Foundation for Timed Systems

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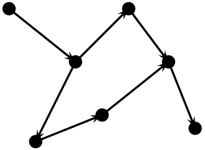
Model-checking

Does the system satisfy the property?

Modelling



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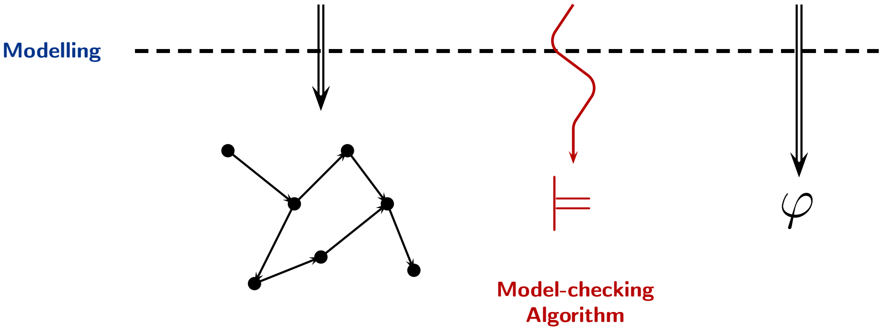

Time!

Context: verification of embedded critical systems

Time

- naturally appears in real systems
- appears in properties (for ex. bounded response time)

➜ Need of models and specification languages integrating timing aspects

Outline

1. **About time semantics**
2. Timed automata, decidability issues
3. Some extensions of the model
4. Implementation of timed automata
5. Conclusion

Adding timing informations

- **Untimed case:** sequence of observable events
  a: send message b: receive message

$$a\ b\ a\ b\ a\ b\ a\ b\ a\ b\ \cdots = (a\ b)^\omega$$

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$$(a, d_1)\ (b, d_2)\ (a, d_3)\ (b, d_4)\ (a, d_5)\ (b, d_6)\ \cdots$$

  d_1: date at which the first a occurs
  d_2: date at which the first b occurs, ...

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  - **Discrete-time semantics:** dates are *e.g.* taken in N
    Ex: $(a, 1)(b, 3)(c, 4)(a, 6)$
  - **Dense-time semantics:** dates are *e.g.* taken in Q^+, or in R^+
    Ex: $(a, 1.28).(b, 3.1).(c, 3.98)(a, 6.13)$

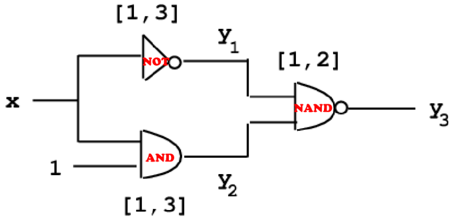
A case for dense-time

Time domain: discrete (*e.g.* N) or dense (*e.g.* Q^+)

- Dense-time is a more general model than discrete time
- A compositionality problem with discrete time
- But, can we not always discretize?

A digital circuit [Alur 91]

Discussion in the context of reachability problems for asynchronous digital circuits [Brzozowski, Seger 1991]



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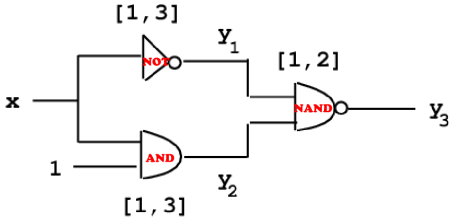

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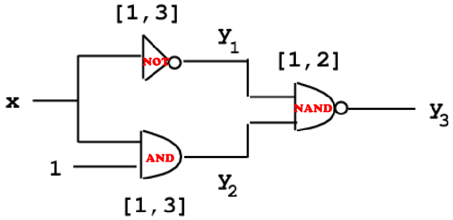

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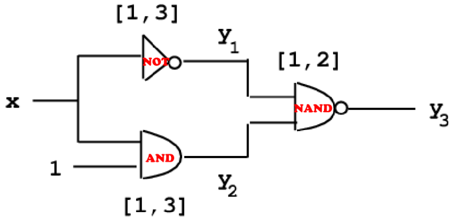

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However, many possible behaviours, e.g.

$$[101] \xrightarrow[1.2]{y_2} [111] \xrightarrow[2.5]{y_3} [110] \xrightarrow[2.8]{y_1} [010] \xrightarrow[4.5]{y_3} [011]$$

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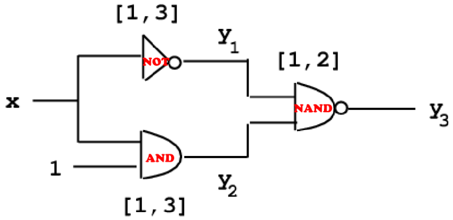

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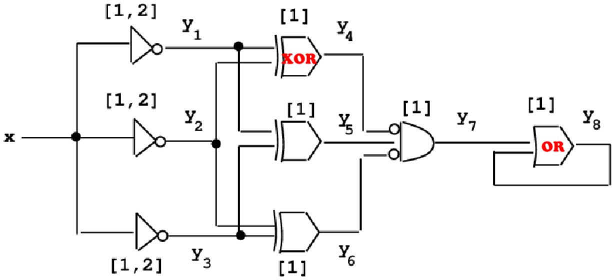
Reachable configurations: {[101], [111], [110], [010], [011], [001]}

Is discretizing sufficient? An example [Alur 91]



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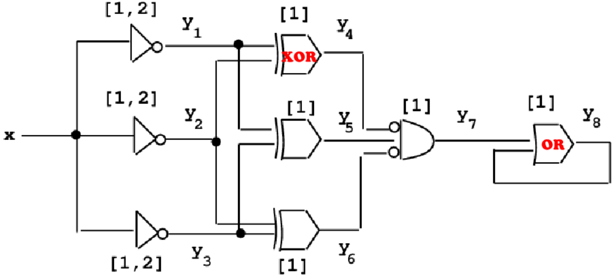

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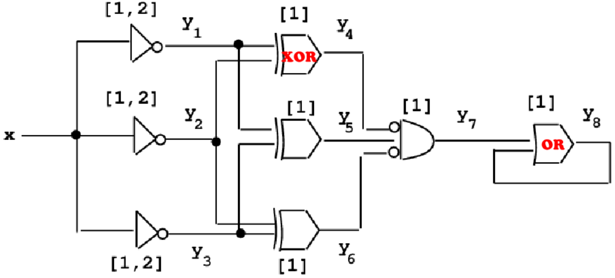

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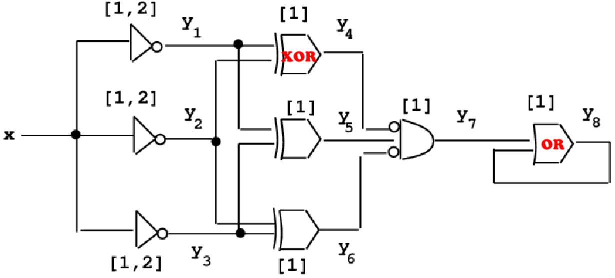

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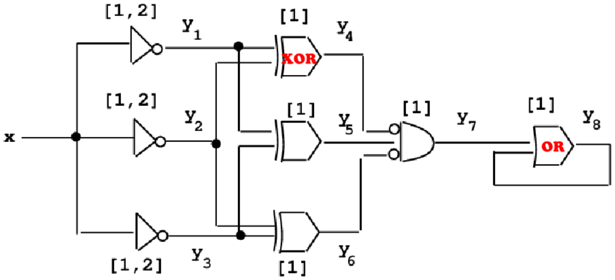

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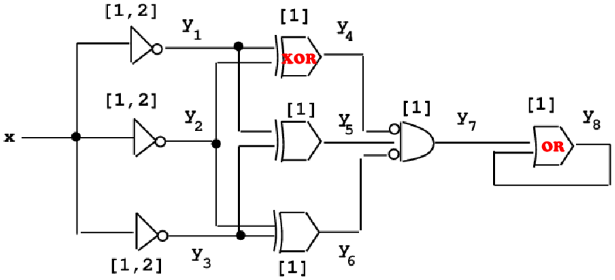

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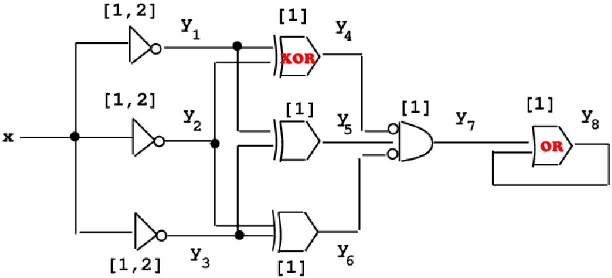

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Theorem [Brzozowski Seger 1991]

For every $k \geq 1$, there exists a digital circuit such that the reachability set of states in dense-time is strictly larger than the one in discrete time (with granularity $\frac{1}{k}$).

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Claim

Finding a correct granularity is as difficult as computing the set of reachable states in dense-time.

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Claim

Finding a correct granularity is as difficult as computing the set of reachable states in dense-time.

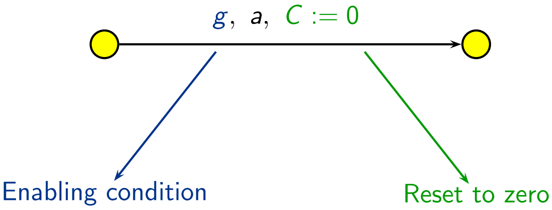
Going further... There exist systems for which no granularity exists.

(see later)

Outline

Timed automata [Alur & Dill 90's]

- A finite control structure + variables (clocks)
- A transition is of the form:



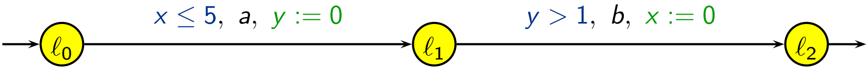
- An enabling condition (or guard) is:

$$g \quad ::= \quad x \sim c \quad | \quad g \wedge g$$

where $\sim \in \{<, \leq, =, \geq, >\}$

Timed automata (example)

x, y : clocks



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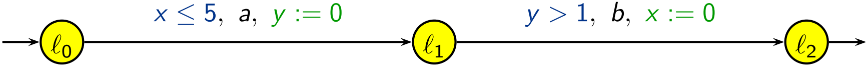

$$
\begin{array}{cccccccccc}
 & \ell_0 & \xrightarrow{\delta(4.1)} & \ell_0 & \xrightarrow{a} & \ell_1 & \xrightarrow{\delta(1.4)} & \ell_1 & \xrightarrow{b} & \ell_2 \\
x & 0 & & 4.1 & & 4.1 & & 5.5 & & 0 \\
y & 0 & & 4.1 & & 0 & & 1.4 & & 1.4
\end{array}
$$

Timed automata (example)

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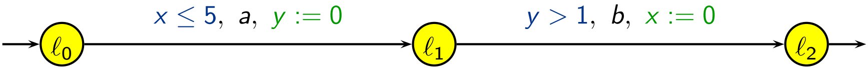
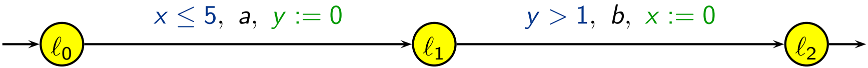


	ℓ_0	$\xrightarrow{\delta(4.1)}$	ℓ_0	$\xrightarrow{a}$	ℓ_1	$\xrightarrow{\delta(1.4)}$	ℓ_1	$\xrightarrow{b}$	ℓ_2
x	0		4.1		4.1		5.5		0
y	0		4.1		0		1.4		1.4

(clock) valuation

Timed automata (example)

x, y : clocks



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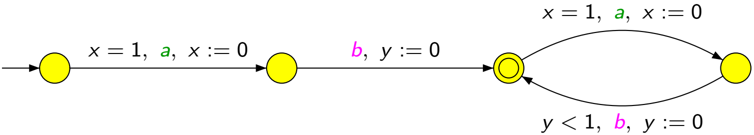
(clock) valuation

➜ timed word $(a, 4.1)(b, 5.5)$

Timed automata semantics

- $\mathcal{A} = (\Sigma, L, X, \longrightarrow)$ is a TA
- **Configurations:** $(\ell, v) \in L \times T^X$ where T is the time domain
- **Timed Transition System:**
  - **action transition**: $(\ell, v) \xrightarrow{a} (\ell', v')$ if $\exists \ell \xrightarrow{g,a,r} \ell' \in \mathcal{A}$ s.t. $\begin{cases} v \models g \\ v' = v[r \leftarrow 0] \end{cases}$
  - **delay transition**: $(\ell, v) \xrightarrow{\delta(d)} (\ell, v + d)$ if $d \in T$

Discrete *vs* dense-time semantics



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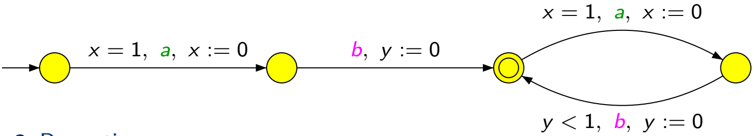

- Dense-time:
  $L_{dense} = \{((ab)^\omega, \tau) \mid \forall i,\ \tau_{2i-1} = i \text{ and } \tau_{2i} - \tau_{2i-1} > \tau_{2i+2} - \tau_{2i+1}\}$

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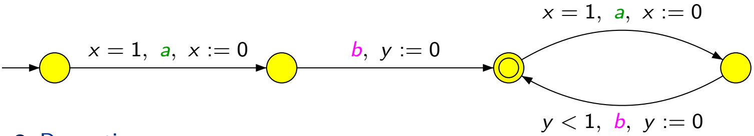

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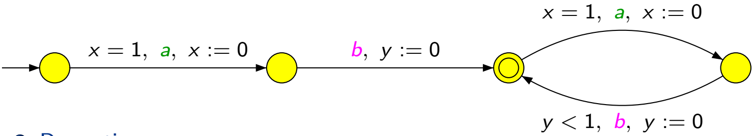

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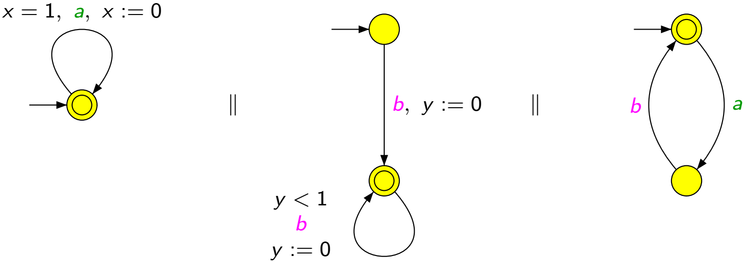
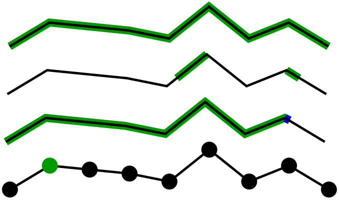


Classical verification problems

- reachability of a control state
- $\mathcal{S} \sim \mathcal{S}'$: bisimulation, etc...
- $L(\mathcal{S}) \subseteq L(\mathcal{S}')$: language inclusion
- $\mathcal{S} \models \varphi$ for some formula φ: model-checking
- $\mathcal{S} \parallel A_T$ + reachability: testing automata
- ...

Classical temporal logics

Path formulas:



$G\phi$ « Always »

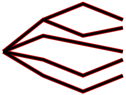
$F\phi$ « Eventually »

$\phi U \phi'$ « Until »

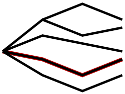
$X\phi$ « Next »

State formulas:

$A\psi$



$E\psi$



➜ LTL: Linear Temporal Logic **[Pnueli 1977]**,
CTL: Computation Tree Logic **[Emerson, Clarke 1982]**

Adding time to temporal logics

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- Temporal logics with **subscripts**. ex: $CTL + \left| \begin{array}{l} E\varphi U_{\sim k}\psi \\ A\varphi U_{\sim k}\psi \end{array} \right.$

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$$AG(\text{problem} \Rightarrow AF_{\leq 20}\ \text{alarm})$$

- Temporal logics with **clocks**.

$$AG(\text{problem} \Rightarrow (x\ \mathtt{in}\ AF(x \leq 20 \wedge \text{alarm})))$$

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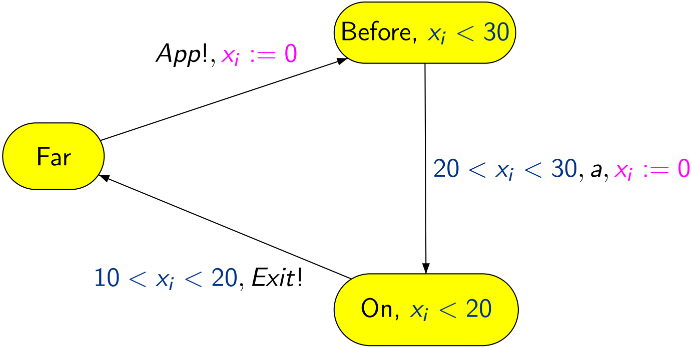
➜ **TCTL**: Timed CTL **[ACD90,ACD93,HNSY94]**

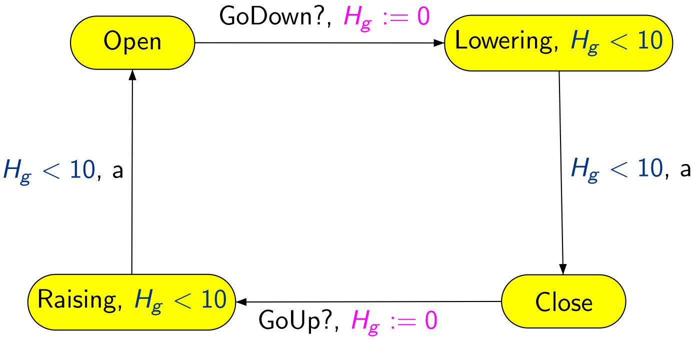
The train crossing example (1)

Train$_i$ with $i = 1, 2, \ldots$



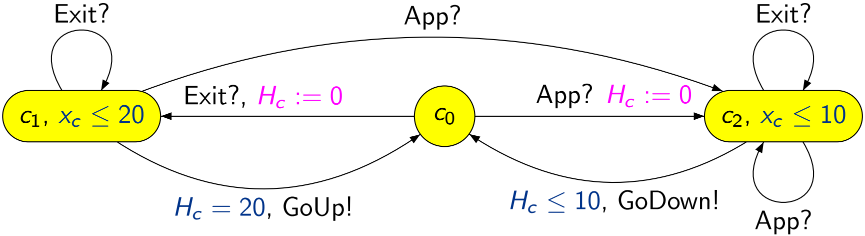
The train crossing example (2)

The gate:



The train crossing example (3)

The controller:



The train crossing example (4)

We use the synchronization function f:

Train_1	Train_2	Gate	Controller	
$App!$	.	.	$App?$	App
.	$App!$	.	$App?$	App
$Exit!$	.	.	$Exit?$	$Exit$
.	$Exit!$	.	$Exit?$	$Exit$
a	.	.	.	a
.	a	.	.	a
.	.	a	.	a
.	.	$GoUp?$	$GoUp!$	$GoUp$
.	.	$GoDown?$	$GoDown!$	$GoDown$

to define the parallel composition $(\text{Train}_1 \parallel \text{Train}_2 \parallel \text{Gate} \parallel \text{Controller})$

NB: the parallel composition does not add expressive power!

The train crossing example (5)

Some properties one could check:

- Is the gate closed when a train crosses the road?

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$$AG\ AF_{<5\text{min}}(\neg\text{gate.Close})$$

Verification

Emptiness problem: is the language accepted by a timed automaton empty?

- reachability properties (final states)
- basic liveness properties (Büchi (or other) conditions)

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The emptiness problem for timed automata is decidable.
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Note: This is also the case for the discrete semantics.

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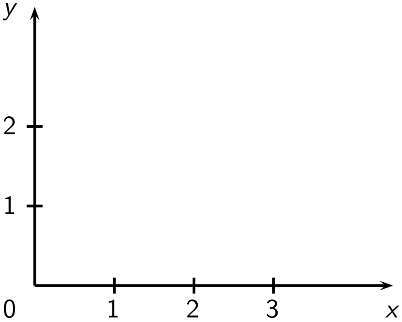
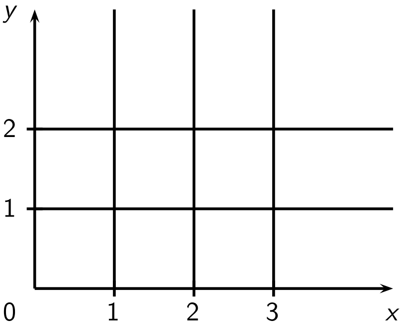
Method: construct a finite abstraction

The region abstraction



Equivalence of finite index

The region abstraction



Equivalence of finite index

- "compatibility" between regions and constraints

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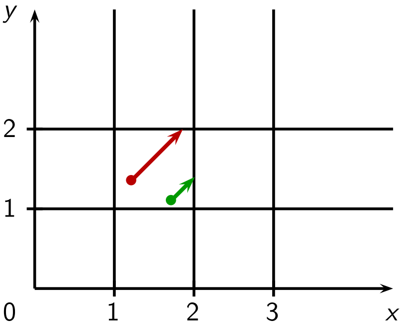

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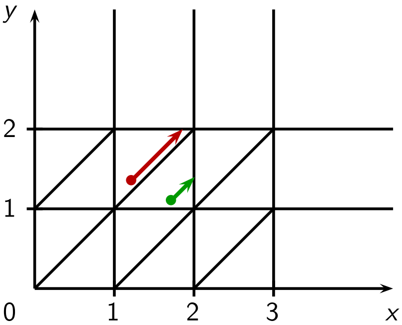

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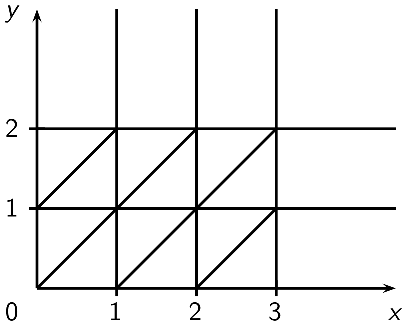

Equivalence of finite index

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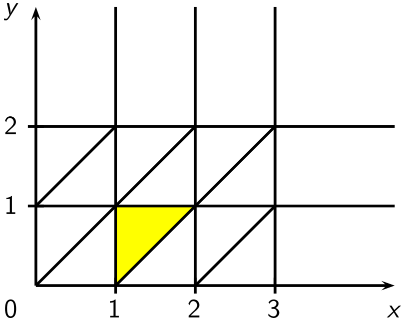
➜ a bisimulation property

The region abstraction



Equivalence of finite index

region defined by

$I_x =]1;2[$, $I_y =]0;1[$

$\{x\} < \{y\}$

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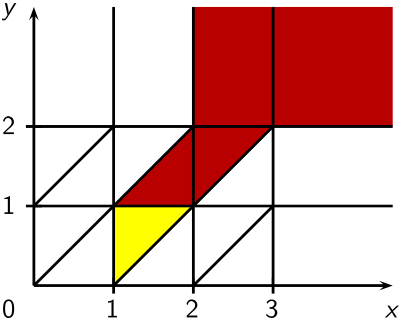


Equivalence of finite index

region defined by

$I_x =]1;2[,\ I_y =]0;1[$

$\{x\} < \{y\}$

successor regions

- "compatibility" between regions and constraints
- "compatibility" between regions and time elapsing

➜ a bisimulation property

The region automaton

timed automaton $\otimes$ region abstraction

$\ell \xrightarrow{g,a,C:=0} \ell'$ is transformed into:

$(\ell, R) \xrightarrow{a} (\ell', R')$ if there exists $R'' \in \mathrm{Succ}_t^*(R)$ s.t.

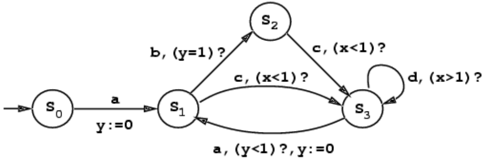
- $R'' \subseteq g$
- $[C \leftarrow 0]R'' \subseteq R'$

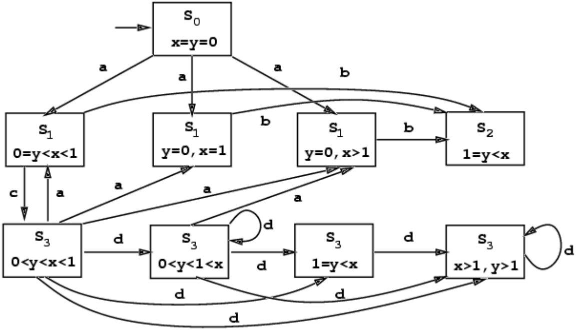
➜ **time-abstract bisimulation**

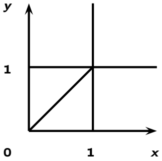
$\mathcal{L}$(reg. aut.) $=$ UNTIME($\mathcal{L}$(timed aut.))

where $\mathrm{UNTIME}((a_1, t_1)(a_2, t_2)\dots) = a_1 a_2 \dots$

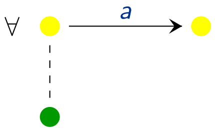
An example [AD 90's]







Time-abstract bisimulation



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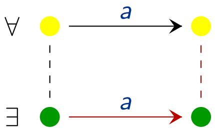

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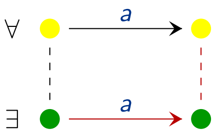

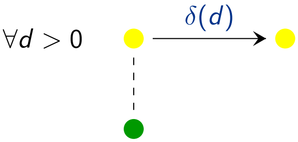

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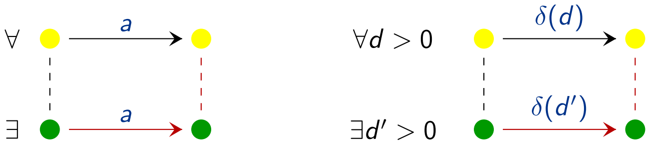

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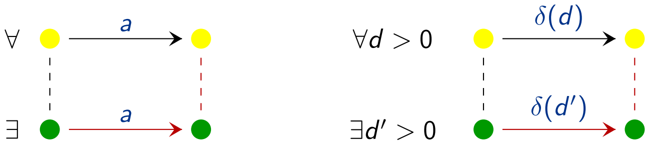

$$(\ell_0, v_0) \xrightarrow{a_1, t_1} (\ell_1, v_1) \xrightarrow{a_2, t_2} (\ell_2, v_2) \xrightarrow{a_3, t_3} \dots$$

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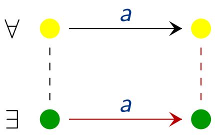
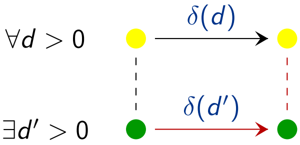
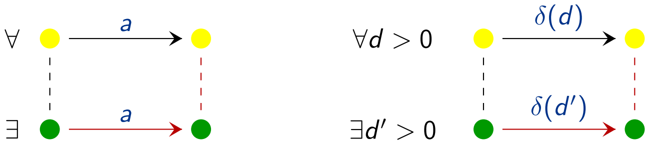




$$
\begin{tikzcd}
(\ell_0, v_0) \arrow[r, "a_1, t_1"] \arrow[d] & (\ell_1, v_1) \arrow[r, "a_2, t_2"] \arrow[d] & (\ell_2, v_2) \arrow[r, "a_3, t_3"] \arrow[d] & \dots \\
(\ell_0, R_0) \arrow[r, "a_1"] & (\ell_1, R_1) \arrow[r, "a_2"] & (\ell_2, R_2) \arrow[r, "a_3"] & \dots
\end{tikzcd}
$$

with $v_i \in R_i$ for all i.

Time-abstract bisimulation



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\updownarrow & & \updownarrow & & \updownarrow & & \\
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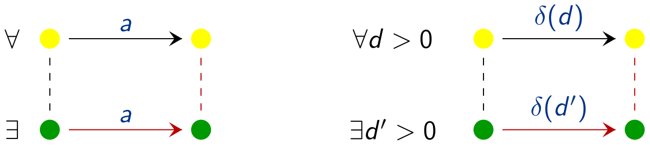

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Remark: Real-time properties can not be checked with a time-abstract bisimulation. For TCTL, a clock associated with the formula needs to be added.

PSPACE-easiness

¡ The size of the region graph is in $\mathcal{O}(|X|!.2^{|X|})$!

- **One configuration:** a discrete location + a region

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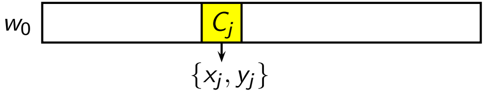
➜ in NPSPACE, thus in PSPACE

PSPACE-hardness

$$\left.\begin{array}{c}\mathcal{M}\ \text{LBTM}\\ w_0 \in \{a,b\}^*\end{array}\right\} \rightsquigarrow \begin{array}{l} A_{\mathcal{M},w_0} \text{ s.t. } \mathcal{M} \text{ accepts } w_0 \text{ iff the final state}\\ \text{of } A_{\mathcal{M},w_0} \text{ is reachable}\end{array}$$



C_j contains an "a" if $x_j = y_j$
C_j contains a "b" if $x_j < y_j$

(these conditions are invariant by time elapsing)

➜ proof taken in **[Aceto & Laroussinie 2002]**

PSPACE-hardness (cont.)

If $q \xrightarrow{\alpha,\alpha',\delta} q'$ is a transition of $\mathcal{M}$, then for each position i of the tape, we have a transition

$$(q,i) \xrightarrow{g,r:=0} (q',i')$$

where:

- g is $x_i = y_i$ (resp. $x_i < y_i$) if $\alpha = a$ (resp. $\alpha = b$)
- $r = \{x_i, y_i\}$ (resp. $r = \{x_i\}$) if $\alpha' = a$ (resp. $\alpha' = b$)
- $i' = i + 1$ (resp. $i' = i - 1$) if δ is right and $i < n$ (resp. left)

Enforcing time elapsing: on each transition, add the condition $t = 1$ and clock t is reset.

Initialization: init $\xrightarrow{t=1,r_0:=0} (q_0, 1)$ where $r_0 = \{x_i \mid w_0[i] = b\} \cup \{t\}$

Termination: $(q_f, i) \longrightarrow$ end

Consequence of region automata construction

Region automata: correct finite abstraction for checking reachability/Büchi-like properties

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However, everything can not be reduced to finite automata...

A model not far from undecidability

- Universality is undecidable **[Alur & Dill 90's]**
- Inclusion is undecidable **[Alur & Dill 90's]**
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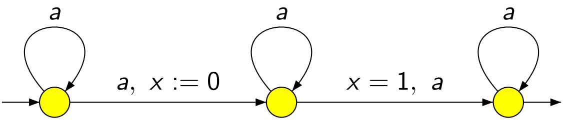
An example of non-determinizable/non-complementable timed aut.:



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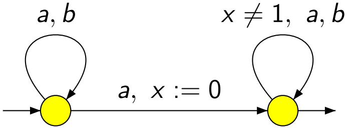
An example of non-determinizable/non-complementable timed aut.: [Alur,Madhusudan 2004]



UNTIME $\left(\overline{L} \cap \{(a^* b^*, \tau) \mid \text{all } a's \text{ happen before 1 and no two } a's \text{ simultaneously}\}\right)$ is not regular **(exercise!)**

Partial conclusion

➜ a timed model interesting for verification purposes

Numerous works have been (and are) devoted to:

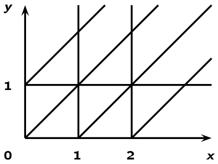
- the "theoretical" comprehension of timed automata (*cf* **[Asarin 2004]**)
- extensions of the model (to ease modelling)
  - expressiveness
  - analyzability
- algorithmic problems and implementation

Outline

Role of diagonal constraints

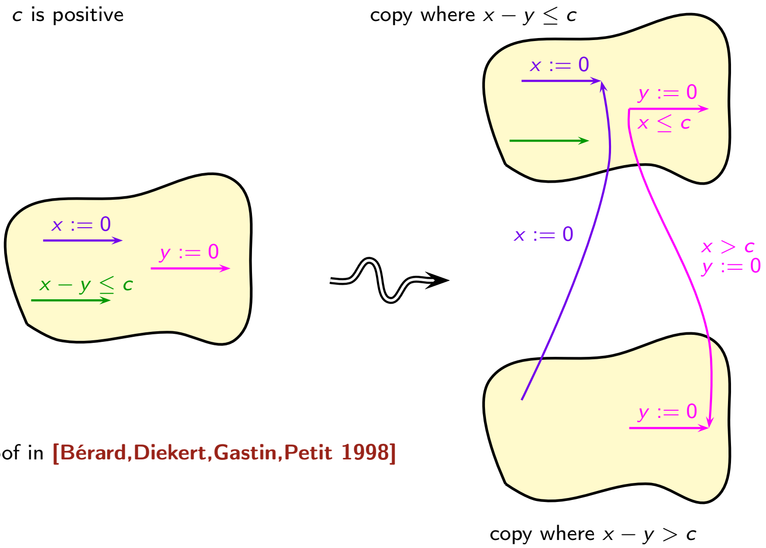
$$\boxed{x - y \sim c \quad \text{and} \quad x \sim c}$$

- **Decidability:** yes, using the region abstraction



- **Expressiveness:** no additional expressive power

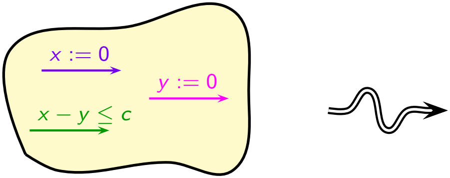
Role of diagonal constraints (cont.)



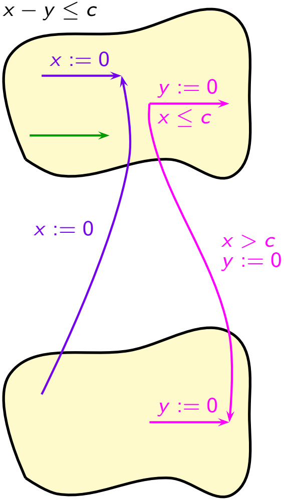
➜ proof in **[Bérard,Diekert,Gastin,Petit 1998]**

Role of diagonal constraints (cont.)

c is positive



copy where $x - y \leq c$



copy where $x - y > c$

- ➜ proof in **[Bérard,Diekert,Gastin,Petit 1998]**
- ➜ exponential blowup unavoidable in general **[Bouyer,Chevalier 2005]**

Adding silent actions

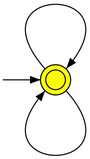
$$\xrightarrow{g,\ \varepsilon,\ C := 0}$$

[Bérard,Diekert,Gastin,Petit 1998]

- **Decidability:** yes

  (actions have no influence on region automaton construction)

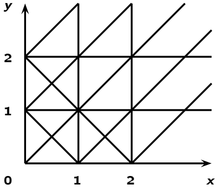
- **Expressiveness:** strictly more expressive!



Adding constraints of the form $x + y \sim c$

$\boxed{x + y \sim c \quad \text{and} \quad x \sim c}$ **[Bérard,Dufourd 2000]**

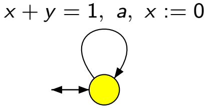
- **Decidability:** - for two clocks, decidable using the abstraction



  - for four clocks (or more), undecidable!

- **Expressiveness:** more expressive! (even using two clocks)

$$\{(a^n, t_1 \dots t_n) \mid n \geq 1 \text{ and } t_i = 1 - \frac{1}{2^i}\}$$



The two-counter machine

Definition

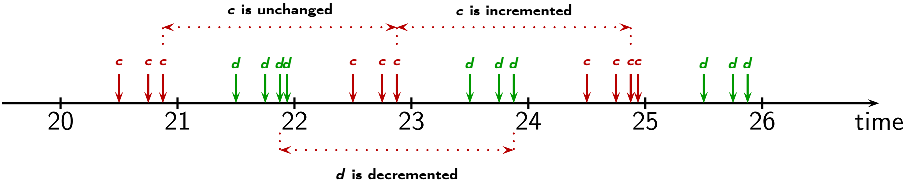
A two-counter machine is a finite set of instructions over two counters (x and y):

- Incrementation:
  (p): $x := x + 1$; goto (q)
- Decrementation:
  (p): if $x > 0$ then $x := x - 1$; goto (q) else goto (r)

Theorem [Minsky 67]

The halting problem for two counter machines is undecidable.

Undecidability proof



➜ simulation of
- decrementation of a counter
- incrementation of a counter

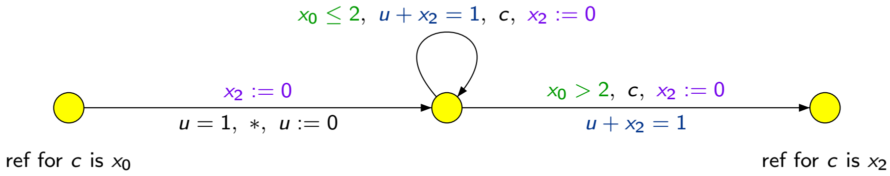
We will use 4 clocks:
- u, "tic" clock (each time unit)
- x_0, x_1, x_2: reference clocks for the two counters

"x_i reference for c" $\equiv$ "the last time x_i has been reset is the last time action c has been performed"

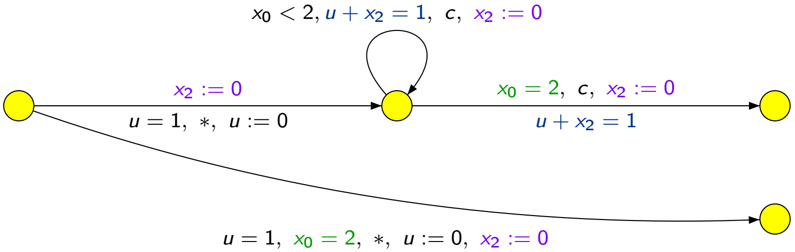
[Bérard,Dufourd 2000]

Undecidability proof (cont.)

- **Incrementation of counter c:**

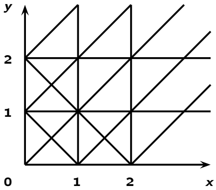


- **Decrementation of counter c:**



Adding constraints of the form $x + y \sim c$

- Two clocks: decidable using the abstraction



- Four clocks (or more): undecidable!

Adding constraints of the form $x + y \sim c$

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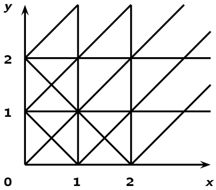

- **Three clocks: open question**

  We only know that the coarsest time-abstract bisimulation respecting these constraints is infinite. **[Robin 2004]**

- Four clocks (or more): undecidable!

Adding new operations on clocks

Several types of updates: $x := y + c$, $x :< c$, $x :> c$, etc...

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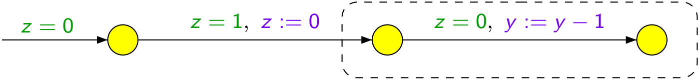
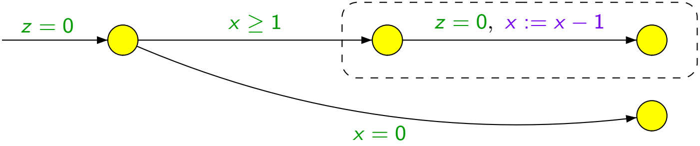
- Only decrementation also leads to undecidability

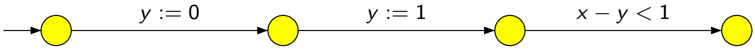
  - **Incrementation of counter x**



  - **Decrementation of counter x**



Decidability



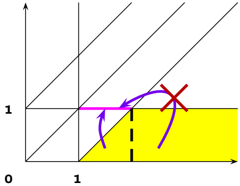


image by $y := 1$

➜ the bisimulation property is not met

The classical region automaton construction is not correct.

Decidability (cont.)

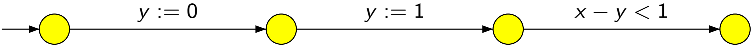
$$\begin{aligned} \mathcal{A} &\leadsto \text{Diophantine linear inequations system} \\ &\leadsto \text{is there a solution?} \\ &\leadsto \text{if yes, belongs to a decidable class} \end{aligned}$$

Examples:

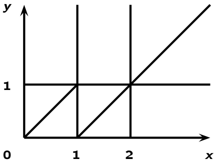
- constraint $x \sim c$ — $c \leq \max_x$
- constraint $x - y \sim c$ — $c \leq \max_{x,y}$
- update $x :\sim y + c$ — $\max_x \leq \max_y + c$
  and for each clock z, $\max_{x,z} \geq \max_{y,z} + c$, $\max_{z,x} \geq \max_{z,y} - c$
- update $x :< c$ — $c \leq \max_x$
  and for each clock z, $\max_z \geq c + \max_{z,x}$

The constants $(\max_x)$ and $(\max_{x,y})$ define a set of regions.

Decidability (cont.)



$$\left\{\begin{array}{l} \max_y \geq 0 \\ \max_x \geq 0 + \max_{x,y} \\ \max_y \geq 1 \\ \max_x \geq 1 + \max_{x,y} \\ \max_{x,y} \geq 1 \end{array}\right. \quad \text{implies} \quad \left\{\begin{array}{l} \max_x = 2 \\ \max_y = 1 \\ \max_{x,y} = 1 \\ \max_{y,x} = -1 \end{array}\right.$$

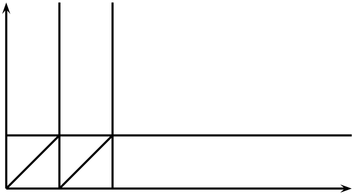


The bisimulation property is met.

What's wrong when undecidable?

Decrementation $x := x - 1$

$\max_x \leq \max_x - 1$



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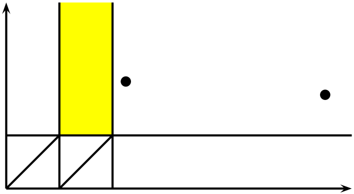

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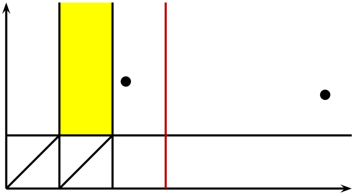

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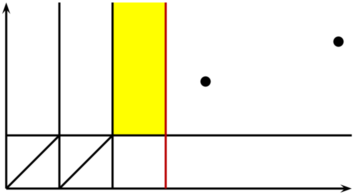

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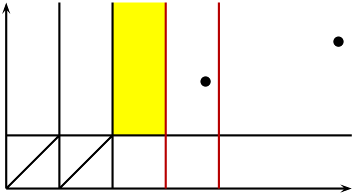

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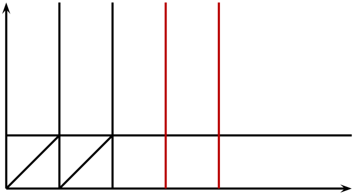

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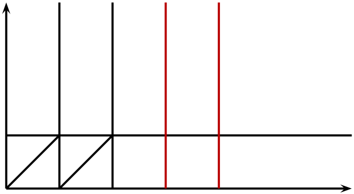

etc...

Decidability (cont.)

<table>
<tr><th></th><th>Diagonal-free constraints</th><th>General constraints</th></tr>
<tr><td>$x := c,\ x := y$</td><td rowspan="3">PSPACE-complete</td><td>PSPACE-complete</td></tr>
<tr><td>$x := x + 1$</td><td rowspan="3">Undecidable</td></tr>
<tr><td>$x := y + c$</td></tr>
<tr><td>$x := x - 1$</td><td>Undecidable</td></tr>
<tr><td>$x :< c$</td><td rowspan="4">PSPACE-complete</td><td>PSPACE-complete</td></tr>
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<tr><td>$x :\sim y + c$</td></tr>
<tr><td>$y + c <: x :< y + d$</td></tr>
<tr><td>$y + c <: x :< z + d$</td><td>Undecidable</td></tr>
</table>

[Bouyer,Dufourd,Fleury,Petit 2000]

Other extensions which have been considered

- New operations on clocks **[Bouyer, Dufourd, Fleury, Petit 2004]**

$$x := y + c,\ x :< c,\ x :> c,\ \text{etc...}$$

- Alternation **[Lasota, Walukiewicz 2005]** **[Ouaknine, Worrell 2005]**
  - One-clock alternating timed automata are decidable.
  - n-clock alternating timed automata are undecidable ($n \geq 2$).

- Slopes of variables: "Linear hybrid automata" **[Henzinger 1996]** **[Henzinger,Kopke,Puri,Varaiya 98]**
  - Almost everything is undecidable.
  - The class of LHA with clocks and only one variable having possibly two slopes $k_1 \neq k_2$ is undecidable.
  - The class of *stopwatch* automata is undecidable.
  - One of the "largest" classes of LHA which are decidable is the class of initialized rectangular automata

Outline

Notice

The region automaton is not used for implementation:

- suffers from a combinatorics explosion
  (the number of regions is exponential in the number of clocks)
- no really adapted data structure

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...but **on-the-fly technics** are prefered.

Reachability analysis

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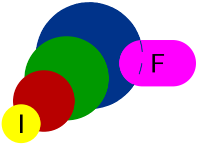

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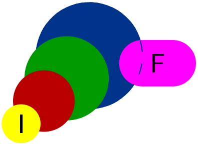

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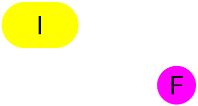

Reachability analysis

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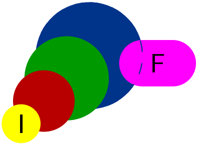

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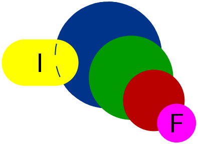

Note on the backward analysis of TA

$$\ell \xrightarrow{g,\ a,\ C := 0} \ell'$$

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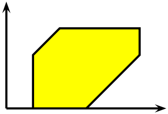

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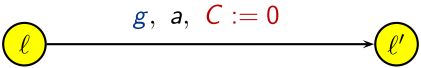

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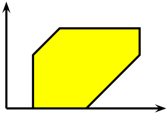

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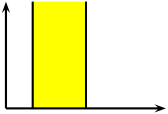

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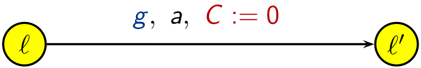

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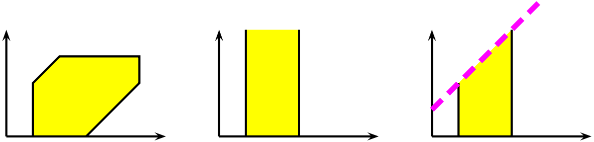

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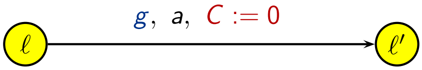

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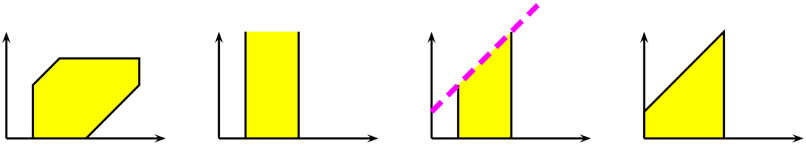

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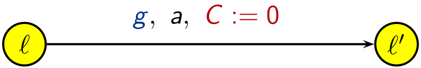

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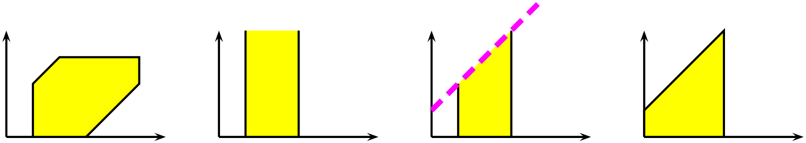

Z $\qquad$ $[C \leftarrow 0]^{-1}(Z \cap (C = 0))$ $\qquad$ $\overleftarrow{[C \leftarrow 0]^{-1}(Z \cap (C = 0)) \cap g}$

The exact backward computation terminates and is correct!

Note on the backward analysis (cont.)

If $\mathcal{A}$ is a timed automaton, we construct its corresponding set of regions.

Because of the bisimulation property, we get that:

"Every set of valuations which is computed along the backward computation is a finite union of regions"

Note on the backward analysis (cont.)

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Let R be a region. Assume:

- $v \in \overleftarrow{R}$ (for ex. $v + t \in R$)
- $v' \equiv_{\mathbf{reg.}} v$

There exists t' s.t. $v' + t' \equiv_{\mathbf{reg.}} v + t$, which implies that $v' + t' \in R$ and thus $v' \in \overleftarrow{R}$.

Note on the backward analysis (cont.)

If $\mathcal{A}$ is a timed automaton, we construct its corresponding set of regions.

Because of the bisimulation property, we get that:

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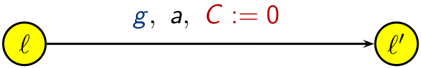
But, the backward computation is not so nice, when also dealing with integer variables...

$$i := j.k + \ell.m$$

Forward analysis of timed automata



zones $\qquad Z \qquad\qquad [C \leftarrow 0](\overrightarrow{Z} \cap g)$

A zone is a set of valuations defined by a clock constraint

$$\varphi \; ::= \; x \sim c \;\mid\; x - y \sim c \;\mid\; \varphi \wedge \varphi$$

Forward analysis of timed automata

$$\ell \xrightarrow{\quad g,\ a,\ C := 0 \quad} \ell'$$

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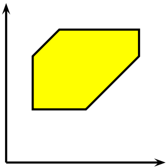

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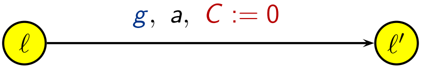

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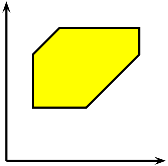
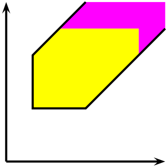
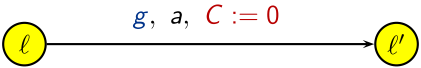


Z



$\overrightarrow{Z}$

Forward analysis of timed automata



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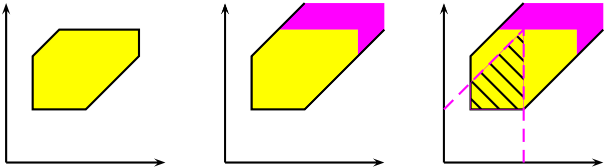

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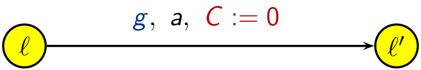

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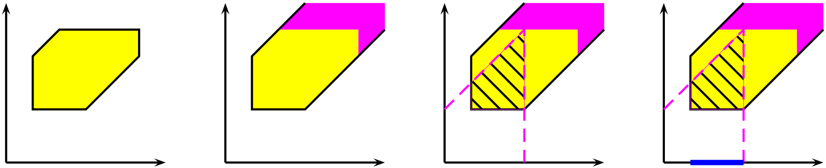

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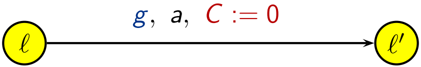

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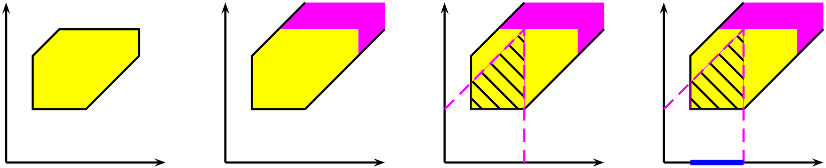

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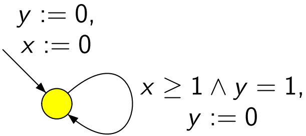
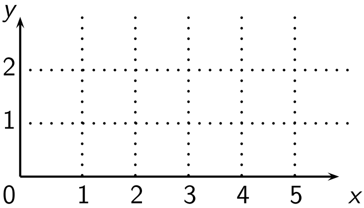
➜ a termination problem

Non termination of the forward analysis





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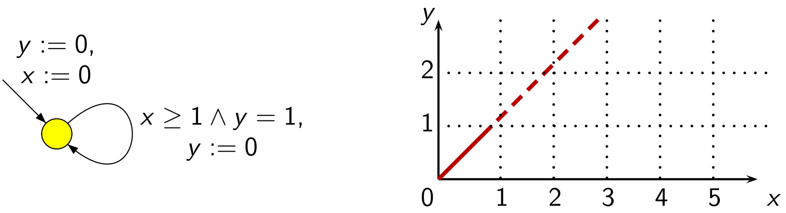

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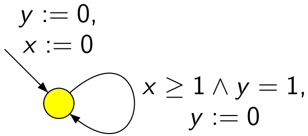

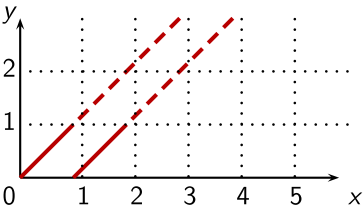

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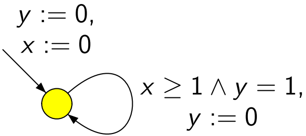

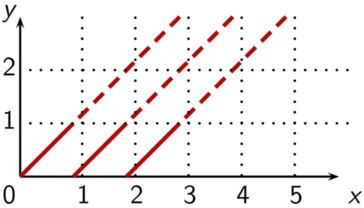

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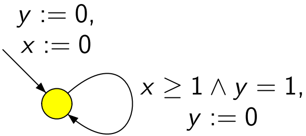

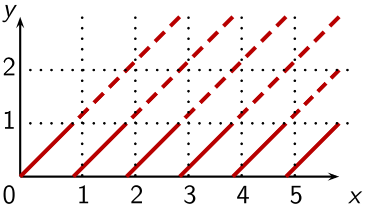

➜ an infinite number of steps...

The DBM data structure

DBM (Difference Bound Matrice) data structure

[Berthomieu, Menasche 1983] [Dill 1989]

$$(x_1 \geq 3) \;\wedge\; (x_2 \leq 5) \;\wedge\; (x_1 - x_2 \leq 4)$$

$$\begin{array}{c|ccc} & x_0 & x_1 & x_2 \\ \hline x_0 & +\infty & -3 & +\infty \\ x_1 & +\infty & +\infty & 4 \\ x_2 & 5 & +\infty & +\infty \end{array}$$

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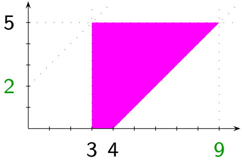
$$\begin{array}{c} \\ x_0 \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{array}{ccc} x_0 & x_1 & x_2 \end{array} \\ \begin{pmatrix} +\infty & -3 & +\infty \\ +\infty & +\infty & 4 \\ 5 & +\infty & +\infty \end{pmatrix} \end{array}$$

- Existence of a normal form



$$\begin{pmatrix} 0 & -3 & 0 \\ 9 & 0 & 4 \\ 5 & 2 & 0 \end{pmatrix}$$

The DBM data structure

DBM (Difference Bound Matrice) data structure

[Berthomieu, Menasche 1983] [Dill 1989]

$$(x_1 \geq 3) \ \wedge \ (x_2 \leq 5) \ \wedge \ (x_1 - x_2 \leq 4)$$

$$\begin{array}{c} \\ x_0 \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{array}{ccc} x_0 & x_1 & x_2 \end{array} \\ \begin{pmatrix} +\infty & -3 & +\infty \\ +\infty & +\infty & 4 \\ 5 & +\infty & +\infty \end{pmatrix} \end{array}$$

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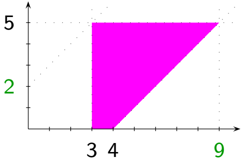

$$\begin{pmatrix} 0 & -3 & 0 \\ 9 & 0 & 4 \\ 5 & 2 & 0 \end{pmatrix}$$

- All previous operations on zones can be computed using DBMs

The extrapolation operator

Fix an integer k ("$*$" represents an integer between $-k$ and $+k$)

$$\begin{pmatrix} * & \boxed{>k} & * \\ * & * & * \\ \boxed{<-k} & * & * \end{pmatrix} \rightsquigarrow \begin{pmatrix} * & \boxed{+\infty} & * \\ * & * & * \\ \boxed{-k} & * & * \end{pmatrix}$$

- "intuitively", erase non-relevant constraints

➜ ensures termination

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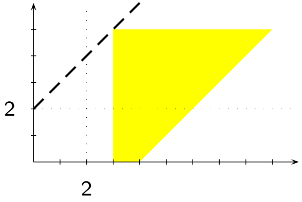

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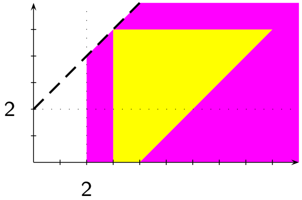

➜ ensures termination

Classical algorithm, focus on correctness

Take k the maximal constant appearing in the constraints of the automaton.

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Theorem

This algorithm is correct for diagonal-free timed automata.

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Theorem

This algorithm is correct for diagonal-free timed automata.

However, this theorem does not extend to timed automata using diagonal clock constraints...

A counter-example

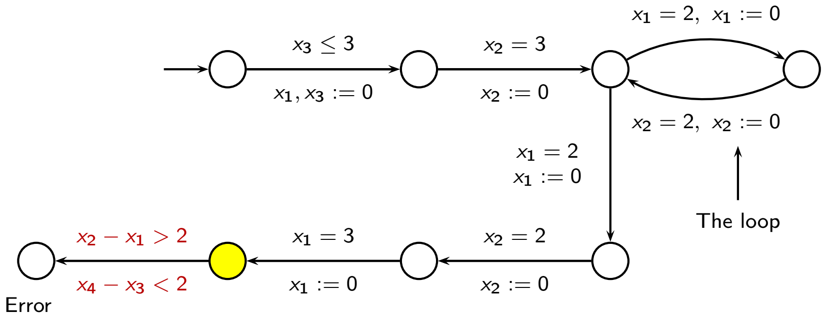
- Implemented in numerous tools:
  - Uppaal, `http://www.uppaal.com/`
  - Kronos, `http://www-verimag.imag.fr/TEMPORISE/kronos/`
  - ...
- Successfully used on many real-life examples since ten years.

Outline

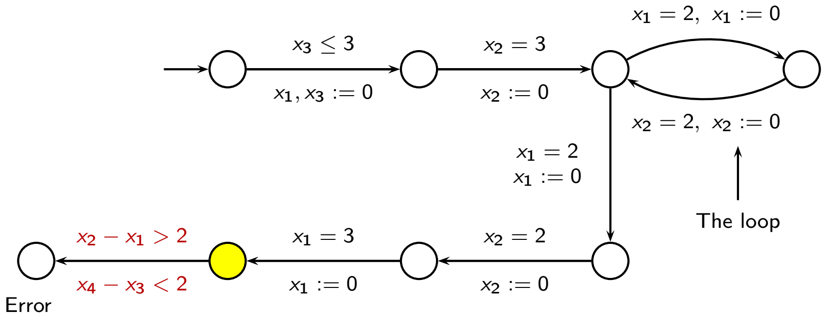
Conclusion & further work

- Decidability is quite well understood.
- There is still some progress which is done for the verification of timed automata. *(see Gerd's talk)*
- Some other current challenges:
  - controller synthesis
  - implementability issues (program synthesis) *(remember Jean-François' talk)*
  - optimal computations
  - ...

A problematic automaton

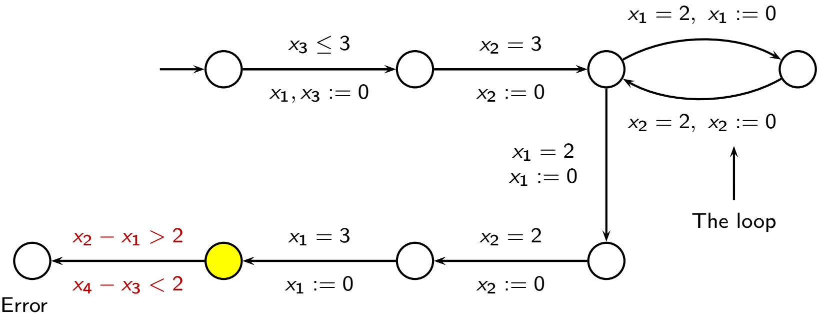


A problematic automaton



$$\begin{cases} v(x_1) = 0 \\ v(x_2) = d \\ v(x_3) = 2\alpha + 5 \\ v(x_4) = 2\alpha + 5 + d \end{cases}$$

A problematic automaton



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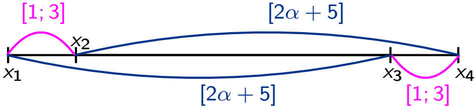
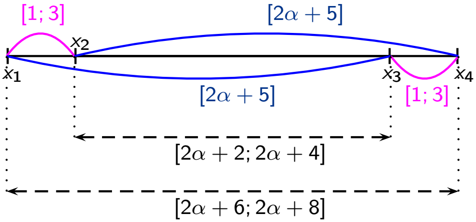
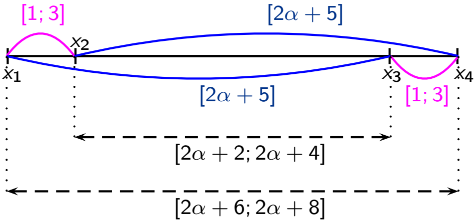


The problematic zone



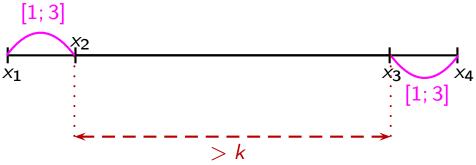
implies $x_1 - x_2 = x_3 - x_4$.

The problematic zone



implies $x_1 - x_2 = x_3 - x_4$.

If α is sufficiently large, after extrapolation:



does not imply $x_1 - x_2 = x_3 - x_4$.

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