Probabilities in Timed Automata

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Based on joint works with Christel Baier (TU Dresden, Germany),
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Nicolas Markey (LSV, France) and Marcus Größer (TU Dresden, Germany)

Outline

Motivations

- Timed automata, an idealized mathematical model for real-time systems

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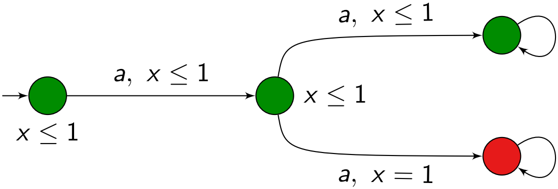
Aim: Use probabilities to "relax" the semantics of timed automata

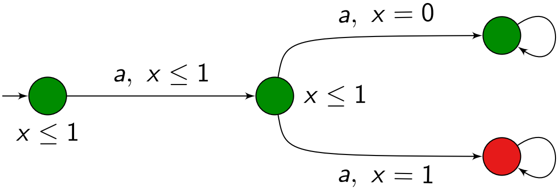
Initial example



Intuition: from the initial state,

this automaton *almost-surely* satisfies "$\mathbf{G}$ green"

A maybe less intuitive example



Does it *almost-surely* satisfy "$\mathbf{F}$ red"?

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Our proposition

- $\pi(s \xrightarrow{e_1} \ldots \xrightarrow{e_n})$: symbolic path from s firing edges $e_1, \ldots, e_n$

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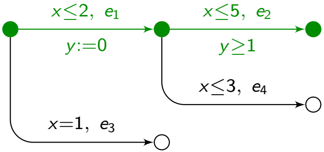

$$\pi(s_0 \xrightarrow{e_1}\xrightarrow{e_2}) = \{s_0 \xrightarrow{\tau_1, e_1} s_1 \xrightarrow{\tau_2, e_2} s_2 \mid \tau_1 \leq 2,\ \tau_1 + \tau_2 \leq 5,\ \tau_2 \geq 1\}$$

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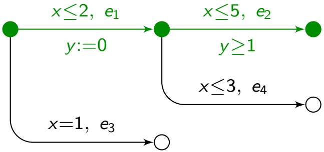

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- Idea:

From state s_0:

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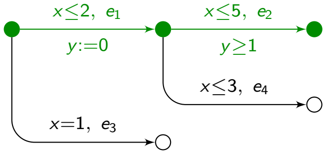

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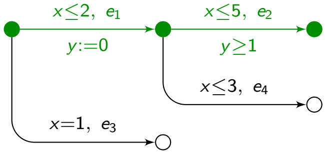

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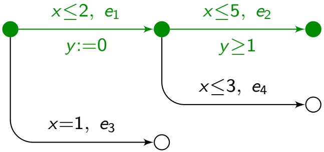

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- Idea:

From state s_0:

- randomly choose a delay
- then randomly select an edge
- then continue

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symbolic path: $\pi\left(s \xrightarrow{e_1} \cdots \xrightarrow{e_n}\right) = \left\{s \xrightarrow{\tau_1, e_1} s_1 \cdots \xrightarrow{\tau_n, e_n} s_n\right\}$

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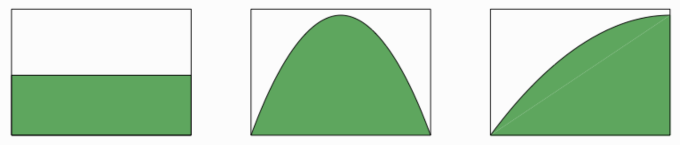

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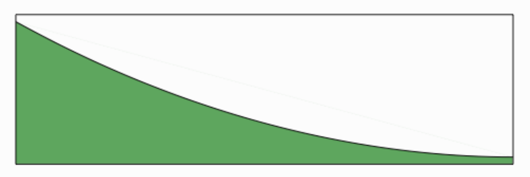

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- Can be viewed as an n-dimensional integral
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  $$\pi_{\mathcal{C}}\big(s \xrightarrow{e_1} \cdots \xrightarrow{e_n}\big) = \{s \xrightarrow{\tau_1, e_1} s_1 \cdots \xrightarrow{\tau_n, e_n} s_n \mid (\tau_1, \cdots, \tau_n) \models \mathcal{C}\}$$

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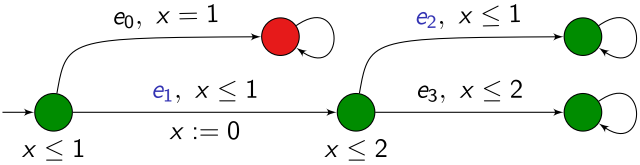
- Example:
  - $$\mathsf{Zeno}(s) = \bigcup_{M \in \mathbb{N}} \bigcap_{n \in \mathbb{N}} \bigcup_{(e_1, \cdots, e_n) \in E^n} \mathsf{Cyl}(\pi_{\Sigma_i \tau_i \leq M}(s \xrightarrow{e_1} \cdots \xrightarrow{e_n}))$$

An example of computation (with uniform distributions)



The probability of the symbolic path $\pi(s_0 \xrightarrow{e_1}\xrightarrow{e_2})$ is $\frac{1}{4}$.

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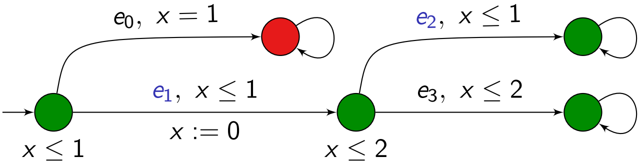

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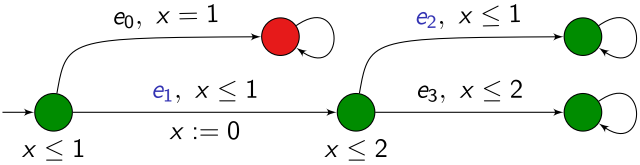

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$$\begin{aligned}
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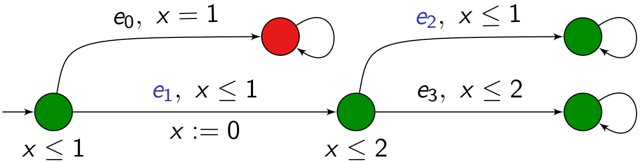

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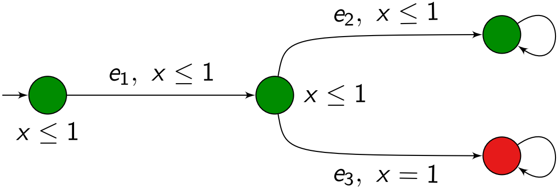
&= \int_0^1 \int_0^1 \left(\frac{\mathbb{P}\big(\pi(s_2)\big)}{2}\mathrm{d}\mu_{s_1}(u)\right) \mathrm{d}\mu_{s_0}(t) \\
&= \int_0^1 \int_0^1 \left(\frac{1}{2}\,\frac{\mathrm{d}u}{2}\right)\mathrm{d}t \quad = \frac{1}{4}
\end{aligned}$$

Back to the first example



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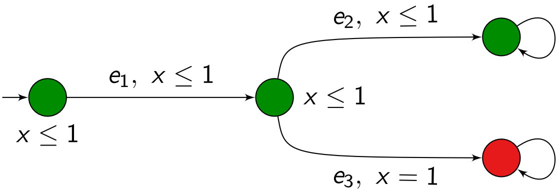

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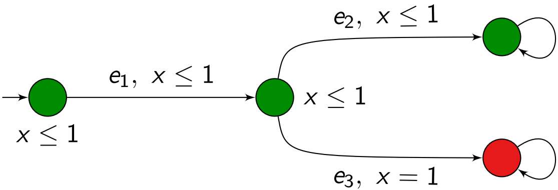

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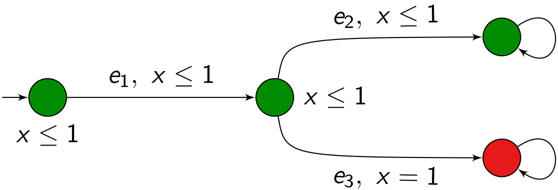

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- $\mathbb{P}\Big(\pi\big(s_0 \xrightarrow{e_1}\xrightarrow{e_3}\big)\Big) = 0$
- $\mathbb{P}\Big(\mathbf{G}\ \text{green}\Big) = 1$

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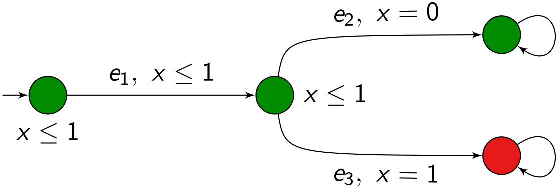

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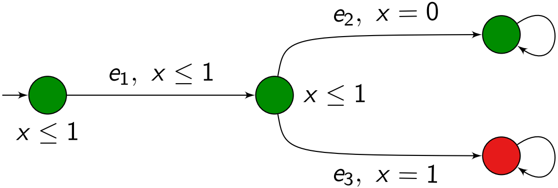

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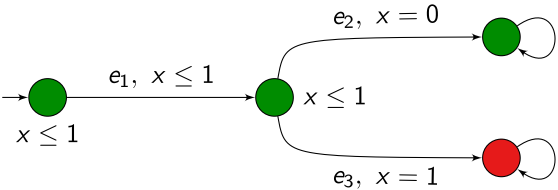

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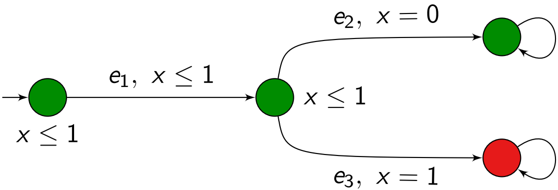

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Almost-sure model-checking

If φ is an LTL (or ω-regular) property,

$$s \approx\!\!\!\!\models \varphi \quad \stackrel{\text{def}}{\Leftrightarrow} \quad \mathbb{P}\Big(\{\varrho \in \mathsf{Runs}(s) \mid \varrho \models \varphi\}\Big) = 1$$

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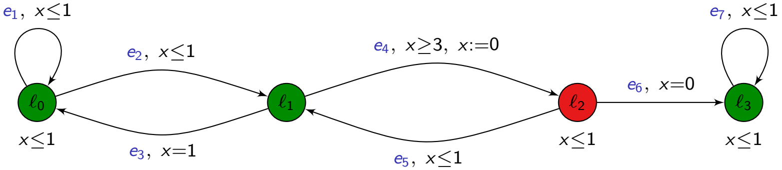
We want to decide the almost-sure model-checking...
(This is a qualitative question)

Outline

An example


e_1, $x \leq 1$
e_2, $x \leq 1$
e_4, $x \geq 3$, $x := 0$
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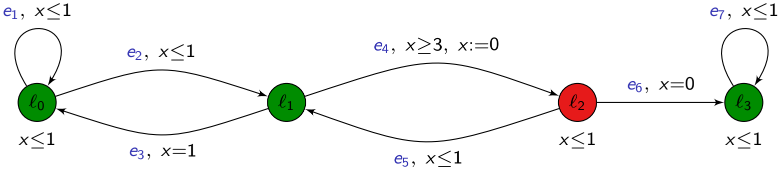

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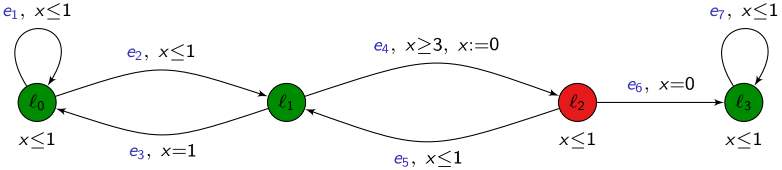

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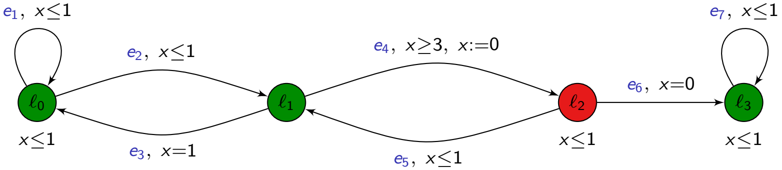

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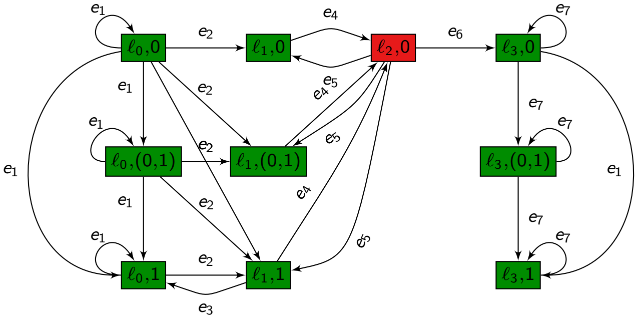
Indeed, almost surely, paths are of the form $e_1^* e_2 \Big(e_4 e_5\Big)^\omega$

The classical region automaton



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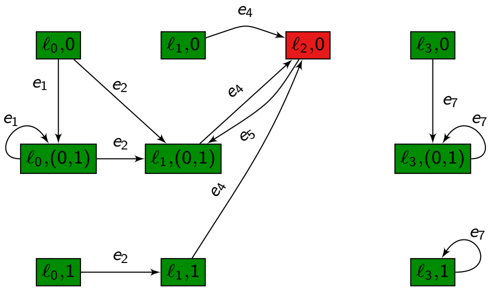

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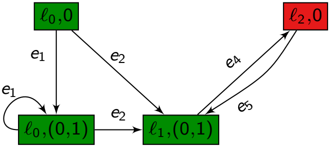

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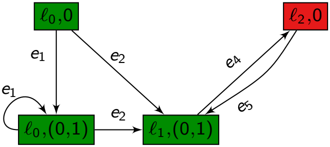

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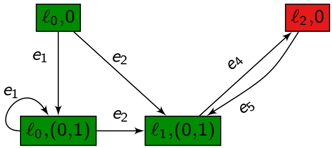

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Theorem

For single-clock timed automata,

$$\mathcal{A} \approx\models \varphi \quad \text{iff} \quad \mathbb{P}(MC(\mathcal{A}) \models \varphi) = 1$$

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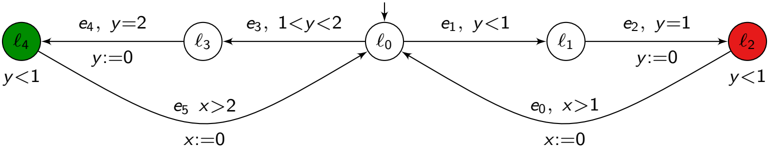
  - link between probabilities and topology thanks to the topological games called Banach-Mazur games

An example with two clocks



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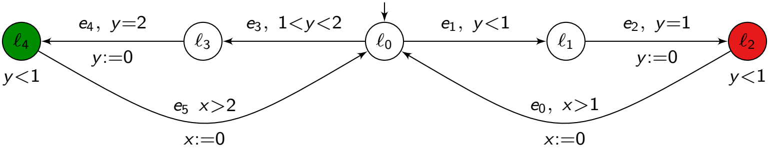

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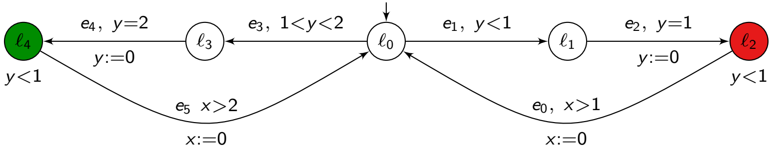

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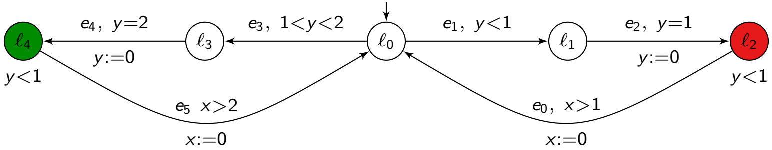

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- There is a *strange* convergence phenomenon: along an execution, if $\delta_i > 0$ is the delay in location ℓ_4, then we have that $\sum_i \delta_i \leq 1$

A note on Zeno behaviours

- The set of Zeno behaviours is measurable:

$$\mathsf{Zeno}(s) \;=\; \bigcup_{M\in\mathbb{N}} \bigcap_{n\in\mathbb{N}} \bigcup_{(e_1,\cdots,e_n)\in E^n} \mathsf{Cyl}(\pi(s \xrightarrow{e_1} \cdots \xrightarrow{e_n}))$$

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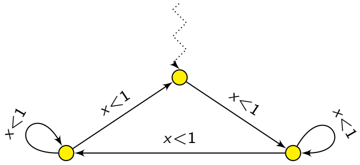

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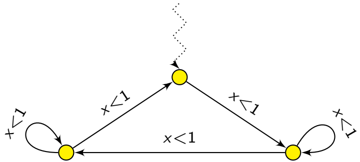

  - an interesting notion of non-Zeno timed automata

Outline

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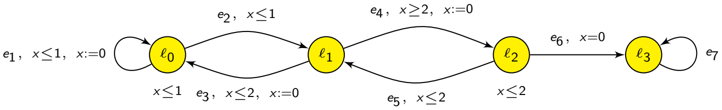
  - we will be able to decide the threshold problem:
    "Given $\mathcal{A}$, φ, $c \in \mathbb{Q}$, and $\sim \in \{<, \leq, =, \geq, >\}$, does $\mathbb{P}\big(s_0 \models \varphi\big) \sim c$ in $\mathcal{A}$?"

An example



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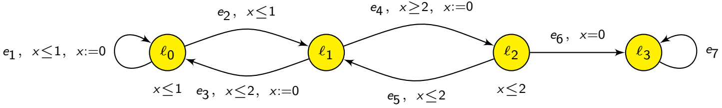

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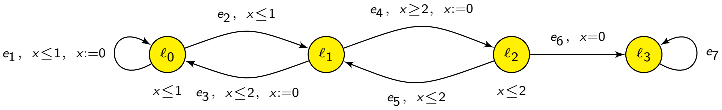

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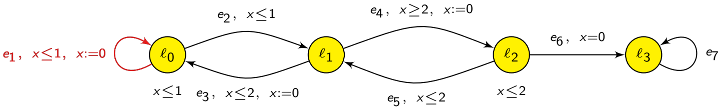

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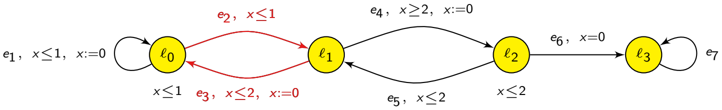

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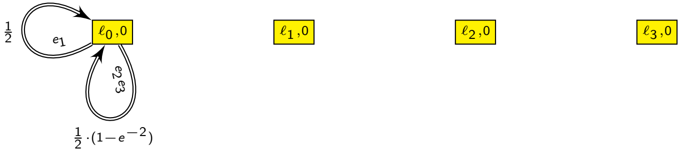

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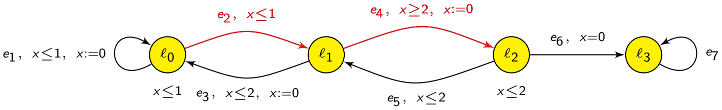

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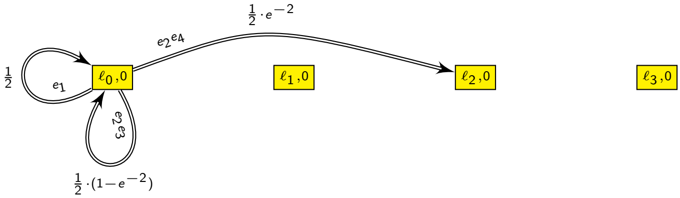

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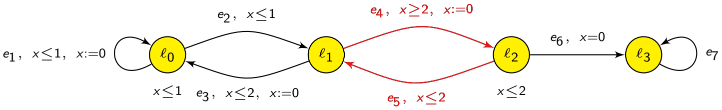

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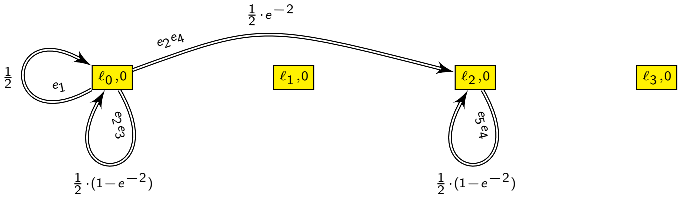

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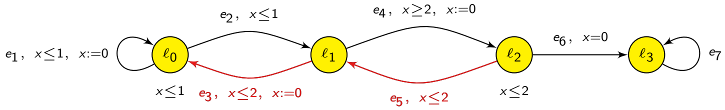

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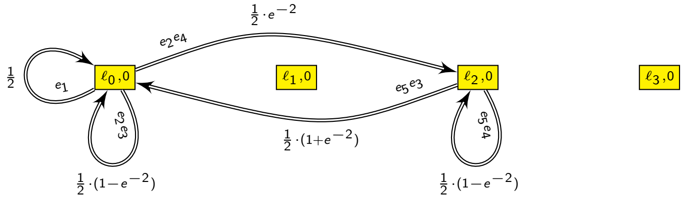

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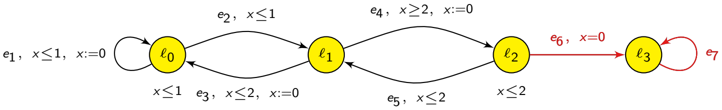

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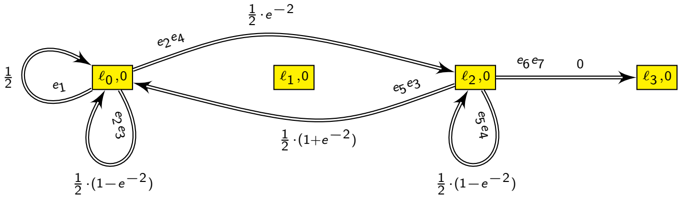

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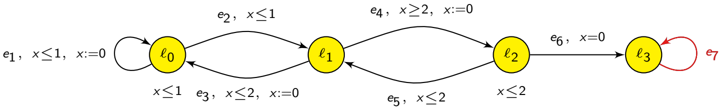

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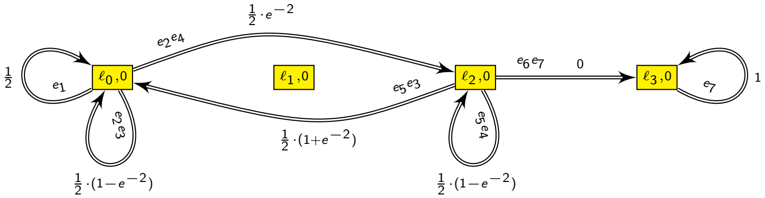

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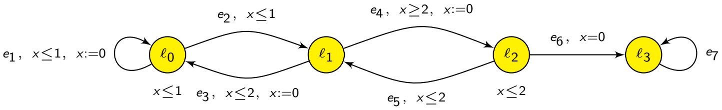
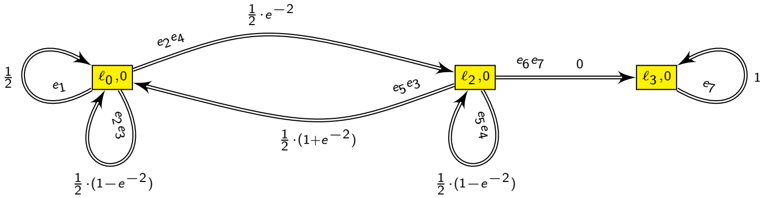


$+$ distributions $\quad \mu_s : t \mapsto e^{-t}$ when $I(s) = \mathbb{R}_+$

μ_s uniform distribution when $I(s)$ is bounded

$+$ uniform weights on transitions

We construct a finite Markov chain $MC'(\mathcal{A})$ with macro-edges:



Correctness of the abstraction

Theorem

Under some hypotheses, for single-clock automaton $\mathcal{A}$ and property φ,

$$\mathbb{P}_{\mathcal{A}}(s_0 \models \varphi) = \mathbb{P}_{MC'(\mathcal{A})}(s_0 \models \Diamond F_\varphi)$$

for some well-chosen set F_φ.

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- **Limits of the abstraction:** there may be no closed form for the values labelling the edges of $MC'(\mathcal{A})$.

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☞ Note: the hypothesis "reset all bounded cycles" is necessary to get this form.

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- When $(a_N, b_N) \subseteq (\alpha, \beta)$, the two sequences $(f(a_i))_{i \geq N}$ and $(f(b_i))_{i \geq N}$ are monotonic and converge to $f\left(e^{-r}\right)$

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  - stop when the under- and the over-approximations are on the same side of the threshold c

Outline

Related works

- Other “probabilistic and timed” (automata-)based models

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  - robust timed automata [GHJ97,HR00]
  - robust model-checking [Puri98,DDR04,DDMR04,ALM05,BMR06,BMR08]

Conclusions

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- qualitative model-checking has a topological interpretation
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Further works

- efficient zone-based algorithm
- apply to relevant examples
- add non-determinism (*à la* MDP)
- handle several clocks
- timed properties
- expected time