Complexité avancée - TD 4

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Exercise 1 A translation result

Show that if $\mathsf{P} = \mathsf{PSPACE}$, then $\mathsf{EXPTIME} = \mathsf{EXPSPACE}$.

Exercise 2 Unary languages

Recall that a *unary* language is any language over a one-letter alphabet.

1. Prove that if a unary language is NP-complete, then $\mathsf{P} = \mathsf{NP}$.
2. Prove that if every unary language in NP is actually in P, then $\mathsf{EXP} = \mathsf{NEXP}$.

Exercise 3 P-choice

A language L is said to be P-*peek*, written $L \in \mathsf{P}_p$, if there is a function $f : \Sigma^* \times \Sigma^* \to \Sigma^*$, computable in polynomial time, such that $\forall x, y \in \Sigma^*$:

- $f(x, y) \in \{x, y\}$,
- if $x \in L$ or $y \in L$ then $f(x, y) \in L$.

In that case, f is called the *peeking function* for L.

1. Show that $\mathsf{P} \subseteq \mathsf{P}_p$.
2. Show that P_p is closed under complementation.
3. Show that if there exists a NP-hard language in P_p then $\mathsf{P} = \mathsf{NP}$.

Exercise 4 Regular language

Let REG denote the set of regular/rational languages.

1. Show that for all $L \in \mathsf{REG}$, L is recognized by a TM running in space 0 and time $n + 1$.
2. Exhibit a language recognized by a TM running in space $\log n$ and time $O(n)$ that is not in REG.

Exercise 5 On the existence of one-way functions

A one-way function is a bijection f from k-bit integers to k-bit integers such that f is computable in polynomial time, but f^{-1} is not. Prove that if there exist one-way functions, then

$$A \stackrel{\text{def}}{=} \{(x, y) \mid f^{-1}(x) < y\} \in (\mathsf{NP} \cap \mathsf{coNP}) \backslash \mathsf{P} \,.$$

Exercise 6 Too fast!

Show that $\mathsf{ATIME}(\log\ n) \neq \mathsf{L}$.