Complexité avancée - TD 3

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Exercise 1 Space hierarchy theorem

Consider two space-constructible functions f and g such that $f(n) = o(g(n))$. Prove that $\mathsf{DSPACE}(f) \subsetneq \mathsf{DSPACE}(g)$.

Hint: You may consider a language $L = \{(M, w') \mid$ the simulation of M on (M, w') rejects $\}$ with an appropriate restriction on the simulation of M.

Exercise 2 Polylogarithmic space

1. Let $\mathsf{polyL} = \cup_{k \in \mathbb{N}} \mathsf{SPACE}(\log^k)$. Show that polyL does not have a complete problem for logarithmic space reduction.[1]
2. We recall that $\mathsf{PSPACE} = \cup_{k \in \mathbb{N}} \mathsf{SPACE}(n^k)$. Does PSPACE have a complete problem for logarithmic space reduction ? Why doesn't the proof of the previous question apply to PSPACE?

Exercise 3 Padding argument

1. Show that if $\mathsf{DSPACE}(n^c) \subseteq \mathsf{NP}$ for some $c > 0$, then $\mathsf{PSPACE} \subseteq \mathsf{NP}$.

   Hint: for $\mathsf{L} \in \mathsf{DSPACE}(n^k)$ one may consider the language $\tilde{L} = \{(x, 1^{|x|^{k/c}}) \mid x \in L\}$.
2. Deduce that $\mathsf{DSPACE}(n^c) \neq \mathsf{NP}$.

[1] From this, we can deduce that $\mathsf{polyL} \neq \mathsf{P}$.