Laboratoire Méthodes Formelles

université PARIS-SACLAY

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école normale supérieure paris—saclay

Parameterized concurrent games

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Based on joint works with Nathalie Bertrand and Anirban Majumdar

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Preliminary results published at FSTTCS'19'20


ETAPS
2018

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How game-theoretic models and technics/tools can help handling parameterized verification and synthesis?

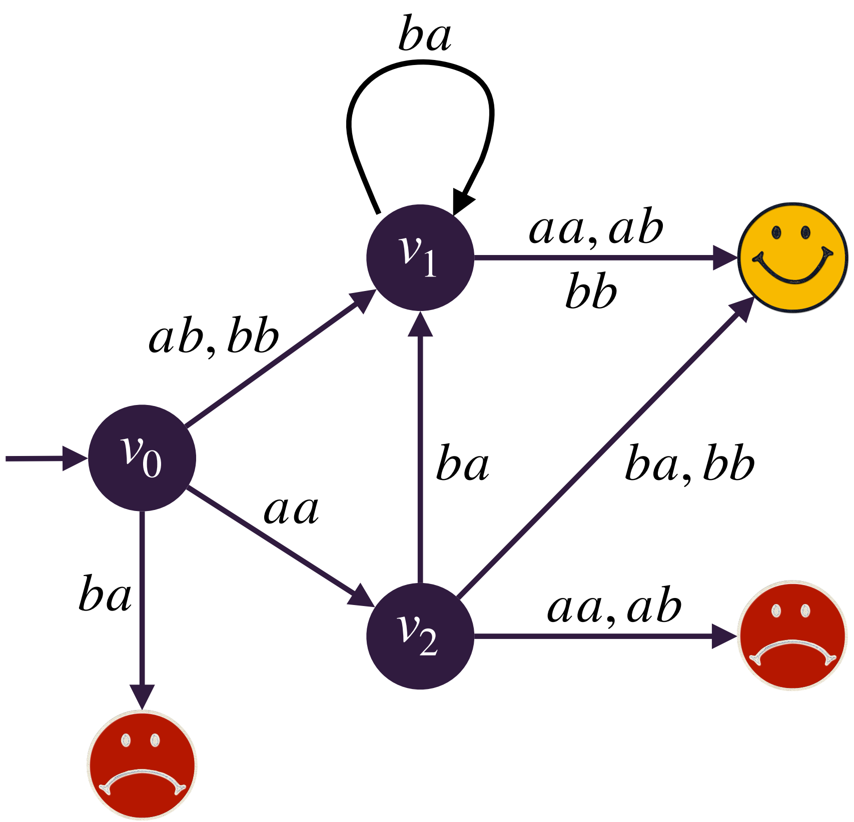
- Propose a new game-based model for parameterized reasoning
- Design synthesis algorithms in two settings:
  - Crowd controller problem
  - Coalition problem

Two-player games as a model for controller synthesis

- Two-player game = model for open systems
- Two players = system *vs* environment
- Winning objective for system player = specification
- Winning strategy for system player = safe controller

[Tho02] W. Thomas : Infinite Games and Verification (CAV'02)
[FOX] Lectures in Game Theory for Computer Scientists (edited by K. R. Apt and E. Grädel)

Two-player (zero-sum) concurrent games



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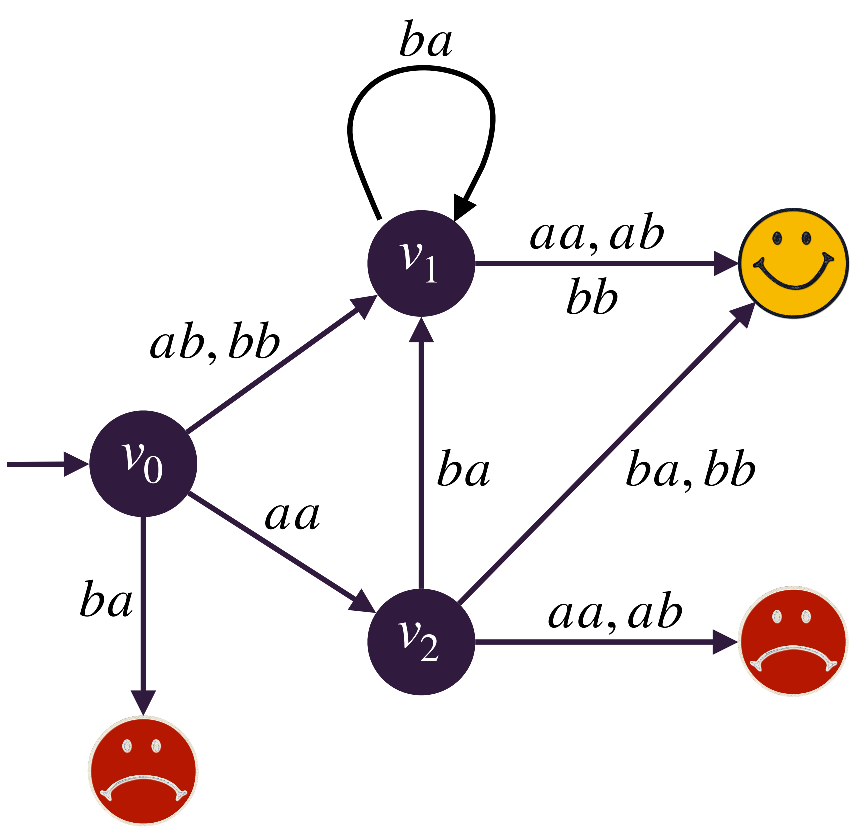

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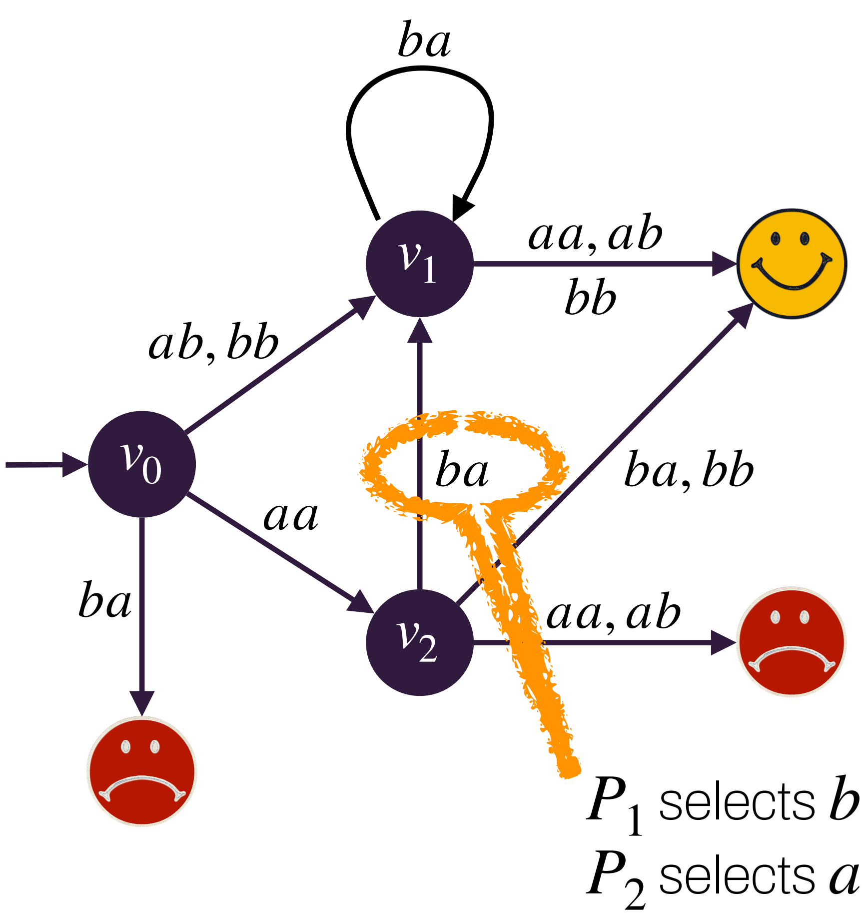

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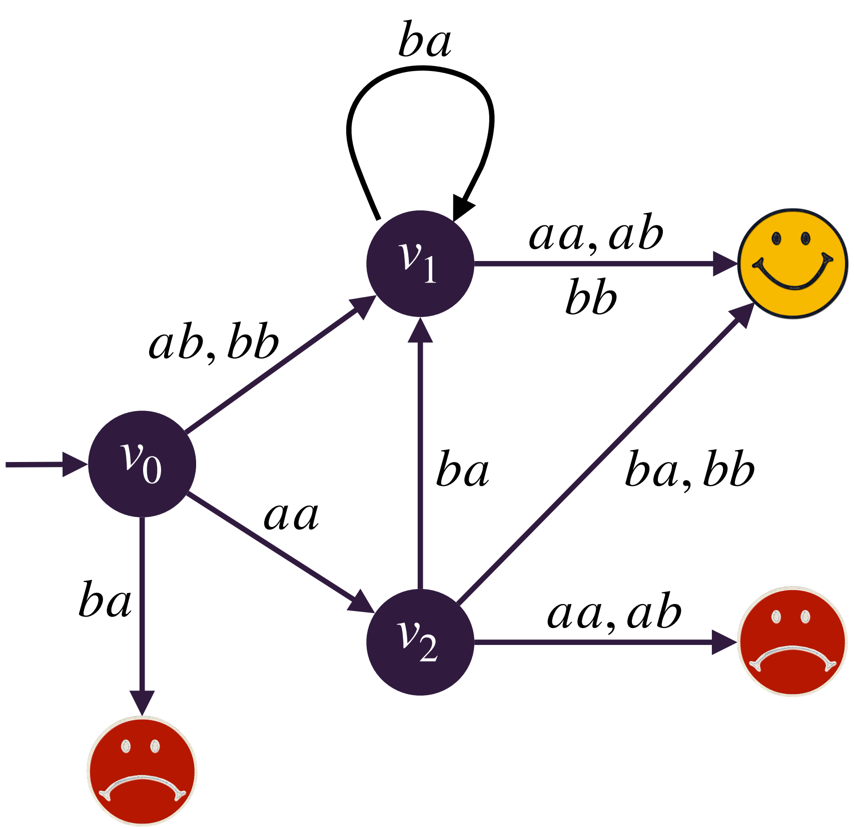

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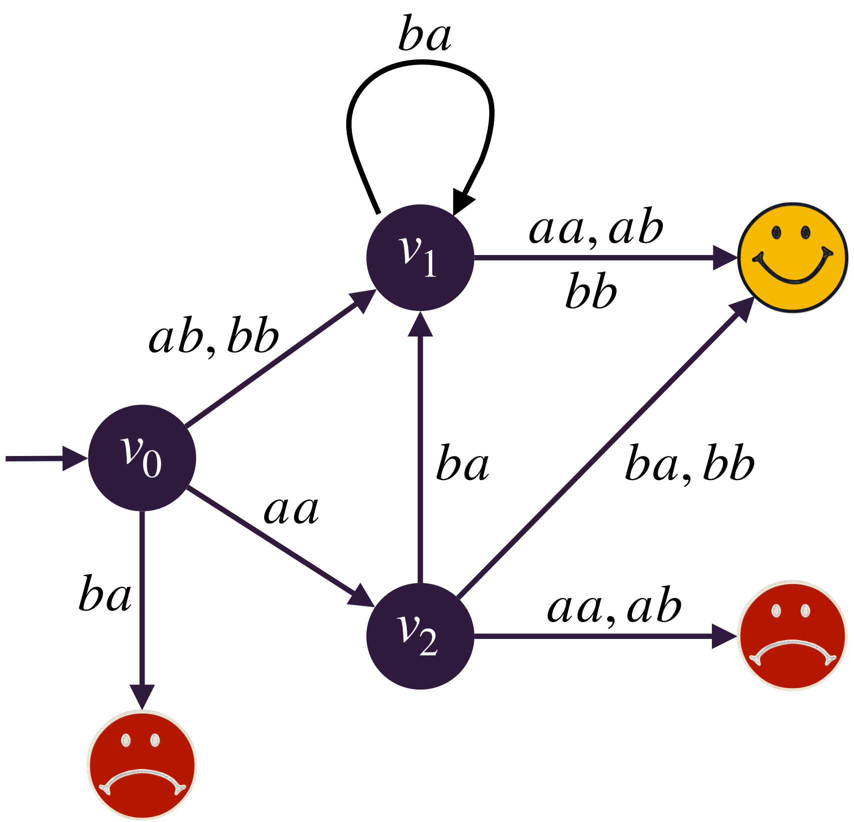

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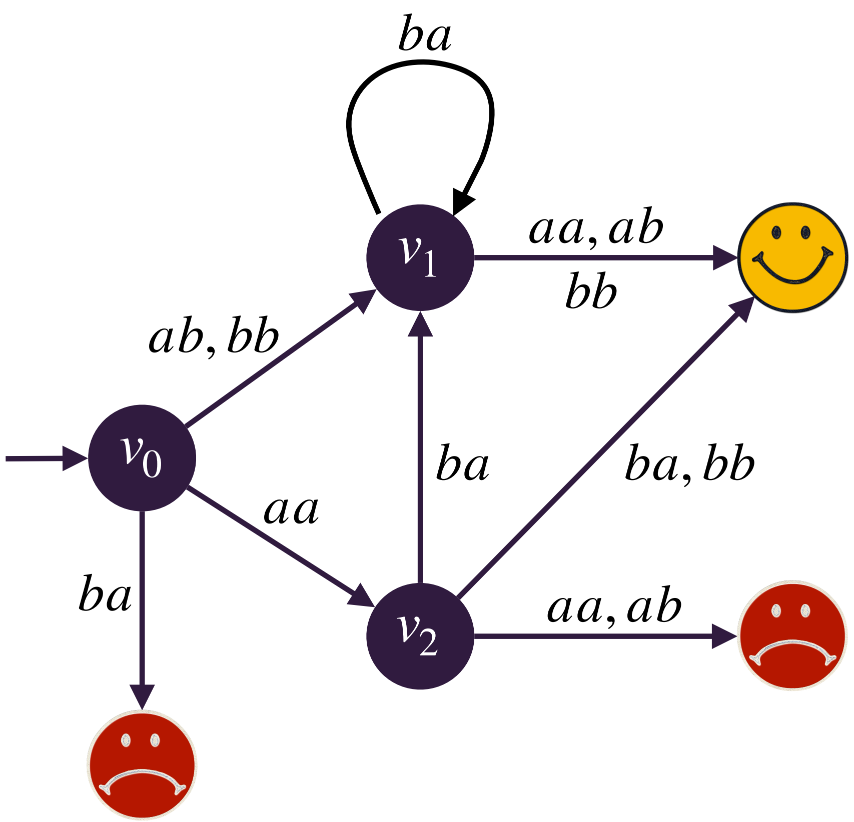

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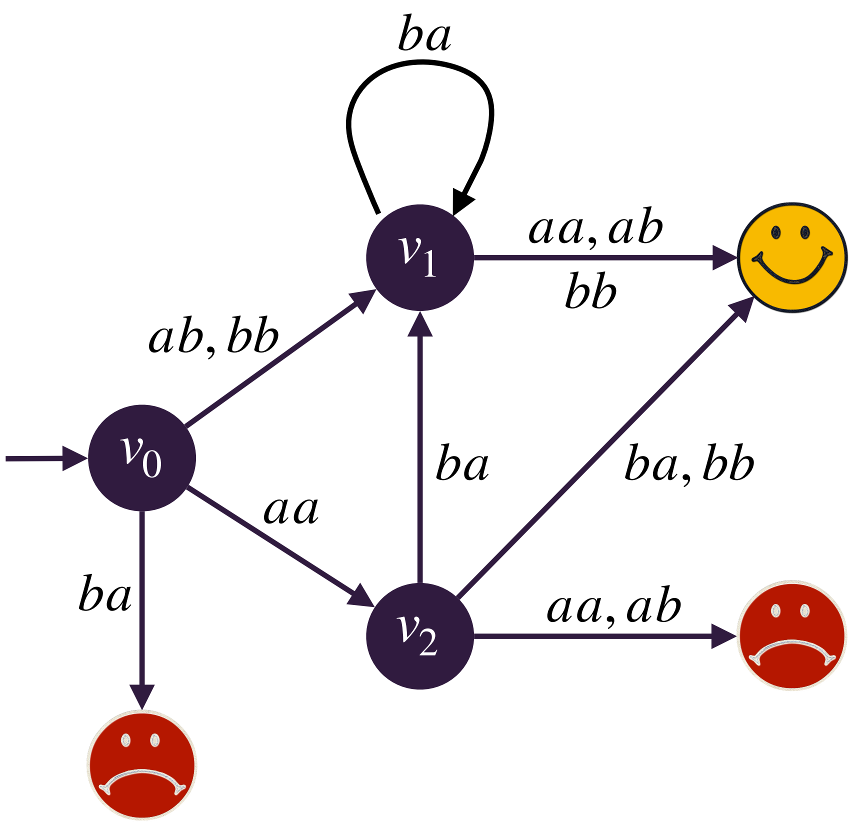

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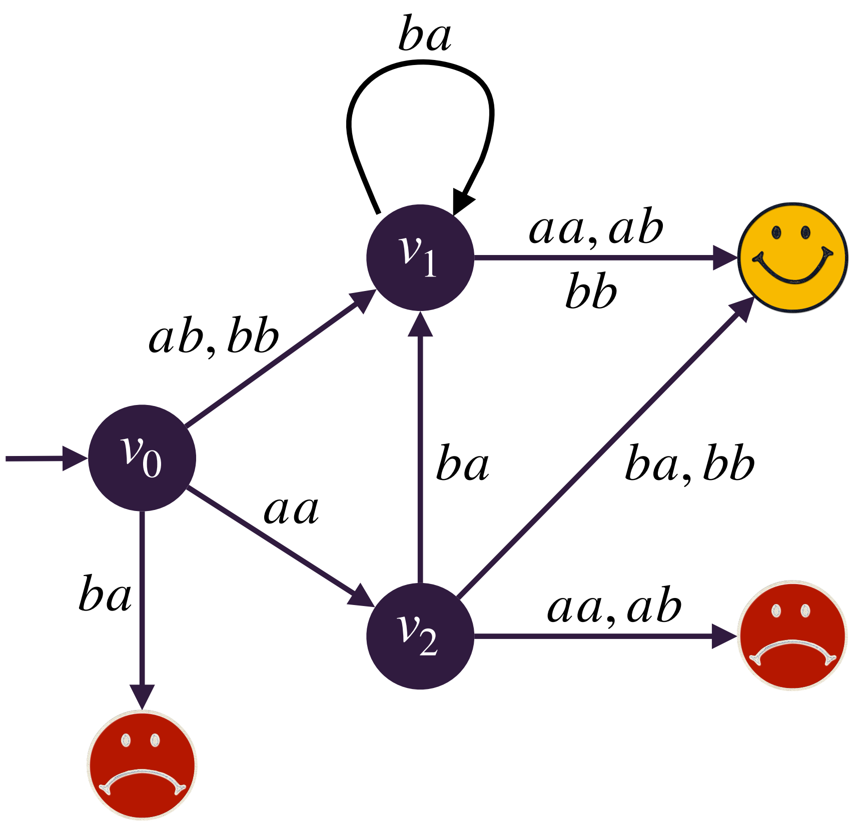

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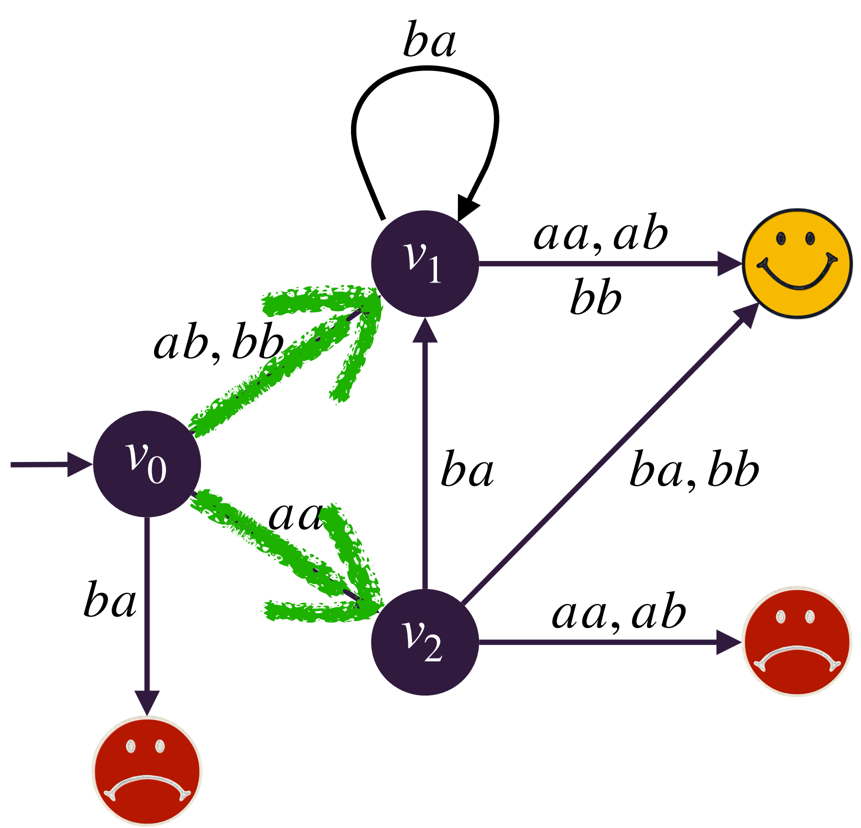

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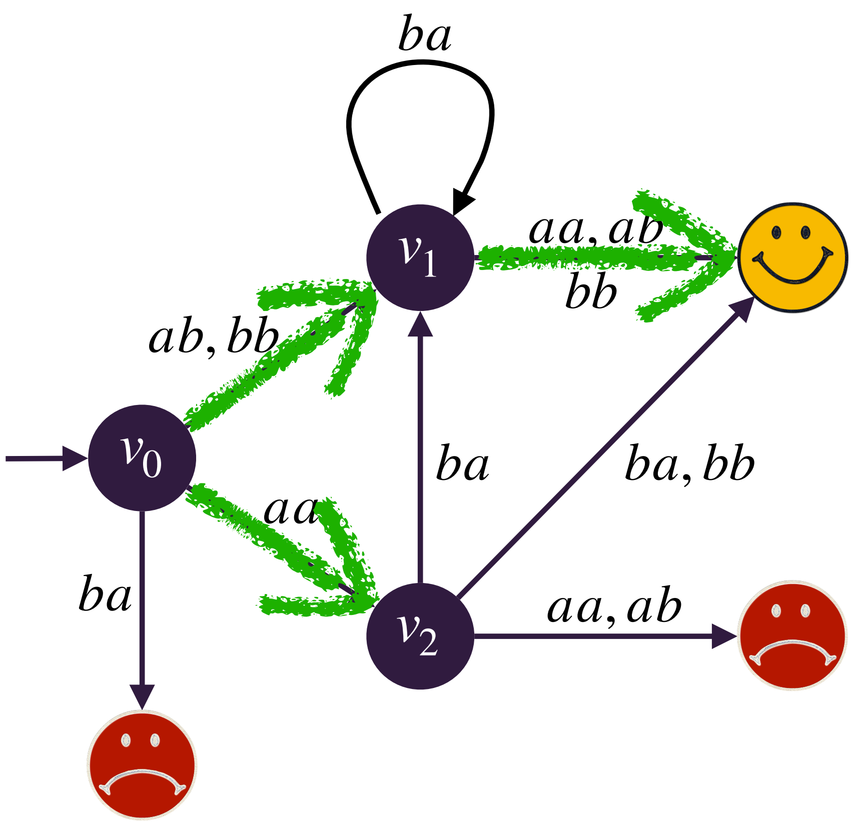

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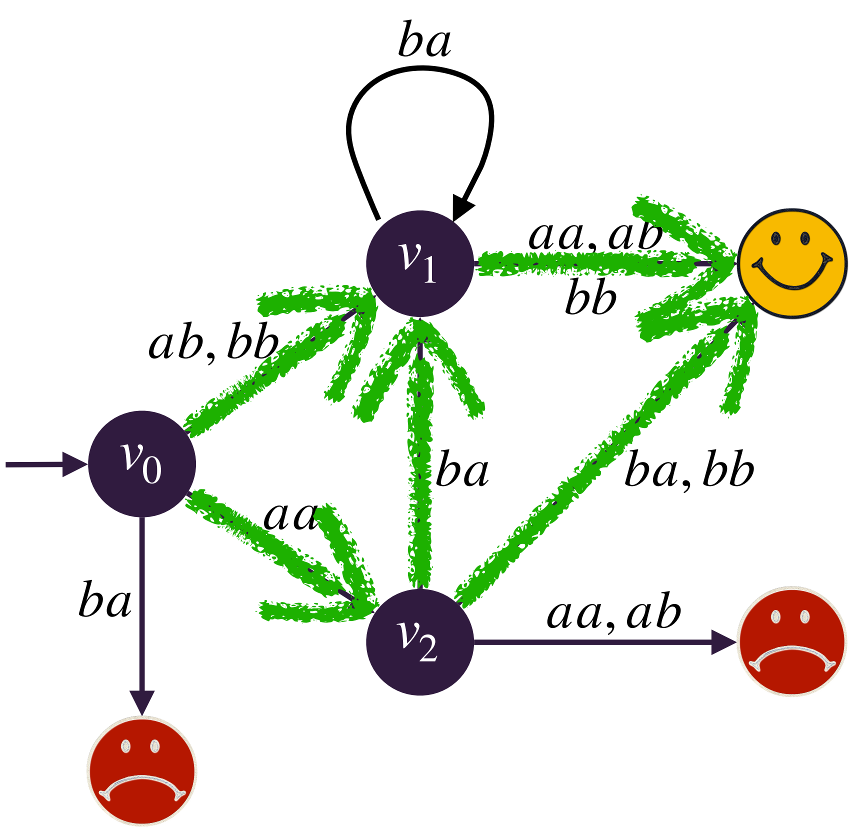

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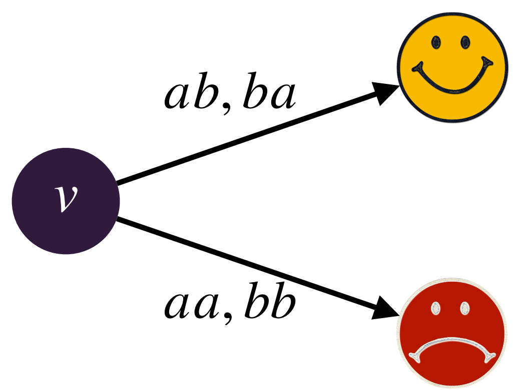
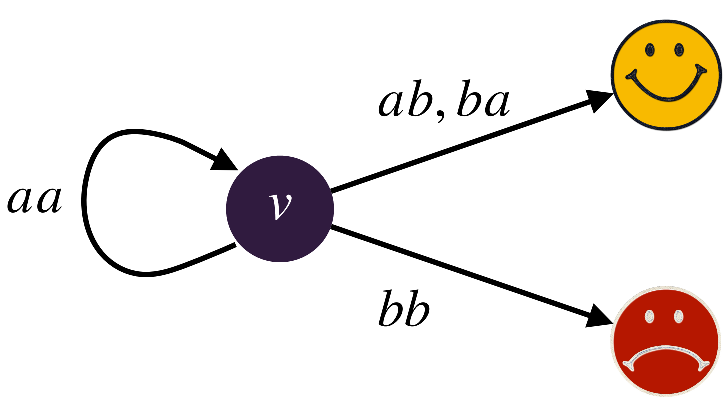
What do we know about those games?

- For many objectives, one can compute winning states and (deterministic) winning strategies for each of the players

- Those games are not determined with deterministic strategies



- They nevertheless have values and almost-optimal winning strategies for both players (Martin's second determinacy results for Blackwell games)



$val_1 = 1$

$\sigma_1(v) = (1 - \varepsilon) \cdot a + \varepsilon \cdot b$ is an ε-optimal strategy

No optimal strategy exists

[Mar98] D. A. Martin. The determinacy of Blackwell games (The Journal of Symbolic Logic'98)

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- Use of these games:
  - For coordination (specific Nature player, and partial observability) *[DMV18]* — linked to distributed synthesis *[MW03]*
  - For rational synthesis (e.g. constrained Nash equilibria) *[BBNM15,KPV16]*

[MW03] S. Mohalik, I. Walukiewicz. Distributed Games (FSTTCS'03)
[BBMU15] P. Bouyer, R. Brenguier, N. Markey, M. Ummels. Pure Nash Equilibria in Concurrent Deterministic Games (LMCS'15)
[KPV16] O. Kupferman, G. Perelli, M. Vardi. Synthesis with Rational Environments (AMAI'16)
[DMV18] D. Berwanger, A.B. Mathew, M. van den Bogaard. Hierarchical Information and the Synthesis of Distributed Strategies (Acta Informatica'18)

Parameterized verification

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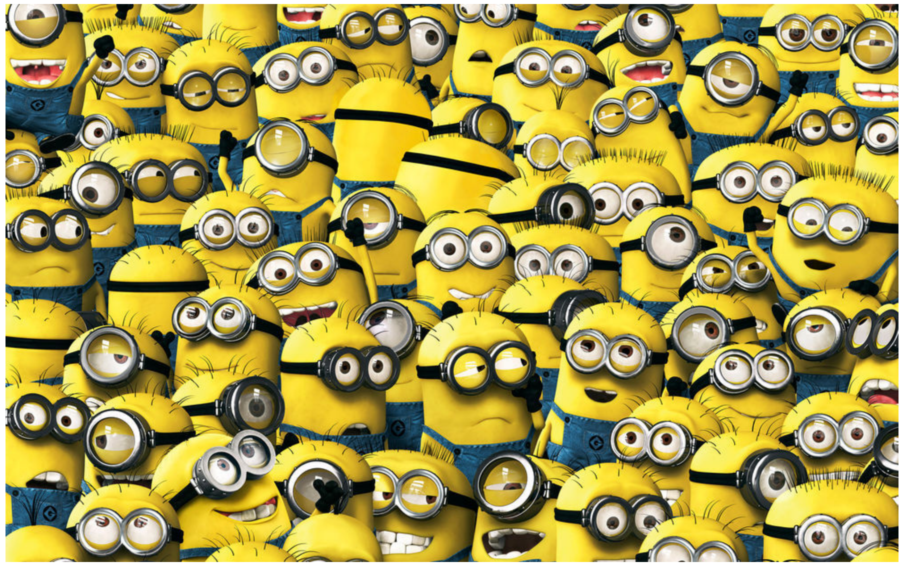

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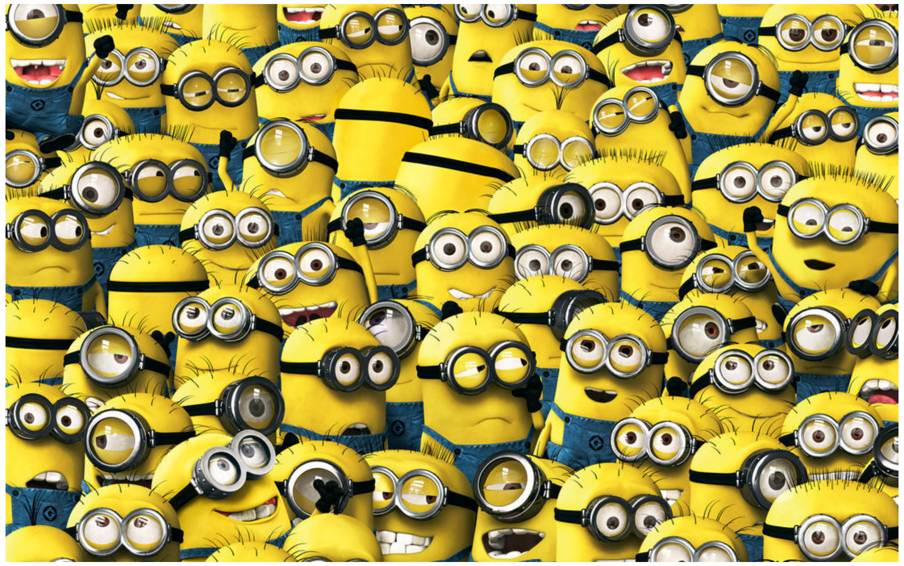
Various kinds of parameters:

- In arithmetic contraints (timed automata, counter automata, hybrid automata, Markov chains, ...)
- Number of agents



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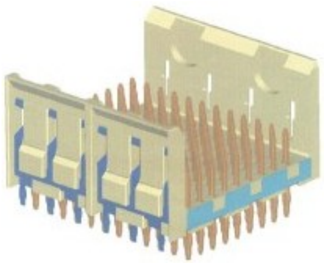

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  - Example of the Futurebus+ cache coherence protocol

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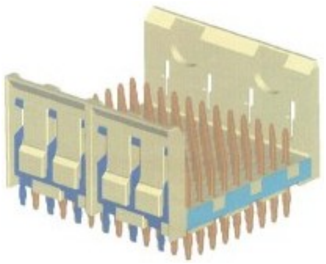

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- Need to design methods for verifying parameterized systems, not only instances

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Parameterized verification of crowds

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- Decidability often relies on:
  - Well-structured transition systems
  - Existence of cutoffs

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From two-player to arbitrarily-many player concurrent games

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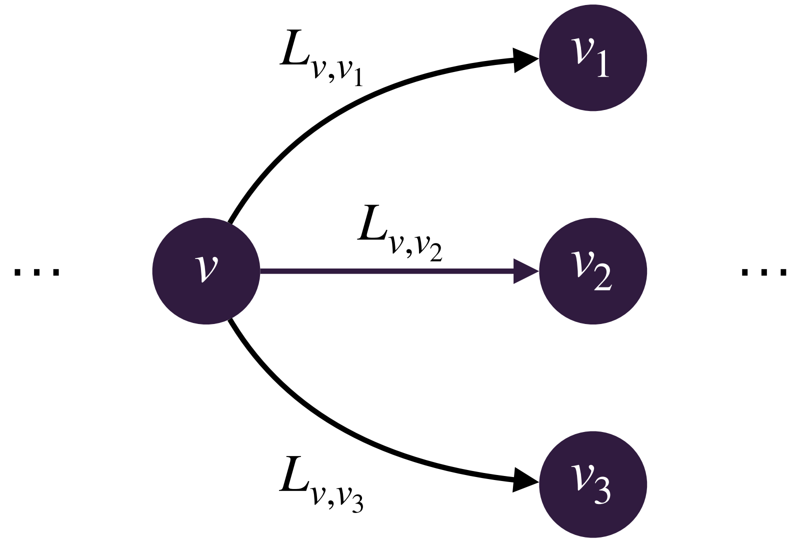
Assumption: for every $(v, v') \in V^2$,

$$\{w \in \Sigma^+ \mid (v, w, v') \in \delta\} \text{ is regular}$$

$$v_0 \xrightarrow{L} v_1$$

with L regular language

How do we play such a game?



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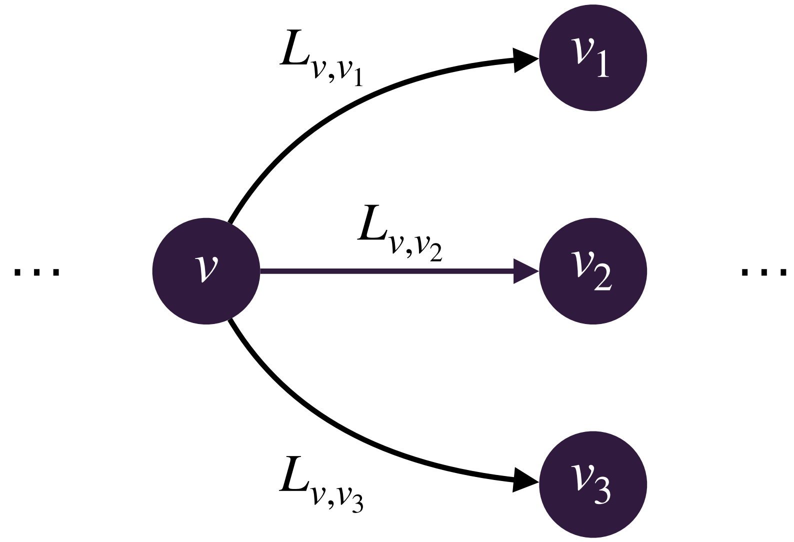

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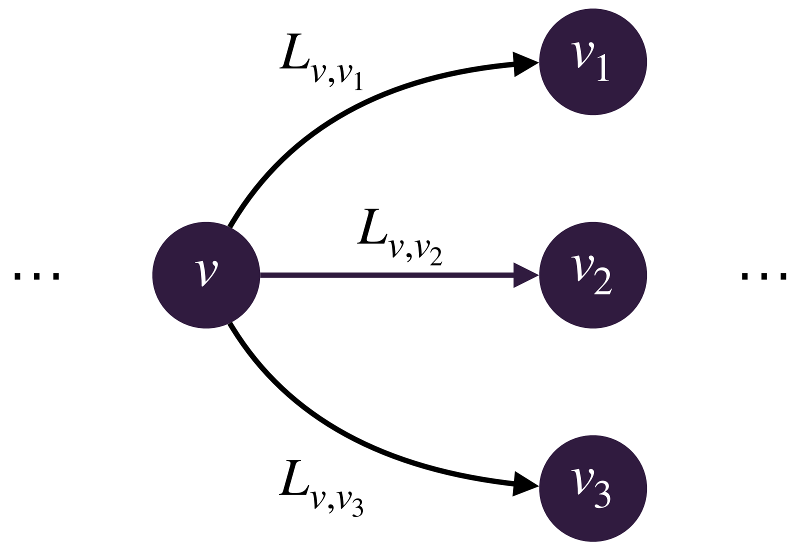

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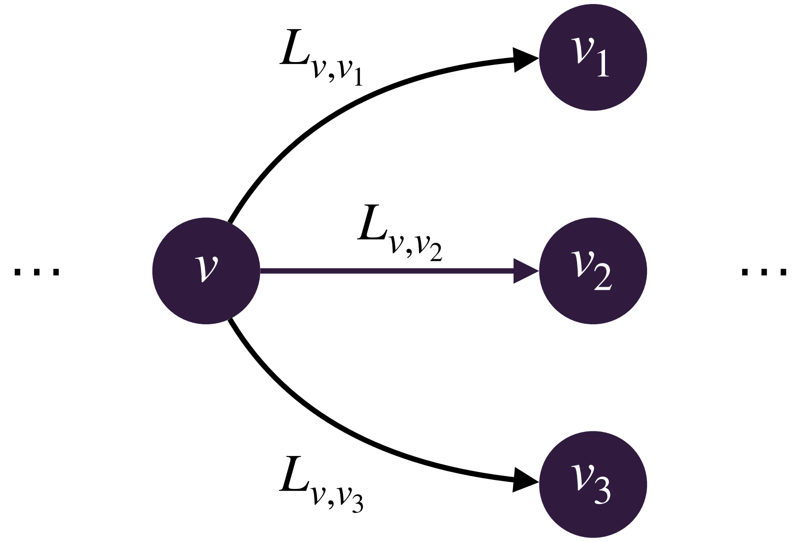

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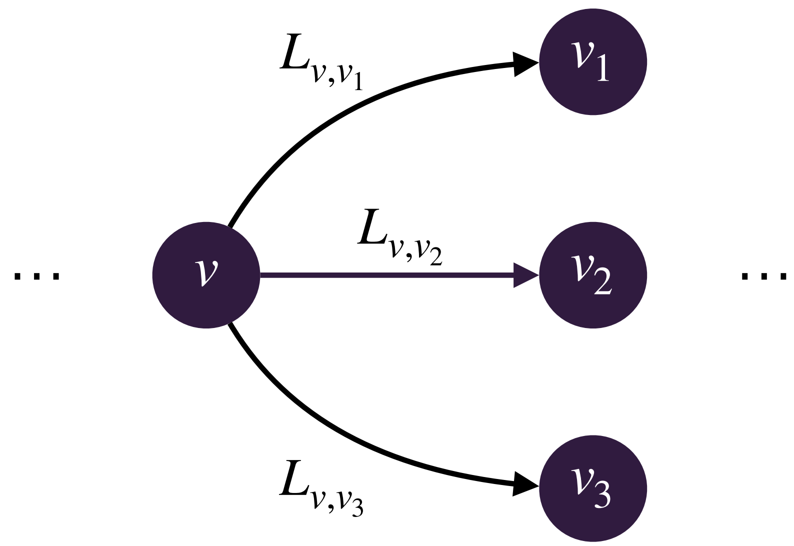

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- The game starts at initial vertex v_0

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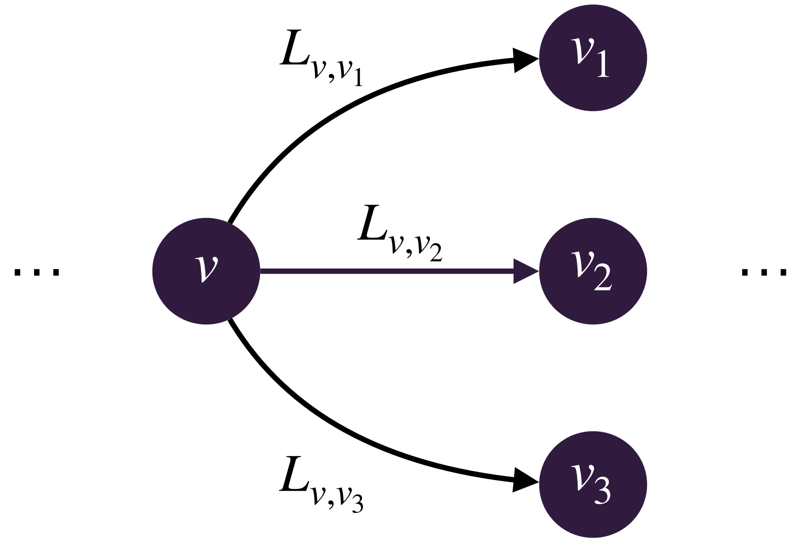

- A coalition of size k is chosen, but is unknown to the players
- Each player P_i knows she is the i-th (implicit identifier)

- The game starts at initial vertex v_0
- Given the history (in V^*v) so far, each player P_i selects an action $a_i \in \Sigma$

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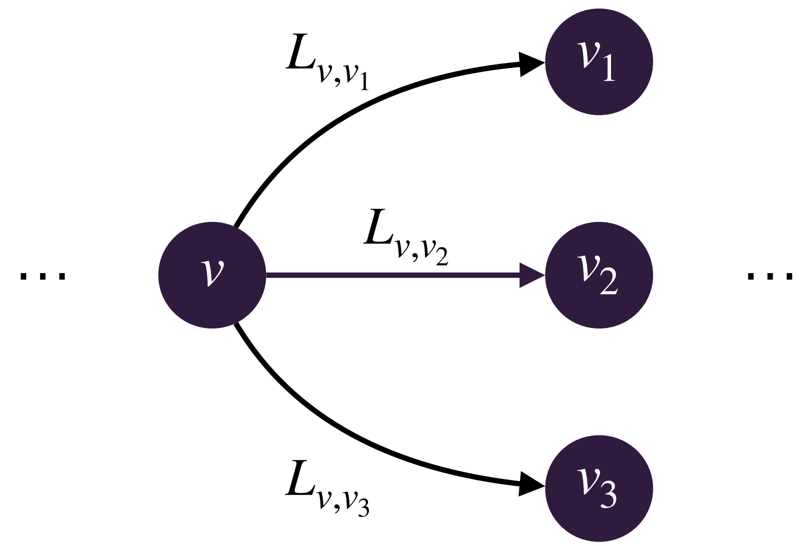

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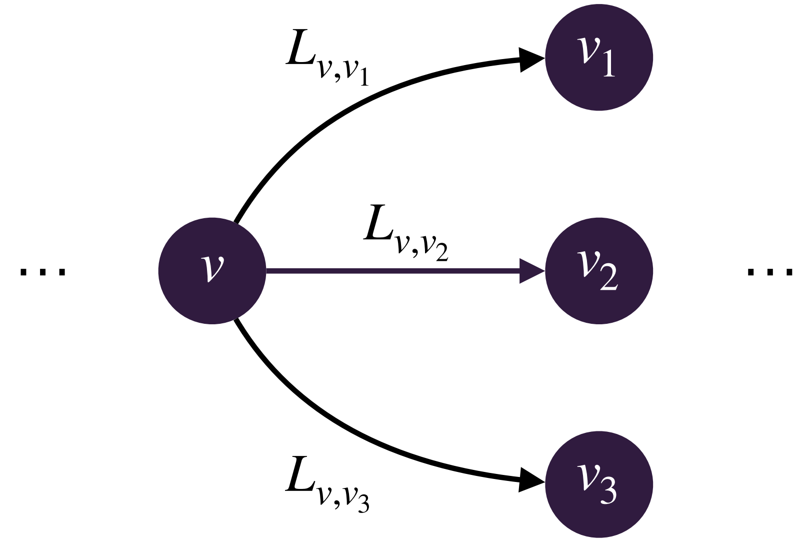

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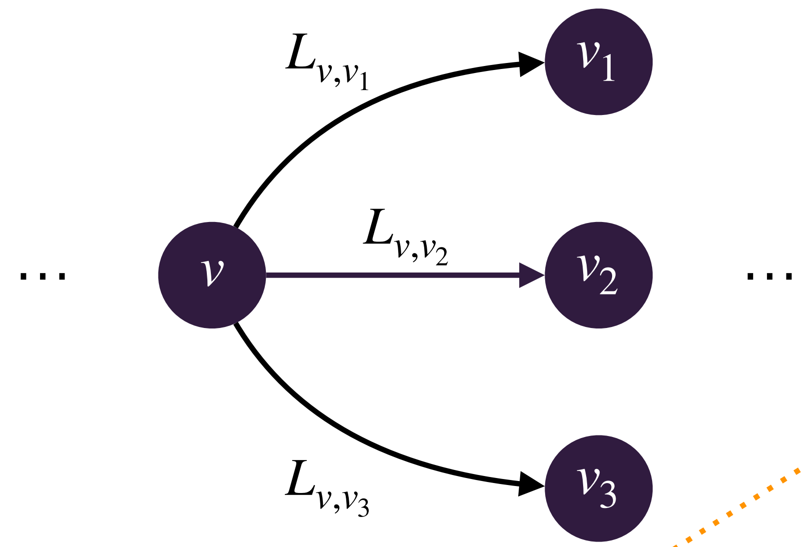

Adversarial choice

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Adversarial choice

Two synthesis problems

Two synthesis problems

The crowd controller problem

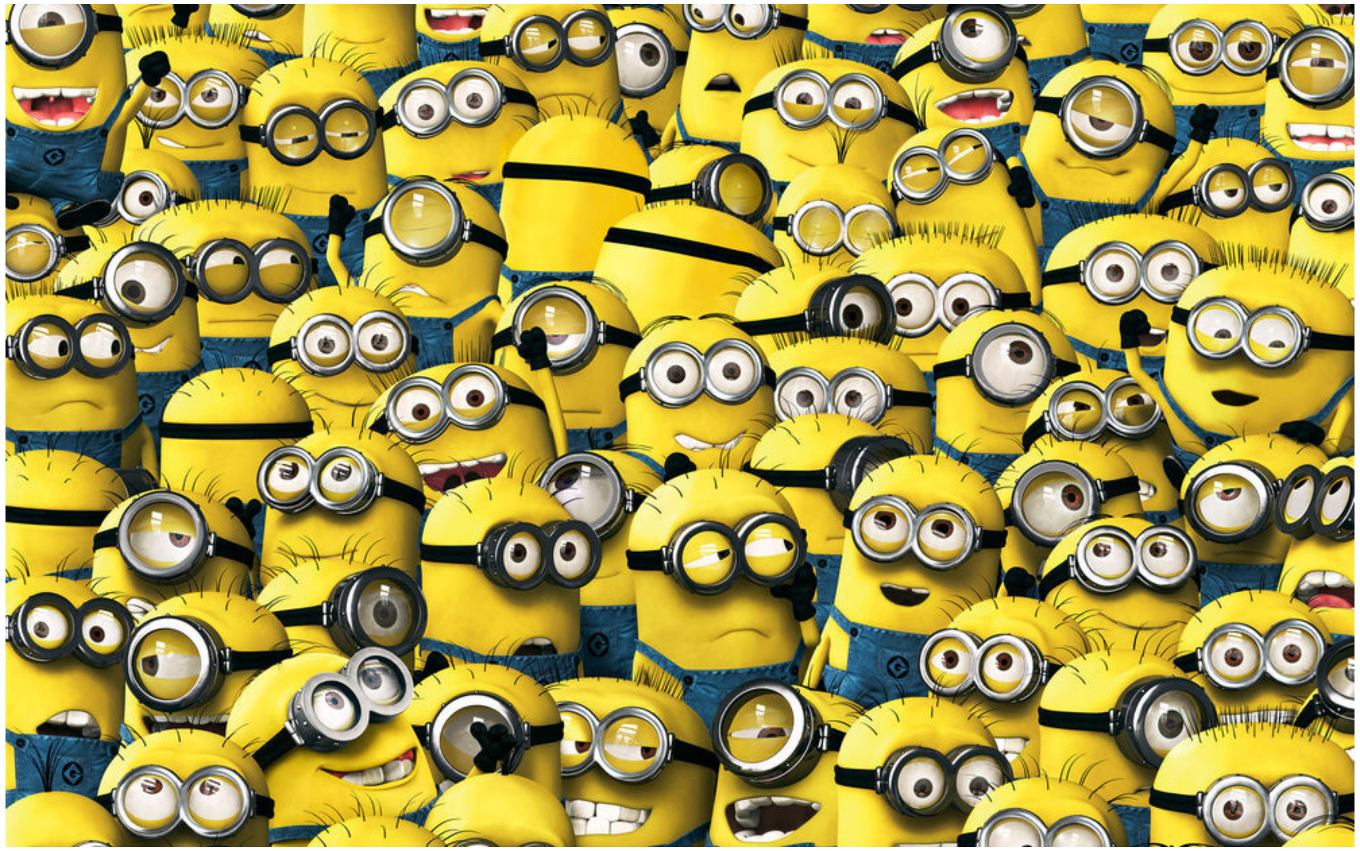
« Gru wants to guide/control the Minions »

Two synthesis problems

The crowd controller problem

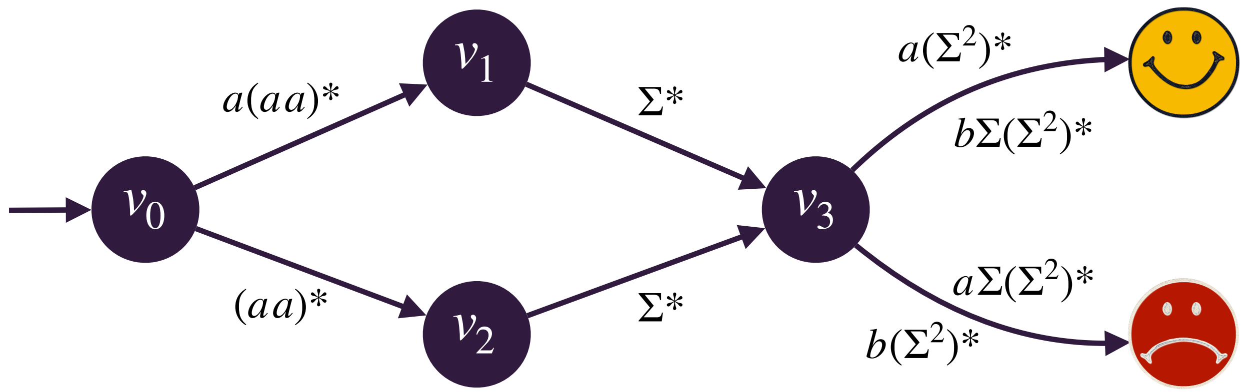
« Gru wants to guide/control the Minions »

The coalition synthesis problem

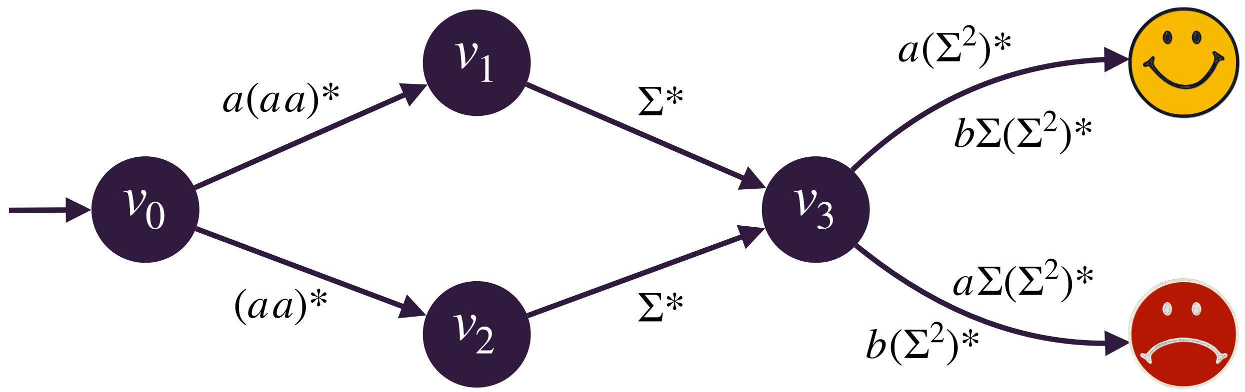


« The Minions want to achieve some goal »

The crowd controller problem

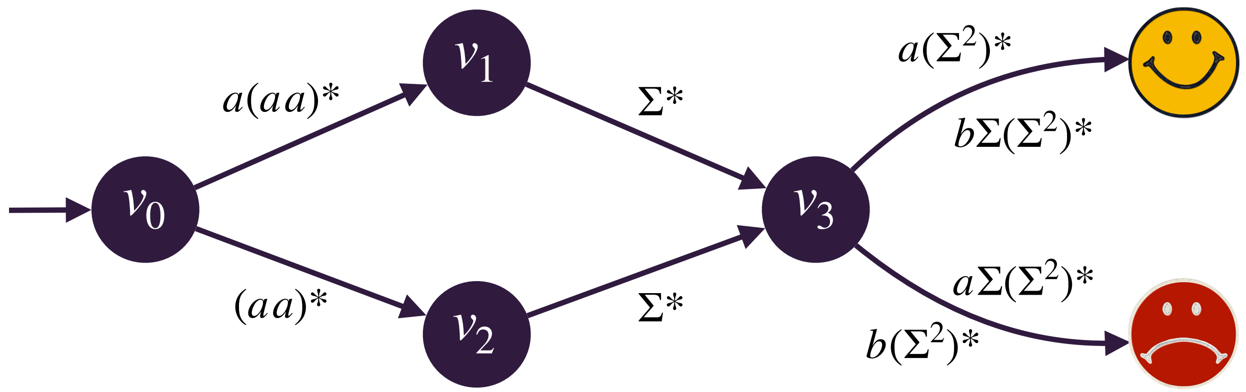


The crowd controller problem



Can player P_1 enforce ?

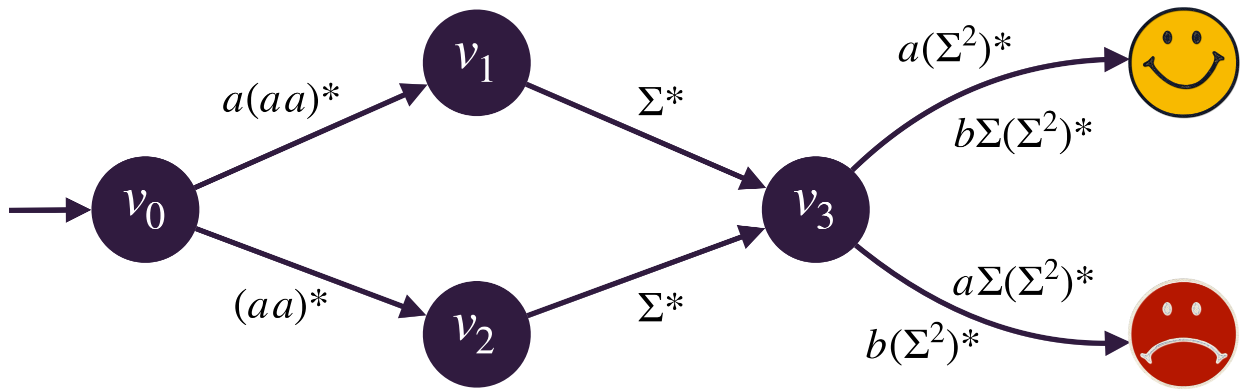
The crowd controller problem



Can player P_1 enforce 🙂 ?

- The number k of players is chosen (but unknown to everyone)

The crowd controller problem



Can player P_1 enforce ?

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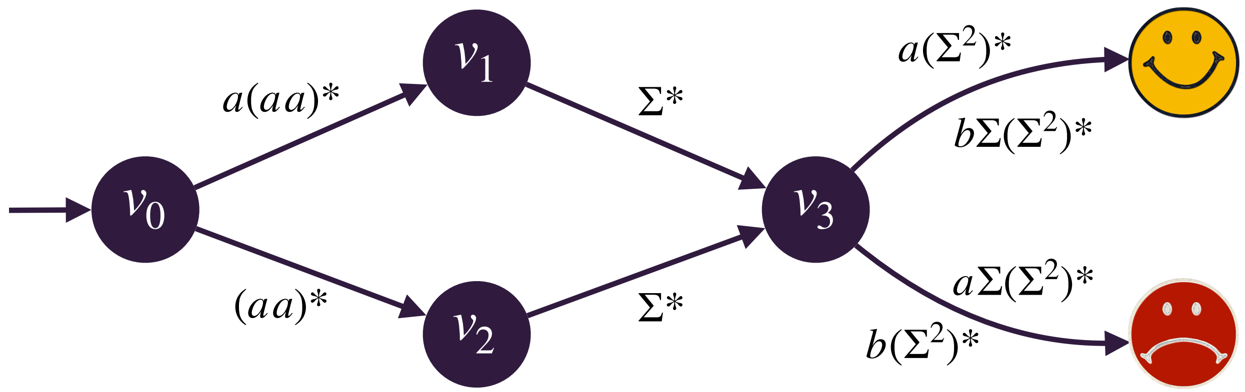
- Assume the following strategy for player P_1:

The crowd controller problem



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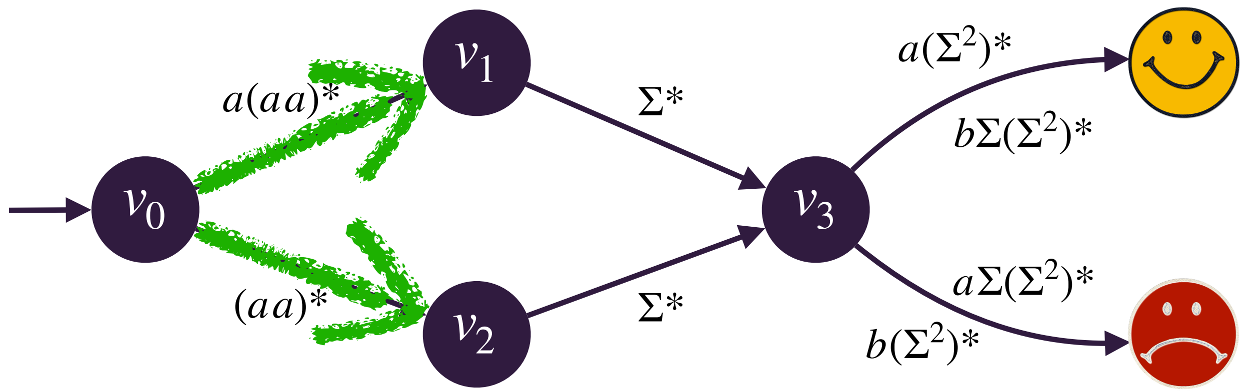
The crowd controller problem



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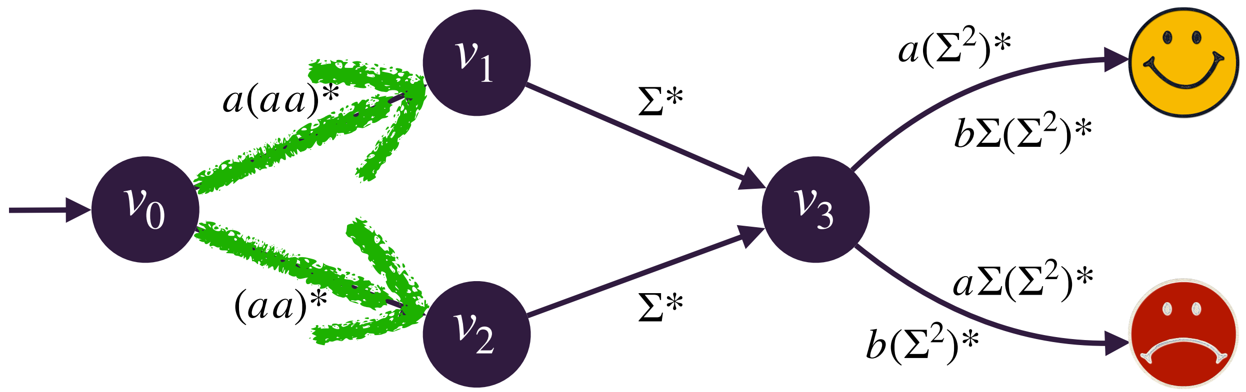

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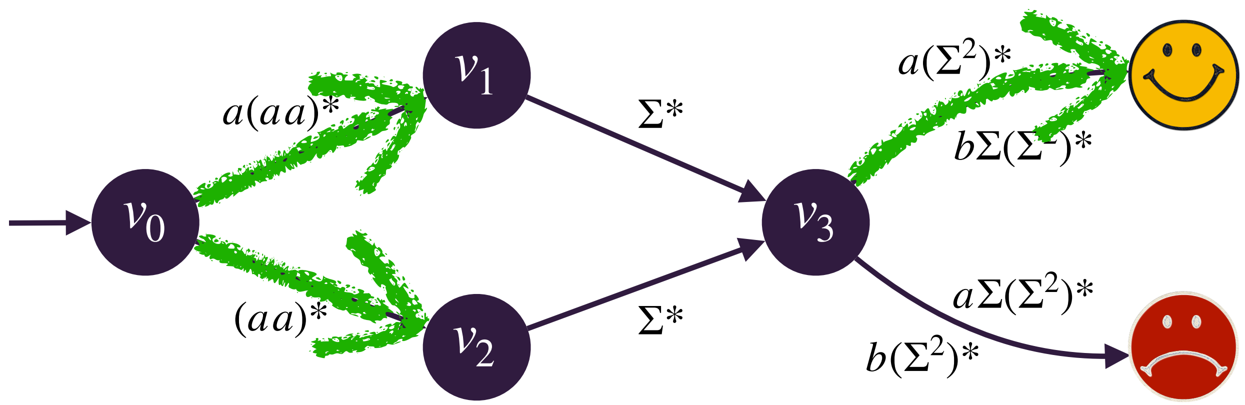
(P_1 learns it when visiting v_1 or v_2)

The crowd controller problem



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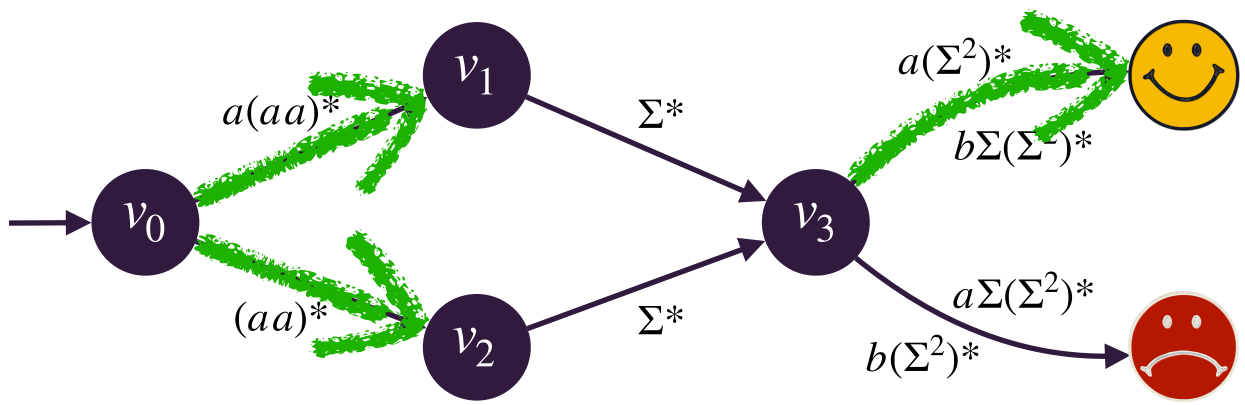

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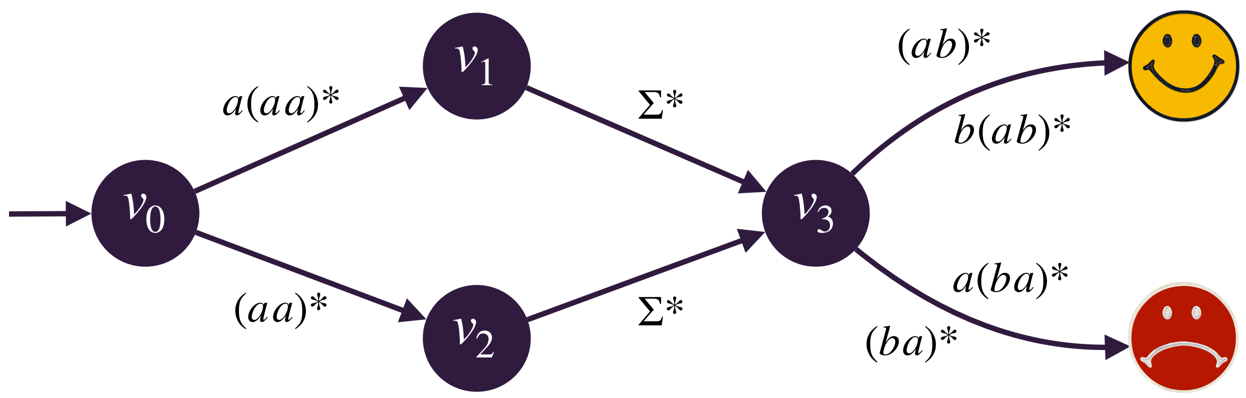
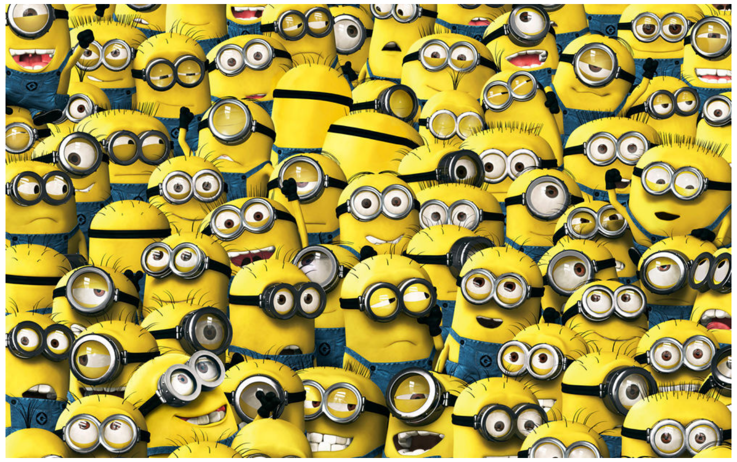
- If k is odd, the game proceeds to v_1
  If k is even, the game proceeds to v_2 (P_1 learns it when visiting v_1 or v_2)
- In both cases, the choice of P_1 at v_3 ensures reaching

The coalition problem





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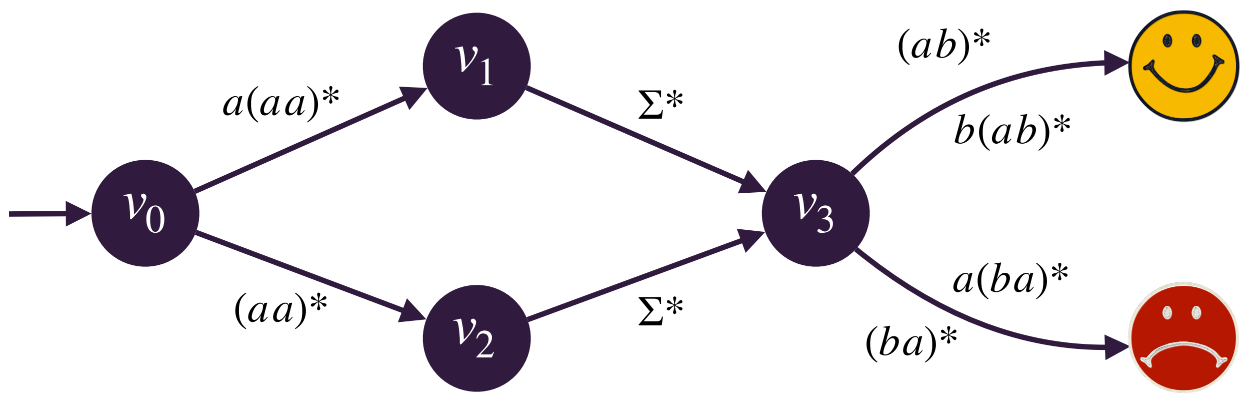
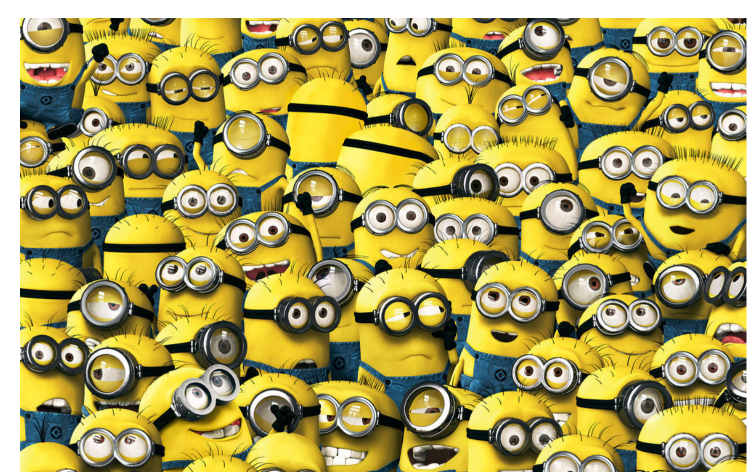


Can any coalition ensure ?



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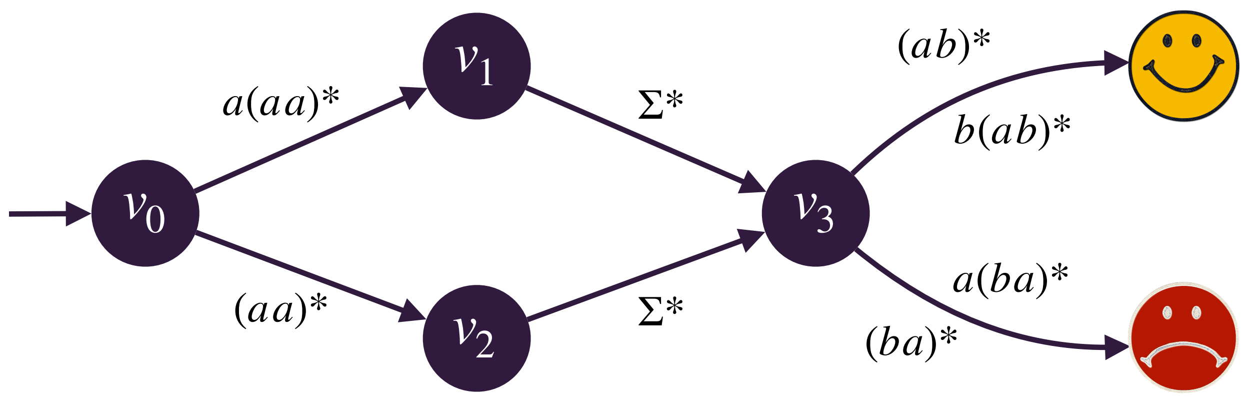
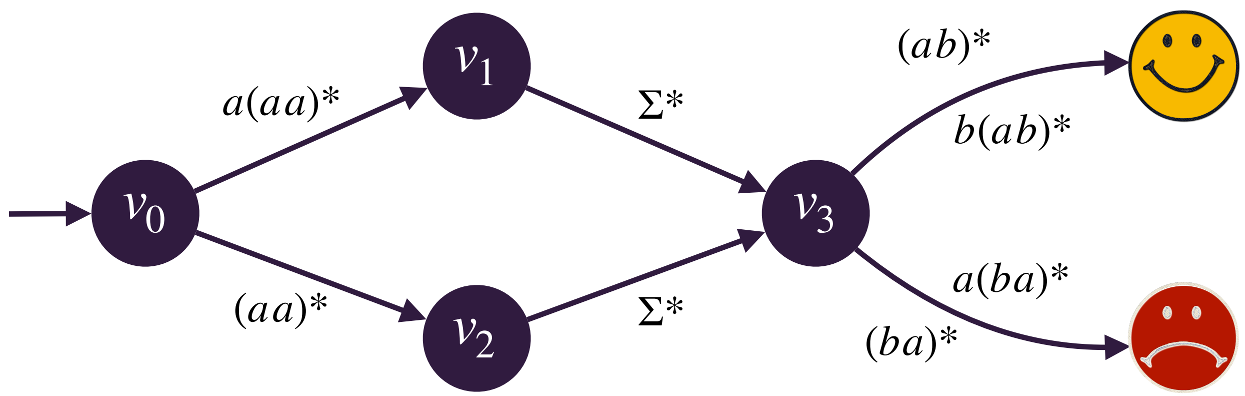


Can any coalition ensure ?

- The number k of players is chosen (but unknown to everyone)

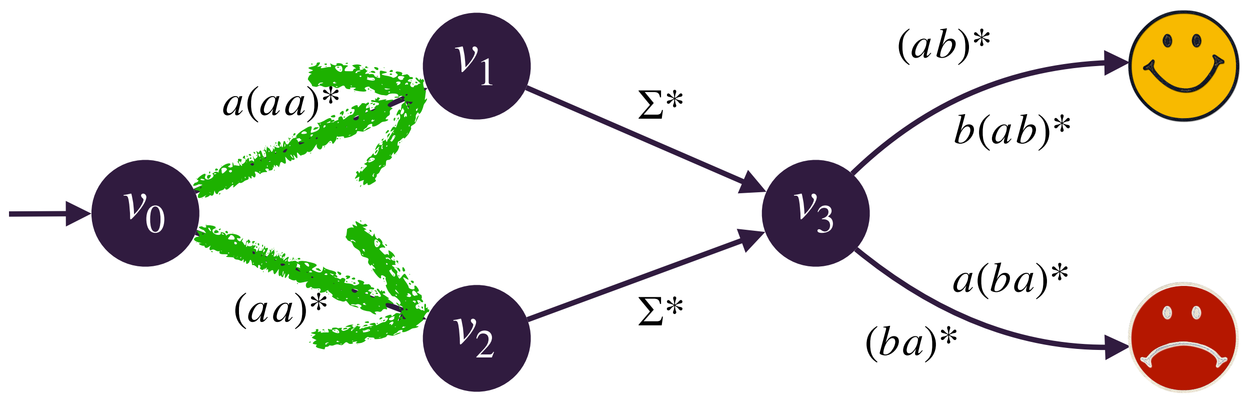
The coalition problem



Can any coalition ensure ?

- The number k of players is chosen (but unknown to everyone)
- At v_0, each player chooses a

The coalition problem



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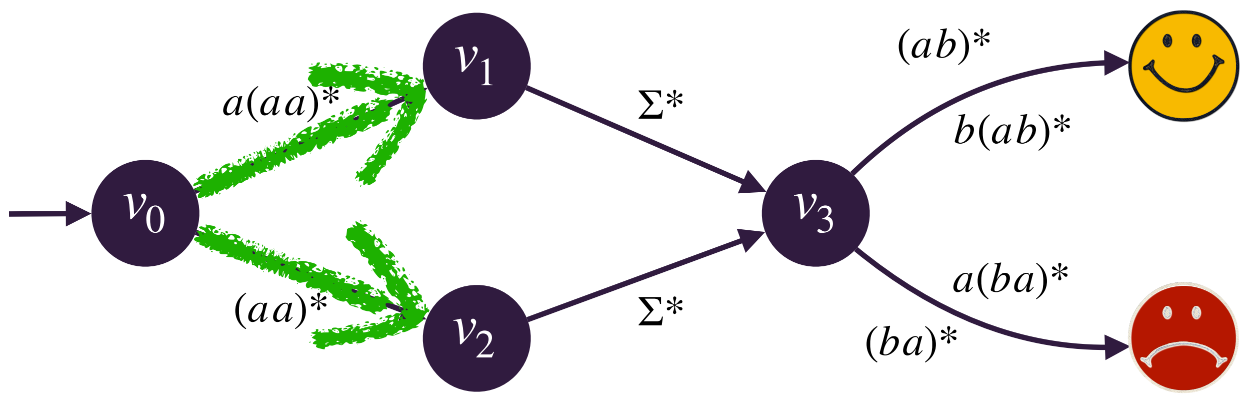

Can any coalition ensure 🙂?

- The number k of players is chosen (but unknown to everyone)
- At v_0, each player chooses a
- If k is odd, the game proceeds to v_1 (all players learn it when visiting v_1 or v_2)
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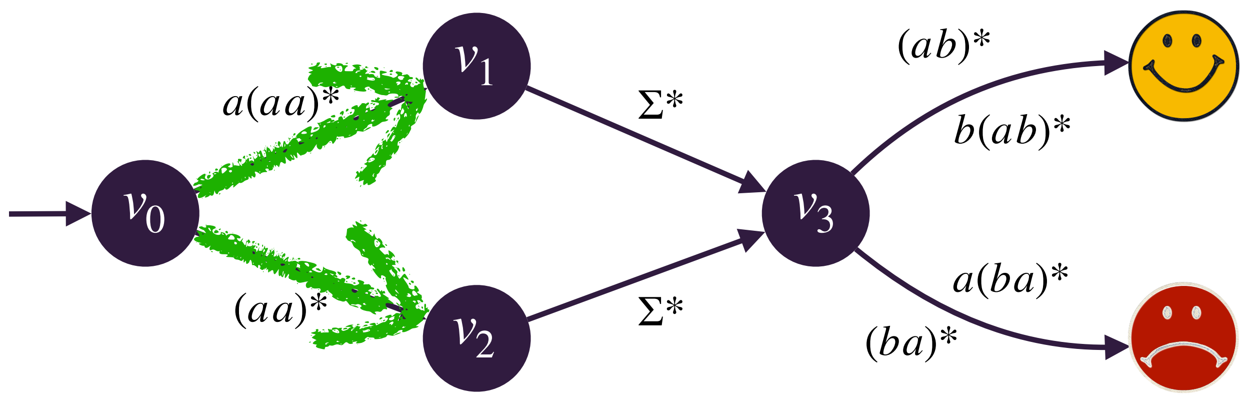
The coalition problem



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- If k is odd, the game proceeds to v_1 (all players learn it when visiting v_1 or v_2)
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- For each i:
  - $\sigma_{2i}(v_0v_1v_3) = a$ and $\sigma_{2i}(v_0v_2v_3) = b$
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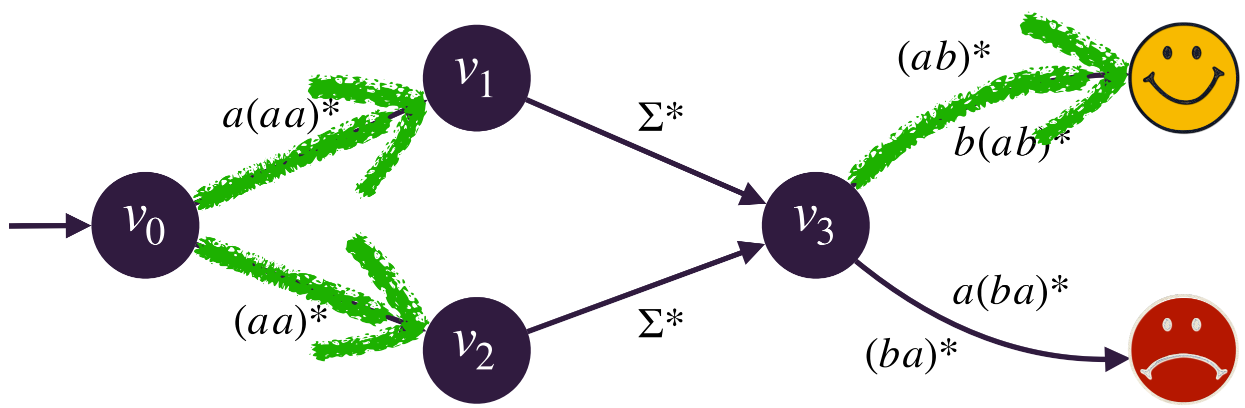

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- For every k, $\mathsf{Out}((\sigma_i)_{1 \le i \le k}) \subseteq V^*$ 🙂

The crowd controller problem

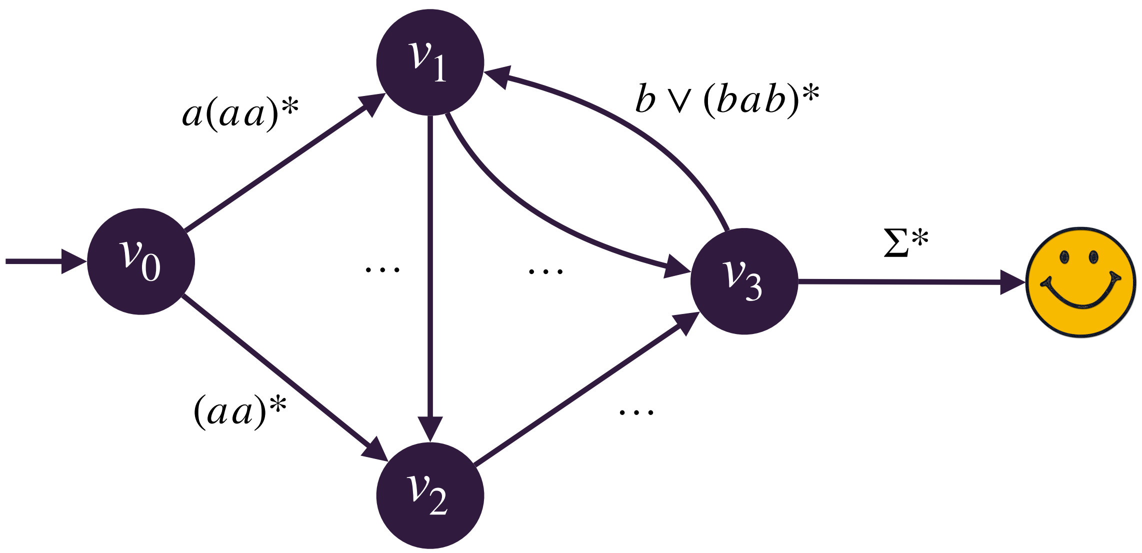
The crowd controller problem

- Input: parameterized game $G = (V, \delta)$ and linear property φ
- Question: does there exist σ_1 s.t. for every k, for every $(\sigma_i)_{2\leq i\leq k}$, for every $\rho \in \mathsf{Out}\big((\sigma_i)_{1\leq i\leq k}\big)$, $\rho \models \varphi$?

P_1 = Gru

From languages to counting

φ objective



From languages to counting

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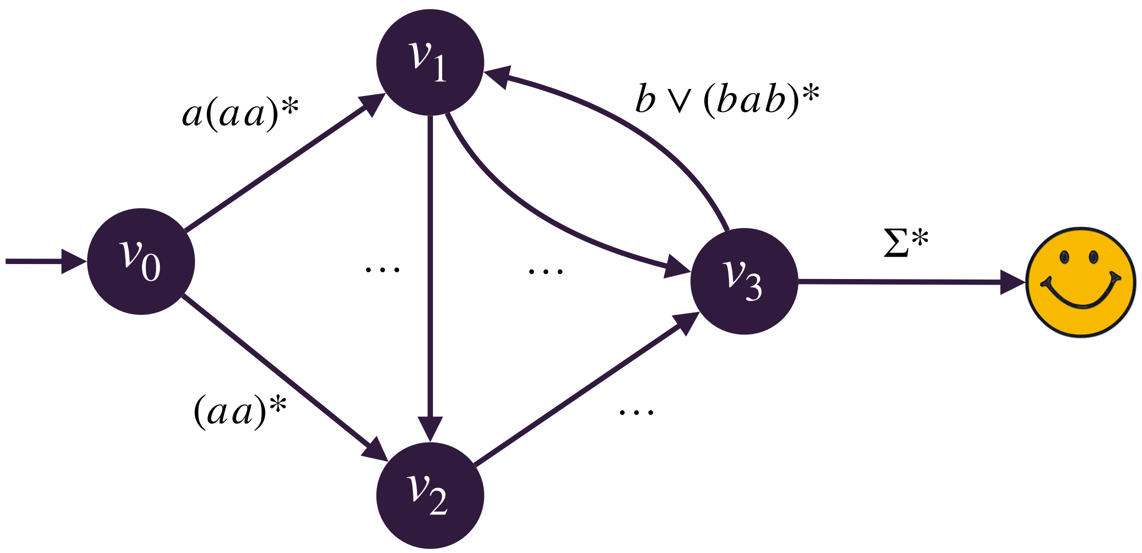
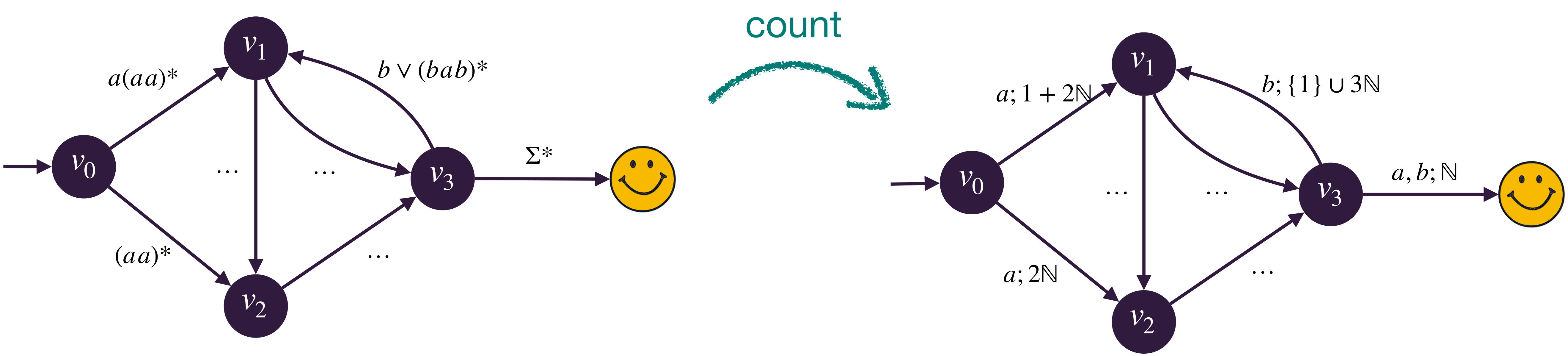


count

From languages to counting

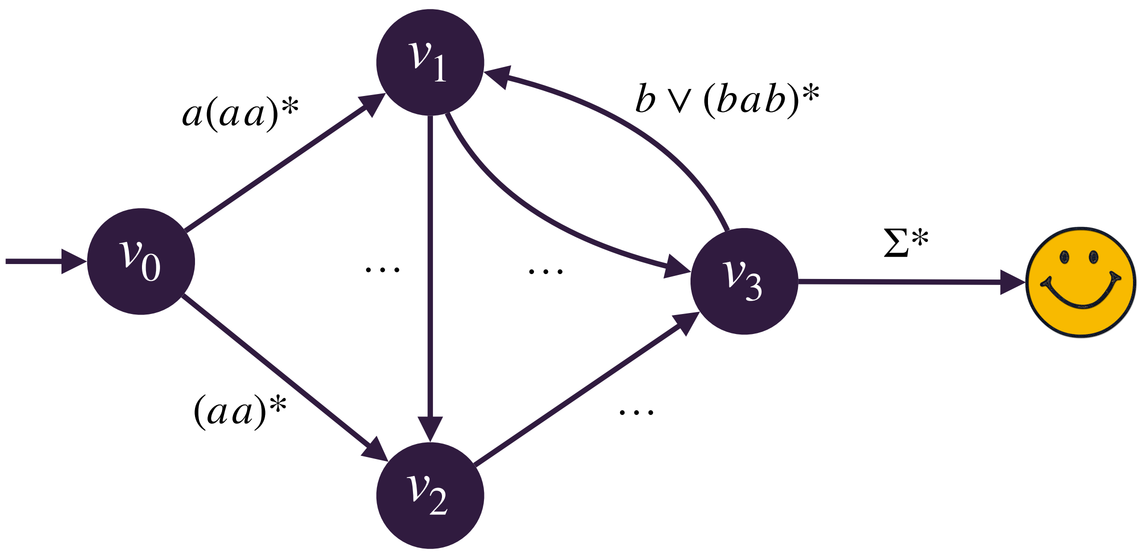
φ objective φ objective



Note : L regular language implies count(L) is a semi linear set

From languages to counting

φ objective



count

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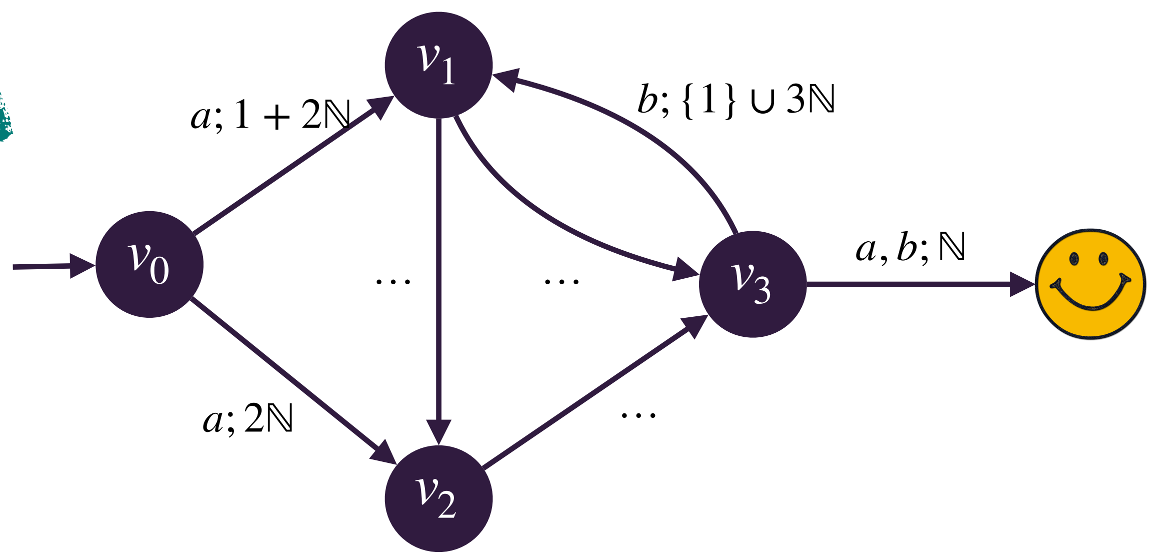
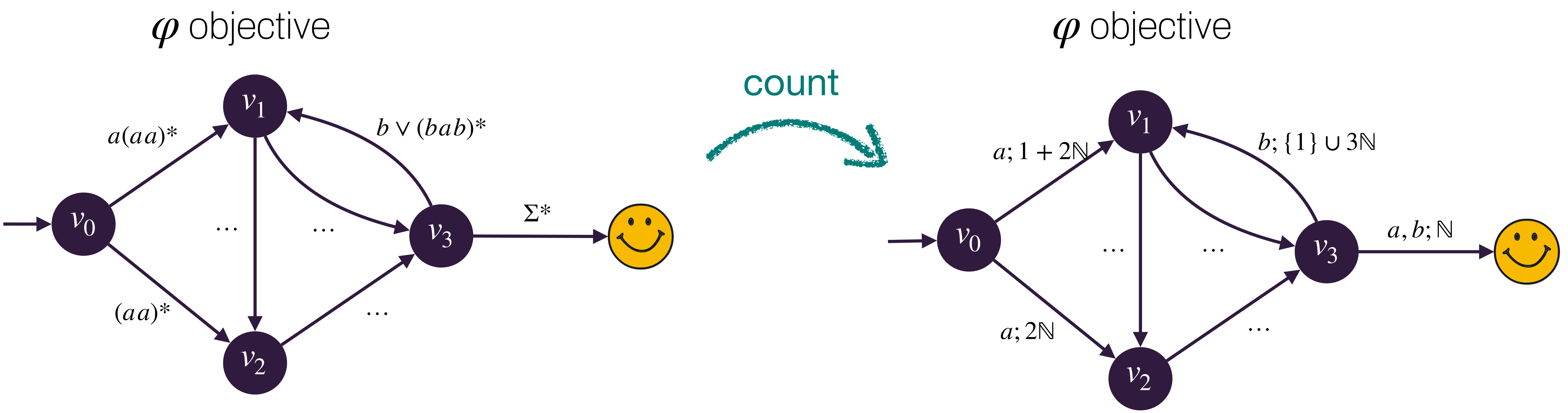


How do we play this new game?

- The game starts at v_0
- The opponent chooses k (unknown to Gru)
- While (true)
  - At vertex v, Gru chooses an action and opponent chooses an edge $v \xrightarrow{a;S} v'$ with $k \in S$
  - The game proceeds from v'

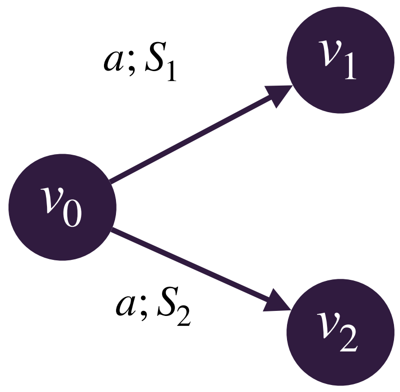
From languages to counting



Gru wins the language/original game iff he wins the counting game

From counting to turn-based knowledge games

φ objective



From counting to turn-based knowledge games

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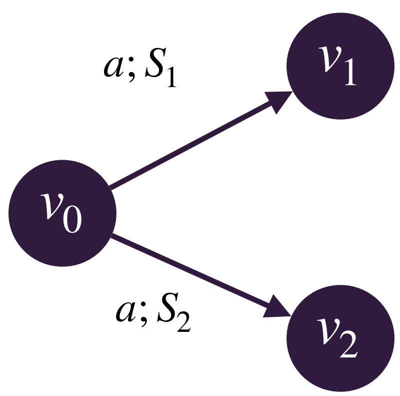

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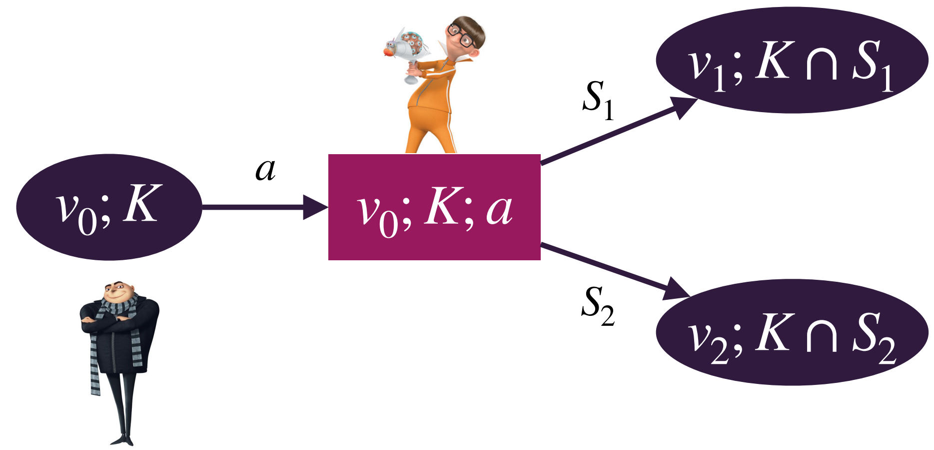

From counting to turn-based knowledge games

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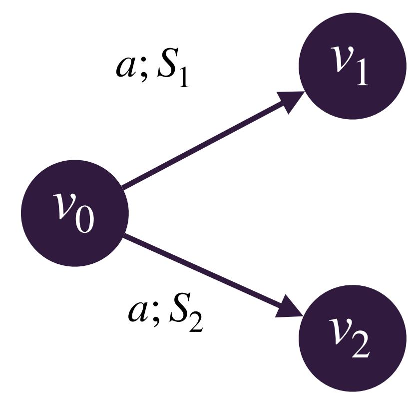

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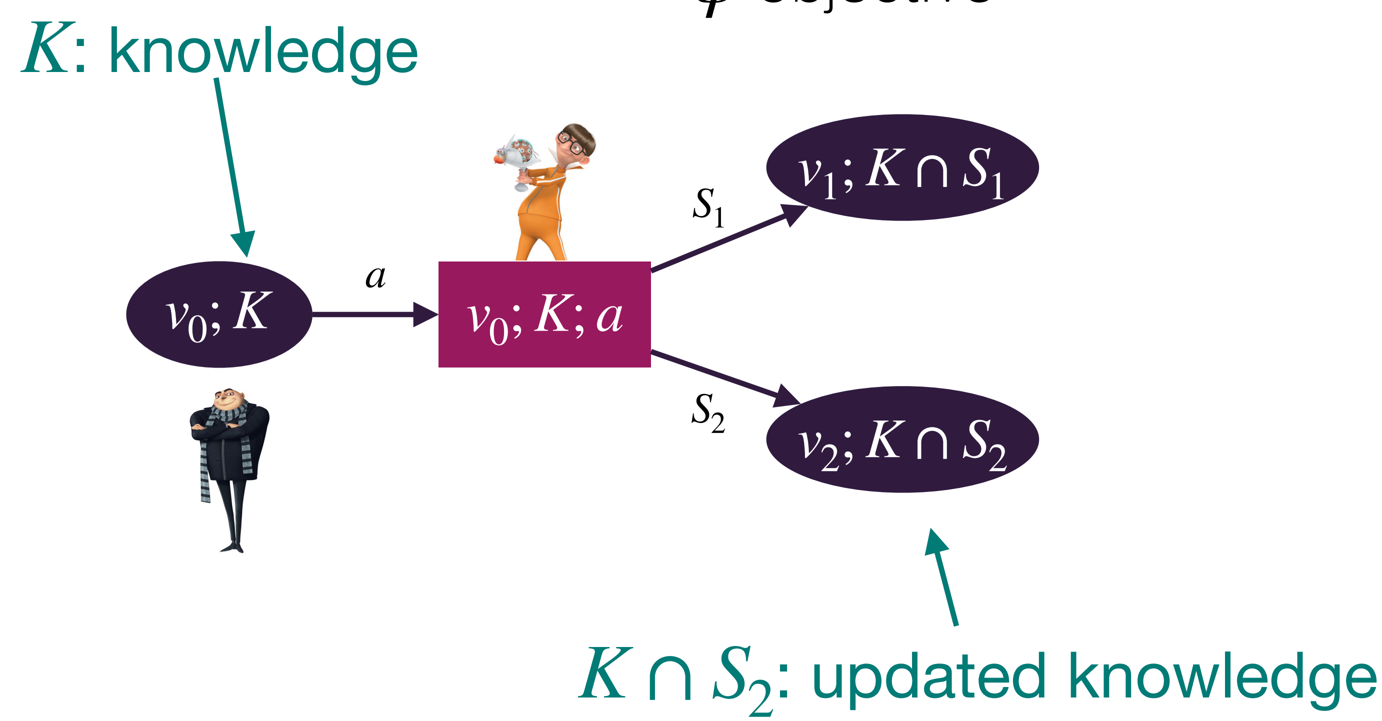

From counting to turn-based knowledge games

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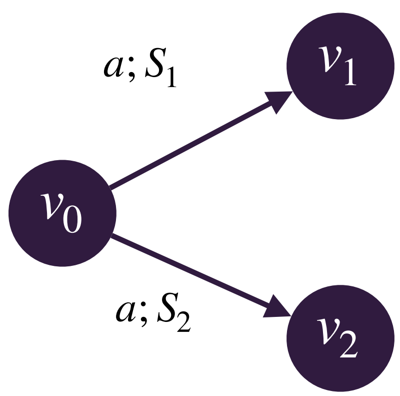

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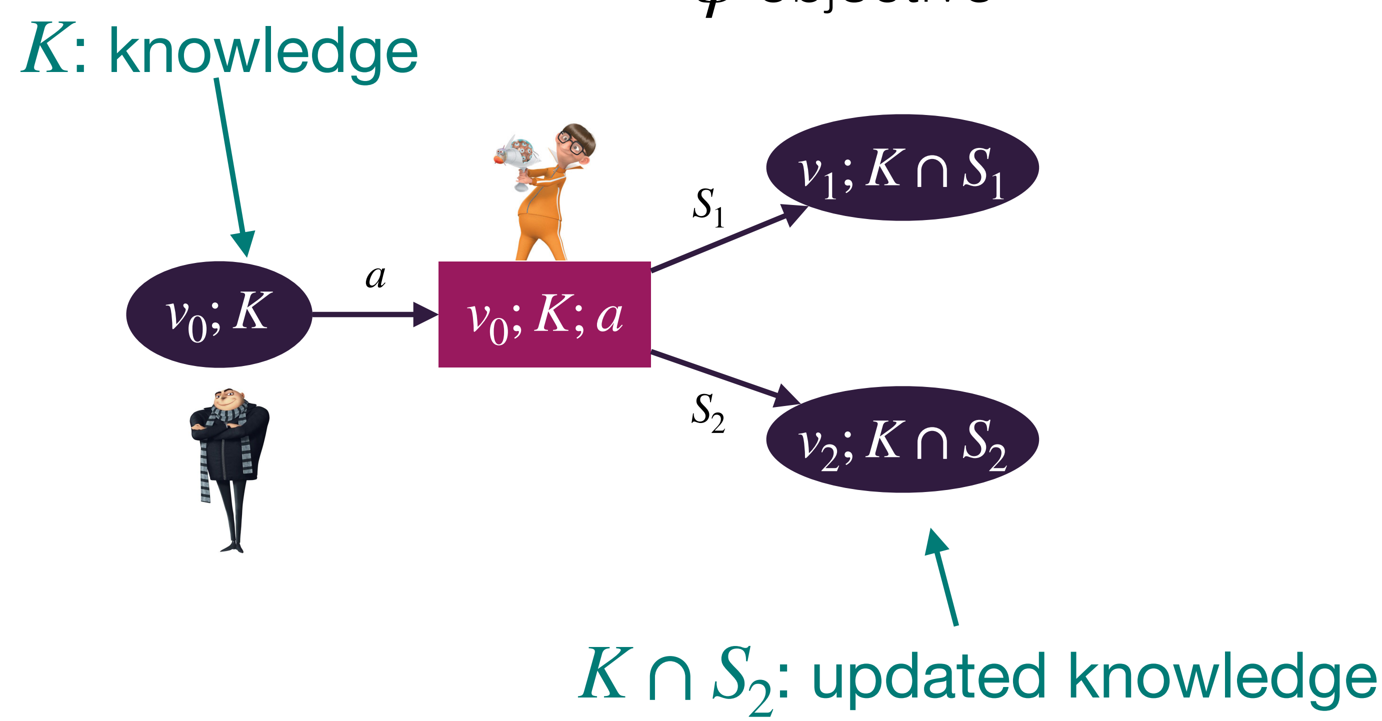
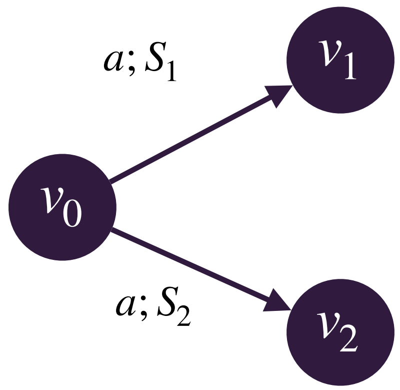


- Gru (Oval) chooses action
- Vector (Box) chooses semi linear set
- The game starts at $(v_0; \mathbb{N})$, and knowledge is updated at each round

From counting to turn-based knowledge games

φ objective



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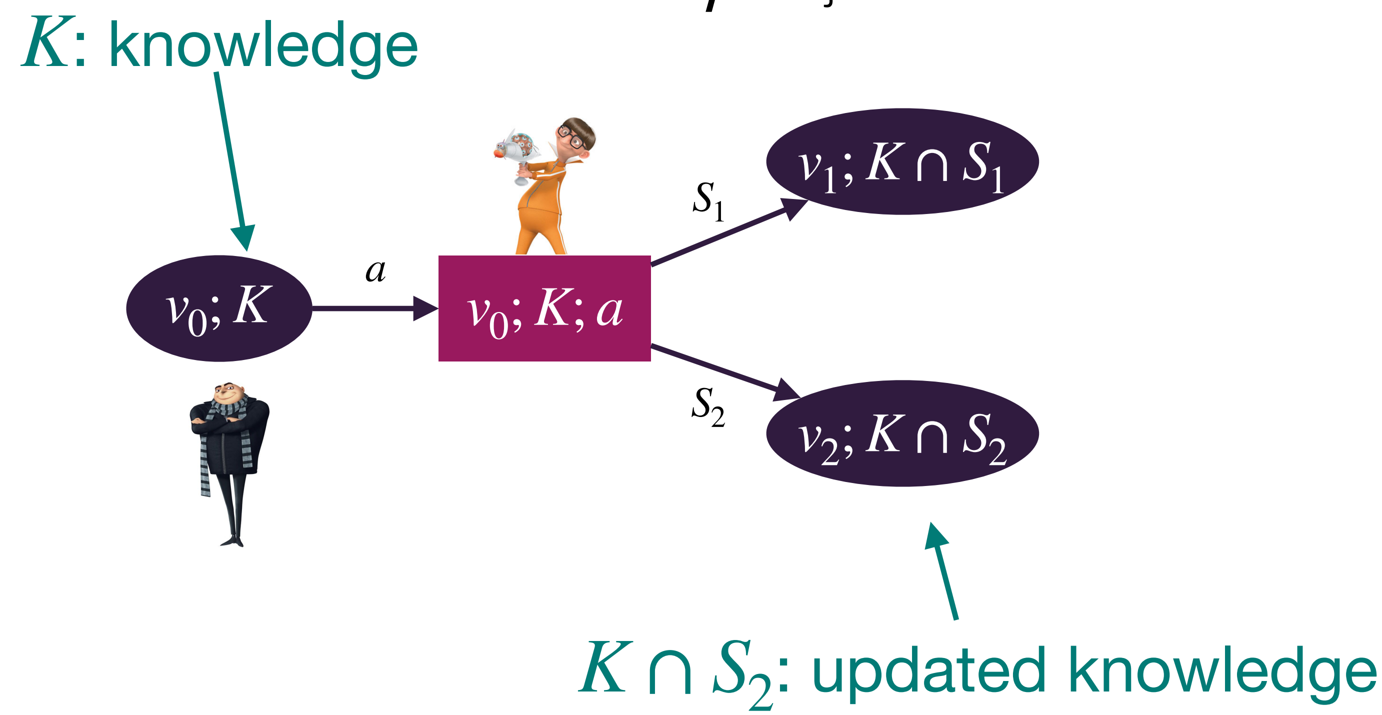
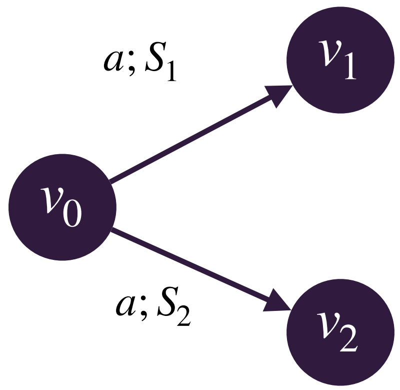


Gru wins the counting game iff he wins the knowledge game

From counting to turn-based knowledge games

φ objective



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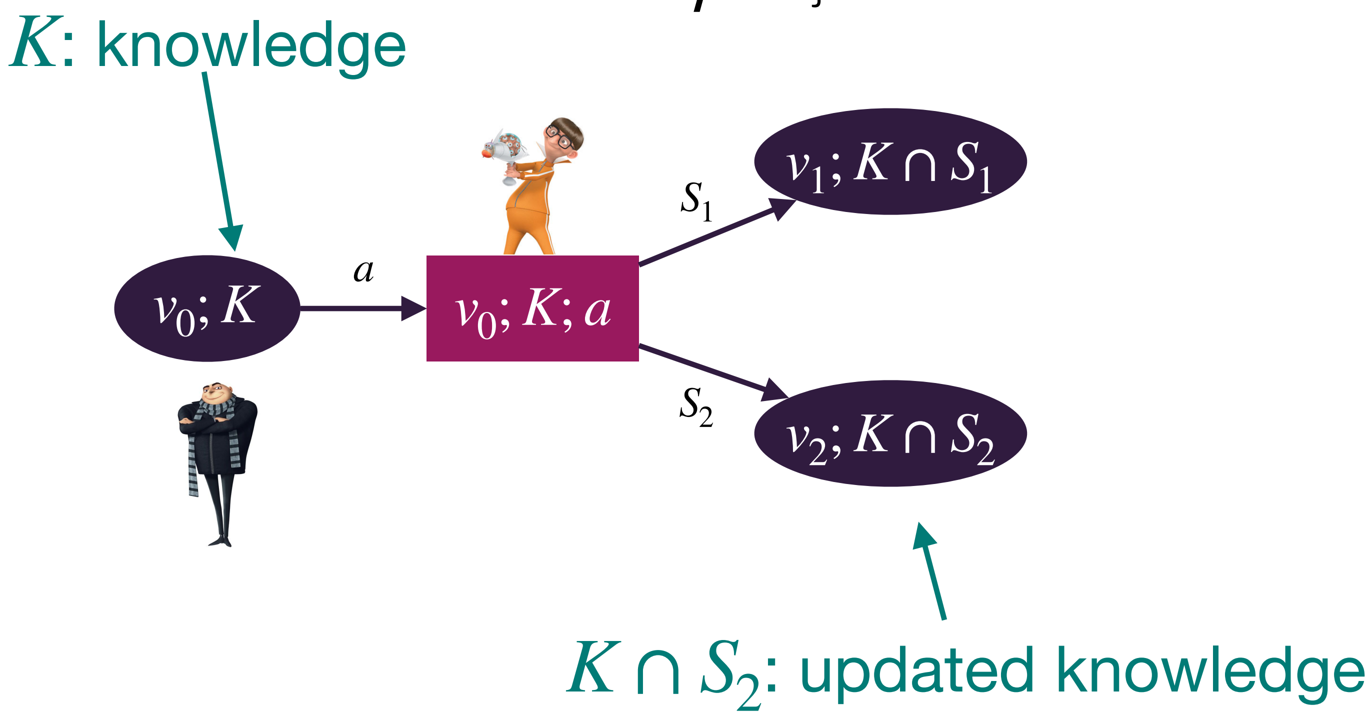
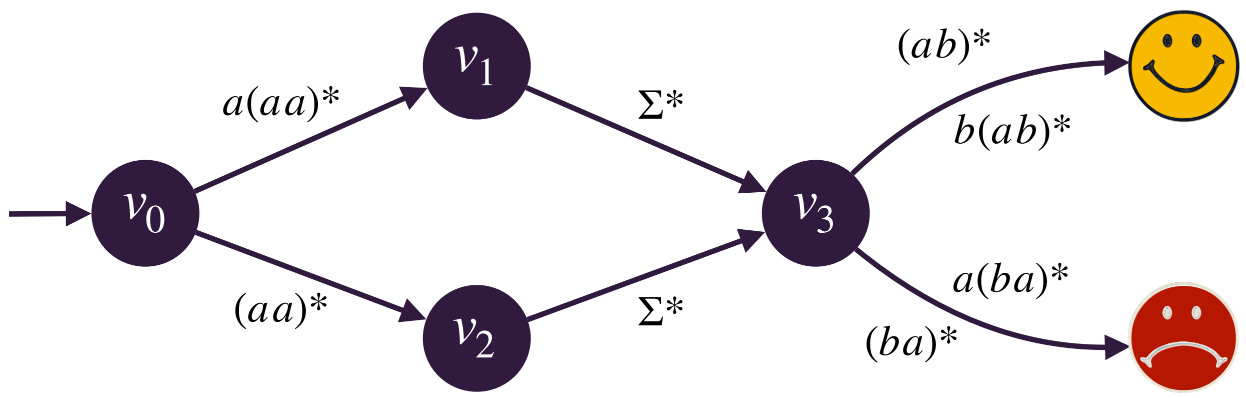


Gru wins the counting game iff he wins the knowledge game

Note : the complexity is that of solving turn-based knowledge games with objective φ
Example: polynomial-time w.r.t. its size for Reachability objectives

An example



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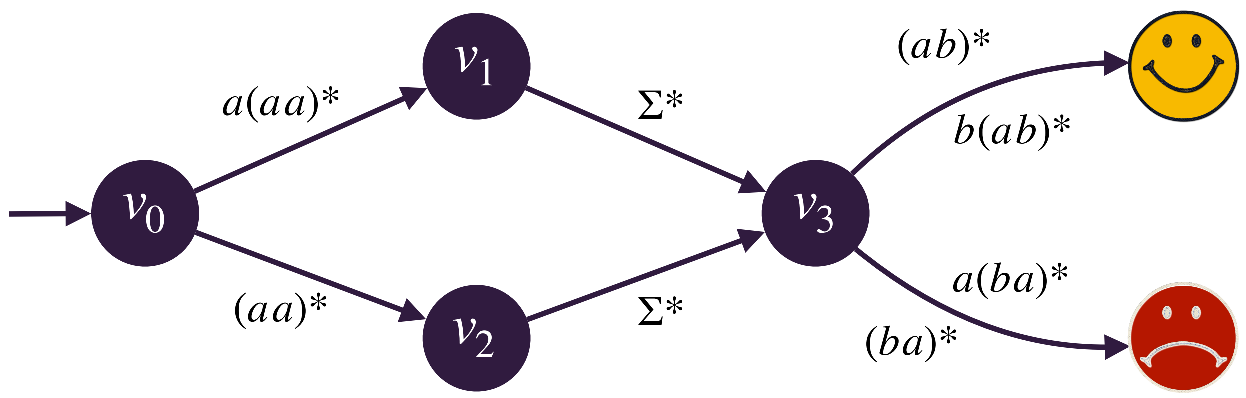
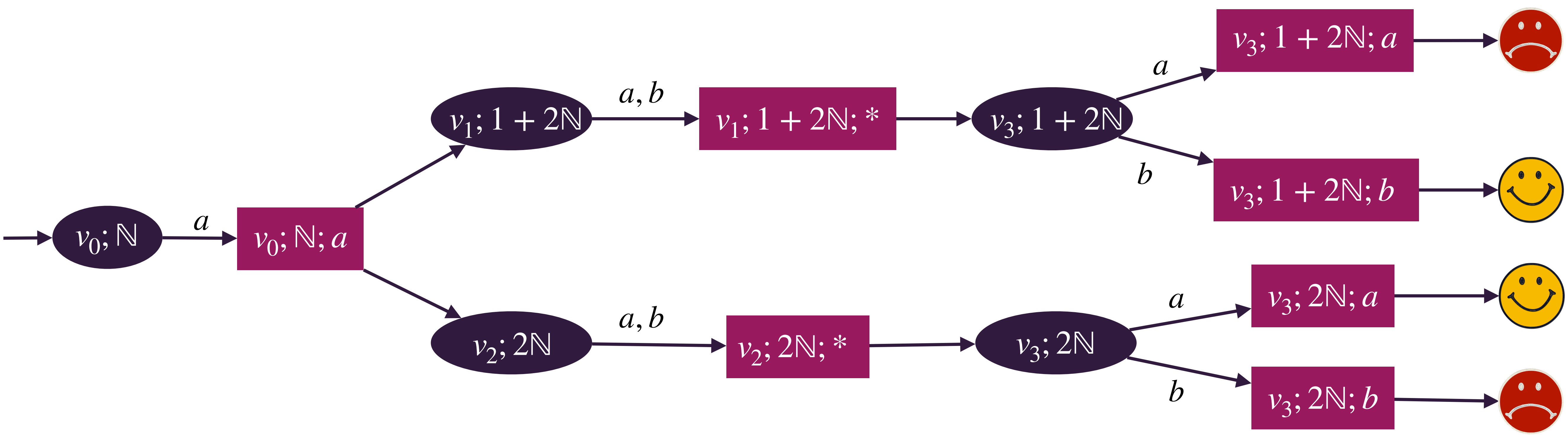
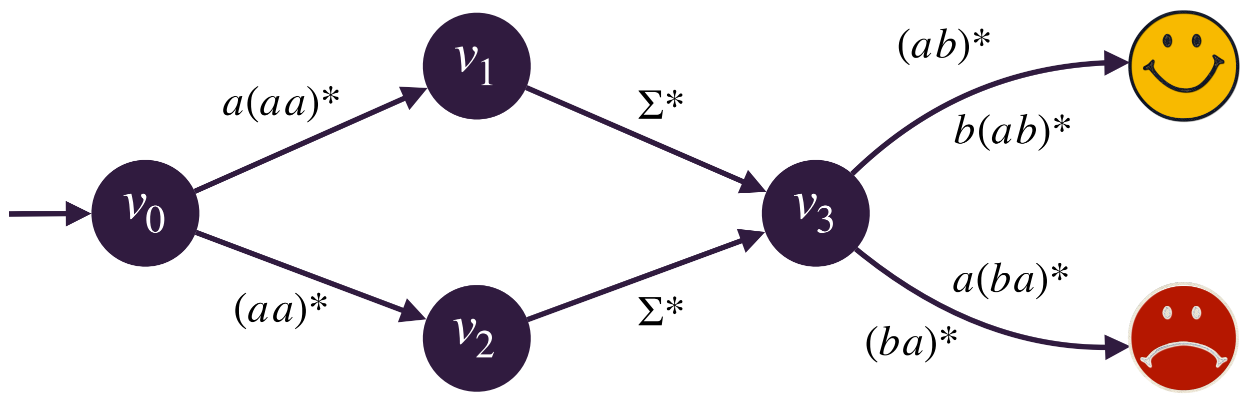


v_0
v_1
v_2
v_3
$a(aa)^*$
$(aa)^*$
Σ^*
Σ^*
$(ab)^*$
$b(ab)^*$
$a(ba)^*$
$(ba)^*$



$v_0; \mathbb{N}$
a
$v_0; \mathbb{N}; a$
$v_1; 1 + 2\mathbb{N}$
a, b
$v_1; 1 + 2\mathbb{N}; *$
$v_3; 1 + 2\mathbb{N}$
a
$v_3; 1 + 2\mathbb{N}; a$
b
$v_3; 1 + 2\mathbb{N}; b$
$v_2; 2\mathbb{N}$
a, b
$v_2; 2\mathbb{N}; *$
$v_3; 2\mathbb{N}$
a
$v_3; 2\mathbb{N}; a$
b
$v_3; 2\mathbb{N}; b$

An example



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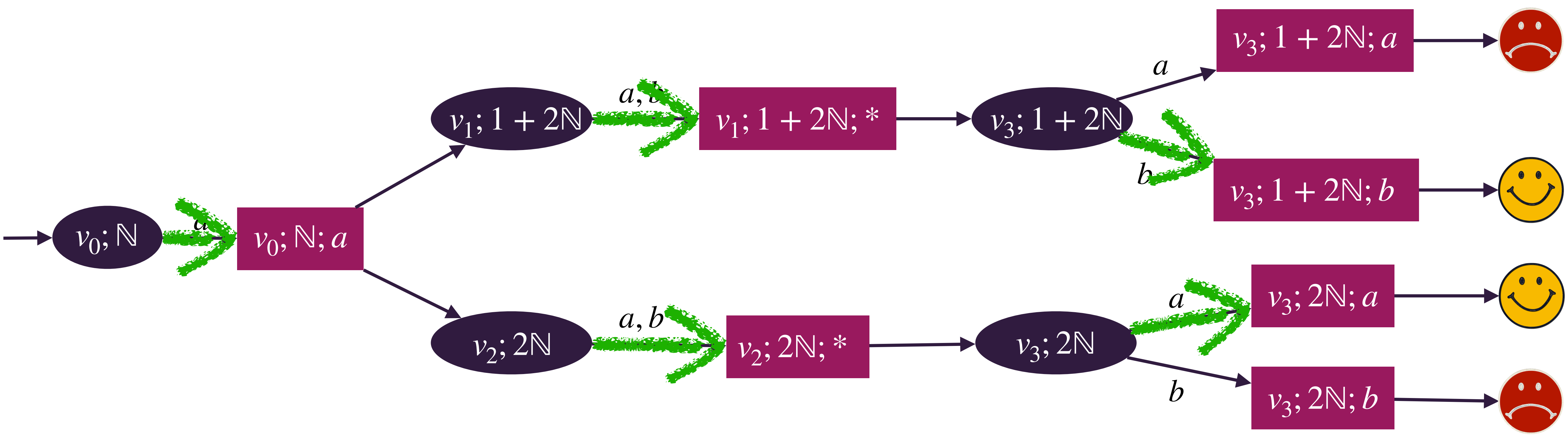

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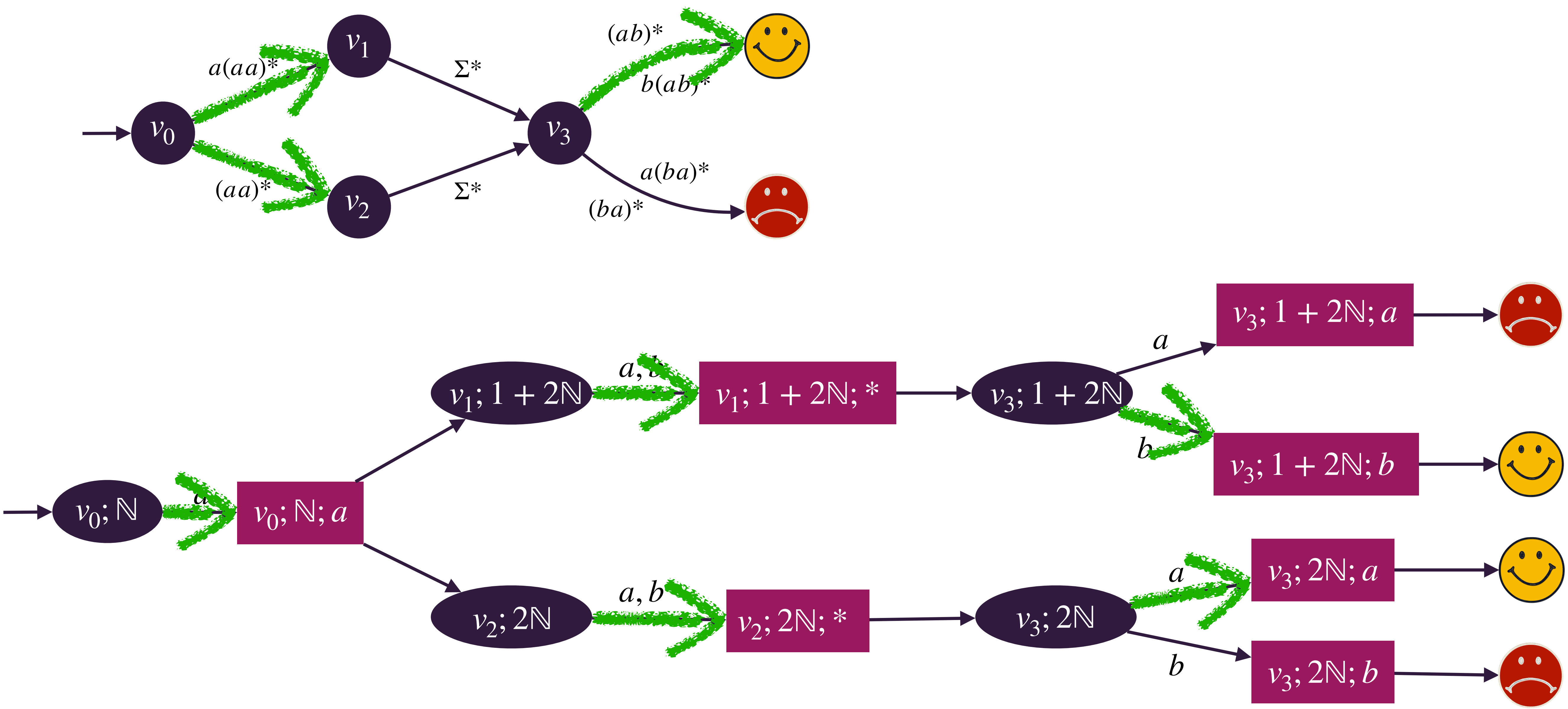
An example



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a, b
$v_1; 1+2\mathbb{N}; *$
$v_3; 1+2\mathbb{N}$
a
$v_3; 1+2\mathbb{N}; a$
b
$v_3; 1+2\mathbb{N}; b$
$v_2; 2\mathbb{N}$
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The results

Complexity results

The crowd controller problem is decidable and has the following complexity for Reachability objectives:

<table>
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<tr><td>Intervals</td><td colspan="2">PTIME-complete</td></tr>
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<tr><td>Reg/CF languages</td></tr>
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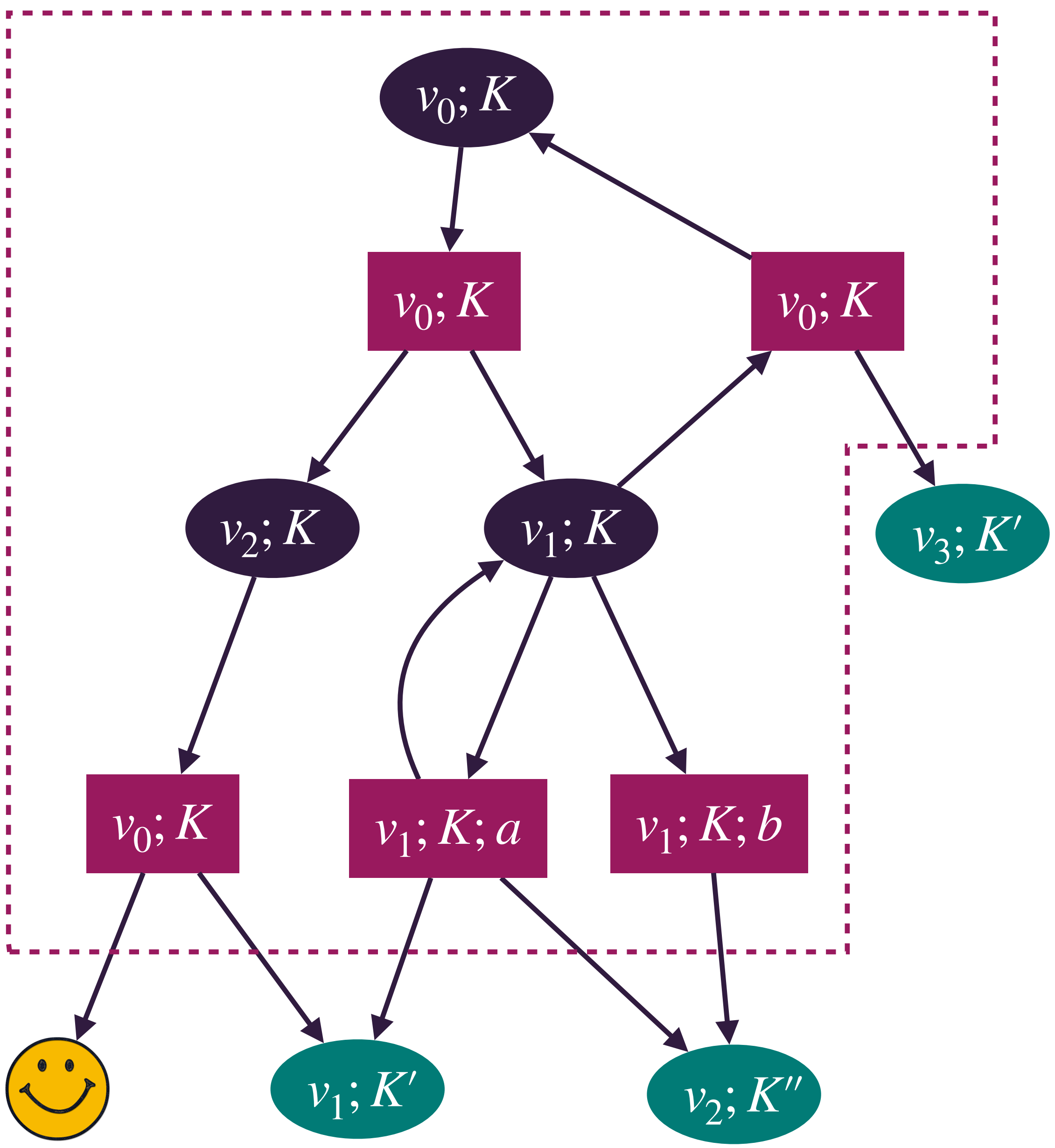
	Deterministic arenas	**Non-deterministic arenas**
Intervals	PTIME-complete	
Finite unions of intervals	NP-complete	PSPACE-complete
Semilinear sets	PSPACE-complete	
Reg/CF languages		

- Each knowledge is an intersection of (atomic) constraints used in the game
- The number of possible knowledges is therefore at most exponential in the number of (atomic) constraints used in the game
  - Semilinear sets: the knowledge game is at most exponential in the number of semilinear sets
- Finite unions of intervals: the knowledge game is at most exponential in the number of endpoints
- Intervals: the knowledge game is quadratic in the number of endpoints of the intervals

PSPACE algorithm - 1

Subgame associated with knowledge K



$K', K'' \subsetneq K$

PSPACE algorithm - 1

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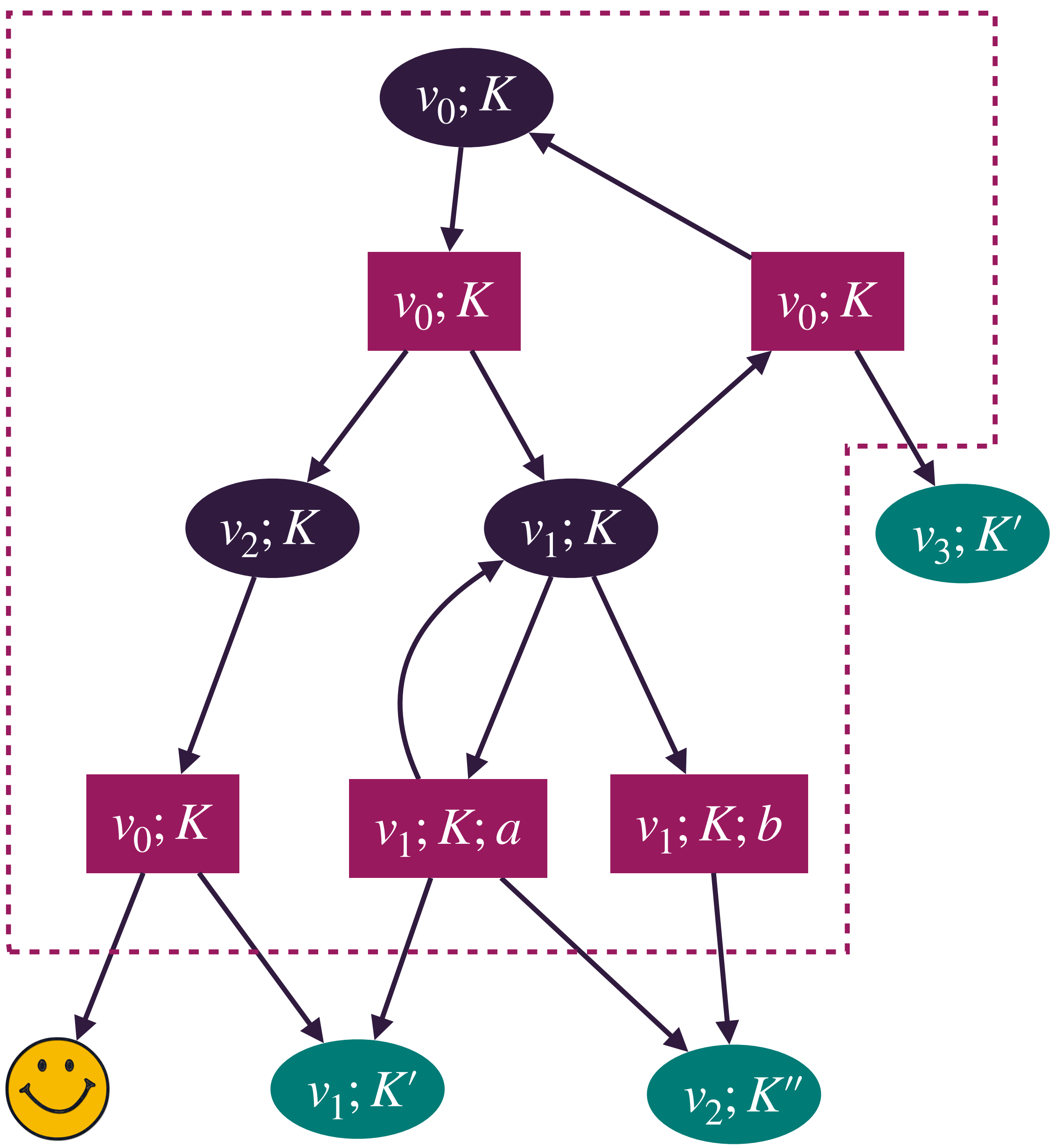

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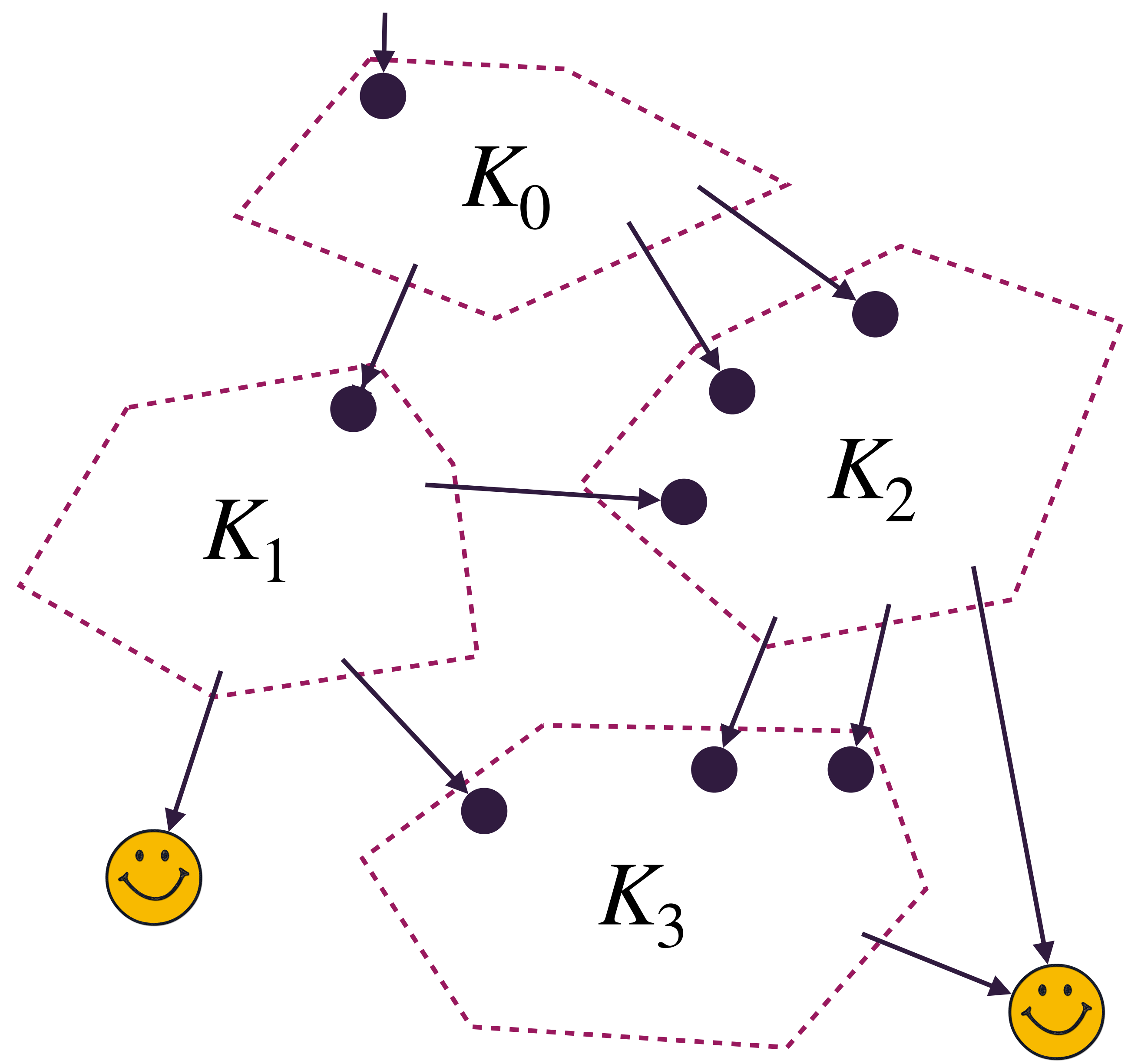
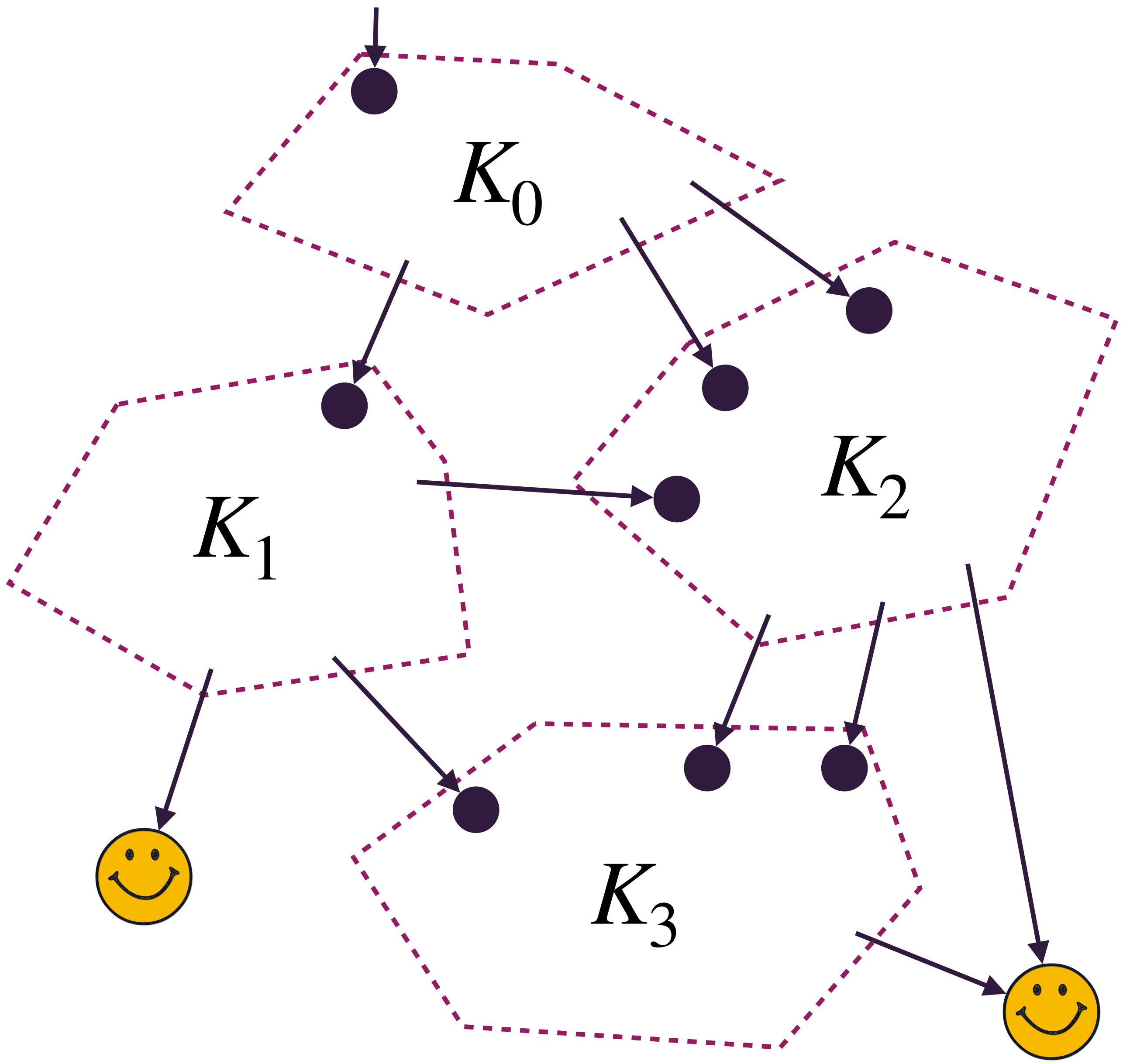


$K_3 \subsetneq K_1, K_2 \subsetneq K_0$

PSPACE algorithm - 2



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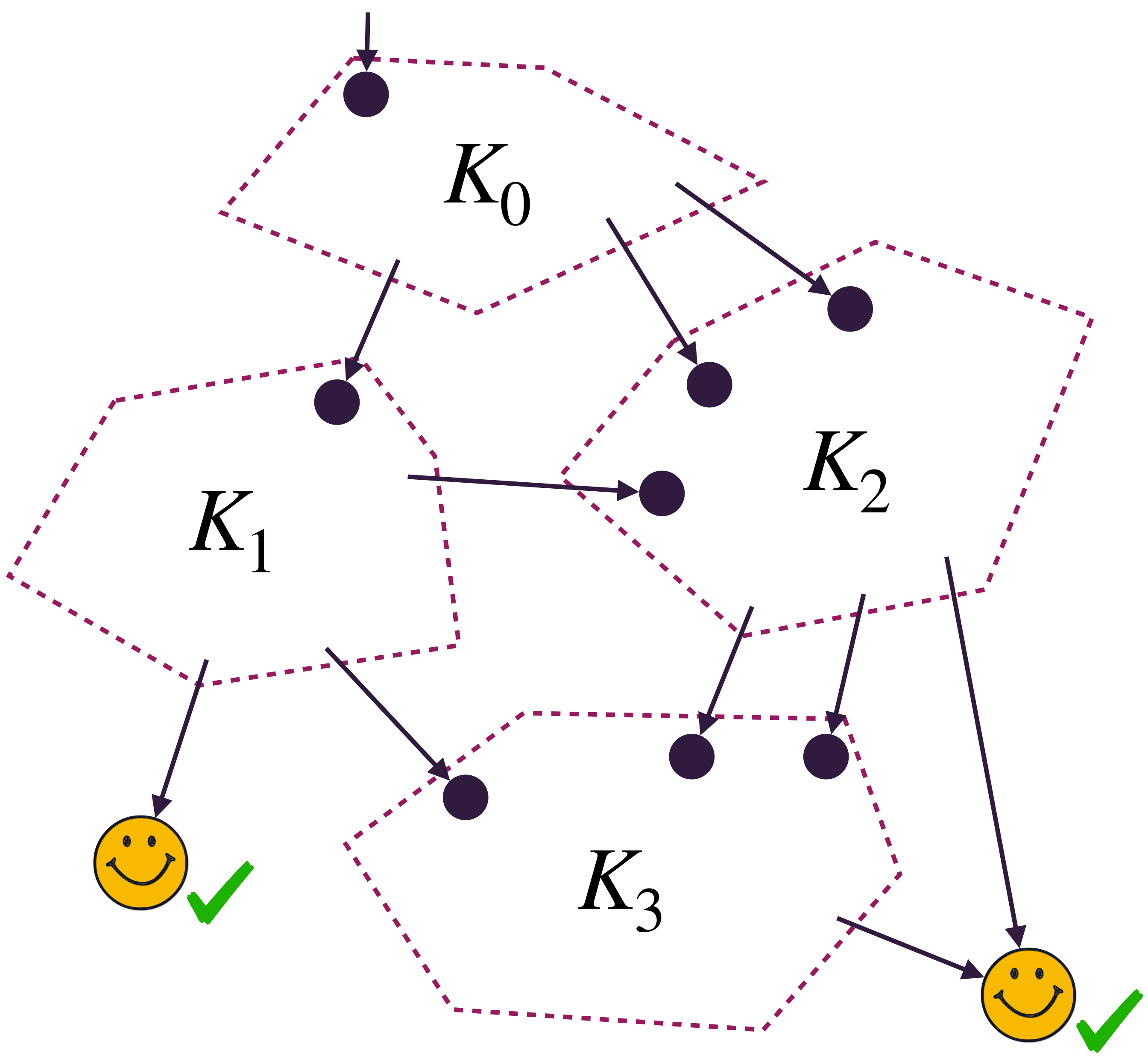
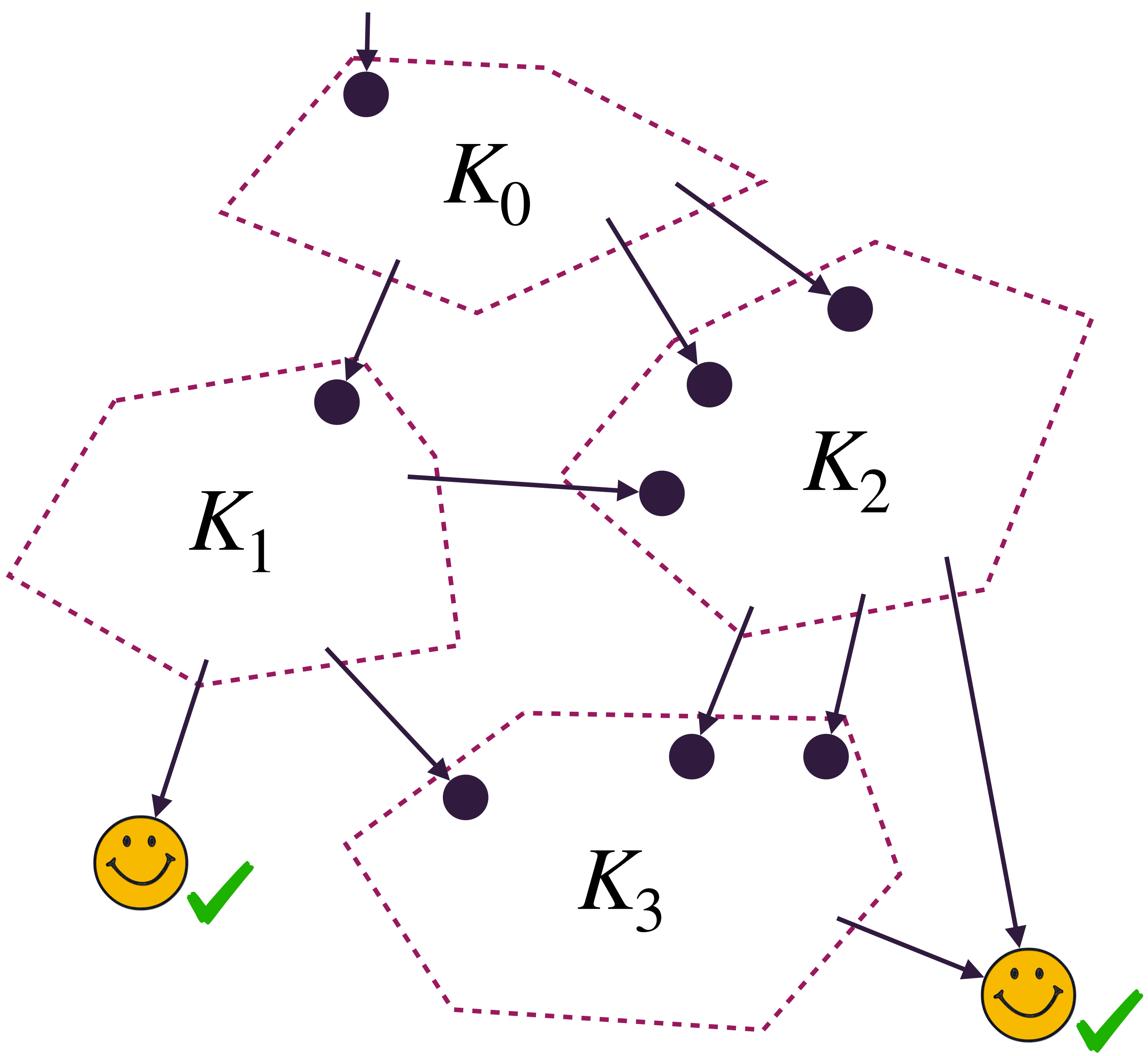


Bottom-up tag of winning states

PSPACE algorithm - 2



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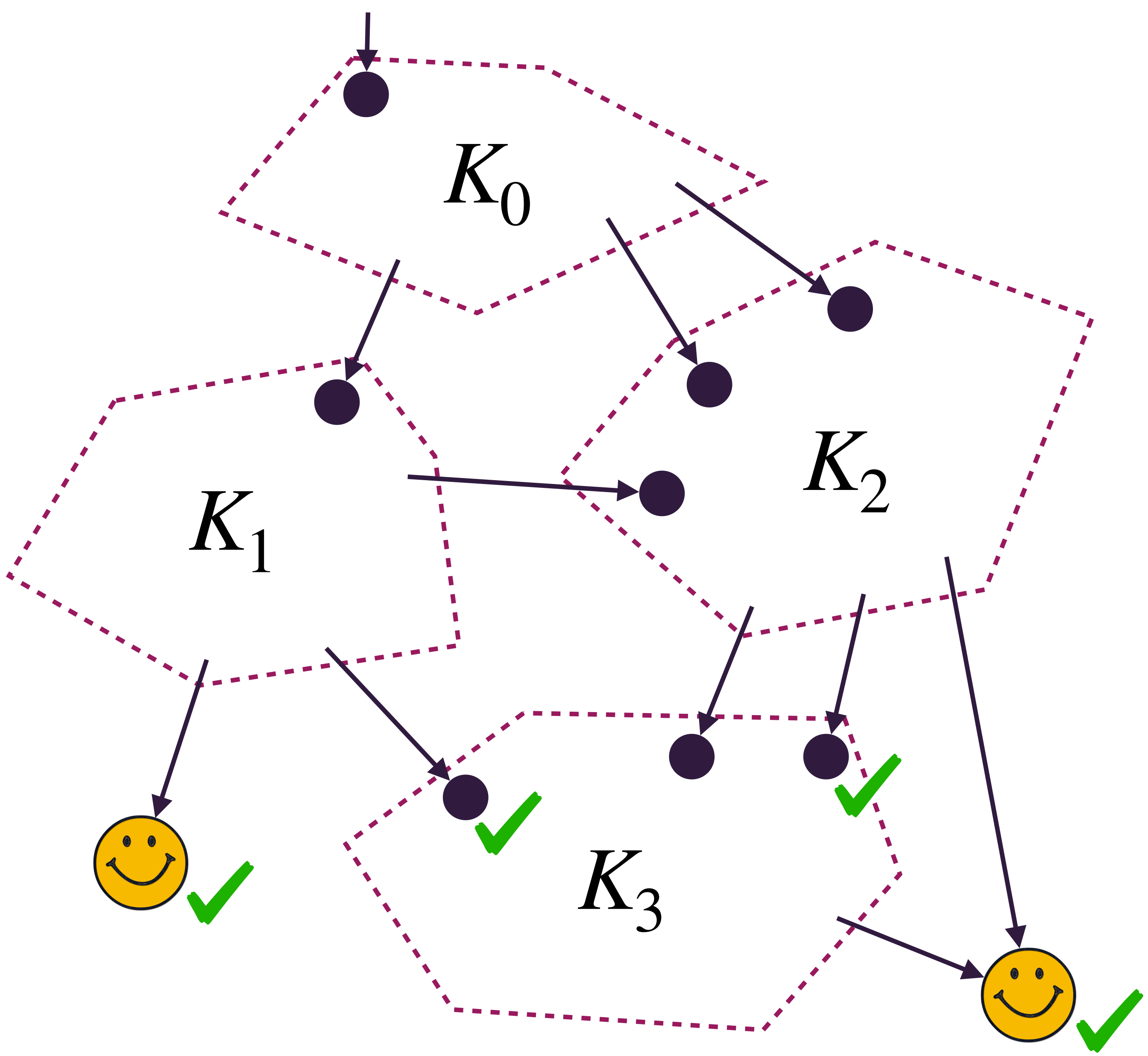

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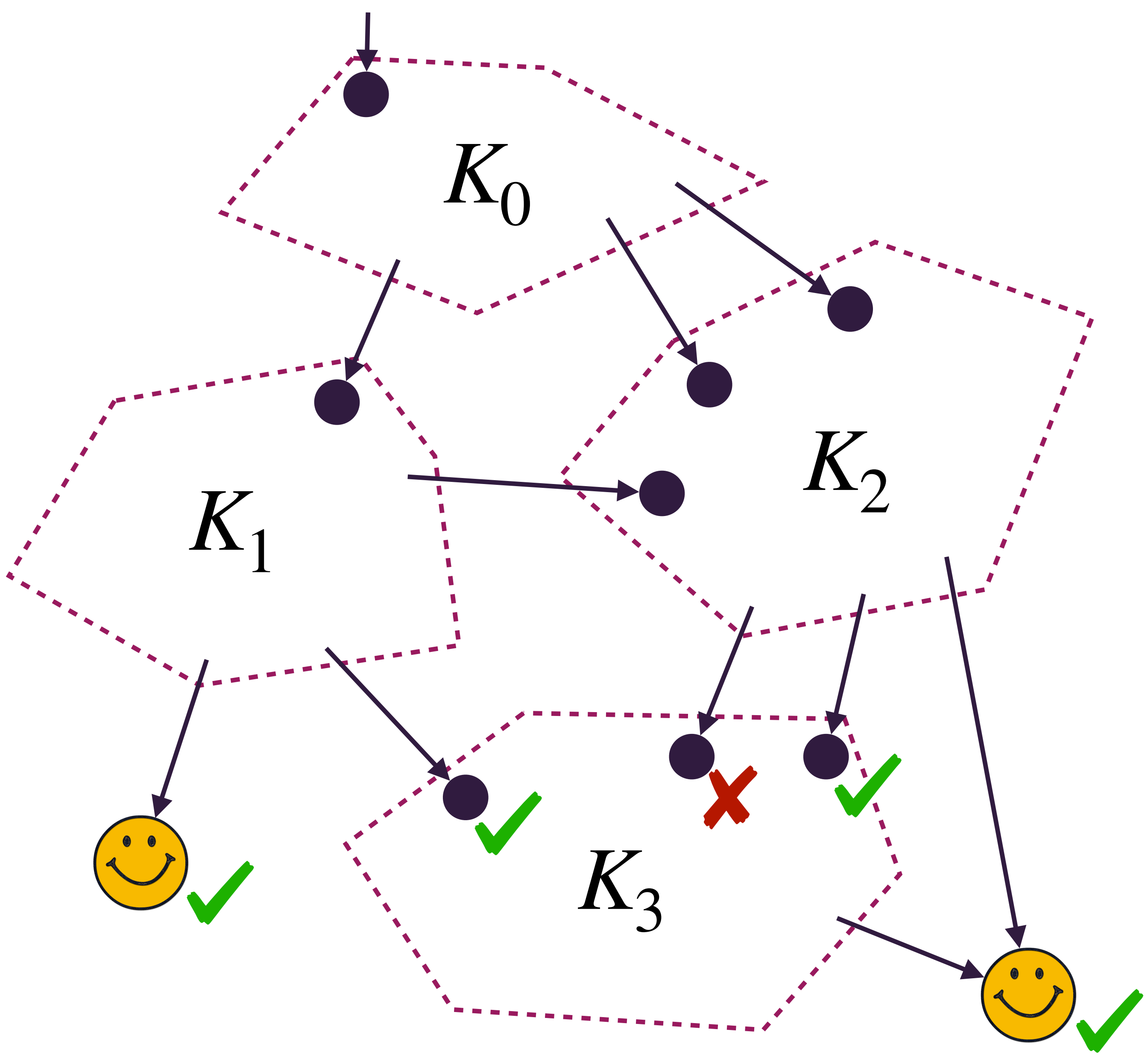
PSPACE algorithm - 2



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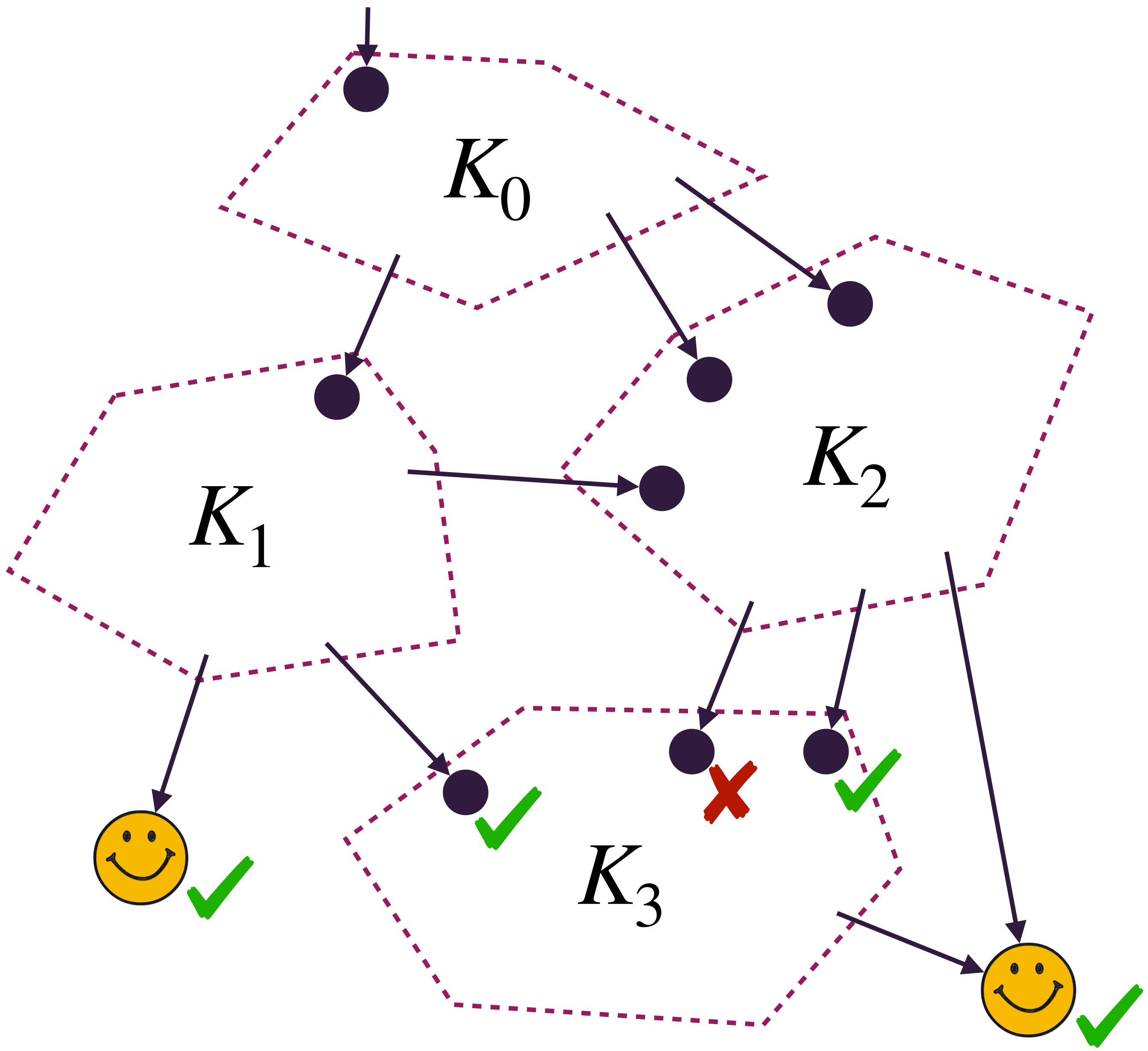

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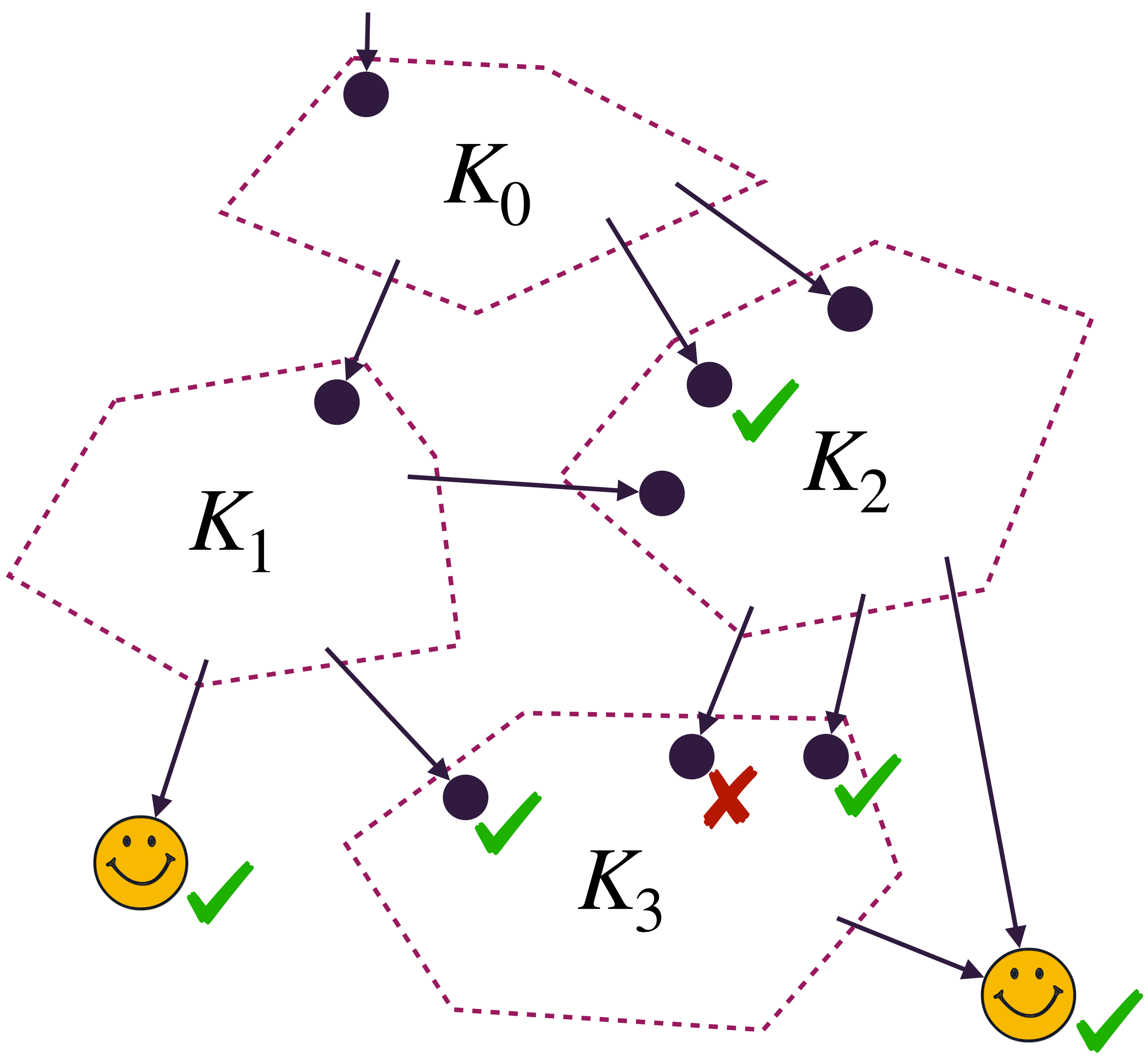

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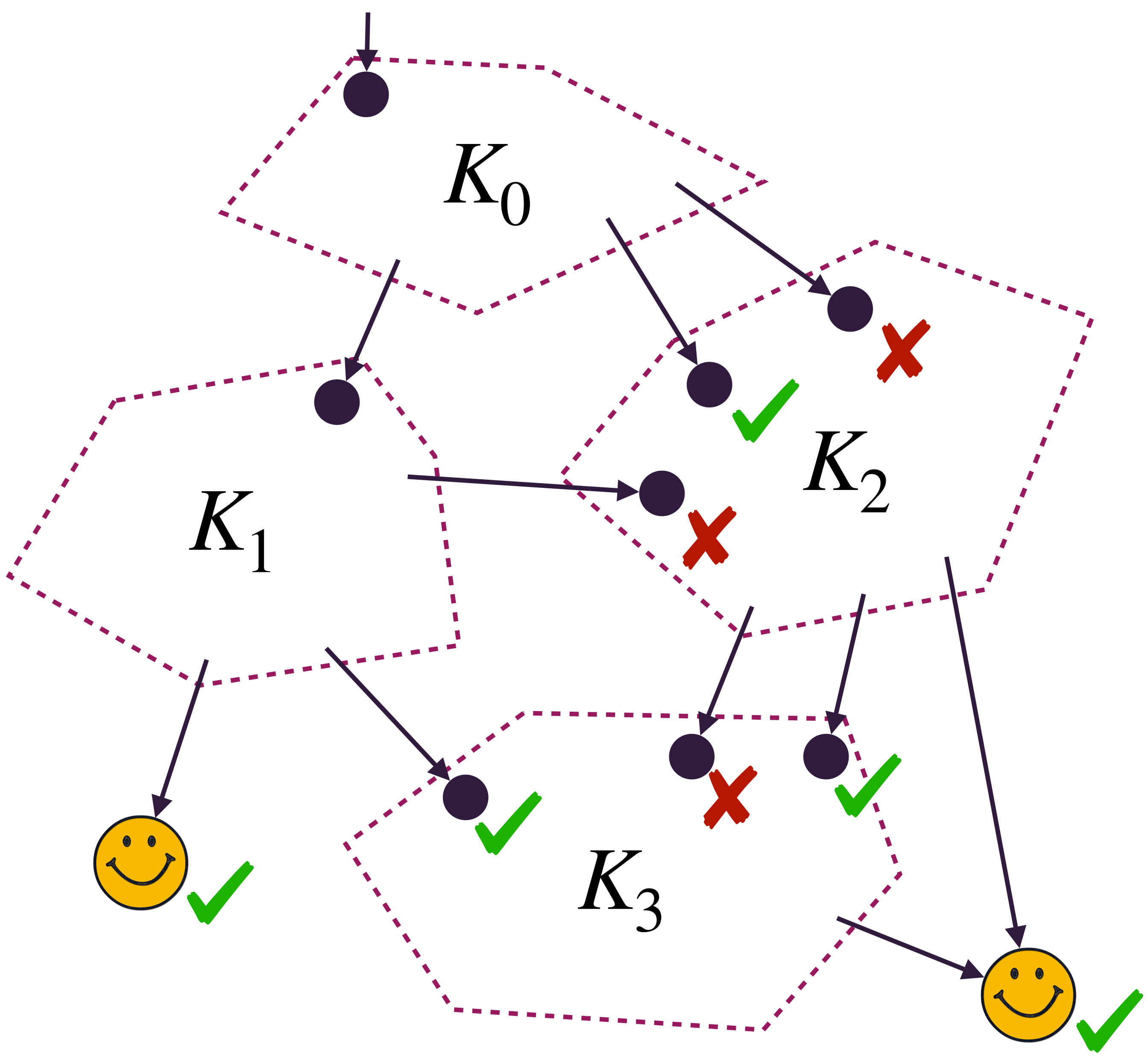

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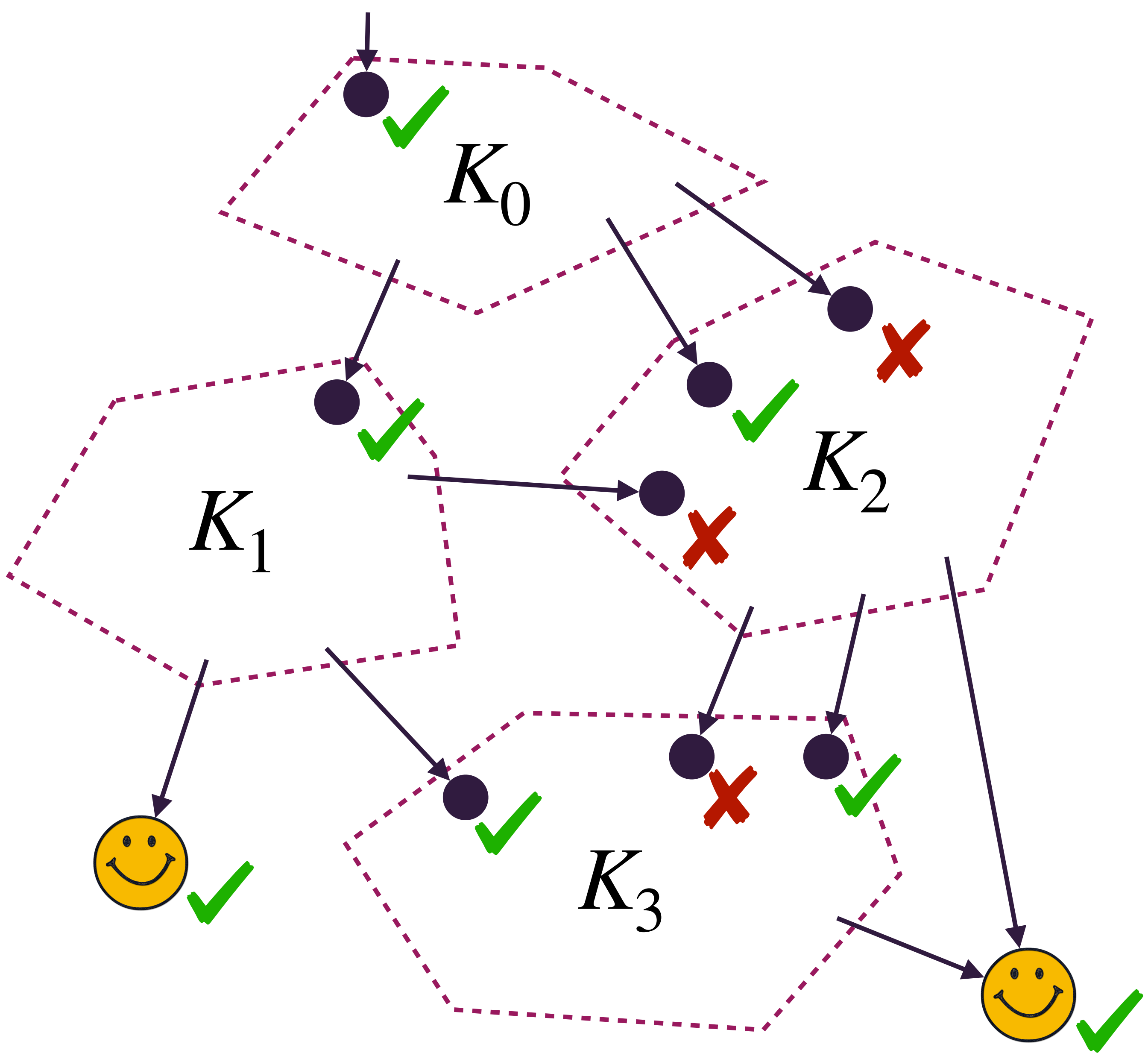

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PSPACE-hardness - 1

QSAT formula $\psi = \exists x_1 \forall x_2 \exists x_3 \forall x_4 \cdot (x_1 \vee \neg x_2 \vee \neg x_3) \wedge (x_2 \vee x_3 \vee \neg x_4)$

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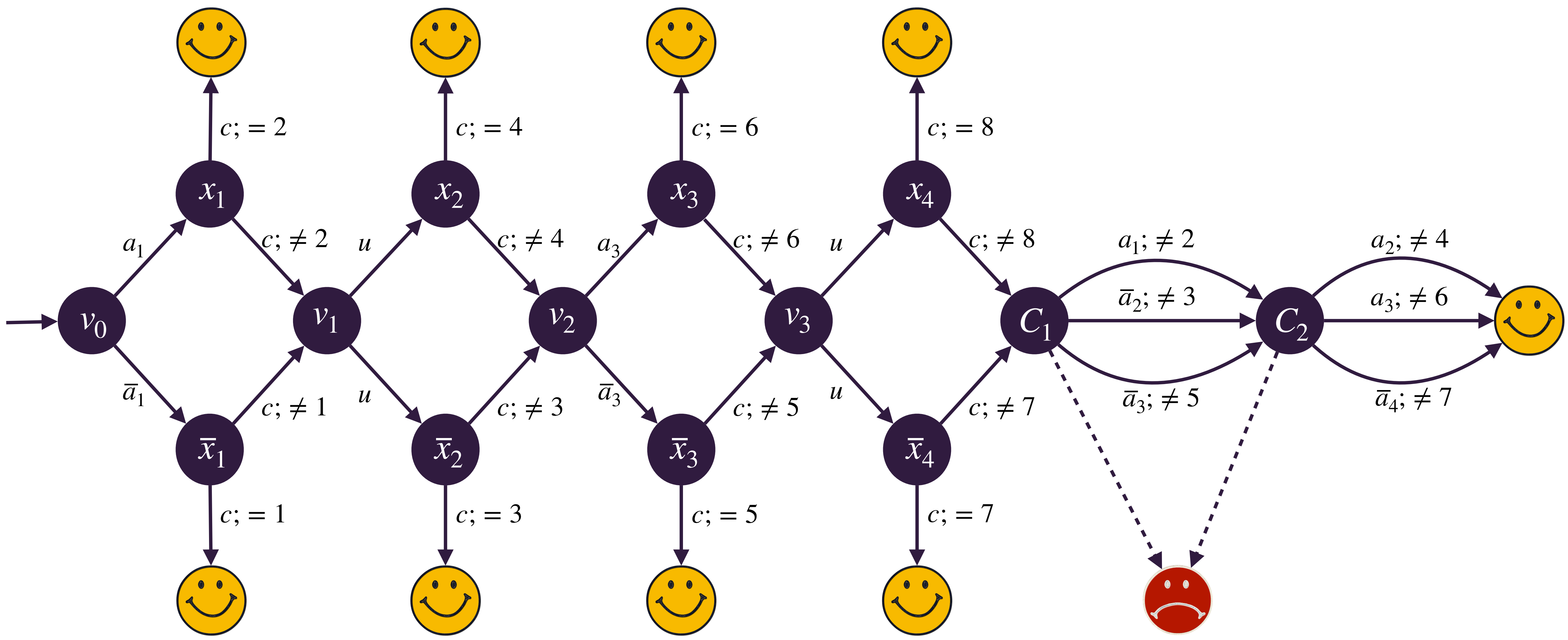

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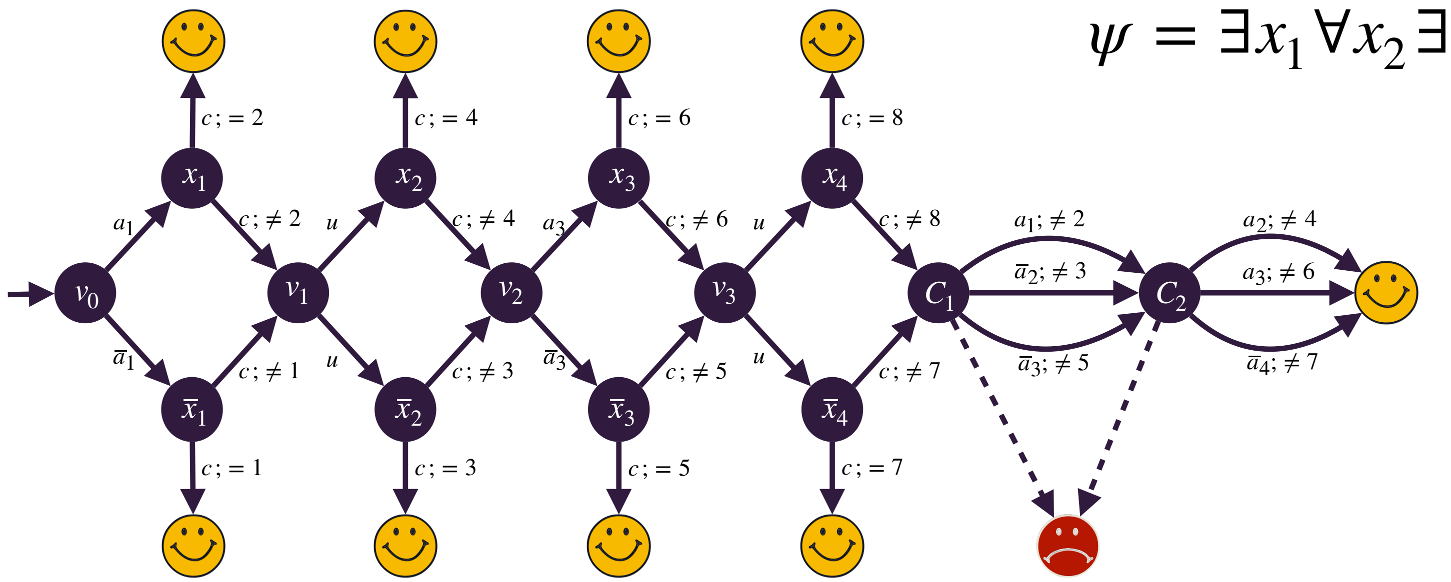

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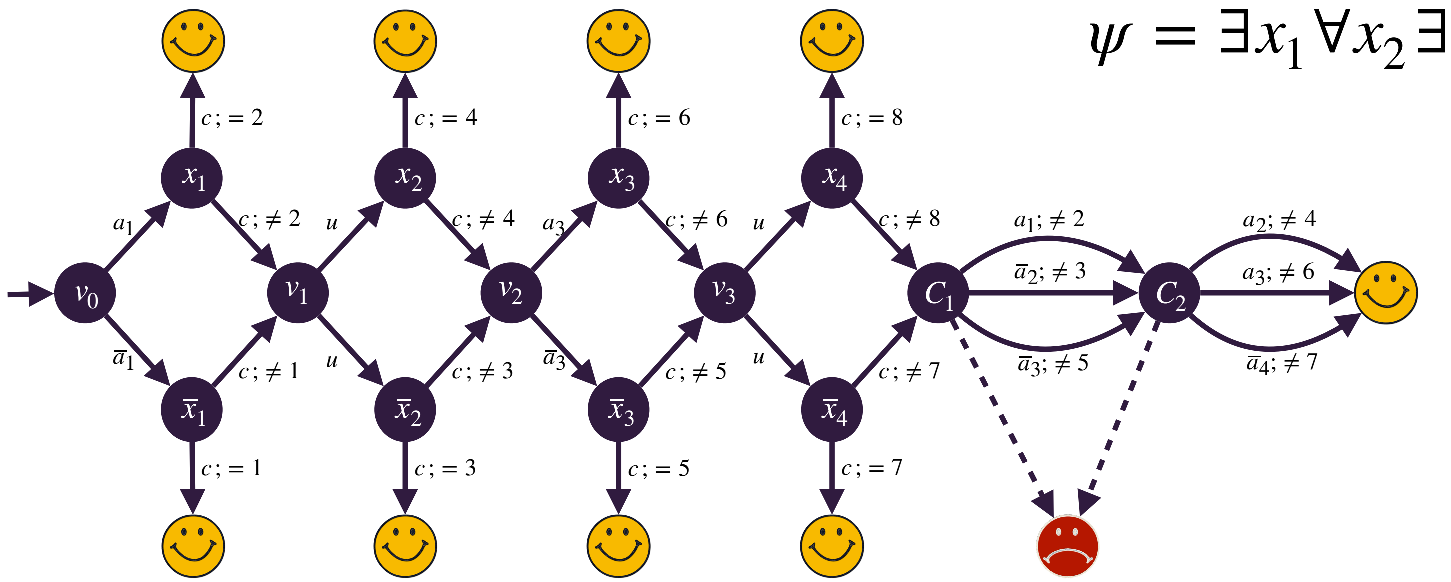
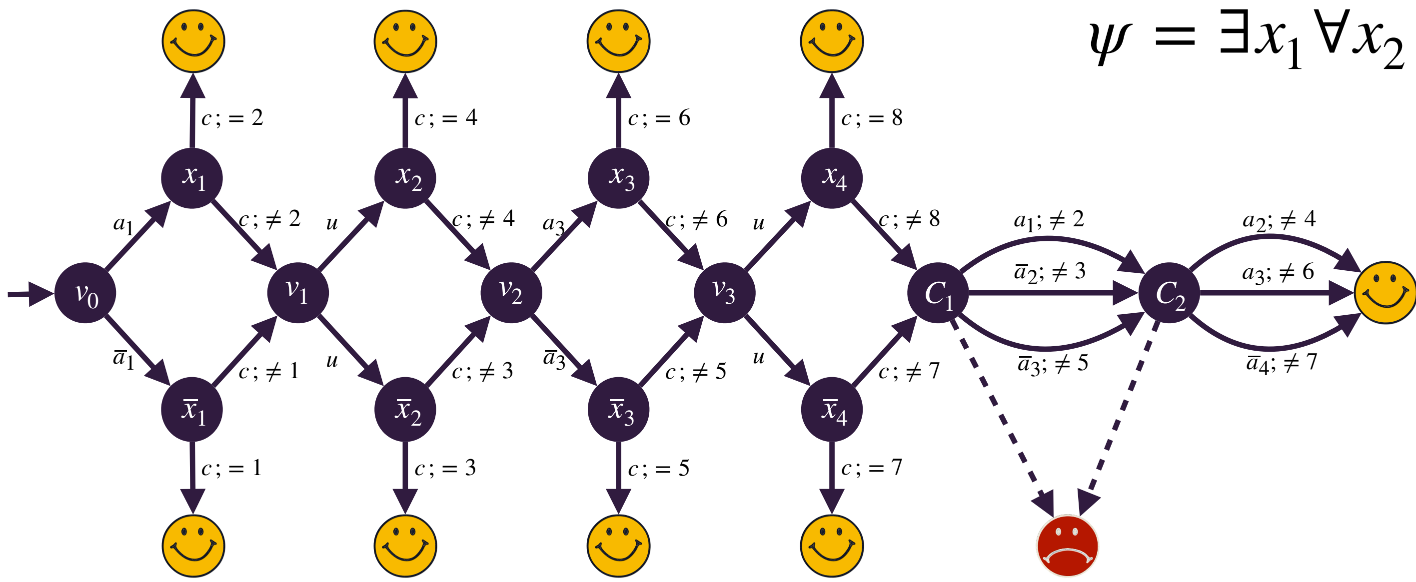


Strategy for Gru if ψ is true

PSPACE-hardness - 2



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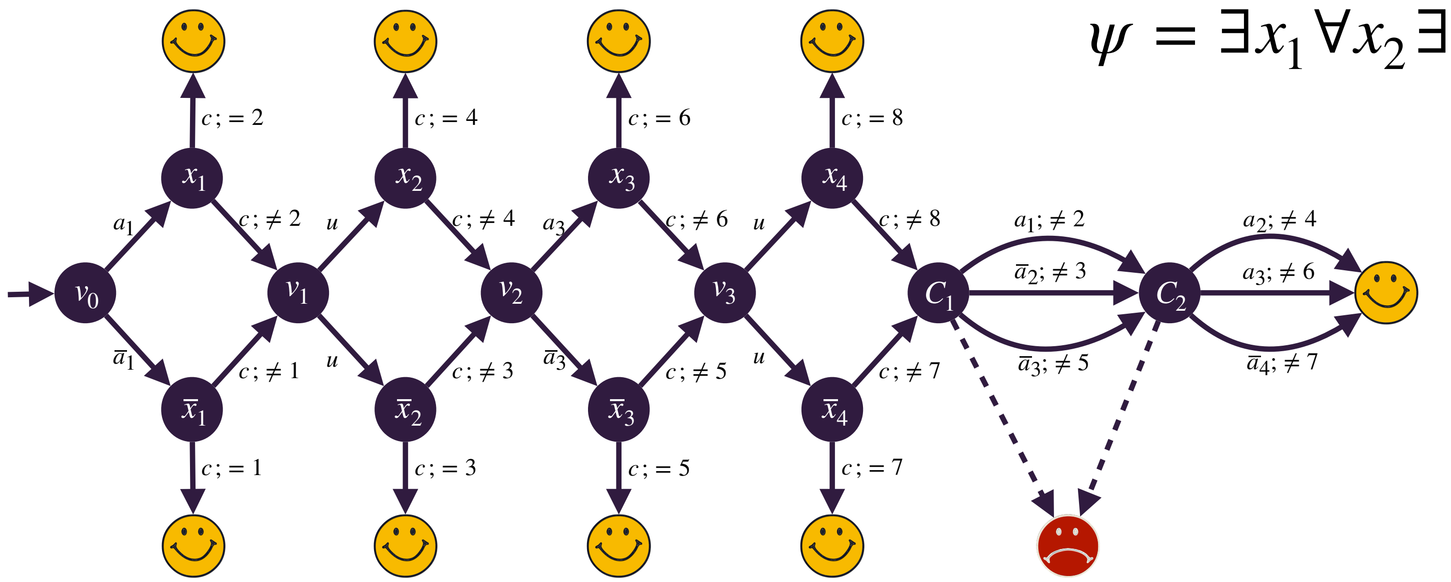

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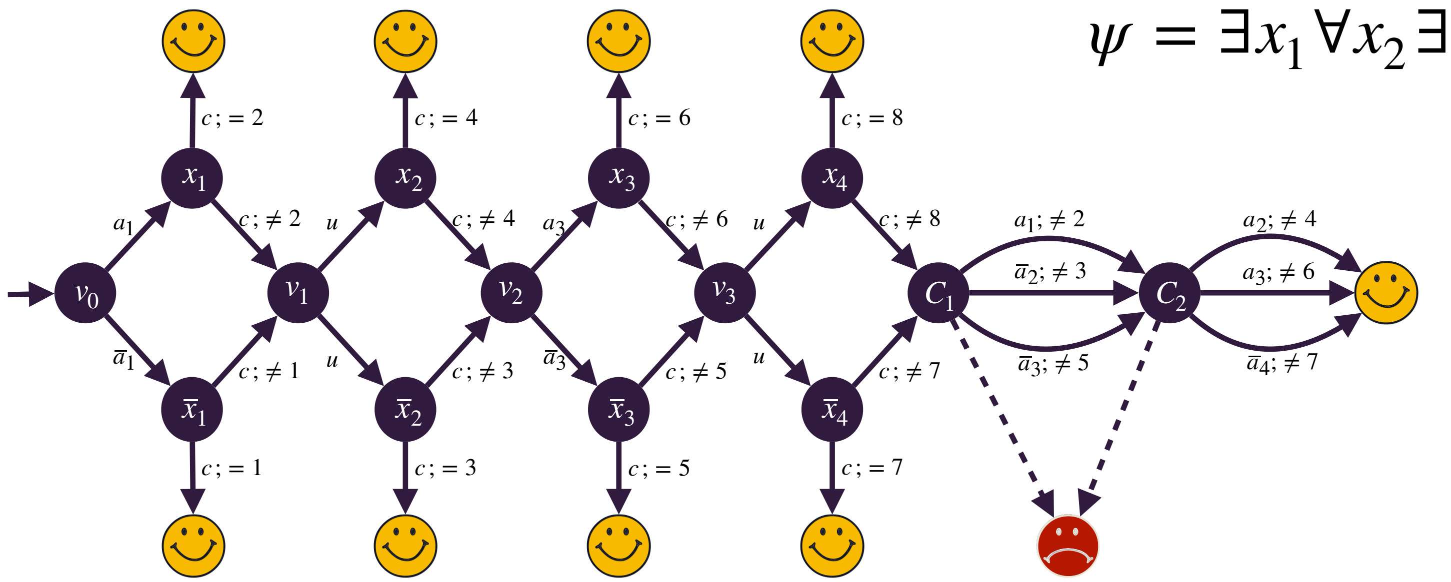

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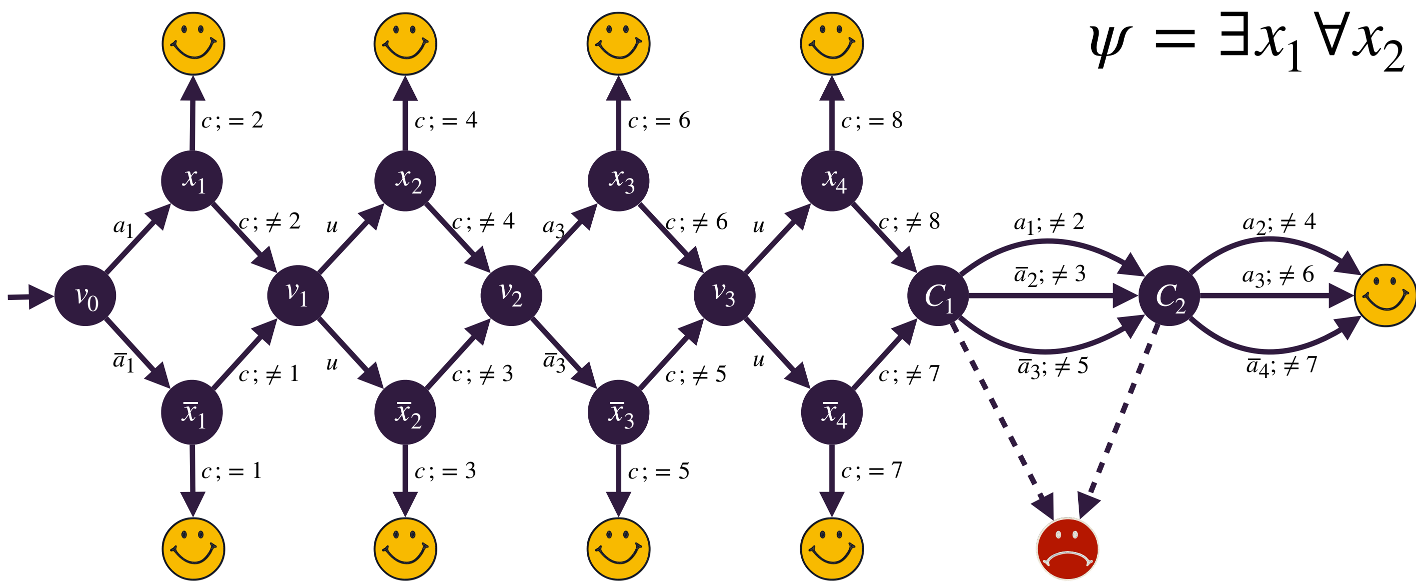

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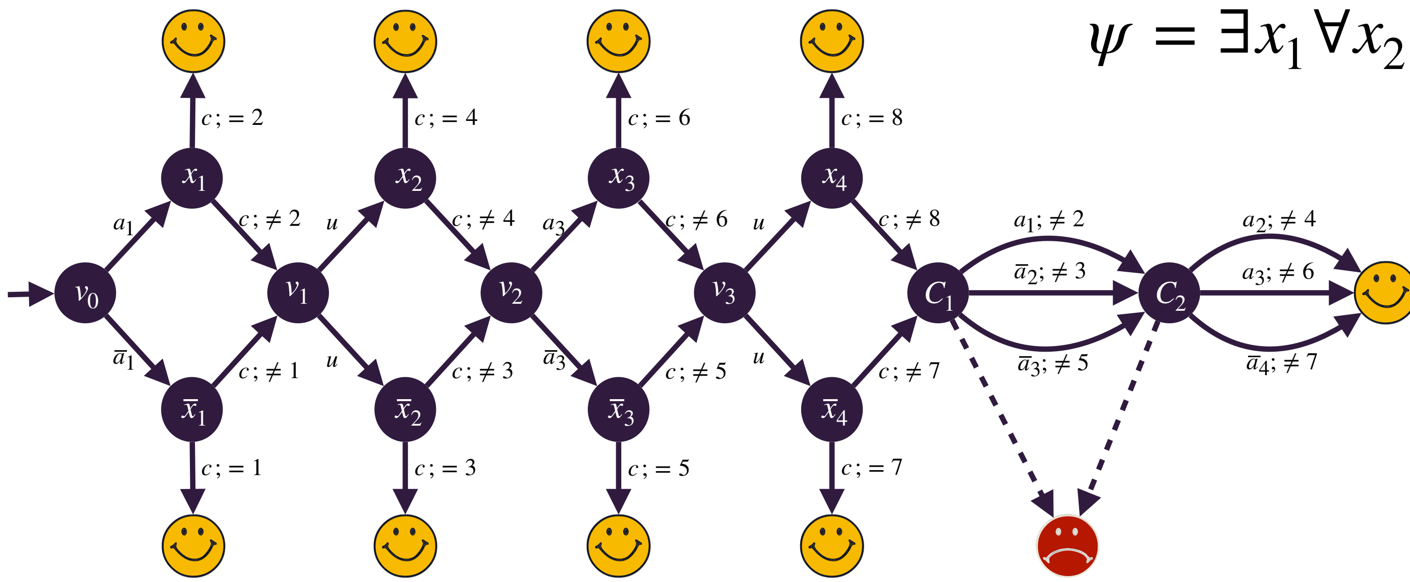

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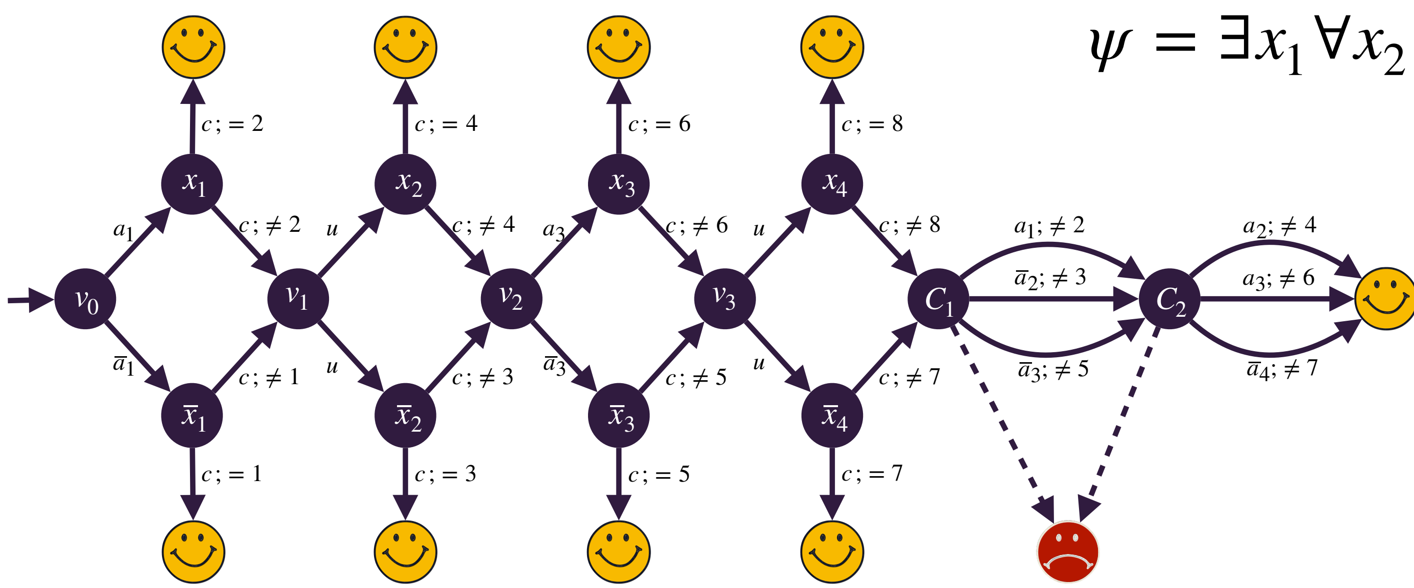
PSPACE-hardness - 2



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Strategy for Gru if ψ is true

- At v_0, play the correct assignment, say false (i.e. $\bar{a}_1$), reaching $\bar{x}_1$
  - If $k = 1$, then go to ☺
  - If $k \neq 1$, the game proceeds to v_1
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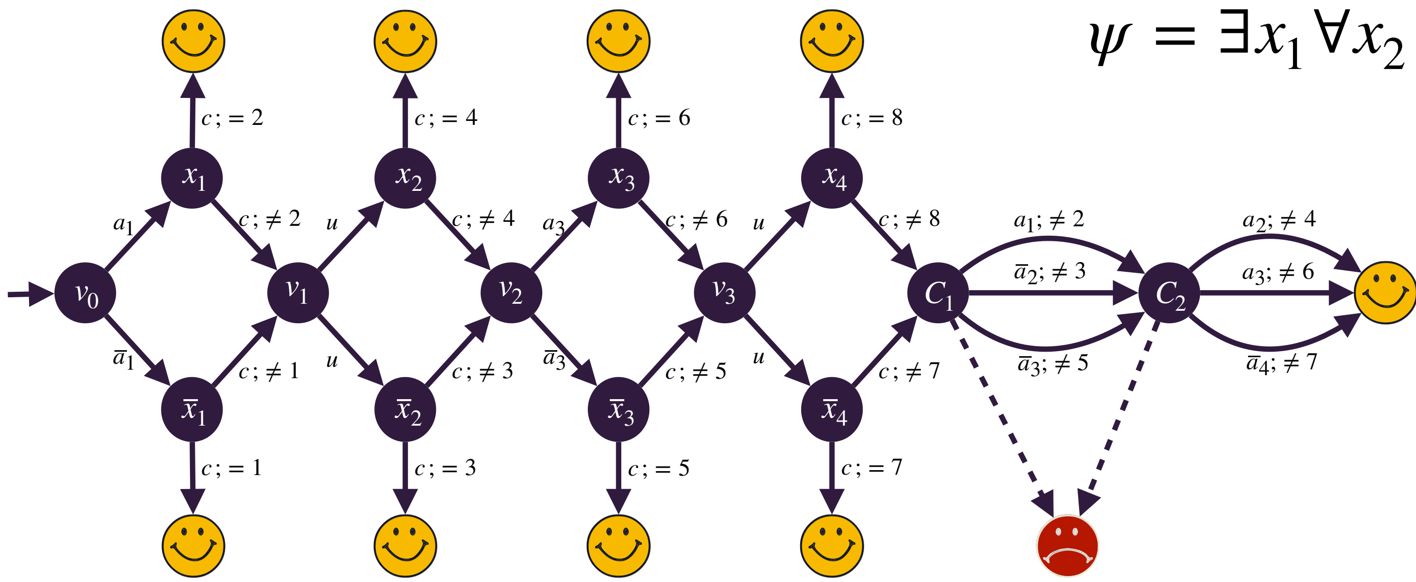
PSPACE-hardness - 2



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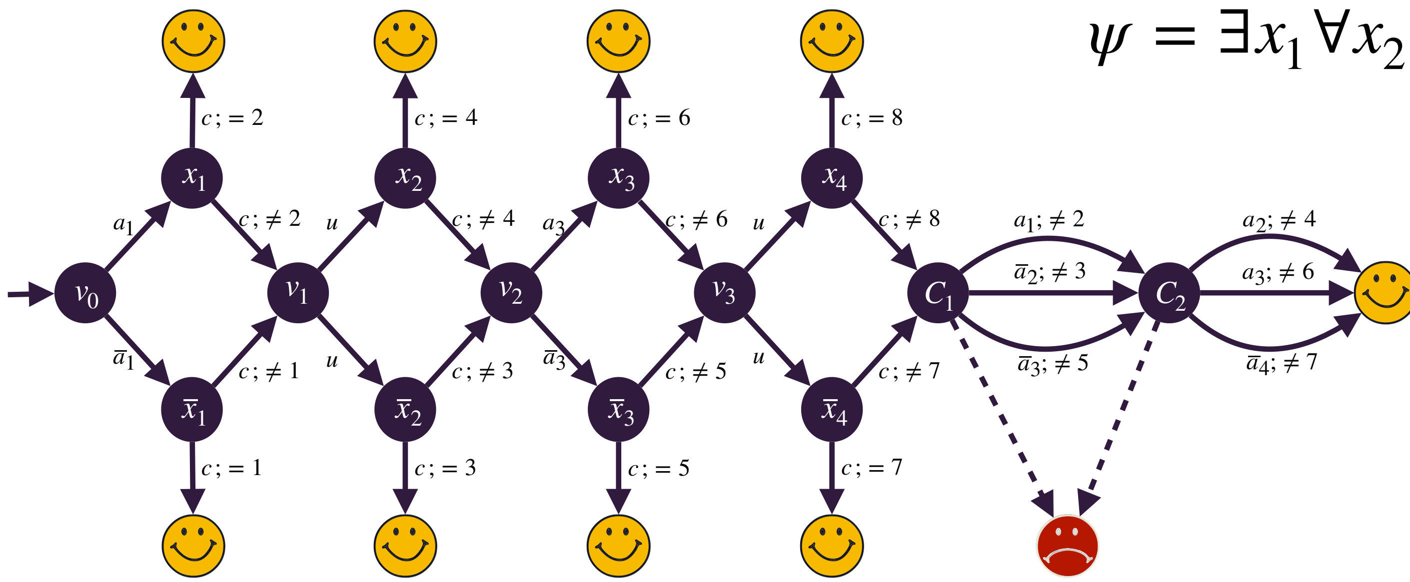

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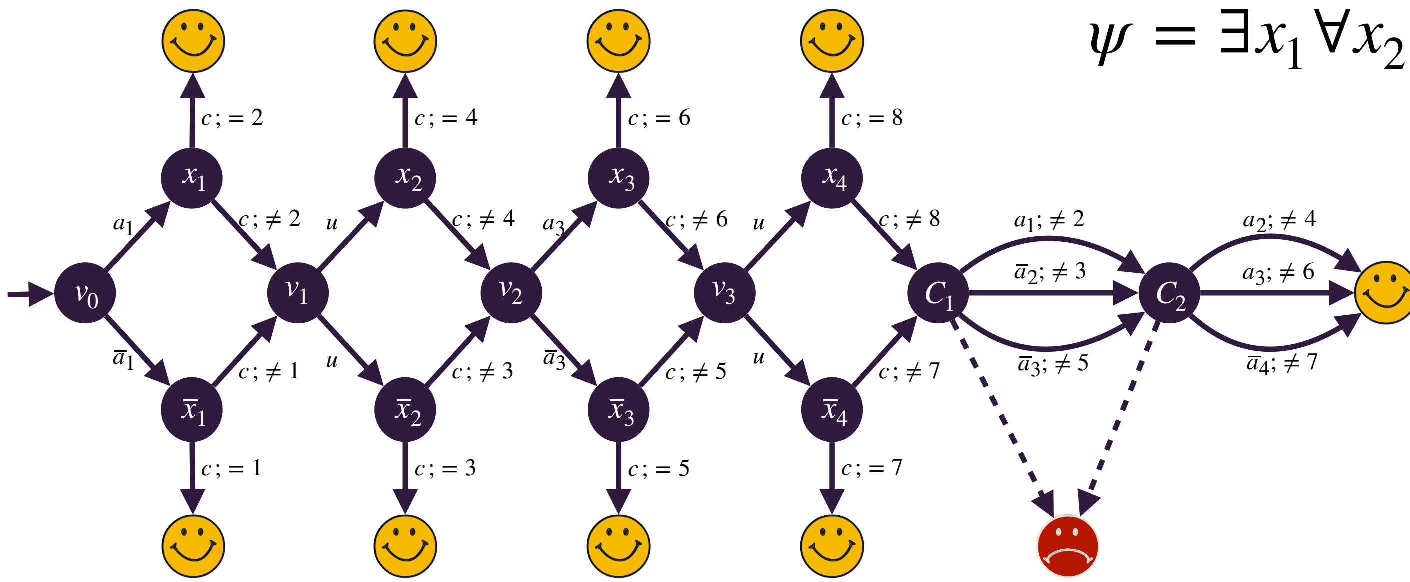
PSPACE-hardness - 2



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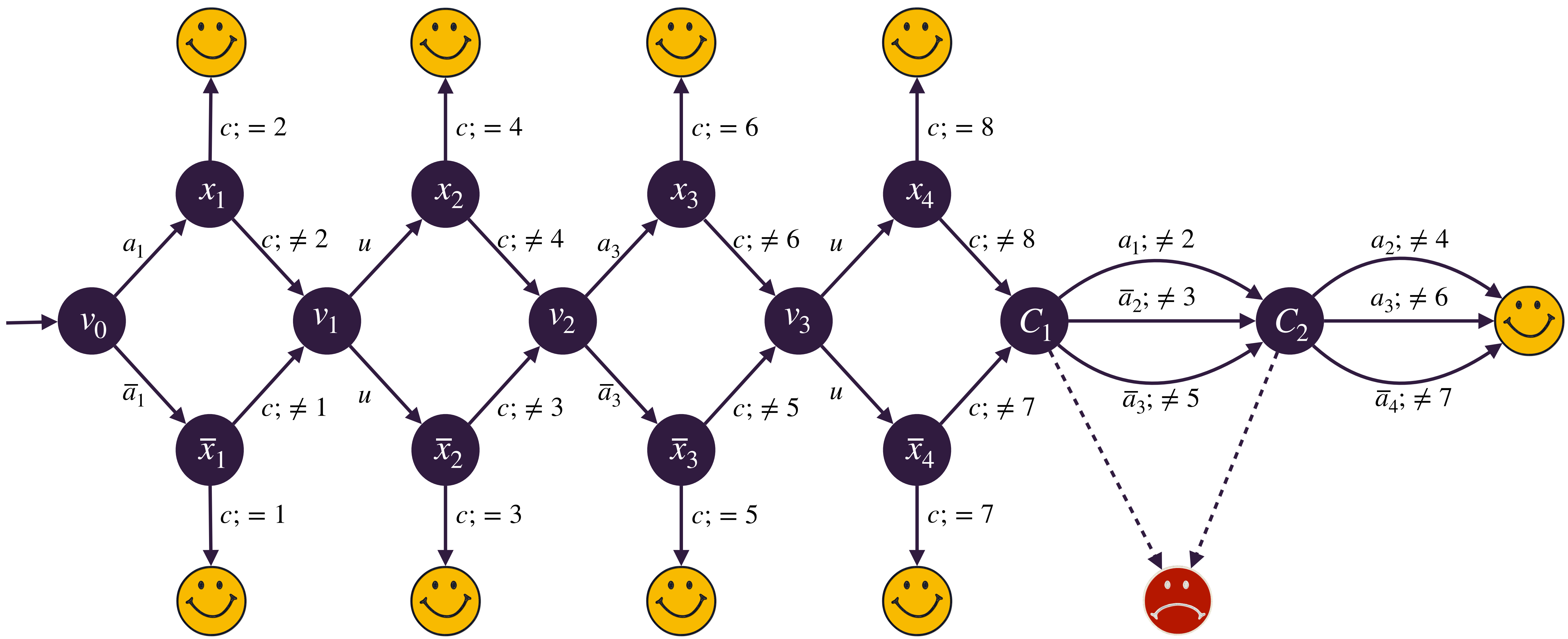
  - Otherwise the game proceeds to v_2
- Etc…
- At C_1, $k \neq 1$, $k \neq 4$ (in the first case above), …. hence Gru can enforce  if and only if the chosen assignment makes the two clauses true

PSPACE-hardness - 3

QSAT formula $\psi = \exists x_1 \forall x_2 \exists x_3 \forall x_4 \cdot (x_1 \vee \neg x_2 \vee \neg x_3) \wedge (x_2 \vee x_3 \vee \neg x_4)$



ψ is true iff Gru has a winning strategy in the above counting game

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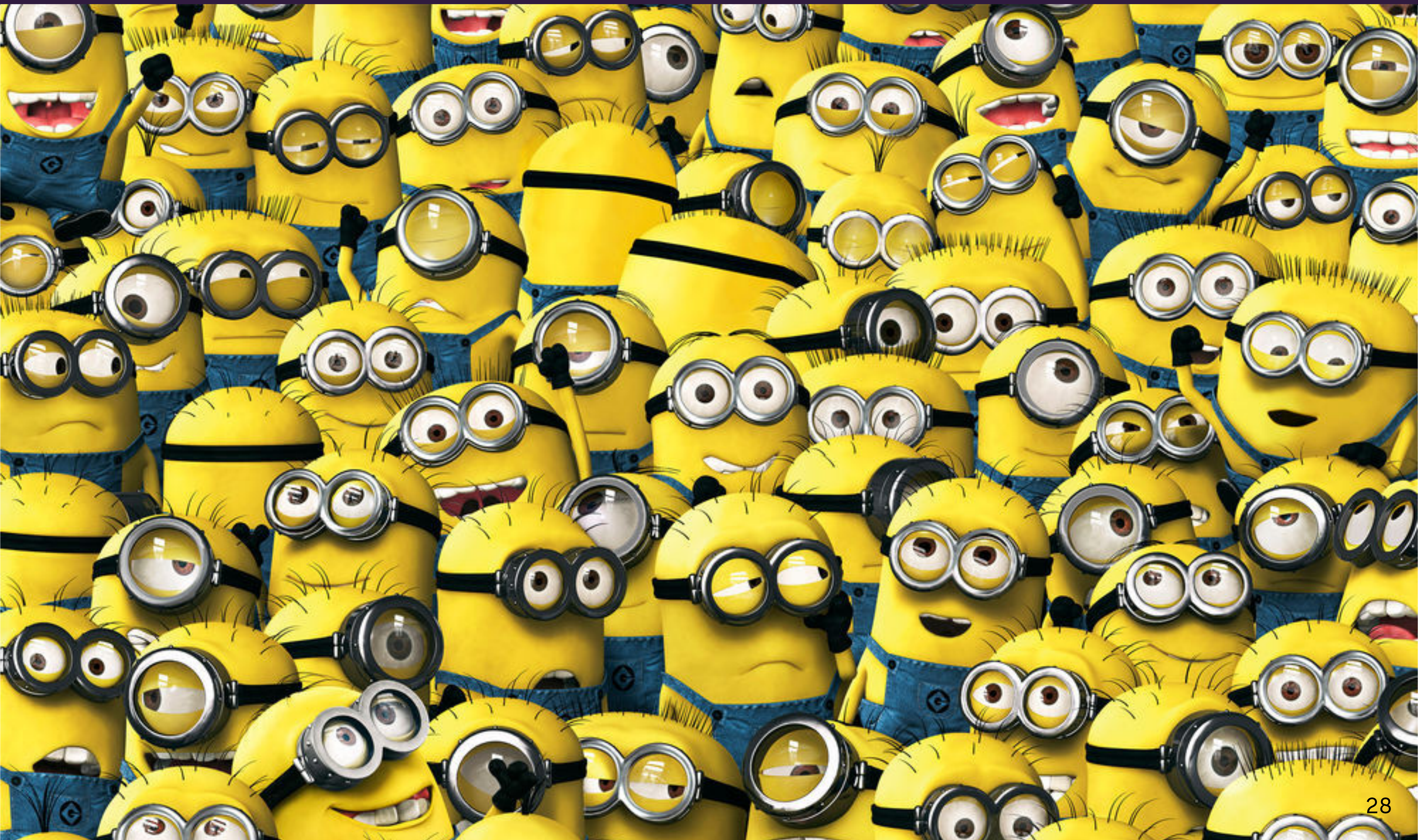
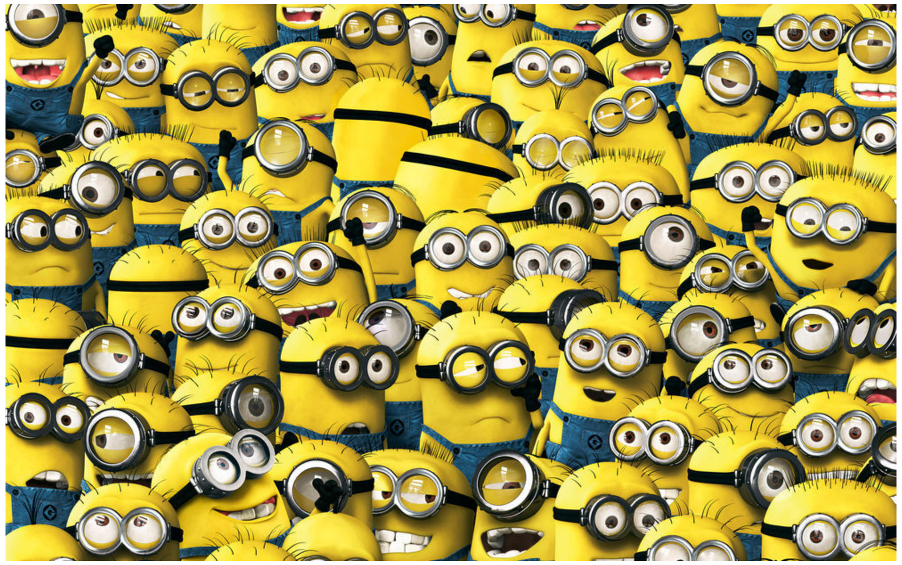
- The previous approach yielding the PSPACE upper bound applies to many other Boolean objectives, as long as solving the corresponding standard games can be achieved in PSPACE
- What about more involved quantitative objectives/payoffs?
- We believe the approach can be extended to « structured » infinite-state systems (e.g. pushdown systems)

The coalition problem

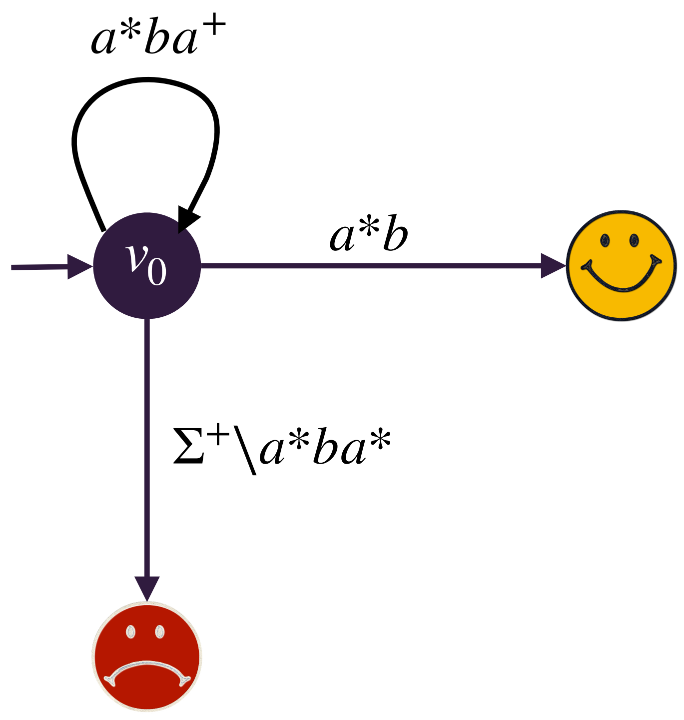


The coalition problem

- Input: parameterized game $G = (V, \delta)$ and linear property φ
- Question: does there exist $(\sigma_i)_{i \geq 1}$ such that for every k, for every $\rho \in \mathsf{Out}\big((\sigma_i)_{1 \leq i \leq k}\big)$, $\rho \models \varphi$?



An intriguing example



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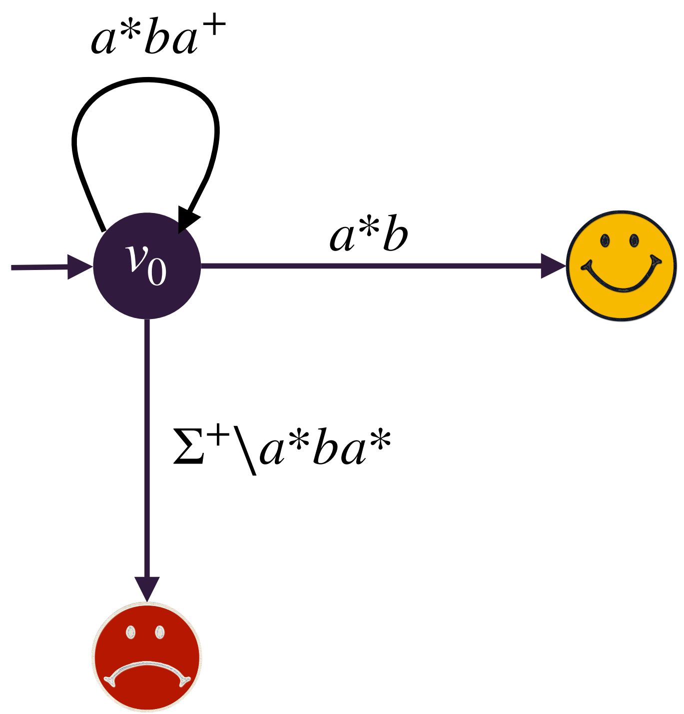

A winning coalition strategy

- At round i:
  - Player i plays b
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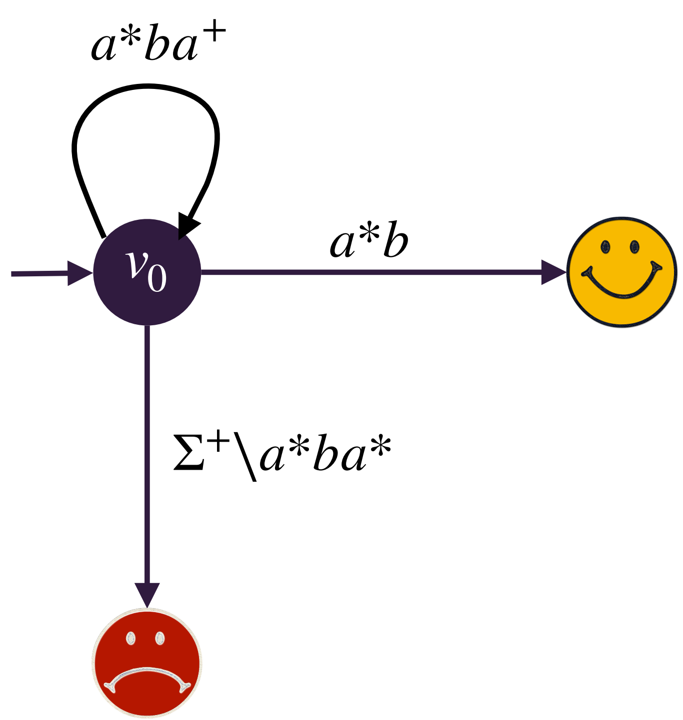

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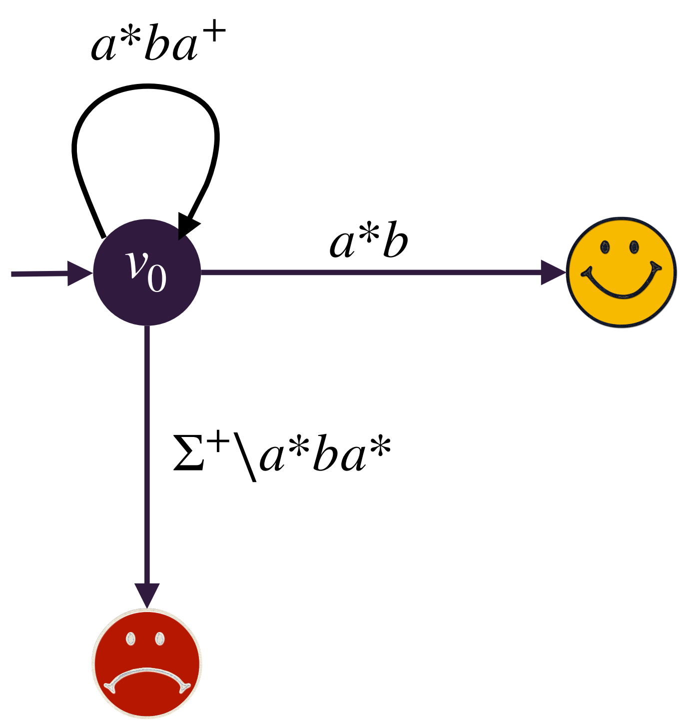
If $k = 1$: $v_0 \xrightarrow{b}$ ☺

If $k = 2$: $v_0 \xrightarrow{ba} v_0 \xrightarrow{ab}$ ☺

If $k = 3$: $v_0 \xrightarrow{baa} v_0 \xrightarrow{aba} v_0 \xrightarrow{aab}$ ☺

⋮

An intriguing example



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- At round i:
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- At round i, coalition plays $a^{i-1}ba^{\omega}$

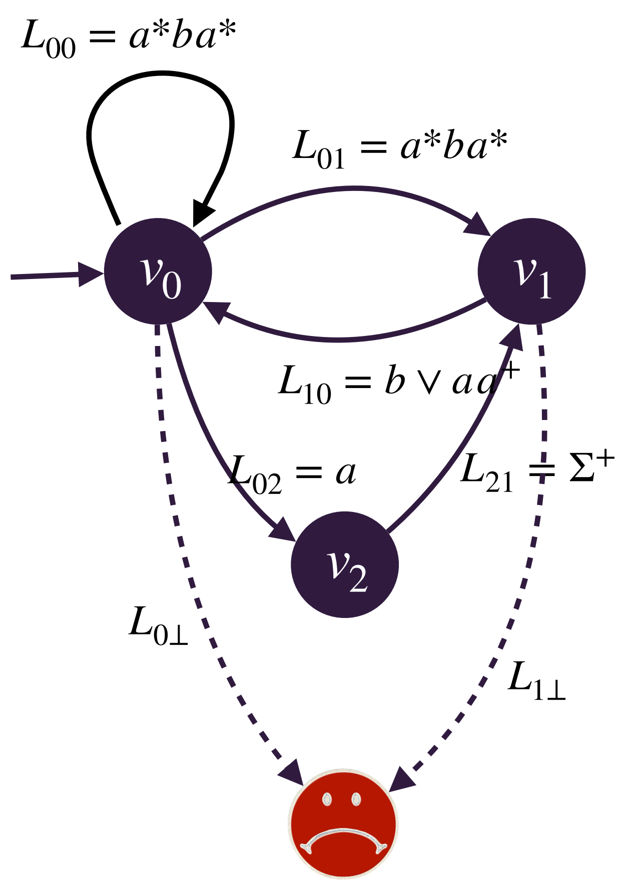
If $k = 1$: $v_0 \xrightarrow{b}$ ☺

If $k = 2$: $v_0 \xrightarrow{ba} v_0 \xrightarrow{ab}$ ☺

If $k = 3$: $v_0 \xrightarrow{baa} v_0 \xrightarrow{aba} v_0 \xrightarrow{aab}$ ☺

$\vdots$

Tree unfolding



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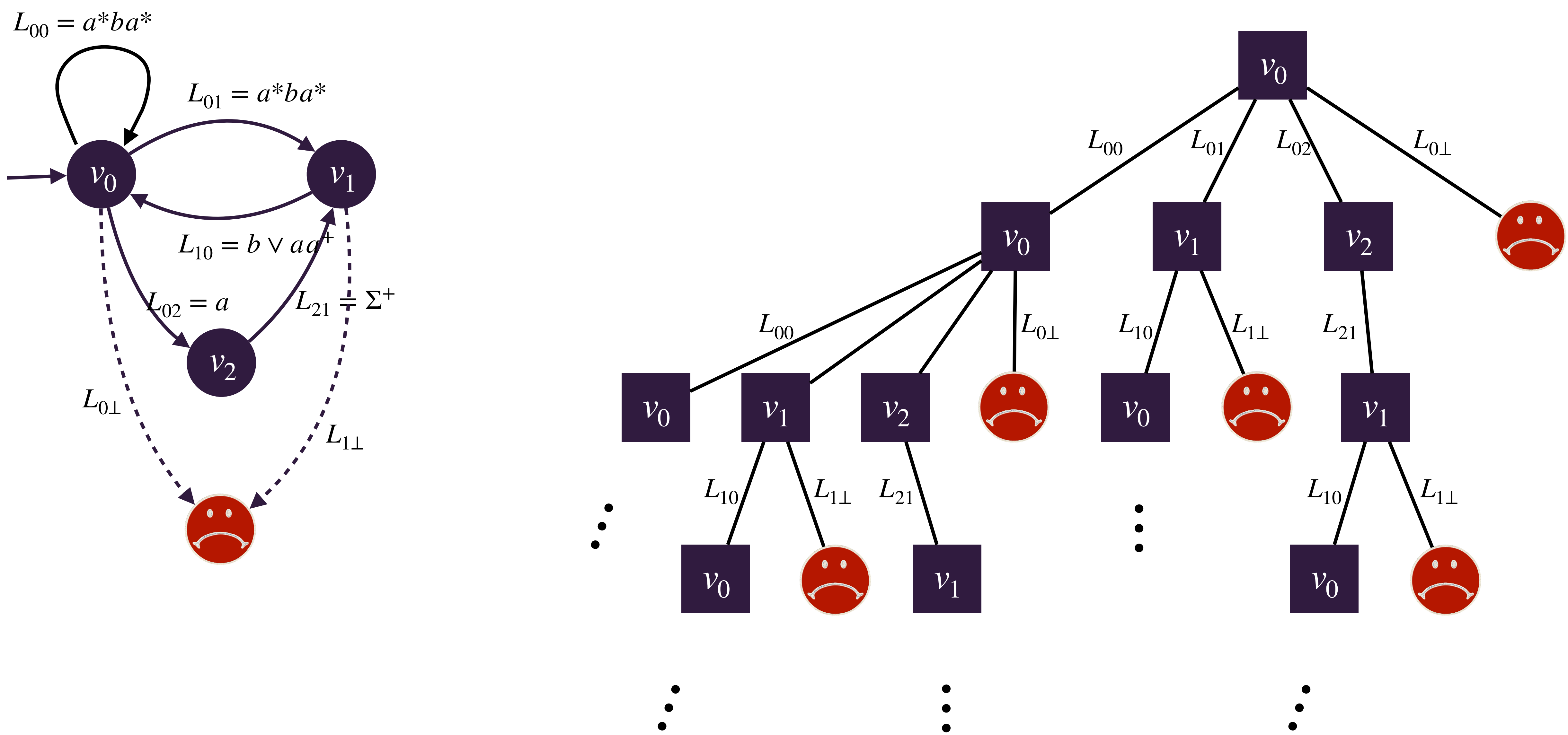

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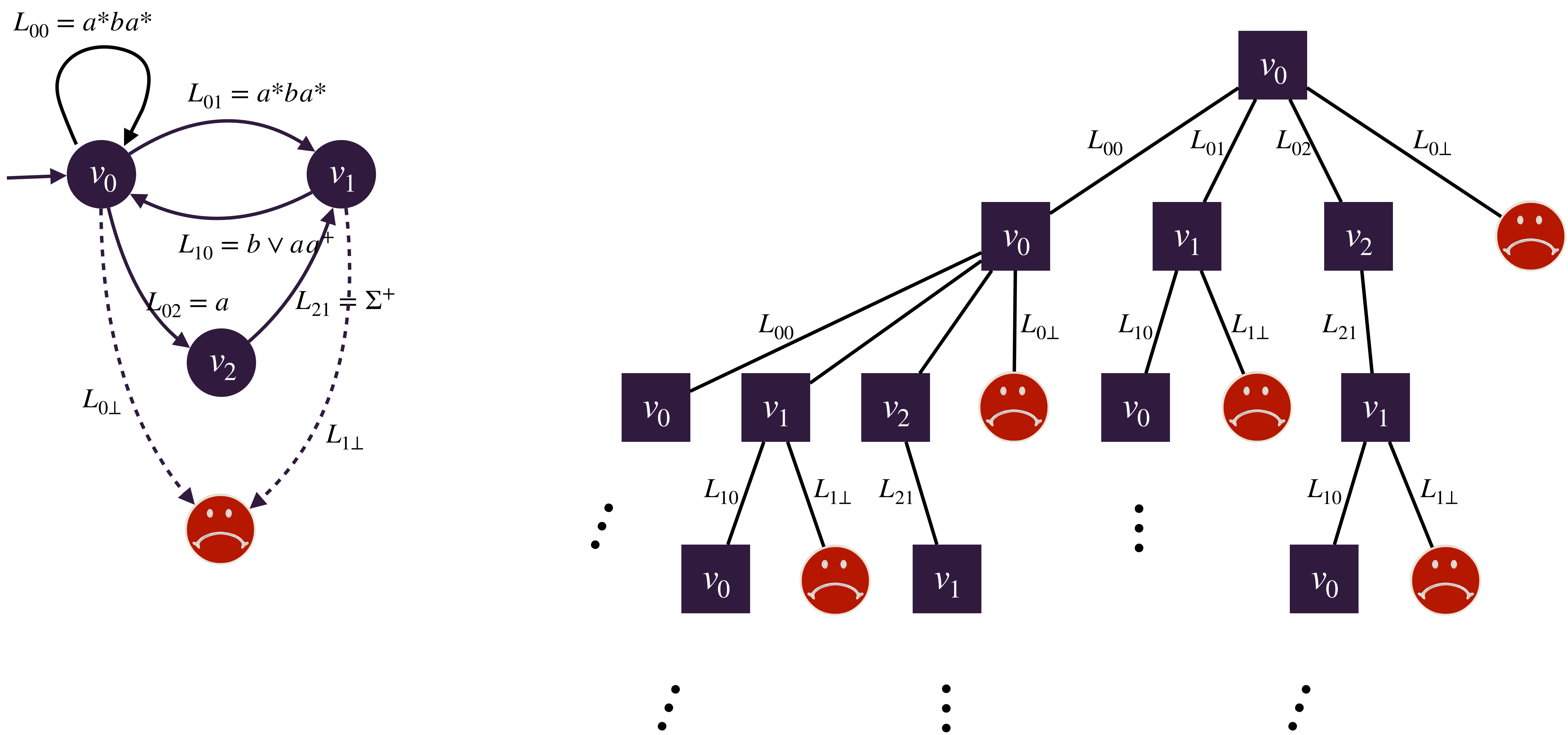
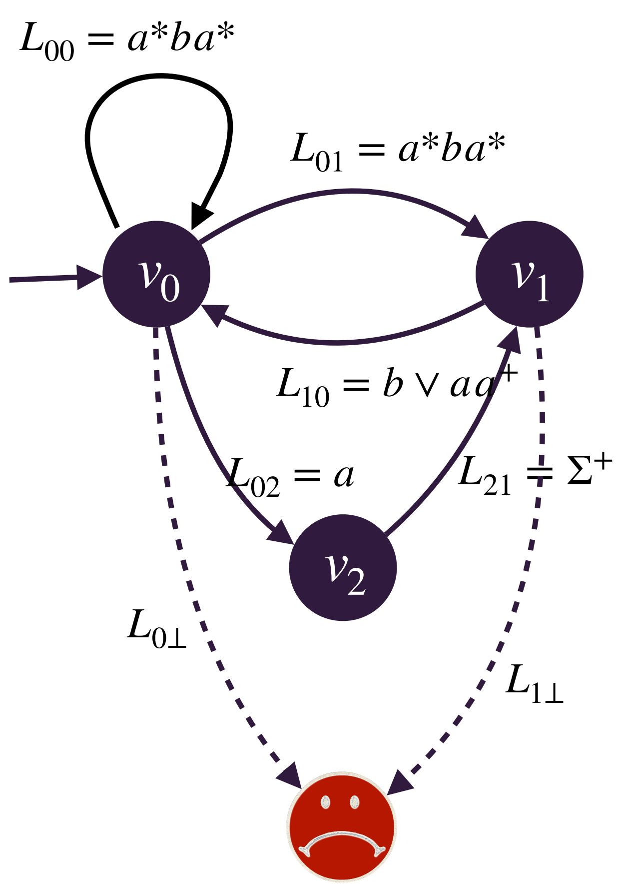


There is a winning coalition strategy in the game iff there is a winning coalition strategy in the unfolding

Tree unfolding — Safety case



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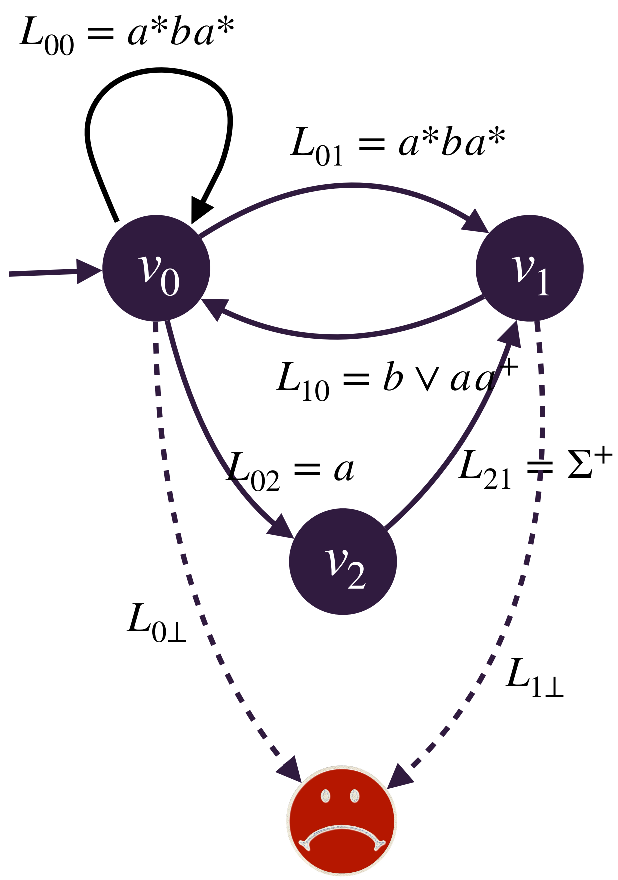
For a safety condition: the unfolding can be pruned



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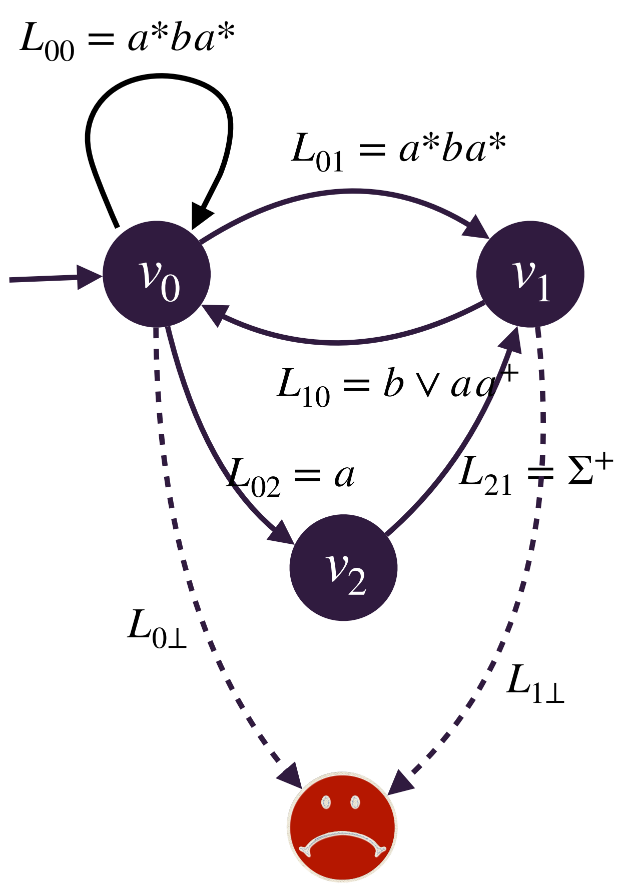

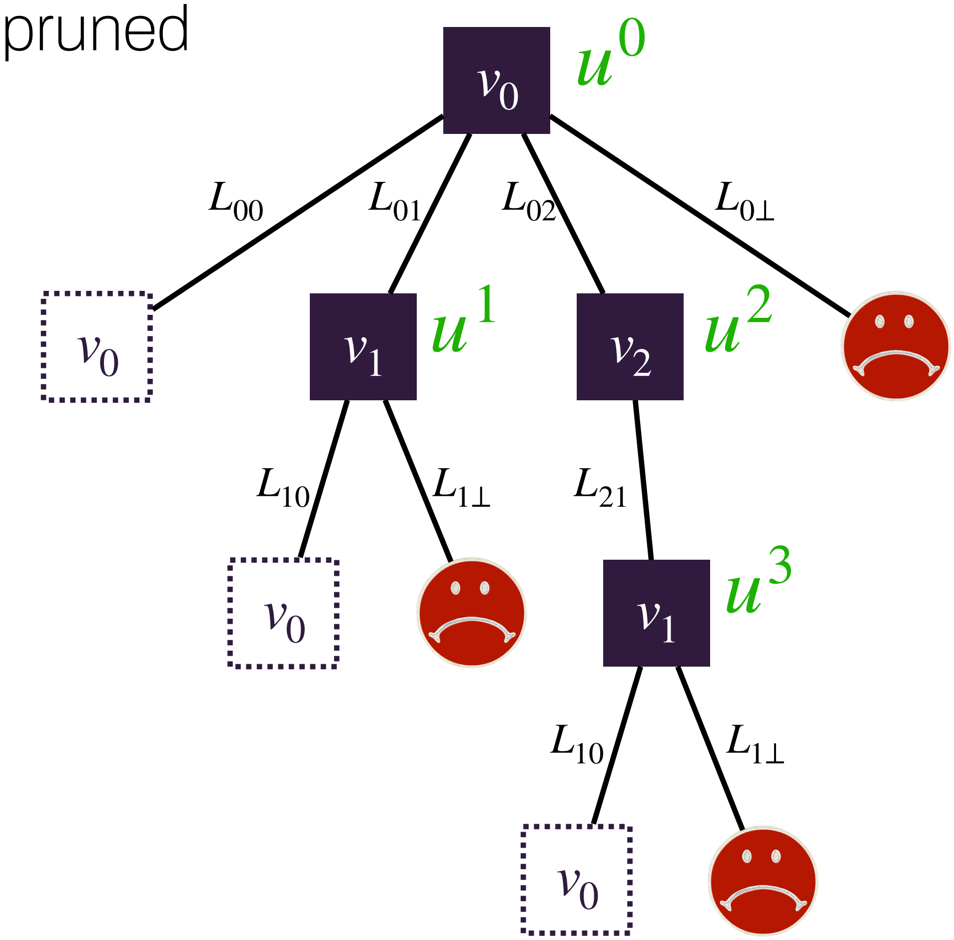

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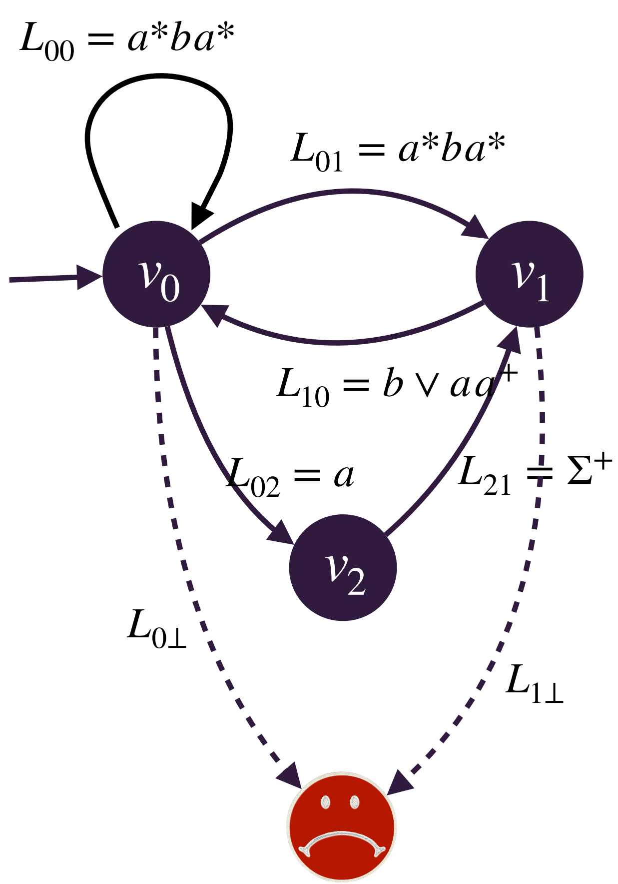

Possible solution:

- $u^0 = aba^\omega$
- $u^1 = a^\omega$
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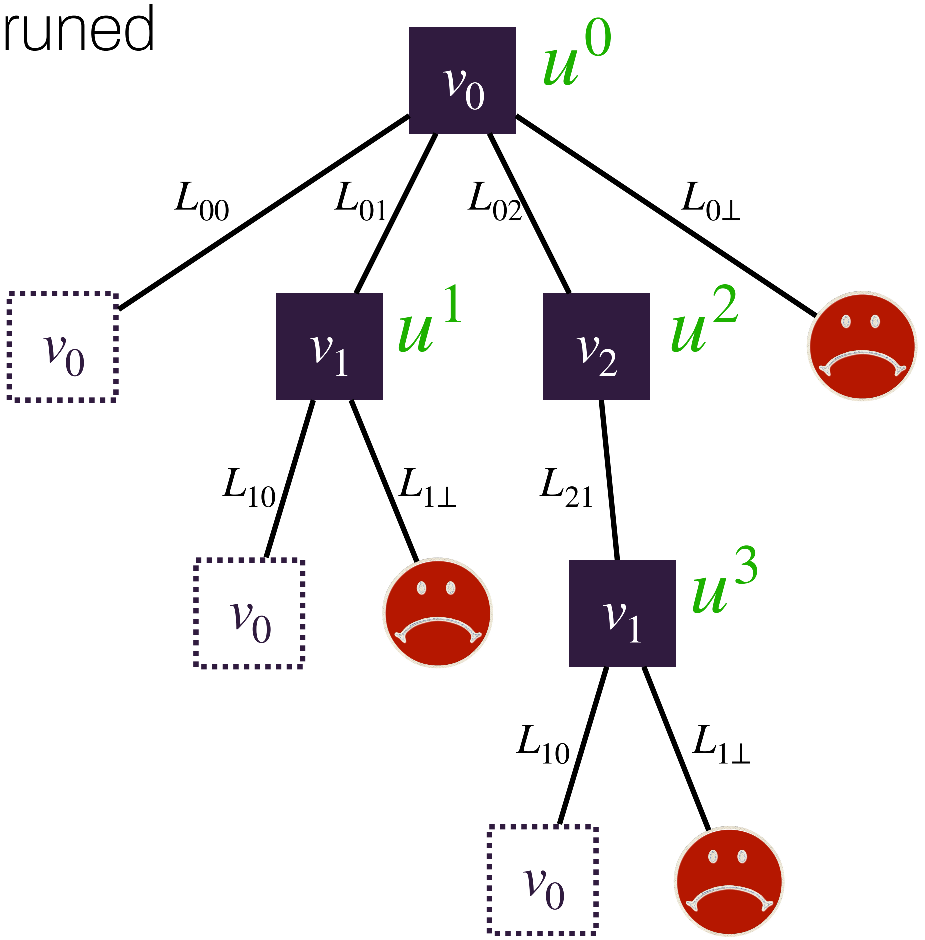
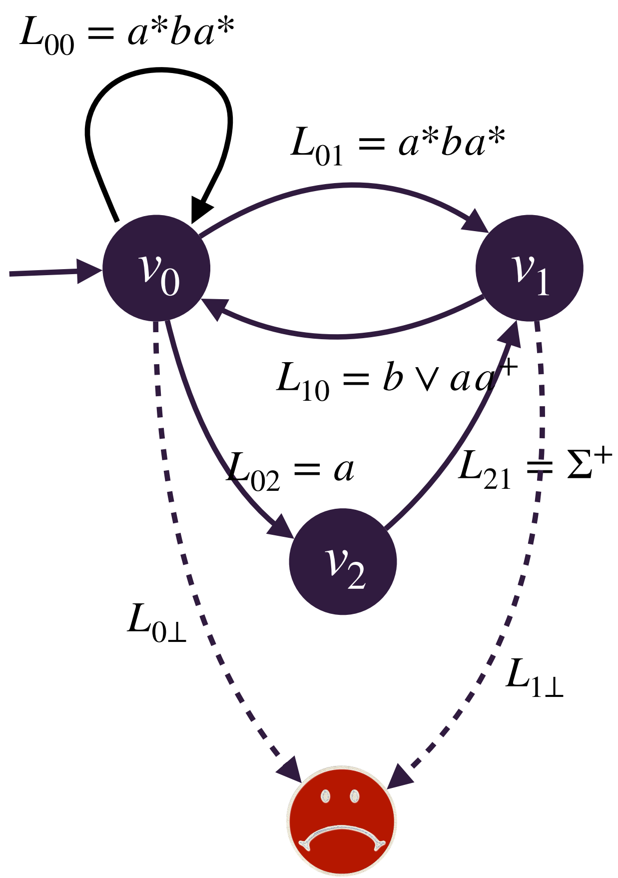


Tree unfolding — Safety case

For a safety condition: the unfolding can be pruned



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▸ $k = 1$: $v_0 \xrightarrow{a} v_2 \xrightarrow{a} v_1 \xrightarrow{b} v_0 \to \ldots$
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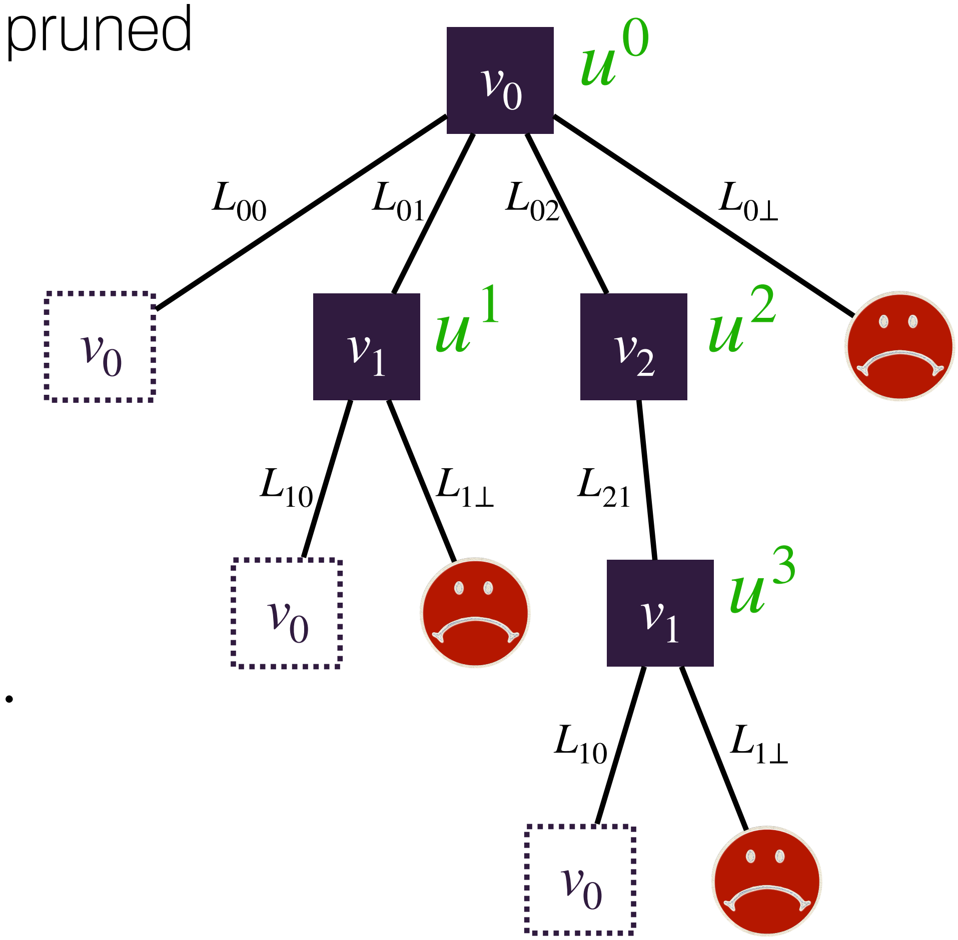

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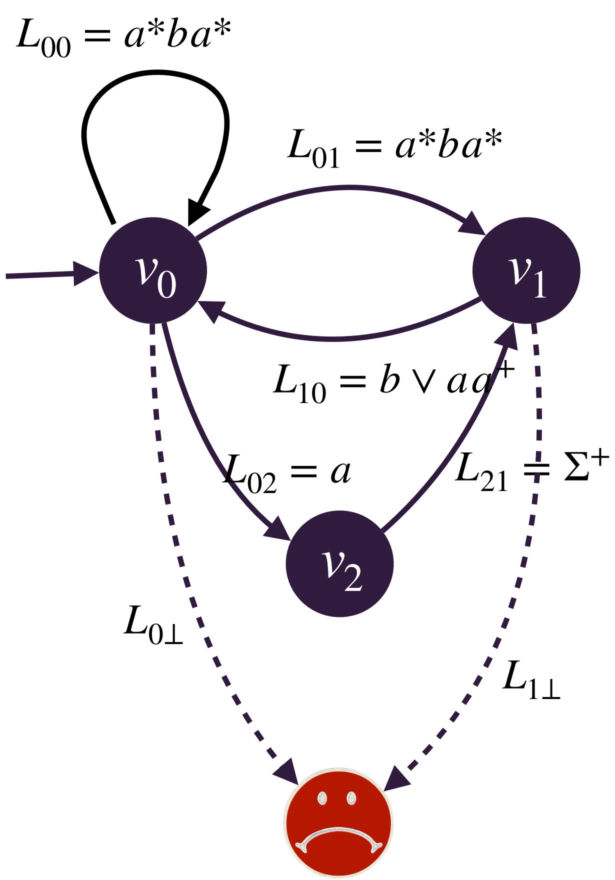

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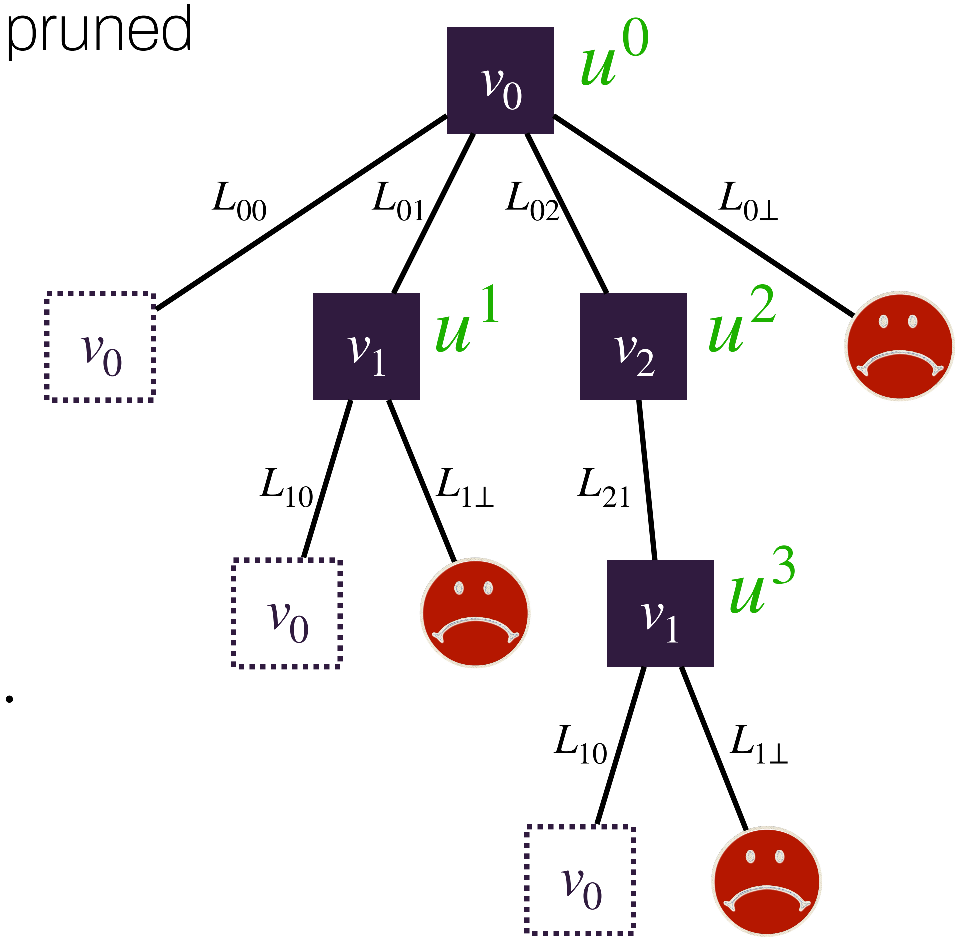

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- $k > 1$: $v_0 \xrightarrow{aba^{k-2}} v_1 \xrightarrow{a^k} v_0 \rightarrow \ldots$
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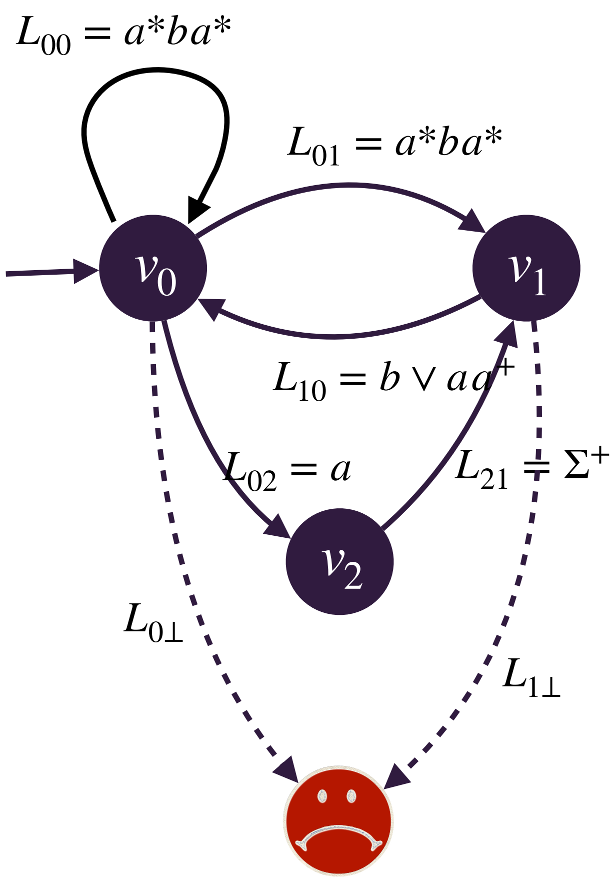

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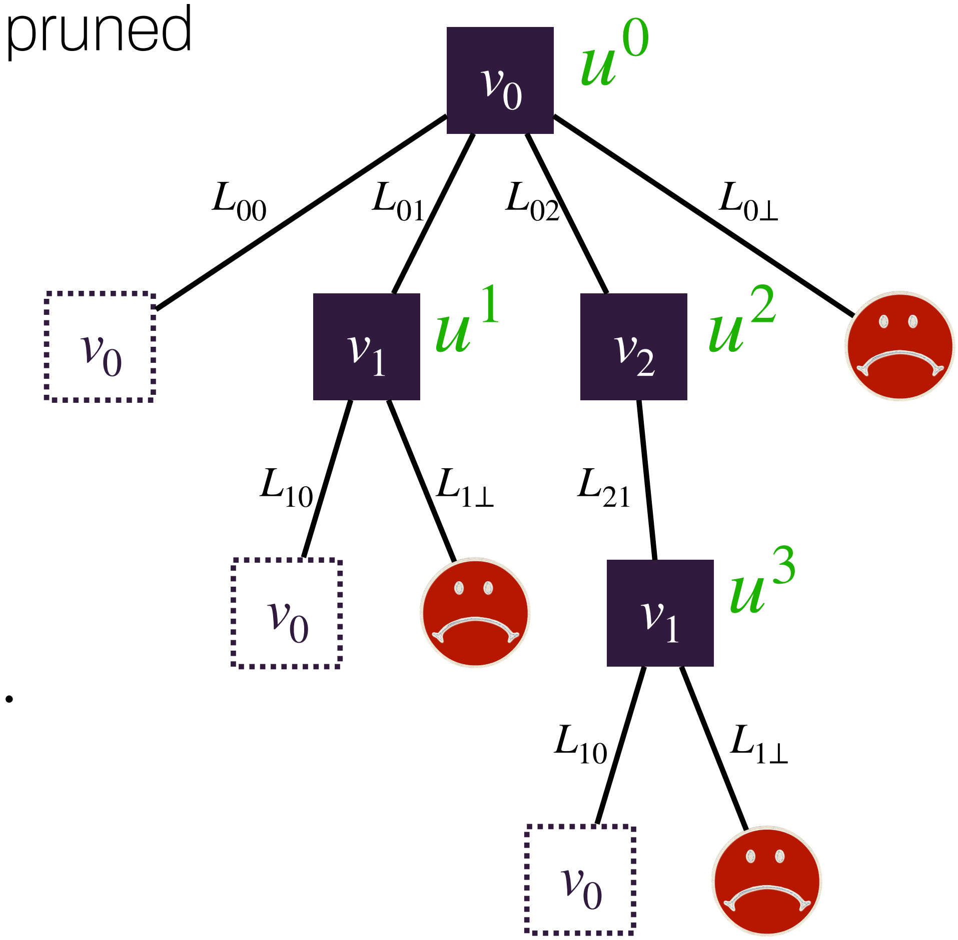

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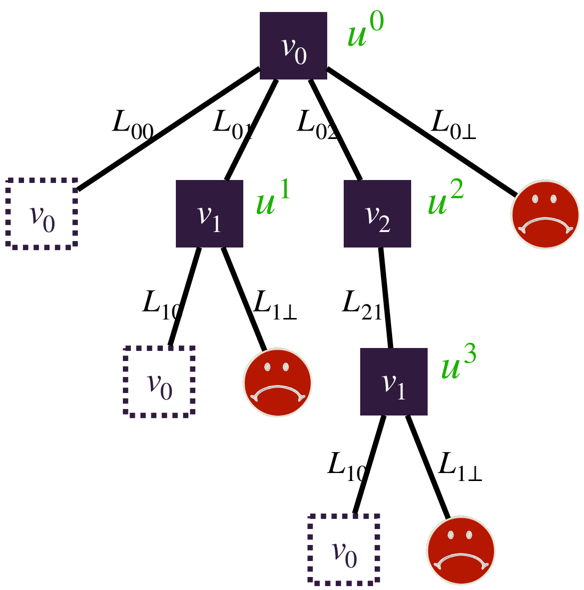
There is a winning coalition strategy in the unfolding iff there are infinite words $(u^i)_i$ s.t. for every $k \geq 1$, playing $u^i_{\leq k}$ at each internal node ensures avoiding ☹

Construction of a finite automaton — 1



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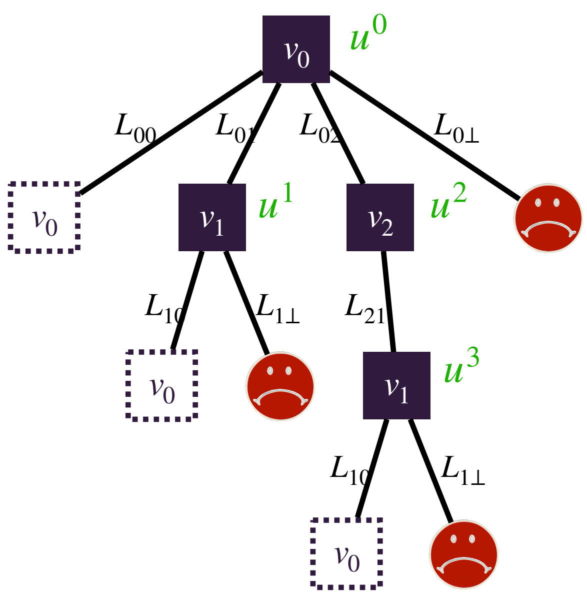
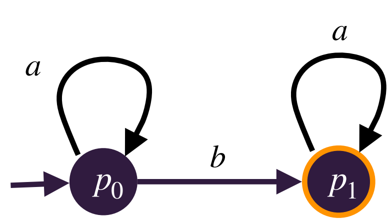
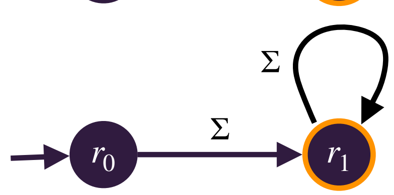


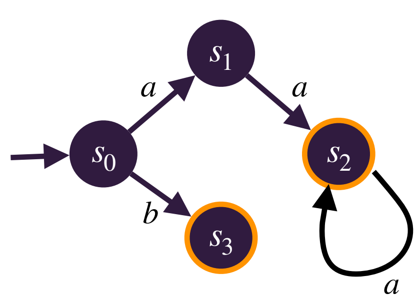


$L_{00} = L_{01}$

L_{02}



L_{21}



L_{10}

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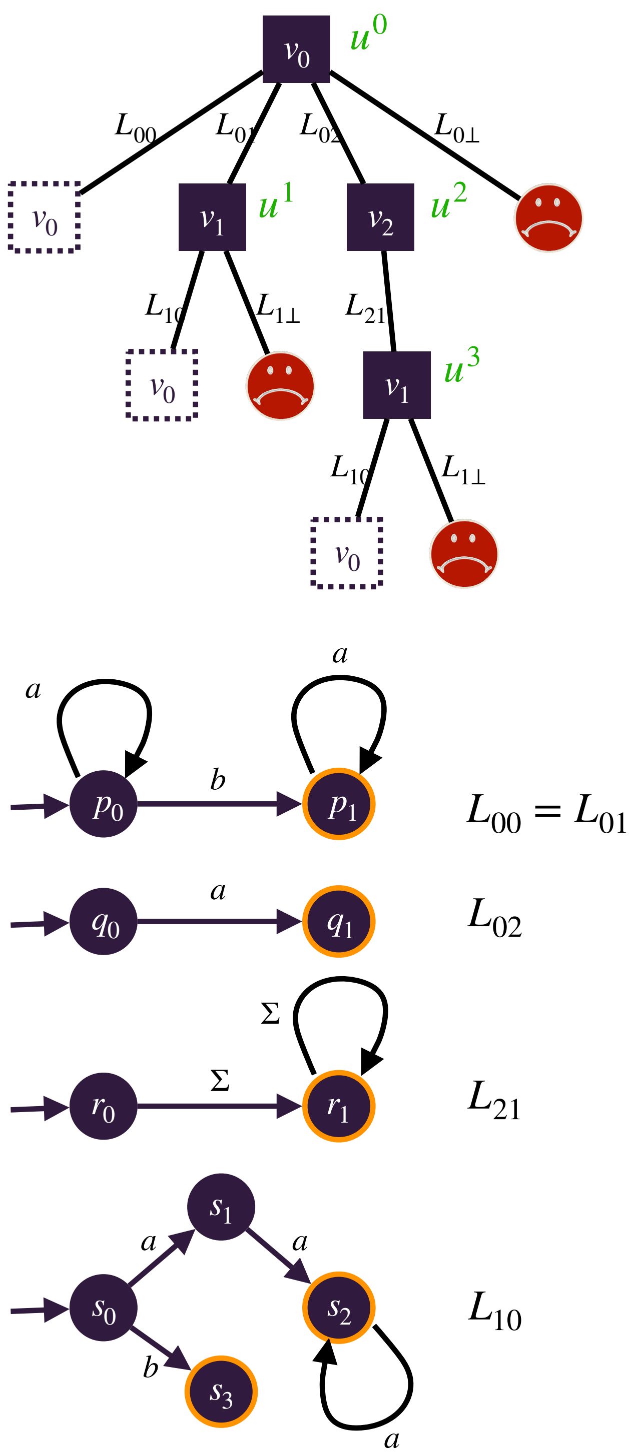

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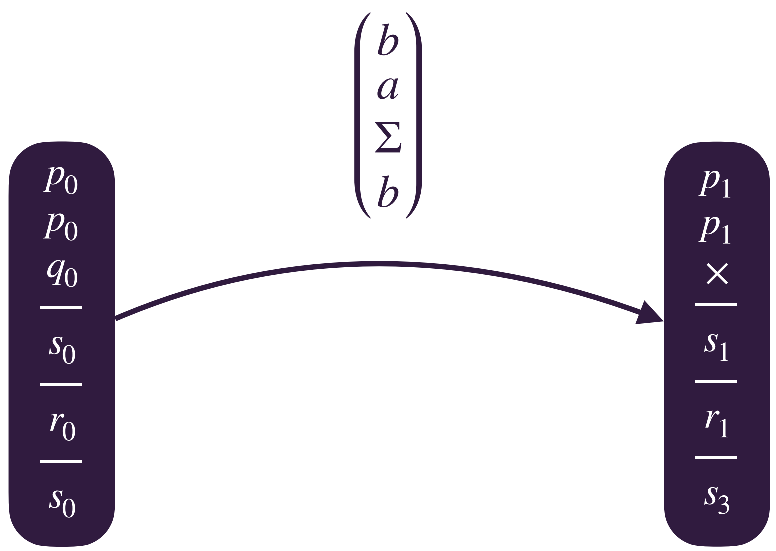

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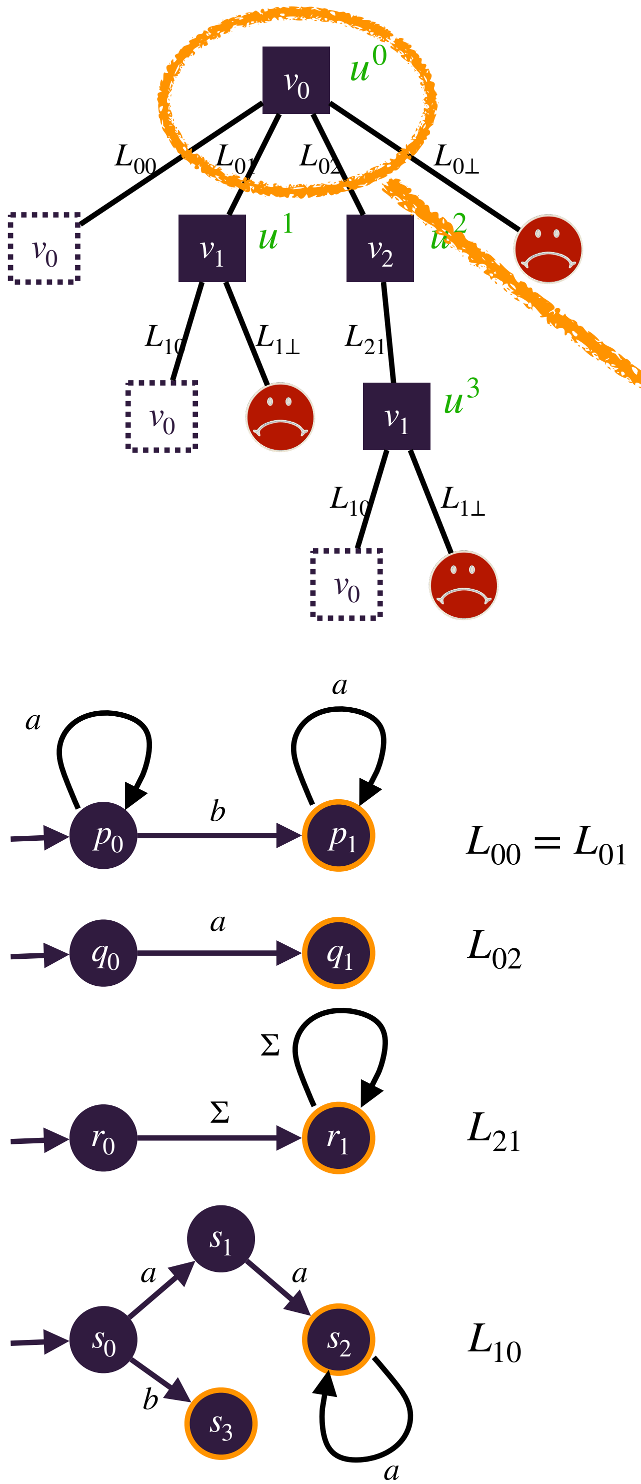

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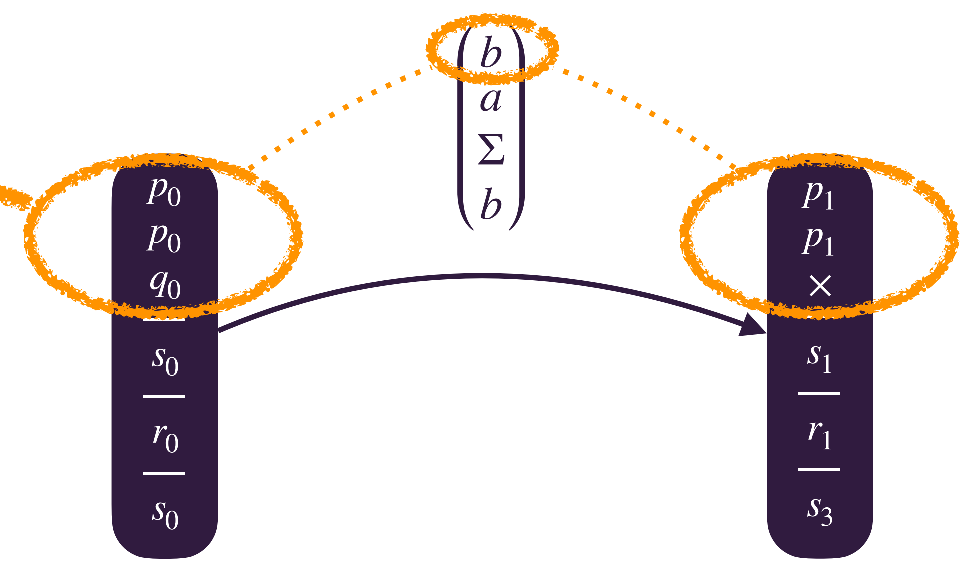

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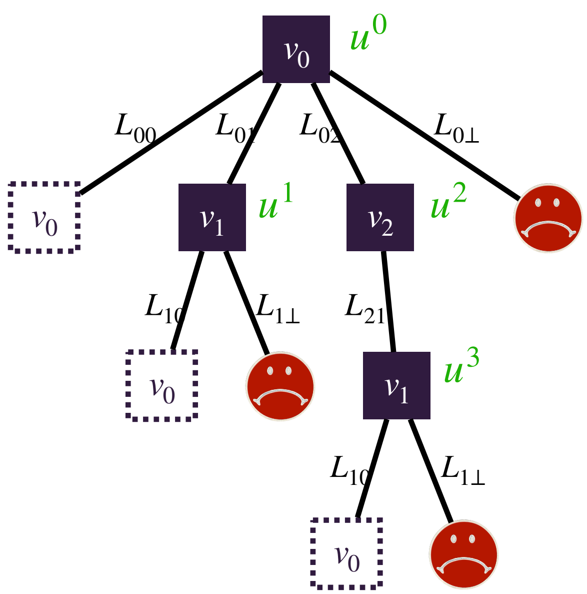

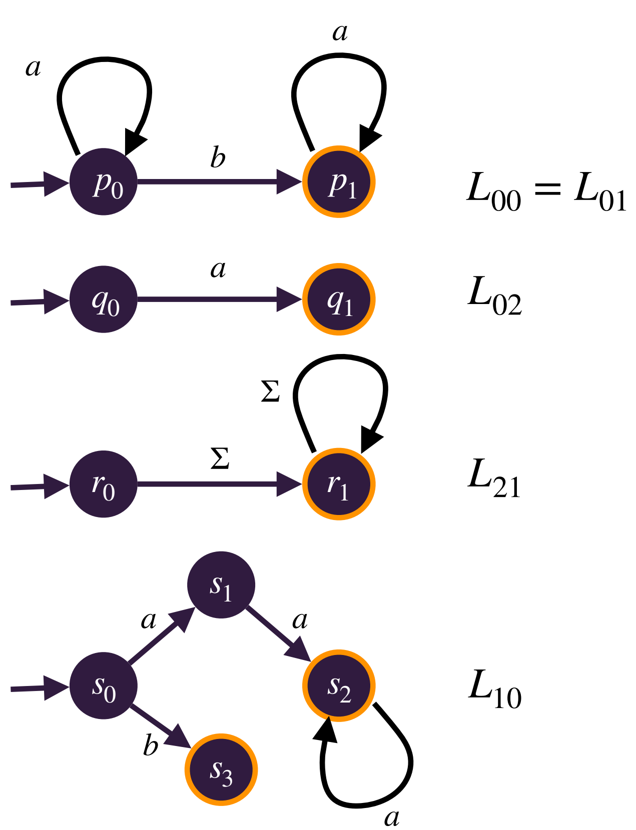

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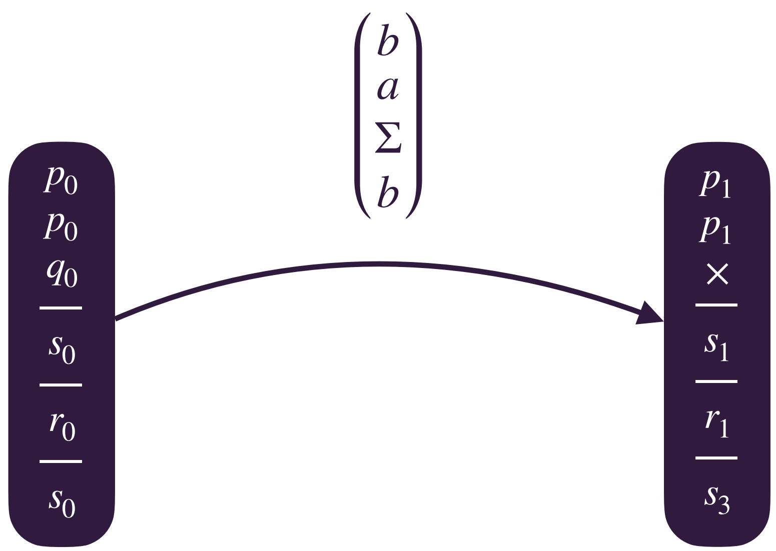

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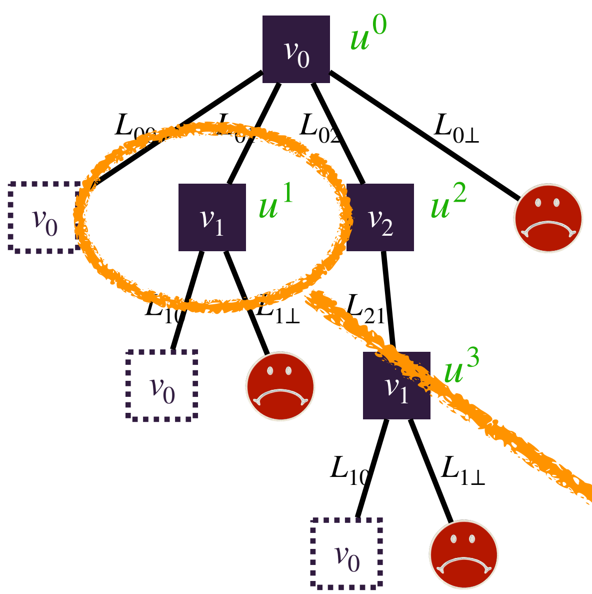

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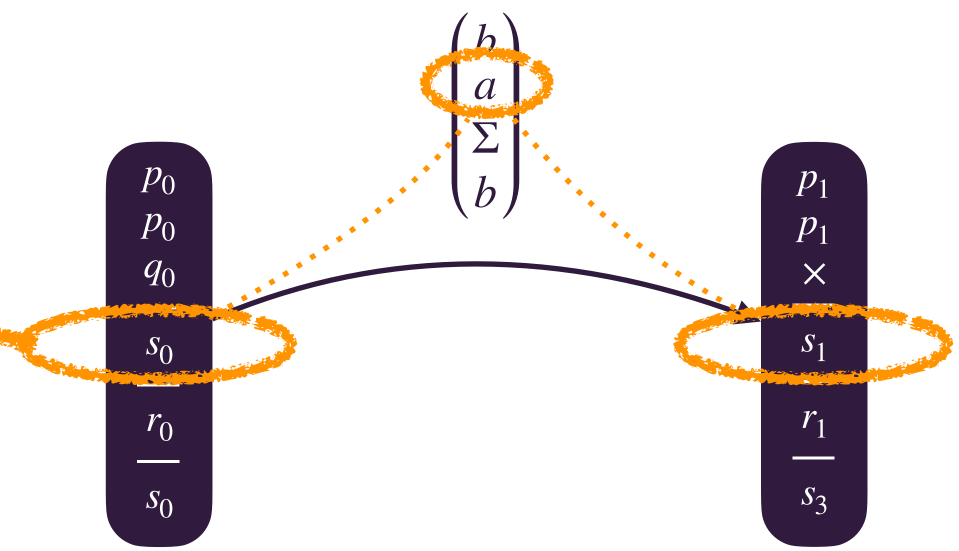

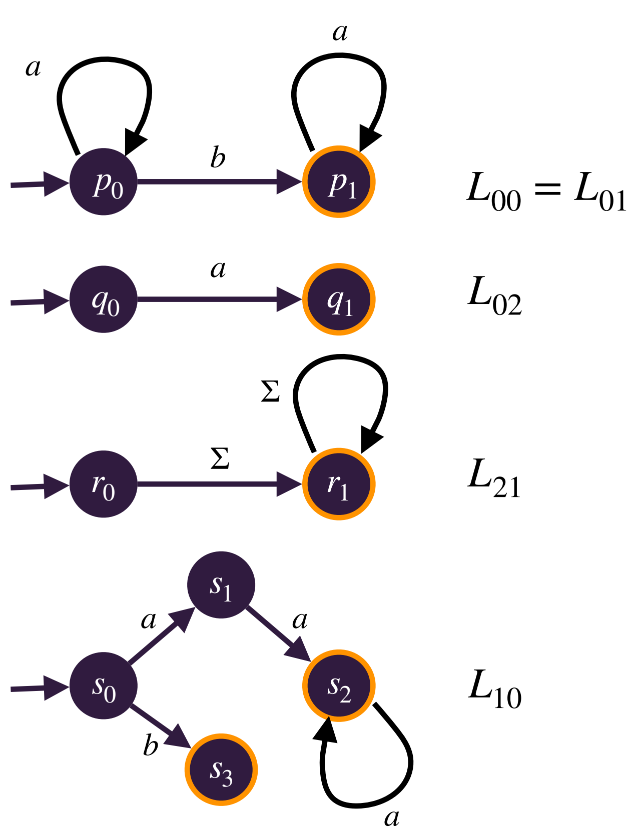

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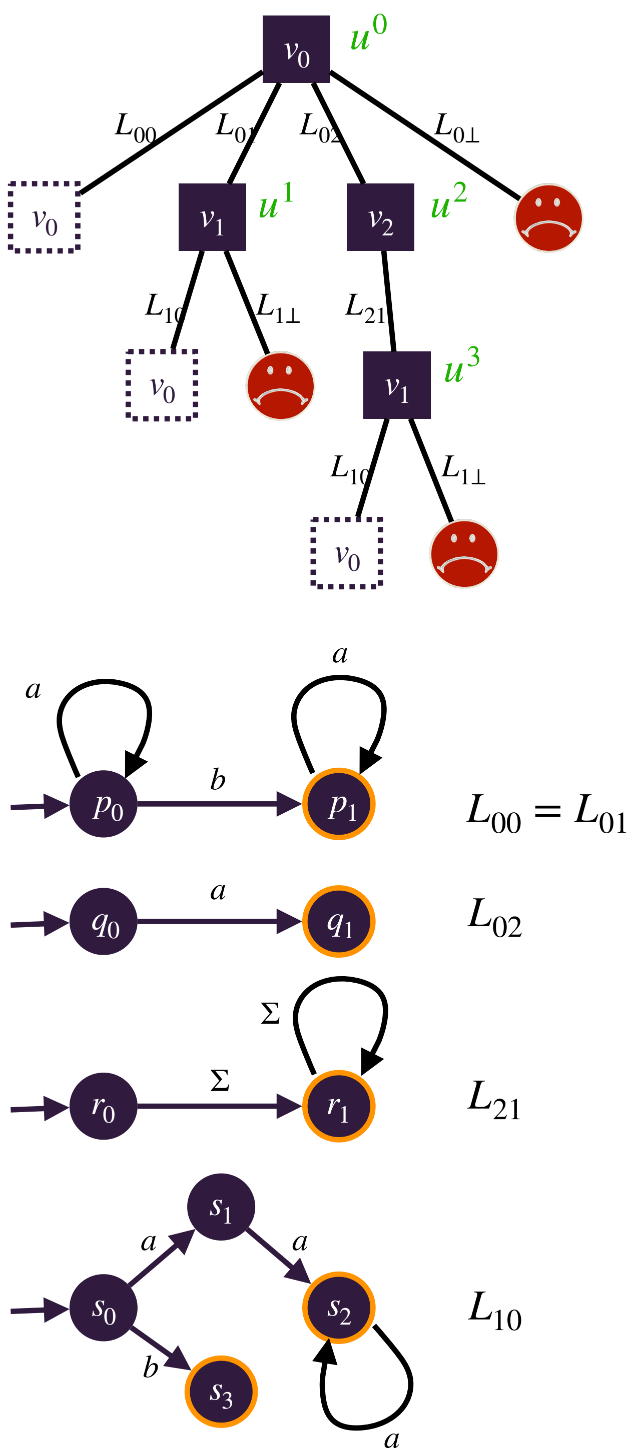

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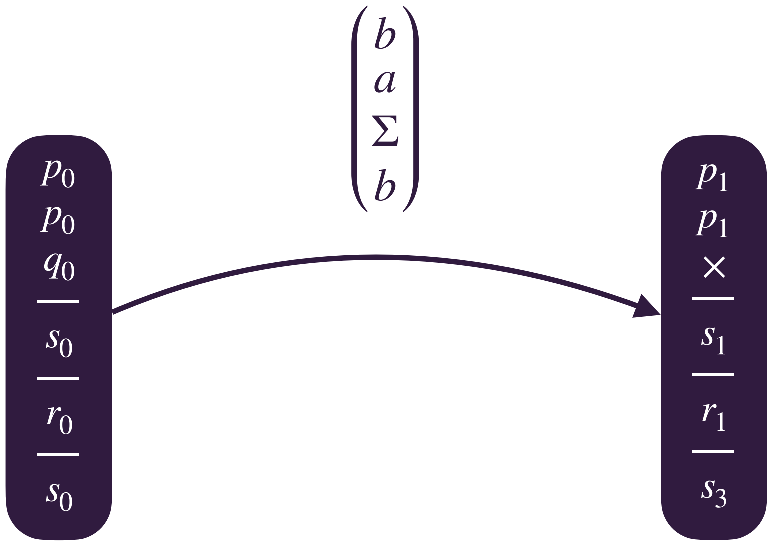

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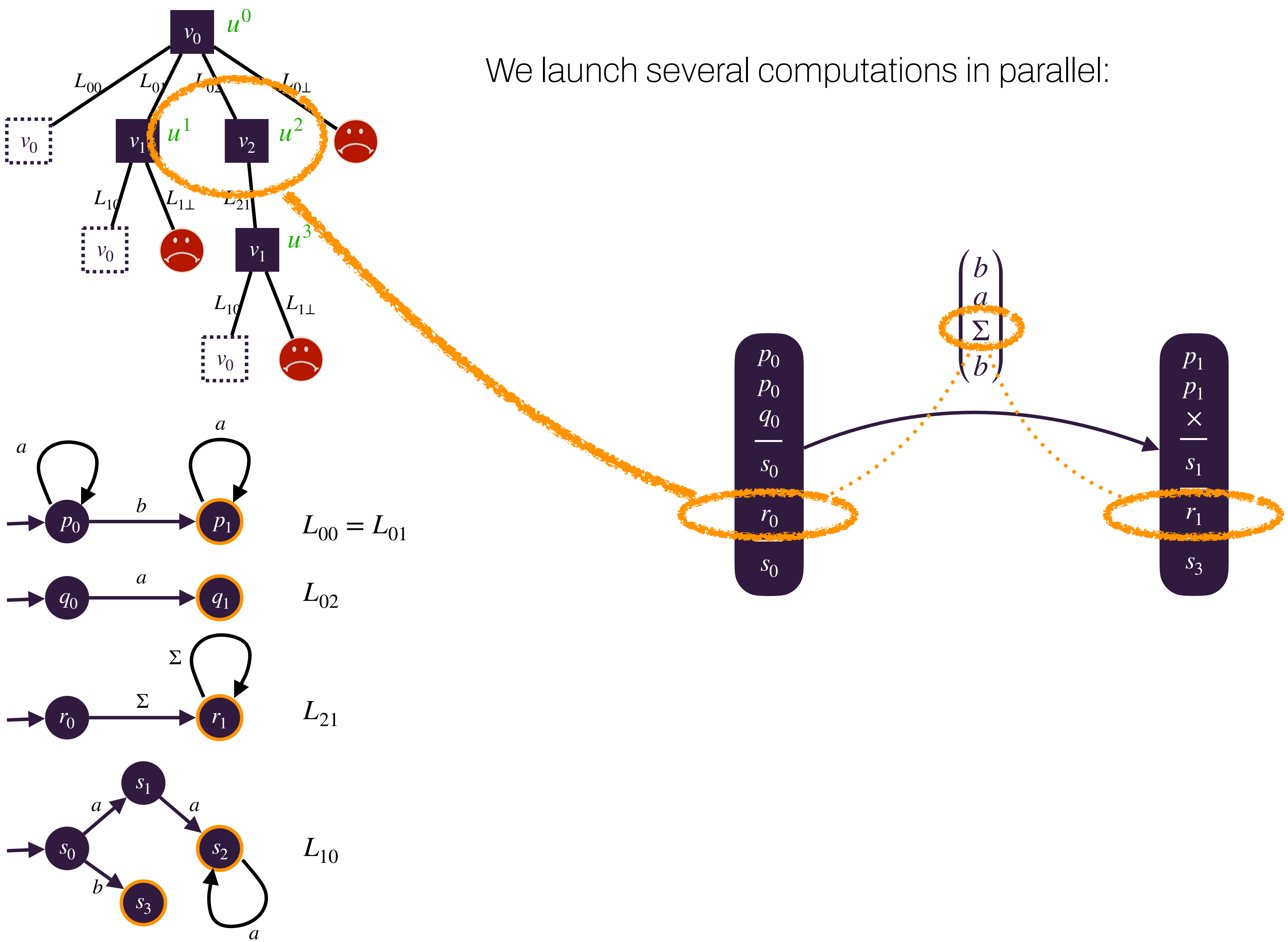

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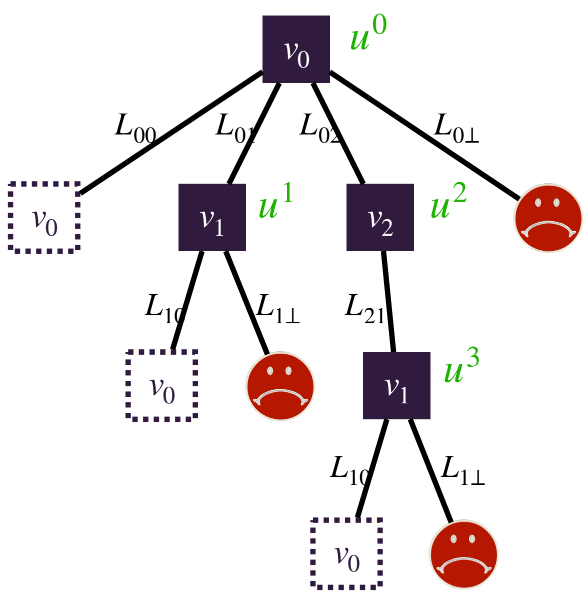

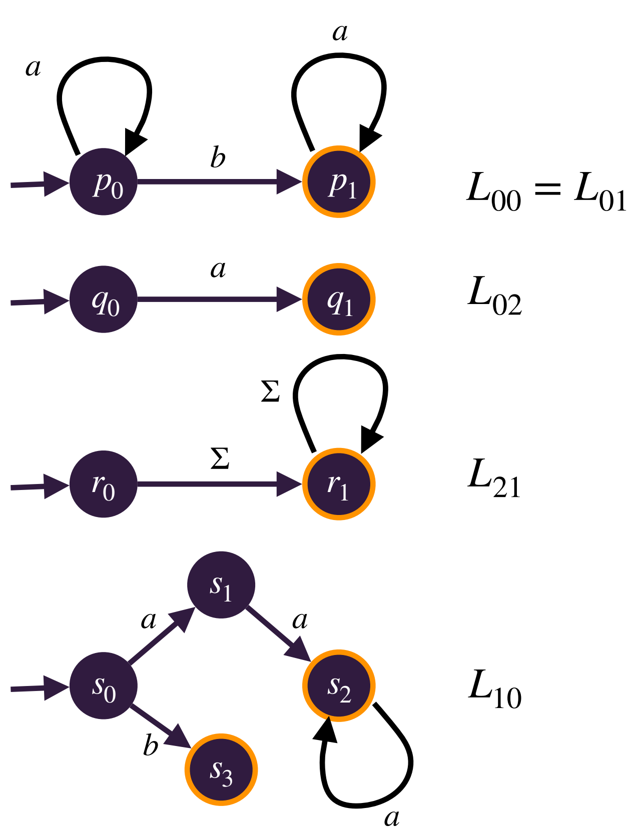

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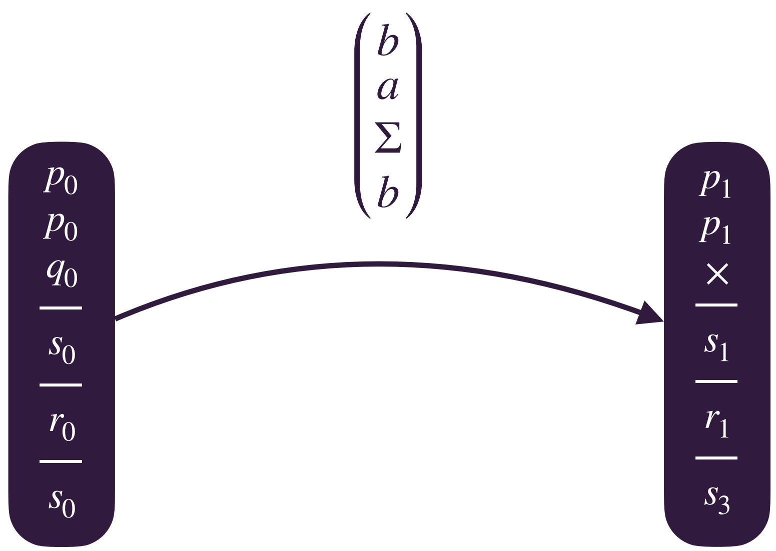

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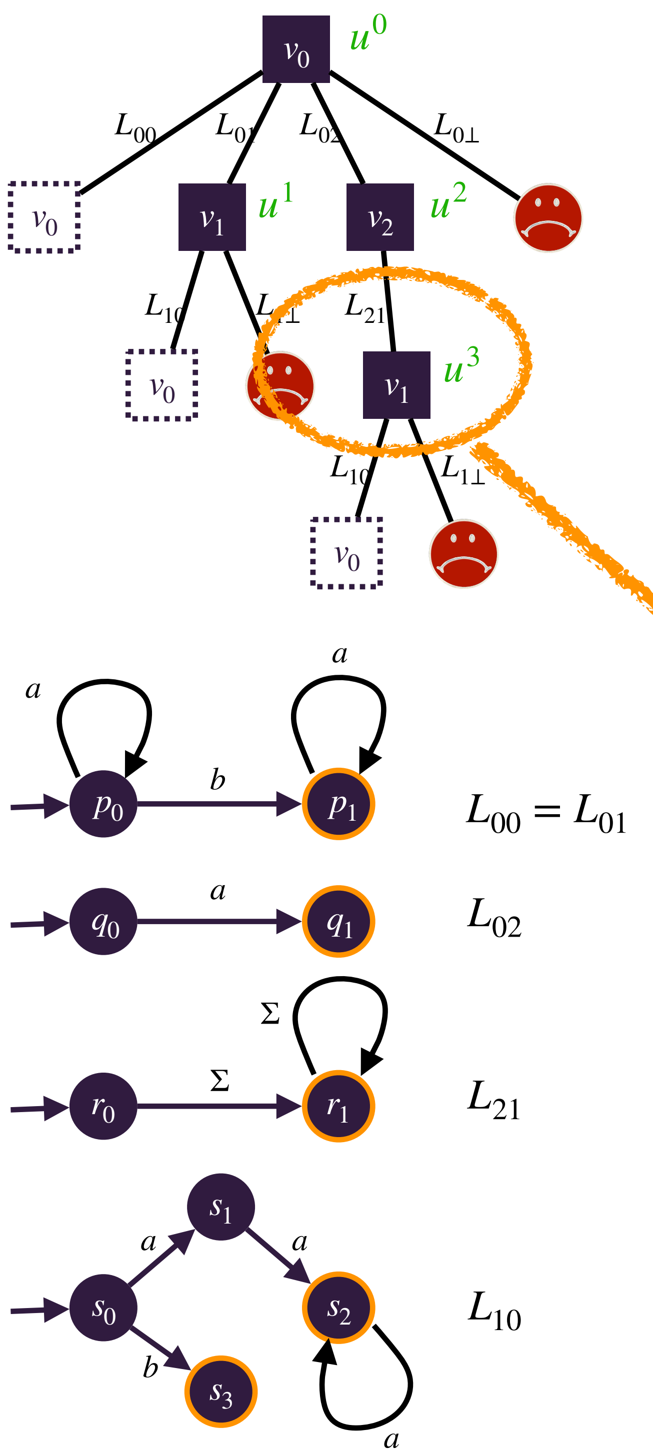

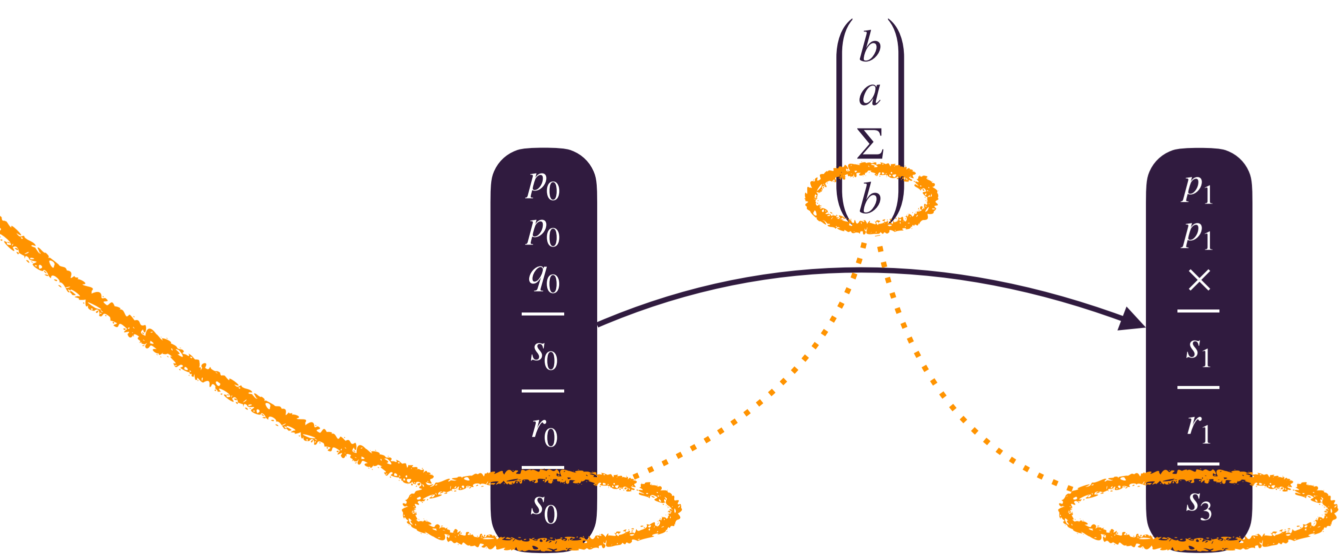

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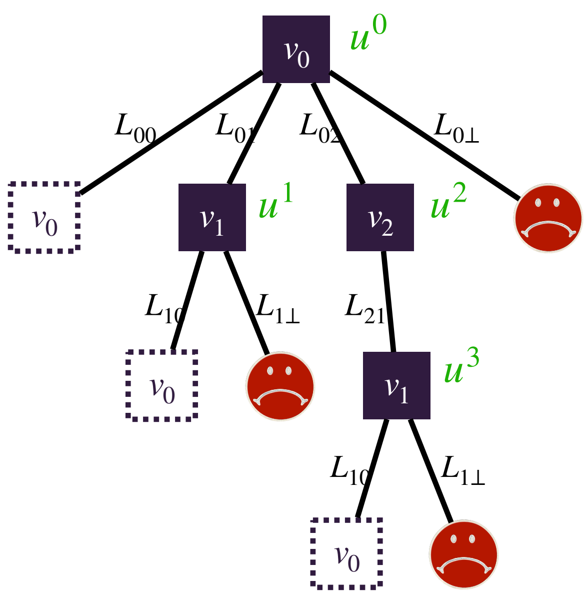

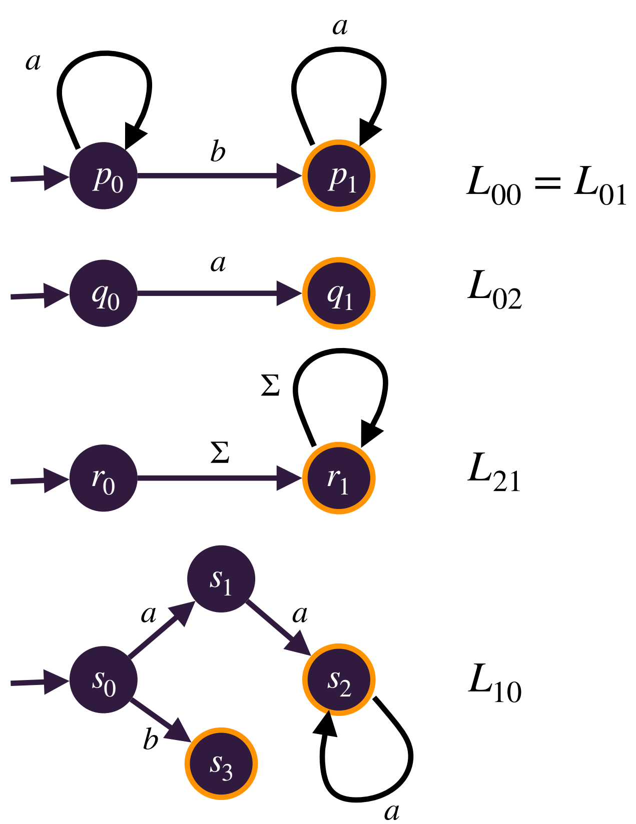

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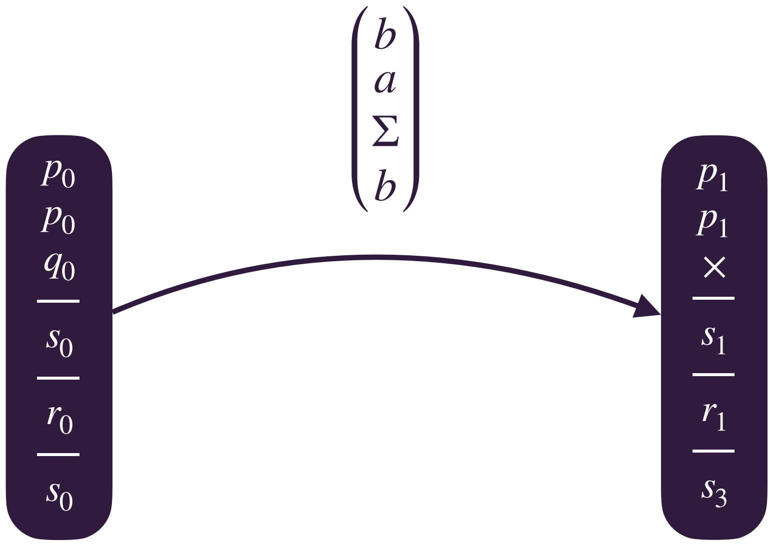

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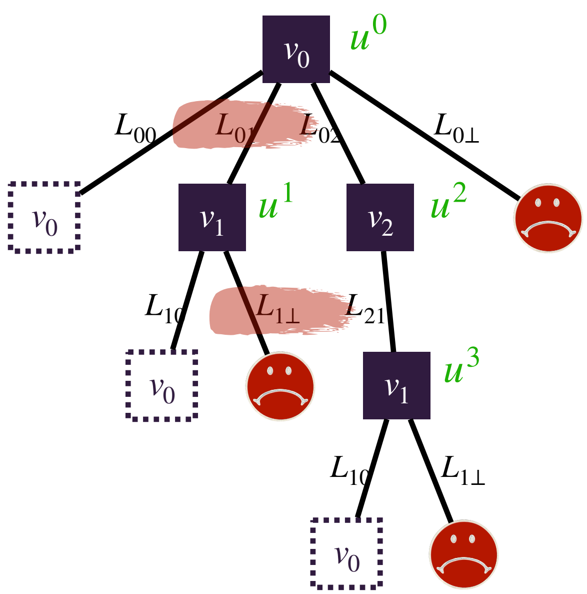

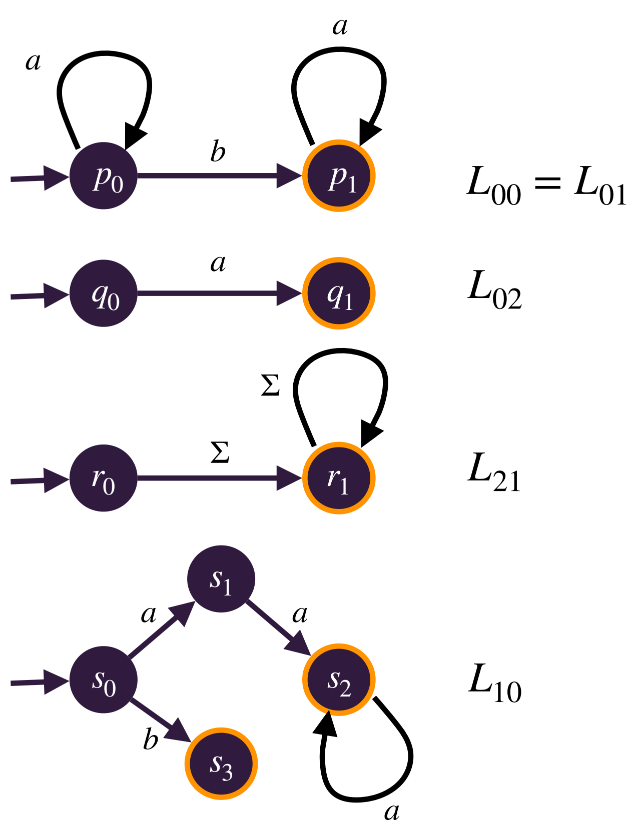

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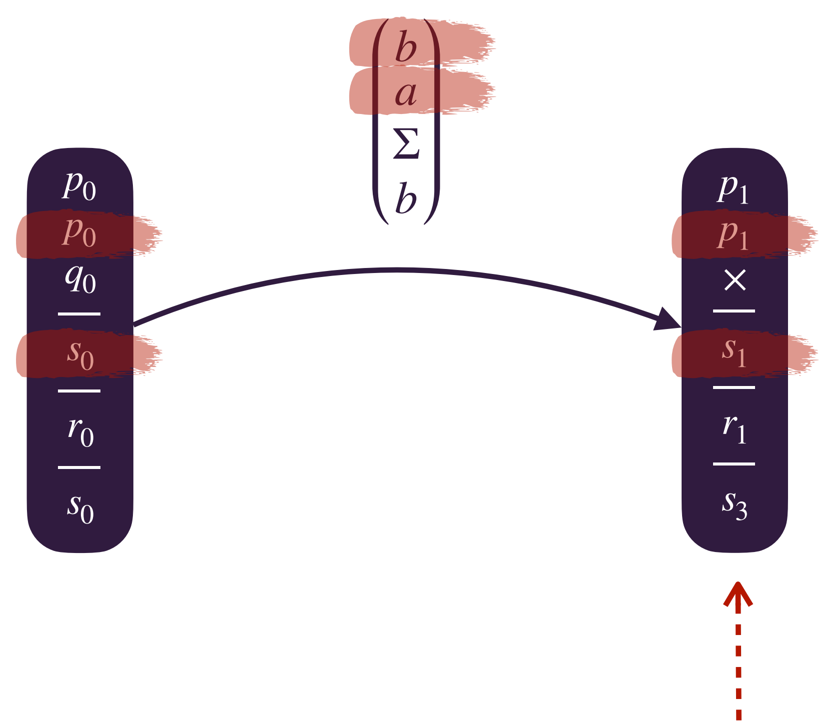
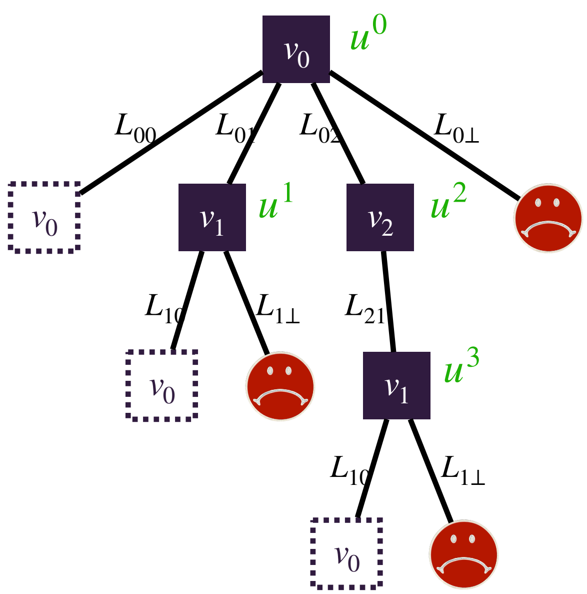


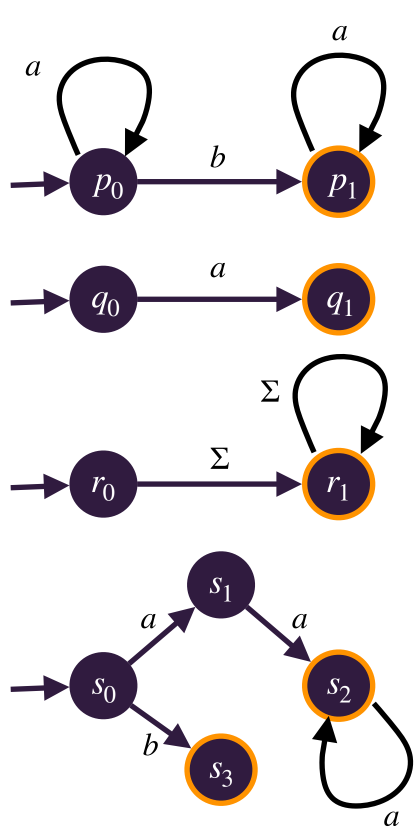
This state is unsafe, due to the branch $v_0 v_1$

$$b \in L_{0,1}, a \in L_{1\perp} = \Sigma^+ \setminus L_{10}$$

(p_1 accepting, s_1 not accepting)

Construction of a finite automaton — 2





$L_{00} = L_{01}$

L_{02}

L_{21}

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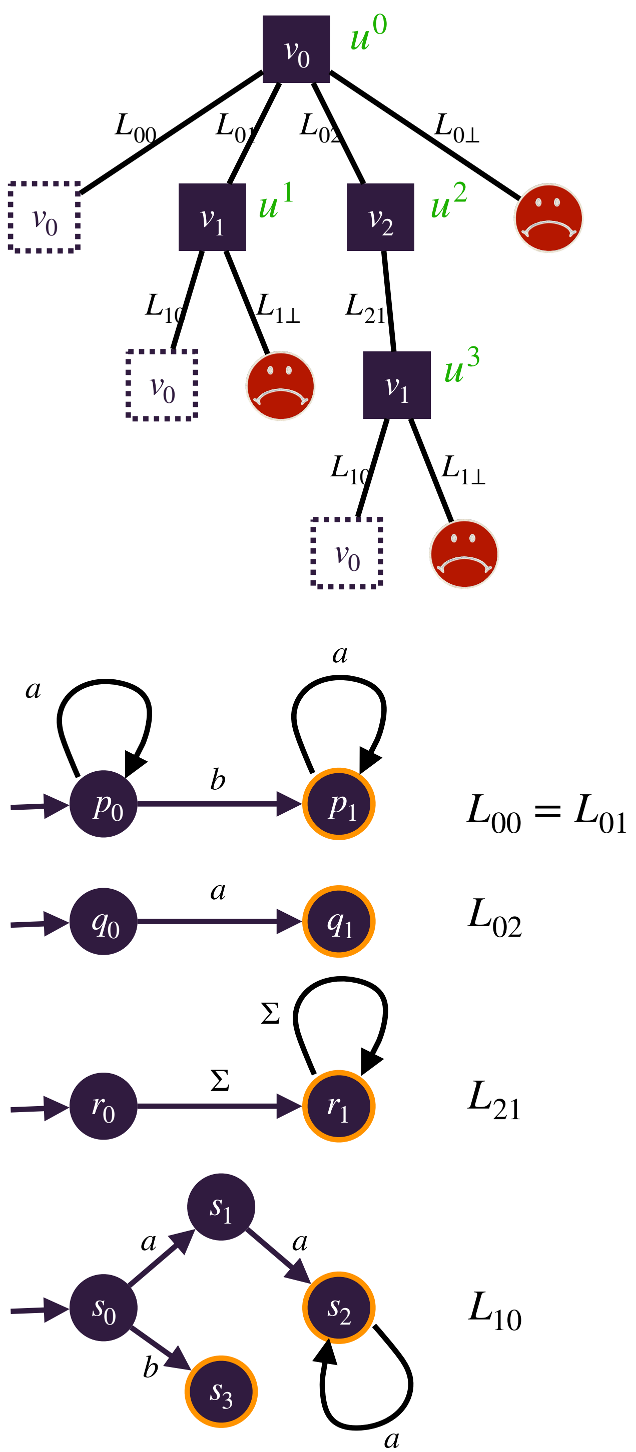
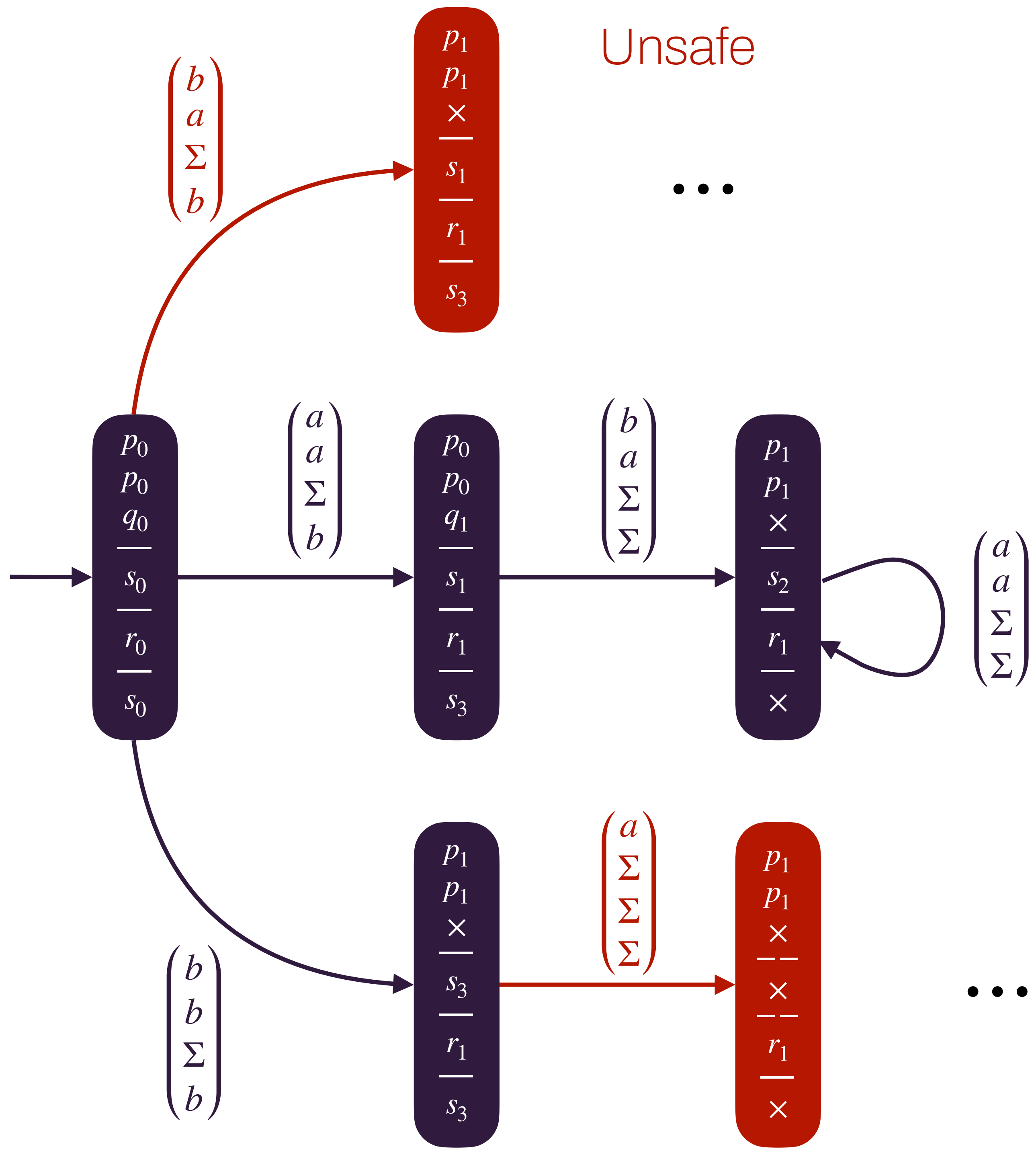
Construction of a finite automaton — 2





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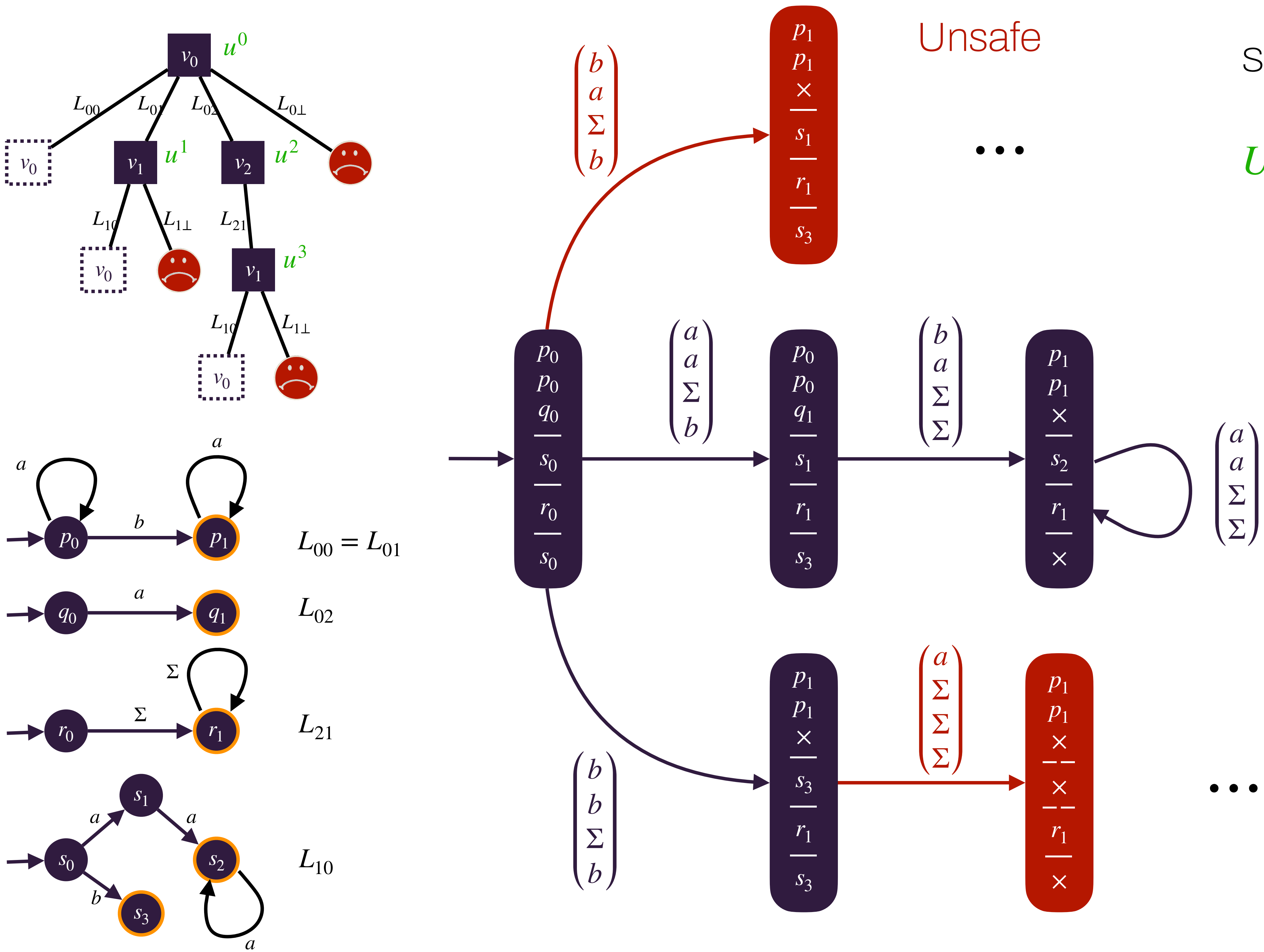
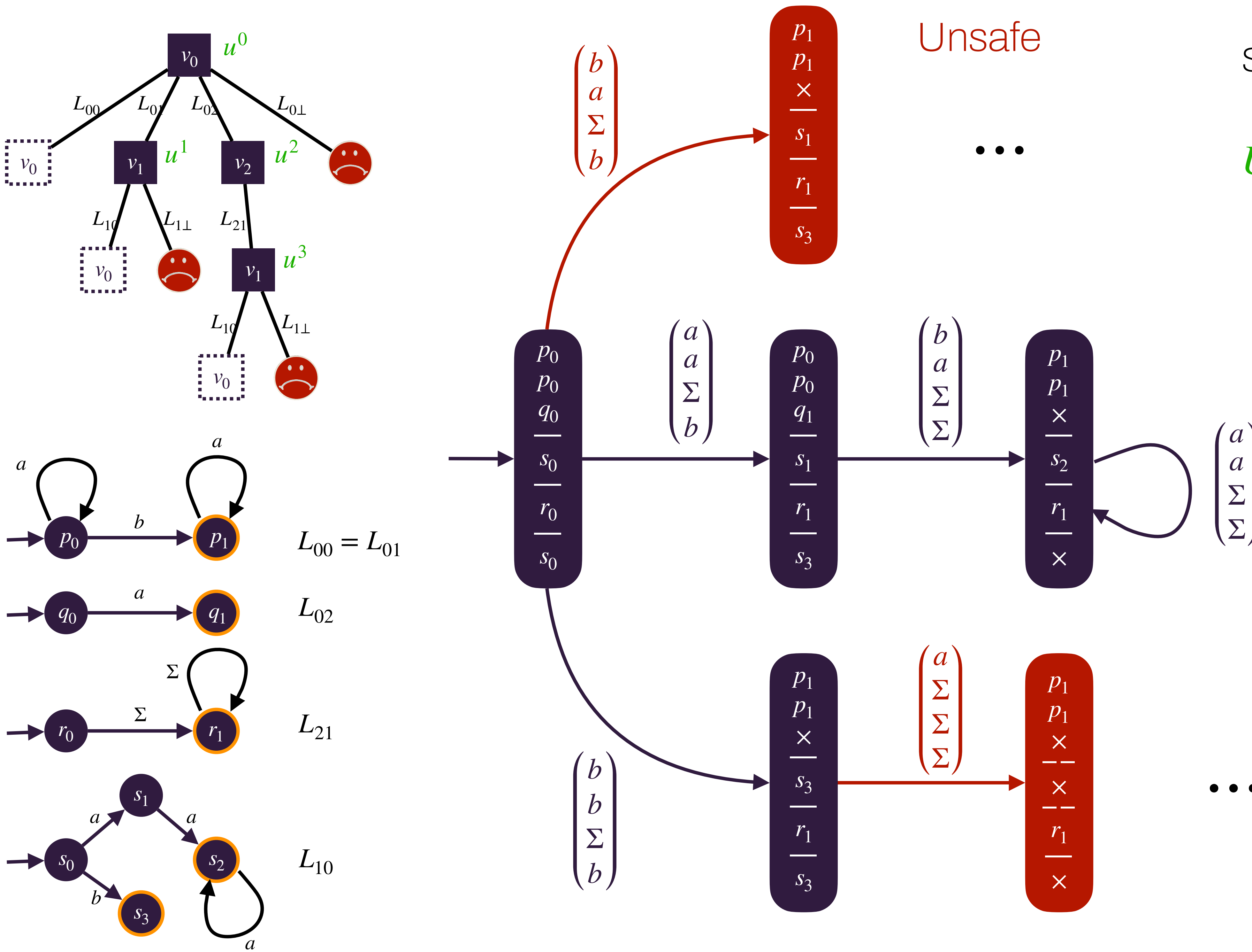


Safe word:

$$U = \begin{pmatrix} a \\ a \\ a \\ b \end{pmatrix} \begin{pmatrix} b \\ a \\ a \\ a \end{pmatrix} \begin{pmatrix} a \\ a \\ a \\ a \end{pmatrix}^{\omega}$$

Construction of a finite automaton — 2



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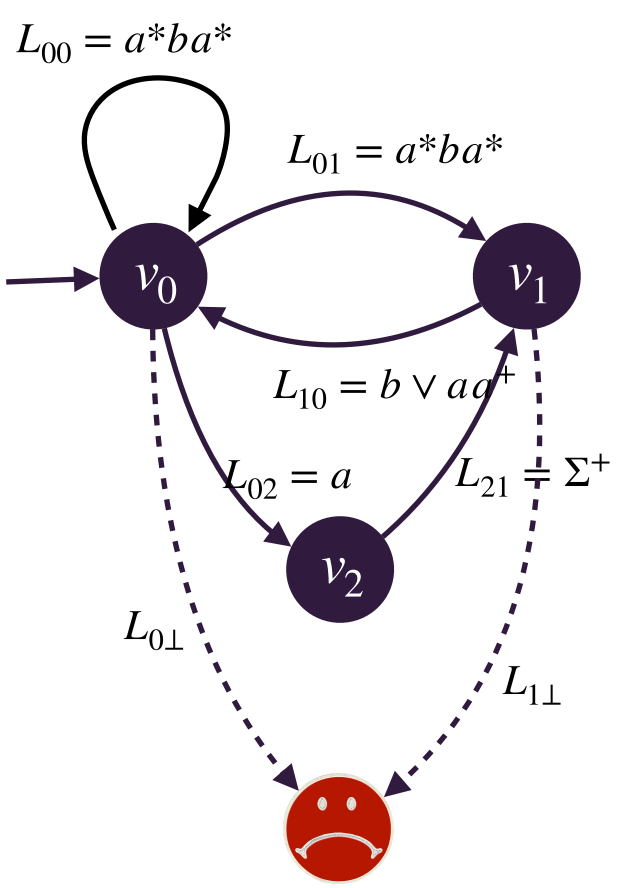
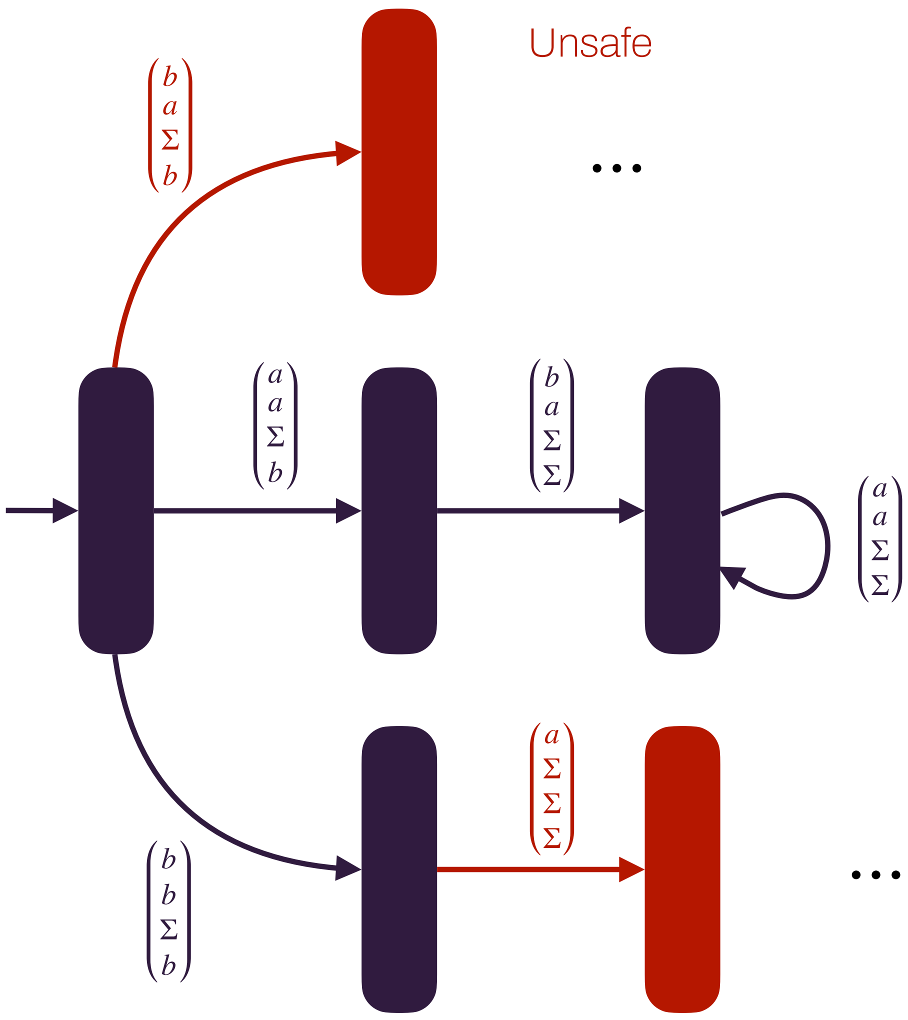
Recap





There is a winning safe coalition strategy in the game iff there is an infinite safe word in the constructed automaton

The result

Decidability/complexity results

The safety coalition problem is decidable in EXPSPACE. It is PSPACE-hard.

- Upper bound: the size of the pruned unfolding can be exponential (and not possible to consider a polynomial-size DAG instead)
- Lower bound: similar reduction as for the strong controller synthesis from QSAT

Going further?

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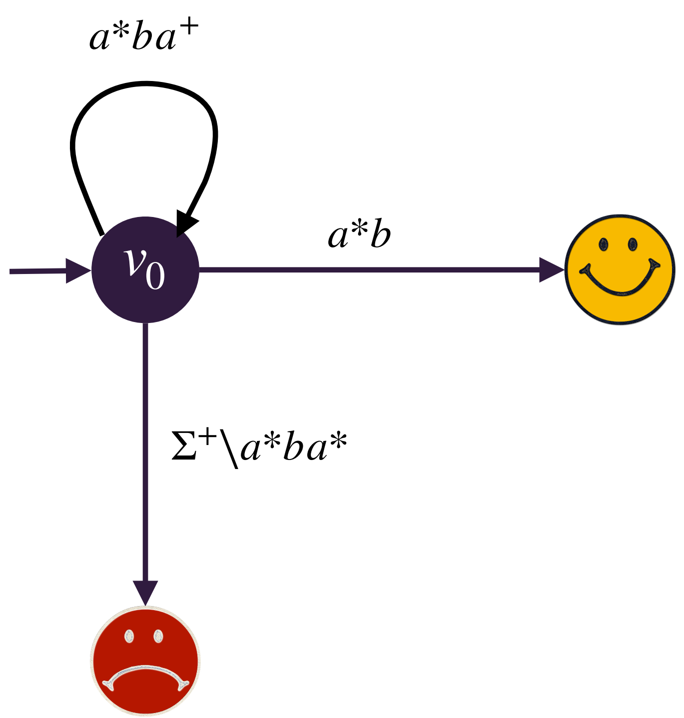
- Understand the case of other objectives, starting with Reachability



A winning coalition strategy

- At round i:
  - Player i plays b
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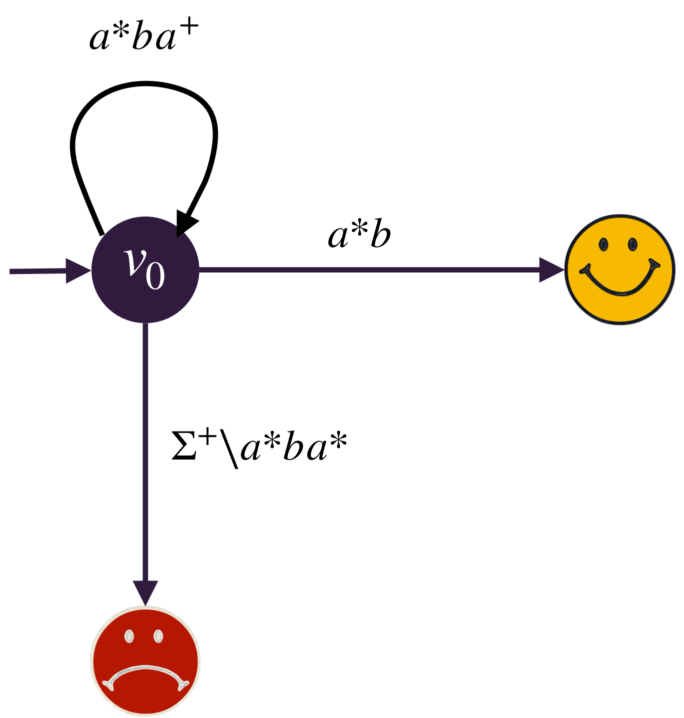

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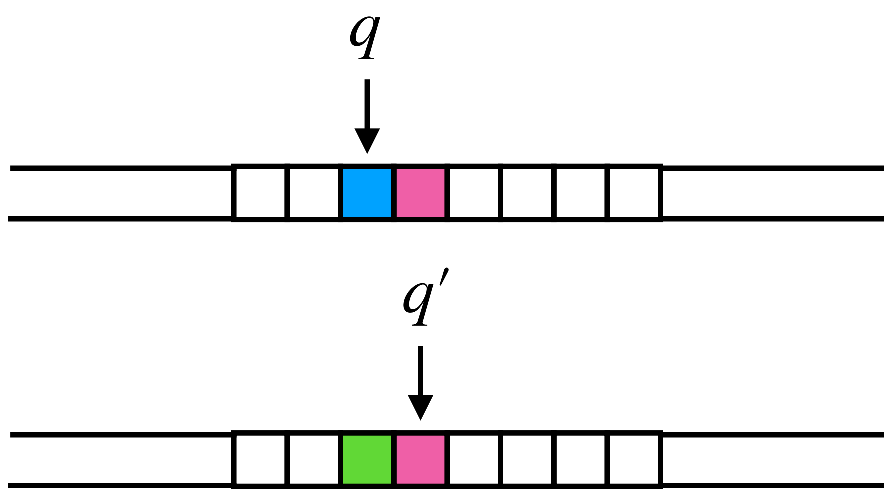
- Limits: undecidability if regular relations instead of regular languages

Undecidability under rational relations

Det. Turing machine $\mathscr{M}$



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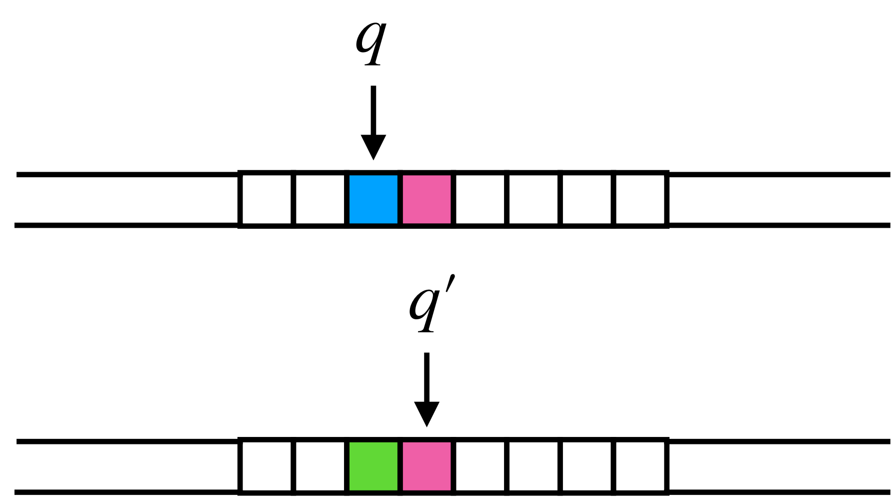

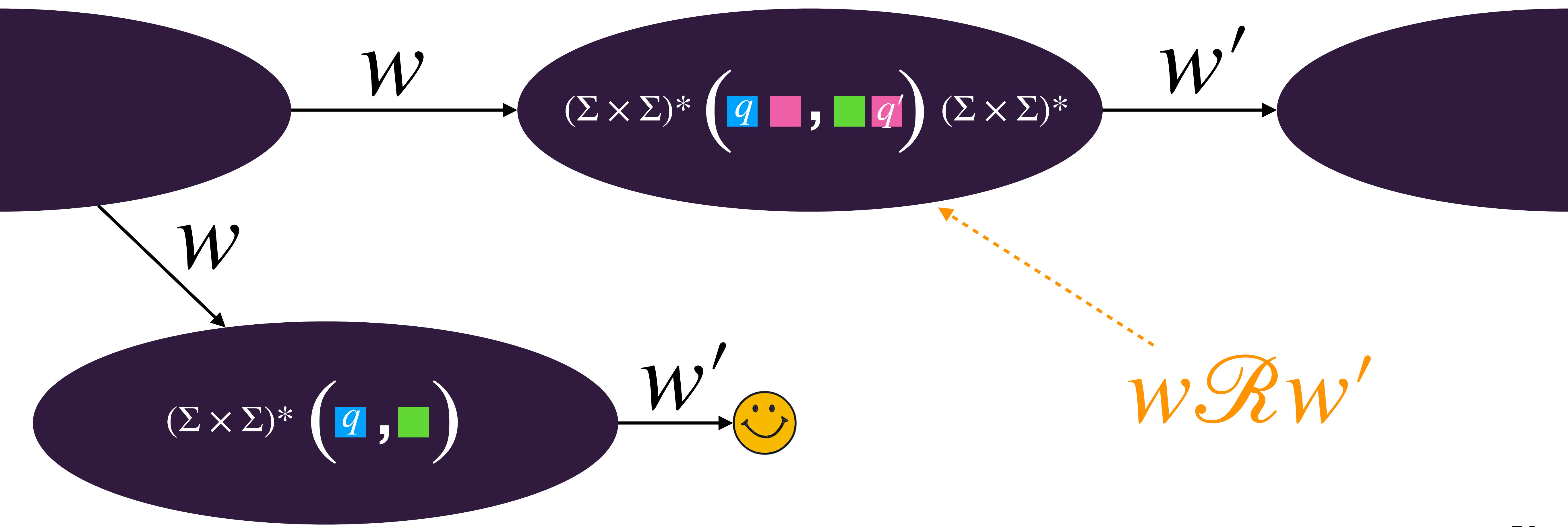

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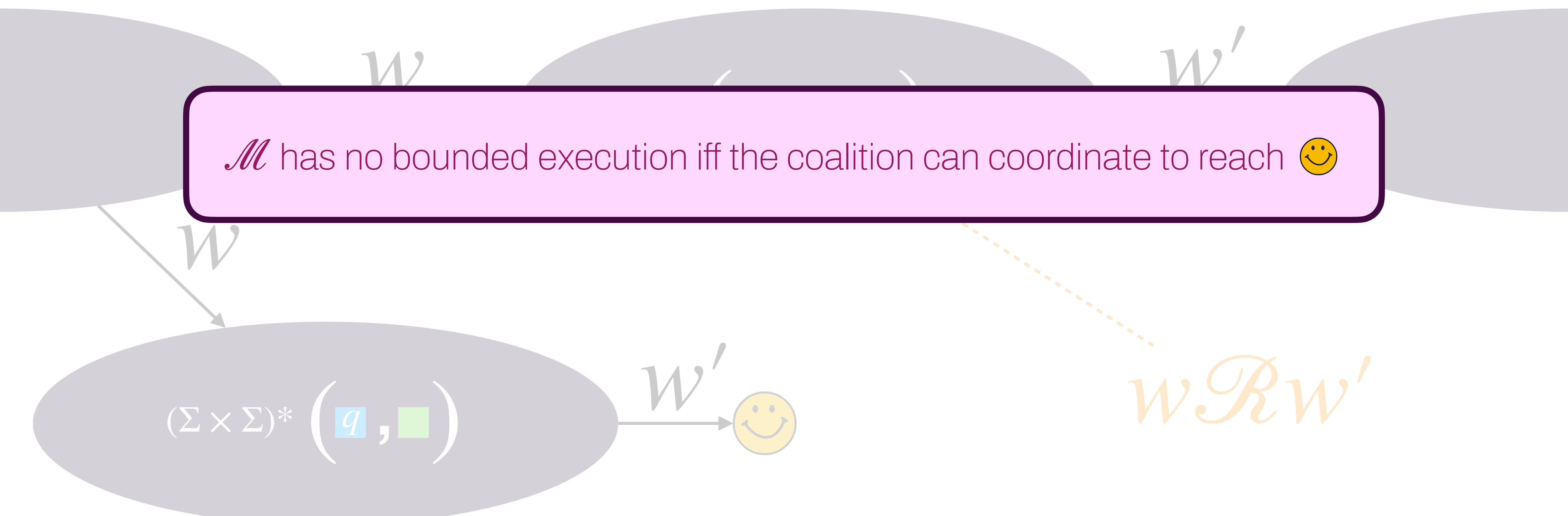

$\mathscr{M}$ has no bounded execution iff the coalition can coordinate to reach 🙂

Conclusion and further work

Summary

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- A concurrent parameterized game model
  - To reason about an unbounded number of agents
  - A natural extension of standard concurrent games

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- A concurrent parameterized game model
  - To reason about an unbounded number of agents
  - A natural extension of standard concurrent games
- Two natural problems under inspection:
  - The crowd controller problem
  - The coalition problem

Further work

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- Some technical further work:
  - Better understand the coalition problem
- Investigate solution concepts relevant to multiplayer games?
  - Various notions of rational behaviors (e.g. equilibria)
- Integrate new features in the model for better modeling power
  - Add partial information?
  - Infinite state space useful?
  - More general structures than words?

Questions?