Vérification de systèmes temporisés et hybrides

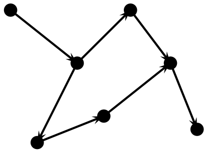
Patricia Bouyer

LSV – CNRS & ENS de Cachan

Model-checking

Does the system satisfy the property?

Modelling



φ

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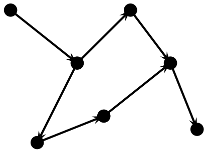


$\models$

φ

Model-checking Algorithm

Time!

Context: verification of embedded critical systems

Time

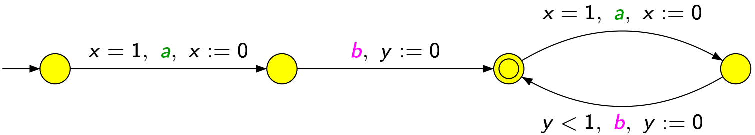
- naturally appears in real systems
- appears in properties (for ex. bounded response time)

➜ Need of models and specification languages integrating timing aspects

A case for dense-time

Time domain: discrete (*e.g.* N) or dense (*e.g.* Q^+)

- A compositionality problem with discrete time
- Dense-time is a more general model than discrete time
- Not all timed systems can be discretized...



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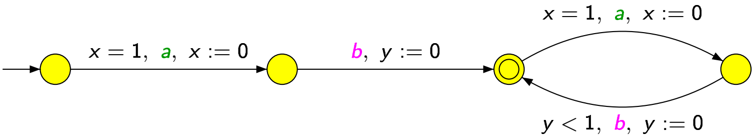

- Dense-time:
  $L_{dense} = \{((ab)^\omega, \tau) \mid \forall i,\ \tau_{2i-1} = i \text{ and } \tau_{2i} - \tau_{2i-1} > \tau_{2i+2} - \tau_{2i+1}\}$

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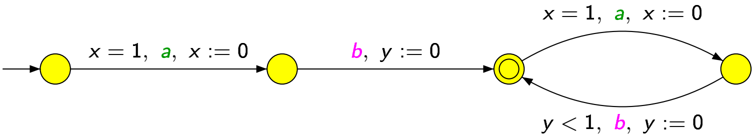

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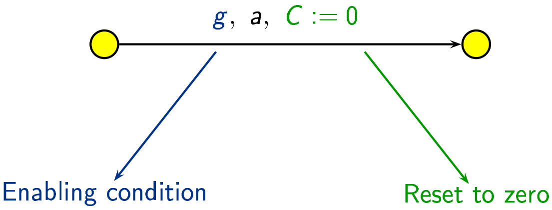
- Discrete-time: $L_{discrete} = \emptyset$

Outline

Timed automata [Alur & Dill 90's]

- A finite control structure + variables (clocks)
- A transition is of the form:



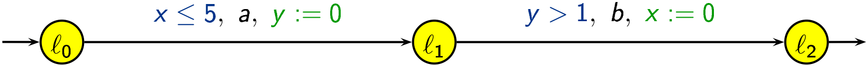
- An enabling condition (or guard) is:

$$g \quad ::= \quad x \sim c \quad | \quad g \wedge g$$

where $\sim \in \{<, \leq, =, \geq, >\}$

Timed automata (example)

x, y : clocks



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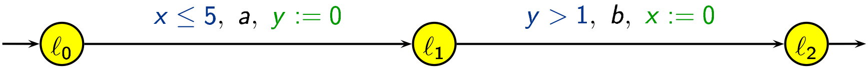

$$\begin{array}{cccccccc} & \ell_0 & \xrightarrow{\delta(4.1)} & \ell_0 & \xrightarrow{a} & \ell_1 & \xrightarrow{\delta(1.4)} & \ell_1 & \xrightarrow{b} & \ell_2 \\ x & 0 & & 4.1 & & 4.1 & & 5.5 & & 0 \\ y & 0 & & 4.1 & & 0 & & 1.4 & & 1.4 \end{array}$$

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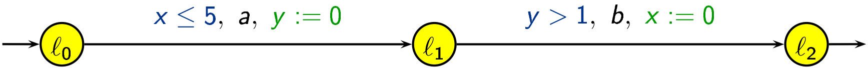

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(clock) valuation

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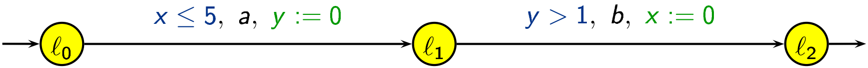

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(clock) valuation

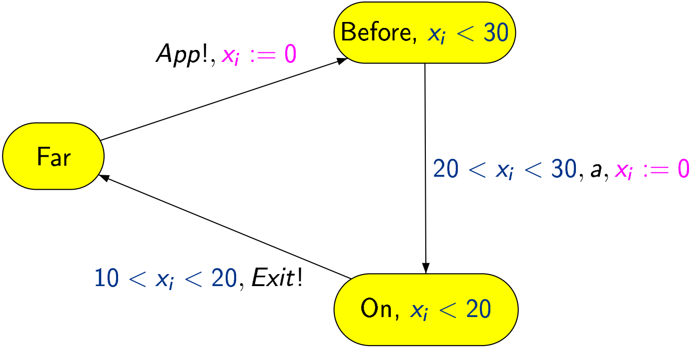
➜ timed word $(a, 4.1)(b, 5.5)$

Timed automata semantics

- $\mathcal{A} = (\Sigma, L, X, \longrightarrow)$ is a TA
- **Configurations:** $(\ell, v) \in L \times T^X$ where T is the time domain
- **Timed Transition System:**
  - action transition: $(\ell, v) \xrightarrow{a} (\ell', v')$ if $\exists \ell \xrightarrow{g,a,r} \ell' \in \mathcal{A}$ s.t. $\begin{cases} v \models g \\ v' = v[r \leftarrow 0] \end{cases}$
  - delay transition: $(\ell, v) \xrightarrow{\delta(d)} (\ell, v + d)$ if $d \in T$

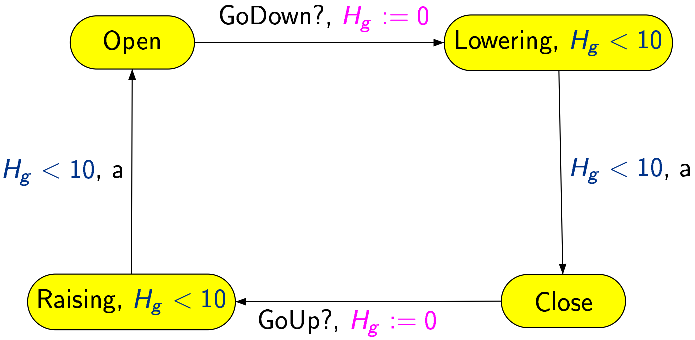
The train crossing example (1)

Train$_i$ with $i = 1, 2, \ldots$



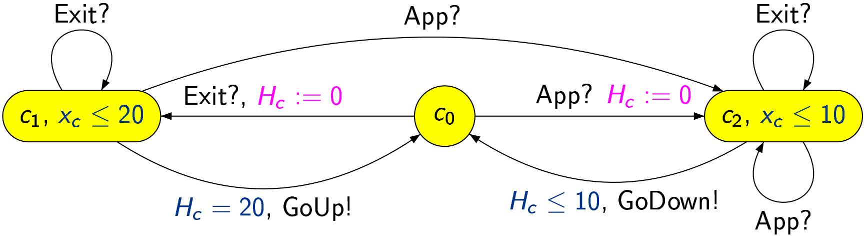
The train crossing example (2)

The gate:



The train crossing example (3)

The controller:



The train crossing example (4)

We use the synchronization function f:

Train_1	Train_2	Gate	Controller	
$App!$	.	.	$App?$	App
.	$App!$	.	$App?$	App
$Exit!$	.	.	$Exit?$	$Exit$
.	$Exit!$	.	$Exit?$	$Exit$
a	.	.	.	a
.	a	.	.	a
.	.	a	.	a
.	.	$GoUp?$	$GoUp!$	$GoUp$
.	.	$GoDown?$	$GoDown!$	$GoDown$

to define the parallel composition $(\mathsf{Train}_1 \parallel \mathsf{Train}_2 \parallel \mathsf{Gate} \parallel \mathsf{Controller})$

NB: the parallel composition does not add expressive power!

The train crossing example (5)

Some properties one could check:

- Is the gate closed when a train crosses the road?

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$$AG(\text{train.On} \Rightarrow \text{gate.Close})$$

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$$\neg EF(\text{gate.Close} \wedge (\text{gate.Close}\ U_{>5min}\ \neg\text{gate.Close}))$$

Outline

Classical verification problems

- reachability of a control state
- $\mathcal{S} \sim \mathcal{S}'$: bisimulation, etc...
- $L(\mathcal{S}) \subseteq L(\mathcal{S}')$: language inclusion
- $\mathcal{S} \models \varphi$ for some formula φ (*e.g.* in a timed extension of classical temporal logics): model-checking
- $\mathcal{S} \parallel A_T$ + reachability: testing automata
- ...

Verification

Emptiness problem: is the language accepted by a timed automaton empty?

- reachability properties (final states)
- basic liveness properties (Büchi (or other) conditions)

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- **Problem:** the set of configurations is infinite
  ➜ classical methods can not be applied

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Theorem: The emptiness problem for timed automata is decidable.
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Note: This is also the case for the discrete semantics.

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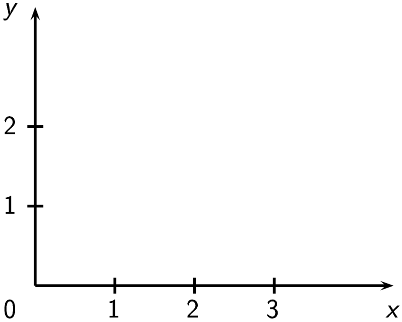
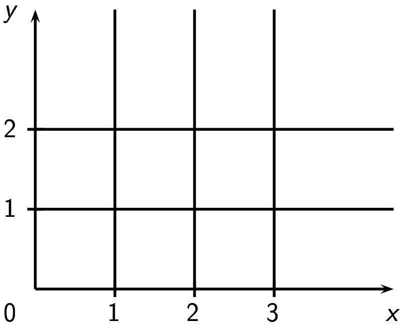
Method: construct a finite abstraction

The region abstraction



Equivalence of finite index

The region abstraction



Equivalence of finite index

- "compatibility" between regions and constraints

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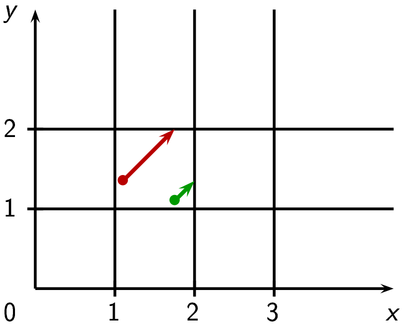

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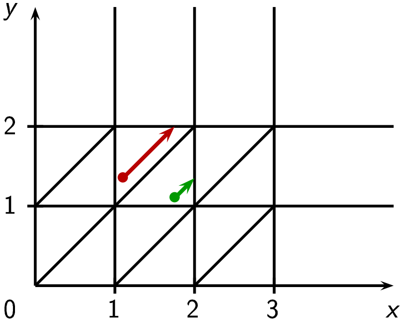

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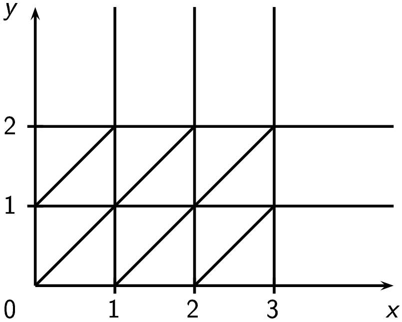

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➜ a bisimulation property

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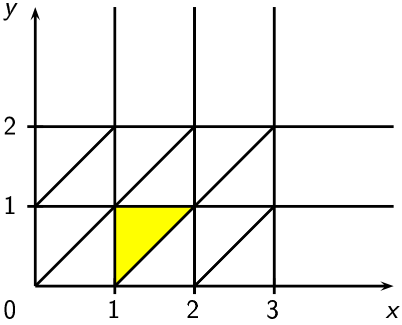

Equivalence of finite index

region defined by

$I_x =]1;2[, \ I_y =]0;1[$

$\{x\} < \{y\}$

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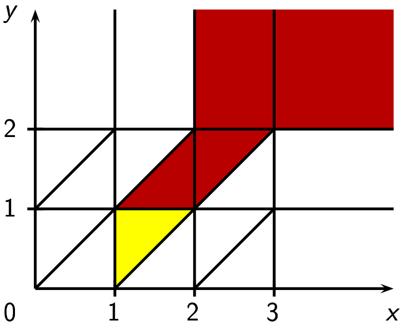

Equivalence of finite index

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successor regions

- "compatibility" between regions and constraints
- "compatibility" between regions and time elapsing

➜ a bisimulation property

The region automaton

timed automaton $\otimes$ region abstraction

$\ell \xrightarrow{g,a,C:=0} \ell'$ is transformed into:

$(\ell, R) \xrightarrow{a} (\ell', R')$ if there exists $R'' \in \mathrm{Succ}_t^*(R)$ s.t.

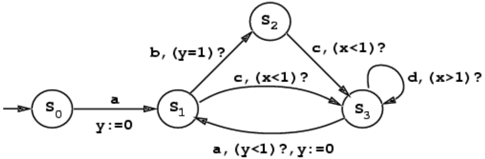
- $R'' \subseteq g$
- $[C \leftarrow 0]R'' \subseteq R'$

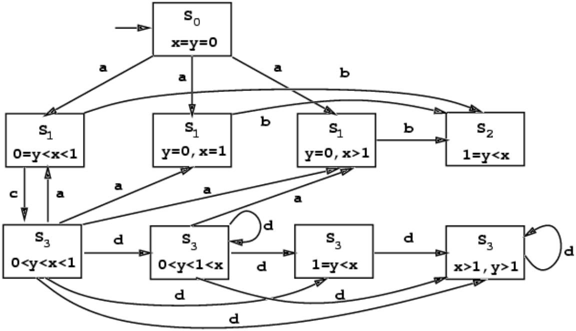
➜ **time-abstract bisimulation**

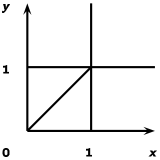
$\mathcal{L}$(reg. aut.) = UNTIME($\mathcal{L}$(timed aut.))

where $\mathrm{UNTIME}((a_1, t_1)(a_2, t_2)\dots) = a_1 a_2 \dots$

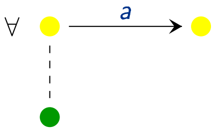
An example [AD 90's]







Time-abstract bisimulation



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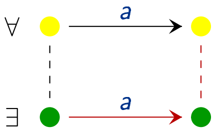

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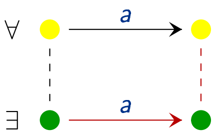

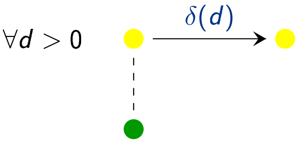

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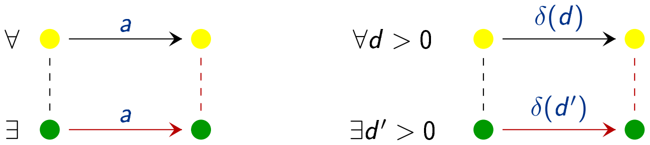

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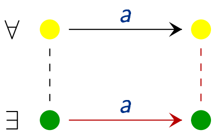
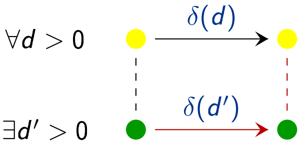
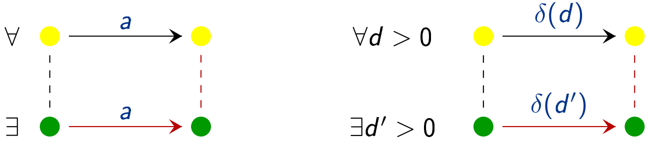




$$(\ell_0, v_0) \xrightarrow{a_1, t_1} (\ell_1, v_1) \xrightarrow{a_2, t_2} (\ell_2, v_2) \xrightarrow{a_3, t_3} \dots$$

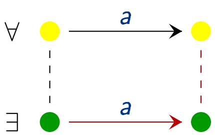
Time-abstract bisimulation

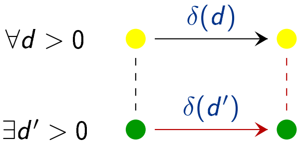


$$
\begin{tikzcd}
(\ell_0, v_0) \arrow[r, "a_1, t_1"] \arrow[d] & (\ell_1, v_1) \arrow[r, "a_2, t_2"] \arrow[d] & (\ell_2, v_2) \arrow[r, "a_3, t_3"] \arrow[d] & \ldots \\
(\ell_0, R_0) \arrow[r, "a_1"] & (\ell_1, R_1) \arrow[r, "a_2"] & (\ell_2, R_2) \arrow[r, "a_3"] & \ldots
\end{tikzcd}
$$

with $v_i \in R_i$ for all i.

Time-abstract bisimulation





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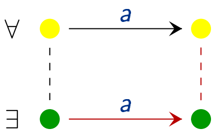

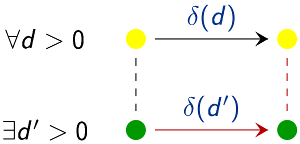

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$$

with $v_i \in R_i$ for all i.

Remark: Real-time properties can not be checked with a time-abstract bisimulation. An extended construction needs to be used.

Consequence of region automata construction

Region automata: correct finite abstraction for checking reachability/Büchi-like properties

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Region automata: correct finite abstraction for checking reachability/Büchi-like properties

However, everything can not be reduced to finite automata...

A model not far from undecidability

- Universality is undecidable [Alur & Dill 90's]
- Inclusion is undecidable [Alur & Dill 90's]
- Determinizability is undecidable [Tripakis 2003]
- Complementability is undecidable [Tripakis 2003]
- ...

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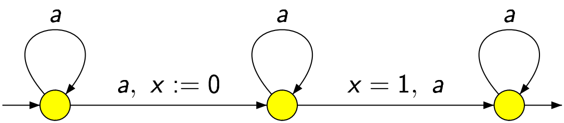
An example of non-determinizable/non-complementable timed aut.:



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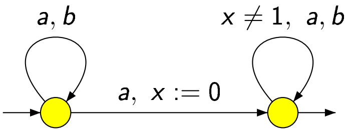
[Alur,Madhusudan 2004]



UNTIME $\left(\overline{L} \cap \{(a^*b^*, \tau) \mid \text{all } a's \text{ happen before 1 and no two } a's \text{ simultaneously}\}\right)$ is not regular **(exercise!)**

Partial conclusion

➜ a timed model interesting for verification purposes

Numerous works have been (and are) devoted to:

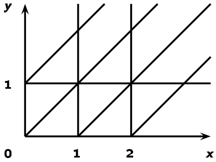
- the "theoretical" comprehension of timed automata (*cf* **[Asarin 2004]**)
- extensions of the model (to ease modelling)
  - expressiveness
  - analyzability
- algorithmic problems and implementation

Outline

Role of diagonal constraints

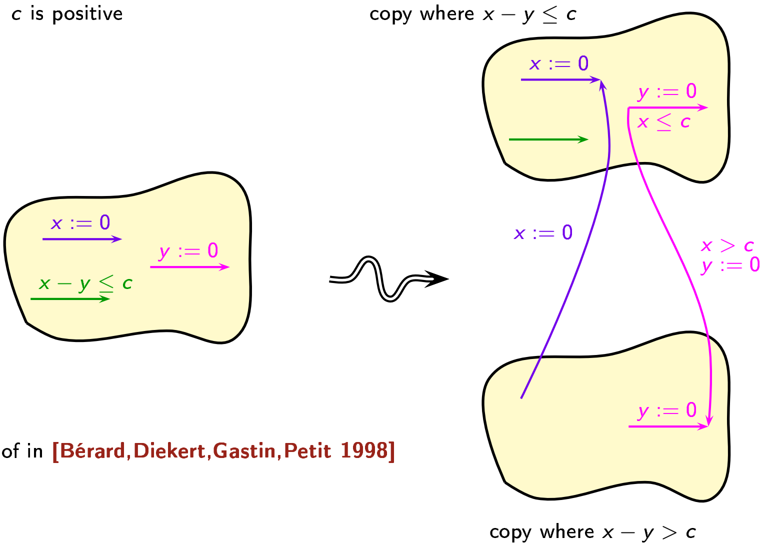
$$x - y \sim c \quad \text{and} \quad x \sim c$$

- **Decidability:** yes, using the region abstraction



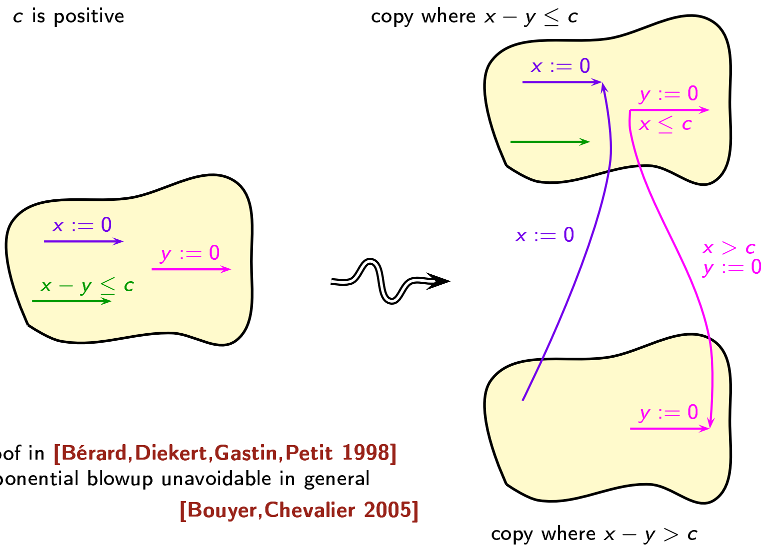
- **Expressiveness:** no additional expressive power

Role of diagonal constraints (cont.)



➜ proof in [Bérard,Diekert,Gastin,Petit 1998]

Role of diagonal constraints (cont.)



➜ proof in **[Bérard,Diekert,Gastin,Petit 1998]**
➜ exponential blowup unavoidable in general **[Bouyer,Chevalier 2005]**

Adding silent actions

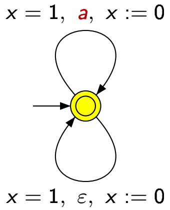
$$\xrightarrow{g,\,\varepsilon,\,C := 0}$$

[Bérard,Diekert,Gastin,Petit 1998]

- **Decidability:** yes

  (actions have no influence on region automaton construction)

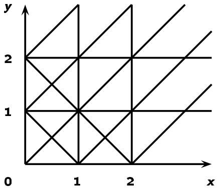
- **Expressiveness:** strictly more expressive!



Adding constraints of the form $x + y \sim c$

$x + y \sim c$ and $x \sim c$ **[Bérard, Dufourd 2000]**

- **Decidability:** - for two clocks, decidable using the abstraction

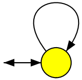


  - for four clocks (or more), undecidable!

▸ The proof

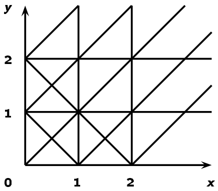
- **Expressiveness:** more expressive! (even using two clocks)

$$\{(a^n, t_1 \dots t_n) \mid n \geq 1 \text{ and } t_i = 1 - \frac{1}{2^i}\}$$



Adding constraints of the form $x + y \sim c$

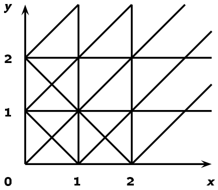
- Two clocks: decidable using the abstraction



- Four clocks (or more): undecidable!

Adding constraints of the form $x + y \sim c$

- Two clocks: decidable using the abstraction

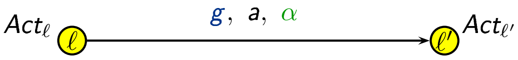


- Three clocks: open question!

- Four clocks (or more): undecidable!

Linear hybrid automata [Henzinger 1996]

- A finite control structure + a set X of **dynamical variables**
- A transition is of the form:

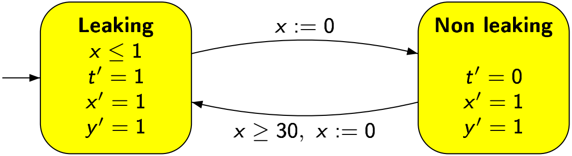


- g is a linear constraint on variables
- α is a jump condition, *i.e.* an affine update of the form $X' = A.X + B$
- in each state, an activity function assigning a slope to each variable (for each $x \in X$, $Act(x) \in [\ell, u]$)

Linear hybrid automata (example)

The **gas burner** may leak. [ACHH93]

- each time a leakage is detected, it is repaired or stopped in less than 1s
- two leakages are separated by at least 30s



Is it possible that the gas burner leaks during a time greater than $\frac{1}{20}$ of the global time after the 60 first minutes?

$$AG(y \geq 60 \Rightarrow 20t \leq y)$$

What about decidability?

➜ almost everything is undecidable
[Henzinger,Kopke,Puri,Varaiya 98]

Theorem. The class of LHA with clocks and only one variable having possibly two slopes $k_1 \neq k_2$ is undecidable.

Theorem. The class of *stopwatch* automata is undecidable.

One of the "largest" classes of LHA which are decidable is the class of initialized rectangular automata.

Outline

Notice

The region automaton is not used for implementation:

- suffers from a combinatorics explosion
  (the number of regions is exponential in the number of clocks)
- no really adapted data structure

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...but **on-the-fly technics** are prefered.

Reachability analysis

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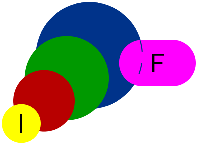

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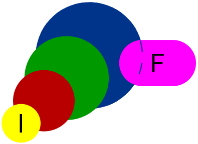

- **backward analysis algorithm:**
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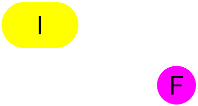

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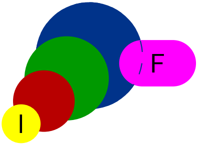

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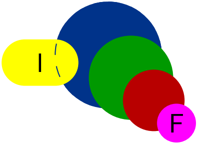


Symbolic representations

Linear hybrid automata: polyhedra, defined by inequations of the form

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n \bowtie b$$

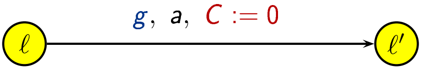
➜ tool HyTech

`http://www-cad.eecs.berkeley.edu:80/~tah/HyTech/`

Timed automata: particular kind of polyhedra called zones

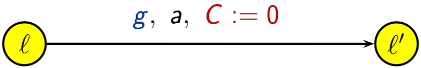
$$x_i - x_j \bowtie c \qquad \text{or} \qquad x_i \bowtie c$$

Note on the backward analysis of TA



$\overleftarrow{[C \leftarrow 0]^{-1}(Z \cap (C = 0)) \cap g}$ $\qquad Z$

Note on the backward analysis of TA



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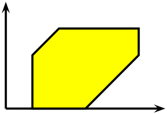

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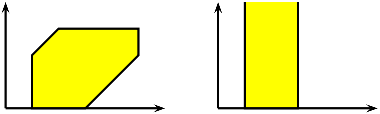

Z $\qquad\qquad$ $[C \leftarrow 0]^{-1}(Z \cap (C = 0))$

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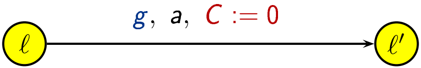

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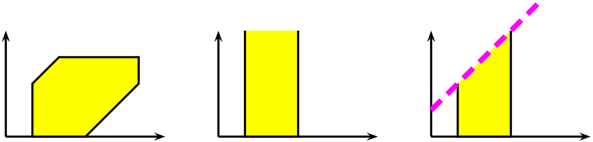

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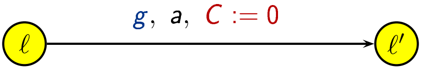

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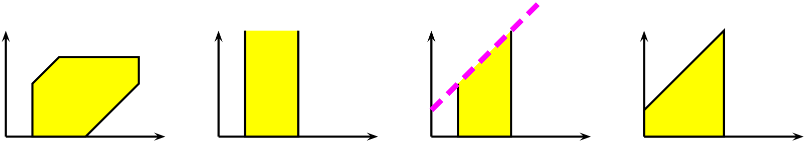

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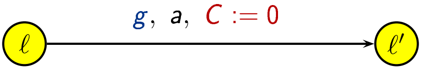

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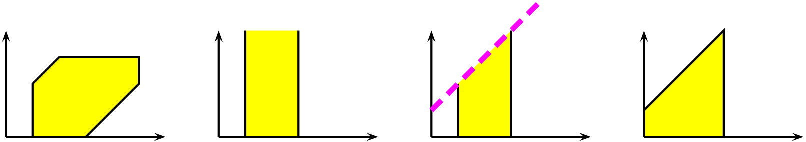
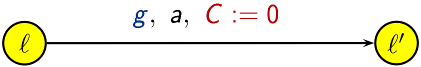


Z $\quad$ $[C \leftarrow 0]^{-1}(Z \cap (C = 0))$ $\quad$ $\overleftarrow{[C \leftarrow 0]^{-1}(Z \cap (C = 0)) \cap g}$

The exact backward computation terminates and is correct!

▸ The proof

Forward analysis of timed automata



zones $\quad Z \quad [C \leftarrow 0](\overrightarrow{Z} \cap g)$

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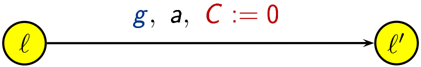

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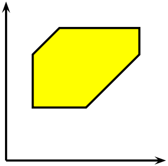

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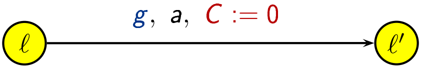

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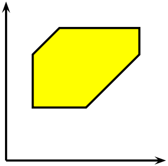

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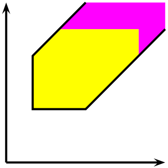

$\overrightarrow{Z}$

Forward analysis of timed automata

$$\ell \xrightarrow{g,\ a,\ C := 0} \ell'$$

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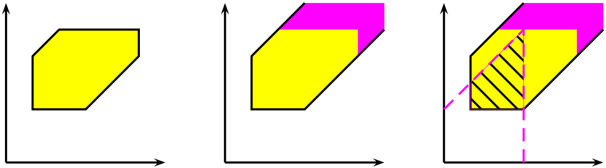

$Z \quad\quad \overrightarrow{Z} \quad\quad \overrightarrow{Z} \cap g$

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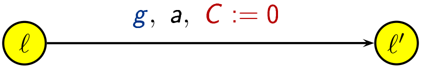

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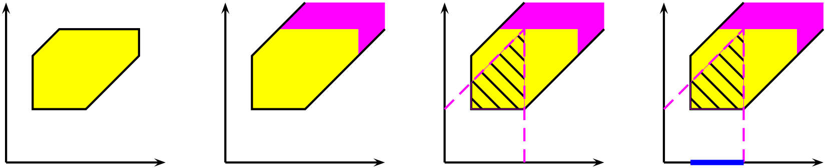

$Z \quad \overrightarrow{Z} \quad \overrightarrow{Z} \cap g \quad [y \leftarrow 0](\overrightarrow{Z} \cap g)$

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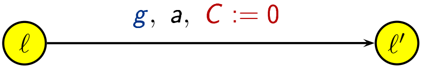

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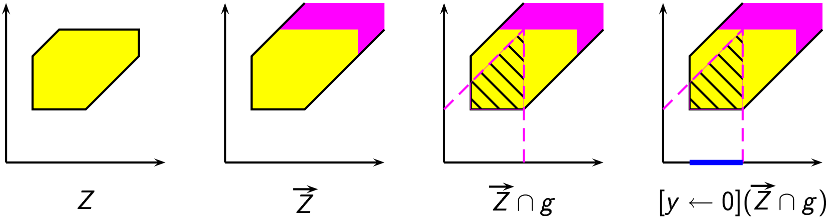
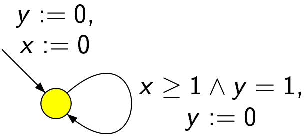
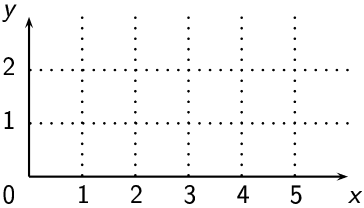


➜ a termination problem

Non termination of the forward analysis





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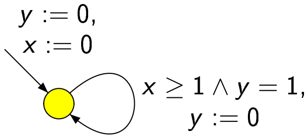

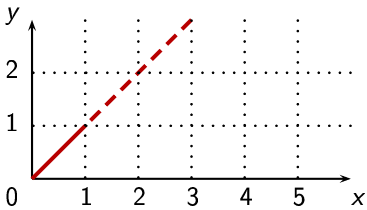

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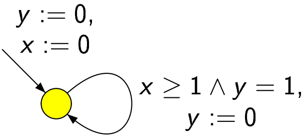

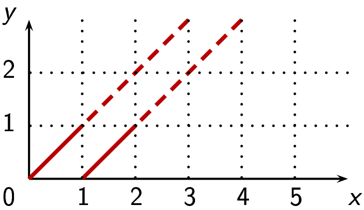

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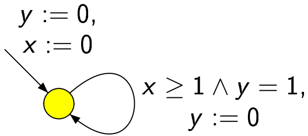

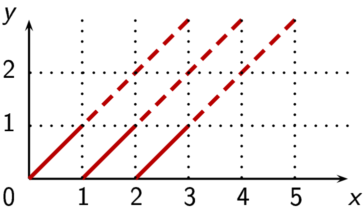

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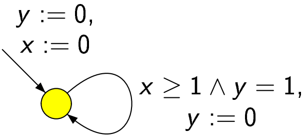

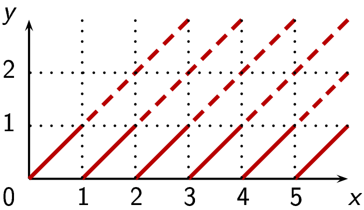

➜ an infinite number of steps...

"Solutions" to this problem

(f.ex. in **[Larsen,Pettersson,Yi 1997]** or in **[Daws,Tripakis 1998]**)

- **inclusion checking**: if $Z \subseteq Z'$ and Z' already considered, then we don't need to consider Z

➜ correct w.r.t. reachability

...

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- **inclusion checking**: if $Z \subseteq Z'$ and Z' already considered, then we don't need to consider Z

  ➜ correct w.r.t. reachability

- **activity**: eliminate redundant clocks [Daws,Yovine 1996]

  ➜ correct w.r.t. reachability

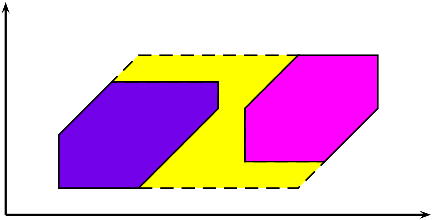
$$q \xrightarrow{g,a,C:=0} q' \quad \text{implies} \quad \mathrm{Act}(q) = \mathrm{clocks}(g) \cup (\mathrm{Act}(q') \setminus C)$$

...

"Solutions" to this problem (cont.)

- **convex-hull approximation**: if Z and Z' are computed then we overapproximate using "$Z \sqcup Z'$".

➜ "semi-correct" w.r.t. reachability



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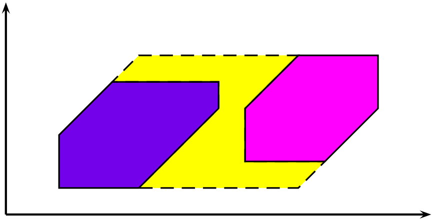

- **extrapolation**, a widening operator on zones

The DBM data structure

DBM (Difference Bounded Matrice) data structure

[Berthomieu, Menasche 1983] [Dill 1989]

$$(x_1 \geq 3) \ \wedge \ (x_2 \leq 5) \ \wedge \ (x_1 - x_2 \leq 4)$$

$$\begin{array}{c c} & \begin{array}{ccc} x_0 & x_1 & x_2 \end{array} \\ \begin{array}{c} x_0 \\ x_1 \\ x_2 \end{array} & \begin{pmatrix} +\infty & -3 & +\infty \\ +\infty & +\infty & 4 \\ 5 & +\infty & +\infty \end{pmatrix} \end{array}$$

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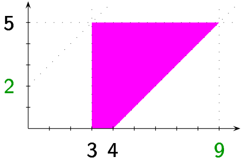
- Existence of a normal form



$$\begin{pmatrix} 0 & -3 & 0 \\ 9 & 0 & 4 \\ 5 & 2 & 0 \end{pmatrix}$$

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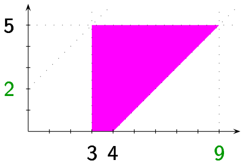

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- All previous operations on zones can be computed using DBMs

The extrapolation operator

Fix an integer k ("$*$" represents an integer between $-k$ and $+k$)

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- "intuitively", erase non-relevant constraints

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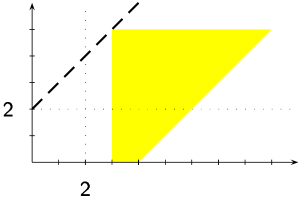

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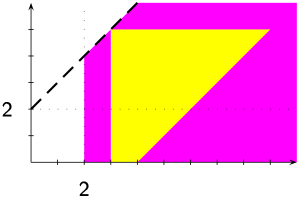

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Classical algorithm, focus on correctness

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However, this theorem does not extend to timed automata using diagonal clock constraints...

A counter-example

- Implemented in numerous tools:
  - Uppaal, `http://www.uppaal.com/`
  - Kronos, `http://www-verimag.imag.fr/TEMPORISE/kronos/`
  - ...
- Successfully used on many real-life examples since ten years.

Outline

Discussion on complexity

[Alur 1991, Alur Henzinger 1994, Alur Courcoubetis Dill 1993, Aceto Laroussinie 2002]

	Kripke structures S	Timed automaton A
Reachability	NLOGSPACE-complete	
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To be continued...

The two-counter machine

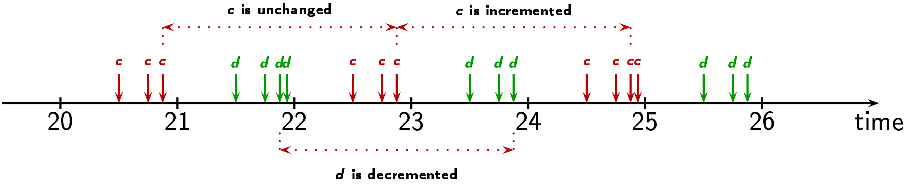
Definition. A two-counter machine is a finite set of instructions over two counters (x and y):

- Incrementation:
  $\texttt{(p)}: \quad x := x + 1;\ \texttt{goto (q)}$

- Decrementation:
  $\texttt{(p)}: \quad \texttt{if } x > 0 \texttt{ then } x := x - 1;\ \texttt{goto (q) else goto (r)}$

Theorem. **[Minsky 67]** The halting problem for two counter machines is undecidable.

Undecidability proof



➜ simulation of
- decrementation of a counter
- incrementation of a counter

We will use 4 clocks:

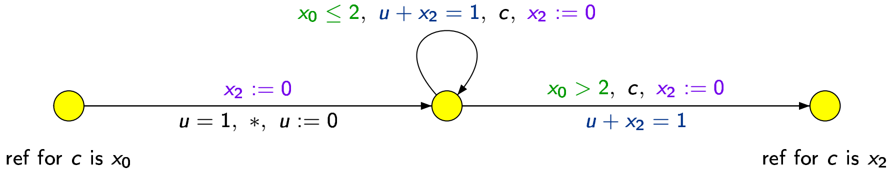
- u, "tic" clock (each time unit)
- x_0, x_1, x_2: reference clocks for the two counters

"x_i reference for c" $\equiv$ "the last time x_i has been reset is the last time action c has been performed"

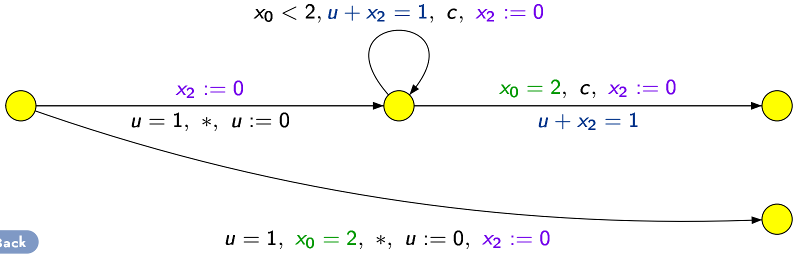
[Bérard,Dufourd 2000]

Undecidability proof (cont.)

- **Incrementation of counter c:**



- **Decrementation of counter c:**



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Note on the backward analysis (cont.)

If $\mathcal{A}$ is a timed automaton, we construct its corresponding set of regions.

Because of the bisimulation property, we get that:

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Let R be a region. Assume:

- $v \in \overleftarrow{R}$ (for ex. $v + t \in R$)
- $v' \equiv_{\mathbf{reg.}} v$

There exists t' s.t. $v' + t' \equiv_{\mathbf{reg.}} v + t$, which implies that $v' + t' \in R$ and thus $v' \in \overleftarrow{R}$.

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If $\mathcal{A}$ is a timed automaton, we construct its corresponding set of regions.

Because of the bisimulation property, we get that:

> "Every set of valuations which is computed along the backward computation is a finite union of regions"

But, the backward computation is not so nice, when also dealing with integer variables...

$$i := j.k + \ell.m$$

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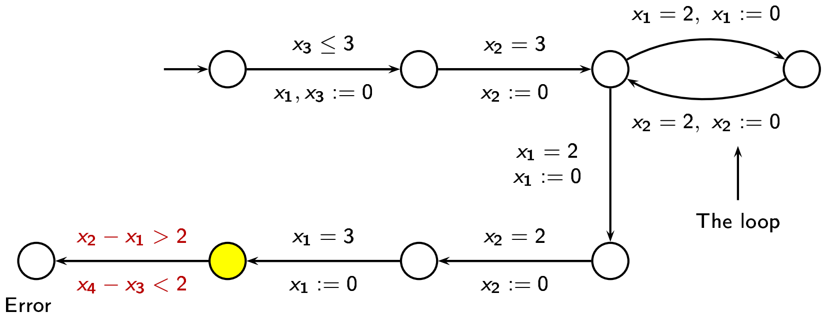
A problematic automaton



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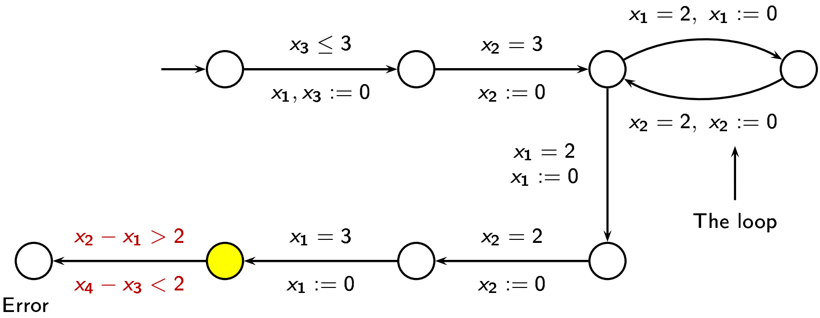

$$\begin{cases} v(x_1) = 0 \\ v(x_2) = d \\ v(x_3) = 2\alpha + 5 \\ v(x_4) = 2\alpha + 5 + d \end{cases}$$

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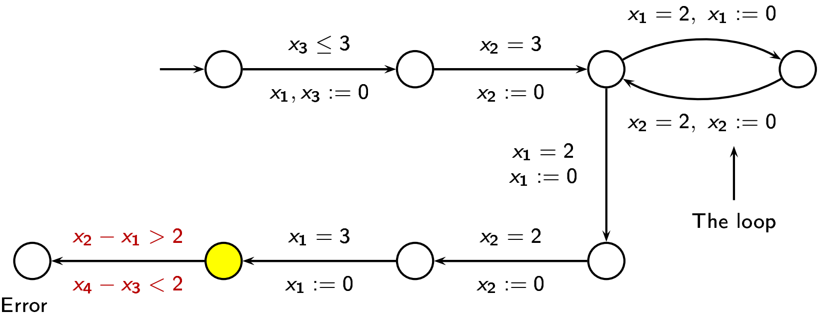

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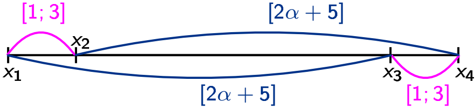
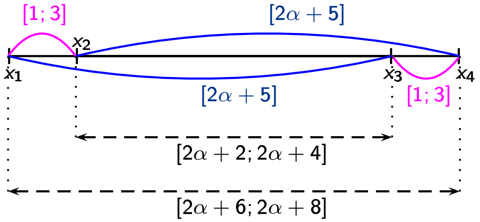


The problematic zone



implies $\quad x_1 - x_2 = x_3 - x_4.$

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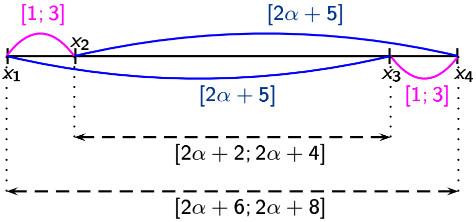
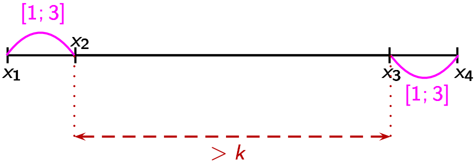


implies $x_1 - x_2 = x_3 - x_4$.

If α is sufficiently large, after extrapolation:



does not imply $x_1 - x_2 = x_3 - x_4$.

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