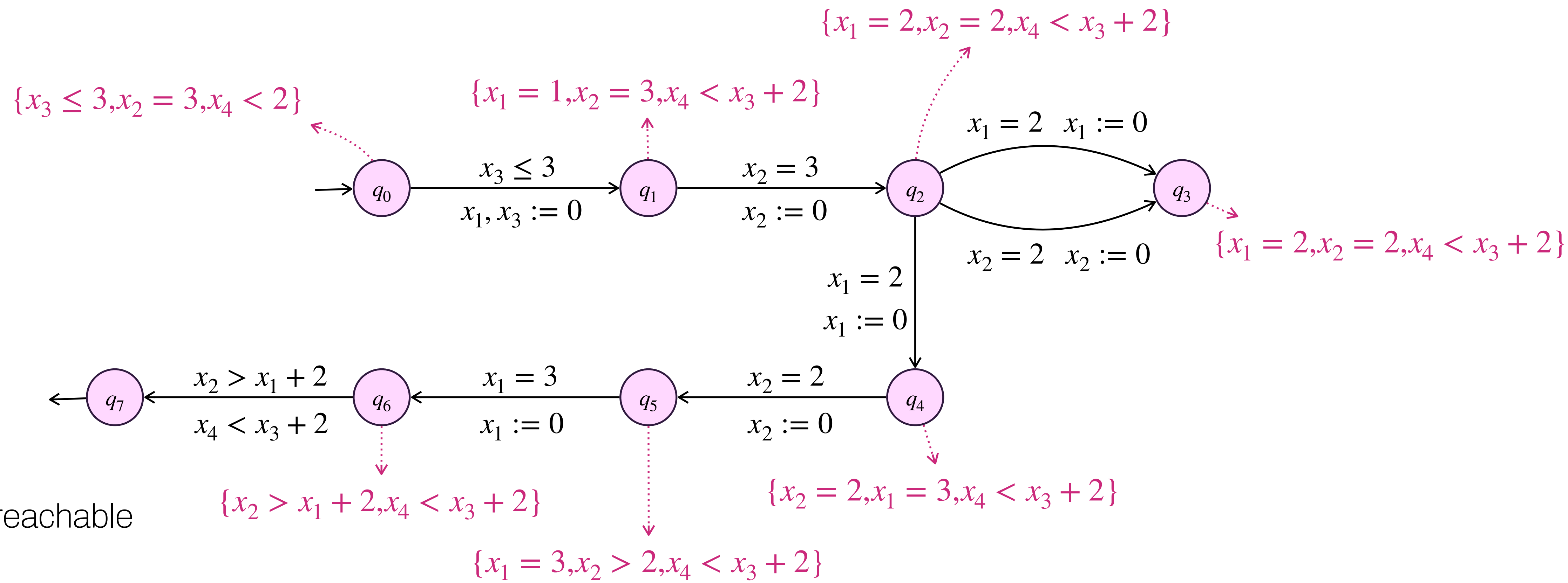
Theorem

- $\preceq_{\mathcal{G}}$ is a simulation relation
- It satisfies the finite-chain property on zones

An example



q_7 is not reachable

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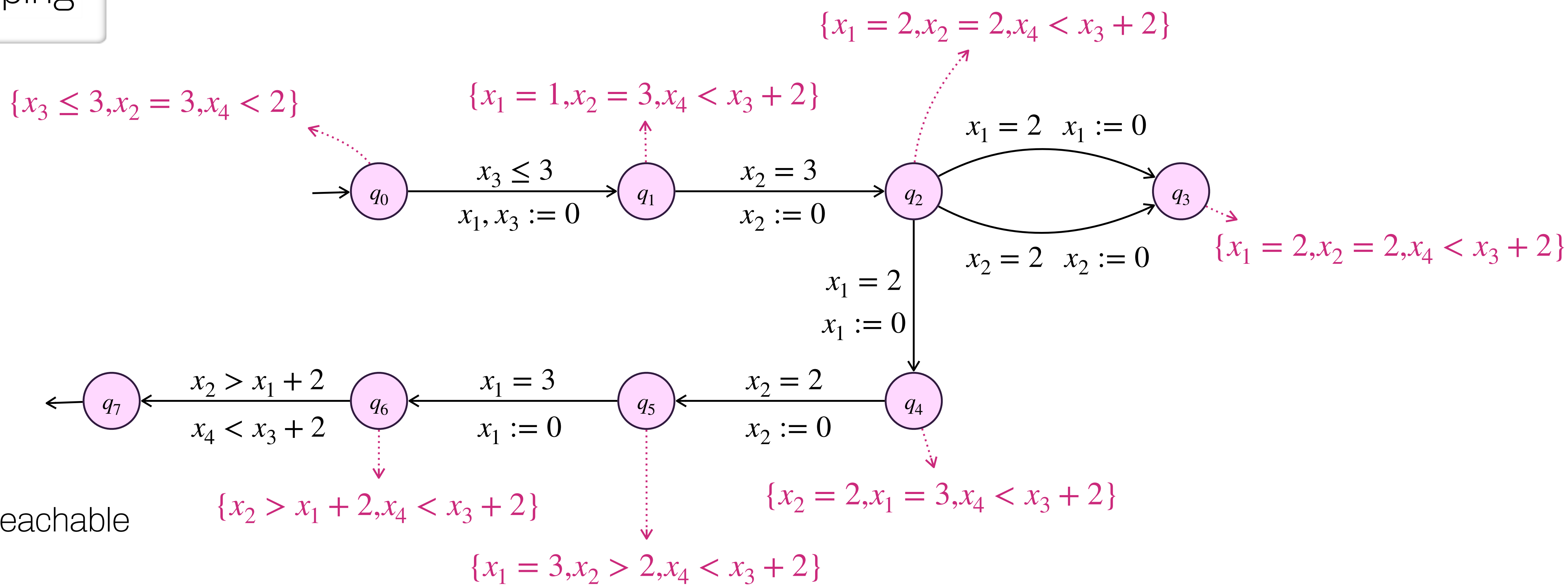

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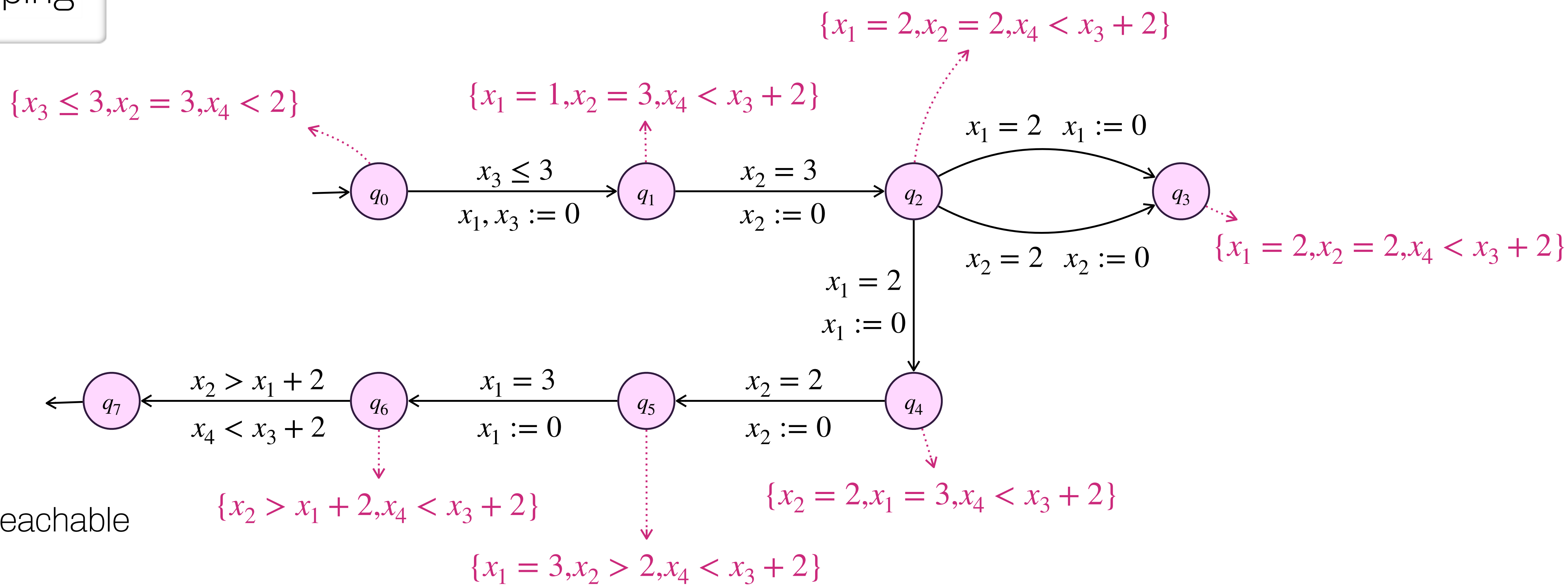

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- On this automaton, any extrapolation-based method fails [Bou04]
- The $\preceq_{\mathscr{G}}$-simulation approach terminates at the second iteration

Going further

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[BCM16] Bouyer, Colange, Markey: Symbolic Optimal Reachability in Weighted Timed Automata (CAV'16)
[HSTW20] Herbreteau, Srivathsan, Tran, Walukiewicz: Why Liveness for Timed Automata Is Hard, and What We Can Do About It (ACM Trans. Comput. Log)
[AGP21] Akshay, Gastin, Prakash: Fast Zone-Based Algorithms for Reachability in Pushdown Timed Automata (CAV'21)
[AGGS22] Akshay, Gastin, Govind, Srivathsan: Simulations for Event-Clock Automata (CONCUR'22)

Tools

- Uppaal https://uppaal.org
- Tchecker https://github.com/ticktac-project/tchecker
- Red https://sites.google.com/site/redlibtw/
- Pat https://pat.comp.nus.edu.sg
- Rabbit https://www.sosy-lab.org/people/beyer/Rabbit/
- MCTA http://gki.informatik.uni-freiburg.de/tools/mcta/
- ...

Tool UPPAAL

https://uppaal.org

- Developed since 1995
- Successfully used in the industry, with many case studies
- Many extensions:
  - games, weighted timed automata, testing, statistical model-checking, ...
- Implements extrapolation-based algorithms

Tool TChecker

https://github.com/ticktac-project/tchecker

- Developed since a couple of years, under development
- Fully open-source verification tool for timed automata
- Implements extrapolation and simulation-based algorithms
- Made also as a framework to develop new verification algorithms or data structures

Conclusion

What next?

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- Much **algorithmic effort** has been made to reduce the impact of the timing aspects (reduce the number of zones to visit)
  - Need to push the ideas to larger classes of models
  - In each case, one of the the difficulties lies in the proof of efficiency of inclusion « up-to »

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- A major bottleneck: the **state explosion** due to control states
  - Use of BDD/SAT technics, bounded model-checking, …
    → No technics overwrites the other, they are useful and complementary
  - Local-time semantics + POR (talk of Sri at SNR) [GHSW22]

[GHSW22] Govind, Herbreteau, Srivathsan, Walukiewicz: Abstractions for the local-time semantics of timed automata: a foundation for partial-order methods (LICS'22)

What next?

- **Domain-specific** algorithms:
  - Funnel automata for robotic systems [BMPS15,BMPS17]

[BMPS15] Bouyer, Markey, Perrin, Schlehuber-Caissier:Timed-Automata Abstraction of Switched Dynamical Systems Using Control Funnels (FORMATS'15)
[BMPS17] Bouyer, Markey, Perrin, Schlehuber-Caissier:Timed-automata abstraction of switched dynamical systems using control invariants (Real Time Syst.)

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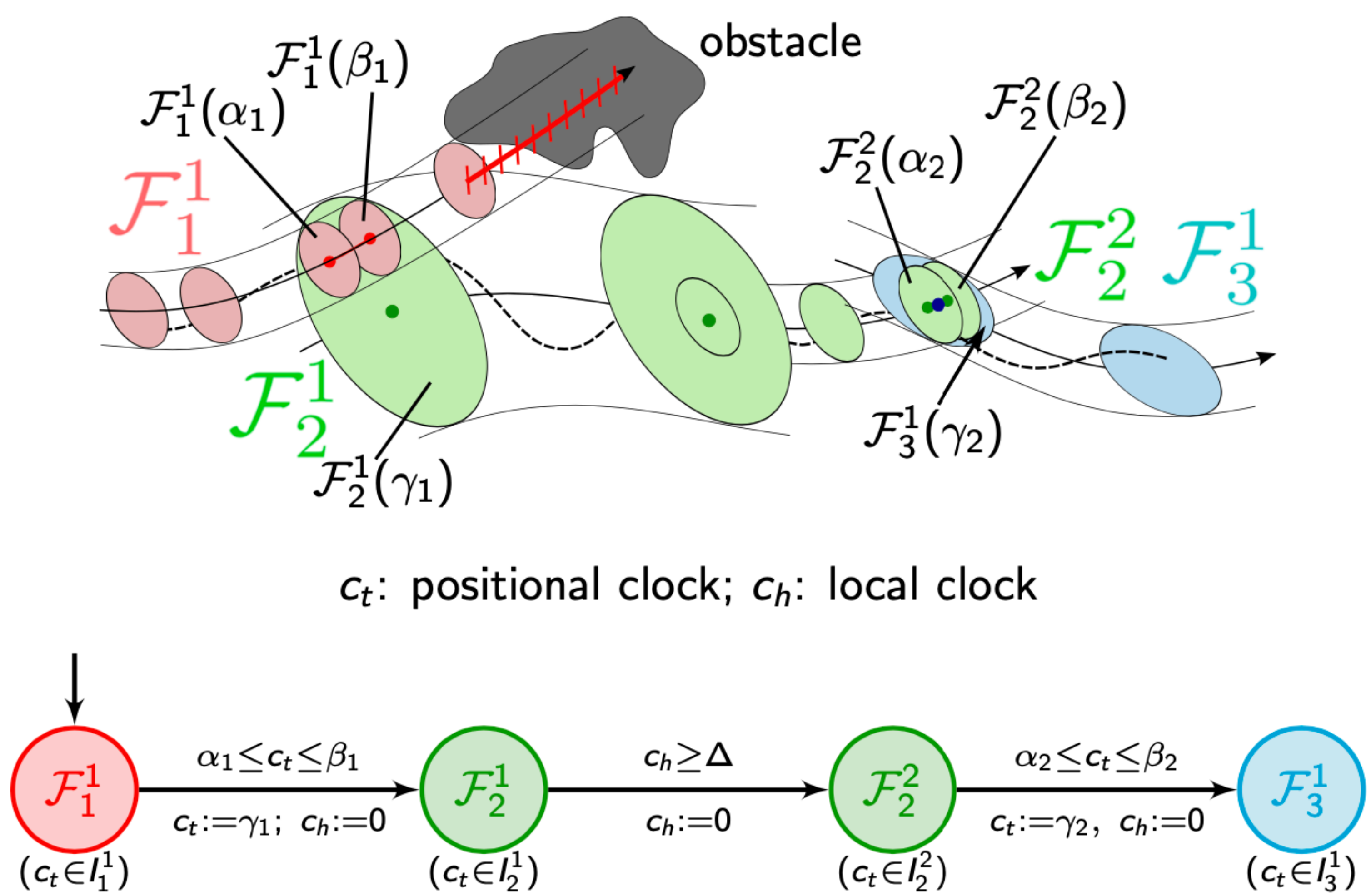

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Thank you for your attention!